The non-planar topologies in EW$\otimes$QCD corrections to W boson production

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April 26, 2024

Outline

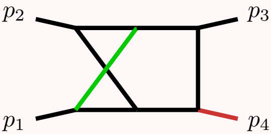
1. The planar topologies in EW$\otimes$QCD corrections to W boson production

2. Convention

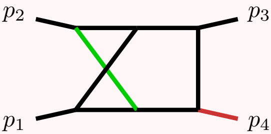
3. **Canonical Differential Equations**

1. The planar topologies in EW$\otimes$QCD corrections to W boson production
   - Topology Left
2. Convention
3. **Canonical Differential Equations**

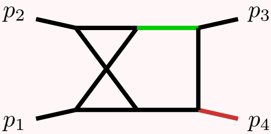
Topology Left one-mass



1ML1

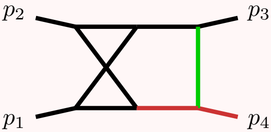


1ML2

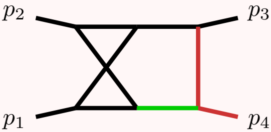


1ML3

Topology Left two-mass

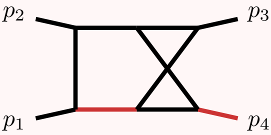


2ML1

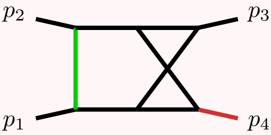


2ML2

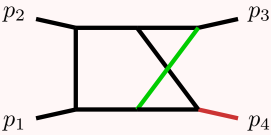
Topology Right one-mass



1MR1

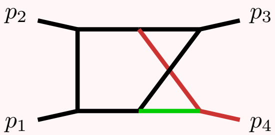


1MR2



1MR3

Topology Right two-mass



2MR1

Convention

- Feynman integral

$$F(\alpha_1, ..., \alpha_9) = (m_t^2)^{\alpha - d} \int \widetilde{d}^d l_1 \widetilde{d}^d l_2 \frac{1}{P_1^{\alpha_1} ... P_9^{\alpha_9}}, \qquad (\alpha_i \in \mathbb{Z}, \quad i = 1, ..., 9)$$

with

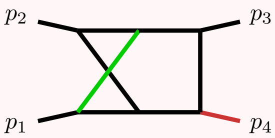
$$\widetilde{d}^d l_i = \frac{d^d l_i}{(2\pi)^d} \left(\frac{i S_\epsilon}{16\pi^2} \right)^{-1}, \qquad S_\epsilon = (4\pi)^\epsilon \, \Gamma(1+\epsilon).$$

- Kinematic variables

$$(p_1 + p_2)^2 = s, \quad (p_2 + p_4)^2 = t, \quad (p_1 + p_4)^2 = u, \quad s + t + u = m_W^2$$

Left one mass

The left one mass 1



Two square roots for 56 MIs

$$r_1^2 = 4stm_W^2m_Z^2 + (-sm_Z^2 - tm_Z^2 + st)^2,$$
$$r_2^2 = tm_Z^2(4s(-m_W^2 + s + t) + tm_Z^2).$$

Letters

$$x = -\frac{s}{m_Z^2}, \quad y = -\frac{t}{m_Z^2}, \quad z = -\frac{m_W^2}{m_Z^2}$$

$$\begin{aligned}
\omega_1 &= x & \omega_2 &= y \\
\omega_3 &= z & \omega_4 &= x-1 \\
\omega_5 &= x+1 & \omega_6 &= y+1 \\
\omega_7 &= x+y & \omega_8 &= z-y \\
\omega_9 &= x-z & \omega_{10} &= x-z+1 \\
\omega_{11} &= y-z+1 & \omega_{12} &= x+y-z \\
\omega_{13} &= x+y-z+1 & \omega_{14} &= xy+x-z \\
\omega_{15} &= -xz+x+y & \omega_{16} &= x^2+x(y-z+1)+y \\
\omega_{17} &= -4x^2-4xy+4xz+y & \omega_{18} &= -x^2+x(-y+z-1)+z \\
\omega_{19} &= xy+x+y^2-yz+y & \omega_{20} &= x^2(y+1)^2+2xy(y-2z+1)+y^2 \\
\omega_{21} &= -\frac{r_1+x(y-1)+y}{r_1+x(-y)+x-y} & \omega_{22} &= \frac{-r_1+x(-y)+x+y}{r_1+x(-y)+x+y} \\
\omega_{23} &= \frac{-r_1+xy+x-y}{r_1+xy+x-y} & \omega_{24} &= \frac{-r_1+xy+x+y}{r_1+xy+x+y} \\
\omega_{25} &= \frac{y-r_2}{r_2+y} & \omega_{26} &= \frac{-r_2+2x+y}{r_2+2x+y} \\
\omega_{27} &= \frac{-r_2-2xy+y}{r_2-2xy+y}
\end{aligned}$$

Rationalization

However, a simple method of rationalization hasn't been found. The complicate changing of variables will turn the letter into a quartic polynomial, rendering the weight of the MPLs extremely complex.

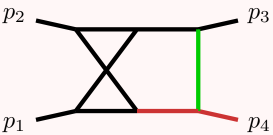
$$
\begin{aligned}
z = \frac{1}{16w^2xy^3}\Big[&- \left(w^4+4\right)y^4 + 2x^2y^2\left(w^2(y(y+6)+1) - 4(y-1)^2\right) \\
&+ 4xy^3\left(3w^2y + w^2 + 2y - 2\right) - x^4(y-1)^4 + 4x^3y(y-1)^3\Big]
\end{aligned}
$$

This is not the better way to rationalize the roots.

Or maybe we just don't need to rationalize them.

Left two mass

The left two mass 1

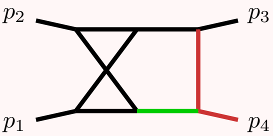


Two square roots for 50 MIs

$$r_1^2 = m_Z^4 - 4m_W^2 m_Z^2$$
$$r_2^2 = s^2(m_Z^2 + s + t)^2 - 2sm_W^2(m_Z^2(s-t) + (s+t)^2) + m_W^4(s+t)^2$$
$$r_3^2 = s(4m_W^2(t(s+t) - sm_Z^2) + sm_Z^4 - 4tm_W^4)$$
$$r_4^2 = m_W^4(2sm_Z^2 + s^2 + 4st + t^2) + s^2(t - m_Z^2)^2 - 2sm_W^2(s+t)(m_Z^2 + t) \\ - 2m_W^6(s+t) + m_W^8$$

Left two mass

The left two mass 2



Two square roots for 49 MIs

$$
\begin{aligned}
r_1^2 &= m_Z^2(m_Z^2 - 4m_W^2) \\
r_2^2 &= -sm_Z^2(-4m_W^2(s+t) + sm_Z^2 + 4t(s+t)) \\
r_3^2 &= t^2(m_Z^2 - s)^2 + 2st(s(s - m_Z^2) + 2m_W^2 m_Z^2) + s^4 \\
r_4^2 &= -2m_W^2(t(m_Z^4 + s^2) + sm_Z^2(m_Z^2 + s)) + (st - m_Z^2(s+t))^2 \\
&\quad + m_W^4(m_Z^2 + s)^2
\end{aligned}
$$

Thanks!