

# COG260: Data, Computation, and The Mind

## Categories

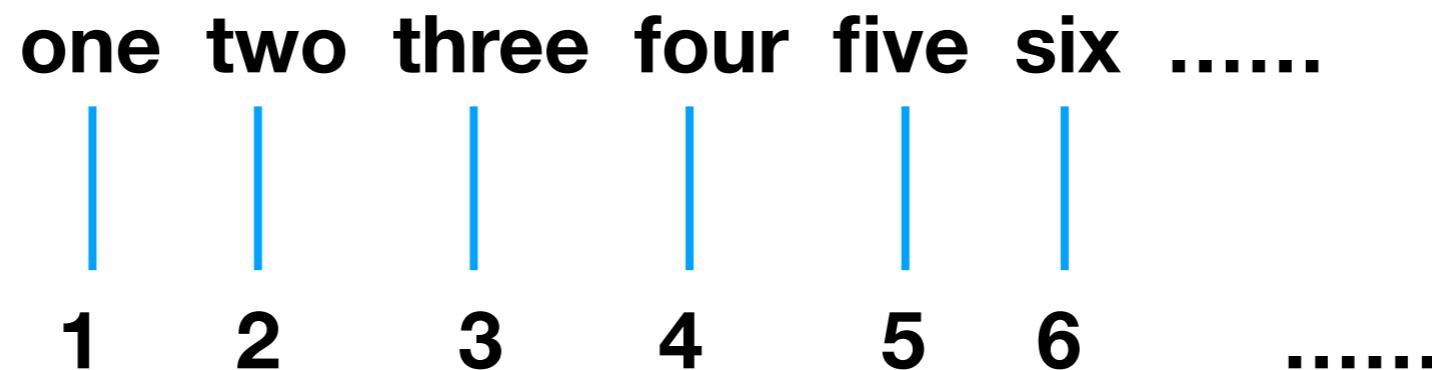
# Categories

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## Lecture 2: Numerals



# Categories

- Categories are everywhere, and we already came across a few:

## Lecture 3: Object categories

**cat**



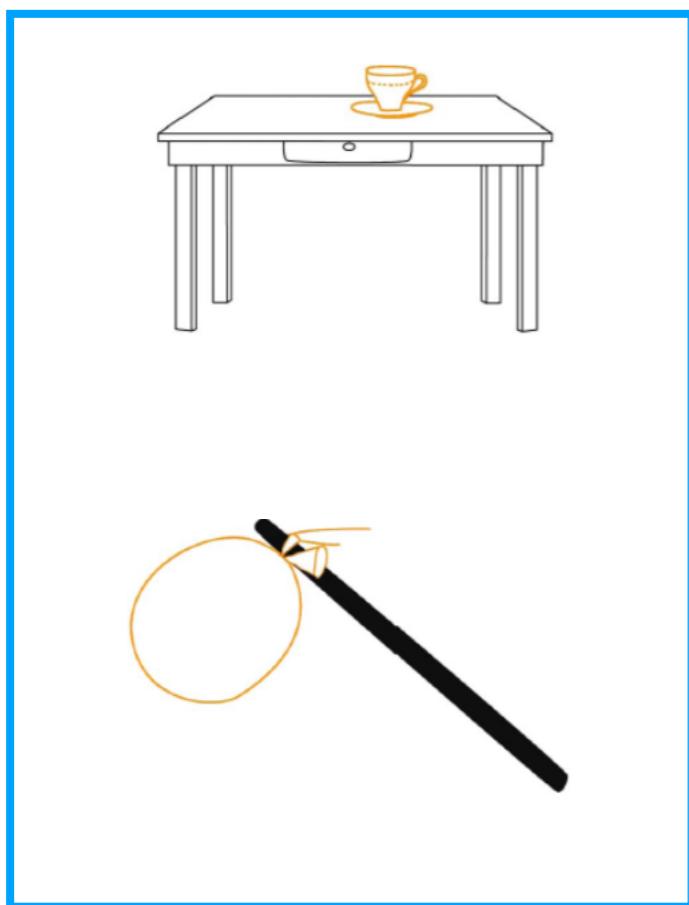
**dog**



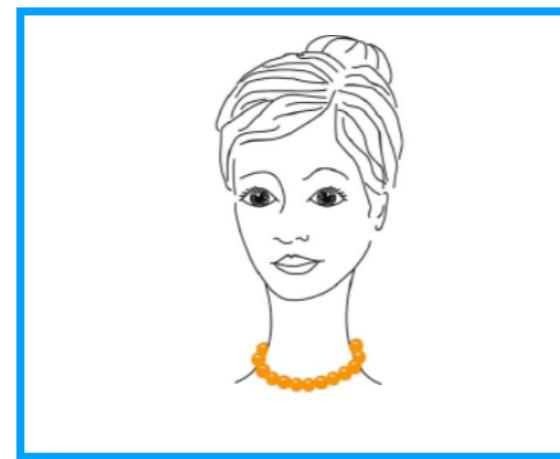
# Categories

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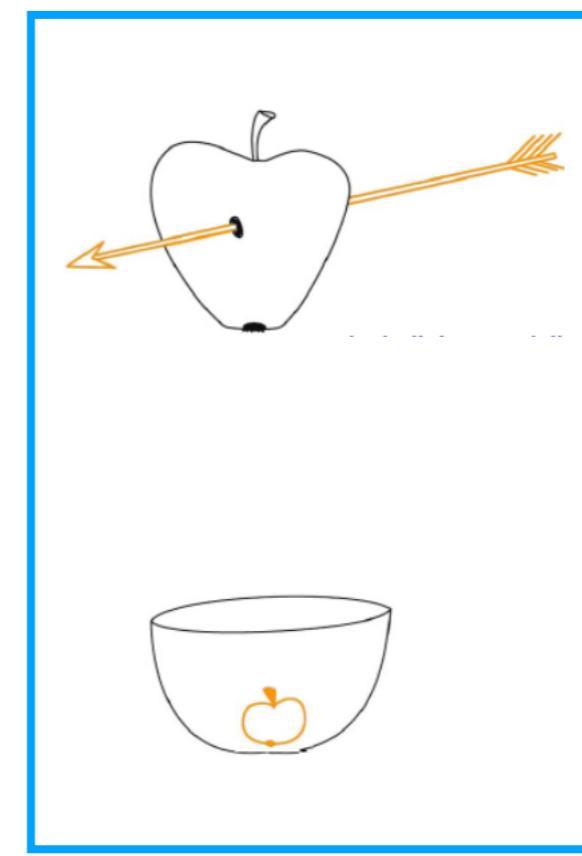
## Lecture 4: Spatial categories



**ON**



**AROUND**



**INSIDE**

# Discussion

- What makes a category?
- What are categories for?
- How are categories formed?

# Outline

- What makes a category?
  - Membership, similarity, and features —> Prototypicality
- What are categories for?
  - Cognitive economy
- \* How are categories formed?
  - Next week

# What makes a category

- Definition: Category = Grouping of entities in the world.

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# What makes a category

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  - Features: Perceived structure or function in the world

# Similarity

- Entities that are similar tend to be grouped together (Gestalt)



# Similarity

- Entities that are similar tend to be grouped together (Gestalt)



Why aren't  
these pizzas  
?



# Features

- Respects for similarity matter (Medin et al., 1993):
  - Pizzas and coins are similar in shape
  - However, pizzas and pineapples are similar in function (i.e. food)

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- Respects for similarity matter (Medin et al., 1993):
  - Pizzas and coins are similar in shape
  - However, pizzas and pineapples are similar in function (i.e. food)

**i.e. similar entities are not always grouped into the same category**

# Features

- Respects for similarity matter (Medin et al., 1993):
  - Pizzas and coins are similar in shape
  - However, pizzas and pineapples are similar in function (i.e. food)
- Respects → Features:
  - Perceived structure: shape (e.g., circular)
  - Function: food vs currency

# Feature representation



**Feature 1:**  
**Is it circular?**

**Feature 2:**  
**Is it food?**

**Feature 3:**  
**Is it heavy?**

# Feature representation



Feature 1:  
Is it circular?

30/30

Feature 2:  
Is it food?

30/30

Feature 3:  
Is it heavy?

10/30

0/30

30/30

15/30

30/30

0/30

0/30

# Feature representation



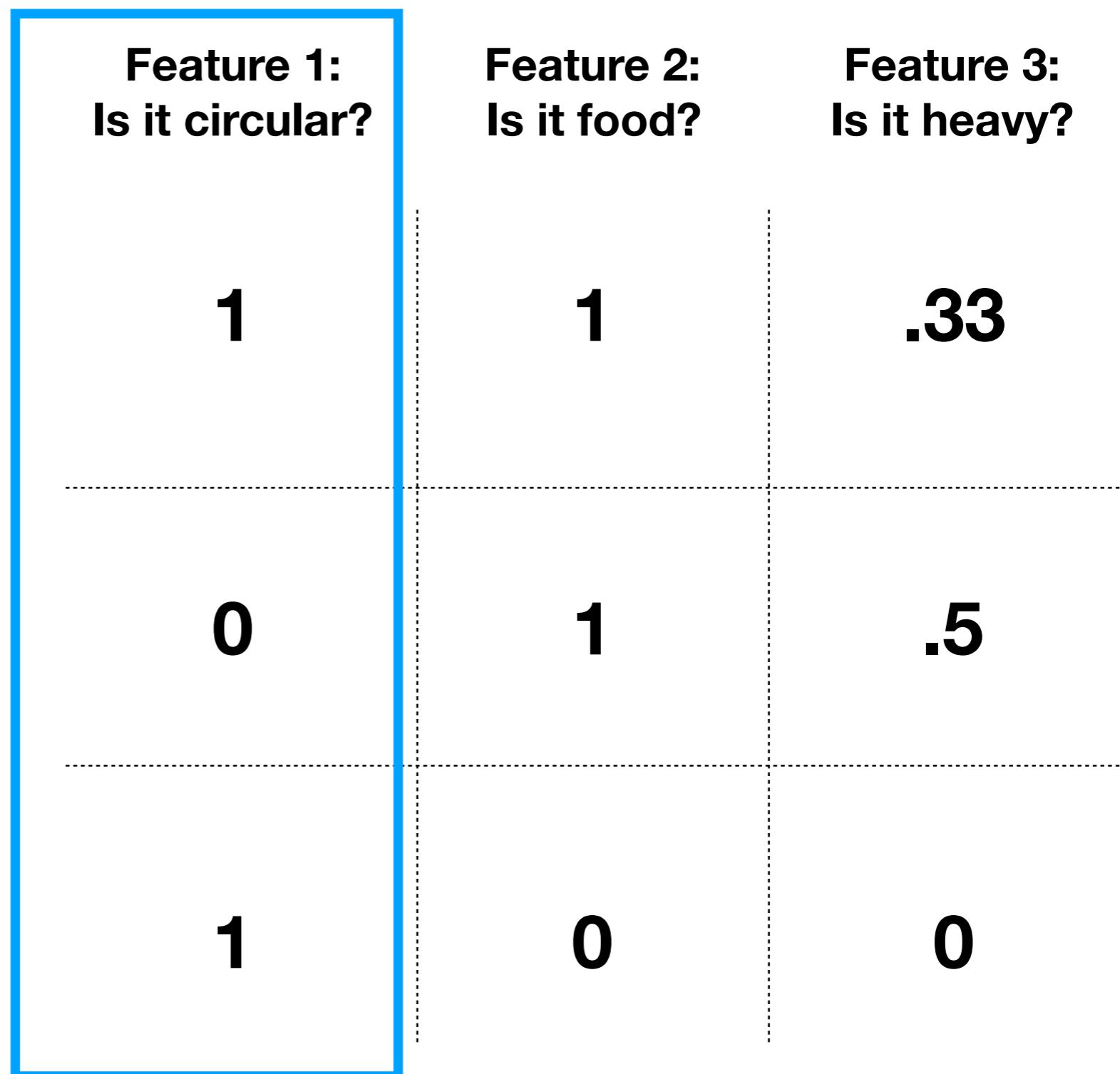
**Feature 1:**  
**Is it circular?**

1                    1                    .33

0                    1                    .5

1                    0                    0

# Feature representation



# Feature representation



Feature 1:  
Is it circular?

1

0

1

Feature 2:  
Is it food?

1

1

0

Feature 3:  
Is it heavy?

.33

.5

0

# Discuss: Which two are more similar—why?



Feature 1: Is it circular?	Feature 2: Is it food?	Feature 3: Is it heavy?
1	1	.33
0	1	.5
1	0	0

# From features to similarity

- Pizza = [ 1, 1, .33 ]
  - Pineapple = [ 0, 1, .5 ]
  - Coin = [ 1, 0, 0 ]
- Which are more similar?
- 
- Pizza - Coin  
OR  
Pizza - Pineapple

# From features to similarity

- Pizza = [ 1, 1, .33 ]
  - Pineapple = [ 0, 1, .5 ]
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- Which are more similar?
- Pizza - Coin  
OR  
Pizza - Pineapple

Similarity (Pizza, Coin) > / < Similarity (Pizza, Pineapple) ?

