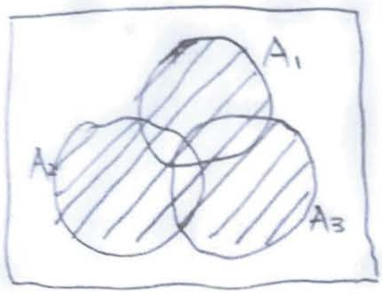
10.

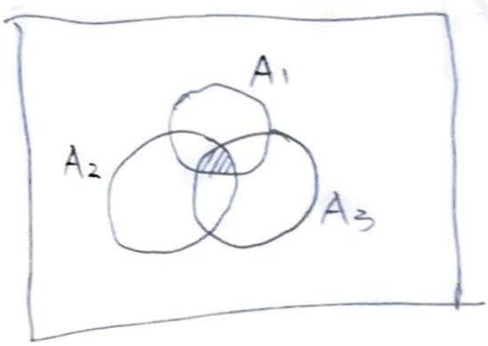
A construction firm is currently working on 3 different buildings. Let A_i denote the event that the i^{th} building is completed by the contract date. Draw a Venn diagram for each of the following scenarios.

a. At least one building is completed by the contract date



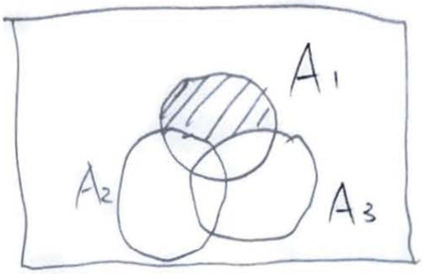
$A_1 \cup A_2 \cup A_3$

b. All buildings are completed by the contract date



$A_1 \cap A_2 \cap A_3$

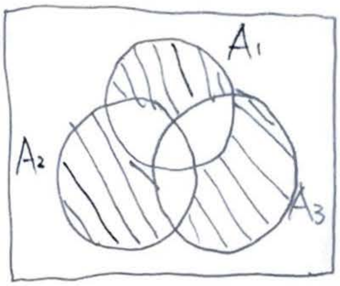
c. Only the first building is completed by the contract date



$$A_1 \cap (A_2 \cup A_3)'$$

$\hookrightarrow$ A_1 AND (neither A_2 or A_3)

d. Exactly one buiding is completed by the contract date



" $(A_1 \cup A_2 \cup A_3)$ but not any of the intersections "

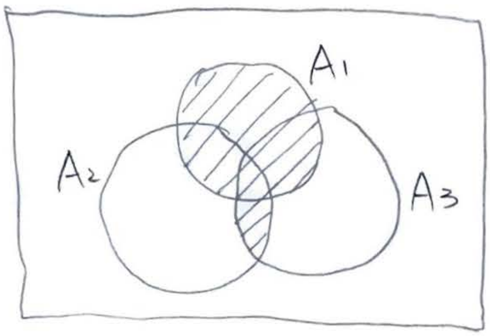
$$\Rightarrow P(\text{exactly one}) = P(A_1 \cup A_2 \cup A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + 2P(A_1 \cap A_2 \cap A_3)$$

$=$

e. Either the first building or both of the other two buildings are completed by the contract date

$\hookrightarrow$ "A_1 <u>OR</u> $(A_2 \cap A_3)$"

$\Rightarrow \quad A_1 \cup (A_2 \cap A_3)$



Recall: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(\text{required}) = P(A_1) + P(A_2 \cap A_3) - P(A_1 \cap A_2 \cap A_3)$

50.

A department store sells sport shirts in three sizes (small, medium, and large), three patterns (plaid, print, and stripe), and two sleeve lengths (long and short). The accompanying tables give the proportions of shirts sold in the various category combinations.

Short Sleeved

	Pattern		
Size	Pl	Pr	St
S	0.04	0.02	0.05
M	0.08	0.07	0.12
L	0.03	0.07	0.08

$\Sigma = 0.56$

Long Sleeved

	Pattern		
Size	Pl	Pr	St
S	0.03	0.02	0.03
M	0.10	0.05	0.07
L	0.04	0.02	0.08

$\Sigma = 0.44$

a. Prob that the next shirt sold is a medium, long-sleeved, print shirt?

b. Prob that the next shirt sold is a medium, print shirt?

d. Prob that the size of the next shirt sold is medium?

f. Given that the shirt just sold was a medium, plaid, what is the prob that it was short-sleeved?

Note: these categories are mutually exclusive

a. Read off table:

$P(L, M, Pr) = 0.05$

b.

