

# Q1

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## Assignment 2: Question 1

A) Suppose we decide to collect 30 observations ( $n = 30$ ). Calculate Power of this test.

```
n = 30
sigma_square = 16
sigma = sqrt(sigma_square)
alpha = 0.05
mu_null = 5
mu_alpha = 4
significance = alpha
```

$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

$$\bar{X} = Z_\alpha * \sigma / \sqrt{n} + \mu_0$$

$$\bar{X} = Z_{0.05} * (4 / \sqrt{30}) + 5$$

$$\bar{X} = -1.644854 * 0.73029674334 + 5$$

$$\bar{X} = 3.79876848$$

$$Power = P[\bar{X} < 3.79876848 \text{ Under the } H_\alpha]$$

$$P[Z \leq \frac{\bar{X} - \mu_\alpha}{\sigma / \sqrt{n}}] = 1 - P[Z > \frac{3.79876848 - 4}{4 / \sqrt{30}}] = 1 - pnorm(-0.27554760696) = 1 - 0.3914478 = 0.6085521$$

$$Power = 0.6085521$$

```
sigma = sqrt(sigma_square)
x_bar = qnorm(significance) * (sigma / sqrt(n)) + mu_null
probability = (x_bar - mu_alpha) / (sigma / sqrt(n))
power = 1 - pnorm(probability)
power
```

```
## [1] 0.6085521
```

1.B) Suppose we want to ensure that the power is 80%. What should be the sample size? ( $n=?$ )

$$\bullet P = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

$$\bullet n = \left( \frac{p * \sigma}{\bar{X} - \mu_\alpha} \right)^2 = \left( \frac{qnorm(1-0.8)*4}{3.79876848-4} \right)^2 = (16.72941)^2 = 279.8732$$

```
power = 0.8  
reverse_probability = qnorm(1 - 0.8)  
estimated_n = ((reverse_probability * sigma)/ (x_bar - mu_alpha))**2  
estimated_n
```

```
## [1] 279.874
```