

**INFS1200/7900**

# **Introduction to Information Systems**

## Functional Dependencies and Normal Forms

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# RiPPLE Feedback Form

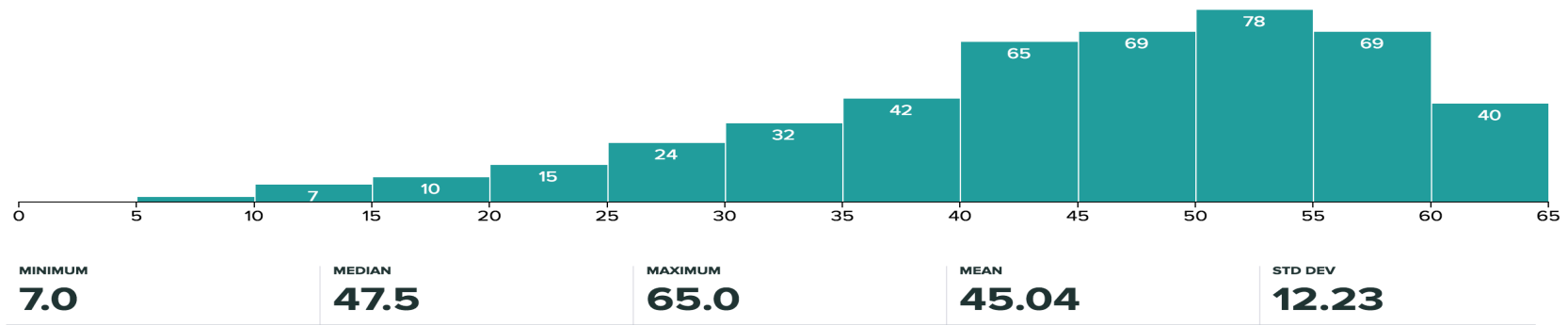
- Please use the form published under **Learning Resources --> Shared Links** to provide feedback on your experience in using RiPPLE.
- We take student feedback very seriously and frequently make changes to RiPPLE based on the responses we receive.

# Notes

- **Lecture:** Continuing with Module 3: Functional Dependencies and Normal Forms.
- **Tutorial:** Tutorial 5 on Normalisation
- **Practical:** Your chance to get help with LDBM, Mini-project and Practical work. Mini-project part 3 has been released
- **Contact:** No contact sessions are scheduled for the next few weeks.
- **Assignment 1:** Due 21 September. Please make sure that you use the provided template.
- **RiPPLE:** Round 2 will be graded soon. Round 3 ends 28 September.

# Quiz 1

- Grades have been released. You can check a soft copy of your exam via Gradescope.



- Check the released sample solution and rubric available on gradescope to determine what you did wrong.
- Remark request procedure described in the email that was sent out.

# Our INFS1200/7900 Journey

1. We've learned that databases are wonderful things.
2. We've learned how to create a *conceptual design* using ER diagrams.
3. We've learned how to create a *logical design* by turning the ER diagrams into a relational schema.
4. In this module we will learn how to *refine* that schema to reduce duplication of information.

# Learning Outcomes

| Description  | Tag                     |
|--|-------------------------|
| Provide examples of modification, insertion, and deletion anomalies.   | Functional-dependencies |
| Explain the term functional dependency.  |                         |
| Given a set F of functional dependencies and a functional dependency fd, determine whether fd can be inferred from F using Armstrong's Axioms.   |                         |
| Given a set of functional dependencies that hold over a table, determine the keys.   |                         |
| Given a set of functional dependencies that hold over a table, determine the superkeys.  |                         |
| Given a set F of functional dependencies and X of attributes of a relation, compute the closure of X, which is X <sup>+</sup>                    |                         |
| Given a relation R, be able to correctly decompose it into two or more relations   | Decomposition           |
| Justify why lossless join decompositions are preferred decompositions.   |                         |
| Explain the benefits of Normalization.   | Normalization           |
| Given a relation schema R and a set of functional dependencies on it, show that R is/isn't in 1NF.   |                         |
| Explain what dependency preservation means.  |                         |
| Given a relation schema R and a set of functional dependencies on it, show that R is/isn't in 3NF.   |                         |
| Given a relation schema R and a set of functional dependencies on it, show that R is/isn't in BCNF.  |                         |
| Describe the process and benefits of Denormalization.  |                         |
| Reason with the logical foundation of the relational data model and understand the fundamental principles of correct relational database design. |                         |

# UQ Active Learn

- UQ Active Learn contains multiple applications including **UQpoll** and **UQwordcloud**



URL: [apps.elearning.uq.edu.au](https://apps.elearning.uq.edu.au)  
ID: 60140

Disclosure: This application allows me to track authors of the responses.

## **Design Guidelines**

Functional Dependencies

Normalization



# Informal Design Guidelines

- Informal measures of relational database schema quality and design guidelines
  - Semantics of the attributes
  - Reducing the redundant values in tuples
  - Reducing the null values in tuples
  - Disallowing spurious tuples