# From features to similarity

- Pizza = [ 1, 1, .33 ]

Which are more similar?

- Pineapple = [ 0, 1, .5 ]



Pizza - Coin

OR

- Coin = [ 1, 0, 0 ]

Pizza - Pineapple

Similarity (Pizza, Coin) > / < Similarity (Pizza, Pineapple) ?



Distance (Pizza, Coin) < / > Distance (Pizza, Pineapple) ?

# From features to similarity

- Pizza = [ 1, 1, .33 ]
  - Pineapple = [ 0, 1, .5 ]
  - Coin = [ 1, 0, 0 ]

Which are more similar?

→

Pizza - Coin  
OR  
Pizza - Pineapple

**Distance(x, y) = Distance( [x1,x2,x3], [y1,y2,y3] ) = ?**

# From features to similarity

- Pizza = [ 1, 1, .33 ]
  - Pineapple = [ 0, 1, .5 ]
  - Coin = [ 1, 0, 0 ]

Which are more similar?

→

Pizza - Coin  
OR  
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$$\text{Distance}(x, y) = \text{Distance}([x_1, x_2, x_3], [y_1, y_2, y_3]) = ?$$

# How about Euclidean distance in vector space?

# From features to similarity

- Pizza = [ 1, 1, .33 ]
  - Pineapple = [ 0, 1, .5 ]
  - Coin = [ 1, 0, 0 ]

Which are more similar?

→

Pizza - Coin  
OR  
Pizza - Pineapple

**Distance(x, y) = Distance( [x1,x2,x3], [y1,y2,y3] ) = ?**

**Distance (Pizza, Coin) =  $\sqrt{(1-1)^2 + (1-0)^2 + (.33-0)^2}$**

Distance (Pizza, Pineapple) =  $\sqrt{(1-0)^2 + (1-1)^2 + (.33-.5)^2}$

# From features to similarity

- Pizza = [ 1, 1, .33 ]

Which are more similar?

- Pineapple = [ 0, 1, .5 ]



Pizza - Coin

- Coin = [ 1, 0, 0 ]

OR

Pizza - Pineapple

Distance( $x, y$ ) = Distance( [ $x_1, x_2, x_3$ ], [ $y_1, y_2, y_3$ ] ) = ?

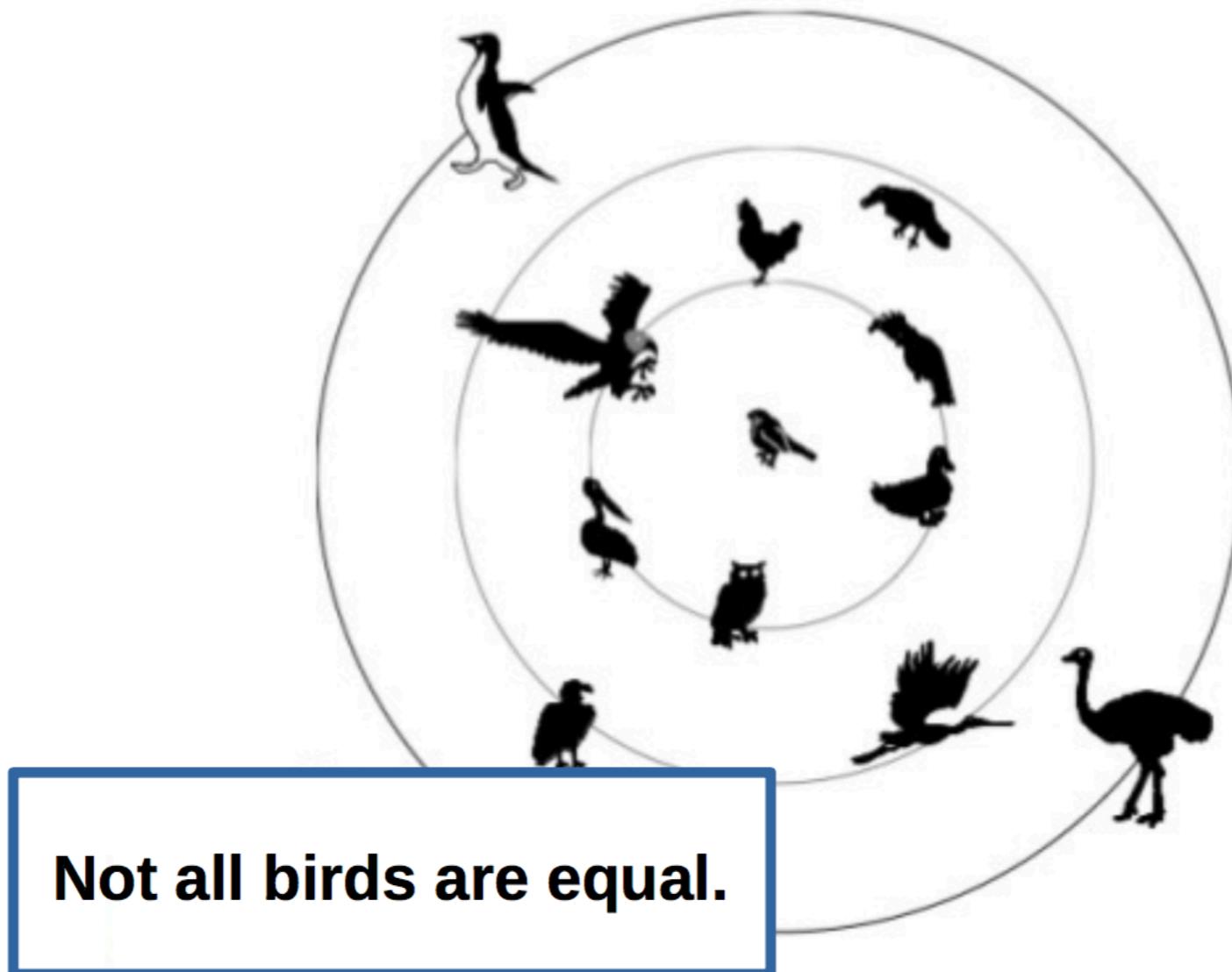
Distance (Pizza, Coin) =  $\sqrt{ (1-1)^2 + (1-0)^2 + (.33-0)^2 }$

Distance (Pizza, Pineapple) =  $\sqrt{ (1-0)^2 + (1-1)^2 + (.33-.5)^2 }$

# **5-minute break**

# Graded category membership

- We have so far assumed that membership is binary: in/out, but is this true? Consider the category of “bird”.



# How “good” is a bird?

Member	Rank	Goodness of example Specific score	Member	Rank	Goodness of example Specific score
Bird					
robin	1	1.02	goldfinch	28	2.06
sparrow	2	1.18	parrot	29	2.07
bluejay	3	1.29	sandpiper	30	2.40
bluebird	4	1.31	pheasant	31	2.69
canary	5	1.42	catbird	32	2.72
blackbird	6	1.43	crane	33	2.77
dove	7	1.46	albatross	34	2.80
lark	8	1.47	condor	35	2.83
swallow	9	1.52	toucan	36	2.95
parakeet	10	1.53	owl	37	2.96
oriole	11	1.61	pelican	38	2.98
mockingbird	12	1.62	geese	39	3.03
redbird	13.5	1.64	vulture	40	3.06
wren	13.5	1.64	stork	41	3.10
finch	15	1.66	buzzard	42	3.14
starling	16	1.72	swan	43	3.16
cardinal	17.5	1.75	flamingo	44	3.17
eagle	17.5	1.75	duck	45	3.24
hummingbird	19	1.76	peacock	46	3.31
seagull	20	1.77	egret	47	3.39
woodpecker	21	1.78	chicken	48	4.02
pigeon	22	1.81	turkey	49	4.09
thrush	23	1.89	ostrich	50	4.12
falcon	24	1.96	titmouse	51	4.35
crow	25	1.97	emu	52	4.38
hawk	26	1.99	penguin	53	4.53
raven	27	2.01	bat	54	6.15

# How “good” is a bird?

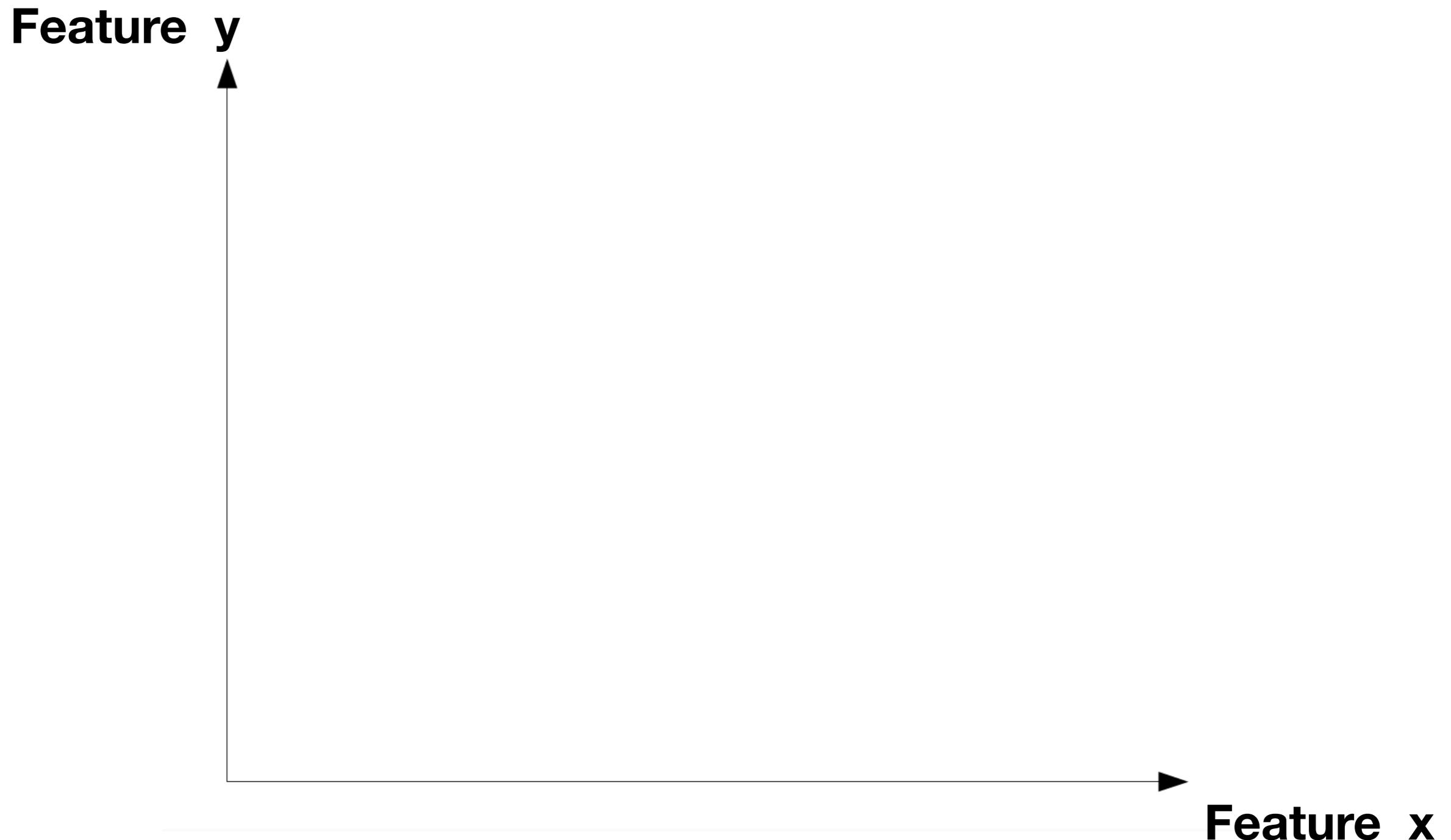
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pigeon	22	1.81		turkey	49	4.09	
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# Prototypicality

