TFTHelper - Probability calculator for TeamFightTactics

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Contents

1 Guide

Hi everyone, This is faultAllForWx, a player in Chinese server and a rookie for CompetitiveTFT with my TFT rolling model.

The motivation for this article is I read some value models about TFT in Chinese bbs nga and reddit recently. Besides, There is no more chosen system again, which means TFT is more linear and predictable and the model more instructive to the games.

However, a tool post from a rookie without good typography would pass quickly with "too long" comments. I will try my best to make my article readable. Moreover, I will provide the application and the well-formatted PDF draft for this article to improve your experience.

Recommended orders for different readers:

1. Wanna intuitions of how it work: omit section "Background" and "Algorithm".
2. Wanna details of how it work, but know binomial distribution and Hypergeometric distribution: omit section "Background".
3. Wanna details of how it work, but know nothing about background:full reading. Or you can skip "background" and turn back when needed.

2 Background

2.1 Discrete random variable and it distribution

Discrete random variable: Random variable taken any of a specified finite or countable list of values (having a countable range). e.g.,"#card drawn for given rolling times X" and "#rolling for roll until get x(given) cards policy" are discrete random variables

Distribution of random variable: A table consisted of every possible value of the r.v. and the corresponding probability

2.2 binomial distribution

Binomial distribution with parameters n and p is the discrete probability distribution of **the number of successes**(X)" in a sequence of n independent experiments with single success prob p. PDF/PMF of it:

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad 0 \le x \le n \tag{1}$$

Binomial distribution is used to get the number of "valid draw" in target tier$c$ pool when rolling.

2.3 Hypergeometric distribution

hypergeometric distribution is a discrete probability distribution that describes the probability of X successes (random draws for which the object drawn has a specified feature) in n draws, without replacement, from a finite population of size M that contains exactly N objects with that feature, wherein each draw is either a success or a failure. PDF/PMF of it:

$$P(X = x) = \frac{\binom{N}{x}\binom{M-N}{n-x}}{\binom{M}{n}} = \frac{\binom{n}{x}\binom{M-n}{N-x}}{\binom{M}{N}}, \quad \max(0, N-M+n) \le x \le \min(n, N) \tag{2}$$

note: names of parameters for HyperGeo is different. Definition here is consistent to library function in the application

HyperGeo is used to get the prob of #target when drawing in the target tier pool.

3 Diagram of TFTHelper

The 2 base questions to solved by the program are:

1. For given rolling times, what's the distribution of the number of target card you get?
2. In the "roll until get x target cards" policy, what's the distribution of the rolling times you need?

The figures below are the process of how it work for those questions.

