45.

$Y|X \sim \text{Unif}(0, x^2)$

a. #1: $E[Y|X=x] = \frac{x^2 - 0}{2} = \frac{x^2}{2}$ (known result)

#2: $= \int_0^{x^2} \frac{t}{x^2-0}\, dt = \frac{t^2}{2x^2}\Big|_0^{x^2} = \frac{(x^2)^2}{2x^2} - \frac{0}{2x^2} = \frac{x^2}{2}$

#1: $V(Y|X=x) = \frac{(b-a)^2}{12} = \frac{(x^2-0)^2}{12} = \frac{x^4}{12}$ (known result)

#2: $E[Y^2|X=x] = \int_0^{x^2} \frac{t^2}{x^2-0}\, dt = \frac{t^3}{3x^2}\Big|_0^{x^2} = \frac{x^4}{3}$

$\Rightarrow V(Y|X=x) = E[Y^2|X=x] - E[Y|X=x]^2$

$= \frac{x^4}{3} - \left(\frac{x^2}{2}\right)^2$

$= \frac{x^4}{3} - \frac{x^4}{4}$

$= \frac{x^4}{12}$

b.

Recall from class $f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)}$

$f(x,y) = f_{Y|X}(y|x)\, f_X(x)$

$= \left(\frac{1}{x^2-0}\right)\left(\frac{1}{1-0}\right)$

$= \frac{1}{x^2}$

C.

$$f_Y(y) = \int_{x=-\infty}^{\infty} f(x,y)\,dx$$

$$= \int_0^1 \frac{1}{x^2}\,dx$$

$$= -\frac{1}{x}\Big|_0^1$$

$$= -\frac{1}{1} + \frac{1}{0} \leftarrow \text{problem}$$

Notice that since $Y|X \sim Unif(0, x^2)$

$X \sim Unif(0, 1)$

We have the relationship

$$0 < y < x^2 < 1$$

$\Rightarrow$ So the bounds would change to $\int_{\sqrt{y}}^{1}$

$$\Rightarrow f_Y(y) = \int_{\sqrt{y}}^{1} \frac{1}{x^2}\,dx$$

$$= -\frac{1}{x}\Big|_{\sqrt{y}}^{1}$$

$$= -1 + \frac{1}{\sqrt{y}}$$

26.

a.

xy	0	25	50
prob	$P(X=0 \text{ or } Y=0)$ $= \text{sum}$ $= 0.25$	$P(X=5, Y=5)$ $= 0.15$	$P((5,10) \text{ or } (10,5))$ $= 0.2 + 0.15$ $= 0.35$

xy	$75\ (5 \times 15)$	$150\ (10 \times 15)$	$100\ (10 \times 10)$
prob	0.1	0.01	0.14

$$E[XY] = 44.25$$

$$E[X] = (0 \quad 5 \quad 10) \begin{pmatrix} \text{row 1 sum} \\ \text{row 2 sum} \\ \text{row 3 sum} \end{pmatrix}$$

$$= 5 \times 0.49 + 10 \times 0.31$$

$$= 5.55$$

$$E[Y] = (0 \quad 5 \quad 10 \quad 15) \begin{pmatrix} \text{column 1 sum} \\ \vdots \\ \text{column 4 sum} \end{pmatrix}$$

$$= 5 \times 0.36 + 10 \times 0.36 + 15 \times 0.21$$

$$= 8.55$$

$$Cov(X,Y) = E[XY] - E[X]E[Y] = 44.25 - (5.55)(8.55)$$

$$= -2.9025$$

b. $$\rho = \frac{Cov(X,Y)}{\sqrt{Var(X)\,Var(Y)}}$$

$$E[X^2] = 5^2 \times 0.49 + 10^2 \times 0.31 = 43.25$$

$$E[Y^2] = 5^2 \times 0.36 + 10^2 \times 0.36 + 15^2 \times 0.21$$

$$= 92.25$$

$$\Rightarrow \rho = \frac{-2.9025}{\sqrt{(43.25 - 5.55^2)(92.25 - 8.55^2)}}$$

$$= -0.04296542$$