10.

b. $\int_{\theta}^{\infty} \frac{k\theta^k}{x^{k+1}}\,dx$ (=1 is the goal)

$= \frac{k\theta^k}{-k} x^{-k} \Big|_{\theta}^{\infty}$

$= \frac{k\theta^k}{-k} [0 - \theta^{-k}]$

$= 1$

c.

$P(X \leq b) = F(b)$

$= \int_{\theta}^{b} \frac{k\theta^k}{x^{k+1}}\,dx$

$= \frac{k\theta^k}{-k} [x^{-k}]_{\theta}^{b}$

$= -\theta^k [b^{-k} - \theta^{-k}]$

$= 1 - \left(\frac{\theta}{b}\right)^k$

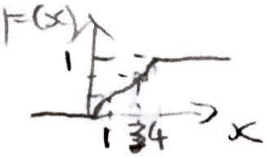
d. $P(a \leq X \leq b) = F(b) - F(a)$

$= \int_{a}^{b} \frac{k\theta^k}{x^{k+1}}\,dx$

$= -\theta^k [b^{-k} - a^{-k}]$

$= \left(\frac{\theta}{a}\right)^k - \left(\frac{\theta}{b}\right)^k$

16.



$F(x) = \frac{x}{4}\left[1 + \log\left(\frac{4}{x}\right)\right], \quad 0 \le x \le 4$

a. $P(X \le 1) = F(1)$

$= \frac{1}{4}\left[1 + \log\left(\frac{4}{1}\right)\right] = 0.597$

b. $P(1 \le X \le 3) = F(3) - F(1)$

$= \frac{3}{4}\left[1 + \log\left(\frac{4}{3}\right)\right] - \frac{1}{4}\left[1 + \log\left(\frac{4}{1}\right)\right]$

$= 0.369$

c. $f(x) = \frac{d}{dx} F(x)$

$= \frac{d}{dx}\left(\frac{x}{4} + \frac{x\log(4) - x\log(x)}{4}\right)$

$= \frac{1}{4} + \frac{\log(4)}{4} - \frac{x}{4}\frac{1}{x} - \frac{1}{4}\log(x)$

$= \frac{\log(4) - \log(x)}{4}$

$= \frac{\log\left(\frac{4}{x}\right)}{4}$

56.

$X \equiv$ weight of parcels

$X \sim N(\mu = 12, \sigma^2 = 3.5^2)$

We want c such that $P(X \leq c - 1) = 99\%$

$$\Rightarrow P\Big(\underbrace{\frac{X-12}{3.5}}_{\sim N(0,1)} \leq \frac{c-1-12}{3.5}\Big) = 99\%$$

$$\Rightarrow \Phi\Big(\frac{c-13}{3.5}\Big) = 99\%$$

$$\Rightarrow c = 3.5\,\underbrace{\Phi^{-1}(0.99)}_{\text{qnorm}(0.99) = 2.326348} + 13$$

$$= 21.142218$$