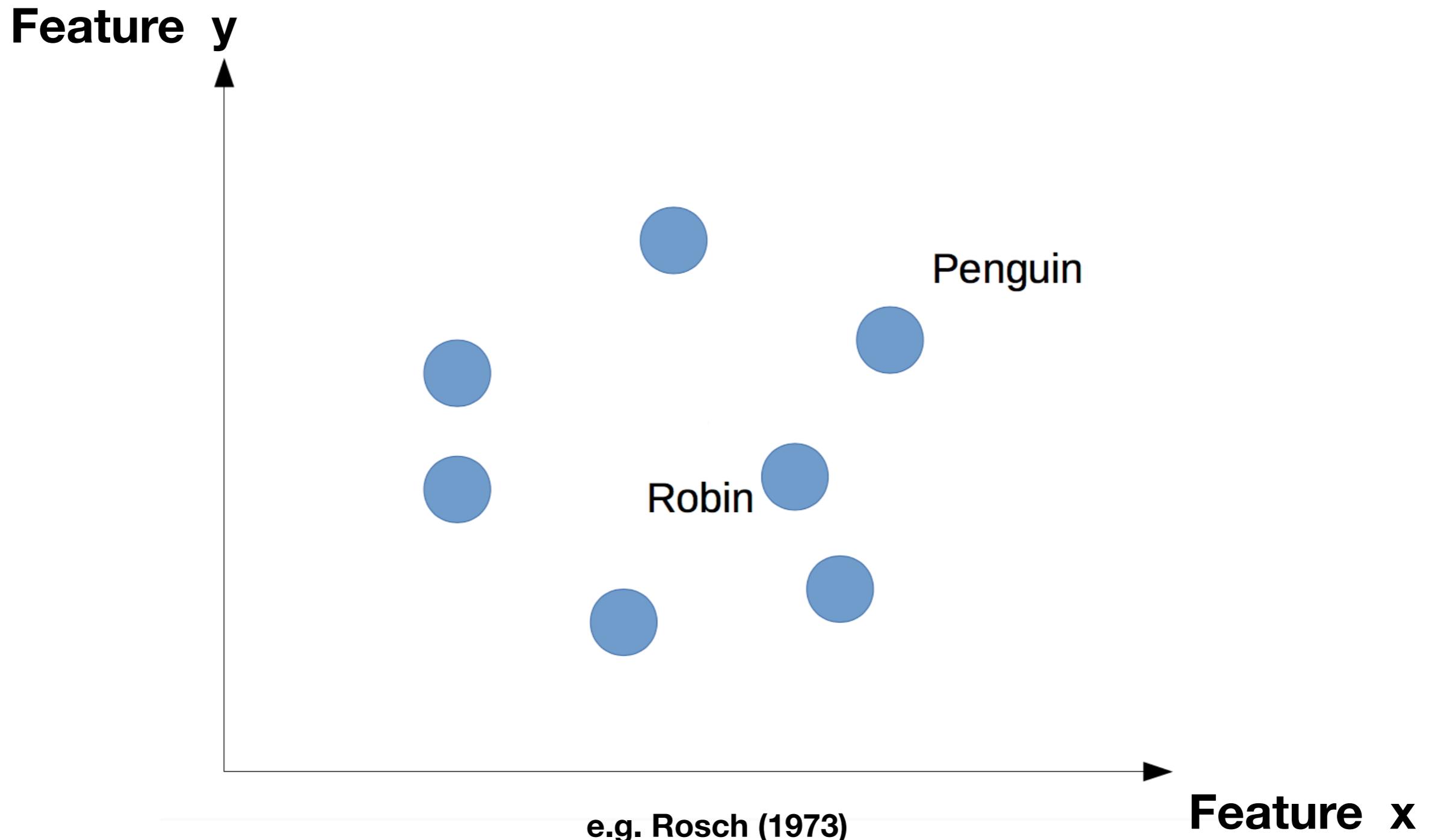
- A descriptive terminology for graded category membership.
- Judgment of how prototypical or salient an entity is as a member for a category, e.g., a prototypical “red” (Rosch 73)
- A prototype does not need to be a member of a category.

cf. Rosch (1978)

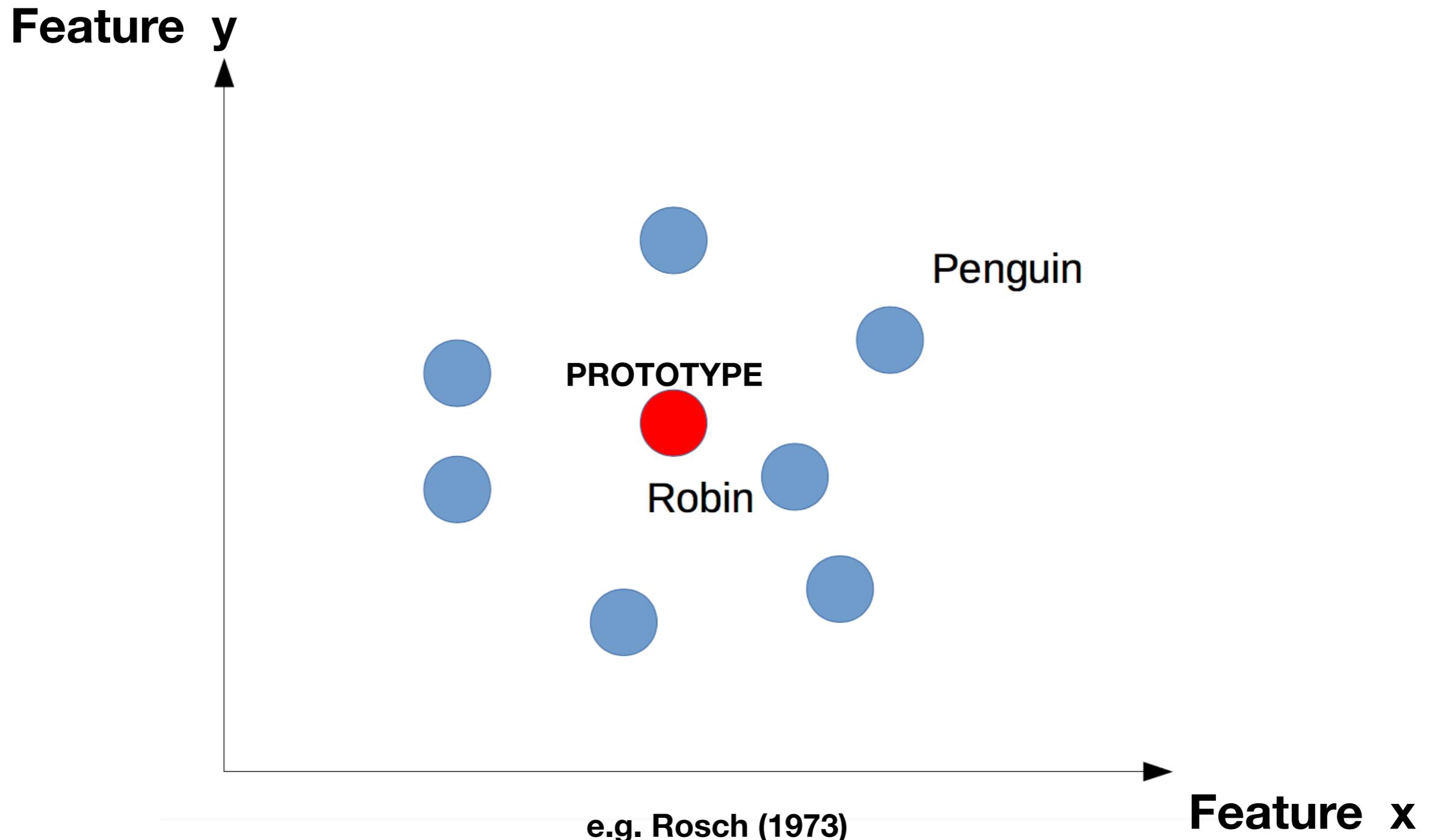
# Linking features, similarity, and prototypicality



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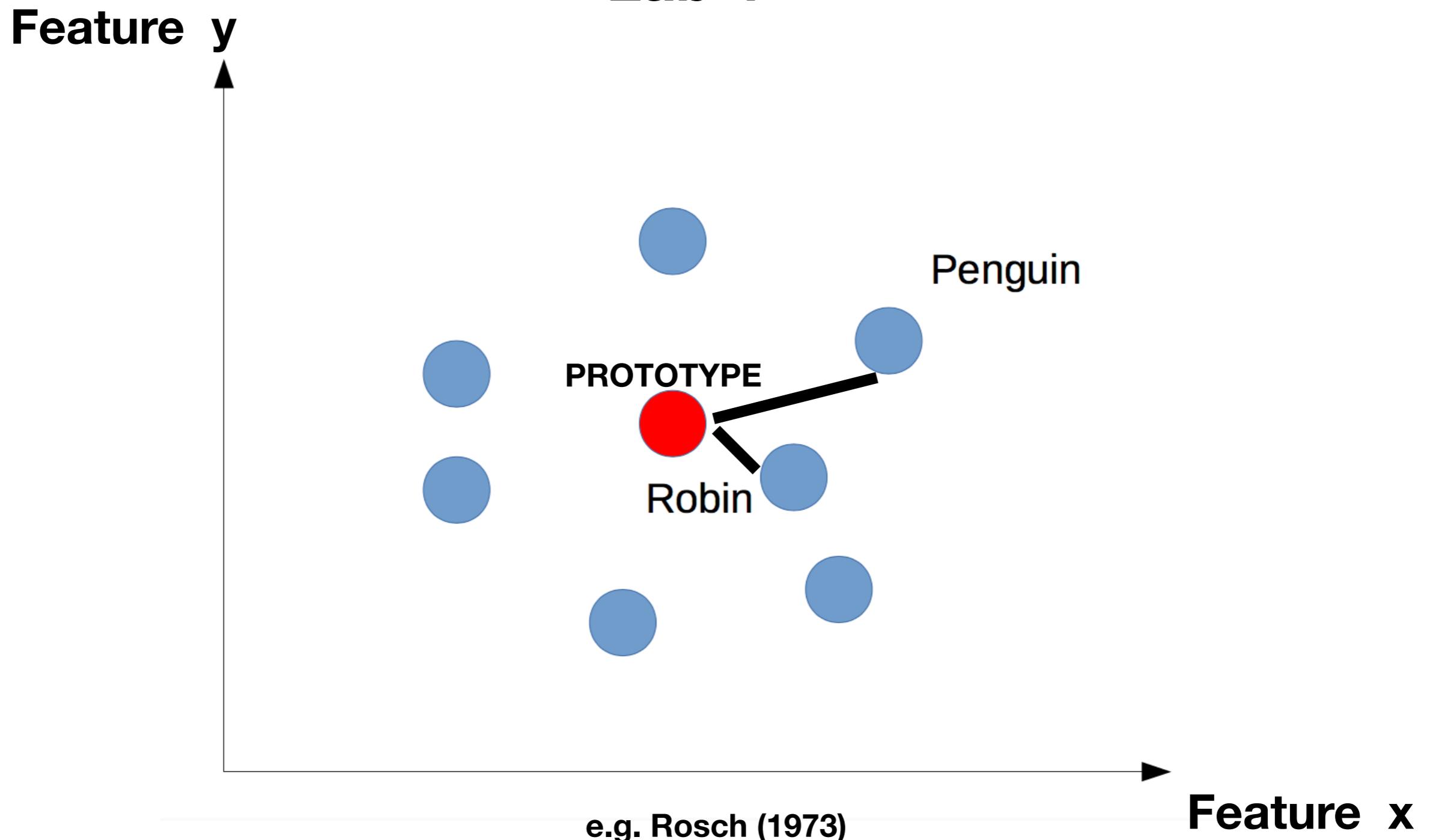


