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Machine learning with Belief Rule-Based Expert Systems to predict stock price movements

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ABSTRACT

Price prediction of financial assets has been a key interest for researchers over the decades. Numerous techniques to predict the price movements have been developed by the researchers over the years. But a model loses its credibility once a large number of traders start using the same technique. Therefore, the traders are in continuous search of new and efficient prediction techniques. In this research, we propose a novel machine learning technique using technical analysis with Belief Rule-Based Expert System (BRBES), and incorporating the concept of *Bollinger Band* to forecast stock price in the next five days. A *Bollinger Event* is triggered when the closing price of the stock goes down the *Lower Bollinger Band*. The BRBES approach has never been applied to stock markets, despite its potential and the appetite of the financial markets for expert systems. We predict the price movement of the Swedish company *TELIA* as a proof of concept. The knowledge base of the initial BRBES is constructed by simulating the historical data and then the learning parameters are optimized using MATLAB's *fmincon* function. We evaluate the performance of the trained BRBES in terms of Accuracy, Area Under ROC Curve, Root Mean Squared Error, type I error, type II error, R^2 value, and profit/loss ratio. We compare our proposed model against a similar rule-based technique, Adaptive Neuro-Fuzzy Inference System (ANFIS), to understand the significance of the improved rule base of BRBES. We also compare the performance against Support Vector Machine (SVM), one of the most popular machine learning techniques, and a simple heuristic model. Finally, the trained BRBES is compared against recent state-of-the-art deep learning approaches to show how competitive the performance of our proposed model is. The results show that the trained BRBES produces better performance than the non-trained BRBES, ANFIS, SVM, and the heuristic approaches. Also, it indicates better or competitive performance against the deep learning approaches. Thus BRBES exhibits its potential in predicting financial asset price movement.

1. Introduction

The stock market is an open public exchange, where investors trade stocks with each other, and companies can access new capital. Usually, companies issue their stocks on the market to enlarge their business areas and also to gather financial resources. Traders around the globe can buy any company's stocks at an agreed price and can enjoy yearly dividends from that company for their owned stocks. The stock market is instrumental in the contemporary world order as an investment venue and is growing in importance. Till June 2021, the top ten stock exchanges have a total capitalization of around \$90.68 trillion (Statista, 2021).

The stock market is vulnerable and full of uncertainty. Factors unknown to most investors may overturn a stock's price drastically. This does not only include changes in the operations of the company but also investors that suddenly may execute their intentions of selling or buying, thus pushing the price up or down. Predicting future stock price by analyzing previous price movement rather than a fundamental understanding of the underlying stock has been a key interest of the investors over the years, and this is known as technical analysis. But predicting the stock price movement has not been that easy, especially short-term price prediction for its complicated and uncertain nature.

Researchers and industry analysts are continuously trying to predict the nature of this volatile market for the last few decades. Since the

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emergence of machine learning approaches, these techniques are being used in various areas (Afroze et al., 2021; Gupta et al., 2019, 2021; Hossain et al., 2021). Several machine learning models have been used to predict the price of the stock market (Battula & Prasad, 2013; Dey et al., 2021). Also, few other techniques have been used to predict other financial assets (e.g., foreign exchange currency) (Islam, Hossain, Rahman, 2020; Saiful Islam & Hossain, 2020). So, why do we need a new approach? While there are many approaches to machine learning (ML) and artificial intelligence (AI) in finance, there is a constant competition where investors try to find new ways to outperform the market. If a certain technique is applied by a large group of traders, the model gets saturated and it loses its predictive capability. Therefore, novel approaches are constantly in demand since their uniqueness may result in that they work in particular niches.

Although several machine learning models have been used in this domain, researchers have never utilized the potential of the Belief Rule-Based Expert System (BRBES) technique. Previously, researchers showed that BRBES has a very good predictive capability in different domains (Hossain et al., 2017; Kabir et al., 2020; Ul Islam et al., 2015). The opaqueness of many AI and ML models is a challenge in the finance industry (Jaeger et al., 2020). Hence, BRBES is an approach worth furthering in finance, but the first step is to show whether and how it is capable of prediction in the area of stock prices. That is one of the objectives of this research.

Moreover, predicting stock price movement is rather easy when the market is considerably stable and follows the previous pattern. However, it becomes trickier and more complicated when certain political or economical situations affect the world trading systems. For example, the current conflict between Russia and Ukraine has made the stock market significantly unstable and it is almost uncertain to predict what is going to happen the next day (Reuters, 2022; The Guardian, 2022; The New York Times, 2022). Therefore, another objective of this work is to predict the price when a stock does not follow its usual pattern. For this purpose, we used a statistical concept, called *Bollinger Band*, which is the 3-standard deviations away from the simple moving average of the price. *Bollinger Band* has two bounds: upper bound and lower bound. The upper bound (and lower bound) is calculated by adding (and subtracting) a 3-standard deviation value to the moving average. Our proposed model predicts the stock price when it goes down the lower bound of the *Bollinger Band*.

The main contributions of this research are two-fold. Firstly, we show that BRBES has good potential in learning and predicting financial assets. Secondly, using the concept of *Bollinger Band*, we demonstrate that BRBES can predict the stock price movement even when the market is unstable due to global political or economical crisis. For predicting price movement, we considered the data of Swedish telephone company *TELIA* as a proof of concept, that is, to determine the feasibility of our proposed method. BRBES is used to predict the closing price of this stock in the upcoming five days considering its previous pricing history. We aim to show that not only is the BRBES a possible approach but that it also is competitive when compared to other machine learning and deep learning approaches.

The rest of the article is organized in the following way: Section 2 discusses the relevant works in stock price forecasting techniques, Section 3 contains an overview of a Belief Rule-Based Expert System as well as discusses about some background concepts, Section 4 illustrates our proposed BRBES methodology including its machine learning technique, and Section 5 discusses the results of our proposed model and also compares the performance against other approaches. Finally, Section 6 concludes our findings and also indicates the future work.

2. Related works

Two conventional approaches are applied in stock market trends prediction: fundamental analysis and technical analysis (Vui et al., 2013). Fundamental analysis considers microeconomics, industrial environment, financial conditions, financial news, etc. (Chen et al., 2017; Islam, Hossain, Rahman, 2020). On the other hand, technical analysis is composed of a set of tools to predict the future price of a financial asset taking into account the historical market data, especially traded volume and stock price movement (Park & Irwin, 2007; Wei et al., 2011; Yamamoto, 2012; Zhu & Zhou, 2009). This paper concentrates on technical analysis, although this approach may in principle be worth applying also to predict fundamental changes. Technical analysis is a form of probabilistic thinking. It does not try to predict an event with 100% accuracy, but rather finds regularities that are non-random. Yet the practical application of the technical analysis may in a single case produce loss, but consistent application over a sufficient number of events will be profitable. Conventional technical analysis is nowadays enhanced with statistical models, machine learning, and artificial intelligence. These techniques include Artificial Neural Networks, Recurrent Neural Networks (Islam, Hossain, Rahman, 2020; Saiful Islam & Hossain, 2020), Genetic Algorithms, Evolutionary Computing, Fuzzy Systems, etc. (Bisoi & Dash, 2014; Kazem et al., 2013; Ticknor, 2013; Wei et al., 2011). Some practitioners may distinguish between a technical analysis approach and quantitative finance (statistics and AI belonging to the latter). Since the literature we base our work on all deals mathematically with concepts from technical analysis, we see no need for this distinction.

Prediction of stock return has been a key interest over decades and is a subject that encompasses many fields such as corporate finance, investment analysis, financial econometrics, and behavioral finance (Chavarnakul & Enke, 2009). Therefore, a keyword search has been conducted in renowned academic databases and publishing websites such as Elsevier, Springer, IEEE Xplore, MDPI, JStor, Taylor & Francis, etc. to limit the literature range of this study. More specifically the following keywords are utilized in our searches: (a) stock prediction, (b) technical analysis, (c) trend prediction, (d) trading, and (e) stock market. Besides, papers on fundamental analysis are excluded in this paper as our study concentrates on technical analysis only.

Carl Gold proposed a neural network (NN) model to predict high-frequency FOREX price (Gold, 2003). The neural network is trained by Recurrent Reinforcement Learning (RRL) where the first author compared the performance of 1-layer and 2-layers NN. The author also demonstrated the effect of neural network weights on the price prediction.