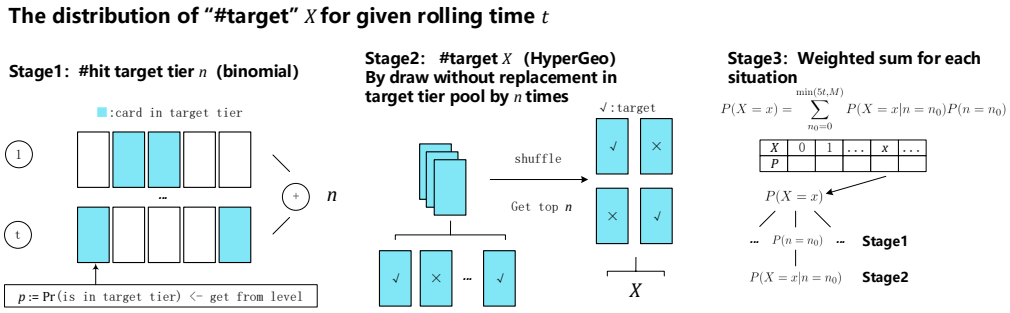

Figure 1: process to get the distribution#target for given #rolling

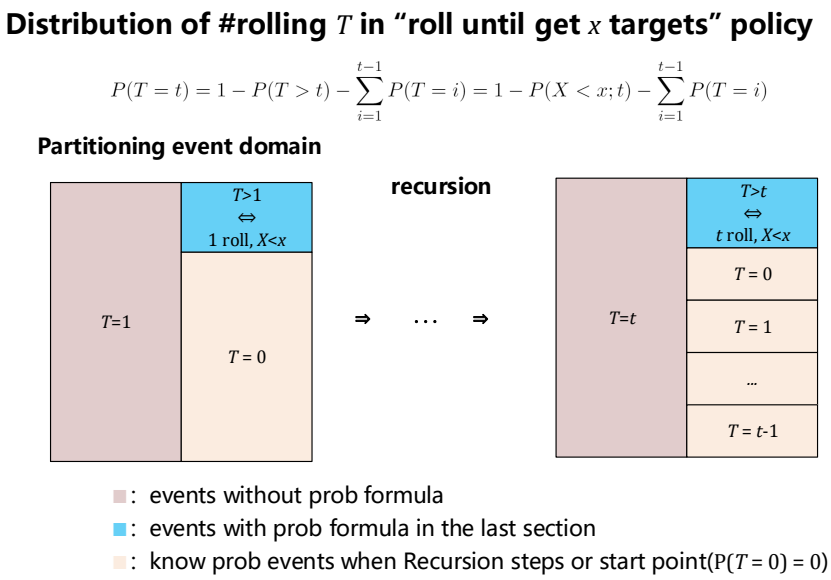

Figure 2: process to get the distribution#rolling for given #target

4 Algorithm

4.1 Parameter definition

Table 1: Parameter definition

Name	Stands for	Name	Stands for
X	#target cards drawn(r.v.)	x	#target cards drawn
T	#rolling(r.v.)	t	#rolling
n	#cards in target tier(#effective drawing, r.v.)	n_0	#effective drawing
p	prob of drawing card in target tier		
M	pool size of target tier		
N	#target card		

4.2 Distribution of #target (X) for given rolling times t

As the figure show above, "target pick by other players" and "pick out same tier when rolling" dynamically change the pool size and induce my 3 stage model for the complex situation.

1. Rolling t times means draw out $5t$ cards. But the prob of each card of the $5t$ slots is in the same tier of the target is level-specific prob p. So, the #effective drawing is a binomial distribution($B(5t, p)$) r.v.

2. If #effective drawing n is given(condition to n), we can draw in pool of target tier$c$ without replacement by n times(i.e. "pick out same tier when rolling" trick). The #target **in this case** follows HyperGeo $H(M, N, n)$, where

   The total pool size M is "#Tier c Species * #single cards - #All Tier c drawn" ; taret pool size N is "#single cards - #target drawn"

3. Synthesiz stage1 and 2 by weighted sum of n(Law of total probability)

$$P(X = x) = \sum_{n_0=0}^{\min(5t,M)} P(X = x|n = n_0)P(n = n_0) \tag{3}$$

4.3 Distribution of #rolling needed (T) in "roll until get x targets" policy

In this case, rolling times T turn from given parameter to r.v.. But the pdf of it can be derived easily with the last subsection. Actually, "stop at rolling t" is the complementary event of the union of "roll t times but get less than x($P(X < x; t)$, known formula)" and "stop before rolling t". That means:

$$\begin{aligned}
&P(T = 0) = 0 \\
&P(T = 1) = 1 - P(T > 1) = 1 - P(X < x; t = 1) \\
&P(t = 2) = 1 - P(T > 2) - P(T = 1) = 1 - P(X < x; t = 2) - P(T = 1) \\
&\qquad \dots \\
&P(T = t) = 1 - P(T > t) - \sum_{i=1}^{t-1} P(T = i) = 1 - P(X < x; t) - \sum_{i=1}^{t-1} P(T = i)
\end{aligned} \tag{4}$$

The possible values r.v. T are natural numbers. But in the application, we should cut down the pdf array when the precison is meet to avoid infinite loop. The feasibility(convergence analysis) and error analysis of truncation are shown in the appendix.

4.4 Error analysis

Briefly enumerate the inconsistencies between the model assumptions and the actual situation:

1. Bench limitation. All the same tier, not target card are drawn out in the model. However, the bench size limitation makes you put them back in some cases.

2. If you got 3*A, A would not appear in your shop anymore. This is tested in pbe 1v0. It disturbs when we come to many 3* comp.

5 Partly conclusion

5.1 Expectation of #rolling at least a target in different cases

As the reference posts show, we first list the expectation of #rolling for a single target card in different levels and different targets drawn before:

Tier 1	0	1	2	3	6	9	12
4	5	5.14	5.3	5.46	6.05	6.8	7.8
5	6.01	6.19	6.38	6.58	7.3	8.21	9.44
6	7.61	7.83	8.08	8.34	9.26	10.44	12.02
7	13.36	14.08	14.53	15.01	16.71	18.88	21.79
8	25.58	26.38	27.24	28.16	31.38	35.52	41.04
9	25.58	26.38	27.24	28.16	31.38	35.52	41.04