# Guideline 1

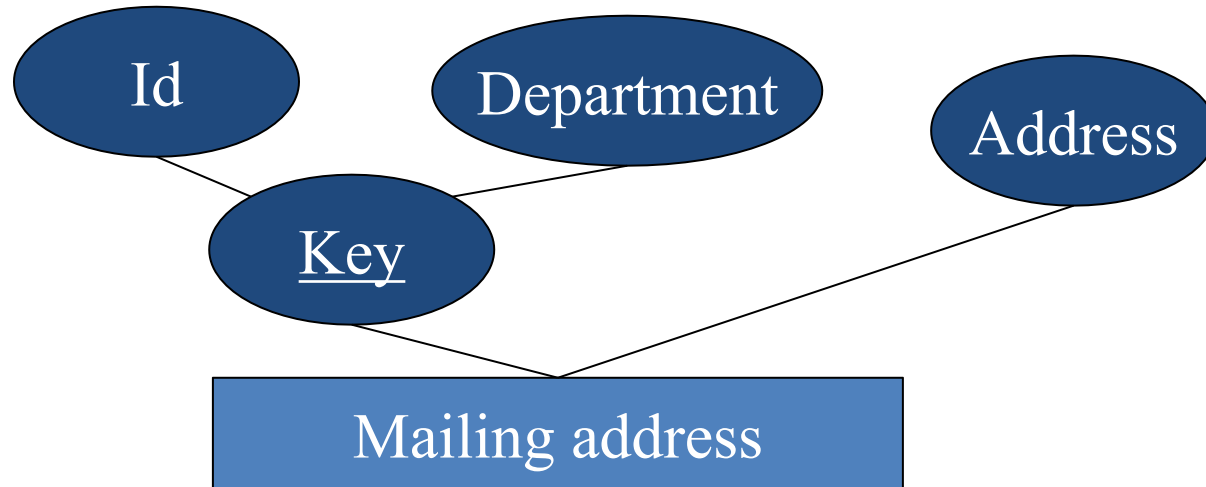
- Design each relation so that it is easy to explain its meaning
- Do not combine attributes from multiple entity types and relationship types into a single relation

# Redundant Values in Tuples

- One design goal is to minimize the storage space that base relations occupy
- The way the grouping of attributes into relation schemas is done, has a significant effect on storage space
- In addition, an incorrect grouping may cause *update anomalies* which may result in inconsistent data or even loss of data

# A Motivating Example

- Imagine that we have created the following entity for mailing addresses at a University



Meets all the criteria that we have discussed for an entity so far

# What Would an Instance Look Like?

| <u>Id</u> | <u>Department</u> | Address          |
|-----------|-------------------|------------------|
| 100       | Computer Science  | 78-101 Main Mall |
| 104       | Computer Science  | 78-101 Main Mall |
| 104       | Math              | 69-201 Main Mall |
| 105       | Physics           | 50-205 Main Mall |

- **Modification anomaly:** data inconsistency that results from data redundancy.
  - Example: Updating the mailing address of a department.
- **Deletion anomaly:** loss of certain attributes because of the deletion of other attributes.
  - Example: Deleting 105 would lead to deletion of the address of Physics.
- **Insertion anomaly:** Lack of ability to insert some attributes without the presence of other attributes.
  - Example: Storing the mailing address of a department that has no faculty members.

# Guideline 2

- Design the base relation schema so that no insertion, deletion, or modification anomalies occur in the relations
- If any do occur, ensure that all applications that access the database update the relations in such a way as to not compromise the integrity of the database

# Guideline 3

- As far as possible, avoid placing attributes in a base relation whose values may be null
- If nulls are unavoidable, make sure that they apply in exceptional cases only and that they do not apply to a majority of tuples in the relation

# Decomposing a Relation

- A **decomposition** of R replaces R by two or more relations such that:
  - Each new relation contains a subset of the attributes of R (and no attributes not appearing in R)
  - Every attribute of R appears in at least one new relation.

- Example:

**Mailing Address (Id, Department, Address)**

**Academics(Id, Department)**

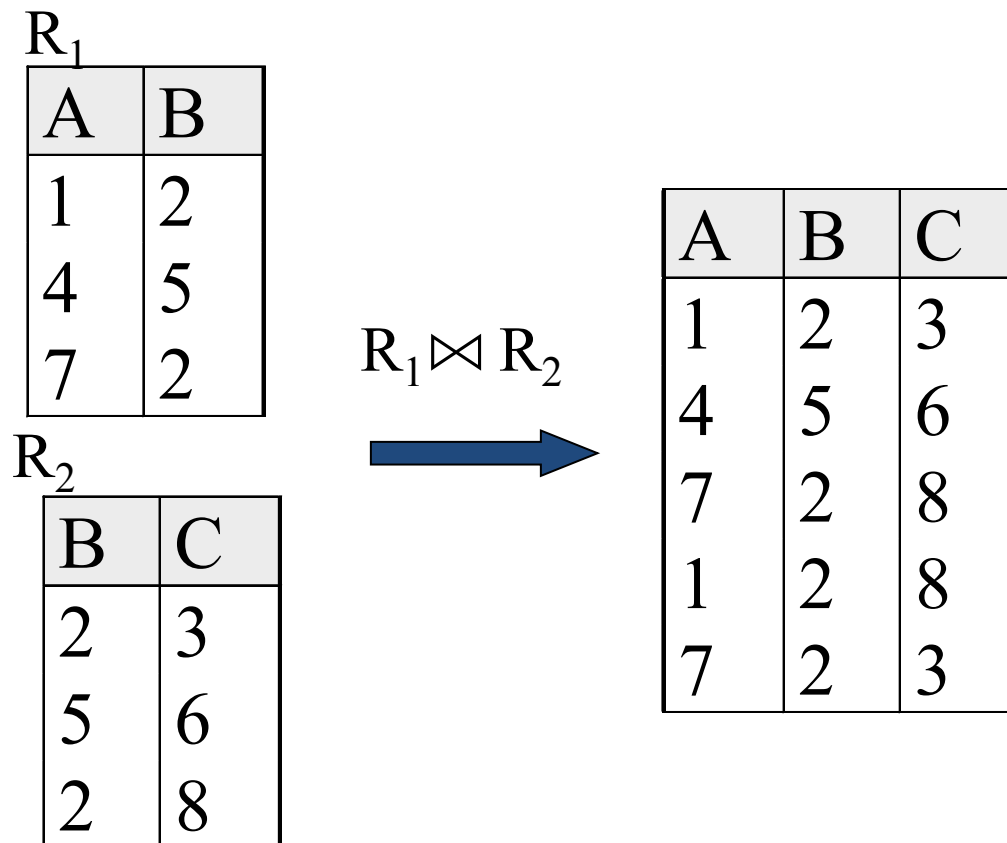
**Mailing Address(Department, Address)**

How should we decompose tables to remove anomalies?



