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Advanced Underwater Acoustic Signal Processing

Underwater Acoustic Modem using OFDM

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2023

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Executive Summary

The purpose of this report is to demonstrate an under water acoustic system using Orthogonal Frequency Division Multiplexing (OFDM). The system has been tested in multiple environments (controlled pool, calm sea) at various system hyperparameters. The report will contain advance digital signal processing methods and algorithms to improve bit error rate (BER). The main experiment took place off the coast of Israel on May 30, 2023, and was carried out by The Underwater Acoustic & Navigation Lab, University of Haifa, Israel.

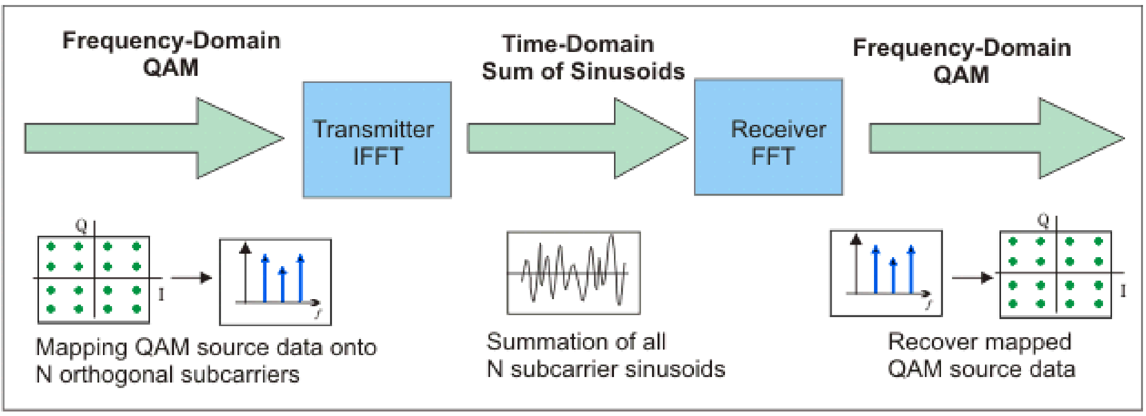
Model

Introduction to OFDM

Existing coherent underwater communication uses single carrier transmission and relies on linear or non-linear equalization techniques to suppress inter-symbol interference (ISI). As the data rate increases, the symbol duration decreases, and thus a channel with the same delay spread contains more channel taps when converted to the baseband discrete time model. This imposes great challenges for the channel equalizer, whose complexity will prevent rate improvement with the existing single-carrier approach.

Multicarrier modulation is an attractive alternative to single carrier broadband modulation on channels with frequency-selective distortion. It is based on the idea of dividing the total available bandwidth into many narrow subbands, such that the channel transfer function appears constant (ideal) within each subband. By doing so, the need for time-domain channel equalization is eliminated. Instead, the subbands must be separated in the frequency domain, which is efficiently performed using only the Fast Fourier Transform (FFT).

This efficient implementation using FFT can be executed for multicarrier modulation and detection when used with rectangular pulse shaping. The modulation technique on which the project is based is the Orthogonal Frequency Division Multiplexing (OFDM). OFDM achieves an adequate transmitter/receiver complexity, appropriate capacity, and provides numerous possibilities for channel compensation. OFDM is a frequency-division multiplexing (FDM) scheme utilized as a digital multi-carrier modulation method. Many closely spaced orthogonal subcarriers are used to carry data. Data is divided into several parallel data streams or channels, one for each subcarrier. Each subcarrier is modulated with a conventional modulation scheme (QAM, PSK,..).



Simplified OFDM System Block Diagram

Mathematical Description

OFDM is formed by blocks each containing the transmission for the K subcarriers. Each block duration contains the effective symbol time and the guard interval :

$$T = T' + T_g$$

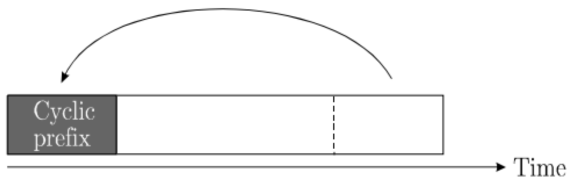
where T_g stands for the guard time and it must be longer than channel impulse response length to prevent Inter-Symbol Interference (ISI) between two consecutive OFDM blocks. T' stands for the effective symbol duration and it is defined as $T' = K/B$ where B is the overall system bandwidth and K is the number of subcarriers, the spacing between adjacent subcarriers is $\Delta f = \frac{1}{T'}$. So, the subcarrier frequencies are:

$$f_k = f_0 + k\Delta f \qquad k = 0,..,K-1$$

Where f_0 is the chosen carrier frequency for the transmission. Each of the subcarriers contain a QAM symbol; we tested 4-QAM and 16-QAM modulations on the UWA channel. The data bits are being mapped to symbols (i.e in 4-QAM each 2 bits are one symbol), and each one of the mapped symbols is modulated with one subcarrier, that is if the symbol on the k-subcarrier is called d_k then the baseband expression of the modulated signal for only one block is

$$b_s(t) = \sum_{k=0}^{K-1} d_k e^{j2\pi k\delta f t} \qquad t = 0, \dots, T'$$

Which is essentially an IFFT calculation. To transmit the signal, a Cyclic Prefix (CP) has been added to preserve the FFT circularity.



A cyclic prefix is a copy of the last part of the OFDM symbol which is prepended to the transmitted symbol. This makes the transmitted signal periodic, which plays a decisive role in avoiding intersymbol and intercarrier interference.

Then, the OFDM system with N subcarriers, a bandwidth of W Hz and symbol length of T seconds, of which T_{cp} seconds is the length of the CP, the transmitted waveforms become

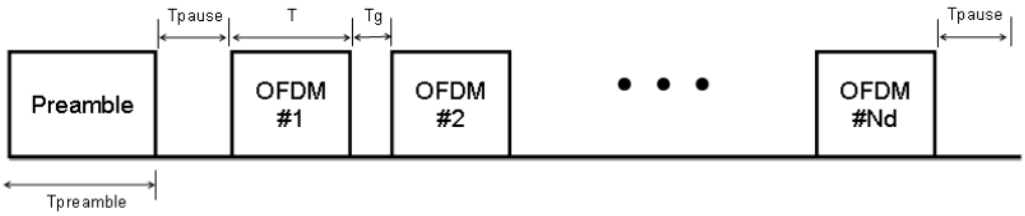
$$\varphi_k(t) = \begin{cases} \frac{1}{\sqrt{T - T_{cp}}} e^{\frac{j2\pi W}{N} k(t - T_{cp})} & if\ t \in [0, T] \\ 0 & else \end{cases}$$

We assume that the support of the impulse response $g(\tau; t)$ of the physical channel is restricted to the interval $\tau \in [0, T_{cp}]$ (the length of the cyclic prefix). The received signal becomes

$$r(t) = (g * s)(t) = \int_0^{T_{cp}} g(\tau; t) s(t - \tau) d\tau + n(t)$$

Where $n(t)$ is complex pink channel noise (for underwater transmission).

Packet Structure



The guard time is a sequence of zeros which has a duration of $\boldsymbol{T_g}$ After guard time is inserted, the frequency adjustment is performed, and the signal is shifted to the passband. As a result, a block is generated which has duration ($\boldsymbol{T}$) of $\frac{\boldsymbol{K}}{\boldsymbol{B}}$ and a guard time of $\boldsymbol{T_g}$. In a OFDM block each subcarrier has a frequency spacing of $\Delta \boldsymbol{f} = \mathbf{1}/\boldsymbol{T}$, so the $\boldsymbol{k}$**th** subchannel of the block will be

$$f_k = f_0 + k\Delta f \quad k = 0, .., K-1$$

where f_0 denotes the lowest subchannel frequency. The generated OFDM block at passband is given by

$$s(t) = Re\left\{\left[\sum_{k=0}^{K-1} s(k)\, e^{j2\pi k\Delta f t}\right] e^{j2\pi k f_{low} t}\right\}, \qquad t \in [0, T + T_g]$$

Where $s(k)$ denotes the data symbol at the kth subchannel. For the *preamble* common use is a pre-selected random symbol, we eventually used a chirp signal with known $[f_{start}, f_{end}]$ at the operational frequency range of the acoustic modem.

Hyperparameters

16 packets were generated and transmitted at the IOLR pool to check for best performance.

Tx Sampling Frequency	250 kHz			
Rx Sampling Frequency	192 kHz			
Amplitude	Normalized to 1, Modem “Level 1” was applied			
Frequencies Range	49 kHz – 77 kHz			
Preamble Chirp	Duration = 50ms, $f = [60k - 70k]Hz$			
Guard period	25ms		50ms	
QAM	4		16	
Bandwidth	2kHz	4kHz	6kHz	8kHz
FFT size	512	256	128	64
Cyclic prefix length	25% of block length			

Analysis

Digital Data

The analysis will focus on 2 types of binary data that will be encoded into symbols:

- *Binary counting* [00,01,10,11] to form a series of '00011011' that is been repeated to dynamic length to accommodate the chosen fft size and cyclic prefix length

$$length\ require = ((fft_{size} * .75) - 1) * 4$$

Example: '00011011000110110001101100011011000110110001101100011011....00011011'
We will use this series later for Viterbi transition matrix.

- Random bits generated from *binomial* distribution

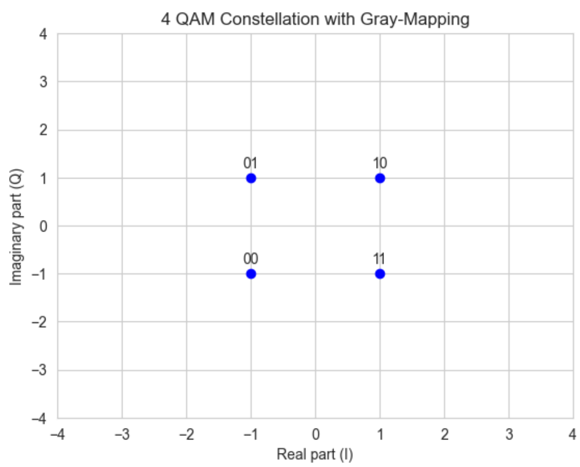
$$X \sim Binom(n = 1, p = 0.5)$$

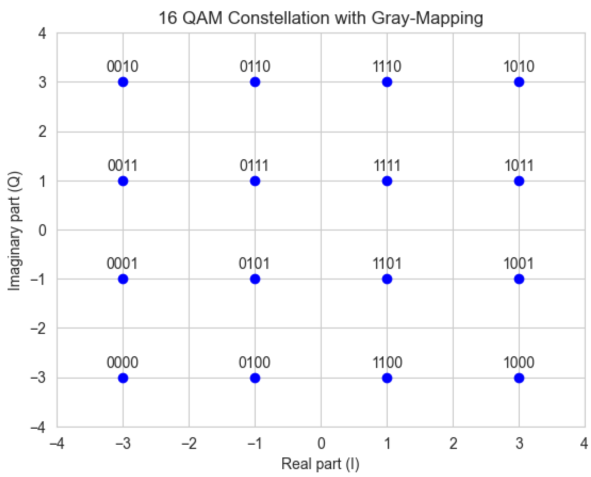
Modulation

The system has been tested with 2 modulation schemes, 4-QAM and 16-QAM [1].

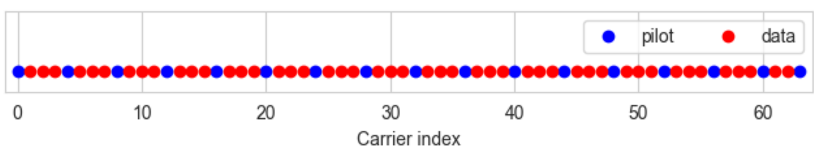
4-qam is a digital modulation scheme that encodes data by adjusting both the amplitude and phase of a carrier signal, with each symbol representing two bits. It offers higher data transmission rates but is more vulnerable to noise and interference compared to binary modulation schemes. With 4-qam, we chose the pilot value to be $1 + 1j$. In 16-qam, each symbol represents 4 bits with pilot value $3 + 3j$.

As the QAM constellation size increases, such as transitioning from 4-QAM to 16-QAM or higher, the main challenge lies in the increased susceptibility to noise and interference, which can lead to a higher probability of symbol errors and reduced overall system performance.





Pilot subcarriers are fixed and transmitted along with data-carrying subcarriers to aid in channel estimation and equalization. They provide reference symbols that help the receiver estimate the channel response and compensate for channel distortions, improving the accuracy of data recovery.



TX Packet

To prepare the packet, a 100ms chirp signal ranging from $60kHz$ to $70kHz$ is added as padding. The data is then divided into 10 OFDM blocks with $25ms$ padding between them, while the final OFDM block is padded with an additional $50ms$ for sanity checks.

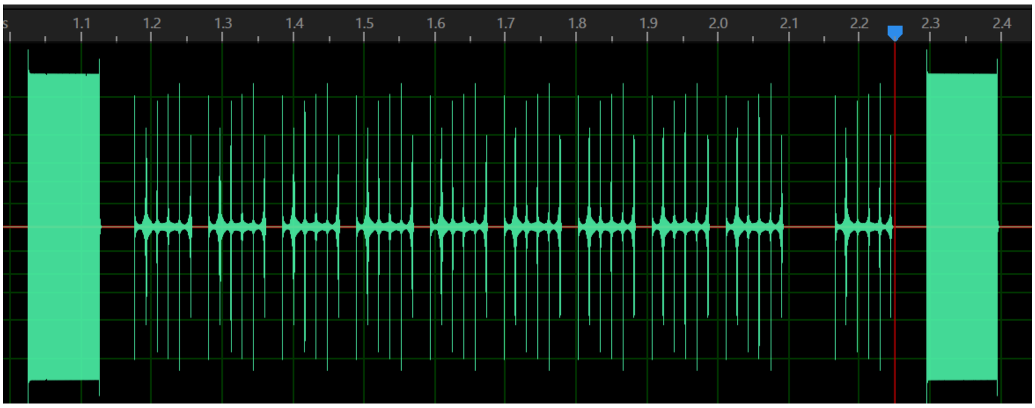


Figure – Packet, Time Domain

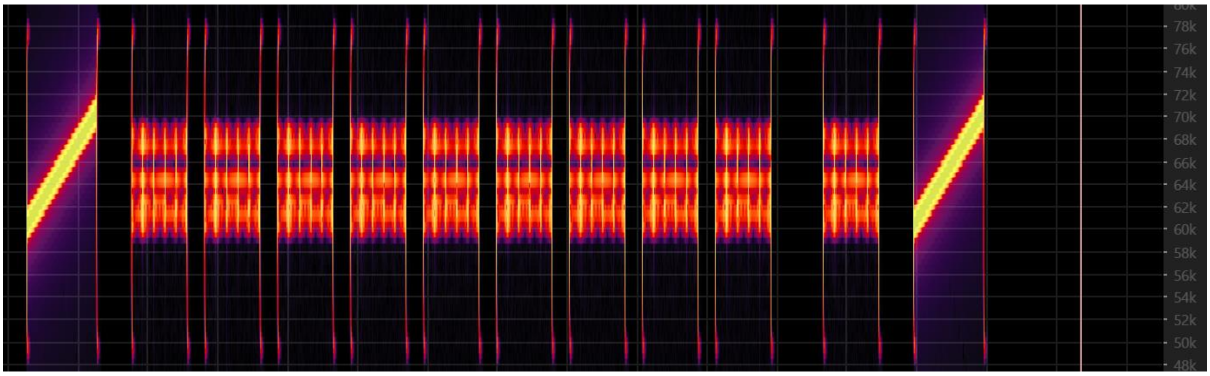


Figure – Packet, Frequency Domain

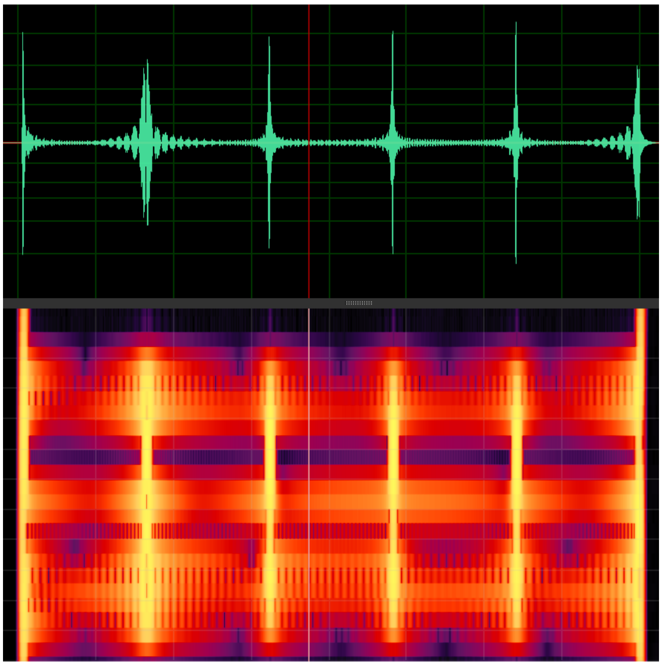


Figure – OFDM block, Frequency Domain

Tx Packet Permutation

As described above we prepared permutations of the transmitted packets with the following hyper parameters: *FFT size* (512 \ 256 \ 128 \ 64), OFDM *block padding* (25ms\50ms), *bandwidth* (2\4\6\8 *kHz*), 4-QAM\16-QAM *modulation* and different *data* of random bits \ counting bits.

