STA255 Week-10 (day-1)

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Review of week-9

- Confidence interval for population proportion
- Sample size calculation
- Idea of test of hypothesis
- Null vs Alternative hypothesis
- Type-I error
- Critical Region
- Testing $\mu = \mu_0$ vs $\mu \neq \mu_0$ under Normal distribution
- Two sided vs one sided test
- Type-II error and power of a test.

Learning goals

- Testing population proportion, $p = p_0$ (Chapter 9.3)
- Idea of p-value. (Chapter 9.4)

Testing population proportion, $p = p_0$

- Suppose we are interested in testing whether the true proportion of something equals some value or not.
  - For example, say we want to test, the true proportion of COVID-19 cases in Canada is 3%. Same as saying $H_0 : p = 0.03$

- Last week, we looked at a natural estimator of p, which was $\frac{X}{n}$
  - where, n be the number of individuals tested.
  - X be the number of confirmed cases out of n individuals.

- We say $\hat{p} = \frac{X}{n}$ is an estimator of p.

Recall:

- Since X is the number of success in n trials with probability of success being p in each trial,
$$X \sim Bin(n, p)$$
- We know, $E[X] = np$ and $V[X] = np(1-p)$
- Therefore, $E[X/n] = p$ and $V[X/n] = \frac{p(1-p)}{n} \implies SE[X/n] = \sqrt{\frac{p(1-p)}{n}}$
- If the null hypothesis is believed to be true, $E[X/n] = p_0$ and $SE[X/n] = \sqrt{\frac{p_0(1-p_0)}{n}}$
- For Large n,
$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \to N(0, 1)$$
- We can use $Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ as the test statistic

A numerical example

According to public health agency of Canada, so far the National Microbiology Laboratory has tested 1008 individual. 37 of them tested positive. Based on this observed data, at $\alpha = 0.05$, test $H_0 : p = 0.03$ vs $H_a : p \neq 0.03$.

- $p_0 = 0.03$
- At $\alpha = 0.05$, two-sided test, Rejection region $\implies (-\infty, -1.96) \cup (1.96, \infty)$
- $\hat{p} = 37/1008 = 0.03670635$
- $Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.03670635-0.03}{\sqrt{\frac{0.03(1-0.03)}{1008}}} = 1.248159$
- Since, our calculated test stat does not fall in the rejection region, we fail to reject the null hypothesis. We don't have the evidence to say that the true proportion of cases of COVID-19 in Canada is not 0.03.

Home work: Using this same data, at $\alpha = 0.05$, test $H_0 : p = 0.03$ vs $H_a : p > 0.03$. (hint: only the rejection region will change)

P-value

- P-value approach is another way of conducting test of hypothesis.
  - In my personal opinion, it's more like a different way of reporting the result of a test of hypothesis.
- In Chapter 9.1-9.3, we have started our hypothesis testing using a given value of α.
- This allowed us to construct a rejection region even before observing the samples.
- In this approach whatever conclusion we make, we make it for that fixed α.
- For example, in the previous page, we failed to reject H_0 at $\alpha = 0.05$. Our conclusion could have been different for a different choice of α.

- For that example, we got the test stat, $Z = 1.248159$
- Let's consider some series of α values and check whether we reject the H_0 or not.
  - At $\alpha = 0.9$, RR: $(-\infty, -0.1256613) \cup (0.1256613, \infty) \implies \textit{reject } H_0$
  - At $\alpha = 0.8$, RR: $(-\infty, -0.2533471) \cup (0.2533471, \infty) \implies \textit{reject } H_0$
  - At $\alpha = 0.5$, RR: $(-\infty, -0.6744898) \cup (0.6744898, \infty) \implies \textit{reject } H_0$
  - At $\alpha = 0.25$, RR: $(-\infty, -1.150349) \cup (1.150349, \infty) \implies \textit{reject } H_0$
  - At $\alpha = 0.22$, RR: $(-\infty, -1.226528) \cup (1.226528, \infty) \implies \textit{reject } H_0$
  - At $\alpha = 0.212$, RR: $(-\infty, -1.248085) \cup (1.248085, \infty) \implies \textit{reject } H_0$

  - At $\alpha = 0.21$, RR: $(-\infty, -1.253565) \cup (1.253565, \infty) \implies \textit{fail to reject } H_0$
- $\alpha = 0.212$ is the minimum value of α for which we fail to reject H_0
  typo, should be: we reject H0
- For any $\alpha < 0.212$, we will reject H_0, based on this current data.
  typo,should be: fail to reject H0
- For this particular test, we say **p-value=0.212**

With this here is the **formal definition** of p-value:

The p-value is the probability, calculates assuming the null hypothesis is true, of obtaining a value of the test statistic that is at least as extreme as the value of the calculated test stat.

Use of p-value

- Once a study reports back its p-value, now the reader has the option to compare it to any α that he she has in mind and make a conclusion for him/herself.
  - For example, if I know p-value=0.212, I know that I would have rejected this particular H_0 at $\alpha = 0.05$ or $\alpha = 0.1$ without actually doing the test again.

Calculation of p-value

- Calculation of p-value depends on three key things:
  - What is the value of the test stat that we have calculated.
  - What distribution we are dealing with.
  - Is the alternative hypothesis left sided ($\mu < \mu_0$) or right sided ($\mu > \mu_0$) or two sided ($\mu \neq \mu_0$)

Two sided Alternative

- For our example, we observed a test statistic, Z=1.248159 and the distribution is N(0,1).
- A two sided p-value $= 2 * (1 - \Phi(|1.248159|))$

One Sided Alternative

- If our alternative was $H_0 : p > 0.03$,

$$p - value = 1 - \Phi(1.248159)$$

- If our alternative was $H_0 : p < 0.03$,

$$p - value = \Phi(1.248159)$$

- Here, $\Phi()$ is the cdf of Standard Normal distribution.
- If the distribution to be used was t, we would have used the CDF of a t-distribution.

Rule of thumb

- Generally a low p-value suggests evidence against the null hypothesis.
- Learned this from one of my student,“If p-value is low, H_0 must go”… * If the problem doesn’t specify an $alpha$, we report the p-value and as an example we say “… this mean at $\alpha = 0.05$, we would have rejected, or would have failed to reject this null”.

Chapter 9.5 not needed.

Homework

Chapter 9.3

37(a), 38, 41, 44(a)

Chapter 9.4

45-52, 53(a)