The planar topologies in EW$\otimes$QCD corrections to W boson production

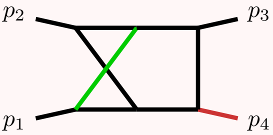
Wenjie He

March 3, 2024

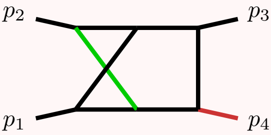
Outline

1. The planar topologies in EW$\otimes$QCD corrections to W boson production
2. Convention
3. **Canonical Differential Equations**

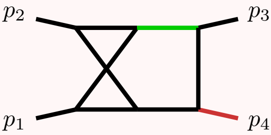
Topology Left one-mass



1ML1

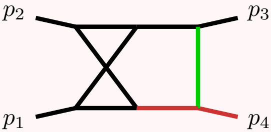


1ML2

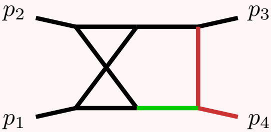


1ML3

Topology Left two-mass

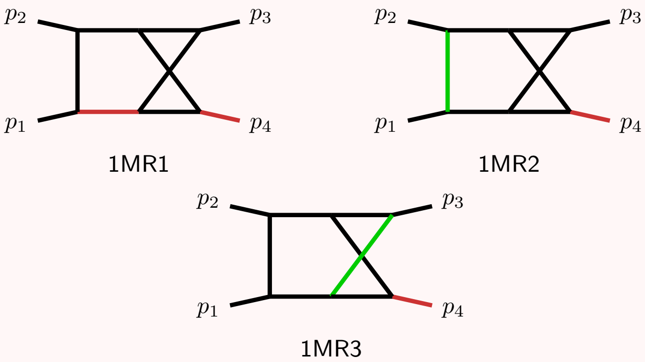


2ML1

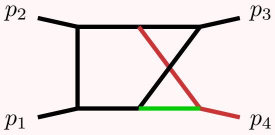


2ML2

Topology Right one-mass



Topology Right two-mass



2MR1

Convention

- Feynman integral

$$F(\alpha_1, ..., \alpha_9) = (m_t^2)^{\alpha-d} \int \widetilde{d}^d l_1 \widetilde{d}^d l_2 \frac{1}{P_1^{\alpha_1} ... P_9^{\alpha_9}}, \qquad (\alpha_i \in \mathbb{Z}, \quad i = 1, ..., 9)$$

with

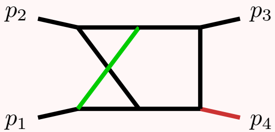
$$\widetilde{d}^d l_i = \frac{d^d l_i}{(2\pi)^d} \left(\frac{i S_\epsilon}{16\pi^2} \right)^{-1}, \qquad S_\epsilon = (4\pi)^\epsilon \, \Gamma(1+\epsilon).$$

- Kinematic variables

$$(p_1+p_2)^2 = s, \quad (p_2+p_4)^2 = t, \quad (p_1+p_4)^2 = u, \quad s+t+u = m_W^2$$

Left one mass

The left one mass 1



Two square roots for 56 MIs

$$
\begin{aligned}
r_1^2 &= 4stm_W^2m_Z^2 + (-sm_Z^2 - tm_Z^2 + st)^2, \\
r_2^2 &= tm_Z^2(4s(-m_W^2 + s + t) + tm_Z^2).
\end{aligned}
$$