Adaptive Neuro-Fuzzy Inference System (ANFIS) can be used to predict the short-term trend of the stock market (Atsalakis & Valavanis, 2009). Some critics of technical analysis argue that the market is efficient and therefore it is not possible to find room to earn money on technical analysis. The technical analyst retort that there are at least temporary inefficiencies that technical analysis can identify and exploit. The results challenge the weak version of the Efficient Market Hypothesis (EMH) (Malkiel & Fama, 1970) by providing much better and improved results than other techniques to determine the next day's trend. We found this research interesting and use this approach as a benchmark for our experimental work.

Ten data mining techniques were applied to predict the future movement direction of the Hang Seng index of the Hong Kong stock market (Ou & Wang, 2009) where they showed that SVM and LS-SVM produce better predictive performance compared with other predicting models as these two techniques do not require prior information about data and can reach the global optimal point.

Kannan et al. used different data mining methods considering the historical price to predict stock market price movement (Kannan et al., 2010). Among Typical Price (TP), Moving Average (MA), Relative Strength Index (RSI), Chaikin Money-flow Indicator (CMI), and *Bollinger Band*, they found that the *Bollinger Band* technique produces the best profitable signals (84.24%) compared with the other four techniques. We have chosen to work from the assumption that Bollinger bands are fruitful to use as parameters for our supervised machine learning.

Nair, Dharini, and Mohandas proposed an automated hybrid decision tree-adaptive neuro-fuzzy system (Nair et al., 2010). Technical analysis was used for feature extraction and a decision tree is used for feature selection. Their system was tested on NASDAQ 100, FTSE 100, BSE-SENSEX, and NIKKEI 225 where the results showed that the proposed hybrid system generates better accuracy than a standalone decision tree technique and standalone ANFIS technique without feature selection and dimensionality reduction.

Cervelló-Royo, Guijarro, and Michniuk proposed risk-adjusted profitable stock trading rules for deciding when to buy or sell (Cervelló-Royo et al., 2015). The results included the probable profit in each transaction and maximum bearable loss when employing the trading rules. They experimented with 91,307 intraday observations from the US Dow Jones index which eliminates the randomness from the result. They parameterized their training rules by combining 96 configurations and the results are tested over three subperiods. They also replicated their analysis in the British FTSE and the German DAX which shows that the return by the proposed trading rules is higher in the European index than in the US index.

Tharavanij, Siraprapasiri, and Rajchamah examine the profitability of technical trading rules in five Southeast Asian stock markets (Tharavanij et al., 2015). Relative Strength Index (RSI), Moving Average Convergence–Divergence, Stochastic Oscillator, On Balance Volume and Directional Movement Indicator were investigated to evaluate the performance with a buy-and-hold method. Their results show that technical trading rules produce significant returns in Singapore, Indonesia, Malaysia, and the Philippines, but not in Thailand's markets.

Dymova, Sevastjanov, and Kaczmarek proposed a new FOREX (foreign exchange market, currency trading) trading expert system (Dymova et al., 2016) based on new technical analysis indicators combined with a new Rule Based Evidential Reasoning (RBER) which is the combination of Dempster–Shafer theory of evidence (Denoeux & Shenoy, 2020) with fuzzy logic. They preserved information about every membership function representing intersecting fuzzy classes in fuzzy logic rules. The results showed that, for four different currency pairs, and for various time frames (15 min to 4 h), the system provides a high Return on Investment (ROI).

Gao and Xu developed a stock market investment model applying Evidential Reasoning rules to combine analysts' opinions for making decisions in the stock market (Gao & Xu, 2019). Analyst's report from the China Stock Market & Accounting Research (CSMAR) database yields average annualized market return up to 13.9% along with a smaller p-value of 0.92% with five-month investment tenure and two-month report collection. Their results support the effectiveness of Evidential Reasoning rules in investment decision making while also suggesting that the Efficient Market Hypothesis (EMH) (Malkiel & Fama, 1970) may be invalid for the China stock exchange and need further revisions.

Rundo et al. proposed a Markov-based adaptive trading system to predict high-frequency stock prices using historical data (Rundo et al., 2019a). They used two LSTM networks; one for trend prediction and the other for closing price prediction. They validated the performance of the model in terms of trend direction, RMSE, variance, accuracy, and profit-loss ratio. Furthermore, they defined a stochastic method that improves LSTM's prediction accuracy.

Shintate and Pichl developed a random sampling method (RSM) using LSTMNet for predicting high-frequency bitcoin time series in the OkCoin market (Shintate & Pichl, 2019). LSTMNet is built using two LSTM layers with 32 LSTM units in each layer. They showed that their model mitigates the class imbalanced problem of MLP and standalone LSTM.

Rundo et al. developed a grid trading system to predict the price in the 1-min intraday timeframe of the EUR/USD currency pair (Rundo et al., 2019b). They have developed a Scaled Conjugate Gradient (SCG) with backpropagation. Their model demonstrates excellent performance in terms of overall drawdown which confirms its robustness in the FOREX market.

Dey et al. analyzed the use of different types of recurrent neural networks (RNN, LSTM, and GRU) in stock price prediction (Dey et al., 2021). They used three different datasets and predicted the stock price 1-day, 3-days, and 5-days prior to the actual time. Their results show that recurrent neural networks can successfully predict the movement of stock price.

All in all, the literature shows that technical analysis is a valid approach. There are several successful applications of expert systems and machine learning within the field of technical analysis. The *Bollinger Band* metric is an adequate basis for our experiment. We found a few papers which bear a family resemblance with the BRBES approach which we had in our mind, which suggested to us that this article is significant. It also showed that the machine learning and knowledge base in Dymova et al. (2016) and Gao and Xu (2019) are relatively similar to that of the BRBES, and therefore more probable to succeed. None of the papers implemented belief-rule base expert systems (BRBES) that is used in the stock markets.

3. The outline of belief rule-based expert system

As mentioned, the primary focus was to consider a stock's closing prices and study its effect on the future price movement of that stock when it goes down Lower Bollinger Band (*Lower BB*), which can be defined by,

$$LowerBB = SMA - (3 \times STDEV) \tag{1}$$

where *SMA* and *STDEV* indicate the simple moving average and standard deviation of the last 20 days' closing price, respectively.

The Belief Rule-Based Expert System (BRBES) approach has been used widely to evaluate the performance of a real system, which might have a complex structure, or to forecast the assessment of an event under uncertainty (Hossain et al., 2017; Ul Islam et al., 2015). A reason for using BRBES for real-world system monitoring is that those systems are more complex have a large number of factors and might have incomplete data which makes it impossible to predict 100% accurately. Although our present experiment does not have large numbers or incomplete data, this would often be the case in the financial markets. So in the present paper, we will examine the performance in a simple case (time series analysis of stock) but further research that advances the BRBES approach in finance can enjoy the possibilities of handling large numbers of input, even if it is partially incomplete. A BRBES consists of two major components: knowledge base and inference engine (Hossain et al., 2015). The knowledge base comprises of a few belief rules and the inference engine uses Evidential Reasoning (ER) to infer or generate decisions using the knowledge of BRBES (Yang et al., 2006).

Predicting future stock price is crucial since it involves a lot of factors such as OHLC (open, high, low, close) price, stock volume, and the brand value of the company (not part of technical analysis), which cannot be measured with 100% accuracy.[1] Therefore, BRBES is appropriate to use for predicting the price of stock under uncertain scenarios. This section provides an overview of the main components of BRBES, mainly focusing on the knowledge base construction and evidential reasoning technique, which itself consists of four steps: input transformation, activation weight calculation for each rule, updating degree of belief for incomplete data, and rule aggregation.

[1] Traded volume may seem a 100% accurate data, but there are market technicalities which can skew the data, such as trading in dark pools or even illegal trading which is rigged to give an impression of high volume. Other complexities may also lead to uncertainties in different data used for technical analysis. Especially retail traders suffer from imperfect data quality.

