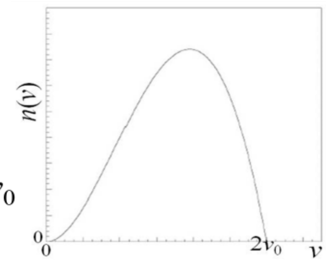
Midterm Examination Fall 2022

Total Possible Score: 140

Name: ______________ Class: ______________ Student number: ____________



1. Consider a gas made up of N molecules.
Suppose the (non-Maxwellian) distribution of the speeds of the gas molecules is given by

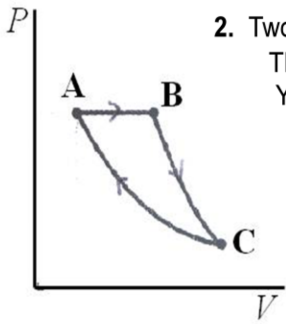
$$n(v) = C\left(\frac{v^4}{v_0^2} - 12.0\frac{v^3}{v_0} + 20.0\,v^2\right) \quad , \quad v \le 2v_0$$

and $n(v) = 0$ for $v > 2v_0$. See plot to the right.
Thus $n(v)dv$ is the number of molecules having speeds in the range v to $v + dv$.

(a) (*4 points*) Find C in terms of N and v_0.

(b) (*6 points*) What is the average speed of the molecules?

(c) (*5 points*) What is the most probable speed for a molecule in this gas?



2. Two moles of a diatomic ideal gas are taken through the cycle shown in the PV-diagram to the left.
The processes **AB**, **BC**, and **CA** are isobaric, adiabatic, and isothermal, respectively.
You are given the following information:

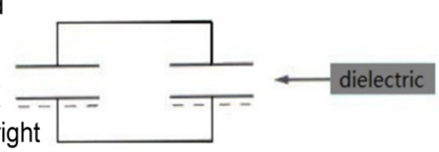
At **A**, the pressure is $5.00\,atm$ and the temperature is $600.0\,K$.
At **B**, the volume is two times what it was at **A**. Assume NO vibrational modes are active.

(a) (*2 points*) What is the volume at **A** and what is the temperature at **B**?
(b) (*2 points*) How much work is done by the gas in process **AB**?
(c) (*4 points*) What is the volume of the gas at **C**?
(d) (*4 points*) What is the work done by the gas in process **BC** and in process **CA**?
(e) (*3 points*) What is the amount of heat absorbed by the gas during each process?
(f) (*3 points*) What is the efficiency of the engine cycle?
(g) (*3 points*) What is the change in entropy during each leg of the cycle?

3. (14 points) Two straight, uniformly-charged rods each have a total charge of Q.
The ends of the first rod are located at $(a, a, 0)$ and $(3a, a, 0)$ – in Cartesian coordinates.
The ends of the second rod are located at $(-3a, a, 0)$ and $(-a, a, 0)$.
What are the direction and magnitude of the electric field at the origin, $(0, 0, 0)$?

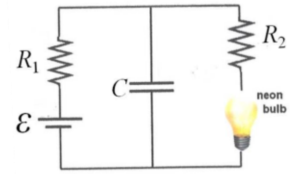
4. A non-conducting sphere or radius R_1 has a charge distribution given by
$\rho(r) = \rho_0\left(1 - r^2 / R_1^2\right)$, where ρ_0 is a constant.

(a) (3 points) What is the total charge, Q, of the sphere?
(a) (*6 points*) In terms of Q, what is the total energy contained in all of space in the electric field of this sphere?
Now (for parts **(c)** and **(d)**), surrounding this sphere and concentric with it, place a conducting spherical shell of inner radius R_1 and outer radius R_2.
(b) (*6 points*) What are the surface charge densities, σ_i and σ_o, on the inner and outer surfaces of the spherical shell?
(c) (*3 points*) Now what is the energy contained in all of space in the electric field due to this configuration?