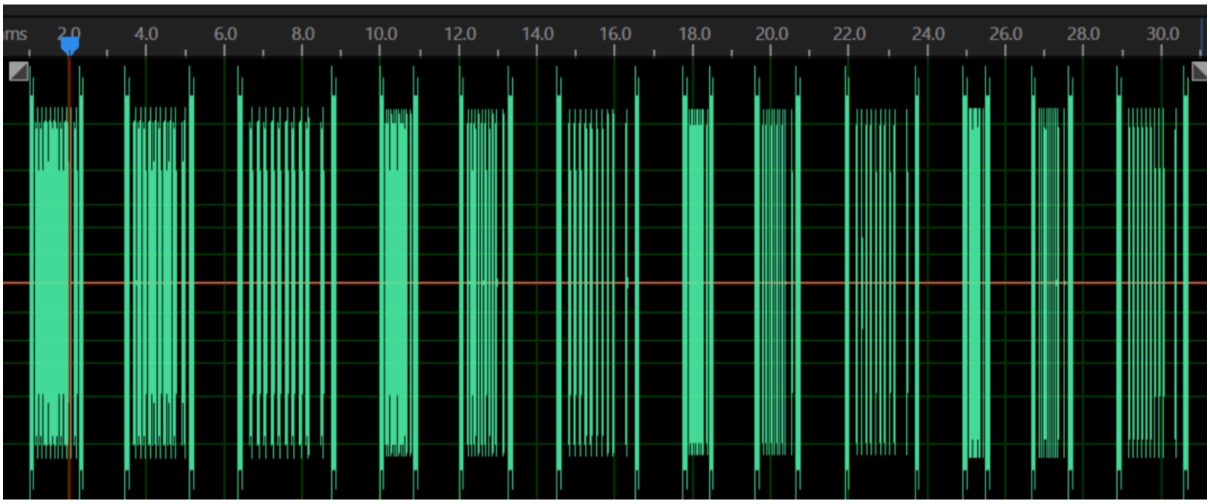


Figure – Example of TX signal, containing 12 packets

Tx Rx Flow

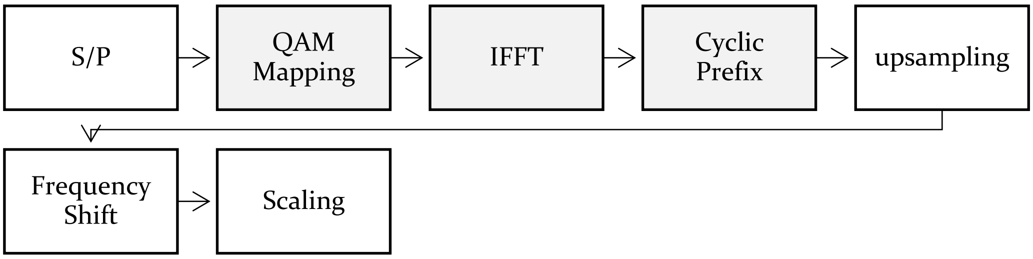


Figure – Transmitter – 1 OFDM block system model

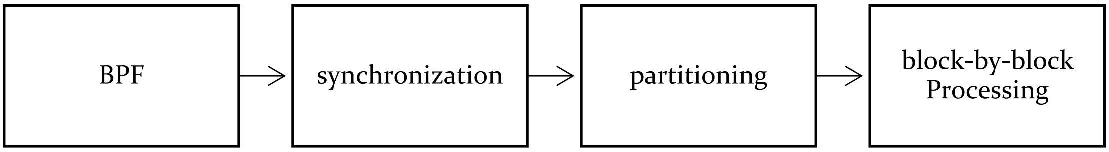


Figure – Receiver – system model

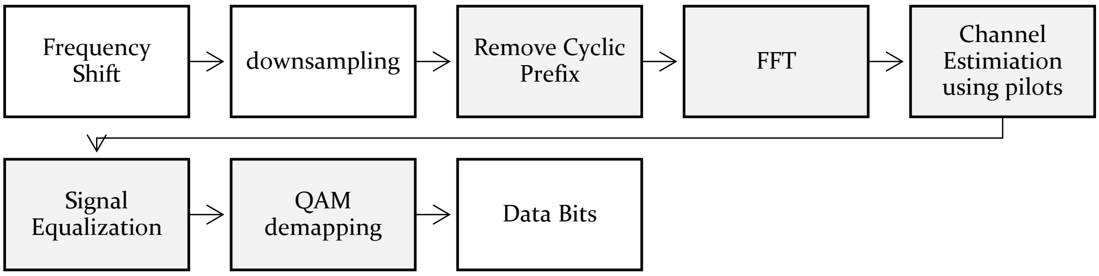


Figure – Receiver – 1 OFDM block system model

BER

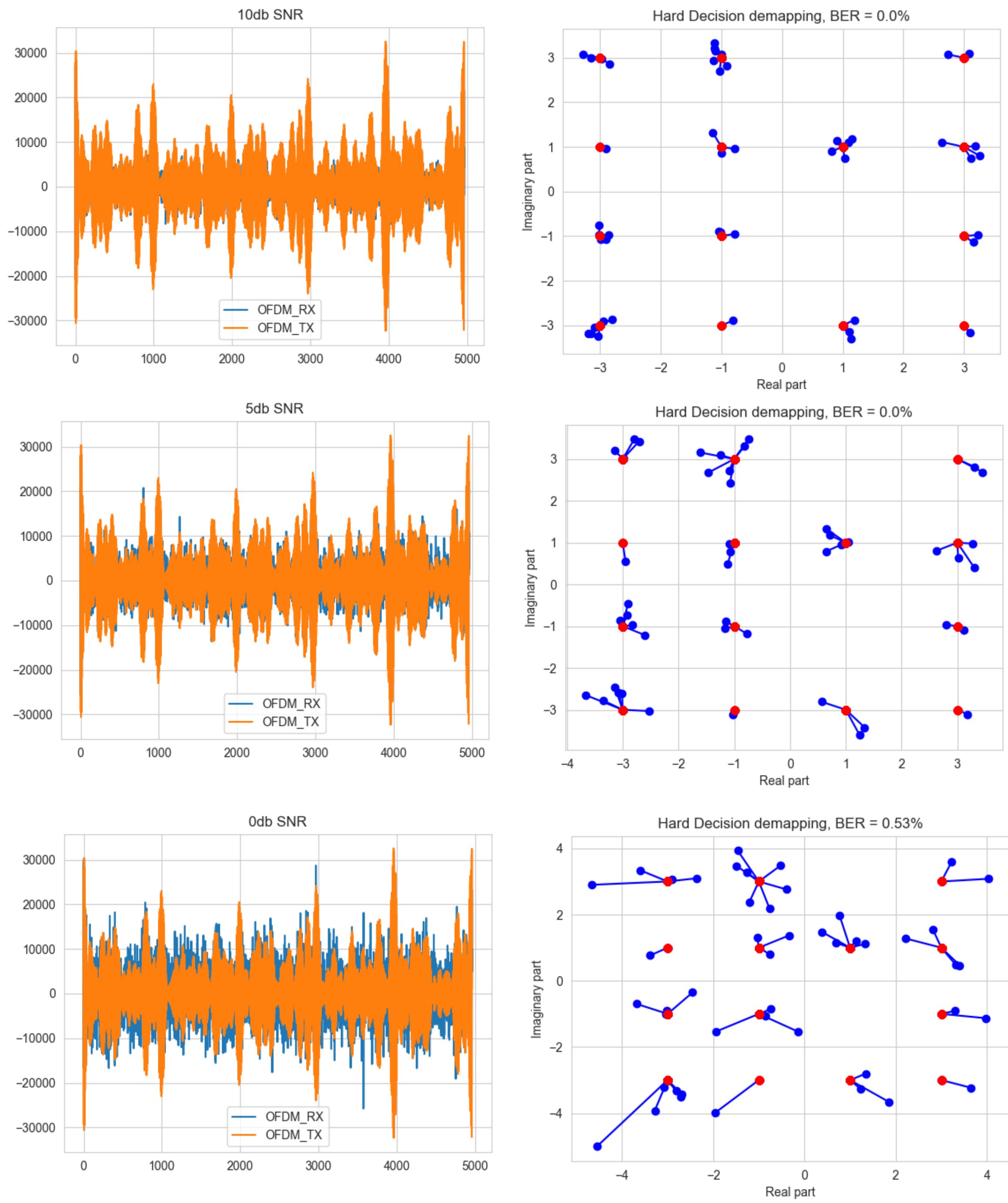
The Bit Error Rate (BER) is a measure of the accuracy or quality of a digital communication system. It represents the ratio of the number of bits that are received incorrectly to the total number of bits transmitted over a communication channel. A lower BER indicates a higher level of accuracy and reliability in the transmission, while a higher BER suggests a higher rate of errors.

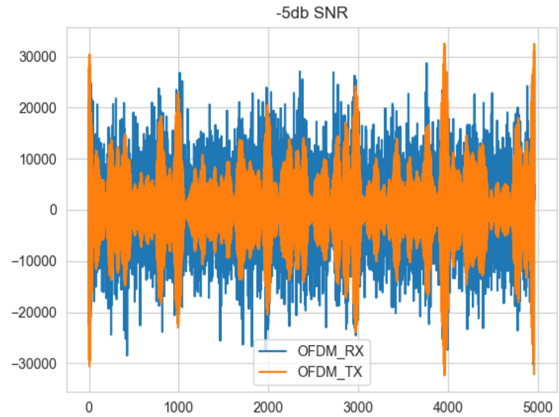
$$BER = \frac{\sum I\{x_i = y_i\}}{N} \qquad N - data\ length;\ x_i - rx\ bit;\ y_i - tx\ bit$$

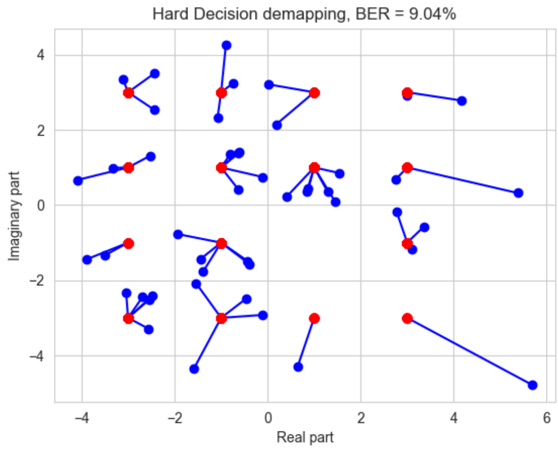
BER Comparison

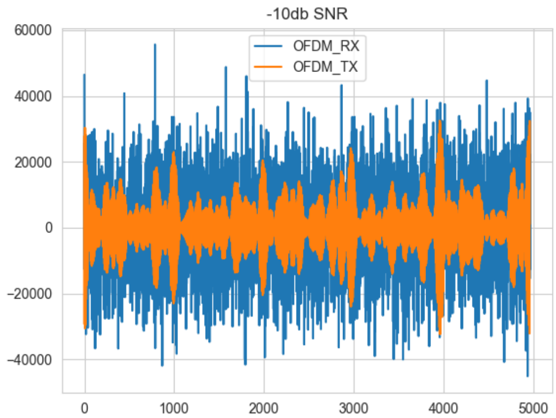
1. Additive white Gaussian noise – Random binomial bits

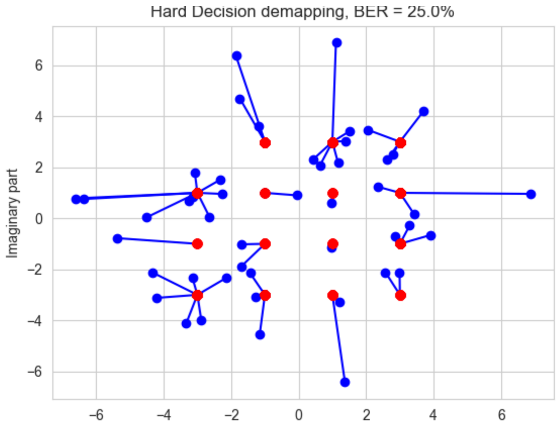
Applied with channel response of $[1, 0, 0.3 + 0.3j]$, FFT=64, bandwidth=$4kHz$

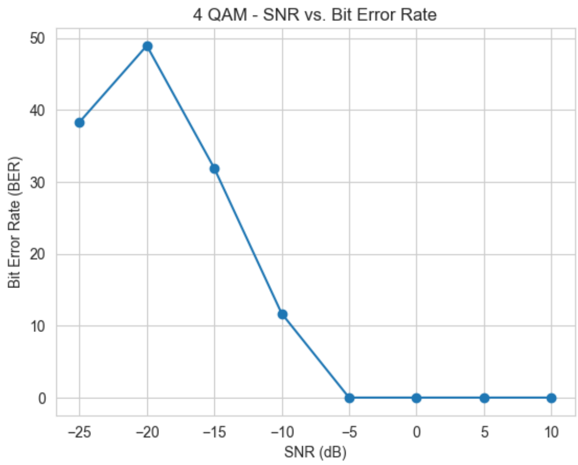


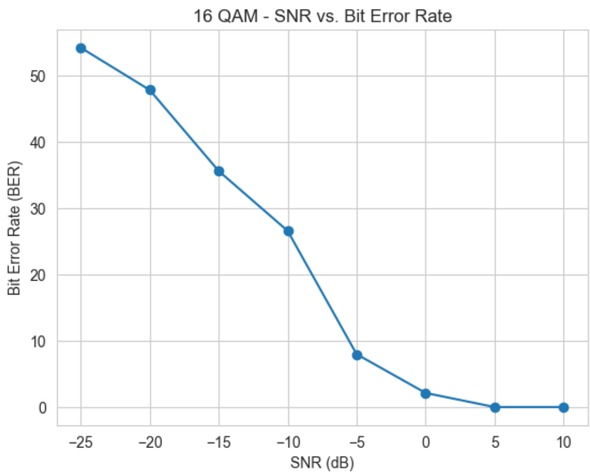




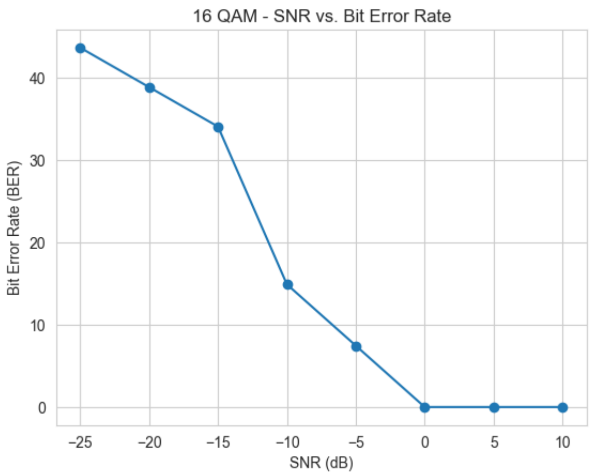


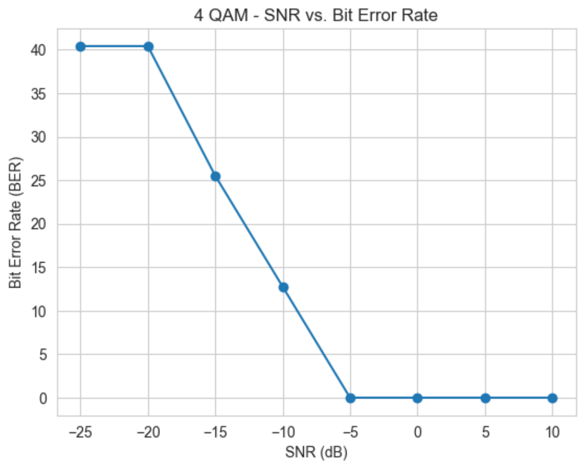






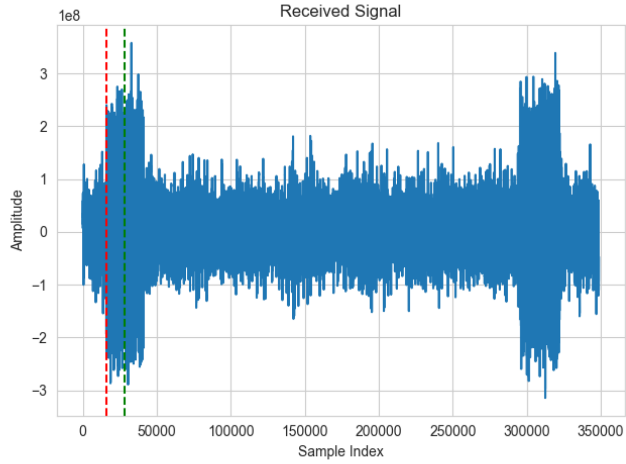
2. Additive white Gaussian noise – *Counting* bits

