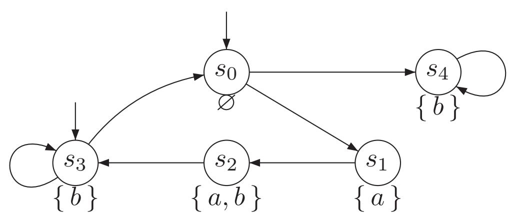
Assignment 4

Due on Friday March 1st, 2013 before 10pm

Problem 1

(6 points) Consider the transition system TS outlined below:

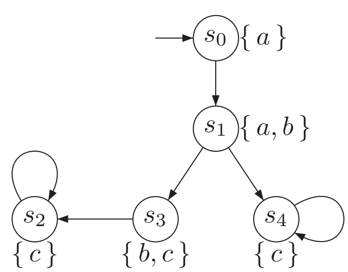


For each of the following CTL formulae determine the set $Sat(\Phi_i)$ and whether $TS \models \Phi_i$:

- $\Phi_1 = \forall(a \,\mathsf{U}\, b) \vee \exists \bigcirc (\forall \Box b)$
- $\Phi_2 = \forall \Box \forall (a \,\mathsf{U}\, b)$
- $\Phi_3 = \forall \Box \exists \Diamond \Phi_1$

Problem 2

(10 points) Consider the two CTL formulas $\Phi_1 = \exists \Diamond \forall \Box c$ and $\Phi_2 = \forall (a \,\mathsf{U}\, \forall \Diamond c)$, and the transition system TS outlined below.



Decide whether $TS \models \Phi_i$ (for $i = 1, 2$) using the CTL model checking algorithm taught in class. Sketch the main steps; by this, we mean identifying the sub-formulas and their Sat sets. You do not have to show the detail of your work for computing individual Sat sets.

Problem 3

(7 points) Let TS be a finite transition system (over AP) without terminal states (i.e. every state has an outgoing transition), and Φ and Ψ be CTL state formulae (over AP). Prove or disprove $TS \models \exists(\Phi \,\mathsf{U}\, \Psi)$ if and only if $TS' \models \exists \Diamond \Psi$ where TS' is obtained from TS by eliminating all outgoing transitions from states s such that $s \models \Psi \vee \neg \Phi$.

Problem 4

(7 points) Which one of the following equivalences is true?

- $\forall \bigcirc \forall \Diamond \Phi \equiv \forall \Diamond \forall \bigcirc \Phi$
- $\exists \bigcirc \exists \Diamond \Phi \equiv \exists \Diamond \exists \bigcirc \Phi$
- $\forall \bigcirc \forall \Box \Phi \equiv \forall \Box \forall \bigcirc \Phi$
- $\exists \bigcirc \exists \Box \Phi \equiv \exists \Box \exists \bigcirc \Phi$

Provide counterexamples for the false ones. Prove one of the true ones correct (fully), and justify the rest using a high level argument (to show your way of coming up to the conclusion).

Bonus Problem

(10 points) Prove that there does not exist an equivalent LTL formula for the CTL formula $\Phi = \forall \Diamond \exists \bigcirc \forall \Diamond \neg a$. Remember that a CTL formula Φ and an LTL formula ϕ are equivalent iff for all transition systems TS, we have

$$TS \models \Phi \iff TS \models \phi$$

Hint: argument by contraposition.