# The join

- Definition:  $R_1 \bowtie R_2$  is the (natural) join of the two relations
  - each tuple of  $R_1$  is concatenated with every tuple in  $R_2$  having the same values on the common attributes.



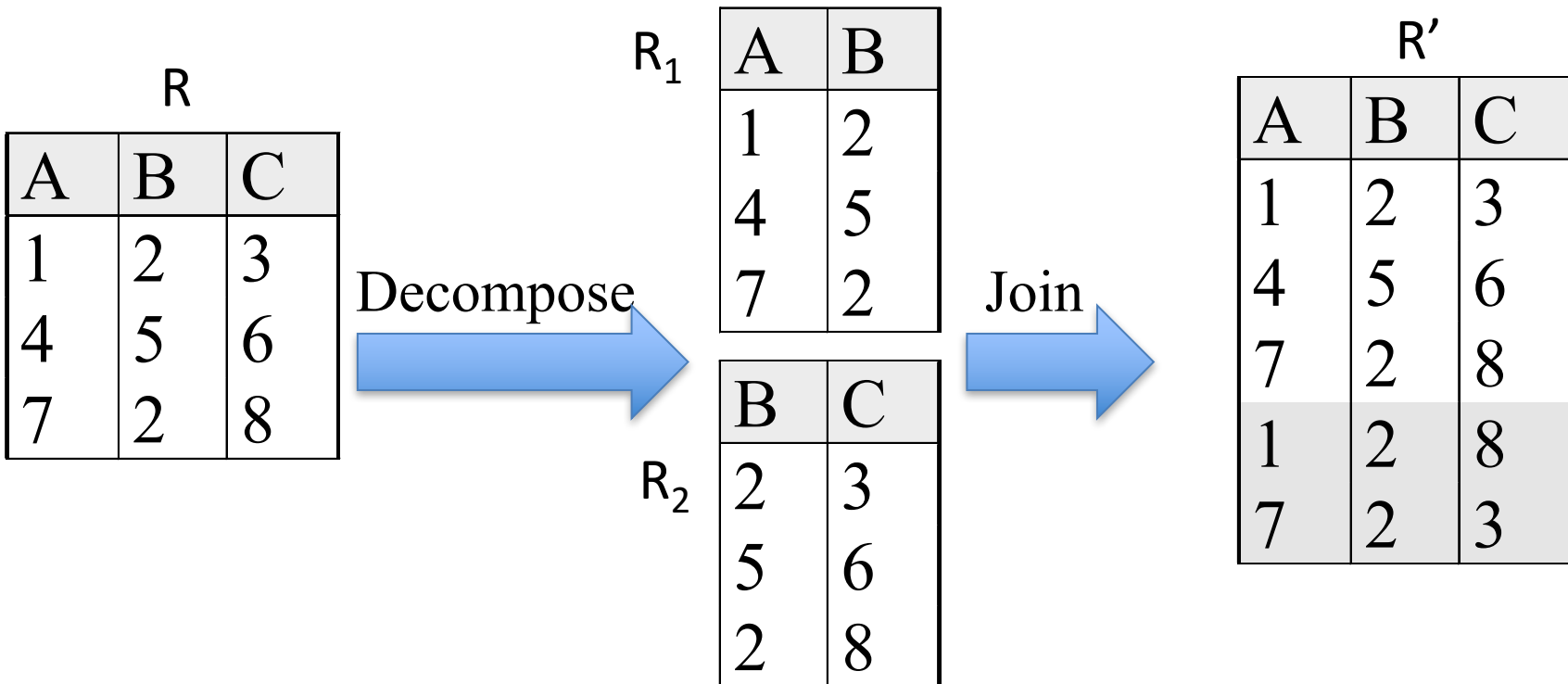
# Lossless-Join Decompositions: Definition

Decomposition of  $R$  into  $R_1$  and  $R_2$  is a lossless-join **w.r.t. a set of FDs  $F$**  if, for every instance  $r$  that satisfies  $F$ :

$$R = R_1 \bowtie R_2$$

- **Informally:** If we break a relation,  $R$ , into bits, when we put the bits back together, we should get exactly  $R$  back again.

# Example Lossy-Join Decomposition



- The word loss in **lossless** refers to **loss of information**, not to loss of tuples. Maybe a better term here is “**addition of spurious information**”.
- Last two rows are not in the original. Would this have happened if B determined C?