# Linking features, similarity, and prototypicality

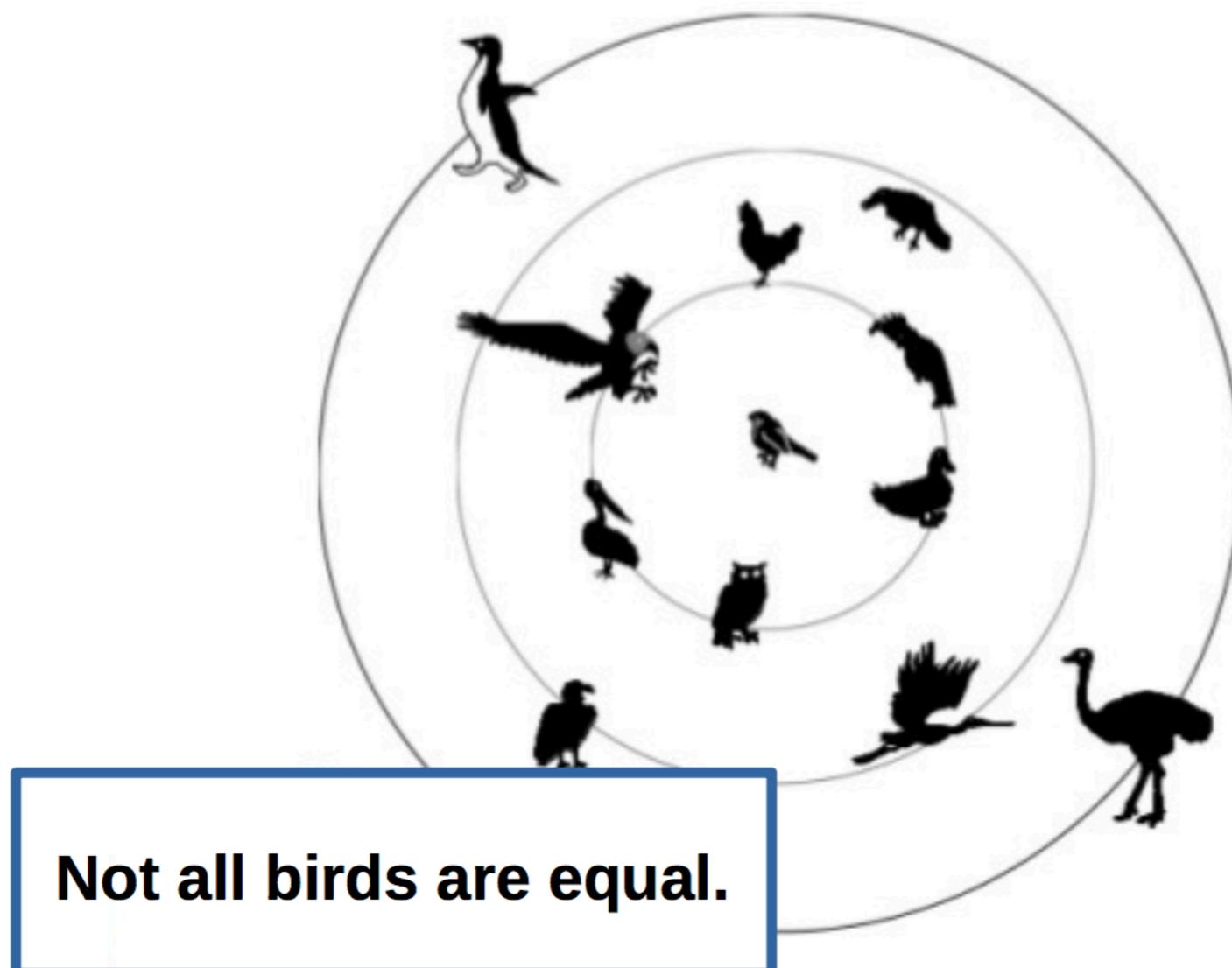


# Similarity-to-prototype → Prototypicality?

## Lab 4



**Discuss:**  
Propose alternative variable(s) that might explain prototypicality



# Outline

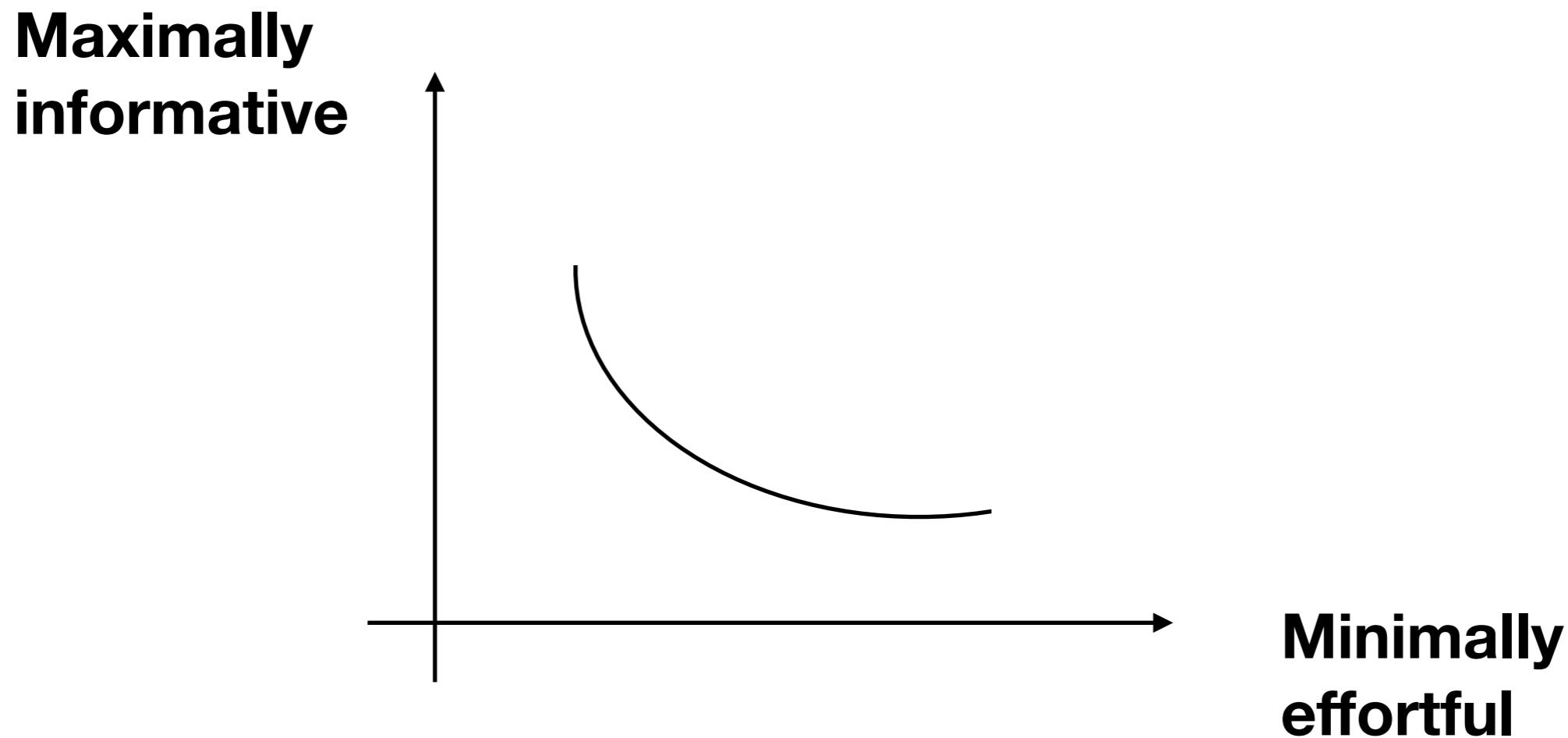
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# Cognitive economy

- The task of category systems is to provide *maximum information with the least cognitive effort* (Rosch, 1978).

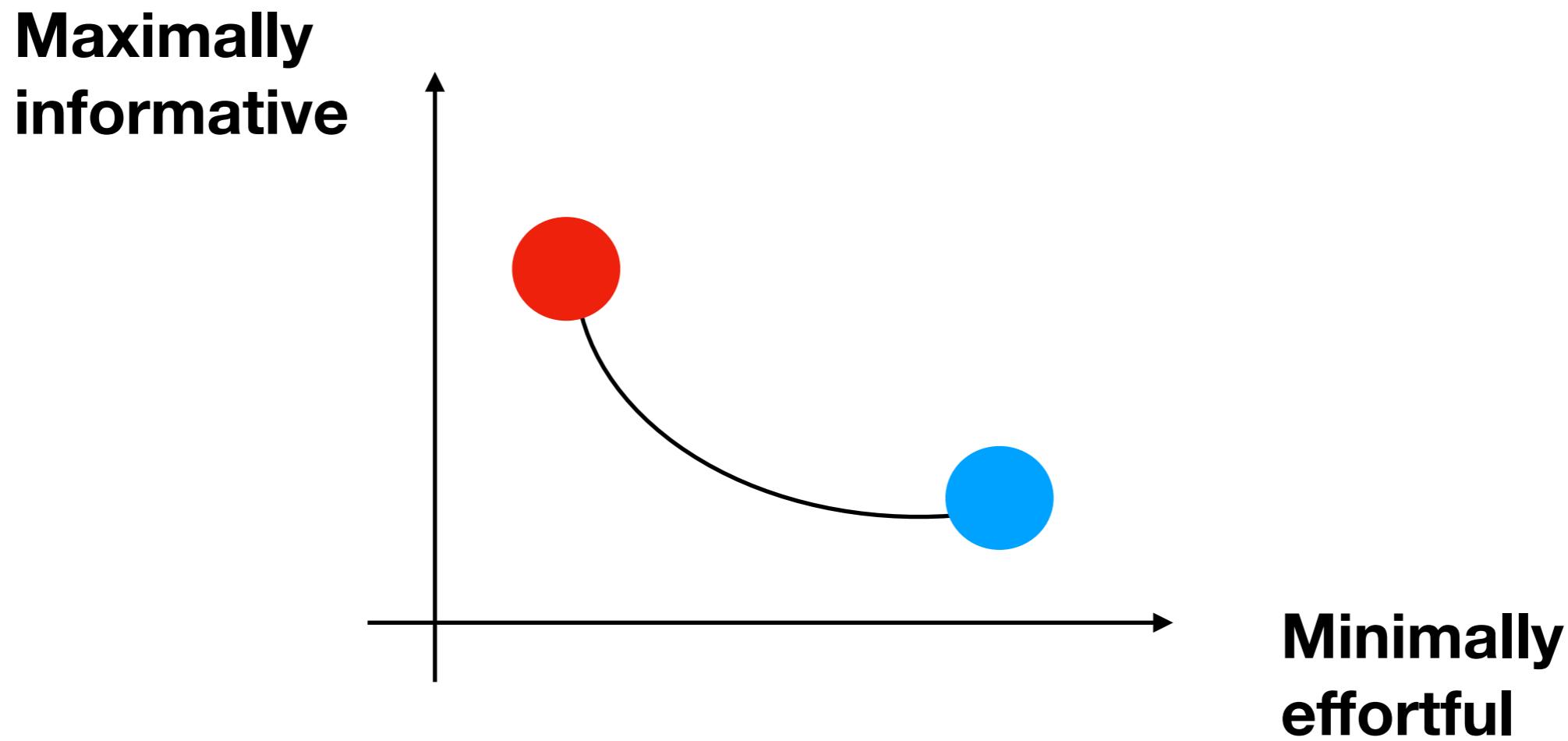
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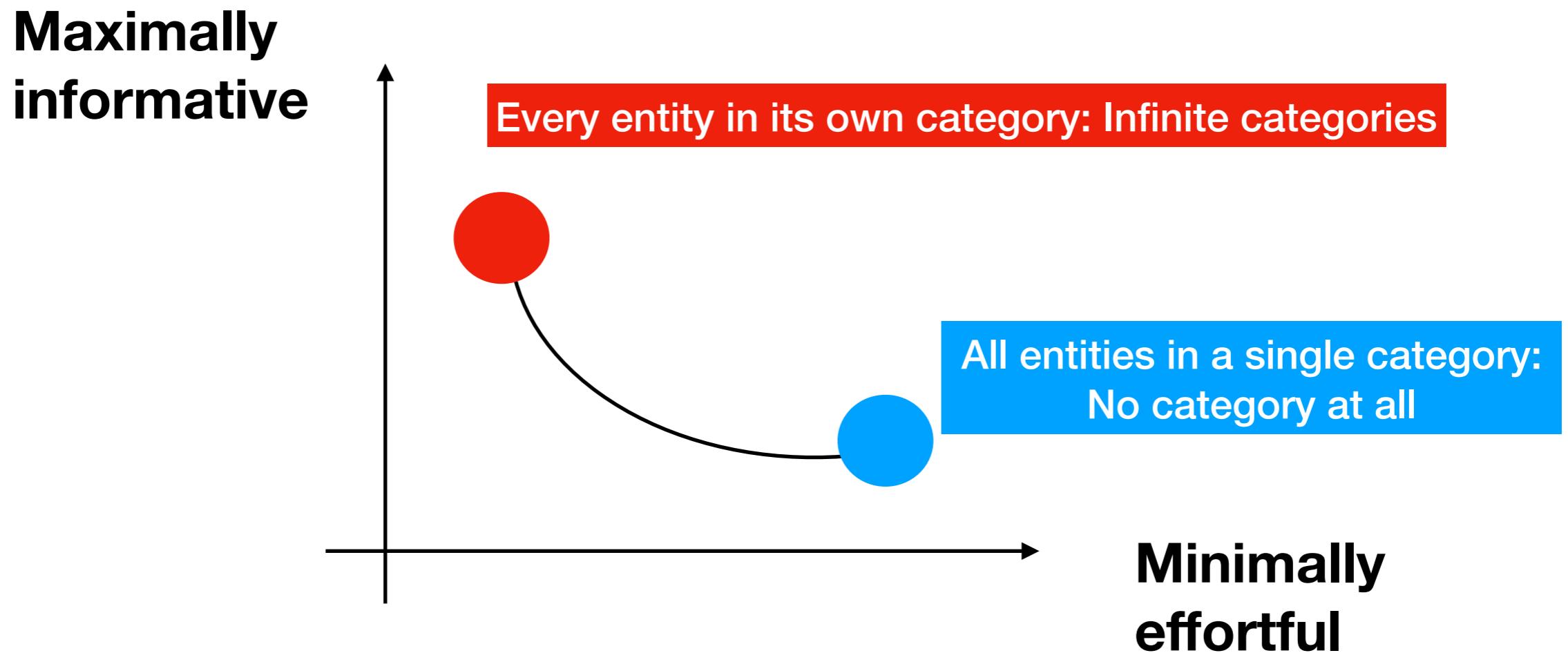
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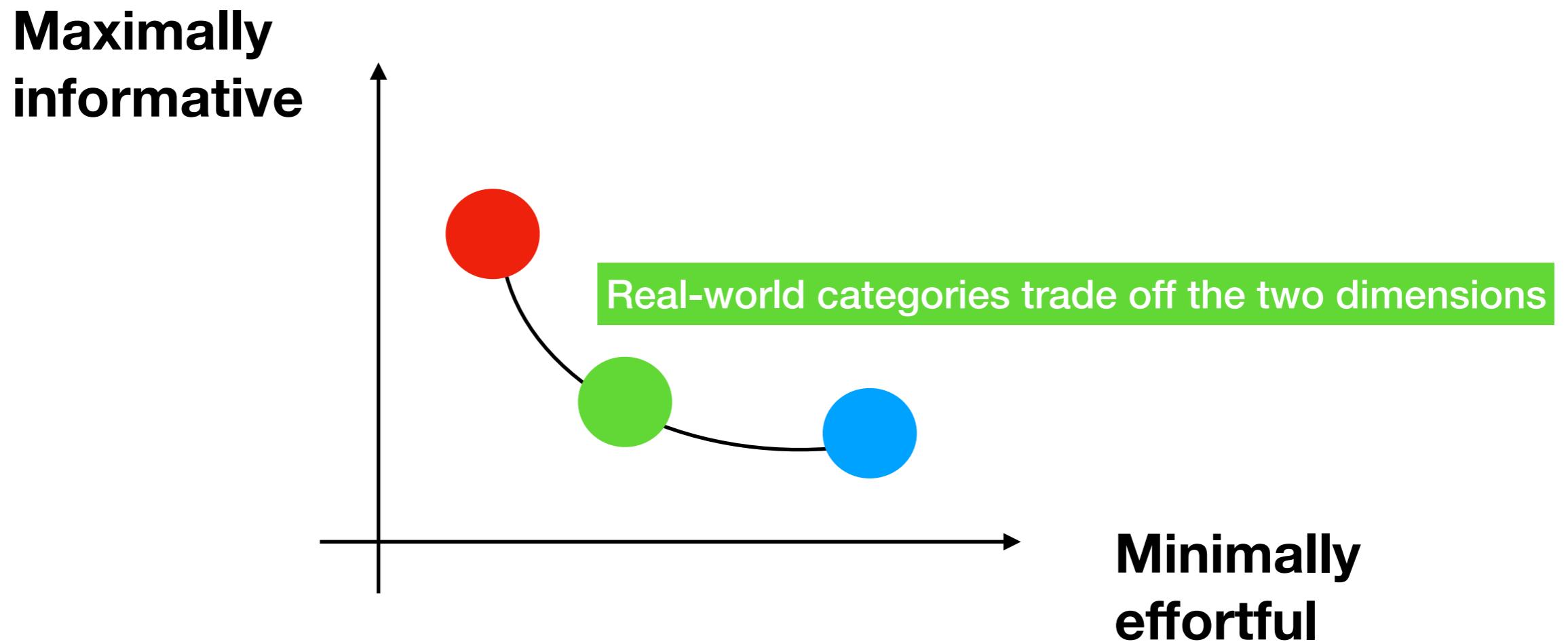
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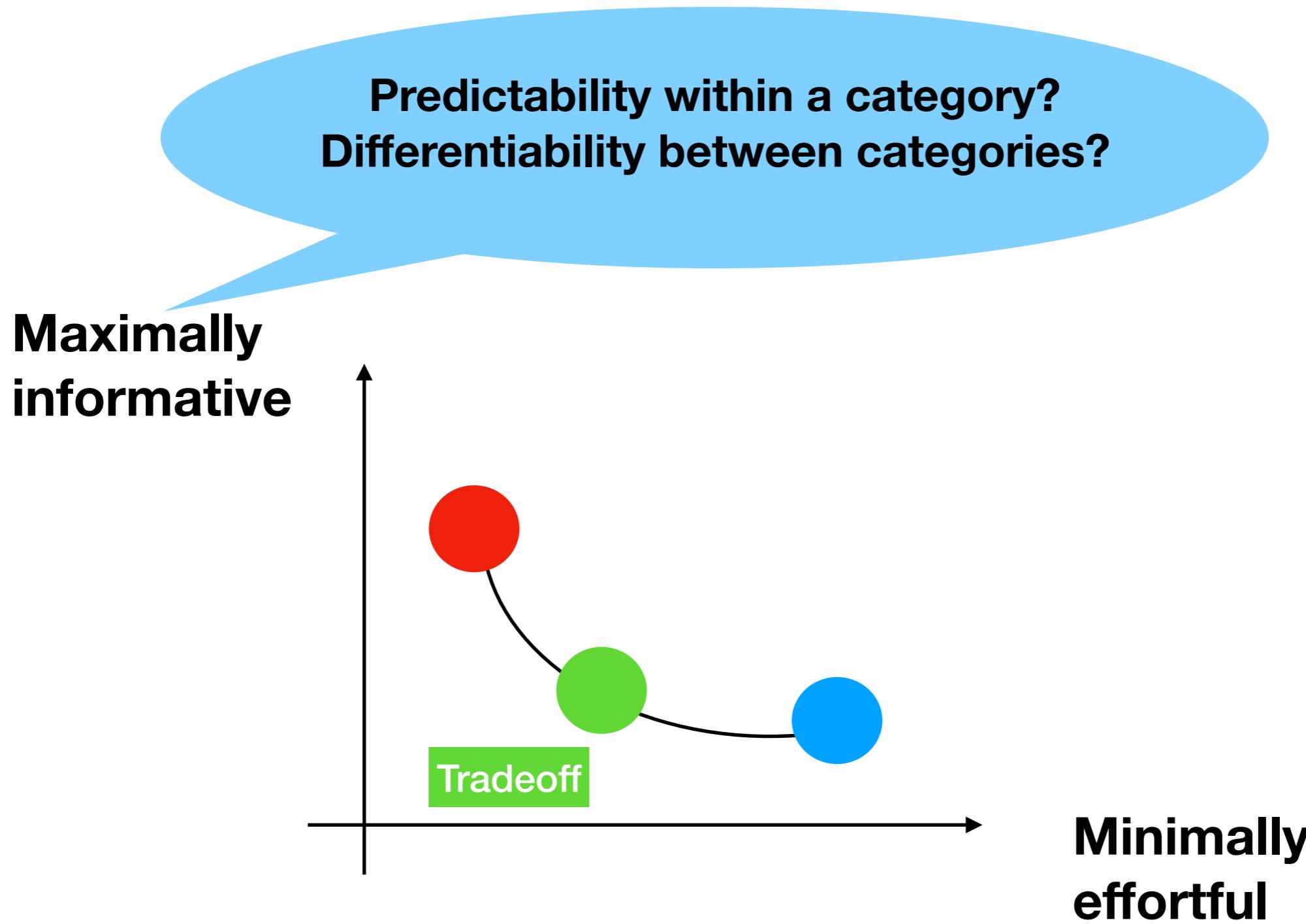