Tier 2	0	1	2	3	6	9	12
4	8.72	9.07	9.45	9.87	11.42	13.63	17.06
5	7.97	8.28	8.63	9.01	10.42	12.43	15.55
6	7.54	7.83	8.16	8.52	6.05	9.85	14.68
7	7.54	7.83	8.16	8.52	6.05	9.85	14.68
8	10.38	10.8	11.26	11.76	13.62	16.28	20.39
9	17.03	17.72	18.48	19.32	22.43	26.86	33.71

Tier 3	0	1	2	3	6	9	12
4	16.88	17.72	18.66	19.72	23.87	30.52	42.85
5	12.76	13.4	14.1	14.89	18.01	22.99	32.24
6	10.29	10.8	11.36	12	14.49	18.47	25.88
7	8.65	9.07	9.54	10.07	12.14	15.46	21.63
8	7.47	7.83	8.24	8.69	10.47	13.31	18.6
9	8.65	9.07	9.54	10.07	12.14	15.46	21.63

Tier 4	0	1	2	3	6	9
5	102.63	110.32	119.4	130.31	181.72	310.27
6	41.29	44.37	48	52.36	72.93	124.35
7	14.04	15.06	16.27	17.73	24.58	41.72
8	8.59	9.2	9.93	10.8	14.91	25.19
9	7.23	7.74	8.34	9.07	12.49	21.06

Tier 5	0	1	2	3	6	9
7	147.56	160.28	175.83	195.27	300.24	720.2
8	29.84	32.38	35.49	39.38	60.37	144.36
9	10.22	11.07	12.1	13.4	20.39	48.39

Figure 3: Expectation of #rolling at least a target in different cases

5.2 Some questions can be explained by the model

Q: Fast8, roll 50 gold but no Tier I want, is that unlucky?

A: Reasonable with approx 20% failure rate. In my opinion, force to a specific 5cost carry comp is bad, unless it's a 1* win often 2* autowin carry(like s1 Pantheon, s2 Yi, s3 Xerath in their own patch)

Q: Hyper roll at lv4 or slow roll at lv5, swhich is better for 3*1cost comp?

A: Take this case for example: 4 non-target 1cost cards are drawn on average, but the #target you owned is changing

Lv/#targetOwned	0	3	4	5	6
4	38.57	28.43	24.59	20.47	16.03
5	47.06	34.66	29.97	24.93	19.50

Table 2: Expectation of #rolling to reach 1cost 3* in different case

When you free get 5+ target in 3-1, Hyper roll in lv4 is feasible in the view of expectation. Of course, you should also consider your economy and health and comp.

#rolling	5	10	15	20
Pr(not get the carry)	69.03%	47.14%	31.83%	21.23%
Expectation	0.36	0.73	1.09	1.45

Table 3: draw specific carry at Lv7

There is a criterion in reddit reference post, but you can derive your own criterion with the help of my TFTHelper(like take standard deviation in consider).

Q: Is it feasible for forcing 4 cost carry in 3-5 with long loss streak?

(I don't know whether I describe this play style properly. The direct translation of this play style is called "Empty Fort Strategy" in Chinese server. It's popular for Mord carry last week.)

A: Take this case for example: total 8 non-target 4cost cards are drawn, none of target is drawn out.

A: Pessimistically, even you roll 20 times(which means bad economy in the late game), 20% chance that we don't get the target card. On the other hands, There is a decent success rate when you just roll 5-10 times. So when there is a broken 4cost carry induced 2-3 loss streak style players, the prob of at least one player succeed and become your final rival is very high.

The experience of a loss streak comp is not only shown in the lucky rolling, but unlucky cases and the conuter play of it.

5.3 Misc

Actually, the insights derived by the calculator are almost indirect conclusion expect for 3* 1cost. In a real game, we don't roll for a single unit, unless it is your decided comp carry. But the basic assumption is rolling for a single unit. The deep analysis I want is to build a added value described by "mean, std and other statistics of #rolling". By adding the added value to the comp value on book, we can derive a more precise Price–Performance ratio of each comp(performance can be derived by data from the meta website like lolchess.gg). Because the assumption of "rolling for a single unit" make the price of some flexible built comp abnormally high, this part of analysis is put off.

If you have other ideas on how to use these results, please comment below.

6 Download link and soucre code

Download in github

The application starts from a tiny python script. Then it turns out a command line tool among my friends. And now, It's released here with simple ui. there were a lot of thoughtlessness due to the rushed development. If there are some bugs of TFTHelper, please comment below or post issue in the repository. New function and demand(Like input the comp to get it's added value by rolling, Website or Mobile terminal app of TFTHelper) are also welcome.