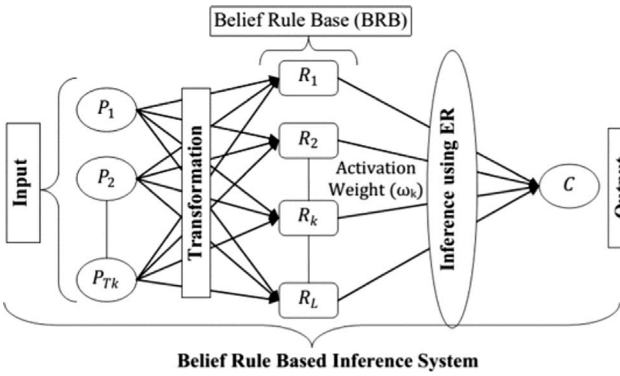

Fig. 1. BRBES inference procedure.

3.1. Knowledge representation in BRBES

The knowledge base of a BRBES consists of some belief rules, which are an extension of conventional IF-THEN rules (Hossain et al., 2015). In a belief rule, the antecedent part may have one or more antecedent attributes along with their corresponding referential values. The consequent part consists of consequent referential values with their belief distribution. When the sum of consequent's degrees of belief of a rule equals 1, then the rule is called complete; and otherwise, incomplete. This incompleteness may arise due to uncertainty or incomplete data. The general structure of the kth rule can be written as:

$$R_k : IF\ P_1\ is\ A_1^k \wedge P_2\ is\ A_2^k \wedge \ldots \wedge P_{T_k}\ is\ A_{T_k}^k\ \ THEN\ (C_1, \beta_{1k}), (C_2, \beta_{2k}), (C_3, \beta_{3k}), \ldots, (C_N, \beta_{Nk}) \tag{2}$$

where $P_1, P_2, \ldots, P_{T_k}$ are the antecedent attributes used in kth rule. A_i^k is the referential value of ith attribute of kth rule. $C_1, C_2, \ldots, C_N$ are referential values of consequent and β_{ik} is the belief degree to which C_i is believed to be true.

3.2. BRBES inference procedure

BRB inference procedure uses the Evidential Reasoning (ER) approach, which consists of four steps (Shafkat Raihan et al., 2022): (i) input transformation, (ii) rule activation weight calculation, (iii) belief degree update, and (iv) rule aggregation. Fig. 1 shows the inference procedure of BRBES.

3.2.1. Input transformation

Input transformation involves the distribution of the value of an antecedent P_i into different referential values of that antecedent (Hossain et al., 2017). The following equation assesses an input A_i.

$$H(A_i) = A_{ij}, \alpha_{ij}, j = 1, \ldots, j_i, i = 1, \ldots, N_k \tag{3}$$

where H is the distribution of degrees of belief assigned to the input value of antecedent, j_i is the total number antecedent referential values, α_{ij} is the matching degree of jth referential value of attribute A_i and N_k is the number of attributes used in rule k.

It is possible to assign some utility values h_{ij} to the referential values A_{ij}. For instance, "Low", "Medium" and "High" can be assigned to $h_{i1} = 0$, $h_{i2} = 0.5$ and $h_{i3} = 1$. Input transformation procedure can be elaborated by the Eqs. (4) and (5). When matching degrees (α_{ij}) are assigned to a rule, the rule is said to be activated.

$$if\, h_{i3} \geq A_i \geq h_{i2}, then\ \alpha_{i2} = \frac{h_{i3} - A_i}{h_{i3} - h_{i2}}, \alpha_{i3} = 1 - \alpha_{i2} \tag{4}$$

$$if\, h_{i2} \geq A_i \geq h_{i1}, then\ \alpha_{i1} = \frac{h_{i2} - A_i}{h_{i2} - h_{i1}}, \alpha_{i2} = 1 - \alpha_{i1} \tag{5}$$

3.2.2. Rule activation weight calculation

A rule may consist of multiple antecedent attributes as shown in (2). So, it is necessary to calculate a rule's combined matching degree considering all the antecedent attributes. This can be found by applying a simple multiplicative aggregation function as defined in (6). This multiplicative function can demonstrate the integration among the antecedent attributes of the rule (Hossain et al., 2016; Islam et al., 2018).

$$\alpha_k = \prod_{i=1}^{T_k} (\alpha_i^k)^{\bar{\delta}_{ki}} \tag{6}$$

where $\bar{\delta}_{ki} = \frac{\delta_{ki}}{max_{i=1,\ldots,T_k}(\delta_{ki})}$, is the relative weight of P_i used in the kth rule. This normalizes the value of $\bar{\delta_k}$ in between 0 and 1. The combined matching degree calculated as in (6), is then used to calculate activation weight of the rules in the BRBES (Hossain et al., 2015).

$$\omega_k = \frac{\theta_k \alpha_k}{\sum_{i=1}^{L} \theta_i \alpha_i} \tag{7}$$

where ω_k, θ_k and α_k represent rule-activation weight, rule weight and combined matching degree of kth rule respectively. If ω_k becomes zero for any rule k, then the rule is considered 'not activated' and therefore, has no impact on decision making.

3.2.3. Belief degree update for incomplete data

Ignorance in BRB system can arise when the input value of an antecedent attribute is not completely known because of incomplete data or uncertain scenarios. In these situations, initial belief degrees which are assigned during the development of the BRB, must be updated to avoid the uncertainty in our expert system. According to Yang et al. (2006), belief degrees can be updated by:

$$\beta_{ik} = \bar{\beta}_{ik} \frac{\sum_{t=1}^{T_k} \left(\tau(t,k) \sum_{j=1}^{j_i} \alpha_{tj} \right)}{\sum_{t=1}^{T_k} \tau(t,k)} \tag{8}$$

where $\bar{\beta}_{ik}$ is the original belief degree, β_{ik} is the updated belief degree, α_{tj} is the jth referential value of tth antecedent attribute, T_k is total number of antecedent attributes used in rule k and

$$\tau(t,k) = \begin{cases} 1, & \text{if } P_i \text{ is used in defining } R_k \\ 0, & \text{otherwise} \end{cases}$$

3.2.4. Rule aggregation

The last and final step of ER approach is to aggregate the activated rules of the knowledge base. Using the RIMER technique (Yang et al., 2006), we can achieve aggregated belief degrees of consequent's referential values using (9).

$$\beta_j = \frac{\mu \times \left[\prod_{k=1}^{L} \left(\omega_k \beta_{kj} + 1 - \omega_k \sum_{j=1}^{N} \beta_{kj} \right) - \prod_{k=1}^{L} \left(1 - \omega_k \sum_{j=1}^{N} \beta_{kj} \right) \right]}{1 - \mu \times \left[\prod_{k=1}^{L} (1 - \omega_k) \right]} \tag{9}$$

where β_j is the final belief degree of jth referential value of the consequent, with

$$\mu = \left[\sum_{j=1}^{N} \prod_{k=1}^{L} \left(\omega_k \beta_{kj} + 1 - \omega_k \sum_{j=1}^{N} \beta_{kj} \right) - (N-1) \times \prod_{k=1}^{L} \left(1 - \omega_k \sum_{j=1}^{N} \beta_{kj} \right) \right]^{-1}$$

When aggregated belief degrees of the different referential values ("High", "Medium", "Low") are calculated, we can find the final prediction level of stock price movement by converting it into a crisp value, y_m. This can be achieved by assigning a utility score for each referential value of the consequent and then applying (10).

$$y_m = \sum_{n=1}^{N} \mu(C_n) \times \beta_n \tag{10}$$

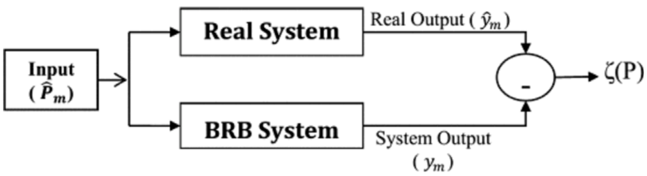

Fig. 2. Optimization model for BRBES.

where y_m is the expected crisp value, β_n is the belief degree of nth referential value of the consequent, and $\mu(C_n)$ is the utility score of that referential value. This crisp value can be used to calculate the final prediction level of stock price movement.

In Algorithm 1, the overall procedure of BRBES is described as pseudocode (Islam, Hossain, Andersson, 2020). Lines 2 to 6 are responsible for input transformation. In the input transformation stage, the matching degree is calculated incorporating the referential values of the antecedent attributes using Eqs. (4) and (5). Line 8 uses the matching degrees to determine the activation weight (Eq. (7)). Using Eq. (8), the belief update computation is carried out on line 9. Line 10 depicts the rule aggregation procedure, which produces a fuzzy output value using Eq. (9). Finally, as illustrated in line 11, the fuzzy output value is transformed to a crisp value using Eq. (10). To calculate the result, lines 8 to 11 are called for each input value in the dataset.

