37.

$$f(x) = \begin{cases} 0.15e^{-0.15x} & x \geq 0 \\ 0 & \text{O.W.} \end{cases}$$

Find the MGF and use it to find mean and variance

Soln:

$$M_X(t) = E[e^{tX}]$$

$$= \int_0^\infty e^{tx} 0.15 e^{-0.15x} \, dx$$

$$= 0.15 \int_0^\infty e^{(t-0.15)x} \, dx$$

$$= \frac{0.15}{t-0.15} e^{(t-0.15)x} \Big|_0^\infty, \quad t < 0.15$$

$$= \frac{-0.15}{t-0.15}$$

$$E[X] = \frac{d}{dt} M_X(t)\Big|_{t=0} = \frac{d}{dt} \frac{-0.15}{t-0.15}\Big|_{t=0} = \frac{0.15}{(t-0.15)^2}\Big|_{t=0}$$

$$= \frac{1}{0.15}$$

$$E[X^2] = \frac{d^2}{dt^2} M_X(t)\Big|_{t=0} = \frac{d}{dt} \frac{0.15}{(t-0.15)^2}\Big|_{t=0}$$

$$= \frac{-0.3}{(t-0.15)^3}\Big|_{t=0} = \frac{0.3}{0.15^3}$$

$$\Rightarrow V(X) = E[X^2] - E[X]^2 = 44.\overline{4}$$