Sum up the cells that satisfy these criteria

$P(\cdot, M, Pr) = 0.07 + 0.05 = 0.12$

ties in to marginal prob

$P(S, M, Pr)$ $P(L, M, Pr)$ $\Rightarrow P(A \cup B) = P(A) + P(B)$ if A&B mutually exclusive

d.

Sum up the rows that satisfy these criteria

$$P(\cdot, M, \cdot) = P(S, M, \cdot) + P(L, M, \cdot)$$

$$= (0.08 + 0.07 + 0.12) + (0.10 + 0.05 + 0.07)$$

$$= 0.49$$

f.

Translate problem into mathematical language

$$P(S, M, Pl \mid \cdot, M, Pl)$$

$$= \frac{P(S, M, Pl \cap \cdot, M, Pl)}{P(\cdot, M, Pl)}$$

$$= \frac{P(S, M, Pl)}{P(\cdot, M, Pl)}$$

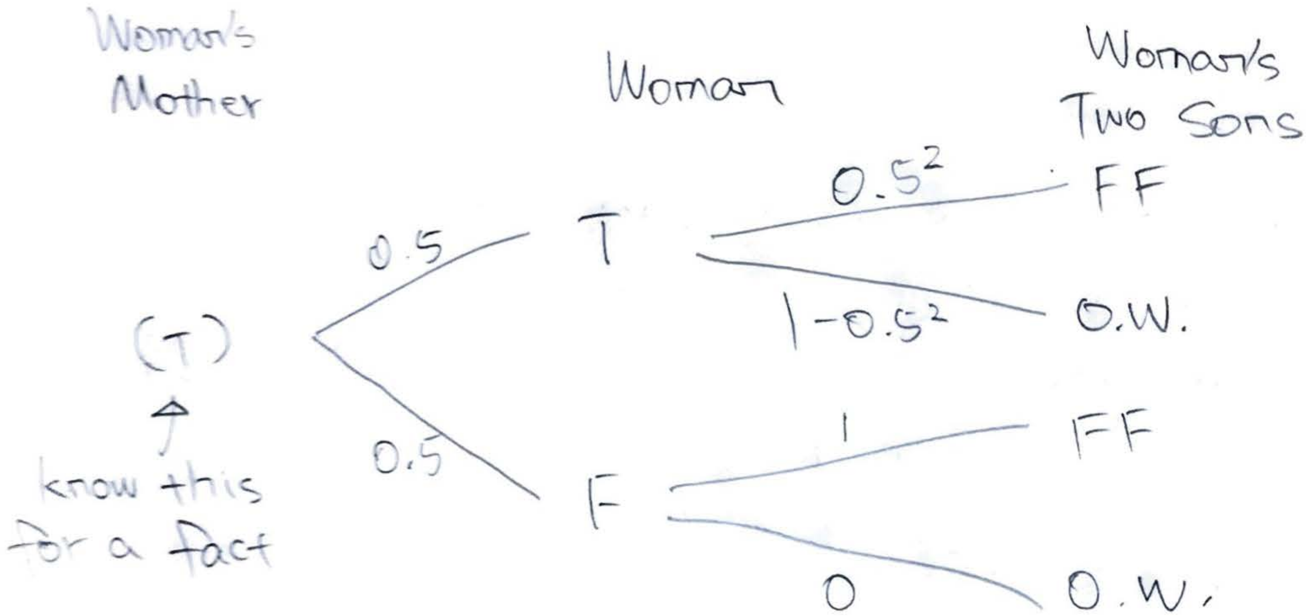
$$= \frac{0.08}{0.08 + 0.1} = \frac{4}{9} \text{ or } 0.\overline{4}$$

106.

Consider a woman whose brother is afflicted w/ hemophilia, which implies that the woman's mother has the hemophilia gene on one of her two X chromosomes. Thus there is a 50% chance that the woman's mother has passed on the bad gene to her. The woman has two sons, each of whom will independently inherit the gene from one of her two chromosomes. If the woman herself has a bad gene, there is a 50% chance that she will pass this on to a son.

Suppose that neither of her two sons is afflicted w/ hemophilia, what then is the probability that the woman is indeed the carrier of the hemophilia gene?

Translate the problem into mathematical language



$$P(\text{woman is carrier} \mid \text{neither sons inflicted})$$

$$= P(T \mid FF)$$

$$= \frac{P(T \cap FF)}{P(FF)}$$

Bayes Rule

$$= \frac{P(FF \mid T)\cdot P(T)}{P(FF \mid T)\cdot P(T) + P(FF \mid F)\cdot P(F)}$$

← denom: Law of Total Prob

$$= \frac{(0.5^2)(0.5)}{(0.5^2)(0.5) + (1)(0.5)}$$

$$= 0.2$$