CSC410 Assignment 4

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1 Question One

1.1 (a) Show that Live Variable Analysis is a bit vector framework.

We know that live variable analysis is defined by the semi-lattice $L = (\mathrm{P(var)}, \cup)$
where P(var) is the all possible subsets of live variables. Thus the property function of live variable analysis agrees with bit vector framework.
Also note that for live analysis, the transfer function for a particular statement st is:
$f_{st}^{LV}(S) = (S \setminus writes(st)) \cup reads(st) = (S \cap writes(st)^c) \cup reads(st)$
Let K = $writes(st)^c$ and G = $reads(st)$ Then live variable analysis is a bit vector framework.

1.2 b) Show that all bit vector frameworks are distributive frameworks.

To prove that analyses are Monotonic Framework we just have to confirm that F has necessary properties:
The function of F are monotonic: Assume that $l \sqsubseteq l'$. Then $(l \cap K) \sqsubseteq (l' \cap K)$ and, therefore $((l \cap K) \cup G) \sqsubseteq ((l' \cap K) \cup G)$ and thus $f(l) \sqsubseteq f(l')$ as required.
let $G_0, K_0 = \emptyset$, since $f = (S \cap G_0) \cup K_0$ is the identity.
The function of F are closed under composition: Suppose $f(l) = (l \cap K) \cup G$ and $f'(l) = (l \cap K') \cup G'$. Then we calculate:
$(f \circ f')(l) = (((l \cap K') \cup G') \cap K) \cup G = (l \cap (K' \cup K)) \cup ((G' \cap K) \cup G)$
So $(f \circ f')(l) = (l \cap K'') \cup G''$ where $K'' = K' \cup K$ and $G'' = (G' \cap K) \cup G)$. The monotonic is proved.

For f in F
f$(l \sqcup l') = f(l \sqcup l') = ((l \sqcup l') \cap K) \cup G = ((l \cap K) \sqcup (l' \cap K)) \cup G = ((l \cap K) \cup G) \sqcup ((l' \cap K) \cup G) = f(l) \sqcup f(l')$
Therefore all bit vector frameworks are distributive frameworks.

1.3 (c) Provide an example of a simple distributive framework that is not a bit vector framework

At each point, determine all the expressions were computed by one control flow path. (the analysis does not analyze whether the operands have been overwritten since or not)

2 Question Two

2.1 (a) With a minor change to the formal definition of available expressions analysis, get a definition for partially available expressions

$$PAE_{entry}(l) = \begin{cases} \emptyset & \text{if } l = init(S_*) \\ \cup\{PAE_{exit}(l') | (l', l) \in flow(S_*)\} & \text{otherwise} \end{cases}$$
$PAE_{exit}(l) = (PAE_{entry}(l) \setminus kill_{PAE}(B^l)) \cup gen_{PAE}(B^l)$ where $B^l \in blocks(S_*)$

$kill_{PAE}([x := a]^l) = \{a' \in AExp_* | x \in FV(a')\}$
$kill_{PAE}([skip]^l) = \emptyset$
$kill_{PAE}([b]^l) = \emptyset$

$gen_{PAE}([x := a]^l) = \{a' \in AExp(a) | x \notin FV(a')\}$
$gen_{PAE}([skip]^l) = \emptyset$
$gen_{PAE}([b]^l) = AExp(b)$

2.2 (b) Anticipability

$$VB_{exit}(l) = \begin{cases} \emptyset & \text{if } l \in final(S_*) \\ \cap\{VB_{entry}(l') | (l', l) \in flow^R(S_*)\} & \text{otherwise} \end{cases}$$
$VB_{entry}(l) = (VB_{exit}(l) \setminus kill_{VB}(B^l)) \cup gen_{VB}(B^l)$ where $B^l \in blocks(S_*)$

$kill_{VB}([x := a]^l) = \{a' \in AExp_* | x \in FV(a')\}$
$kill_{VB}([skip]^l) = \emptyset$
$kill_{VB}([b]^l) = \emptyset$

$gen_{VB}([x := a]^l) = AEexp(a)$
$gen_{VB}([skip]^l) = \emptyset$
$gen_{VB}([b]^l) = AExp(b)$

2.3 (c) Placement Possible Analysis

Property Space:
$L = (P(AExp_*), \cup)$

Transfer Function:
$$PP_{exit}(l) = \begin{cases} \emptyset & \text{if } l \in final(S_*) \\ \cup\{PP_{entry}(l') | (l', l) \in flow^R(S_*)\} & \text{otherwise} \end{cases}$$
$PP_{entry}(l) = (PP_{exit}(l) \setminus kill_{PP}(B^l)) \cup gen_{PP}(B^l)$ where $B^l \in blocks(S_*)$

$kill_{PP}([x := a]^l) = \{a' \in AExp_* | x \in VB(a') \cap PAE(a')\}$
$kill_{PP}([skip]^l) = \emptyset$
$kill_{PP}([b]^l) = \emptyset$

$gen_{PP}([x := a]^l) = \{a' \in AExp | x \in VB(a') \cap PAE(a')\}$
$gen_{PP}([skip]^l) = \emptyset$
$gen_{PP}([b]^l) = AExp(b)$

Initialization Function:
for each block B initialize $PP_{exit}(B) = \cup$ where $\cup$ is the whole universe.

2.4 (d) Provide two equations defining the two key sets insert and delete

According to the transfer function in part(c):

$insert_l = \cup gen_{PP}(B^l)$ where $(l', l) \in flow^R(S_*)$

$delete_l = \cup kill_{PP}(B^l)$ where $(l', l) \in flow^R(S_*)$