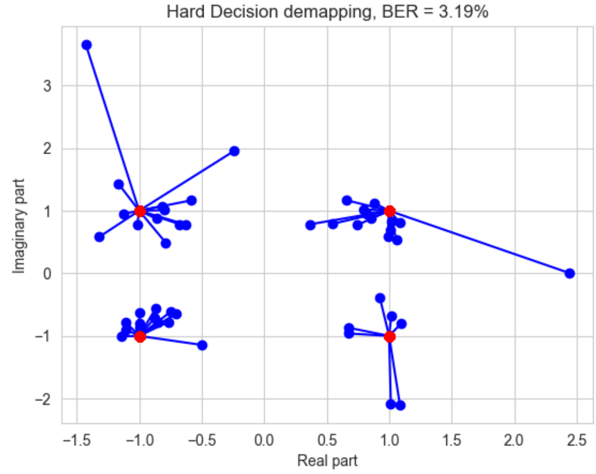




3. Sea recording; 20m – *Random* bits – QAM4

$$fft\ size = 64; bandwidth = 8kHz; QAM = 4; center\ frequency = 64kHz$$



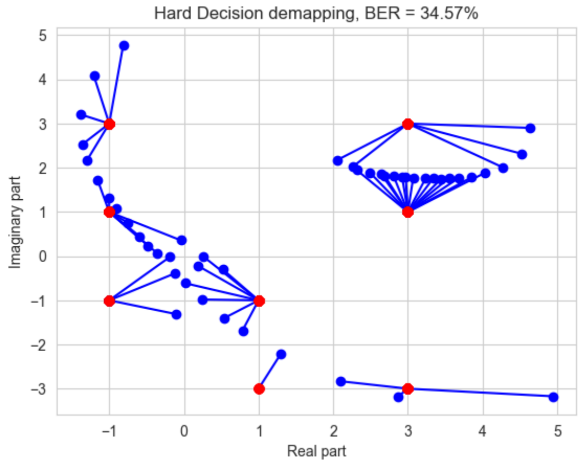


4. Rayleigh distribution noise – *Counting* bits

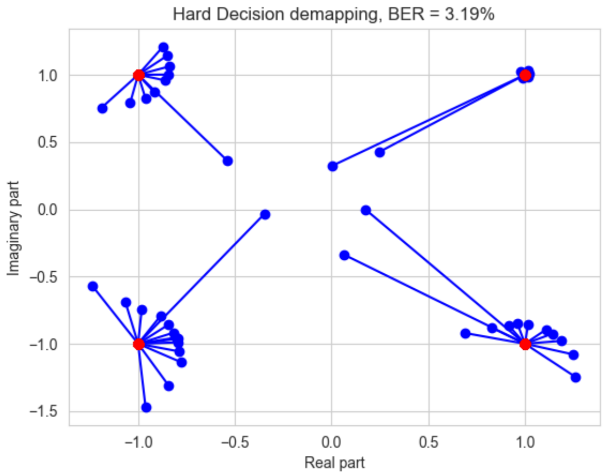
Rayleigh distribution is a useful tool for modeling underwater noise, the actual noise characteristics may deviate from a strict Rayleigh distribution due to the complex nature of underwater environments.

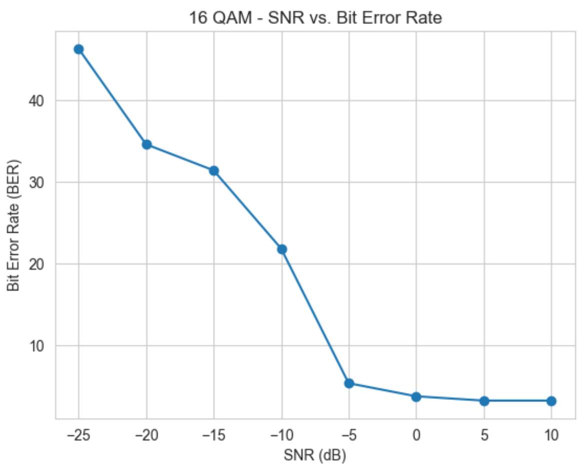
$$f(x;\sigma) = \frac{x}{\sigma^2} e^{-\left(\frac{x^2}{2\sigma^2}\right)}$$

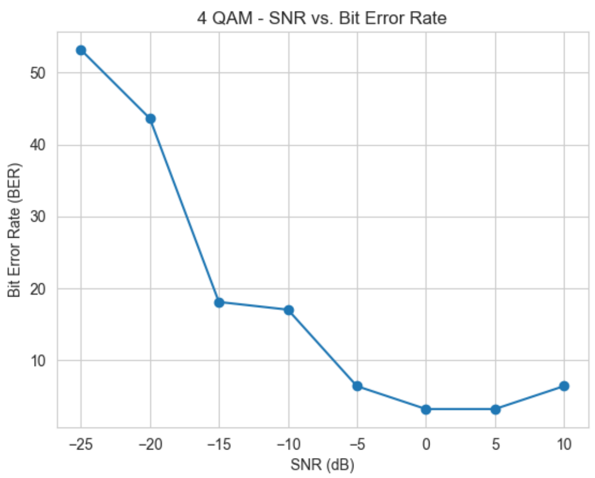
$SNR = -25dB; 16QAM$



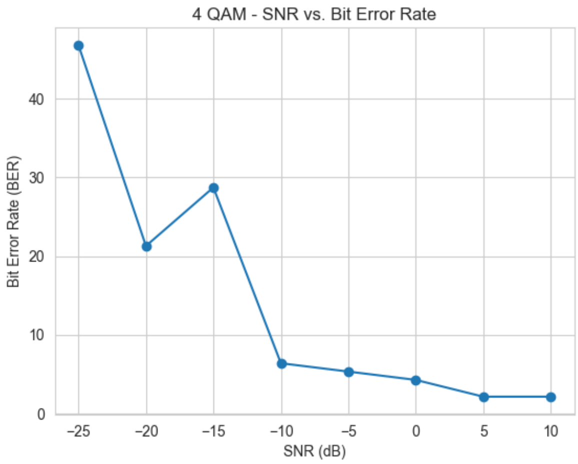
$SNR = 10dB; 4QAM$

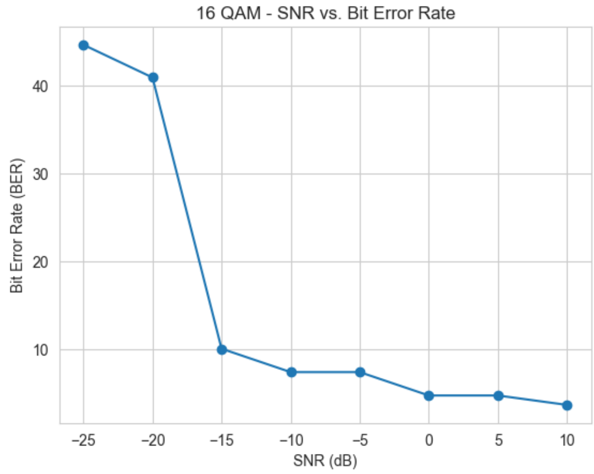






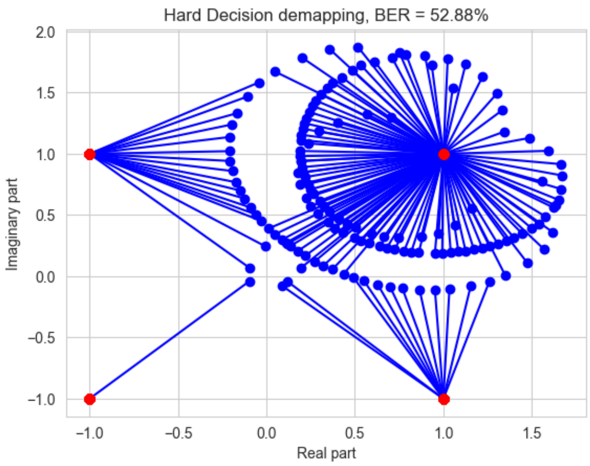
5. Rayleigh distribution noise – *Random* bits

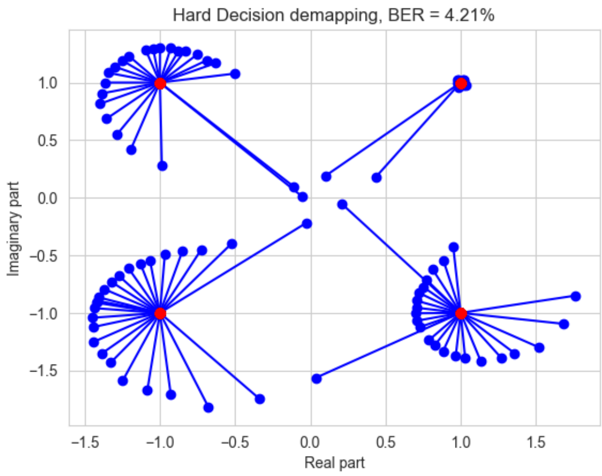


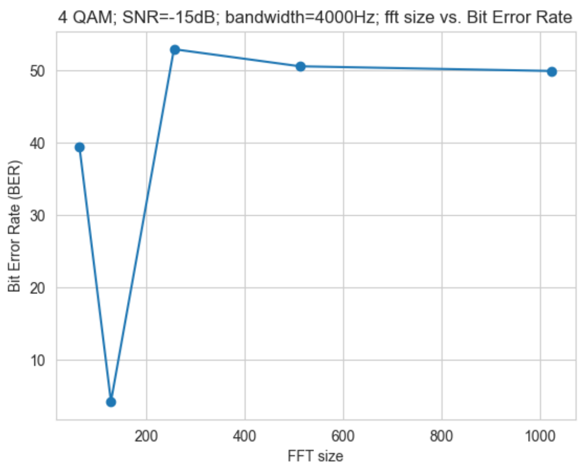


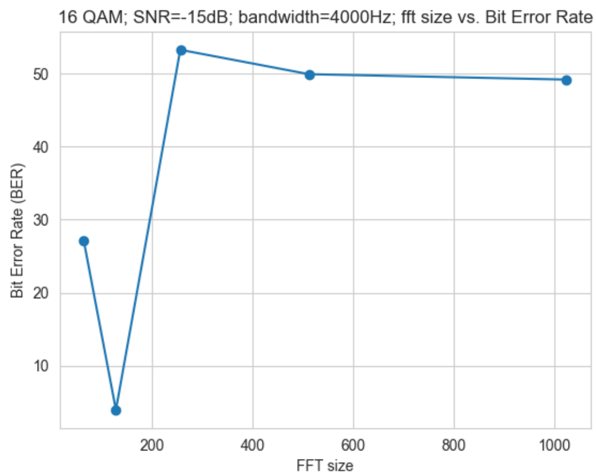
6. Rayleigh distribution noise; *OFDM Hyperparameters; Counting bits*

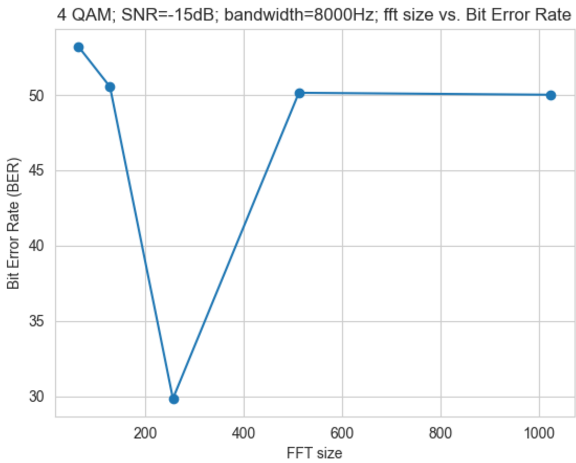
$$SNR = -15dB$$

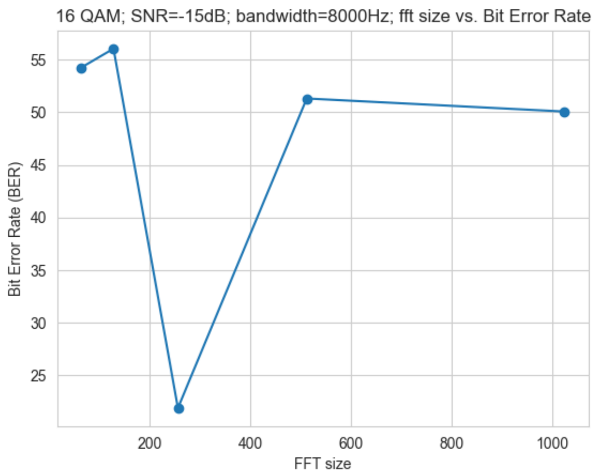


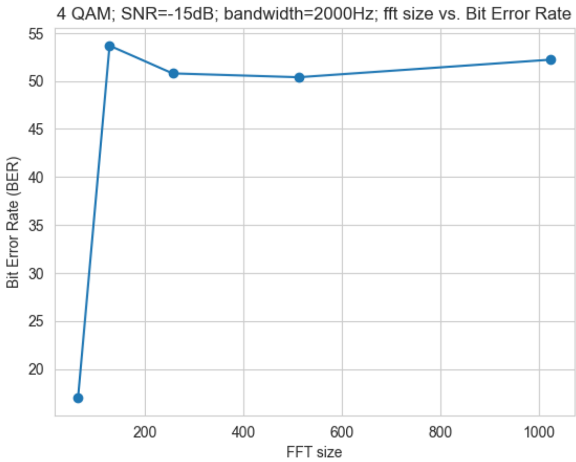


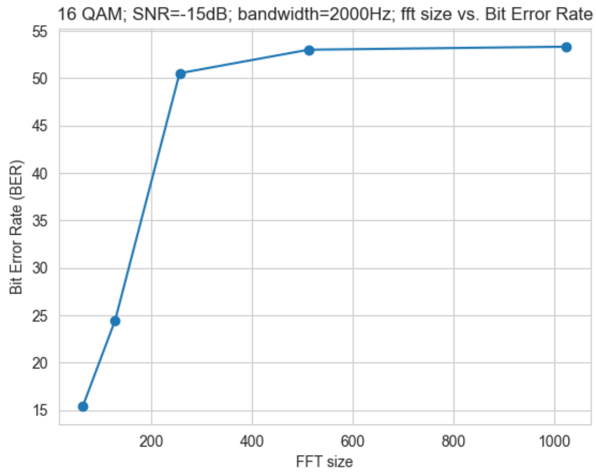


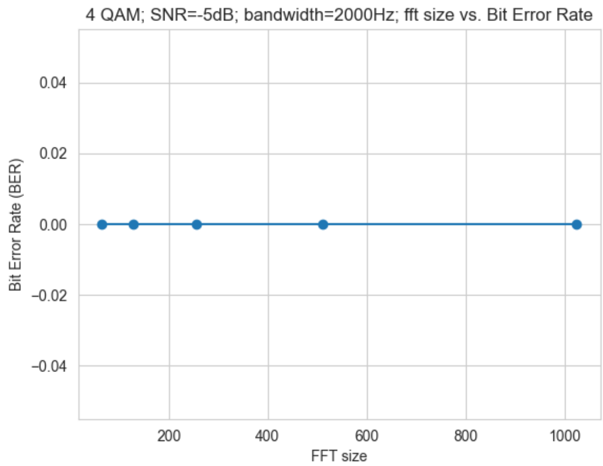


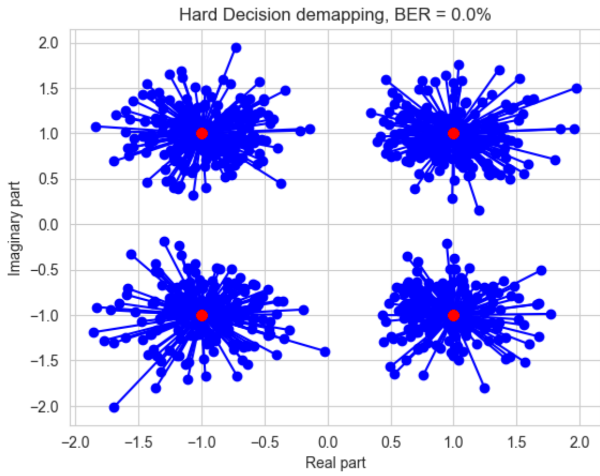












Conclusion – Our approach included incorporating a cyclic prefix, allocating 25% of subcarriers for pilot signals, and excluding permeable for synchronization. Alongside the analysis of sea recordings, we introduced Rayleigh distribution and Normal distribution noise to synthetic packets. Additionally, we conducted an analysis on the Bit Error Rate (BER) over various FFT sizes and determined that the optimal selection for our setup is $\frac{Bandwidth}{2kHz} * 64$.