# Guideline 4

- Design the relation schemas so that they can be (relationally) *joined* with equality conditions on attributes that are either primary keys or foreign keys in a way that guarantees that no spurious tuples are generated

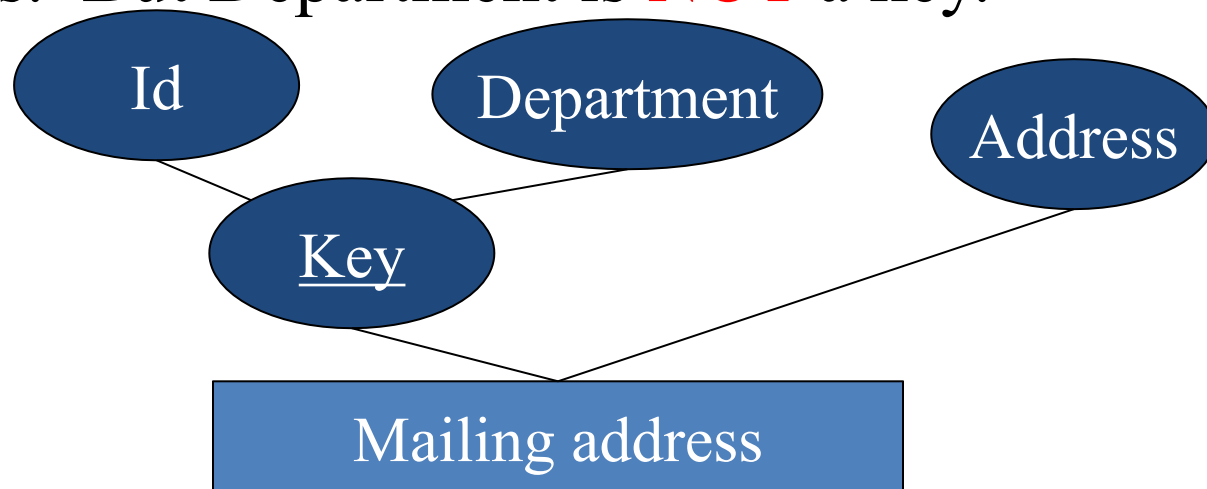
Design Guidelines

**Functional Dependencies**

Normalization

# Functional Dependencies, Informally

- How do I know for sure if departments only have one address?
- Databases allow you to say that one attribute determines another through a **functional dependency**.
- So if Department determines Address but not Name, we say that there is a functional dependency from Department to Address. But Department is **NOT** a key.



# Functional Dependencies, Formally

- A functional dependency  $X \rightarrow Y$  holds if for every legal instance, for all tuples  $t1, t2$ :

$$\text{if } t1.X = t2.X \rightarrow t1.Y = t2.Y$$

- Which means given two tuples in  $r$ , if the  $X$  values agree, then the  $Y$  values must also agree.
- Example:

Department  $\rightarrow$  Address

if  $t1.Department = t2.Department \rightarrow t1.Address = t2.Address$

# Identifying Functional Dependencies

- A FD is a statement about *all* allowable instances.
  - Must be identified by application semantics.
  - Given some instance of R, we can check if it violates some FD  $f$ , but we cannot tell if  $f$  holds over R!

| <u>Id</u> | <u>Department</u> | Address          |
|-----------|-------------------|------------------|
| 100       | Computer Science  | 78-101 Main Mall |
| 104       | Computer Science  | 78-101 Main Mall |
| 104       | Math              | 69-201 Main Mall |
| 105       | Physics           | 50-205 Main Mall |

- Based on this instance alone, we cannot conclude that Department  $\rightarrow$  Address.



# Question

- Consider the relation R with the following instance:

| A | B | C | D |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 2 | 3 | 4 | 6 |
| 6 | 7 | 8 | 9 |
| 1 | 3 | 4 | 5 |

What FDs **cannot** be true given the instance above?

- A.  $B \rightarrow C$
- B.  $B \rightarrow D$
- C.  $D \rightarrow B$
- D. All of the above can be true
- E. None of the above can be true

# Question

- Consider the relation R with the following instance:

| A | B | C | D |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 2 | 3 | 4 | 6 |
| 6 | 7 | 8 | 9 |
| 1 | 3 | 4 | 5 |

What FDs **cannot** be true given the instance above?

A.  $B \rightarrow C$

fine

B.  $B \rightarrow D$

not possible (2<sup>nd</sup> and 4<sup>th</sup>), so correct answer

C.  $D \rightarrow B$

fine

D. All of the above can be true

E. None of the above can be true

# Fixing Anomalies

| <u>Id</u> | <u>Department</u> | Address          |
|-----------|-------------------|------------------|
| 100       | Computer Science  | 78-101 Main Mall |
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| <u>Id</u> | <u>Department</u> | <u>Department</u> | Address          |
|-----------|-------------------|-------------------|------------------|
| 100       | Computer Science  | Computer Science  | 78-101 Main Mall |
| 104       | Computer Science  | Math              | 69-201           |
| 104       | Math              | Physics           | 50-205           |
| 105       | Physics           |                   |                  |

# Fixing Anomalies

| <u>Id</u> | <u>Department</u> |
|-----------|-------------------|
| 100       | Computer Science  |
| 104       | Computer Science  |
| 104       | Math              |
| 105       | Physics           |

| <u>Department</u> | <u>Address</u>   |
|-------------------|------------------|
| Computer Science  | 78-101 Main Mall |
| Math              | 69-201           |
| Physics           | 50-205           |

**Modification anomaly fixed:** Updating the mailing address only in one place does not lead to inconsistencies in the database.