Algorithm 1 BRBES ALGORITHM

Let the input data is denoted by $X_{i,j}(i = 1, \dots, N; j = 1, \dots, TR)$, total amount of data is denoted by N, the total number of attributes is denoted by TR. The q^{th} referential value of p^{th} attribute is denoted by $r_{p,q}$, the belief rule base is denoted by BRB, and $Y_i(i = 1, .., N)$ denotes the predicted output.

Input $X_{i,j}$, r, BRB, N
Output Y_i
1: **procedure** BRBES($X_{i,j}$, $r_{i,j}$, BRB, N)
2: **for each** $i \in N$ **do**
3: **for each** $j \in M$ **do**
4: $X_{i,j}$ is transformed to matching degree, $md_{i,k}(k = 1, \dots, L)$ based on the referential value r
5: **end for**
6: **end for**
7: **for each** $i \in N$ **do**
8: Calculate the activation weight using equation (7)
9: Calculate the belief degree update using equation (8)
10: Calculate the rule aggregation using equation (9)
11: Convert crisp value Y_i from fuzzy value generated from rule aggregation using equation (10)
12: **end for**
13: **end procedure**

3.3. Optimal learning method to train BRBES

The optimal learning model involves determining the best values for different learning parameters of a rule in the rule base; such as rule weight, attribute weight, and belief degrees (θ_k, δ_i, β_{ik}). The value of these parameters is usually obtained from domain experts or generated at random. These settings, however, may not be ideal or completely correct. As illustrated in Fig. 2, the goal of the optimal learning technique is to find the best set of BRB learning parameters (θ_k, δ_i, β_{ik}) for reducing the discrepancy $\zeta(P)$ between BRBES outcomes (y_m) and real system outputs ($\bar{y}_m$). It uses the back-propagation technique to update the parameters so that the error is minimized.

The optimal learning approach for training the *Stock Market Prediction* system was established by combining three key phases (Kong, 2011): (a) creating an objective function; (b) constraining the training parameters; and (c) using a training module to find the best parameter set (θ_k, δ_i, β_{ik}). The number of instances in the training samples is assumed to be M, and the input–output pairings of the M cases are in the form of ($\hat{P}_m$, $\hat{y}_m$) (m = 1, …, M), where $\hat{P}_m$ is the input and $\hat{y}_m$ is the actual output. Section 4.4 explains these three steps in details for our proposed method.

3.4. Cross validation

We used k-fold cross-validation (where k indicates the number of folds) to evaluate the performance of the model. The main purpose of using k-fold cross-validation is to avoid backtest overfitting when training our BRBES. According to Wong (2015), the factors which can affect the accuracy of the k-fold cross-validation technique, are the total number of folds, the number of instances in each fold, the level of average, and the number of repetitions.

The accuracy estimate bias will be smaller if the number of folds is either five or ten, that is $k = 5$ or $k = 10$ (Rodriguez et al., 2010). After determining the number of folds, the next step is to decide the number of instances in each fold. The dataset is randomly partitioned into k disjoint folds with approximately equal size (Wong, 2015). There are two possible options for the level of averaging for accuracy estimation. This can be done fold by fold or over the whole dataset. Both ideas are supported by the researchers, where few researchers supported fold by fold technique (Alpaydin, 2009; Kantardzic, 2011; Witten et al., 2016) and few others supported the idea over the whole dataset (Han et al., 2011; Tan et al., 2005). k-fold cross-validation can be performed repeatedly for obtaining several unbiased estimates to obtain more reliable point of estimation (Alpaydin, 2009; Han et al., 2011; Witten et al., 2016).

4. BRBES to predict stock price movement

In this section, we present our research methodology including the data collection method, and the initial knowledge base of BRBESes. Besides, it also illustrates the k-fold cross-validation and optimized learning technique used in this work.

4.1. Proposed architecture

The problem formulated in this research is to forecast the future stock price of a company for the next five days after a specific event, namely a sharply plunging price. In practical terms it focuses on the research question, does the price recover with any regularity? To solve this problem and answer this question, we used Swedish TELIA company's previous pricing history (Yahoo Finance, 2021) and considered the stock's closing price as the prediction factor. We use *Bollinger Band* as an indicator for predicting when the stock may increase in price. The *Lower Bollinger Band (Lower BB)* describes when the stock price is fluctuating unusually sharply, and the BRBES here investigates whether it tends to revert to the mean of its price movement and whether it is profitable to speculate on that. This is an example of technical analysis, the approach to attempt to predict the future stock price based on its historical price action (movements). A proof of concept in one asset is sufficient for generalizing knowledge about the BRBES since that proves that there at least is a niche for the BRBES. It is chosen as a paradigmatic case (Flyvbjerg, 2006). Even if the TELIA stock is not representative of all assets in the market, there will likely be other indicators where BRBES is competitive, because TELIA is a company that is no outlier in terms of characteristics of stocks and companies, such as volatility, growth, solidity, competition, or faces market disruption. In a similar vein, we do not try to prove that we have found the best indicator or feature. We choose a common indicator, the *Bollinger Band*, to see whether BRBES can be competitive. If it is, then it is quite possible that other indicators may work as well.

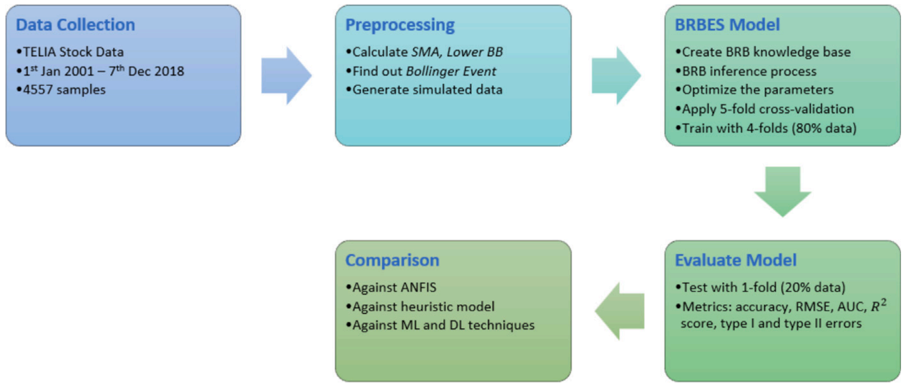

Fig. 3. Proposed methodology to predict stock price movement.

We used technical analysis which predicts the future price of a stock solely based on the previous pricing history of that stock. This pricing history includes *open price, high price, low price, closing price, traded volume per day*, etc. Among all these observed prices, the closing price is considered the most important because it represents the stock's end-of-day value. Moreover, the closing price is used to compute returns analyzed by practitioners and researchers and to calculate mutual fund net asset values (Felixson & Pelli, 1999). That is why we considered closing prices as our prediction factor. We take the simple mean average (SMA) of the previous 20 days' closing price and find the *Lower BB* boundary as in Eq. (1). A sharply plunging event (or simply *Bollinger event*) triggers when the closing price goes below the *Lower BB*. This also justifies our selection of closing price as the predicting feature.

The overall methodology of this research is given in Fig. 3. The data used in this study is from the Swedish TELIA company (a large company with quite high liquidity), which is available on the Yahoo Finance website (Yahoo Finance, 2021). We collected data from 1st January, 2001 to 7th December, 2018 which has a total of 4557 samples. We calculated SMA and *Lower BB* for each day by considering the previous 20 days' closing prices and found a total of 89 cases where *Bollinger event* is triggered. We considered these 89 events and their next five days dataset and generated *simulated TELIA* dataset which contains 1000 samples. Then we developed our proposed BRBES model by constructing the knowledge base and following the BRB inference procedure. Then we optimized the model using *fmincon* function. The optimized BRBES model is trained and tested by using *5*-fold cross-validation, as described in Section 3.4. So the model is trained and tested on the whole dataset in different iterations. However, at each iteration, *4*-folds (80% of the data) are used for training, and the other fold (20% of the data) is used for testing. We validated the performance of the model using several performance metrics: accuracy, root mean squared error (RMSE), R^2 score, area under ROC curve (AUC), and type I and type II errors. To understand how well the model is performing, we evaluated the performance against a similar rule-based system, ANFIS. Moreover, we compared the performance against a popular machine learning model (SVM) and state-of-the-art deep learning techniques. Finally, we compared against a simple heuristic model to understand whether we can do this prediction using some simple statistical methods and whether we actually need all the other complex techniques.