Match Filter

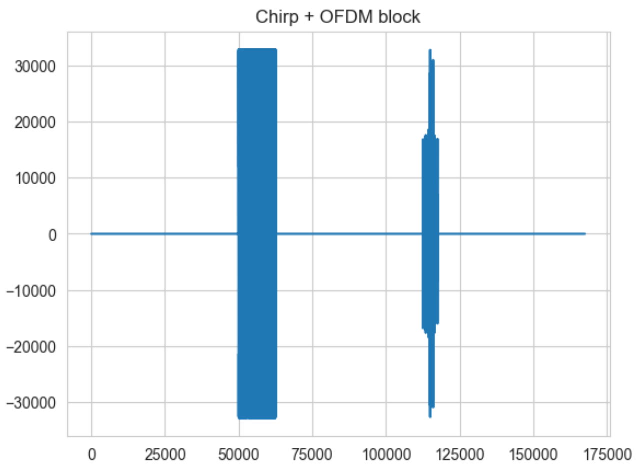
We used a cross-correlation function to implement a match filter, which involved comparing a reference signal (such as a chirp) with a received signal at different time offsets to detect the presence and time location of the reference signal :

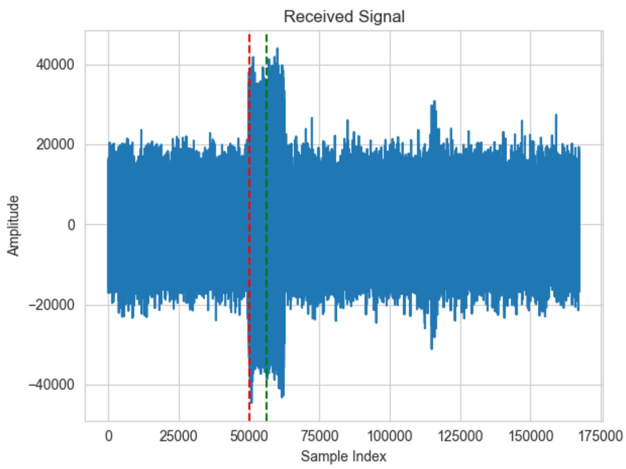
$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t) \cdot y(t\ + \tau)dt$$

Let the miss metric for chirp detection be :

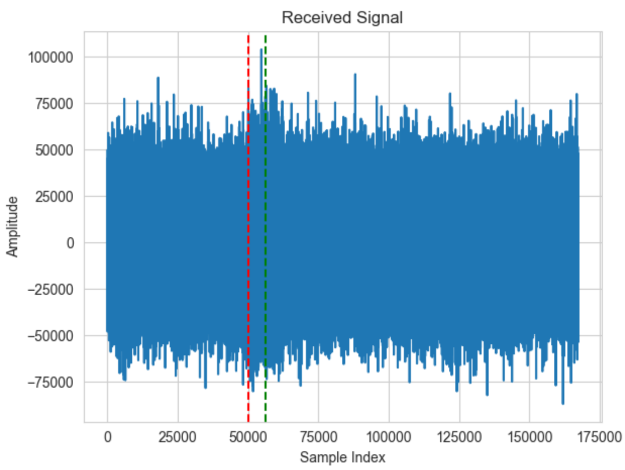
$$Miss_{chirp} = \frac{|\hat{t}_{start} - t_{start}|}{duration}$$

1. **Additive white Gaussian noise; SNR -5db**
   $Miss_{chirp} = 0ms$



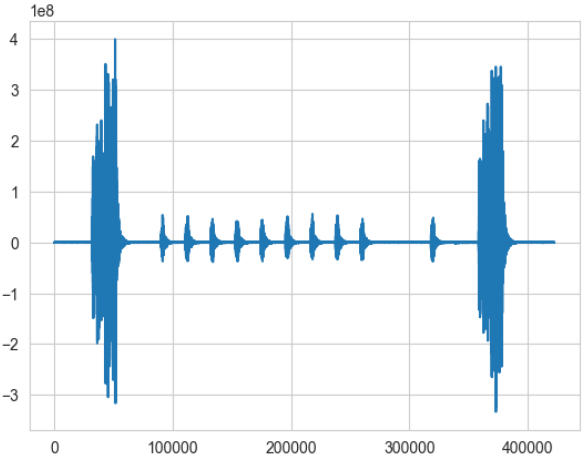


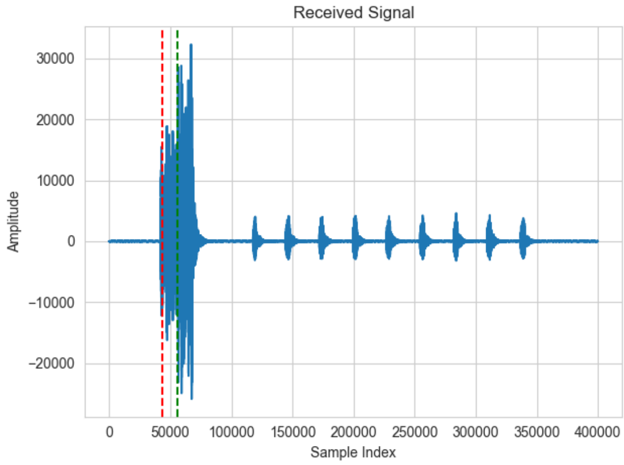
2. **Additive white Gaussian noise; SNR -15db**
   $MR_{chirp} = 0ms$

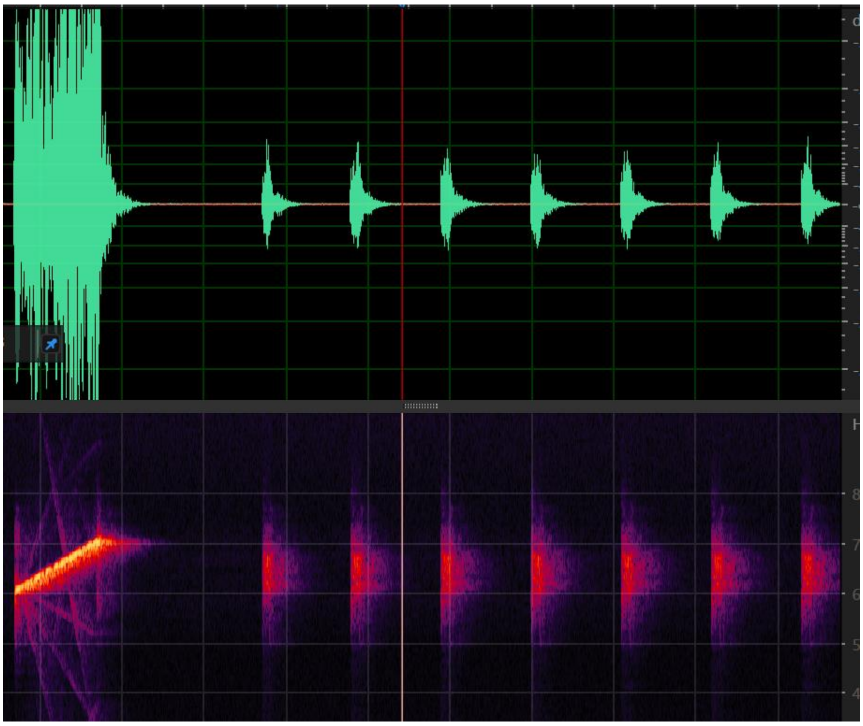


3. IOLR recording; SNR=12db

$$Miss_{chirp} = \frac{|43179 - 42100|}{fs * 0.1} = \frac{1079}{25000} = 4.31\% \; (4.3ms \; over \; 100ms)$$



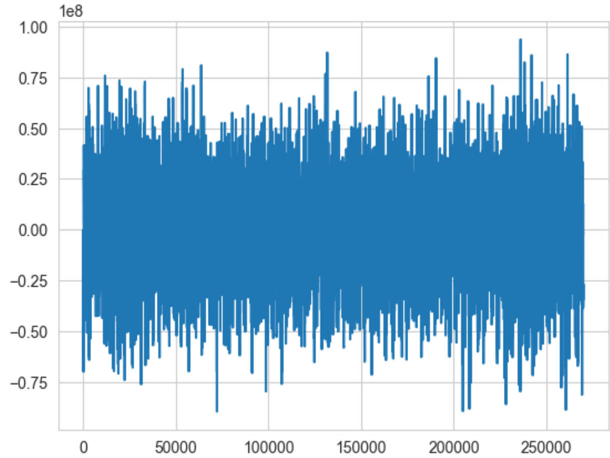


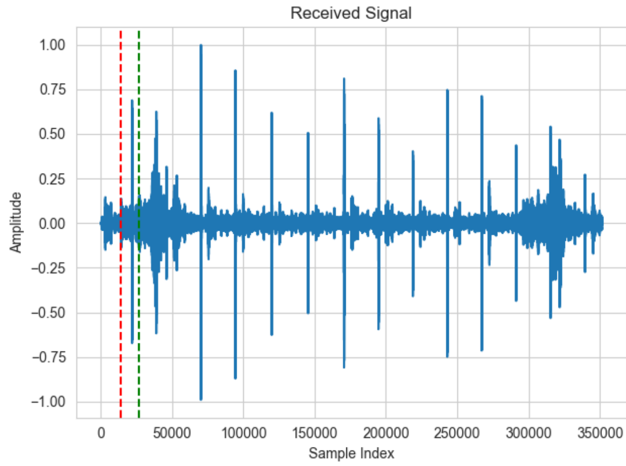


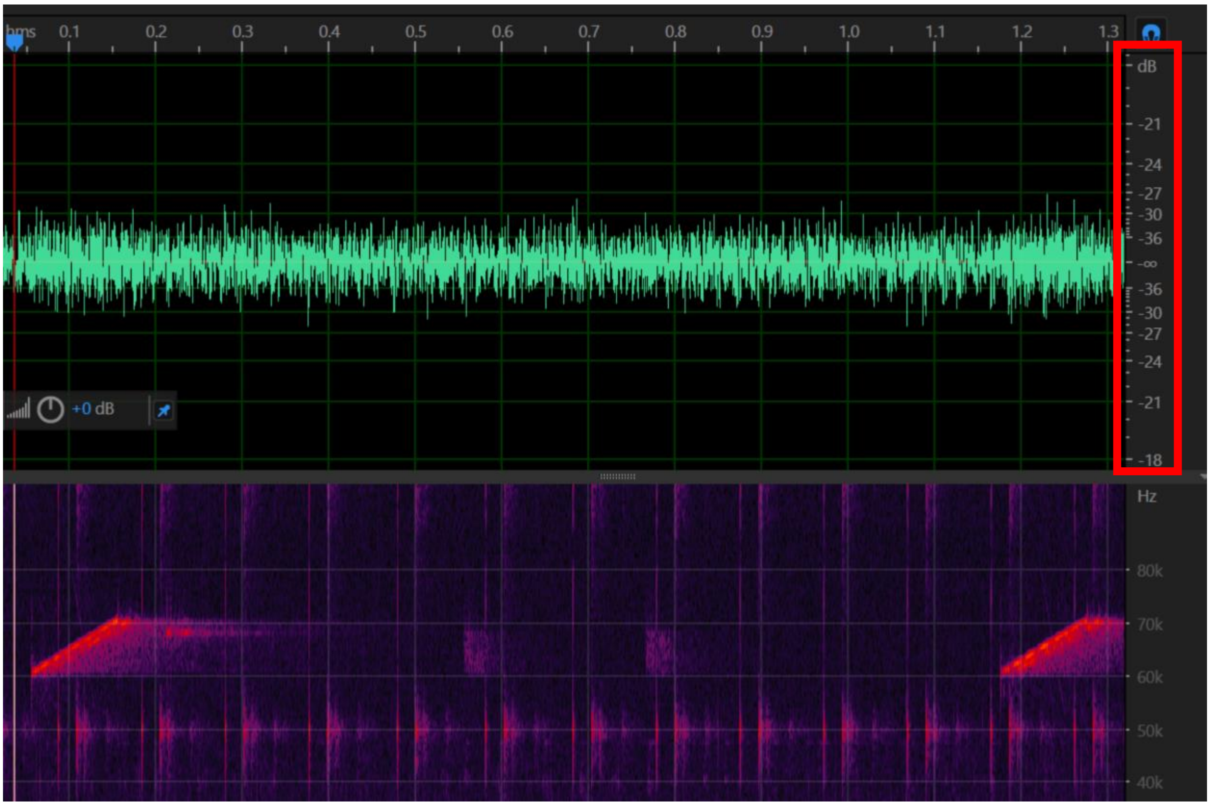
4. Sea recording 60m; SNR= -27dB

$$Miss_{chirp} = \frac{|13870 - 10368|}{fs * 0.1} = \frac{3502}{25000} = 14\% \; (14ms \; over \; 100ms)$$