**Deletion anomaly fixed:** Deleting the row with id 105 does not delete the mailing address of Physics.

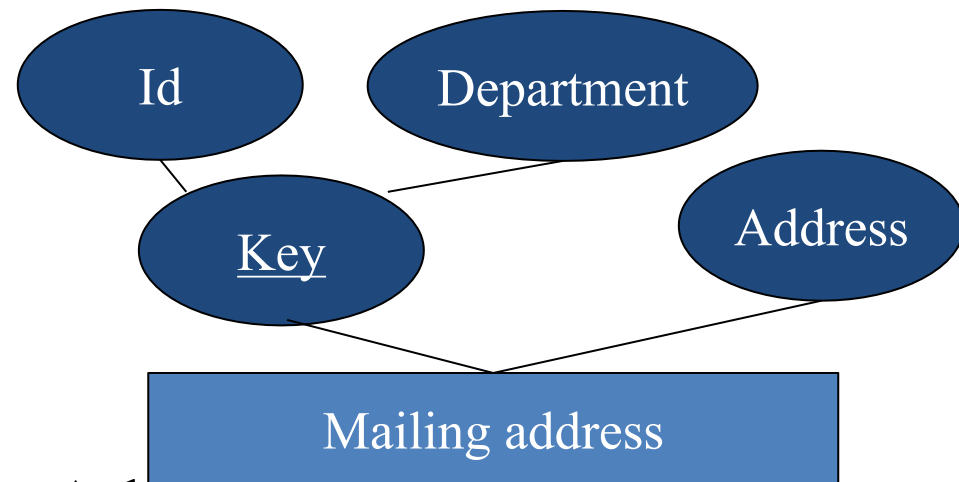
**Insertion anomaly fixed:** It is now possible to store the mailing address of a department that has no faculty members.

# Fixing Anomalies Beyond our Example

- We were able to fix the anomalies by splitting the Mailing address table.
- In the rest of this module, we will discuss how anomalies can be addressed formally.

# Keys

- As a reminder, a key is a **minimal** set of attributes that uniquely identify a relation
  - i.e., a key is a minimal set of attributes that *functionally determines* all the attributes
- A **superkey** for a relation uniquely identifies the relation, but does not have to be minimal
  - i.e.,:  $\text{key} \subseteq \text{superkey}$



- Example
  - key {Id, Department}
  - superkey {Id, Department, Address}

# Clicker question: Possible Keys

- Assume that the following FDs hold for a relation  $R(A,B,C,D)$ :  
 $B \rightarrow C$   
 $C \rightarrow B$   
 $D \rightarrow ABC$

Which of the following is a **key** for the above relation?

- A. B
- B. C
- C. BD
- D. All of the above
- E. None of the above

# Clicker question: Possible Keys

- Assume that the following FDs hold for a relation  $R(A,B,C,D)$ :  
 $B \rightarrow C$   
 $C \rightarrow B$   
 $D \rightarrow ABC$

Which of the following is a **key** for the above relation?

A. B **Does not determine all**

B. C **Does not determine all**

C. BD **Not minimal**

D. All of the above

E. None of the above **The right answer**



# Clicker question: Possible Superkeys

- Assume the same relation  $R(A,B,C,D)$  and FDs

$B \rightarrow C$

$C \rightarrow B$

$D \rightarrow ABC$

Which of the following is a superkey for the above relation?

A. D

B. BD

C. BCD

D. All are superkeys

E. None are superkeys

# Clicker question: Possible Superkeys

- Assume the same relation  $R(A,B,C,D)$  and FDs

$B \rightarrow C$

$C \rightarrow B$

$D \rightarrow ABC$

Which of the following is a superkey for the above relation?

A. D

B. BD

C. BCD

D. All are superkeys

E. None are superkeys

D is a key. Therefore, all of the answers are superkeys

# Explicit and Implicit FDs

- Given a set of (explicit) functional dependencies, we can determine implicit ones

$studentid \rightarrow city, city \rightarrow acode$  implies  $studentid \rightarrow acode$

- A functional dependency fd is implied by a set F of functional dependencies if fd holds whenever all FDs in F hold.

$fd = \{studentid \rightarrow acode\}$

**F**

fd1:  $studentid \rightarrow city$

fd2:  $city \rightarrow acode$

- Closure of F**: the set of all FDs implied by F.

# Armstrong's Axioms

- Armstrong's Axioms ( $X, Y, Z$  are sets of attributes):
  - **Reflexivity**: If  $Y \subseteq X$ , then  $X \rightarrow Y$   
e.g.,  $\text{city, major} \rightarrow \text{city}$
  - **Augmentation**: If  $X \rightarrow Y$ , then  $XZ \rightarrow YZ$  for any  $Z$   
e.g., if  $\text{sid} \rightarrow \text{city}$ , then  $\text{sid, major} \rightarrow \text{city, major}$
  - **Transitivity**: If  $X \rightarrow Y$  and  $Y \rightarrow Z$ , then  $X \rightarrow Z$   
 $\text{sid} \rightarrow \text{city}$ ,  $\text{city} \rightarrow \text{areacode}$  implies  $\text{sid} \rightarrow \text{areacode}$
- These are *sound* and *complete* inference rules.

1.  $\text{studentid} \rightarrow \text{acode}$  fd1, fd2, transitivity

**F**

fd1:  $\text{studentid} \rightarrow \text{city}$

fd2:  $\text{city} \rightarrow \text{acode}$

# Additional Rules

- Couple of additional rules (that follow from axioms):

- **Union**: If  $X \rightarrow Y$  and  $X \rightarrow Z$ , then  $X \rightarrow YZ$   
e.g., if  $\text{sid} \rightarrow \text{acode}$  and  $\text{sid} \rightarrow \text{city}$ , then  $\text{sid} \rightarrow \text{acode,city}$

1.  $X \rightarrow XY$  fd1, augmentation  
2.  $XY \rightarrow ZY$  fd2, augmentation  
3.  $X \rightarrow ZY$  1, 2, transitivity