4.2. Knowledge base construction of BRBES

From the simulated TELIA dataset, we found that the difference between the closing price and the *Lower BB* lies in the range of 0.01 SEK to 3.38 SEK. The price movement range for the next five days is given in Table 1.

The above limits of antecedent and consequents of five BRBs are then normalized into the range [0,1]. We then used these normalized samples to produce 1000 simulated training samples using SPSS 25.0 for constructing the knowledge base of BRBESes (IBM Knowledge Center, 2021). A small portion of simulated data for day 1, day 2, day 3, day 4, and day 5 is given in Table 2.

Table 1
Price movement range for next five days.

Day No.	Lower limit (SEK)	Upper limit (SEK)
Day 1	−2.39	4.80
Day 2	−3.33	3.20
Day 3	−3.91	5.80
Day 4	−2.82	5.60
Day 5	−5.21	4.20

Table 2
Samples of simulated training data for Day 1, Day 2, Day 3, Day 4 and Day 5.

Sl.	Antecedent	Price movement				
	(LBB-CP)	Day 1	Day 2	Day 3	Day 4	Day 5
1.	0.2263	0.3271	0.5022	0.3950	0.3649	0.5903
2.	0.0733	0.1923	0.2941	0.3026	0.1454	0.3778
3.	0.2042	0.2925	0.4011	0.3573	0.3464	0.5271
4.	0.1841	0.4509	0.7240	0.5629	0.3672	0.6282
5.	0.0814	0.3721	0.5546	0.4279	0.3780	0.5106
6.	0.1372	0.2173	0.4541	0.3072	0.3381	0.4920
7.	0.0661	0.5097	0.7292	0.6547	0.6757	0.9009
8.	0.6053	0.1710	0.3286	0.2100	0.1513	0.2807
...	...	...	...	...	...	...
...	...	...	...	...	...	...
...	...	...	...	...	...	...
998.	0.0467	0.4059	0.6195	0.4076	0.5175	0.6298
999.	0.0359	0.5139	0.6872	0.5854	0.5371	0.5756
1000.	0.0376	0.4881	0.7695	0.5236	0.4603	0.6563

The knowledge base of BRB can be constructed in four ways: (i) extracting belief rules by analyzing historical data, (ii) extracting belief rules by consulting with experts of the domain, (iii) extracting new belief rules from previous rule base, and (iv) creating random belief rules with no prior knowledge (Dymova et al., 2016).

In this work, the knowledge base for five BRBs is constructed by analyzing the historical data of TELIA. The antecedent attribute of each BRB has three referential values (*high*, *medium*, *low*), so there are three rules for each BRB and total 15 rules in our BRBESes. We used 5-fold (k = 5) cross-validation technique with 200 instances in each fold. In fold 1, we considered the first 200 instances as validation data and the rest of the 800 instances as training data. In fold 2, instances in the range [201,400] are used as validation samples and the rest as training samples. A similar procedure has been carried out in fold 3, fold 4 and fold 5. Initial knowledge bases of BRB day 1 to BRB day 5 have been created from the training data for each fold. The knowledge bases for fold 1 are given in Tables 3 to 7.

Table 3
Initial knowledge base for BRB Day 1.

Rule ID	Rule weight	IF LowerBB-ClosingPrice	THEN price movement		
			H	M	L
1.	1	High	0.0000	0.7273	0.2727
2.	1	Medium	0.0411	0.5753	0.3836
3.	1	Low	0.0098	0.5237	0.4665

Table 4
Initial knowledge base for BRB Day 2.

Rule ID	Rule weight	IF LowerBB-ClosingPrice	THEN price movement		
			H	M	L
1.	1	High	0.4000	0.0000	0.6000
2.	1	Medium	0.1053	0.5263	0.3684
3.	1	Low	0.0670	0.3918	0.5412

Table 5
Initial knowledge base for BRB Day 3.

Rule ID	Rule weight	IF LowerBB-ClosingPrice	THEN price movement		
			H	M	L
1.	1	High	0.0000	0.7500	0.2500
2.	1	Medium	0.0238	0.5952	0.3810
3.	1	Low	0.0147	0.5333	0.4520

Table 6
Initial knowledge base for BRB Day 4.

Rule ID	Rule weight	IF LowerBB-ClosingPrice	THEN price movement		
			H	M	L
1.	1	High	0.0000	0.7273	0.2727
2.	1	Medium	0.0548	0.4247	0.5205
3.	1	Low	0.0489	0.5433	0.4078

Table 7
Initial knowledge base for BRB Day 5.

Rule ID	Rule weight	IF LowerBB-ClosingPrice	THEN price movement		
			H	M	L
1.	1	High	0.0000	0.8000	0.2000
2.	1	Medium	0.0625	0.5000	0.4375
3.	1	Low	0.1220	0.4134	0.4647

4.3. BRBES inference using ER approach

In this work, the inference engine of our BRBESes uses the Evidential Reasoning approach (Dymova et al., 2016), as described in Section 3.2. An Inference engine in the ER approach works as follows: (i) first, it reads an input data (which in this work, is the difference between *Lower BB* and closing price), (ii) finds the matching degree of given input using Eqs. (4) and (5), (iii) calculates activation weight of every rule in the knowledge base using Eq. (7), (iv) updates the degree of belief of the initial knowledge base for incomplete or uncertain data using Eq. (8), and finally (v) aggregates all rules in the knowledge base using Eq. (9). After completing the aggregation process, a crisp value can be calculated to determine the final assessment of the stock price movement using Eq. (10) where utility scores of the referential values are given in Table 8.

4.4. Optimizing the rule base of BRBES

Section 3.3 introduces the BRBES optimal training model and outlines the three key phases associated with it. The first step of the optimal learning approach is to define an objective or cost function. In this research, we used mean square error (MSE) as the objective function which can be calculated by Eq. (11). The second step is to define any constraints on the training parameters. We set the following constraints in this work: (i) attribute weights, rule weights, and belief degrees of the consequent must be between zero and one; and (ii) if the kth rule is complete, its total degree of belief in the consequent will be equal to one, i.e., $\sum_{i=1}^{N} \beta_{ik} = 1$. Finally, use the training module to find the optimal parameters (θ_k, δ_i, β_{ik}). We have three rules (for high, medium, and low) in the knowledge base and those will have a weight (θ_k) between [0,1]. Similarly, we considered the difference between *Lower BB* and *Closing Price* as the only attribute, it will have a fixed weight (δ_i) and therefore, is ignored from the learning process. Finally, the belief degree (β_{ik}) of the consequent (i.e., *price movement*) will be in the range [0,1] and when the rule is complete, the summation of the total belief degrees will be 1.

$$\zeta(P) = \frac{1}{M}\sum_{m=1}^{M}(\bar{y}_m - y_m)^2 \tag{11}$$

The optimization problem just defined above is solved by using MATLAB's *fmincon* function. The *fmincon* functions finds the optimal value of the parameters under non-linear constraints. The *fmincon* solver uses a sequential quadratic programming (SQP) algorithm where the function solves a quadratic programming (QP) sub-problem at each iteration (MathWorks, 2021). *fmincon* function calls a minimization routine to reduce the MSE (cost function) with three bound constraints defined above for the parameters, to find the optimal rule weights and belief degrees of BRBES within a specified search space. I trained the model for 200 iterations. Fig. 4 shows the convergence of each BRBES (day 1 to day 5) in terms of discrepancies (ζ) between actual and predicted output, as calculated by Eq. (11). As the figure suggests, all BRBES are converged within 80 iterations.

The optimal parameters of BRB day 1 to BRB day 5 for fold 1 are given in Tables 9 to 13. Optimal parameters for the other folds are available on GitHub (Github, 2021). These optimized parameters are used to create the rule bases of trained BRBESes.

Table 8
Utility scores for referential values of BRB Day 1 to BRB Day 5.