BPF

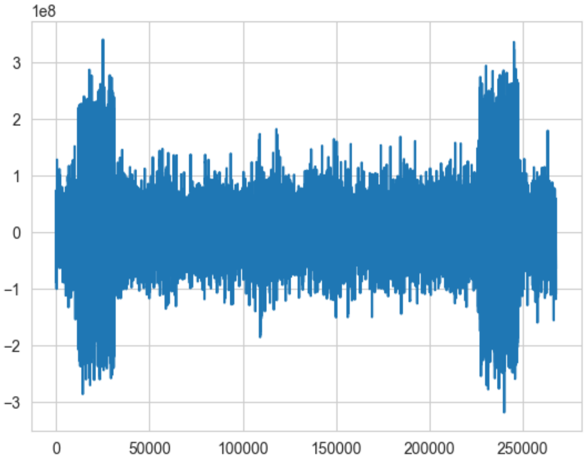


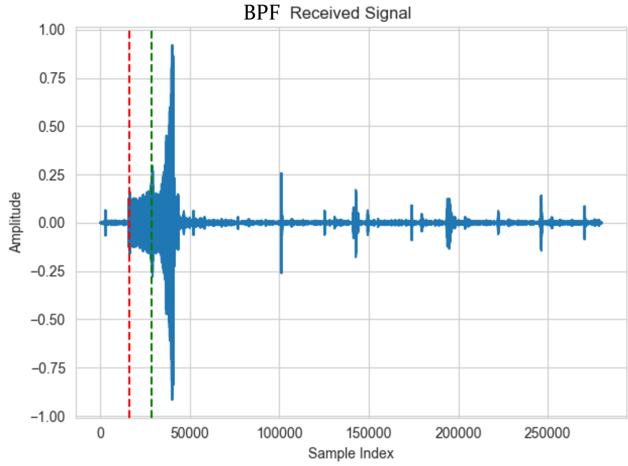


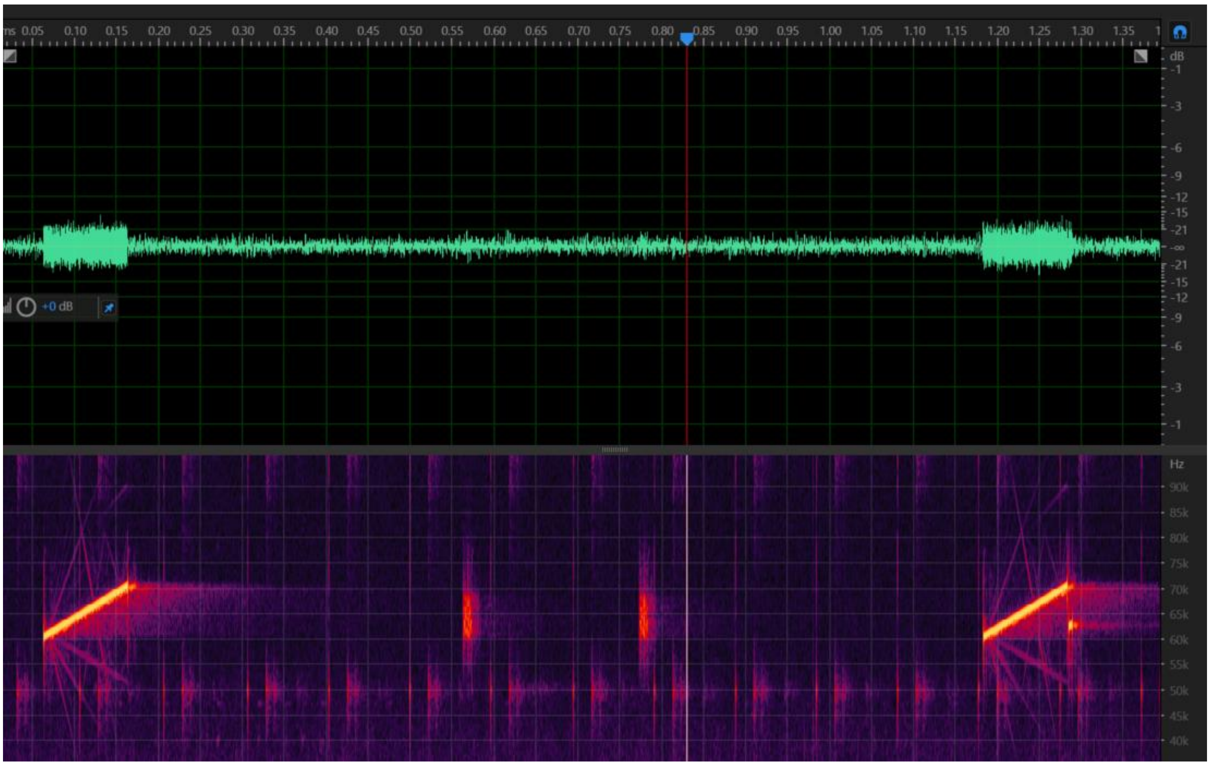


5. Sea recording 20m; SNR= -8.5dB

$$Miss_{chirp} = \frac{|15985 - 15975|}{fs * 0.1} = \frac{10}{25000} = 0.04\% \; (0.04ms \; over \; 100ms)$$







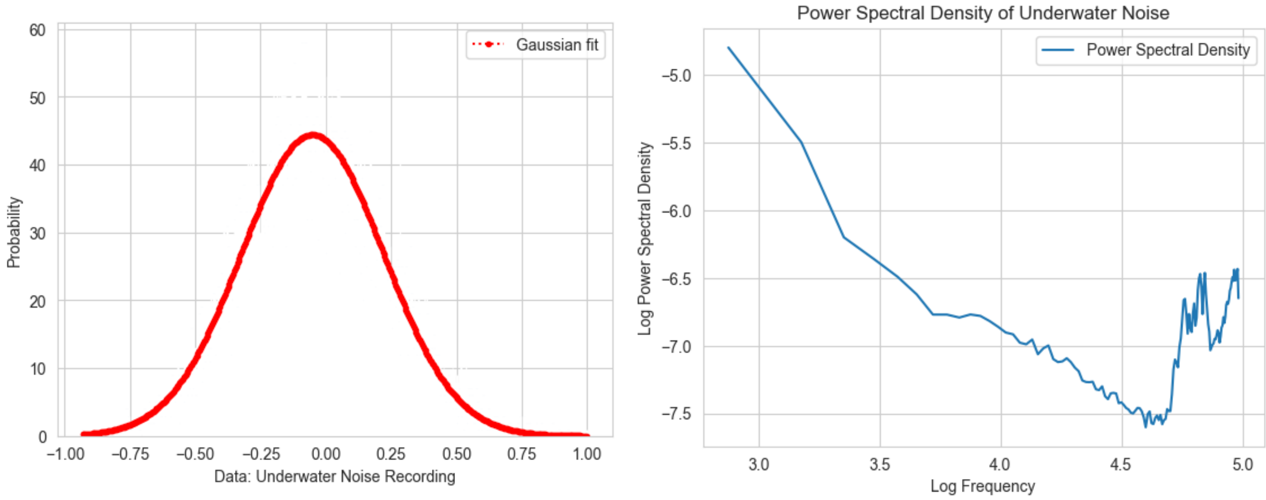
Conclusion – We employed a cross-correlation function as a match filter to compare a reference signal (e.g., chirp) with a received signal at various time offsets. This process allowed us to detect the presence and determine the time location of the reference signal. In our analysis of sea recordings, specifically at distances of 20m and 60m with signal strengths of -8.5dB and -27dB respectively, we observed that the match filter's performance was sufficient for OFDM block synchronization until a signal strength of -10dB. At this level, the miss rate for synchronization was only 0.04%.

Noise Analysis

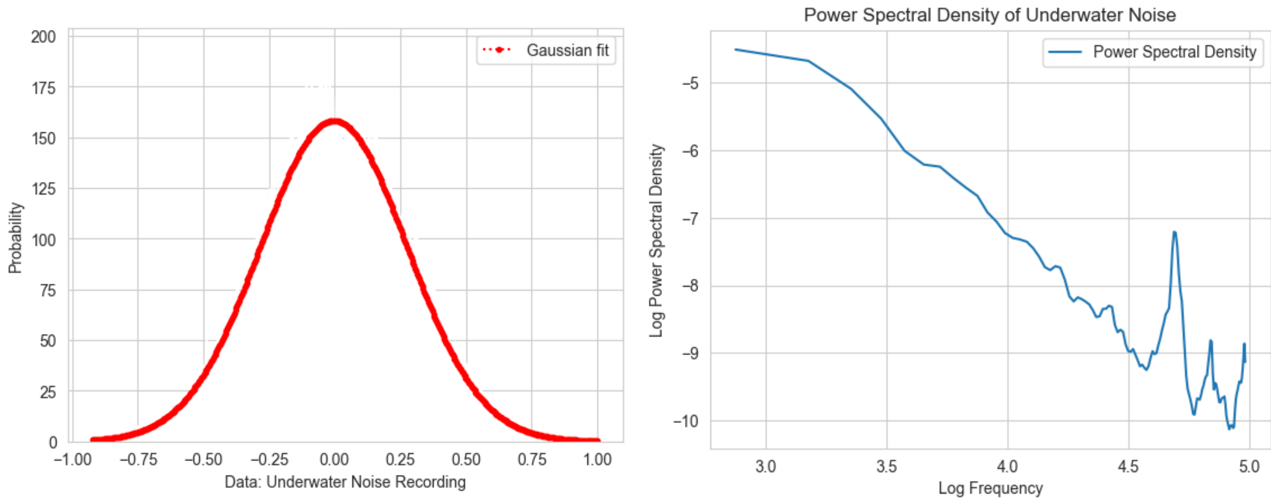
We used *nonlinear least squares* fitting to estimate the underwater noise from the recordings by fitting a Gaussian noise model to the observed data and optimizing the parameters to minimize the differences between the model predictions and the recorded values.

$$SNR = \frac{P_{signal}}{P_{noise}}$$

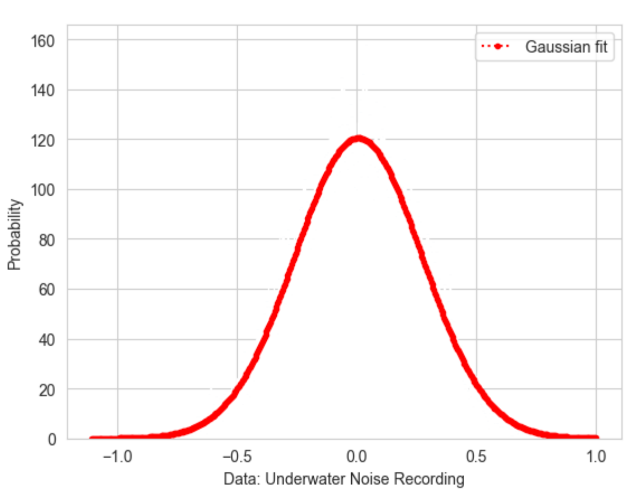
1. **IOLR recording**
   SNR of **12.7dB** was calculated utilizing chirp signal from another transmitted signal.
   Normalized noise distribution was fitted to $Nosie \sim N(-0.05, 0.27)$

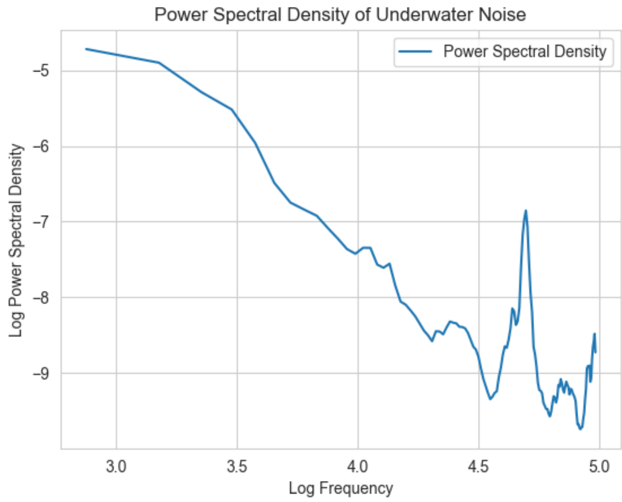


2. **Sea recording – 20m**
   SNR of **-8.5dB** was calculated utilizing chirp signal from another transmitted signal.
   Normalized noise distribution was fitted to $Nosie \sim N(-0.004, 0.27)$

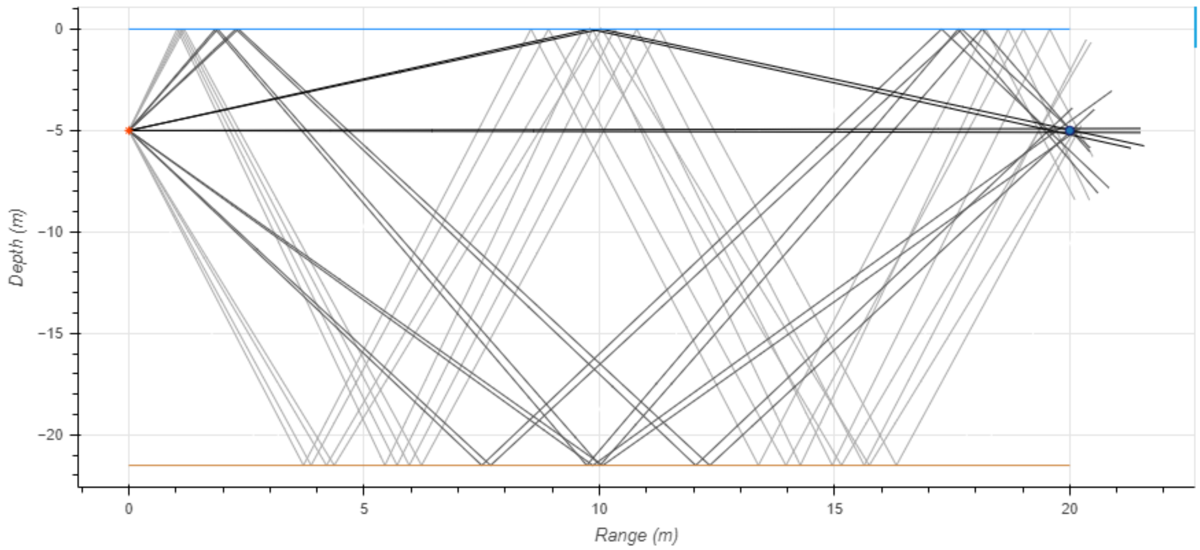


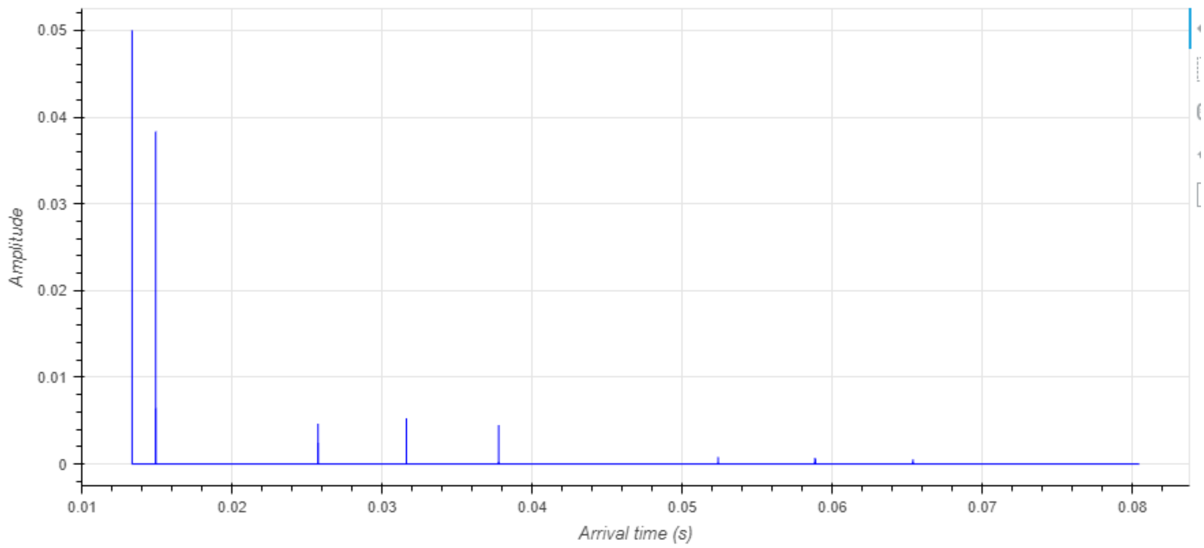
3. **Sea recording – 60m**
SNR of **-27dB** was calculated utilizing chirp signal from another transmitted signal.
Normalized noise distribution was fitted to $Nosie \sim N(0, 0.266)$



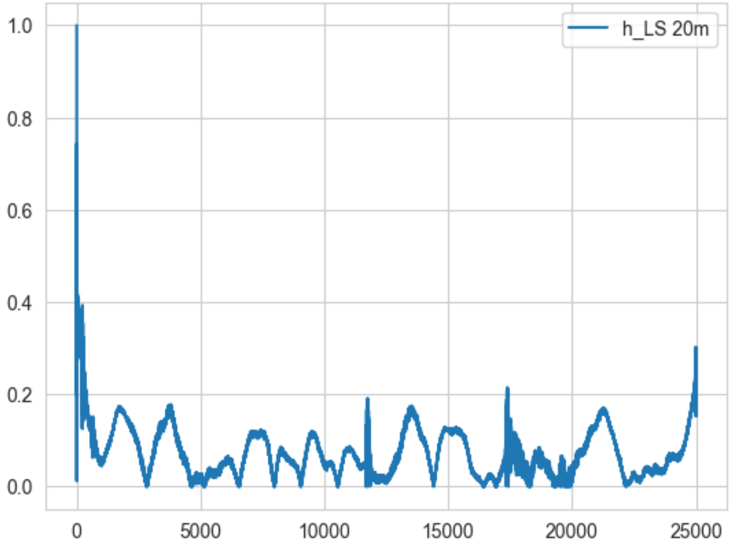


Bellhop
Assuming flat bathometry, we can simulate the channel of the 20m distance with bellhop:

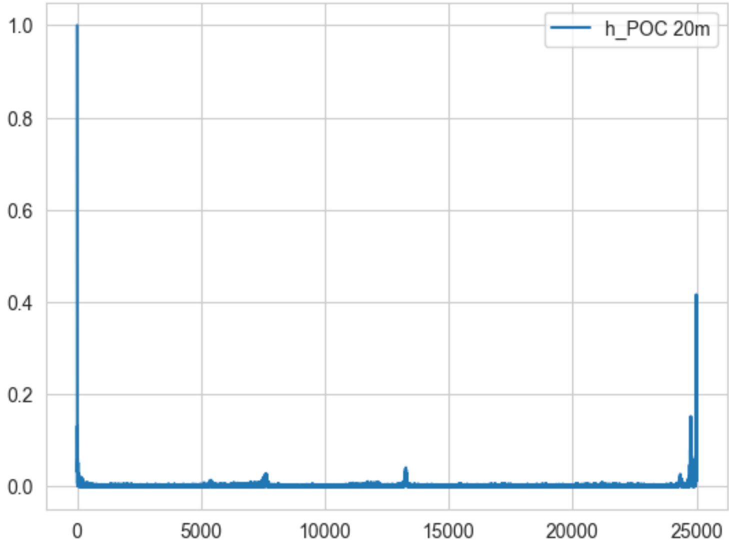




And applying LS (Lease-Square) channel estimation on the extracted chirp (using Matched Filter) results with $MSE(h, h_{LLS}) = 0.0193$:



Estimating with Phase only-correlator (POC) results $MSE(h, h_{LLS}) = 0.000263$:



Conclusion - we successfully extracted a chirp signal from an OFDM symbol transmitted at a 20m distance. We simulated the channel in a sea environment using the *bellhop* simulator. When comparing the performance of the *LLS* (Least Squares) estimator with the *POC* (Phase-Only Correlation) estimator, we observed that the POC estimator exhibited a significant improvement in accuracy, achieving a **100** times lower Mean Squared Error (*MSE*) compared to the LLS estimator.