# Cognitive economy

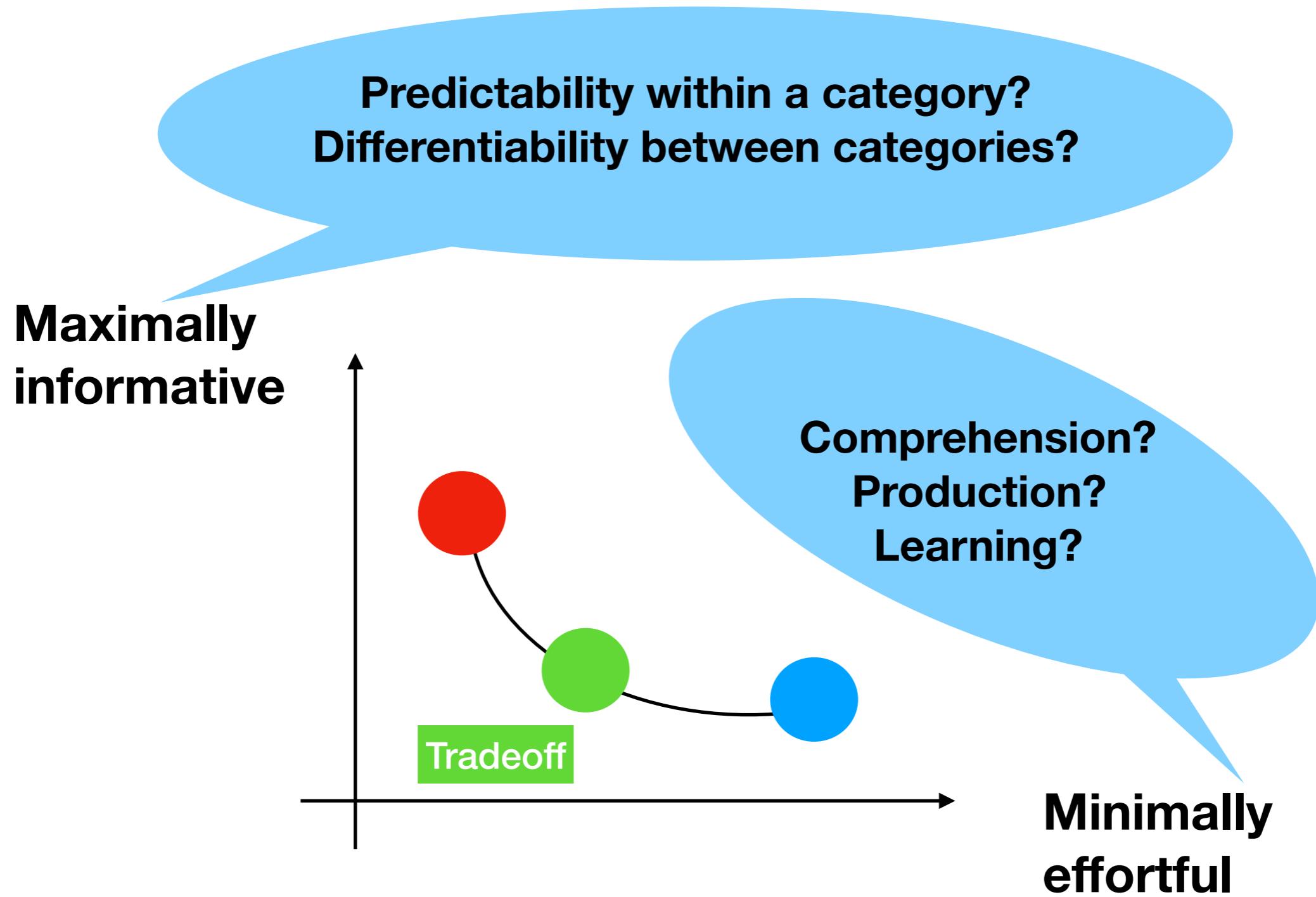
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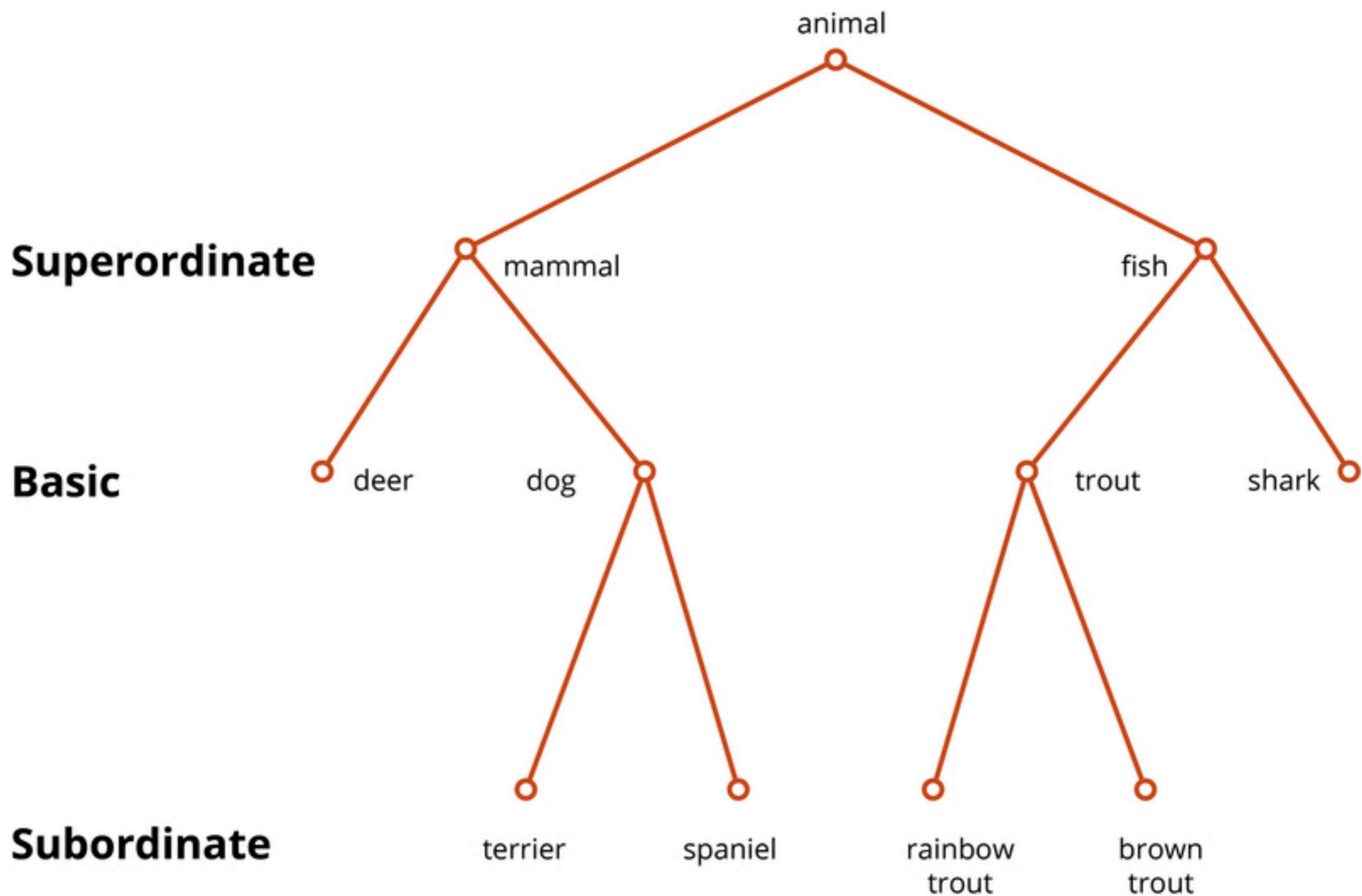
# Nuances of the theory