**F**  
fd1:  $X \rightarrow Y$   
fd2:  $X \rightarrow Z$

- **Decomposition**: If  $X \rightarrow YZ$ , then  $X \rightarrow Y$  and  $X \rightarrow Z$   
e.g., if  $\text{sid} \rightarrow \text{acode,city}$  then  $\text{sid} \rightarrow \text{acode}$ , and  $\text{sid} \rightarrow \text{city}$

1  $YZ \rightarrow Y$  Reflexivity  
2  $YZ \rightarrow Z$  Reflexivity  
3.  $X \rightarrow Y$  fd1, 1, transitivity  
5.  $X \rightarrow Z$  fd1, 2, transitivity

**F**  
fd1:  $X \rightarrow YZ$

# Example: Supplier-Part DB

- Suppliers supply parts to projects.
  - SupplierPart(sname,city,status,p#,pname,qty)
    - supplier attributes: sname, city, status
    - part attributes: p#, pname
    - supplier-part attributes: qty:
- Functional dependencies:
  - fd1: sname  $\rightarrow$  city
  - fd2: city  $\rightarrow$  status
  - fd3: p#  $\rightarrow$  pname
  - fd4: sname, p#  $\rightarrow$  qty

Exercise: Show that (sname, p#) is a superkey

# Supplier-Part Key: Part 1:

Exercise: Show that  $(sname, p\#)$  is a superkey of  
 $SupplierPart(sname, city, status, p\#, pname, qty)$

Proof has two parts:

a. Show:  $sname, p\#$  is a (super)key

- |  |               |               |
|--|---------------|---------------|
| 1. $sname, p\# \rightarrow sname, p\#$                           | reflex        |               |
| 2. $sname \rightarrow city$                                      | fd1           |               |
| 3. $sname \rightarrow status$                                    | 2, fd2, trans |               |
| 4. $sname, p\# \rightarrow city, p\#$                            | 2, aug        |               |
| 5. $sname, p\# \rightarrow status, p\#$                          | 3, aug        |               |
| 6. $sname, p\# \rightarrow sname, p\#, status$                   |               | 1, 5, union   |
| 7. $sname, p\# \rightarrow sname, p\#, status, city$             |               | 4, 6, union   |
| 8. $sname, p\# \rightarrow sname, p\#, status, city, qty$        |               | 7, fd4, union |
| 9. $sname, p\# \rightarrow sname, p\#, status, city, qty, pname$ |               | 8, fd3, union |

fd1:  $sname \rightarrow city$   
fd2:  $city \rightarrow status$   
fd3:  $p\# \rightarrow pname$   
fd4:  $sname, p\# \rightarrow qty$

# Example: Supplier-Part DB

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  - SupplierPart(sname,city,status,p#,pname,qty)
    - supplier attributes: sname, city, status
    - part attributes: p#, pname
    - supplier-part attributes: qty:
- Functional dependencies:
  - fd1: sname  $\rightarrow$  city
  - fd2: city  $\rightarrow$  status
  - fd3: p#  $\rightarrow$  pname
  - fd4: sname, p#  $\rightarrow$  qty

Exercise: Show that (sname, p#) is a key



# Supplier-Part Key: Part 2

b. Show:  $(sname, p\#)$  is a *minimal* key of  
SupplierPart(sname,city,status, p#,pname,qty)

1.  $p\#$  does not appear on the RHS of any FD therefore except for  $p\#$  itself, nothing else determines  $p\#$
3. specifically,  $sname \rightarrow p\#$  does not hold
4. therefore,  $sname$  is not a key
5. similarly,  $p\#$  is not a key

fd1:  $sname \rightarrow city$   
fd2:  $city \rightarrow status$   
fd3:  $p\# \rightarrow pname$   
fd4:  $sname, p\# \rightarrow qty$

$sname, p\# \rightarrow sname, p\#, city, status, pname, qty$

$sname \rightarrow sname, city, status$

$p\# \rightarrow p\#, pname$

# Computing the Closure

- Closure for a set of attributes  $X$  is denoted by  $X^+$
- $X^+$  includes all attributes of the relation IFF  $X$  is a (super) key

Algorithm for finding Closure of  $X$ :

Let Closure =  $X$

Until Closure doesn't change do

if  $a_1, \dots, a_n \rightarrow C$  is a FD and  $\{a_1, \dots, a_n\} \in \text{Closure}$

Then add  $C$  to Closure

- Example
  - $\text{Studentid}^+ = \{\text{Studentid}\}$
  - $\text{Studentid}^+ = \{\text{Studentid}, \text{city}\}$
  - $\text{Studentid}^+ = \{\text{Studentid}, \text{city}, \text{acode}\}$

**F**

fd1:  $\text{studentid} \rightarrow \text{city}$

fd2:  $\text{city} \rightarrow \text{acode}$

# Supplier-Part Key: Closure

Algorithm for finding Closure of X:

Let Closure = X

Until Closure doesn't change do

if  $a_1, \dots, a_n \rightarrow C$  is a FD and  $\{a_1, \dots, a_n\} \in \text{Closure}$

Then add C to Closure

fd1:  $\text{sname} \rightarrow \text{city}$

fd2:  $\text{city} \rightarrow \text{status}$

fd3:  $\text{p\#} \rightarrow \text{pname}$

fd4:  $\text{sname}, \text{p\#} \rightarrow \text{qty}$

Ex: SupplierPart(sname,city,status,p#,pname,qty)

$\{\text{sname}, \text{p\#}\}^+ = \text{sname}, \text{p\#}, \text{city}, \text{status}, \text{pname}, \text{qty}$

