Chapter 7.2

Test 7

30.

a. $L(\theta, \lambda) = \prod_{i=1}^{n} f(x_i \mid \lambda, \theta)$

$= \prod_{i=1}^{n} \lambda e^{-\lambda(x_i - \theta)}$

$= \lambda^n e^{-\lambda \sum_{i=1}^{n}(x_i - \theta)} = \lambda^n e^{-\lambda \sum_{i=1}^{n} x_i} e^{n\lambda\theta}$

$\log L(\theta, \lambda) = n \log(\lambda) - \lambda \sum_{i=1}^{n} x_i + n\lambda\theta$

Remember $x \geq \theta$, and we learned in class that the differentiation technique does not work if the random variable is bounded by the parameter

In this case, without doing differentiation, we can see that $\theta \uparrow$ $\log L(\theta, \lambda) \uparrow$ and $\theta \leq x_i \ \forall i = 1, \ldots, n$

$\Rightarrow \hat{\theta} = \min\{x_1, \ldots, x_n\}$

$$\frac{d \log L(\theta, \lambda)}{d\lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i + n\theta \overset{\text{set}}{=} 0$$

$$\Rightarrow \hat{\lambda} = \frac{n}{\left(\sum_{i=1}^{n} x_i\right) - n\hat{\theta}} \quad \text{or} \quad \frac{n}{\sum_{i=1}^{n}(x_i - \hat{\theta})}$$

b. $\hat{\theta} = \min\{x_1, \ldots, x_{10}\}$

$= \min\{3.11, \ldots, 1.30\}$

$= 0.64$

$$\hat{\lambda} = \frac{10}{\left(\sum_{i=1}^{10} x_i\right) - 10(0.64)}$$

$= 0.202$