Day No.	Low	Medium	High
Day 1	0	0.40	1
Day 2	0	0.60	1
Day 3	0	0.55	1
Day 4	0	0.40	1
Day 5	0	0.60	1

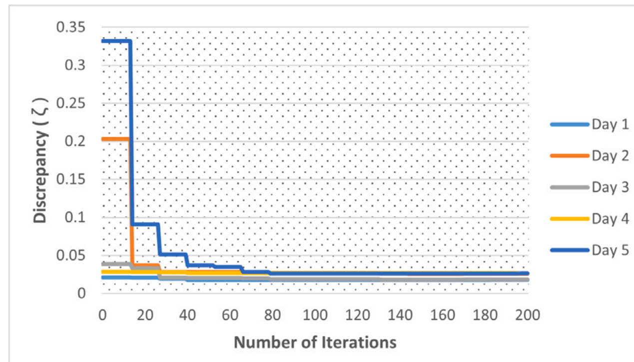

Fig. 4. Convergence of the BRBES.

5. Results and discussion

In Section 4.2 we talked about how we generated the initial knowledge base of BRBES. In Sections 4.3 and 4.4, we discussed the evidential reasoning (ER) approach and the procedure of finding optimal training

Table 9
Optimal parameters of BRB Day 1.

Rule ID	Rule weight	IF LowerBB-ClosingPrice	THEN price movement		
			H	M	L
1.	0.2160	High	0.5432	0.0000	0.4568
2.	0.6549	Medium	0.0000	0.8786	0.1213
3.	0.1663	Low	0.1218	0.5326	0.3456

Table 10
Optimal parameters of BRB Day 2.

Rule ID	Rule weight	IF LowerBB-ClosingPrice	THEN price movement		
			H	M	L
1.	0.9999	High	0.0951	0.9049	0.0000
2.	1.0000	Medium	0.5012	0.0000	0.4988
3.	1.0000	Low	0.0000	0.8529	0.1471

Table 11
Optimal parameters of BRB Day 3.

Rule ID	Rule weight	IF LowerBB-ClosingPrice	THEN price movement		
			H	M	L
1.	0.9686	High	0.0000	0.9163	0.0837
2.	0.0098	Medium	0.0000	0.8036	0.1964
3.	0.0042	Low	0.0000	0.8155	0.1845

Table 12
Optimal parameters of BRB Day 4.

Rule ID	Rule weight	IF LowerBB-ClosingPrice	THEN price movement		
			H	M	L
1.	0.0081	High	0.5369	0.1832	0.2799
2.	0.3123	Medium	0.3437	0.0000	0.6563
3.	0.0781	Low	0.3830	0.0000	0.6170

Table 13
Optimal parameters of BRB Day 5.

Rule ID	Rule weight	IF LowerBB-ClosingPrice	THEN price movement		
			H	M	L
1.	0.4287	High	0.3308	0.6692	0.0000
2.	1.0000	Medium	0.0000	0.9047	0.0953
3.	0.6576	Low	0.5813	0.0000	0.4188

parameters using *fmincon* function. The following Section 5.1 shows the results we achieved following the above methodology, and Sections 5.2–5.4 compare the performance of the proposed model against other machine learning, deep learning, and statistical models. The programs and datasets used in this research are available online and can be downloaded from GitHub (Github, 2021).

5.1. Performance evaluation of trained BRBES

We evaluated our trained BRBESes in terms of Accuracy, RMSE, R^2 value, AUC, Type I error, Type II error, and Profit-Loss ratio. Table 14 shows the results that we achieved for the trained BRBES in each day's price prediction. To avoid overfitting and underfitting the model, we utilized 5-fold cross-validation technique as described in Section 3.4. Fig. 5 shows the average training accuracy of the five folds vs. the average testing accuracy of the five folds. As we can see, the training accuracy and testing accuracy are very close to each other, which indicates that the model is neither overfitted nor underfitted.

We define the accuracy of the model as the percentage of correctly classified samples where BRBES can predict the stock price movement accurately. AUC tells us how much a model is capable of distinguishing between classes. The higher the AUC, the better the model at predicting positive class values as positive and negative class values as negative. Table 14 shows that each BRB achieves more than 91% AUC indicating

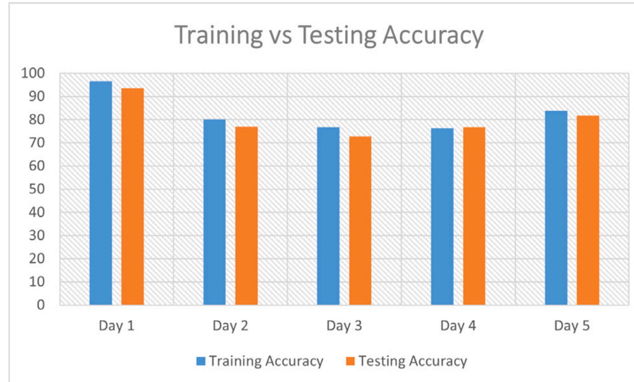

Fig. 5. Average training accuracy vs. average testing accuracy.

that they have excellent capabilities in determining the future price direction of the stock.

R^2 is a statistical measure that represents the percentage of an asset's movements that is based on the movements in a benchmark index which is also used to calculate risk-adjusted return. The R^2 value ranges from 0% to 100%, where 0% indicates that the proposed model cannot predict at all and 100% indicates a perfect prediction. The lesser the value of the R^2 measure, the more the risk associated with the return. Improvement in the regression model increases the R^2 value proportionally. RMSE is an absolute measure of fit, whereas R^2 is a relative measure of fit. Lower RMSE values indicate a more accurate model. In RMSE, the difference between actual and predicted values is squared before taking the average hence it provides high weights for large errors. RMSE is a better choice to evaluate a model where large errors are particularly undesirable and which is why we have considered this metric in this case.

In this study, positive class (P) and negative class (N) are classified by whether the stock price goes upward or downward, respectively. Therefore, a false positive (FP) event occurs when the actual stock price goes down but the model predicts the opposite. Similarly, a false negative (FN) event suggests that the actual price goes up but the model predicts a downward trend. Based on this concept, we have tested the percentage of type I error (α), which is related to false positive, and type II error (β), which is related to a false negative, of our model by using Eqs. (12) and (13). Table 14 shows that our model achieves significant accuracy each day and also demonstrates considerably low type I and type II errors.

$$\alpha = (FP/N) \times 100\% \tag{12}$$

$$\beta = (FN/P) \times 100\% \tag{13}$$

We also evaluated the model in terms of *profit/loss* ratio which indicates whether we will gain profit or incur loss over the long run if we follow the model's recommendation. If the ratio is greater (or less) than 1, that means we will gain profit (or incur a loss).

After analyzing the optimal BRBESes' belief structures of BRB Day 1 to BRB Day 5, the trained BRBES approach believes that when the antecedent value is medium and high, selling on day 5 will produce a better profit. Otherwise, it recommends selling on day 1. This means that if the closing price of Day 0 goes down 1.12 SEK or less amount below the *Lower BB* (antecedent of BRBES is low), BRBES recommends selling stock on Day 1. If the closing price goes down more than 1.12 SEK (antecedent of BRBES is high or medium), then it will suggest selling on Day 5. This proof-of-concept use of the BRBES does not operate with any stop-loss rules. After the trade is entered, it always exits the trade at the predicted optimal day, no matter the price movements until the end of the trade.

Table 14
Performance evaluation of trained BRBES.

Performance metric	BRB Day 1	BRB Day 2	BRB Day 3	BRB Day 4	BRB Day 5
Accuracy (%)	93.50	76.90	72.70	76.80	81.80
RMSE	0.1233	0.1518	0.1283	0.1544	0.1534
R^2 score (%)	48.06	49.32	46.65	35.25	32.57
AUC	0.984	0.991	0.987	0.916	0.924
Type I error (%)	3.77	0.68	29.79	28.53	4.36
Type II error (%)	8.63	28.74	2.47	3.94	30.88

We define profit and loss in the following manner. Suppose, on a certain day TELIA's price closes at 27.73 SEK, 0.11 SEK below the *Lower BB*. So BRBES suggests selling on Day 1 which has an actual closing price of 28.31 SEK. So the trader will earn 0.58 SEK from this trade. Suppose, on another day TELIA's price closes at 52.34 SEK, 1.22 SEK below the *Lower BB*. So BRBES suggests selling on Day 5 which has an actual closing price of 51.18 SEK. So the trader will lose 1.16 SEK from this trade. We find total profits and total losses in all instances and divided them by the number of winning trades and the number of losing trades respectively to find out the average profit and average loss per trade.