Advanced Analysis

Viterbi

The Viterbi algorithm is a dynamic programming algorithm for obtaining the maximum a posteriori probability estimates of the most likely sequence of hidden states—called the Viterbi path—those results in a sequence of observed events, especially in the context of Markov information sources and hidden Markov models (HMM).

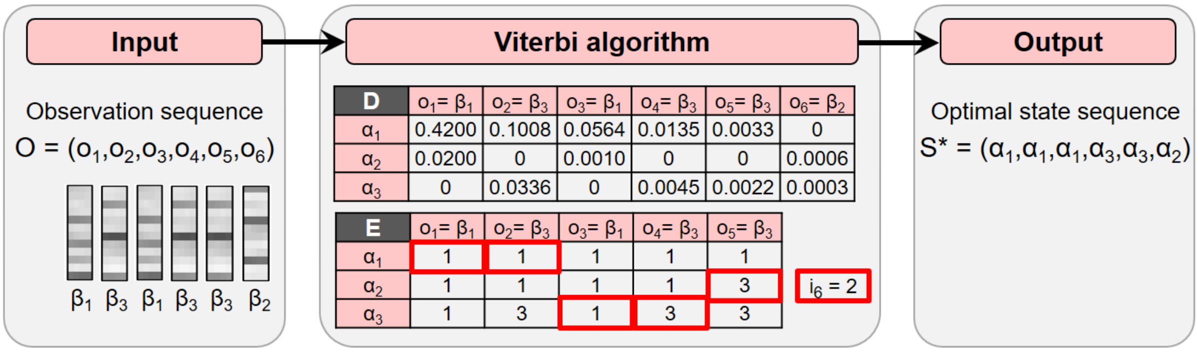


Table 5.2 from [Müller, FMP, Springer 2015]

Algorithm: VITERBI

Input: HMM specified by $\Theta = (\mathcal{A}, A, C, \mathcal{B}, B)$
Observation sequence $O = (o_1 = \beta_{k_1}, o_2 = \beta_{k_2}, \ldots, o_N = \beta_{k_N})$

Output: Optimal state sequence $S^* = (s_1^*, s_2^*, \ldots, s_N^*)$

Procedure: Initialize the $(I \times N)$ matrix $\mathbf{D}$ by $\mathbf{D}(i,1) = c_i b_{ik_1}$ for $i \in [1:I]$. Then compute in a nested loop for $n = 2, \ldots, N$ and $i = 1, \ldots, I$:

$$\begin{aligned} \mathbf{D}(i,n) &= \max_{j\in[1:I]} \left(a_{ji} \cdot \mathbf{D}(j,n-1)\right) \cdot b_{ik_n} \\ \mathbf{E}(i,n-1) &= \operatorname{argmax}_{j\in[1:I]} \left(a_{ji} \cdot \mathbf{D}(j,n-1)\right) \end{aligned}$$

Set $i_N = \operatorname{argmax}_{j\in[1:I]} \mathbf{D}(j,N)$ and compute for decreasing $n = N-1, \ldots, 1$ the maximizing indices

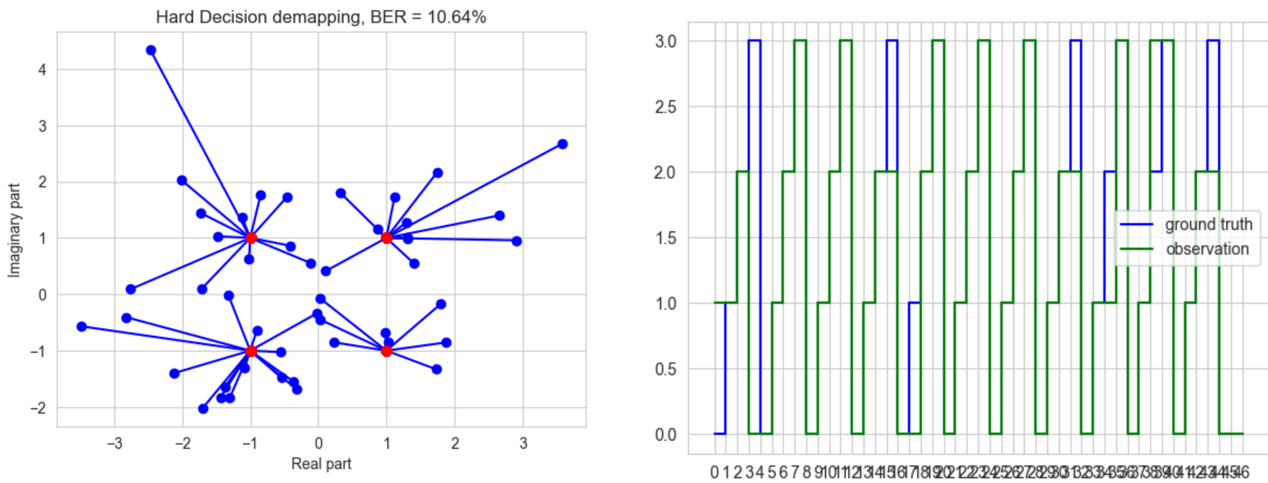
$$i_n = \operatorname{argmax}_{j\in[1:I]} \left(a_{j i_{n+1}} \cdot \mathbf{D}(j,n)\right) = \mathbf{E}(i_{n+1}, n).$$

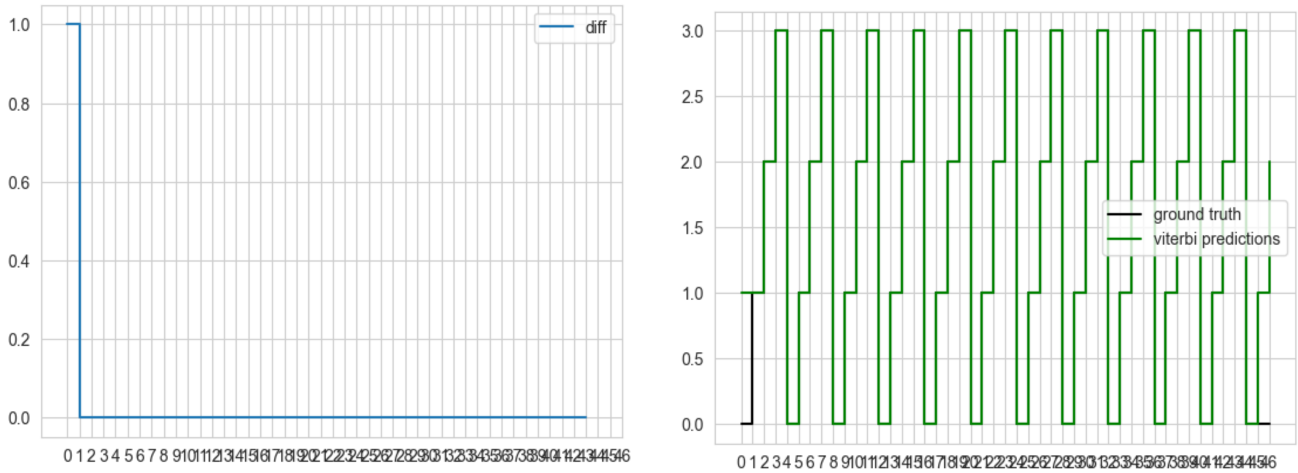
The optimal state sequence $S^* = (s_1^*, \ldots, s_N^*)$ is defined by $s_n^* = \alpha_{i_n}$ for $n \in [1:N]$.

Transition and emission matrix values has been decided to support the demonstration. The approach for error correction (after hard decision) is based on [2].

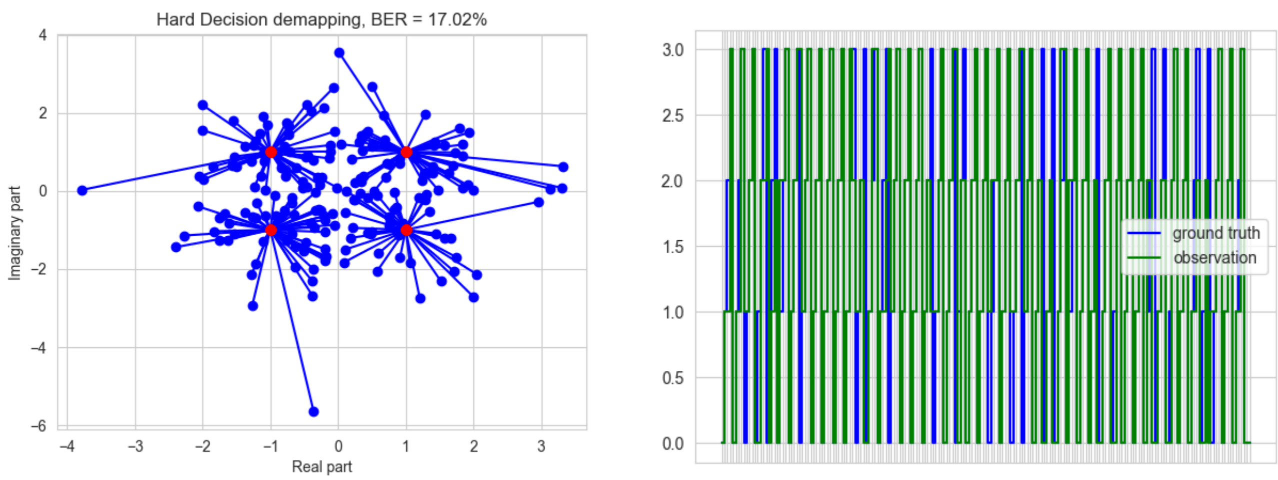
```
states = ('00', '01', '10', '11')
observations = observed
start_probability = {'00': 0.2, '01': 0.4, '10': 0.2, '11': 0.2}
transition_probability = {
    '00': {'00': 0.1, '01': 0.8, '10': 0.1, '11': 0.1},
    '01': {'00': 0.1, '01': 0.1, '10': 0.8, '11': 0.1},
    '10': {'00': 0.1, '01': 0.1, '10': 0.1, '11': 0.8},
    '11': {'00': 0.8, '01': 0.1, '10': 0.1, '11': 0.1}
}
emission_probability = {
    '00': {'00': 0.8, '01': 0.1, '10': 0.1, '11': 0.1},
    '01': {'00': 0.1, '01': 0.8, '10': 0.1, '11': 0.1},
    '10': {'00': 0.1, '01': 0.1, '10': 0.8, '11': 0.1},
    '11': {'00': 0.1, '01': 0.1, '10': 0.1, '11': 0.8}
}
```

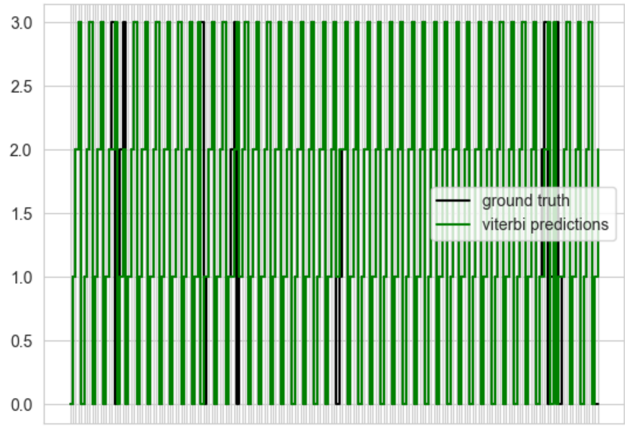
Running on samples with $fft = 64; bandwidth = 2kHz; 4\ QAM; SNR = -15dB$ resolved with an 33% error correction, improving BER from 10.64% to 1.1%.

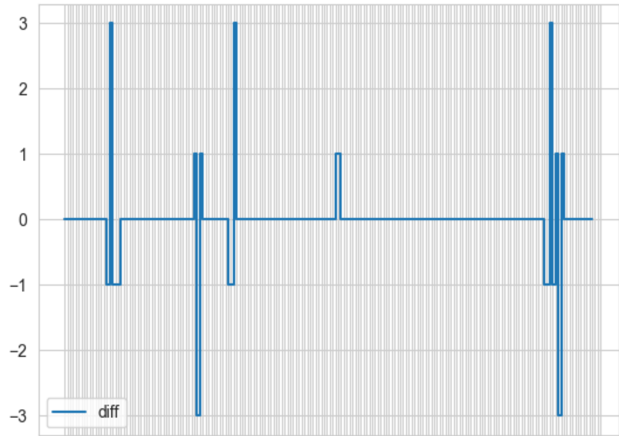




Running on samples with $fft = 256; bandwidth = 2kHz; 4\ QAM; SNR = -15dB$ resolved with an 33% error correction, improving BER from 17.78% to 7.6%.



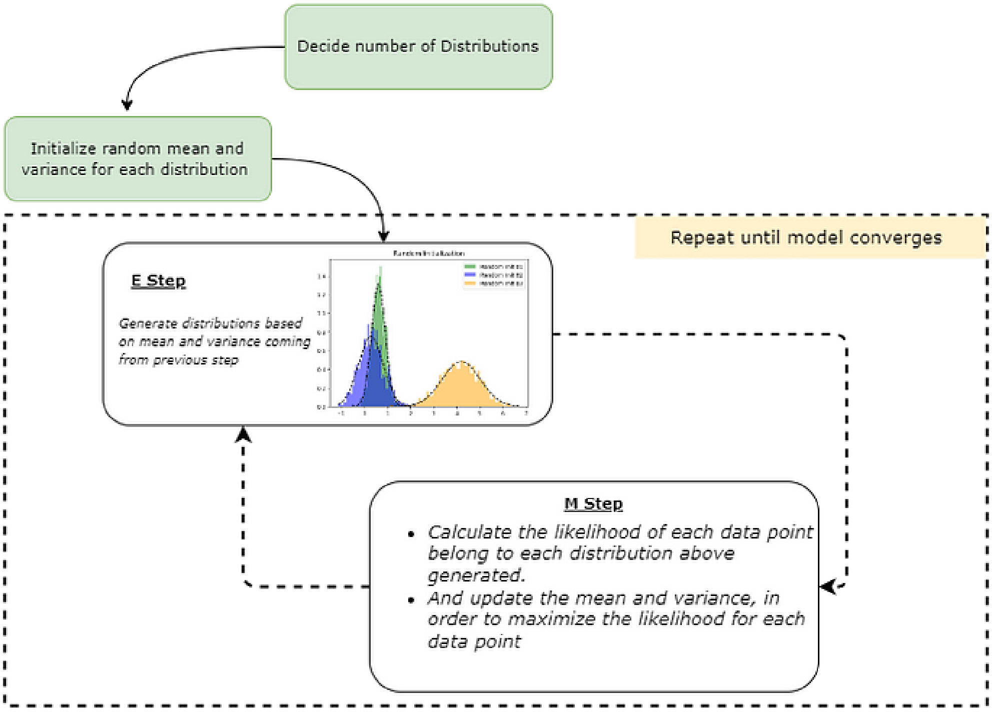




Conclusion - In the given mission to correct the code after hard decisions were made with OFDM modulation, the implementation of the Viterbi algorithm using hidden Markov models has shown significant improvement in the bit error rate (BER). The Viterbi algorithm operates by efficiently decoding the received sequence based on the probabilities of different code sequences. By considering the inherent dependencies and correlations within the transmitted data, the Viterbi algorithm can accurately correct errors introduced during the hard decision process.