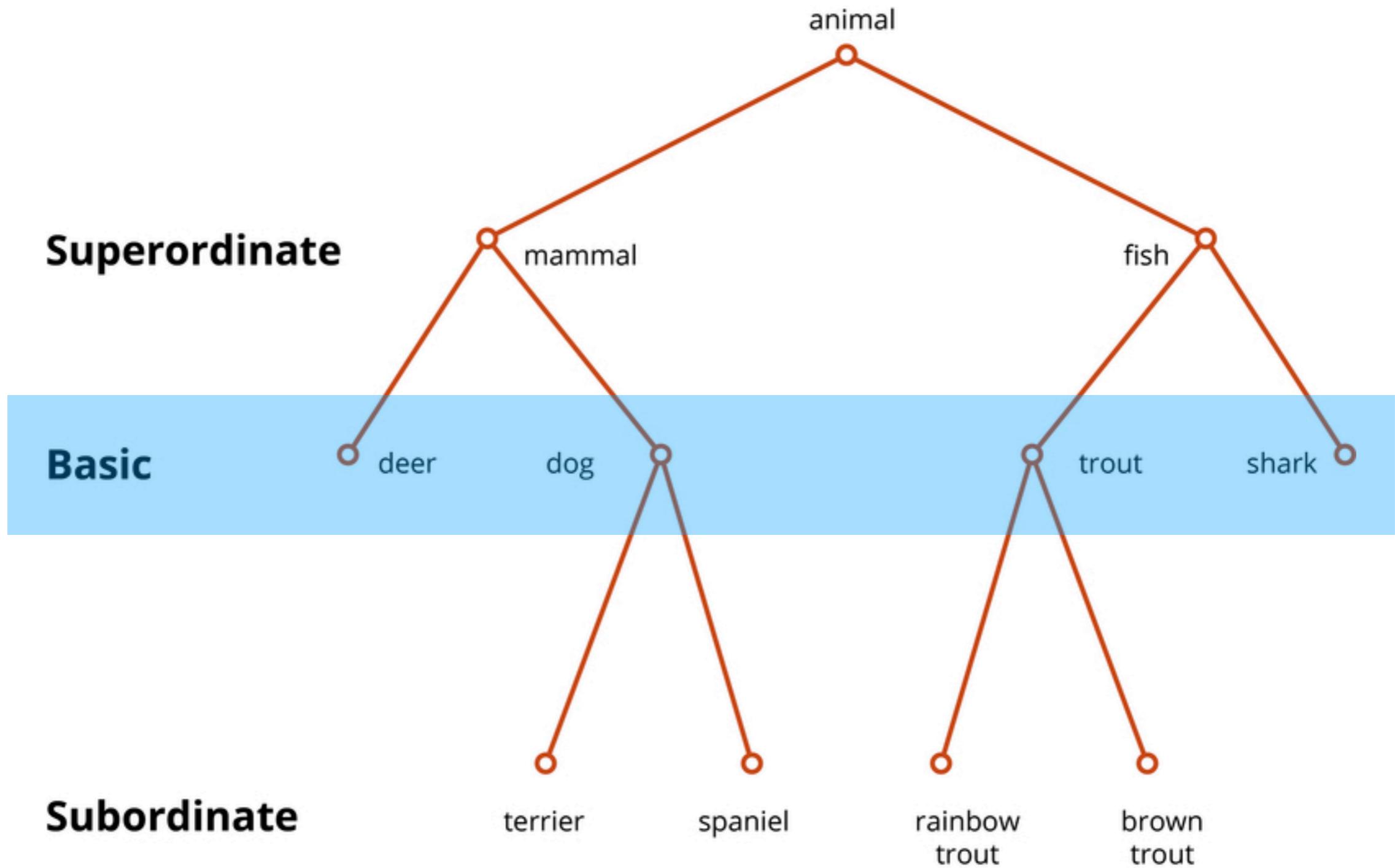
# Nuances of the theory



# Categories at different levels



# The supremacy of basic categories



# Cognitive economy predicts basic-category supremacy?

- Rosch (1978): “Not all possible levels of categorization are equally good or useful; rather, the most basic level of categorization will be the most inclusive (abstract) level at which the categories can mirror the structure of attributes perceived in the world.”

# Cognitive economy predicts basic-category supremacy?

- Rosch (1978): “Not all possible levels of categorization are equally good or useful; rather, the most basic level of categorization will be the most inclusive (abstract) level at which the categories can mirror the structure of attributes perceived in the world.”
- Some evidence for basic-category supremacy:
  - Cue validity:  $p(\text{category} \mid \text{cue/feature})$ , e.g. bw-category shapes are more distinctive for basic than for superordinate —> informative
  - Development: Parents tend to say “dog” but not “Labrador”, and children tend to pick up basic terms the earliest —> low effort
  - Language: Basic categories tend to be coded more in languages, and might possibly have emerged the earliest

# Question of the day

- How can features of a category be learned?

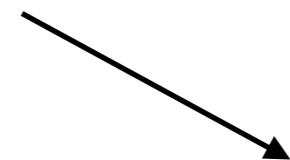
# Readings

## Required reading:

- Eleanor, R. (1978) Principles of Categorization, in Rosch, E. and Lloyd, B. B. (eds), *Cognition and Categorization*. Lawrence Erlbaum, 27–48.

## Technical reference:

- Chapter 7 in *Stats*.



**Pearson (linear) correlation  
Lab 4**

# Optional readings

*Optional readings:*

- Rosch, E. (1973). Natural categories. *Cognitive Psychology*, 4(3), 328–350.
- Rosch, E. (1975). Cognitive representations of semantic categories. *Journal of experimental psychology: General*, 104(3), 192–233.
- Tversky, A. (1977). Features of similarity. *Psychological Review*, 84(4), 327–352.
- Medin, D. L., Goldstone, R. L., and Gentner, D. (1993). Respects for similarity. *Psychological Review*, 100(2), 254–278.

*Recommended book:*

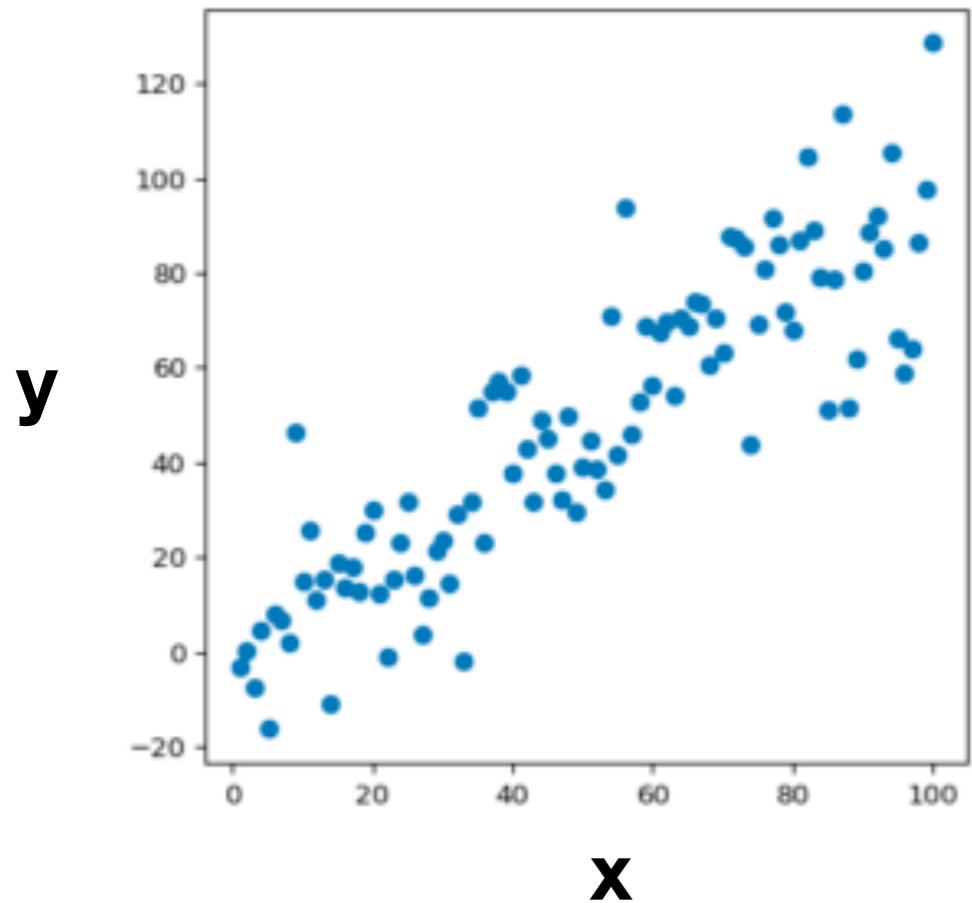
- Murphy, G. (2004). *The big book of concepts*. MIT Press.

# Summary

- Categories are groupings of entities based on similarity, which may be explained in part by the features that entities share
- Some categories show graded membership, with certain entities judged to be more prototypical than others
- Categories are designed to support communicative informativity at low cognitive effort, e.g., basic categories
- To be continued: How are categories formed?

# **5-minute break**

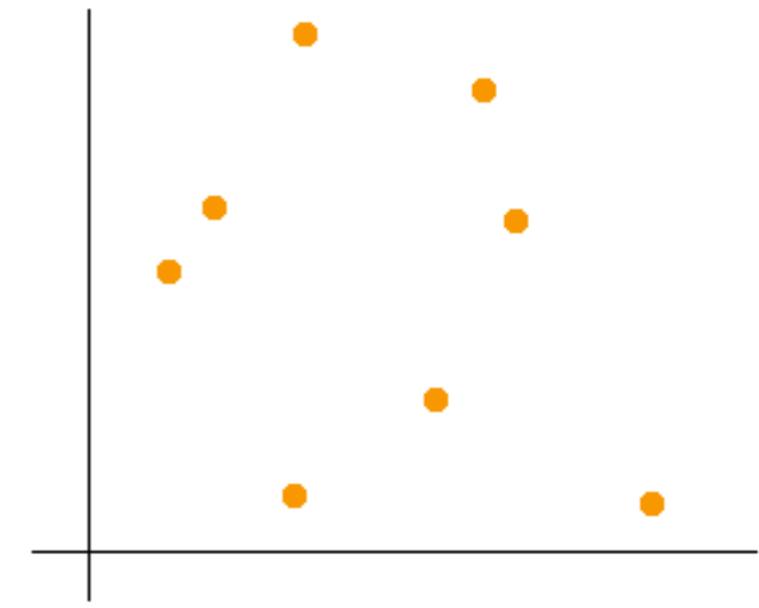
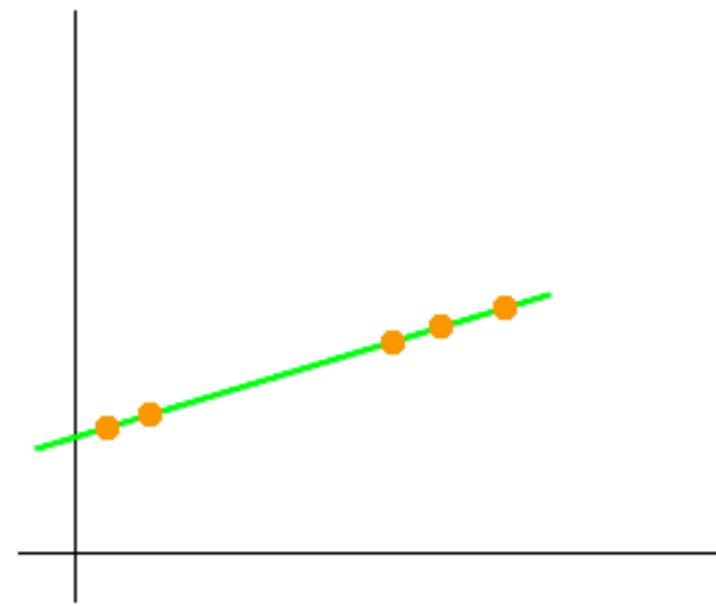
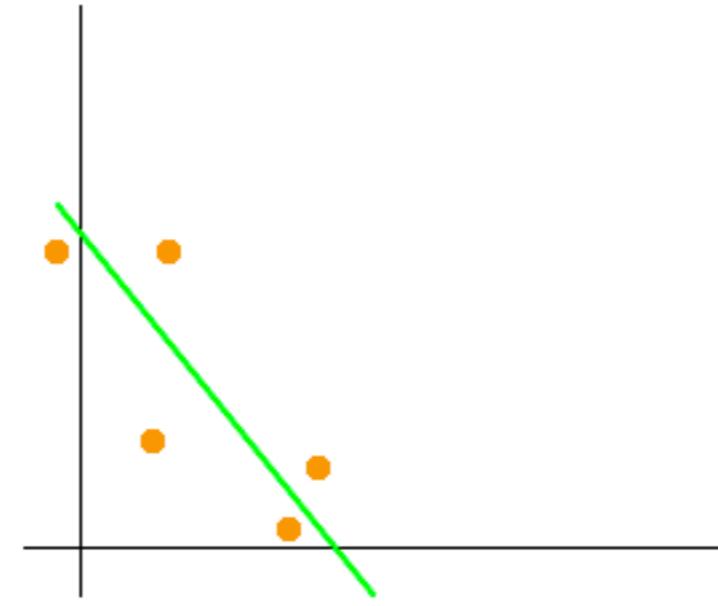
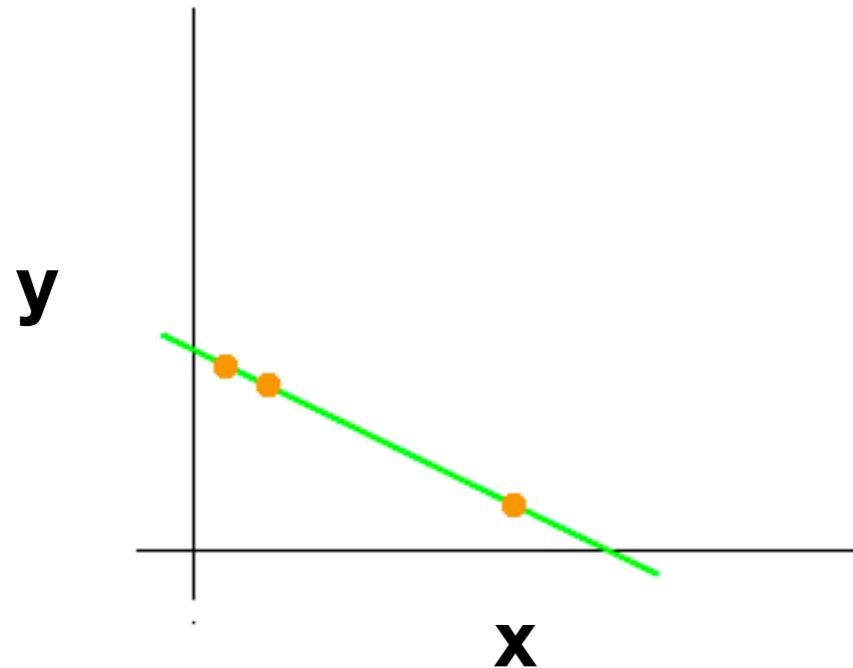
# Pearson correlation