Letters

$$x = -\frac{s}{m_Z^2}, \quad y = -\frac{t}{m_Z^2}, \quad z = -\frac{m_W^2}{m_Z^2}$$

$$\omega_1 = x \qquad \omega_2 = y$$
$$\omega_3 = z \qquad \omega_4 = x - 1$$
$$\omega_5 = x + 1 \qquad \omega_6 = y + 1$$
$$\omega_7 = x + y \qquad \omega_8 = z - y$$
$$\omega_9 = x - z \qquad \omega_{10} = x - z + 1$$
$$\omega_{11} = y - z + 1 \qquad \omega_{12} = x + y - z$$
$$\omega_{13} = x + y - z + 1 \qquad \omega_{14} = xy + x - z$$
$$\omega_{15} = -xz + x + y \qquad \omega_{16} = x^2 + x(y - z + 1) + y$$
$$\omega_{17} = -4x^2 - 4xy + 4xz + y \qquad \omega_{18} = -x^2 + x(-y + z - 1) + z$$
$$\omega_{19} = xy + x + y^2 - yz + y \qquad \omega_{20} = x^2(y+1)^2 + 2xy(y - 2z + 1) + y^2$$
$$\omega_{21} = -\frac{r_1 + x(y-1) + y}{r_1 + x(-y) + x - y} \qquad \omega_{22} = \frac{-r_1 + x(-y) + x + y}{r_1 + x(-y) + x + y}$$
$$\omega_{23} = \frac{-r_1 + xy + x - y}{r_1 + xy + x - y} \qquad \omega_{24} = \frac{-r_1 + xy + x + y}{r_1 + xy + x + y}$$
$$\omega_{25} = \frac{y - r_2}{r_2 + y} \qquad \omega_{26} = \frac{-r_2 + 2x + y}{r_2 + 2x + y}$$
$$\omega_{27} = \frac{-r_2 - 2xy + y}{r_2 - 2xy + y}$$

Rationalization

However, a simple method of rationalization hasn't been found. The complicate changing of variables will turn the letter into a quartic polynomial, rendering the weight of the MPLs extremely complex.

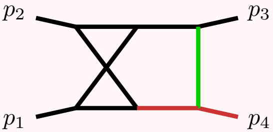
$$z = \frac{1}{16w^2xy^3}\Big[-\left(w^4+4\right)y^4 + 2x^2y^2\left(w^2(y(y+6)+1) - 4(y-1)^2\right) \\ + 4xy^3\left(3w^2y + w^2 + 2y - 2\right) - x^4(y-1)^4 + 4x^3y(y-1)^3\Big]$$

This is not the better way to rationalize the roots.

Or maybe we just don't need to rationalize them.

Left two mass

The left two mass 1

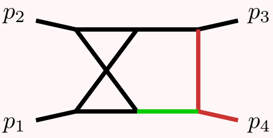


Two square roots for 50 MIs

$$
\begin{aligned}
r_1^2 &= m_Z^4 - 4m_W^2 m_Z^2 \\
r_2^2 &= s^2(m_Z^2 + s + t)^2 - 2sm_W^2(m_Z^2(s-t) + (s+t)^2) + m_W^4(s+t)^2 \\
r_3^2 &= s(4m_W^2(t(s+t) - sm_Z^2) + sm_Z^4 - 4tm_W^4) \\
r_4^2 &= m_W^4(2sm_Z^2 + s^2 + 4st + t^2) + s^2(t - m_Z^2)^2 - 2sm_W^2(s+t)(m_Z^2 + t) \\
&\quad - 2m_W^6(s+t) + m_W^8
\end{aligned}
$$

Left two mass

The left two mass 2



Two square roots for 49 MIs

$$
\begin{aligned}
r_1^2 &= m_Z^2(m_Z^2 - 4m_W^2) \\
r_2^2 &= -sm_Z^2(-4m_W^2(s+t) + sm_Z^2 + 4t(s+t)) \\
r_3^2 &= t^2(m_Z^2 - s)^2 + 2st(s(s-m_Z^2) + 2m_W^2 m_Z^2) + s^4 \\
r_4^2 &= -2m_W^2(t(m_Z^4 + s^2) + sm_Z^2(m_Z^2 + s)) + (st - m_Z^2(s+t))^2 \\
&\quad + m_W^4(m_Z^2 + s)^2
\end{aligned}
$$

Thanks!