Then calculated the profit/loss ratio by diving the average profit per trade by average loss per trade (Investopedia, 2021), as defined in Eq. (14).

$$Profit/LossRatio = \frac{TotalGain}{NWT} \div \frac{TotalLoss}{NLT} \quad (14)$$

where NWT = number of winning trades and NLT = number of losing trades. We used this formula to calculate the profit/loss ratio of 200 instances in each fold and then took the average value of all five folds. Trained BRBESes yield an average profit/loss ratio of 2.13, which means that traders earn on average 2.13 times more profits per trade than losses when following trained BRBESes' recommendation. Many trading books consider having a profit/loss ratio of at least 2 as a significantly profitable trade (Investopedia, 2021).

5.2. Comparison with ANFIS and SVM

Recently, nonparametric models (Jeong et al., 2010) and fuzzy models (Ketsarapong et al., 2012; Precup et al., 2020) are being used in machine learning. Both Fuzzy Logic (FL) and Artificial Neural Network (ANN) are used when creating the architecture of Adaptive Neuro-Fuzzy Inference System (ANFIS) (Avci, 2008; Avci & Akpolat, 2006; Avci et al., 2005; Boyacioglu & Avci, 2010). ANFIS is consist of traditional IF-THEN rules and input–output pairs. Learning methodologies of neural networks are also used for training ANFIS (Avci, 2008; Jang, 1993). Now the question is why it is necessary to compare the performance of BRBES against ANFIS? To be specific, BRBES is a variant of a fuzzy inference system. Where ANFIS uses traditional IF-THEN rules, BRBES uses an extended version of IF-THEN rules which incorporates belief degree in antecedent and consequent parts as shown in Eq. (2). A simple IF-THEN rule in ANFIS can only decide whether the stock price will go up or down. But since BRBES also includes the degree of belief, the model will show how much confidence the model has about each price movement. For example, BRBES may tell us that it is 70% confident that the price will go too high, 20% confident that the price will go slightly high, and it has 10% confidence of price going down. Based on this confidence (or probability), the trader can decide whether to sell, hold, or buy a stock. Therefore, the comparison against ANFIS is important to understand whether including the degree of belief helps to better predict the price or not.

We also compared the performance against one of the most popular machine learning (ML) models: Support Vector Machine (SVM). The reason for choosing SVM is that it is still a very effective ML technique for time series forecasting (Pattanayak et al., 2020; Shao et al., 2020; Singh et al., 2020).

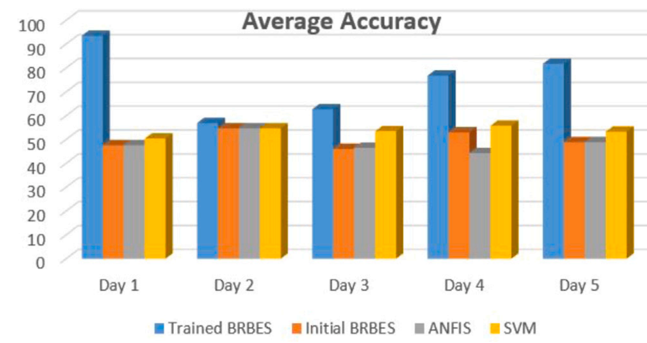

Fig. 6. Average accuracy of five folds.

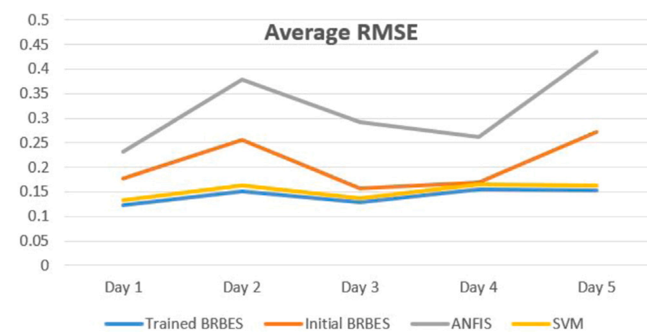

Fig. 7. Average RMSE of five folds.

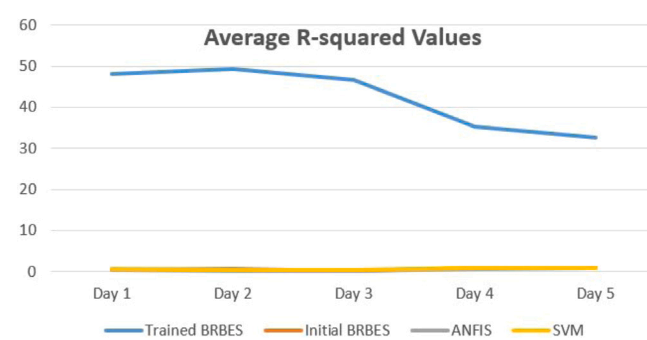

Fig. 8. Average R-squared values of five folds.

We used MATLAB's fuzzy toolbox to create a Mamdani-type ANFIS, one for each day in every fold. Each ANFIS has three gaussian-type membership functions as well as three rules for three referential values (*high*, *medium*, *low*). Fuzzy systems are trained on the training data and then optimized using MATLAB's fuzzy toolbox's backpropagation technique. The SVM regression model is built in R language using *caret* library. SVM model is trained using *svmLinear* method and 5-fold cross-validation (to overcome overfitting and underfitting).

To compare the performance against ANFIS and SVM, we considered average Accuracy, AUC, RMSE, and R^2 value as the evaluation metrics. Figs. 6–9 show the comparative study among trained BRBES,

Table 15
A summary of compared deep learning approaches.

Model	Objective	Proposed technique
Markov ATS by Rundo et al. (2019a)	Proposed an Adaptive Trading System (ATS) applying Markov-based correction for high frequency stock price prediction	Used two LSTM networks: one for stock trend prediction and the other for closing price prediction
GTSbot by Rundo et al. (2019b)	Predicts 1 min intra-day closing price of EUR/USD FOREX currency	Proposed a Grid Trading Strategy (GTS) using Scaled Conjugate Gradient (SCG) approach as regressor network
RSM-LSTMNet by Shintate and Pichl (2019)	Predicts OkCoin bitcoin (USD and CNY) prices for 1 min data	Random Sampling Method (RSM) with LSTMNet as encoder. LSTMNet has two LSTM layers with 32 LSTM units in each layer
NN-RRL by Gold (2003)	Predicts high-frequency FOREX currency with neural network trained vai RRL (Recurrent Reinforcement Learning)	Used neural network (NN) which is trained by Recurrent Reinforcement Learning (RRL). Compares the performance of 1-layer NN with 2-layers NN where 2-layers NN outperforms the other

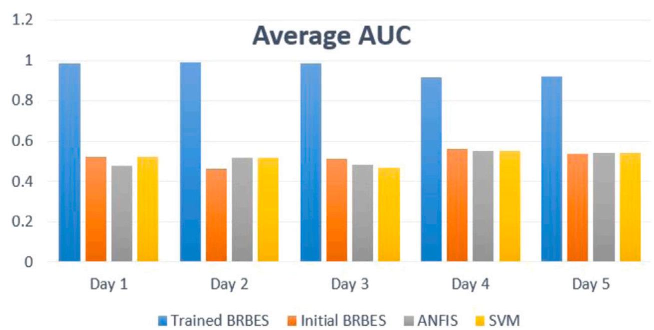

Fig. 9. Average AUC values of five folds.

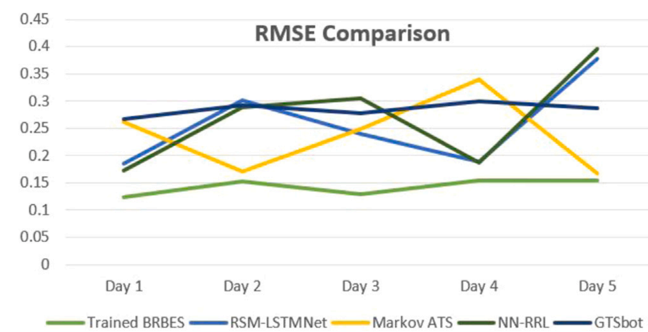

Fig. 11. RMSE values of proposed model and deep learning approaches.