- Pearson correlation:
  - Linear relationship between x,y
  - Coefficient in range  $[-1, +1]$
  - $0 \rightarrow$  Random
  - $-1 \rightarrow$  Perfect negative corr.
  - $+1 \rightarrow$  Perfect positive corr.

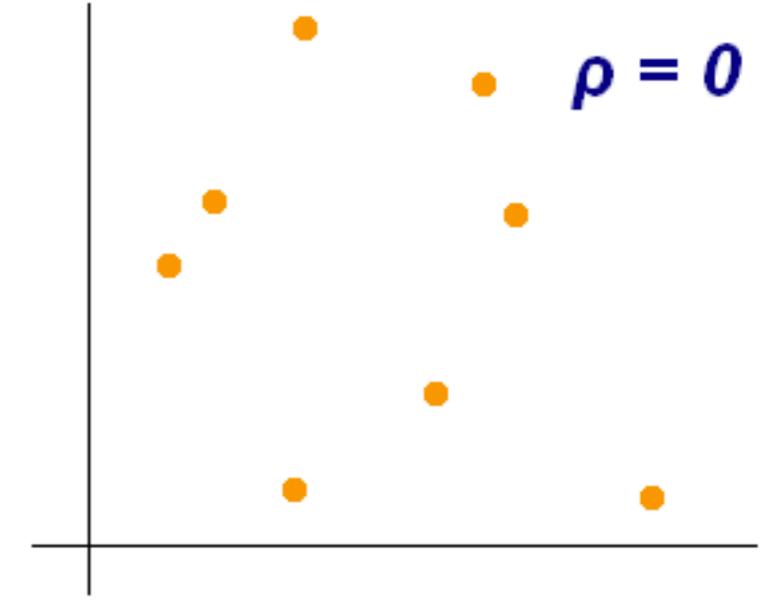
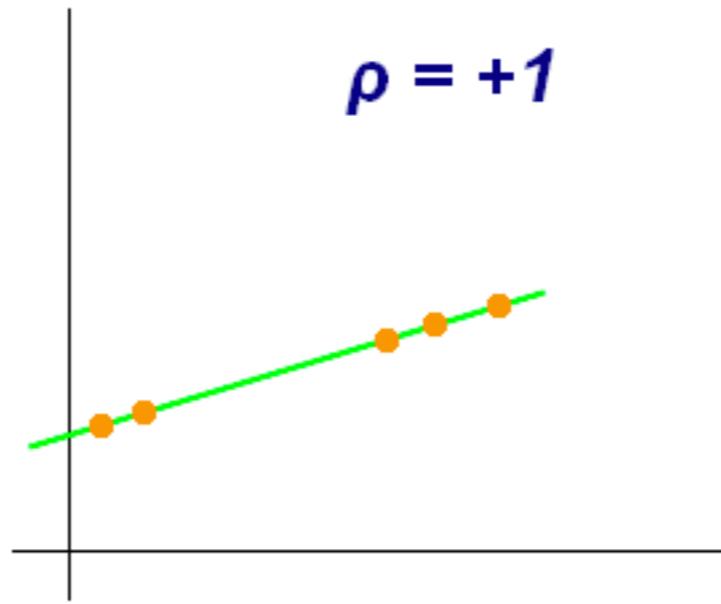
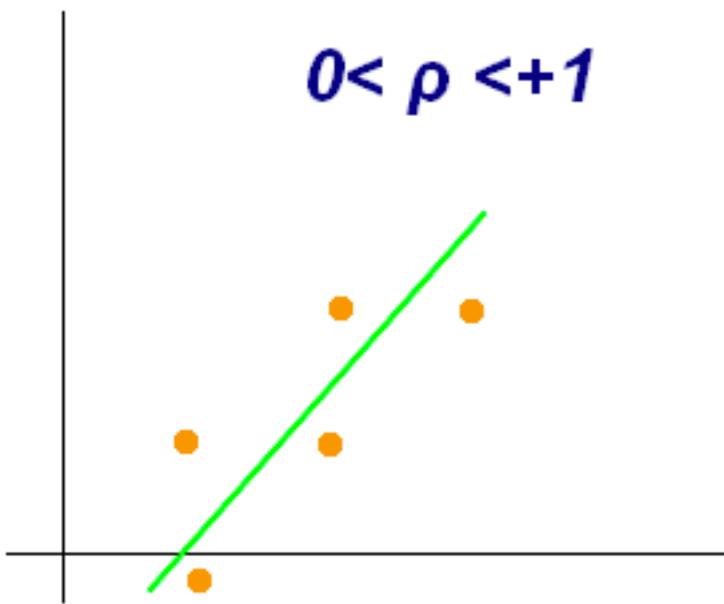
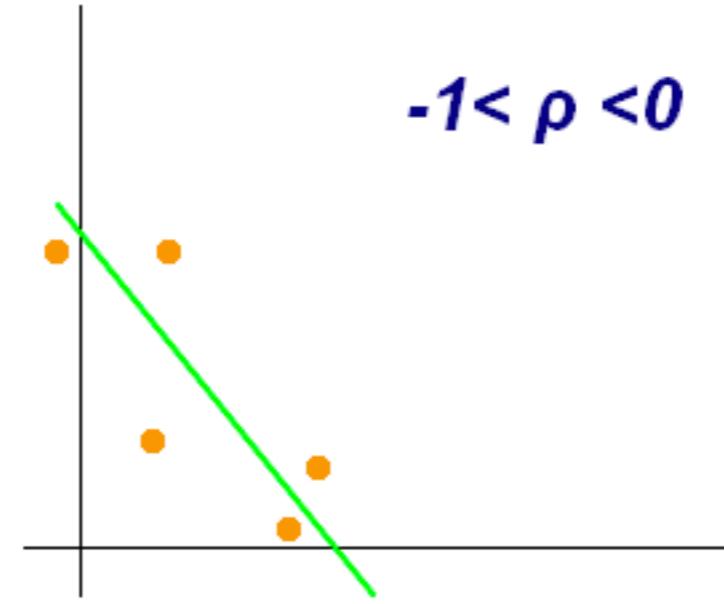
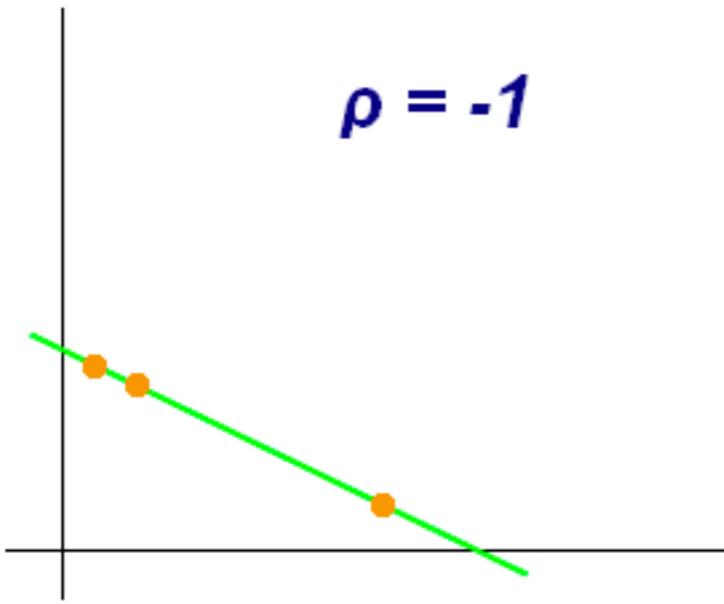
# Some examples

- What range of correlation values would you expect of these?