$\{\text{sname}\}^+ = \text{sname}, \text{city}, \text{status}$

$\{\text{p\#}\}^+ = \text{p\#}, \text{pname}$

So seeing if a set of attributes is a superkey means checking to see if it's closure is all the attributes – pretty simple

# Question: Finding Key

- Which of the following is a key of the Relation  $R(ABCDE)$  with FD's:

|      | <b>F</b>            |
|------|---------------------|
| fd1: | $D \rightarrow C$   |
| fd2: | $CE \rightarrow A,$ |
| fd3: | $D \rightarrow A$   |
| fd4: | $AE \rightarrow D.$ |

- A. ABDE
- B. BCE
- C. CDE
- D. All of these are keys
- E. None of these are keys

# Question: Finding Key

- Which of the following is a key of the Relation  $R(ABCDE)$  with FD's:

|      | <b>F</b>            |
|------|---------------------|
| fd1: | $D \rightarrow C$   |
| fd2: | $CE \rightarrow A,$ |
| fd3: | $D \rightarrow A$   |
| fd4: | $AE \rightarrow D.$ |

A. ABDE Superkey, since  $D \rightarrow A$

B. BCE Key

C. CDE  $CDE \neq CDEA$

D. All of these are keys

E. None of these are keys

# Approaching Normality

- Role of FDs in detecting redundancy:
  - Consider a relation R with 3 attributes, A B C.
    - **No FDs hold:** There is no redundancy here.
    - **Given  $A \rightarrow B$ :** Several tuples could have the same A value, and if so, they'll all have the same B value!
- **Normalization:** the process of removing redundancy from data
- We were able to fix the anomalies by splitting (decomposing) the Mailing address table, but how should we do this more formally? and how is it related to functional dependencies?

Design Guidelines

Functional Dependencies

**Normalization**

# Normalization

- Normalization is a process that aims at achieving better designed relational database schemas using
  - Functional Dependencies
  - Primary Keys
- The normalization process takes a relational schema through a **series of tests** to certify whether it satisfies certain conditions
  - The schemas that satisfy certain conditions are said to be in a given '*Normal Form*'.
  - Unsatisfactory schemas are decomposed by breaking up their attributes into smaller relations that possess desirable properties (e.g., no anomalies)



# Review of “Key” Concepts

- **Superkey** - A set of attributes such that no two tuples have the same values for these attributes
- **Key** - A minimal Superkey, called a Candidate Key if more than one:
  - **Primary key** - A selected candidate key
  - **Secondary key** - Remaining candidate keys
- **Prime Attribute** - An attribute that is a member of any candidate key
- **Non-prime attribute** - An attribute that is not a member of any candidate key

# First Normal Form (1NF)

- A relation schema is in 1NF if domains of attributes include only atomic (simple, indivisible) values and the value of an attribute is a single value from the domain of that attribute
- 1NF disallows
  - having a set of values, a tuple of values, or a combination of both as an attribute value for a single tuple
  - “relations within relations” and “relations as attributes of tuples

| Customer | Order | Items              |
|----------|-------|--------------------|
| Jones    | 123   | Basket, Football   |
| Gupta    | 876   | Hat, Glass, Pencil |

A non-1nf relation

# Relations in 1NF

| Customer | Order | Item     |
|----------|-------|----------|
| Jones    | 123   | Basket   |
| Jones    | 123   | Football |
| Gupta    | 876   | Hat      |
| Gupta    | 876   | Glass    |
| Gupta    | 876   | Pencil   |

Problem with this design:

Redundancy (Jones, 123)

| Customer | Order | Item1  | Item2    | Item3  | Item4 |
|----------|-------|--------|----------|--------|-------|
| Jones    | 123   | Basket | Football | Null   | Null  |
| Gupta    | 876   | Hat    | Glass    | Pencil | Null  |

Problem with this design:

Too many Null values

What if an order has >max (4) items

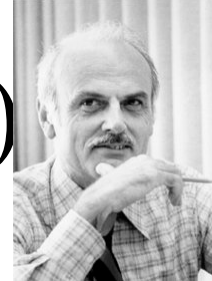
# Second Normal Form (2NF)

- A relation is in 2NF, if for every FD  $X \rightarrow Y$  where  $X$  is a minimal key and  $Y$  is a non-prime attribute, then no proper subset of  $X$  determines  $Y$ .
- e.g., the address relation is not in 2NF:
  - House#, street, postal\_code is a minimal key
  - House#, street, postal\_code  $\rightarrow$  Province
  - Postal\_code  $\rightarrow$  province

$X = \text{House\#, street, postal code}$   
 $Y = \text{province}$



Raymond Boyce



Ted Codd

# Boyce-Codd Normal Form (BCNF)

A relation  $R$  is in BCNF if for all non-trivial dependencies in  $R$ :  
If  $X \rightarrow b$  then  $X$  is a superkey for  $R$

- A dependency is trivial if the LHS contains the RHS, e.g.,  
 $\text{City, Province} \rightarrow \text{City}$  is a trivial dependency
- **Informally:** Whenever a set of attributes of  $R$  determine another attribute, it should determine all the attributes of  $R$ .

# BCNF Example

- Address(House#, Street, City, Province, PostalCode)

House#, Street, PostalCode  $\rightarrow$  City

House#, Street, PostalCode  $\rightarrow$  Province

PostalCode  $\rightarrow$  City

PostalCode  $\rightarrow$  Province

Is Address in BCNF?