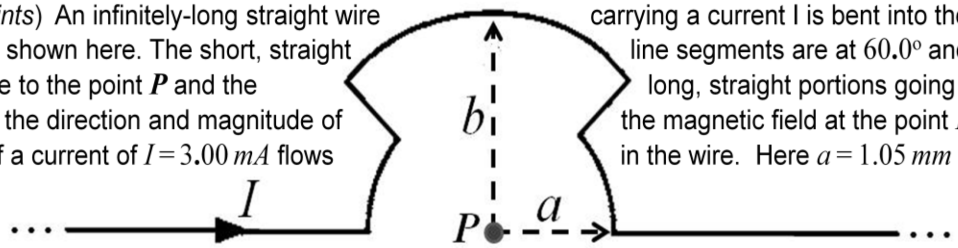


5. (*12 points*) Two capacitors each have two conducting plates of surface area A and an air gap separation of d. They are connected in parallel as shown to the right (note there is no battery). Each capacitor has a charge $+Q$ onits positively-charged plate.
A dielectric slab of width d and area A is inserted between the plates of the capacitor on the right.
The dielectric has a dielectric constant of κ. After the dielectric is inserted, what are the charges, Q_{left}^{new} and Q_{right}^{new}, on the positively-charged plates of the two capacitors?

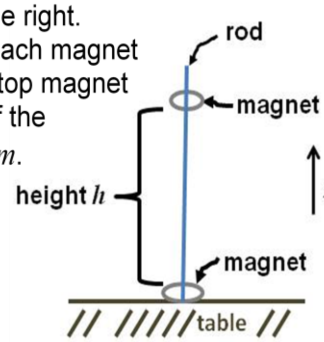
6. (*13 points*) An automobile blinker circuit contains a $\mathcal{E} = 3.00\ V$ battery in series with a resistance of $R_1 = 240.0\ k\Omega$. This pair is connected to a circuit with two arms.
One arm contains a capacitor of capacitance $C = 5.00\ \mu F$. The other arm contains a neon bulb in series with a resistor of resistance $R_2 = 10.0\ k\Omega$. See figure to the right.
The neon bulb goes on when the voltage passing through it exceeds $1.800\ V$, at which time the bulb's resistance effectively goes to zero.
The bulb goes off when the voltage through it is less than $0.600\ V$, at which time its resistance effectively goes to infinity.
How long is the blinker light on and how long is it off?

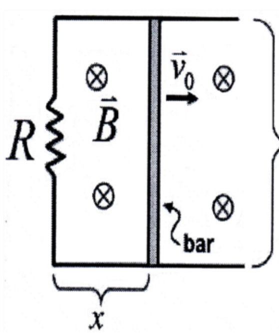


7. (*10 points*) An infinitely-long straight wire carrying a current I is bent into the shape shown here. The short, straight line segments are at 60.0° and 120.0° relative to the point ***P*** and the long, straight portions going off to $\pm\infty$. What are the direction and magnitude of the magnetic field at the point ***P*** due to the wire if a current of $I = 3.00\ mA$ flows in the wire. Here $a = 1.05\ mm$ and $b = 2.10\ mm$.



8. (*13 points*) Consider two very small, thin torroidal (doughnut-shaped) permanent magnets which have a long vertical rod running through them as shown in the figure to the right.
The magnets are able to slide up and down the rod with no friction. Consider each magnet to be a current loop with current flowing clockwise (as seen from above) in the top magnet and current flowing counter-clockwise in the bottom magnet. The magnitude of the magnetic dipole moment of each magnet is μ and the mass of each magnet is m.
What is the height h of the top magnet above the bottom one?
Hint: you will need to use the fact that the two magnets are very small relative to the distance between then to make a wise approximation.





9. A metal bar of mass m slides without friction on two long conducting rails that are a distance b apart. A resistor with resistance R is connected across the rails at one end; compared to this R, the resistance of the bar and the rails can be neglected. A uniform magnetic field $\vec{B}$ is perpendicular to the plane of the rails as shown in the figure to the left.
(a) (*6 points*) At time $t = 0$, the bar is given a velocity v_0 to the right. Find the velocity as a function of time for $t \geq 0$.
(b) (*3 points*) Approximately how far will the bar go if you wait a long, long time?
(c) (*5 points*) Show that energy is conserved by calculating the energy dissipated as heat by this circuit.

10. (*10 points*) A shark  has delicate sensors along the surface of its body permitting it to sense small potential differences. It can sense electrical disturbances created by other fish, and it can also sense the Earth's magnetic field and use this for navigation.
Suppose the shark is swimming horizontally toward the north at $27.0\ km/h$ at a location where the Earth's magnetic field has a strength of $0.470\ Gauss$ pointing downward at an angle of 40° to the vertical. Model the shark as a cylinder with a radius of $25.0\ cm$ (to not confuse you, I intentionally do NOT specify the cylinder's length).
What is the largest indiced emf possible between points on opposite sides of the shark?
Note: neglect the difference between geographical north and the position of the Earth's magnetic pole.