Formula Sheet

- Demorgans Laws: $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; $\quad P(A^c) = 1 - P(A)$; $\quad P(A|B) = \frac{P(A \cap B)}{P(B)}$
- For a series of k disjoint events $A_1, A_2, ..., A_k$ and an event B
  - Law of total probability, $P(B) = P(B|A_1) * P(A_1) + P(B|A_2) * P(A_2) + ... + P(B|A_k) * P(A_k)$
  - Bayes rule, $P(A_j|B) = \frac{P(B|A_j)*P(A_j)}{P(B|A_1)*P(A_1)+P(B|A_2)*P(A_2)+...+P(B|A_k)*P(A_k)}$
- A and B are independent if $P(A|B) = P(A)$ *or* $P(B|A) = P(B)$ *or* $P(A \cap B) = P(A) * P(B)$
- $Bern(p)$: $p_X(x) = p^x(1-p)^{(1-x)}$ where, x =0, 1
- $Bin(n,p)$: $p_Y(x) = \begin{pmatrix} n \\ x \end{pmatrix} p^x(1-p)^{n-x}$ where, $x = 0, 1, 2, ..., n$
- $Geom(p)$: $p_X(x) = (1-p)^x p$ where $x = 0, 1, 2, ...$
- $Pois(\lambda)$: $p_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ where, $x = 0, 1, 2, ...$
- $NB(r,p)$: $p_X(x) = \begin{pmatrix} x+r-1 \\ r-1 \end{pmatrix} (1-p)^x p^r$ where, $x = 0, 1, 2, ...$
- $\begin{pmatrix} n \\ x \end{pmatrix} = \frac{n!}{x!(n-x)!}$
- $Unif(\alpha, \beta)$: $f(x) = \frac{1}{\beta - \alpha}$ $\quad for \ \ \alpha \leq x \leq \beta$
- $Exp(\lambda)$: $f(x) = \lambda e^{-\lambda x}$ $\quad for \ \ x \geq 0 \, and \, \lambda > 0$ another way to write, $Exp(\beta)$: $f(x) = \frac{1}{\beta} e^{-x/\beta}$ $\quad for \ \ x \geq 0 \, and \, \beta > 0$
- $Gamma(\alpha, \lambda)$: $f(x) = \frac{1}{\Gamma(\alpha)} \lambda^\alpha x^{\alpha-1} e^{-\lambda x}$ $\quad for \ \ x \geq 0, \ \alpha > 0 \, and \, \lambda > 0$; if α is integer, $\Gamma(\alpha) = (\alpha - 1)!$
- $N(\mu, \sigma^2)$: $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$ $\quad for \ \ -\infty < x < \infty$; $\sigma^2 > 0$;
- $X \sim N(\mu, \sigma^2)$; $Z = \frac{X - \mu}{\sigma}$, Then $Z \sim N(0,1)$
- $\eta(p)$ is called the $100p^{th}$ percentile of a distribution if $F(\eta(p)) = \int_{-\infty}^{\eta(p)} f(x)dx = p$
- Expectation:
  - Discrete, $E(X) = \sum_i x_i * P(X = x_i) = \sum_i x_i * p(x_i)$; $\quad$ Continuous, $E(X) = \int_{-\infty}^{\infty} x f(x) dx$
  - Joint discrete, $E[h(X,Y)] = \sum_x \sum_y h(x,y) * P(X = x, Y = y)$; joint continuous $E[h(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x,y) f(x,y) dx dy$
  - $E(aX + bY + c) = aE(X) + bE(Y) + c$
- MGF:
  - $M_X(t) = E[e^{tx}]$; $\quad E[X^r] = M^{(r)}(0) = \frac{d^r}{dt^r} M_X(t)\big|_{t=0}$
  - $Y = aX + b \implies M_Y(t) = e^{bt} M_X(at)$
  - X and Y are indep. and $Z = X + Y \implies M_Z(t) = M_X(t) * M_Y(t)$
- Variance:
  - $V(X) = E[(X - E(X))^2] = E[X^2] - (E[X])^2$
  - X and Y are indep. $\implies V[aX + bY + c] = a^2 V[X] + b^2 V[Y]$
- Covariance: $cov(X,Y) = E[XY] - E[X]E[Y]$; $\quad$ Correlation: $\rho(X,Y) = \frac{cov(X,Y)}{\sqrt{V(X)V(Y)}}$
- Conditional distribution,
  - Discrete: $p_{Y|X}(y|x) = \frac{p(x,y)}{p_X(x)} = \frac{\text{join pmf of x and y}}{\text{marginal pmf of x}}$
  - $f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{\text{join pdf of x and y}}{\text{marginal pdf of x}}$
- CLT: as $n \to \infty$; $\bar{X} \xrightarrow{D} N(\mu, \frac{\sigma^2}{n})$
- Some probabilities from N(0,1):
  $\Phi(-1) = 0.1586553$; $\Phi(2.0) = 0.9772499$; $\Phi(2.241) = 0.9875089$
  these values given above are for illustration purposes. The quantiles that you will get in the actual test might be different.
  For example, instead of $\Phi(-1)$, you might see $\Phi(-1.9)$.