Gaussian Mixture Models (GMM) and Expectation Maximization (EM)

It's been showed in [3] that the expectation maximization algorithm is a powerful tool for combating and modeling system impairments, (nonlinearities, I/Q imperfections, and laser linewidth), which significantly distort the signal constellation .Joint carrier frequency and phase, means and noise variance estimation is demonstrated for coherent communication system employing QAM.



We implemented the close solution for 2-GMM and used *sklearn* [1]library for the 4-GMM closed solution.

$$\hat{\omega}_m = \frac{\sum_{t=1}^{T} p\left(m|\vec{x}_t;\theta\right)}{T}$$

$$\hat{\vec{\mu}}_m = \frac{\sum_{t=1}^{T} p\left(m|\vec{x}_t;\theta\right)\vec{x}_t}{\sum_{t=1}^{T} p\left(m|\vec{x}_t;\theta\right)}$$

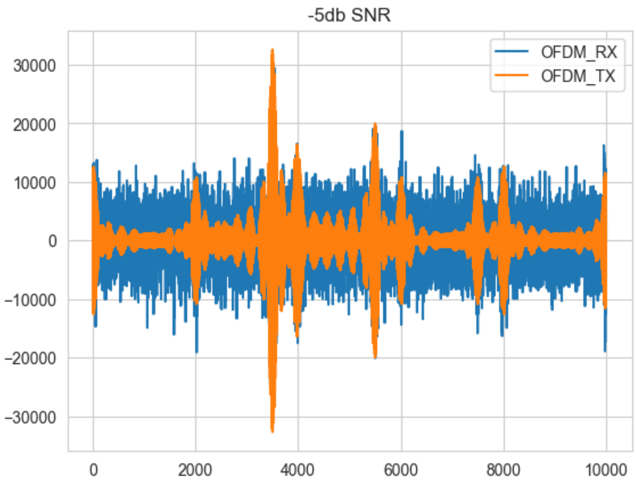
$$\hat{\Sigma}_m = \frac{\sum_{t=1}^{T} p\left(m|\vec{x}_t;\theta\right)\vec{x}_t^2}{\sum_{t=1}^{T} p\left(m|\vec{x}_t;\theta\right)} - \hat{\vec{\mu}}_m^2$$

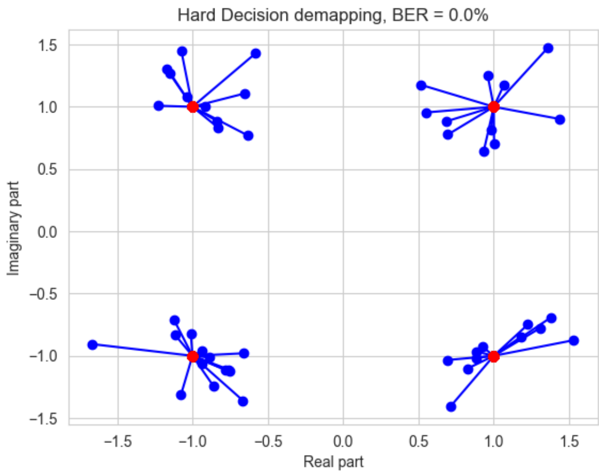
The GMM model has been initiated with 4 mapping points as means for 4 components, each component has its own general covariance matrix. Then, we apply fitting on the data & estimating on the equalize output in the OFDM RX flow.

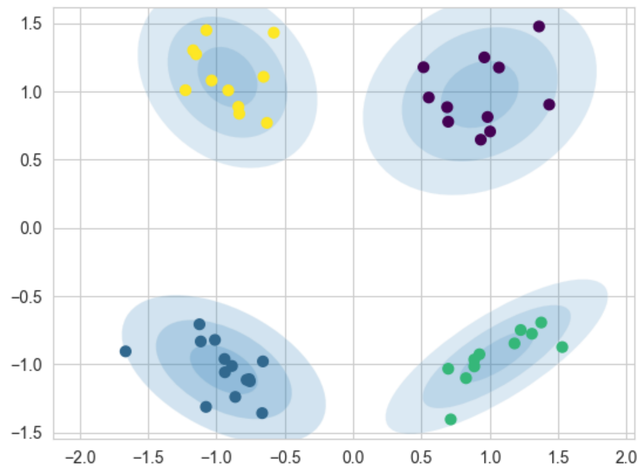
1 Sklearn.mixture.gaussianmixture scikit. Available at: https://scikit-learn.org/stable/modules/generated/sklearn.mixture.GaussianMixture.html#sklearn.mixture.GaussianMixture (Accessed: 05 July 2023).

Estimating with $SNR = -5dB; fft = 512; bandwidth = 8kHz; 4QAM$:

$$GMM\ EM\ BER = 0\%$$

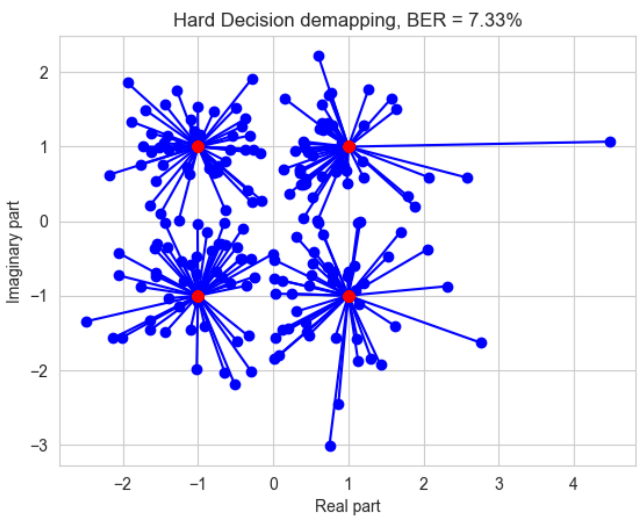


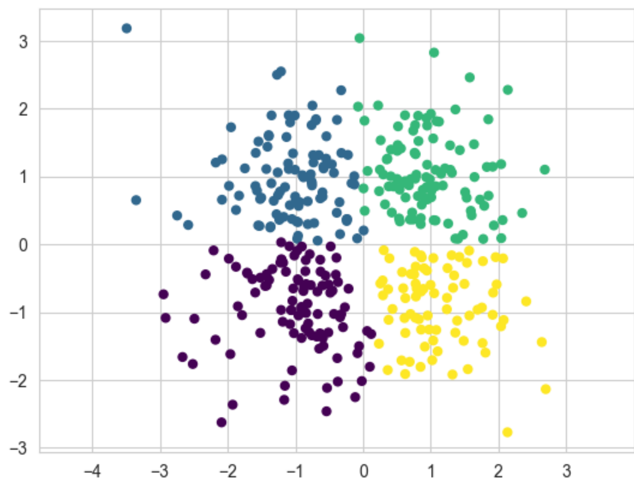




Estimating with $SNR = -15dB; fft = 512; bandwidth = 8kHz; 4QAM$:

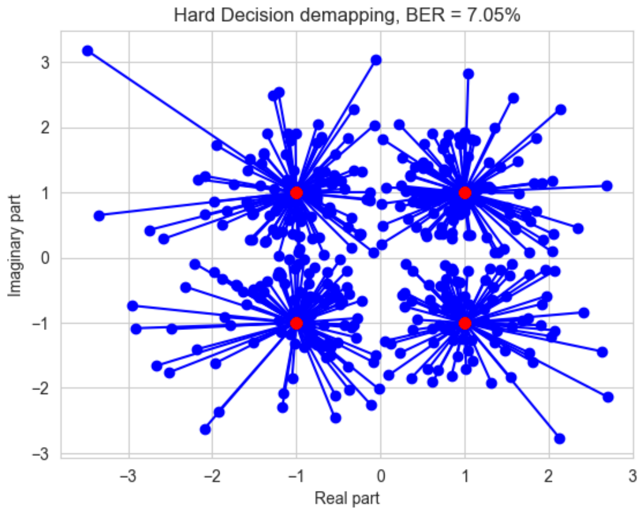
$$GMM\ EM\ BER = 6.4\%$$

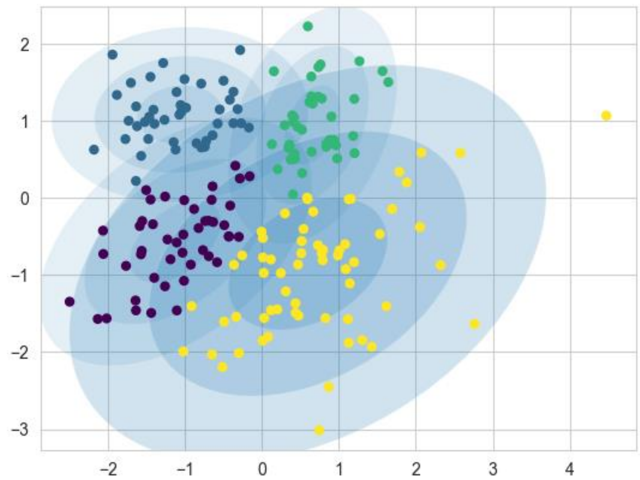




Estimating with $SNR = -15dB; fft = 256; bandwidth = 8kHz; 4QAM$:

$$GMM\ EM\ BER = 13.09\%$$





Conclusion – the process described above involves iteratively estimating the 4 center points of the QAM, even when channel effect and noise are present - using expectation maximization. However, when clutter data is present in the center with sparsity around it, the performance of this method deteriorates. In such cases, the approach resolves into three Gaussian models for the center points and one for the surrounding points. Nevertheless, this approach is intriguing and can prove valuable with modifications and optimization. I have achieved significant success by employing it in another project to cluster noise and data based on the histogram of the power density in the main frequency spectrum.

Exact Method – Subcarrier hyperparameter selection

Exact methods for optimization refer to mathematical algorithms that guarantee finding the optimal solution to a problem. They are typically used for small or medium-sized problems with a well-defined mathematical formulation. However, for large-scale problems, they may be computationally intensive and require a lot of computational resources.

In an OFDM system, there are various parameters that can be optimized, such as the number of subcarriers, the length of the cyclic prefix, the modulation scheme, and the channel coding rate. For example, we could use an exact method to optimize the allocation of subcarriers based on the channel characteristics to maximize the overall data rate, minimize the bit error rate (BER), or joint subcarrier, bit and power allocation problem as shown in [4] and [5].

This problem can be formulated as a combinatorial optimization problem, and it can be solved using methods such as *branch-and-bound* or genetic algorithms.

Problem formalization :

OFDM Underwater Communication System

- OFDM using QAM modulation
- N subcarriers
- Bandwidth = $[B]\ Hz$
- SNR_i : SNR for subcarrier i
- BPS_i : Data rate for subcarrier i

Subcarrier selection optimization:

Select a subset of subcarriers from the available N subcarriers such that the overall data rate is maximized while satisfying the bandwidth and SNR constraints

Maximize:

$$\sum_{i=1}^{N} BPS_i * I_i; \quad I_i = 1\ if\ subacrrier\ i\ is\ selected$$

Subject to:

$$\sum_{i=1}^{N} \frac{B}{N} * I_i; \ \frac{B}{N}\ represents\ the\ bandwidth\ of\ each\ subcarrier$$

$$SNR_i * I_i \leq C;$$
$$C\ is\ a\ constant\ determined\ by\ the\ noise\ and\ interference\ characteristics\ of\ the\ channel$$

Regrettably, I did not reach convergence with the current problem definition, indicating that additional work is required to formalize the correct objectives. It may also be necessary to reconsider and relax certain constraints to make progress.

Energy Detector

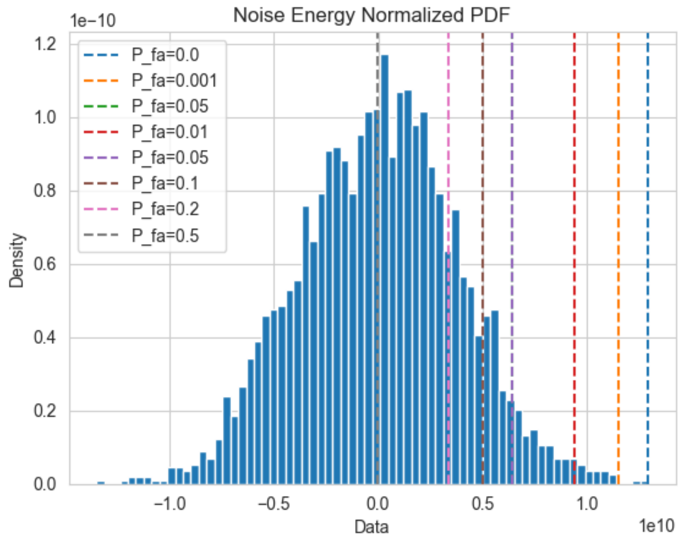
We did the energy detector analysis on synthetic data with gaussian noise. First, we analyze the noise energy and PDF. Those characteristics will help us to build the CFAR energy detector. We chose frame length as follow:

$$T \triangleq frame\ length = \frac{noise\ signal}{\frac{chirp\ length}{20} * fs}$$

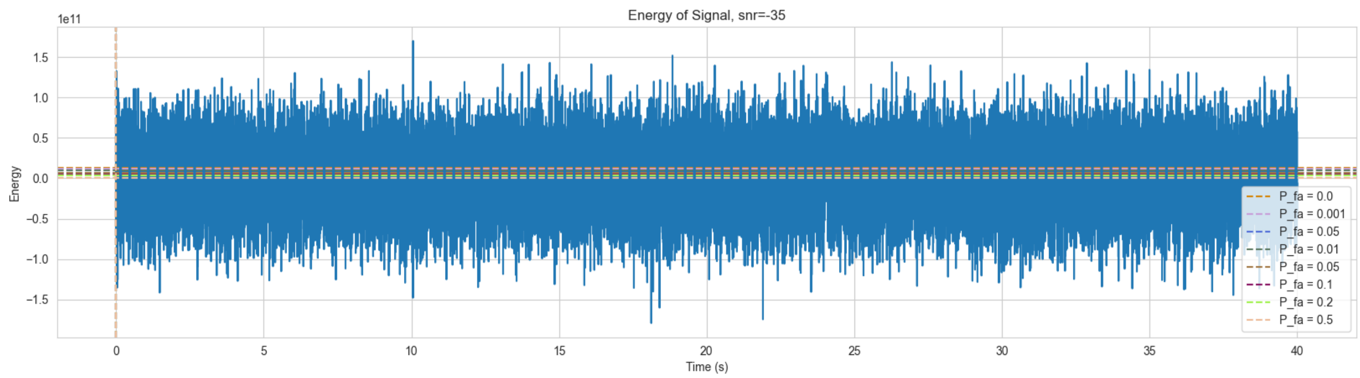
$$noise_j = frame\ j\ of\ the\ noise\ signal$$

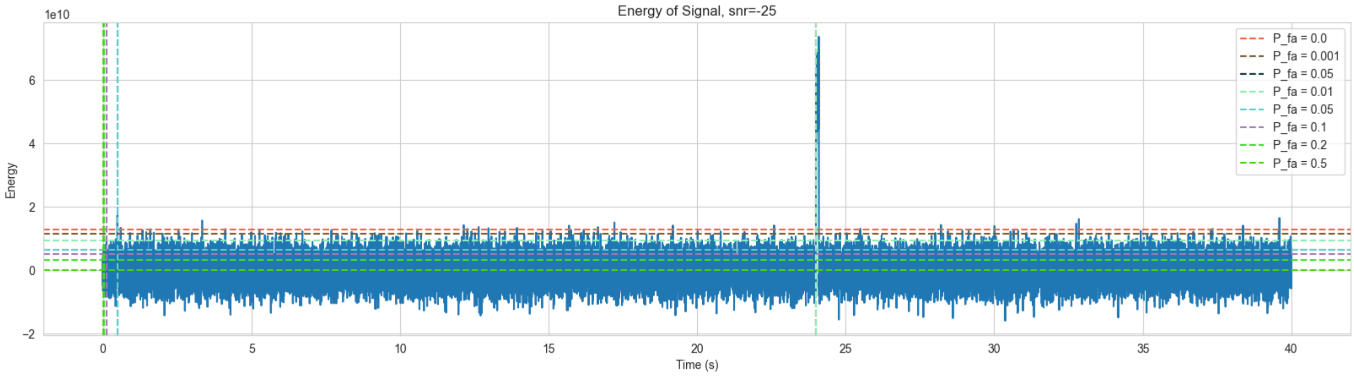
$$E_j = \frac{1}{T}\sum_{i=1}^{n} |noise_j(i)^2|$$