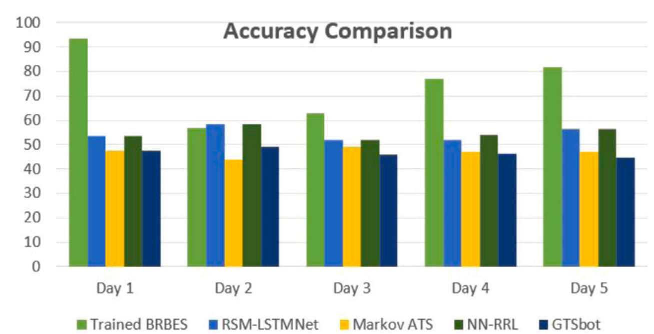

Fig. 10. Accuracy of proposed model and deep learning approaches.

initial BRBES, ANFIS, and SVM models in terms of the four metrics. It is evident that the trained BRBES achieves much higher accuracy, higher AUC, higher R^2 values, and lower RMSE on each day than the other models. This means that our trained BRBES can predict the stock price direction more accurately, producing lesser errors and having a lower risk on return than the other techniques. Thus, the trained BRBES establishes its superiority among all these models.

5.3. Comparison with deep learning approaches

In addition to SVM, ANFIS, and the heuristic models, we have also compared our proposed model against recent deep learning approaches used for time series prediction. Table 15 shows the summary of the deep learning approaches against which we have compared our model. We have labeled each model by a name based on the proposed method for simplicity purposes.

We have compared our proposed trained BRBES model against these aforementioned models considering Accuracy and RMSE metrics. RMSE indicates the actual error in the predicted output by the models. On the other hand, accuracy demonstrates how well a model can predict the direction of the price movements. Figs. 10 and 11 show the models' performance in terms of accuracy and RMSE respectively. The results reflect that although these deep learning approaches have better-predicting capability for high frequency or 1 min data, our proposed model performs better when predicting inter-day prices. The trained BRBES achieves the highest accuracy in stock trend prediction in most of the cases except on day 2. Moreover, it is also evident that our proposed model has the lowest prediction error among all these models.

5.4. Comparison with a simple heuristic model

Many investors still operate without advanced quantitative support in the form of MATLAB or advanced statistical techniques such as the ones discussed in this paper. One example of such a model would be a simple averaging model. The line of reasoning goes like this: It simply buys the stock on day 0, and sells based on which middle point of the forthcoming days has the best average return. We will call this strategy *a simple heuristic approach*. We do so because the BRBES may in principle use many antecedents and rules and can be trained to optimize the decision between these. Simple heuristics will take only one indicator (event triggered by the reach of the *Bollinger Band*) with one rule on when to buy, and will have one rule on when to sell. We have included this comparison to show the practical value of the BRBES.

From the data given in Table 1, the simple heuristic approach suggests that there will be 1.21% profit (which is the middle point between −2.39 and 4.8) if we sell the stock on Day 1. Similarly, the outcome on Day 2, Day 3, Day 4, and Day 5 will be −0.07%, +0.94%, +1.39%, and −0.51% respectively. According to the simple heuristic approach, Day 4 is producing a larger profit than the others. Suppose that the Telia stock plunges on a Thursday and triggers the *Bollinger event*. So, the simple heuristics approach will recommend the investors

Table 16
RMSE values of the simple heuristic model and trained BRBES.

Model	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Average
Trained BRBES	0.1576	0.1653	0.1588	0.1680	0.1684	0.1636
Simple Heuristic	0.1778	0.1727	0.1725	0.1801	0.1763	0.1759

to buy stock on Thursday (Day 0) and sell the stock on Wednesday (Day 4) considering the market remains close on Saturday and Sunday. This is a plausible interpretation of how for instance some retail traders may plan their trading. Let us compare such a strategy with our trained BRBES.

As we mentioned earlier, trained BRBES will recommend selling on day 5 when the antecedent value is medium or high. Otherwise, it recommends selling on day 1. Table 16 shows the comparison of trained BRBES with the heuristic approach in terms of RMSE. Trained BRBES produces a lower RMSE than the heuristic model meaning that higher accuracy can be achieved by following the proposed trained BRBES recommendation.

The BRBES can also take multiple antecedents into account, although we have not showcased it here. An example is that ceteri paribus, stock falls in price when the dividend is paid to the investors (because the value of that dividend is no longer inherent in the stock price). But maybe we can find some market inefficiency in the fall and subsequent recovery of the price during the year. If we find that, we can make additional rules and combine that with the existing *Bollinger* rule. Also, simple heuristic analysts may find such a market inefficiency. However, the simple heuristic can simply not accommodate a combination of several events (*Bollinger Band* plus dividend payment) in a systematic way, instead, the traders will (we speculate) just combine those pieces of knowledge intuitively. Thus, there is also this qualitative advantage of the BRBES approach.

6. Conclusion and future works

In this research, we demonstrated the power of BRBES to predict future stock prices using the concept of the *Bollinger Band*. We developed an initial BRBES by constructing an initial knowledge base of BRBES from the previous pricing history of the stock and subsequently applied an evidential-reasoning approach. Then we optimized the parameters of the expert system and developed trained BRBESes which reduce the Mean Squared Error (MSE) of the system output. We evaluated the performance of our proposed trained BRBES in terms of average Accuracy, type I error, type II error, RMSE, AUC, R^2 value, and profit/loss ratio. Then we compared our trained and initial BRBES with an ANFIS model and an SVM model considering average accuracy, RMSE, AUC, and R^2 value. The trained BRBES outperforms other models in terms of every performance metric. Also, we have compared trained BRBES against recent deep learning approaches where trained BRBES shows promising performance in stock price prediction. Finally, we have validated the trained BRBES approach with a statistical heuristic approach. The results reflect that the trained BRBESes produce less RMSE and hence have better accuracy than the heuristic model.

Is it sufficient for BRBES to be better than the ANFIS, SVM, deep learning approaches, and the simple heuristic approach to be viable? Or are there approaches with better performance? This will be a question for further applied research, where the BRBES gets a full-fledged trading strategy, including rules of where to exit the position in case of drawdown loss. As can be seen in the literature review, the ANFIS is an accepted approach in the field, so BRBES should now be considered an alternative amongst the machine learning/expert systems alternatives for prediction in financial markets. It is quite possible that in some markets and maybe some market conditions, ANFIS will produce better results and in others, the BRBES will be superior. A full comparison would also require that the benchmarked approaches competed with the same data and at the same time in the market, on a specific instrument (stock). Doing this with old data is in our view misleading, since the market is changing and may have become more or less machine-learning friendly, also for specific approaches (e.g. BREBES or neural networks). However, we aimed to demonstrate the viability of the BRBES approach, not to show that this particular *Bollinger Band* strategy is statistically likely to be valid throughout all financial applications for trading purposes. This proof-of-concept did not take transaction costs into account. However, when the profit/loss ratio is sufficiently high, in combination with a rise of the stock by several percent and a small spread, the transaction costs will not render the strategy unprofitable. However, the transaction cost is a factor that needs to be factored in, if the BRBES approach would be used e.g. in intra-minute trading where the percentual gain is very small compared to the underlying asset.

In the future, we will also add other companies' pricing data to develop the BRBES knowledge base. This will produce a more adaptive knowledge base and the new BRBES can be applied to predict the price action of other companies as well. Moreover, apart from the closing price, other prediction factors such as stock volume traded in a day, high price, low price, and opening price of a stock can also be considered in decision making. Furthermore, the proposed BRB expert system model can applied to various fields, including inventory planning (Ketsarapong et al., 2012), signature modeling (Pozna & Precup, 2014), tower crane system modeling (Hedrea et al., 2021), etc.

CRediT authorship contribution statement

Emam Hossain: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Writing - review & editing, Visualization. **Mohammad Shahadat Hossain:** Conceptualization, Methodology, Formal analysis, Investigation, Writing – original draft, Writing - review & editing, Supervision, Project administration. **Pär-Ola Zander:** Conceptualization, Validation, Investigation, Resources, Writing – original draft, Project administration. **Karl Andersson:** Conceptualization, Investigation, Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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