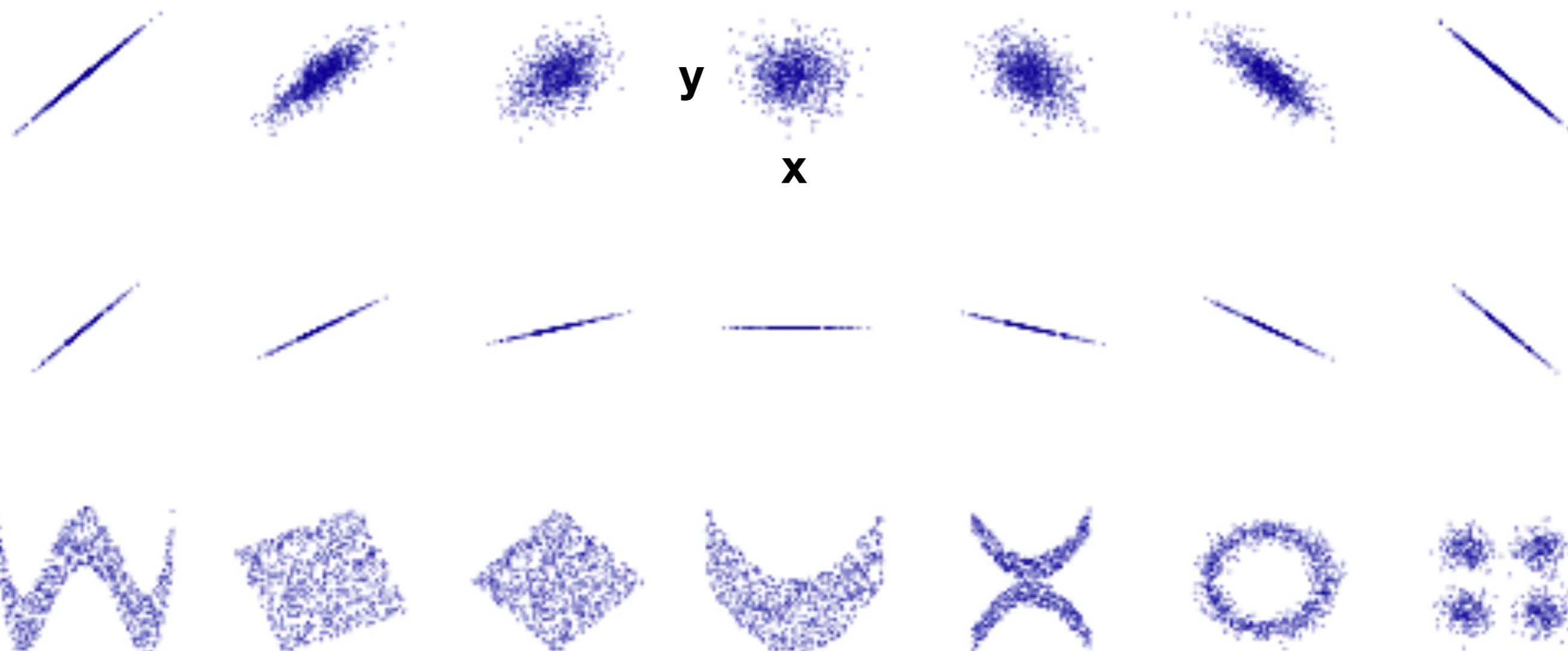
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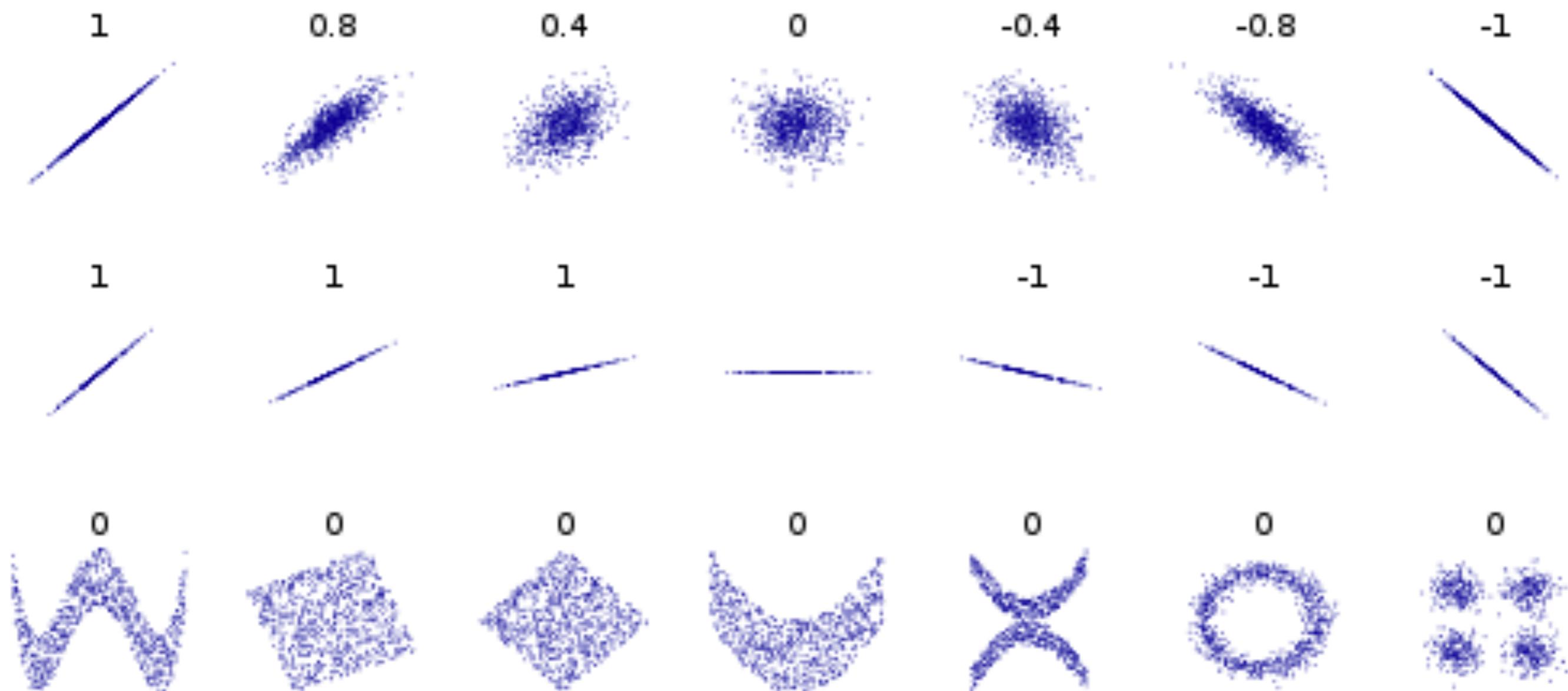
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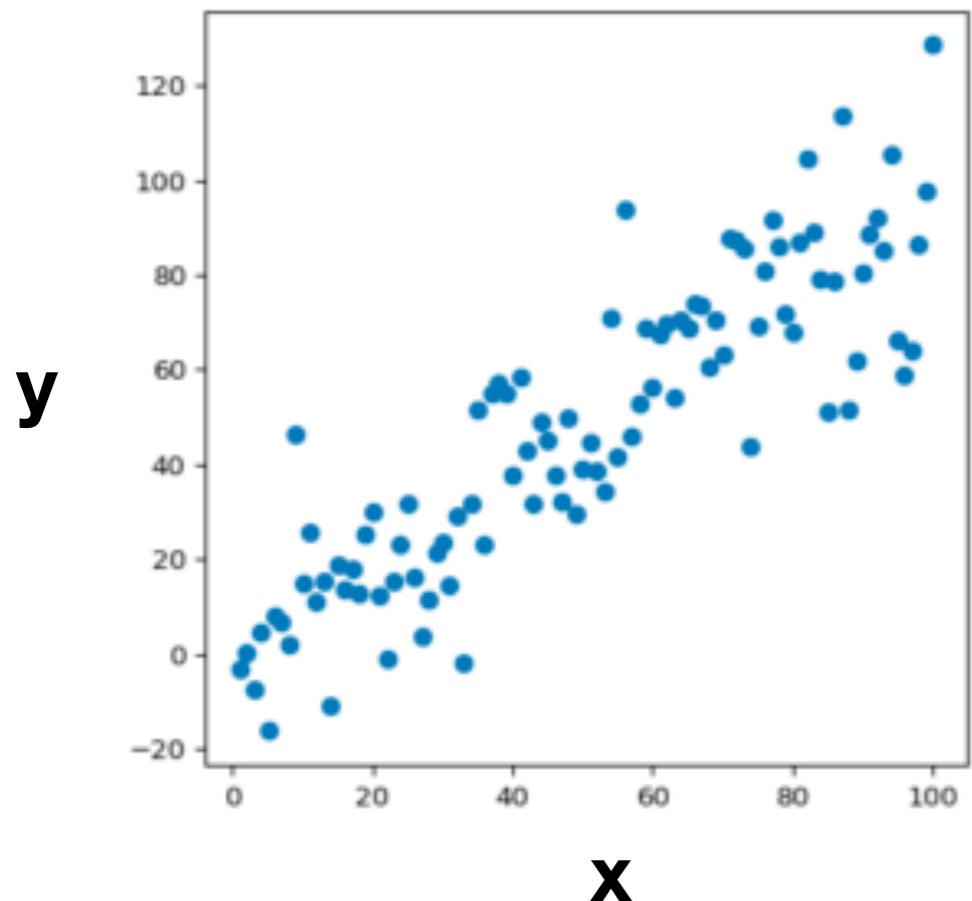
# More examples

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Both examples adapted from Wikipedia

# Formal definition

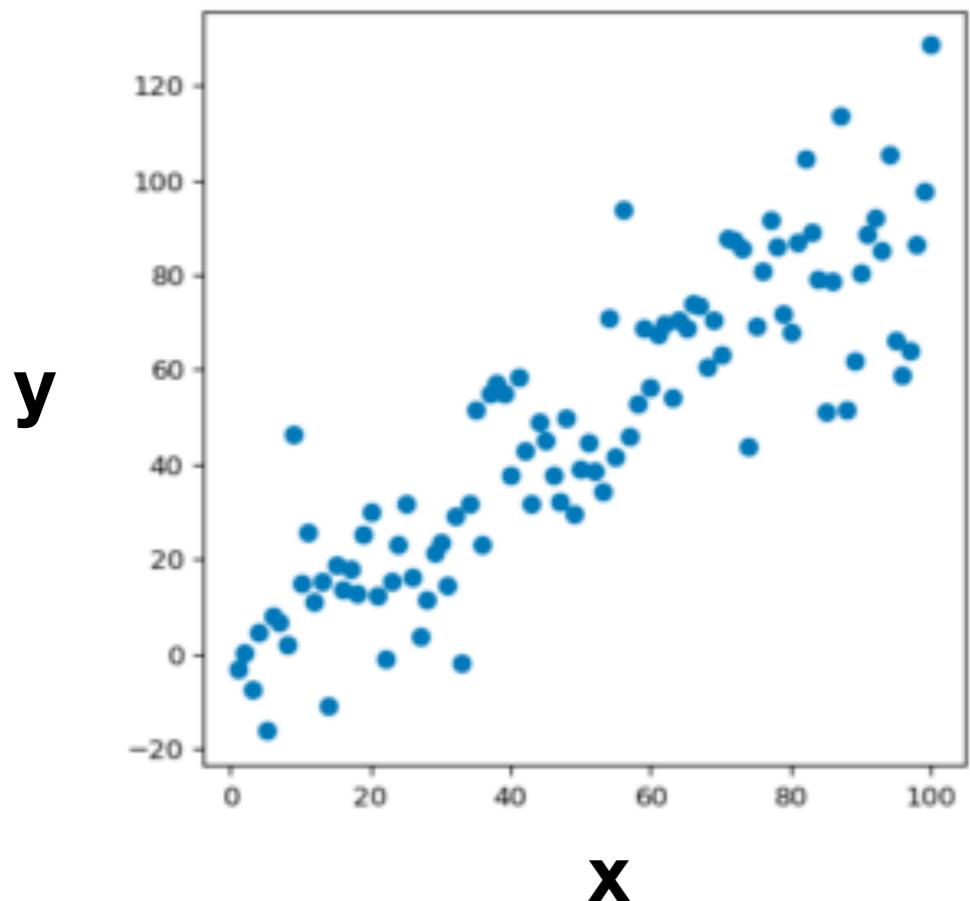


$$\rho_{X,Y} = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y}$$

where:

- cov is the covariance
- $\sigma_X$  is the standard deviation of  $X$
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# Formal definition



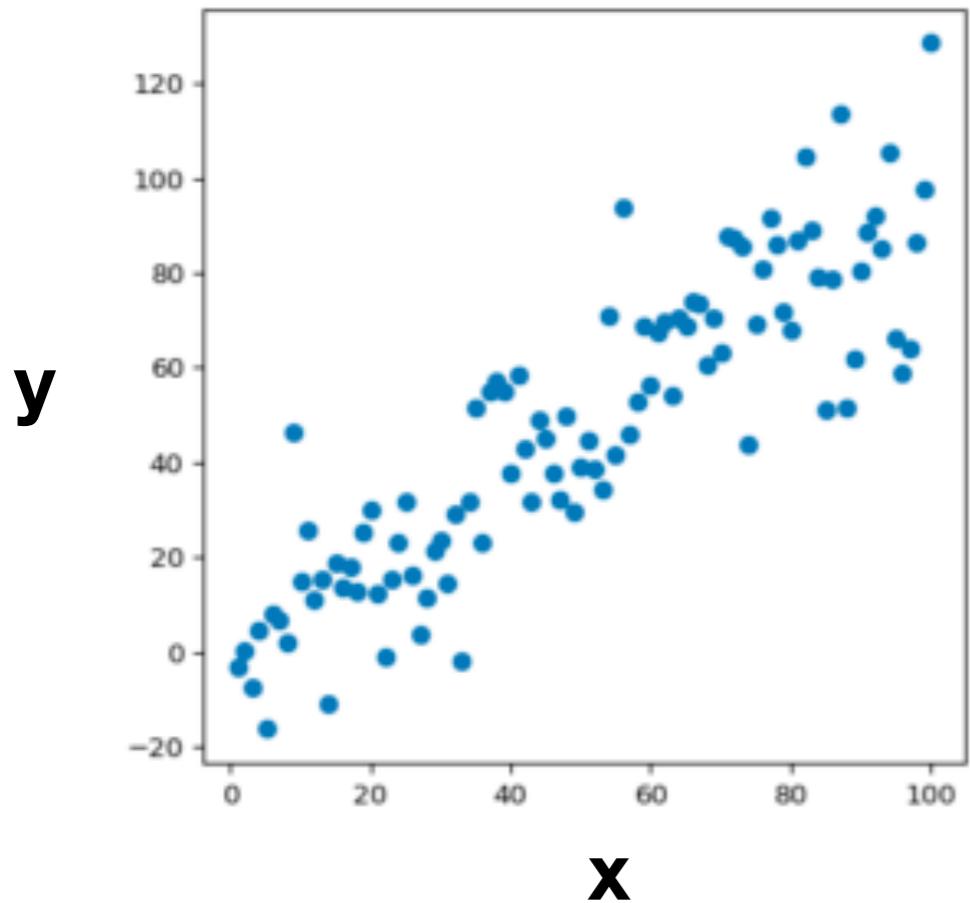
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# Formal definition



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$$\rho_{X,Y} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

# Lab 4: Prototypicality

- Notebook orientation: Data and task explanation
- Useful functions:
  - `dist = spatial.distance.euclidean(x, y)`
  - Pearson (linear) correlation: `corr=scipy.stats.pearsonr(a, b)`
  - In this lab, you'll need to implement Pearson correlation