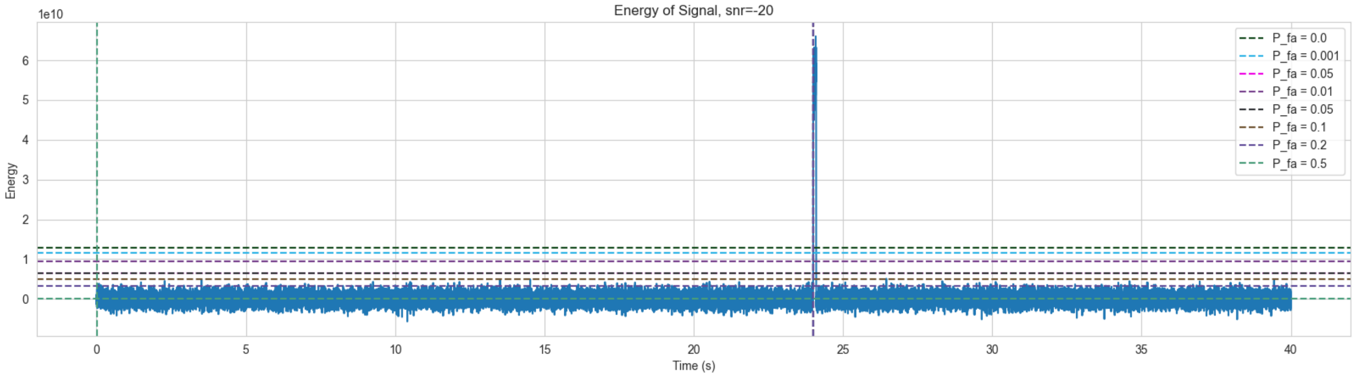
Binned Noise Energy Histogram (PDF):

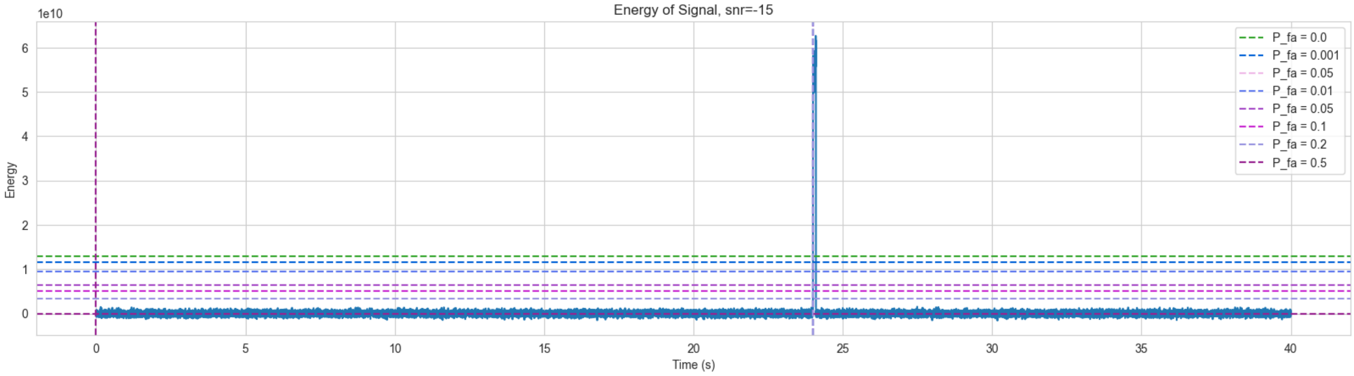


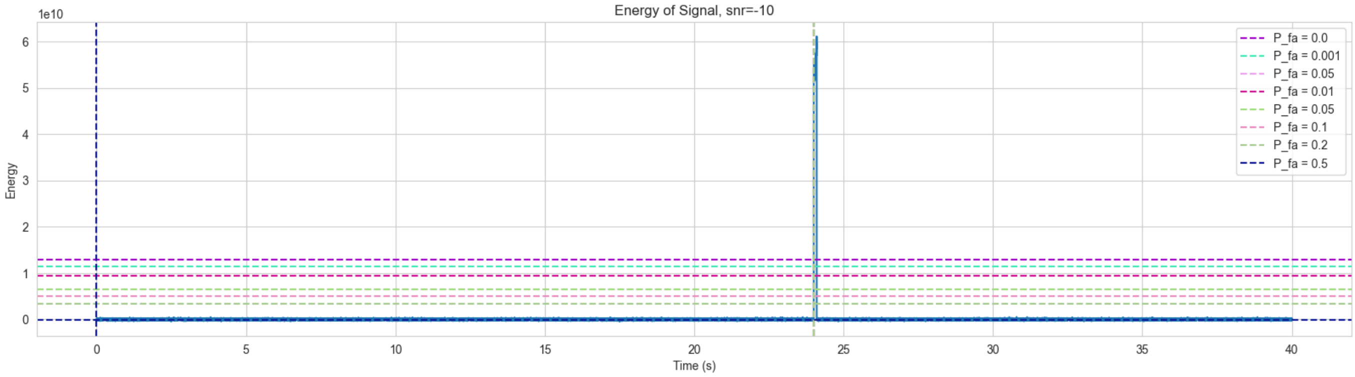
$$CFAR\ Threshold: \quad 1 - P_{fa} = \sum_{i=1}^{m} PDF_{noise}\ , m\ is\ determine\ when\ selecing\ P_{fa}$$

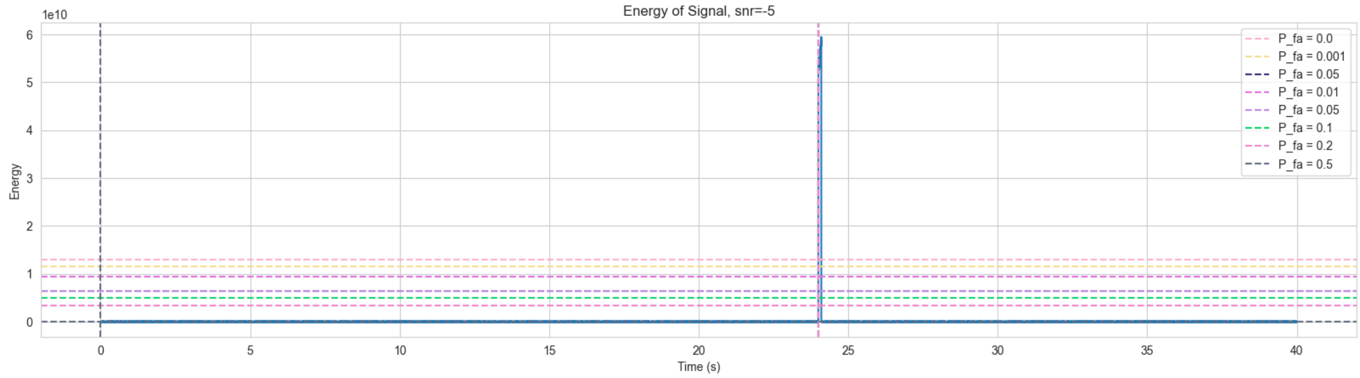


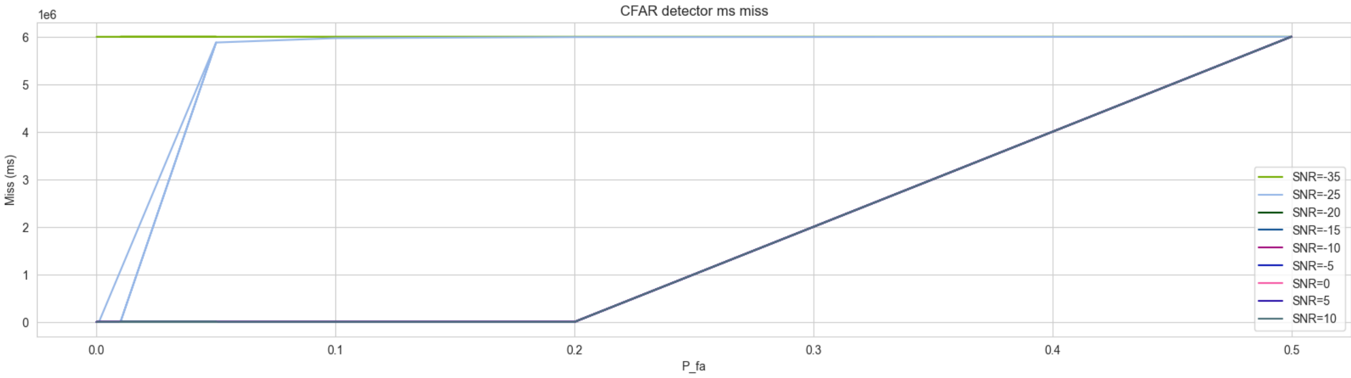

Energy of Signal, snr=-25
1e10
Energy
Time (s)
P_fa = 0.0
P_fa = 0.001
P_fa = 0.05
P_fa = 0.01
P_fa = 0.05
P_fa = 0.1
P_fa = 0.2
P_fa = 0.5


Energy of Signal, snr=-20
1e10
Energy
Time (s)
P_fa = 0.0
P_fa = 0.001
P_fa = 0.05
P_fa = 0.01
P_fa = 0.05
P_fa = 0.1
P_fa = 0.2
P_fa = 0.5


Energy of Signal, snr=-15
1e10
Energy
Time (s)
P_fa = 0.0
P_fa = 0.001
P_fa = 0.05
P_fa = 0.01
P_fa = 0.05
P_fa = 0.1
P_fa = 0.2
P_fa = 0.5


Energy of Signal, snr=-10
1e10
Energy
Time (s)
P_fa = 0.0
P_fa = 0.001
P_fa = 0.05
P_fa = 0.01
P_fa = 0.05
P_fa = 0.1
P_fa = 0.2
P_fa = 0.5


Energy of Signal, snr=-5
1e10
Energy
Time (s)
P_fa = 0.0
P_fa = 0.001
P_fa = 0.05
P_fa = 0.01
P_fa = 0.05
P_fa = 0.1
P_fa = 0.2
P_fa = 0.5



Conclusion –The energy detector performed well in matching the chirp signal. It was tested with SNR values ranging from *-35dB to 10dB*, and the *Probability of False Alarm* was varied from 0.001 to 0.5. In addition to this approach, we can apply a spectrum detector (spectrogram, statistic analysis) when the transmitted signal bandwidth is known and general packet structure. Applying *GMM* on the *spectrum frequency power density histogram* can give us an idea if the current frame is noise or data.

Minimum Likelihood Estimation – Binomial data distribution

To estimate the parameter 'p' of the random data bits transmitted in the OFDM block, we utilize *Minimum Likelihood Estimation* with a Binomial data distribution, when $n = 1$:

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$L(p) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} \left(\frac{n!}{x_i!(n-x_i)!} p^{x_i} (1-p)^{n-x_i} \right) = \left(\prod_{i=1}^{n} \frac{n!}{x_i!(n-x_i)!} \right) p^{x_i} (1-p)^{n-\sum_{i=1}^{n} x_i}$$

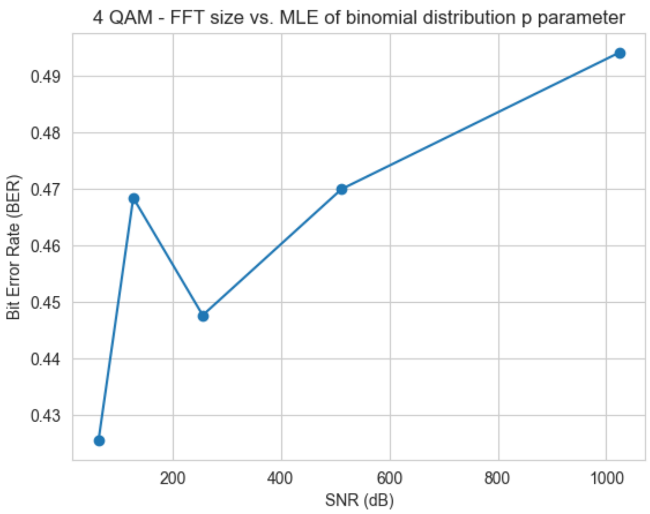
$$\ln L(p) = \sum_{i=1}^{n} x_i \ln p + \left(n - \sum_{i=1}^{n} x_i \right) \ln(1-p)$$

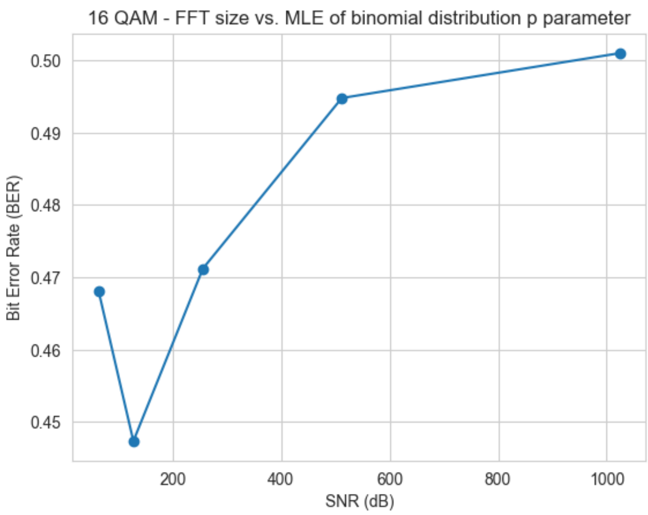
$$\frac{d \ln L(p)}{dp} = \frac{1}{p} \sum_{i=1}^{n} x_i - \left(n - \sum_{i=1}^{n} x_i \right) \frac{1}{1-p} = 0$$

$$\frac{(1-\dot{p}) \sum_{i=1}^{n} x_i - \left(n - \sum_{i=1}^{n} x_i \right) \dot{p}}{\dot{p}(1-\dot{p})} = 0$$

$$\sum_{i=1}^{n} x_i - \dot{p} \sum_{i=1}^{n} x_i - n\dot{p} + \sum_{i=1}^{n} x_i \dot{p} = 0$$

$$\dot{p} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{k}{n}$$





Conclusion – It is expected that the accuracy of the estimate improves as the number of data points increases, which is evident in the graph depicting the 4/16 QAM modulation scheme. This observation aligns with the concept of the closed solution for the Minimum Likelihood Estimation using a Binomial data distribution.

Conclusion

In this report, we presented a demonstration of an underwater acoustic system utilizing *Orthogonal Frequency Division Multiplexing* (OFDM). The system underwent testing in various environments, including a controlled pool and a calm sea, while considering different system hyperparameters. Our focus was on advanced digital signal processing techniques and algorithms to improve the bit error rate (BER) performance.

During our experimentation, we tested 4-QAM and 16-QAM modulations on the underwater acoustic (UWA) channel. We conducted BER analyses on sea recordings and synthetic noise, using both *Rayleigh* and *Normal* distributions. Additionally, we analyzed the optimal FFT size for different bandwidth options.

Our approach incorporated a cyclic prefix, allocated 25% of subcarriers for pilot signals, and excluded permeable for synchronization. We also employed a cross-correlation function as a match filter to detect and locate reference signals. Furthermore, we utilized nonlinear least squares fitting to estimate underwater noise parameters and employed the *Viterbi* algorithm for efficient error correction.

In conclusion, we successfully extracted a chirp signal from an OFDM symbol transmitted at a 20m distance. The performance comparison between the *LLS* (Least Squares) estimator and the *POC* (Phase-Only Correlation) estimator revealed that the POC estimator exhibited significantly improved accuracy. We also observed substantial BER improvement through the implementation of the Viterbi algorithm. Moreover, our iterative estimation process, involving expectation maximization, showed promising results for parameter estimation and clustering of data and noise. Finally, Minimum Likelihood Estimation with a Binomial data distribution was employed to estimate the parameter 'p' of the random data bits transmitted in the OFDM block.

The conducted experiments and analyses were carried out by The Underwater Acoustic & Navigation Lab at the University of Haifa, Israel, with the main experiment taking place off the coast of Israel on May 30, 2023.

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