STA 255 Tut 2

Suppose that an insurance claim occurs according to a Poisson process with rate 1 every 5 months. A month is designated "high activity" if at least 2 claims occur in it.

a) What is the prob of a "high activity" month?

b) Find the prob that in 100 months, at least 3 of the months are "high activity".

c) Find the prob that it takes 100 months to observe 3 "high activity" months

d) Given that it takes 100 months to observe 3 "high activity" months, find the prob that there were no "high activity" months in the first 50 of the 100 months.

Sol'n:

a) Let X be # claims occurs in 1 month

$$X \sim Poi\left(\frac{1}{5}\right)$$

We want $P(X \geq 2)$

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - P(X=0) - P(X=1)$$

$$= 1 - \frac{e^{-\frac{1}{5}}\left(\frac{1}{5}\right)^0}{0!} - \frac{e^{-\frac{1}{5}}\left(\frac{1}{5}\right)^1}{1!}$$

$$= 0.017523$$

b) Let Y be # months deemed as "high activity" in 100 months.

$$Y \sim Bin(100, 0.017523)$$

We want $P(Y \geq 3)$

$$P(Y \geq 3) = 1 - P(Y=0) - P(Y=1) - P(Y=2)$$

$$= 1 - \binom{100}{0}(0.017523)^0(1-0.017523)^{100}$$

$$- \binom{100}{1}(0.017523)^1(1-0.017523)^{99}$$

$$- \binom{100}{2}(0.017523)^2(1-0.017523)^{98}$$

$$= 0.24208$$

c) Let Z be # non "high-activity" months it takes to observe 3 "high activity" months

$$Z \sim NB(r=3,\ p=0.017523)$$

We want $P(Z=97)$ ← 97 failures

$$P(Z=97) = \binom{97+3-1}{3-1}(1-0.017523)^{97}(0.017523)^{3}$$

$$= 0.004517$$

d)

$$P(0 \text{ "h-a" in first } 50 \mid 100 \text{ months to obs } 3 \text{ "h-a"})$$

$$= \frac{P(0 \text{ "h-a" in first } 50 \ \& \ 100 \text{ mths to obs } 3 \text{ "h-a"})}{P(100 \text{ mths to obs } 3 \text{ "h-a"})}$$

$$= \frac{P(0 \text{ "h-a" in } 1^{st}\ 50 \ \& \text{ takes another } 47 \text{ non-"h-a" mths to obs } 3 \text{ "h-a"})}{0.004517}$$

$$= \frac{\binom{50}{0}(0.017523)^{0}(1-0.017523)^{50} \times \binom{47+3-1}{3-1}(1-0.017523)^{47}(0.017523)^{3}}{0.004517}$$

$$= 0.219015$$