# STA255 Tut 6 Week 8

18. $\mu = 11.5$

$\sigma = 4.0$

a. By the Central Limit Theorem,

$\bar{X} \xrightarrow{D} N(\mu, \frac{\sigma^2}{n}) \sim N(11.5, \frac{4.0^2}{50})$

$\hookrightarrow E[\bar{X}] = E[\frac{X_1 + X_2 + X_3 + \cdots + X_n}{n}]$ where $X_i \overset{iid}{\sim} (\mu, \sigma^2)$

$$= \frac{E[\sum_{i=1}^{n} X_i]}{n}$$

$$= \frac{\sum_{i=1}^{n} E[X_i]}{n}$$

$$= \frac{\sum_{i=1}^{n} \mu}{n}$$

$$= \frac{n\mu}{n}$$

$$= \mu$$

$V(\bar{X}) = Var(\frac{X_1 + X_2 + \cdots + X_n}{n})$ where $X_i \overset{iid}{\sim} (\mu, \sigma^2)$

$$= \frac{1}{n^2} Var(\sum_{i=1}^{n} X_i)$$

$$= \frac{1}{n^2} \sum_{i=1}^{n} Var(X_i)$$

$$= \frac{1}{n^2}(n\sigma^2)$$

$$= \frac{\sigma^2}{n}$$

$$P(\bar{X} \geq 12) = P(\frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} \geq \frac{12 - \mu_{\bar{X}}}{\sigma_{\bar{X}}})$$

$$= P(Z \geq \frac{12 - 11.5}{\sqrt{\frac{4.0^2}{50}}})$$

$$= P(Z \geq 0.88388)$$

$$= 0.1884$$

b. Let T be the total amount of gas purchased

Then $T = n\bar{X} = 50\bar{X}$

$E[T] = E[50\bar{X}] = 50E[\bar{X}] = 50 \times 11.5 = 575$

$V(T) = V(50\bar{X}) = 50^2 V(\bar{X}) = 50^2 \frac{4.0^2}{50} = 50(4.0)^2$

$\Rightarrow T \sim N(575, 50(4.0)^2)$

We want to find

$$P(T \leq 600) = P\left(\frac{T-575}{\sqrt{50(4.0)^2}} \leq \frac{600-575}{\sqrt{50(4.0)^2}}\right)$$
$$= P(Z \leq 0.88388)$$
$$= 0.8116$$

C. We want to find $q_{0.95}$ such that

$$P(T \leq q_{0.95}) = 0.95$$
$$\Rightarrow P\left(Z \leq \frac{q_{0.95} - 575}{\sqrt{50(4.0)^2}}\right) = 0.95$$
$$\Rightarrow \frac{q_{0.95} - 575}{\sqrt{50(4.0)^2}} = 1.645 =$$

#1: qnorm(0.95, 0, 1)
#2: find inverse from normal table

$$\Rightarrow q_{0.95} = 621.5$$