$\{\text{PostalCode}\}^+ = \{\text{PostalCode}, \text{City}, \text{Province}\}$

No. PostalCode is not a superkey

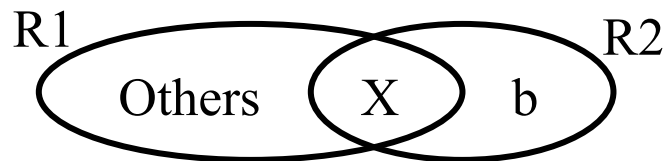
PostalCode  $\rightarrow$  City is violating BCNF

# Decomposing into BCNF Losslessly

1. Add implicit FDs to your list of FDs

e.g. If FDs contain  $A \rightarrow B$  and  $B \rightarrow C$  add  $A \rightarrow C$  to list of FDs

2. Pick any  $f \in \text{FD}$  that violates BCNF of the form  $X \rightarrow b$
3. Decompose  $R$  into two relations:  $R_1(A-b)$  &  $R_2(X \cup b)$



4. Recurse on  $R_1$  and  $R_2$  using FD

Note: answer may vary depending on order you choose. That's okay

# How do we decompose into BCNF losslessly?

- The algorithm terminates since all two attribute relations are in BCNF.

$R(X,Y)$

No FD so no redundancy

$X \rightarrow Y$  so  $X$  is key, so in BCNF

$Y \rightarrow X$  so  $Y$  is key, so in BCNF

$Y \rightarrow X$  and  $X \rightarrow Y$ , both  $X$  and  $Y$  are keys, so in BCNF

**BCNF Definition:** A relation  $R$  is in BCNF if for all non-trivial dependencies in  $R$ : If  $X \rightarrow b$  then  $X$  is a superkey for  $R$



# BCNF Decomposition Example

- Relation:  $R(ABCD)$
- Closure and keys

$$A^+ = A$$

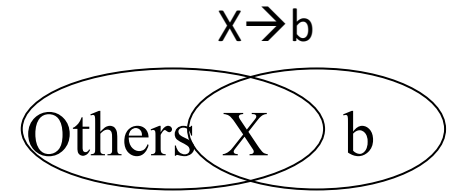
$$B^+ = BC$$

$$C^+ = C$$

$$D^+ = AD$$

$BD^+ = BDCA$  is the only key

fd1:  $B \rightarrow C$   
fd2:  $D \rightarrow A$



Considering  $B \rightarrow C$ , is  $B$  a superkey in  $R$ ?

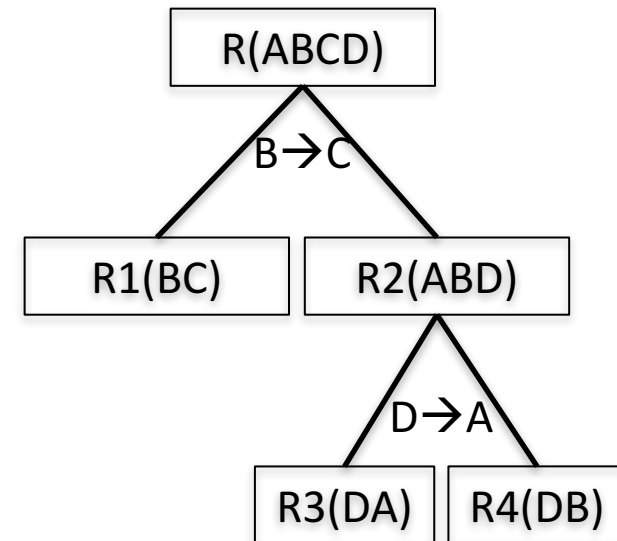
**No. Decompose**

**$R_1(B,C), R_2(A,B,D)$**

Considering  $D \rightarrow A$ , is  $D$  a superkey for  $R_2$ ?

**No. Decompose**

**$R_3(D,A), R_4(D,B)$**



Final answer:  $R_1(B,C), R_3(D,A), R_4(D,B)$ .  
What does this mean?

# Determining Which FDs Apply

For an FD  $X \rightarrow b$ , if the decomposed relation  $S$  contains  $\{X \cup b\}$ , and  $b \in X^+$  then the FD holds for  $S$ .

- For example. Consider relation  $R(A,B,C,D,E)$  with functional dependencies  $AB \rightarrow C$ ,  $BC \rightarrow D$ ,  $CD \rightarrow E$ ,  $DE \rightarrow A$ , and  $AE \rightarrow B$ .
- Project these FD's onto the relation  $S(A,B,C,D)$ .
- Does  $AB \rightarrow D$  hold?
  - First check if  $A$ ,  $B$  and  $D$  are all in  $S$ ? They are
  - Find  $AB^+ = ABCDE$
  - Then yes  $AB \rightarrow D$  does hold in  $S$ .
- Does  $CD \rightarrow E$  hold?
  - No

# Question

- Consider relation  $R(A,B,C,D,E)$  with functional dependencies  $AB \rightarrow C$ ,  $BC \rightarrow D$ ,  $CD \rightarrow E$ ,  $DE \rightarrow A$ , and  $AE \rightarrow B$ . Project these FD's onto the relation  $S(A,B,C,D)$ .
- Which of the following hold in  $S$ ?
  - A.  $A \rightarrow B$
  - B.  $AB \rightarrow E$
  - C.  $AE \rightarrow B$
  - D.  $BCD \rightarrow A$
  - E. None of the above

# Question

- Consider relation  $R(A,B,C,D,E)$  with functional dependencies  $AB \rightarrow C$ ,  $BC \rightarrow D$ ,  $CD \rightarrow E$ ,  $DE \rightarrow A$ , and  $AE \rightarrow B$ . Project these FD's onto the relation  $S(A,B,C,D)$ .
- Which of the following hold in  $S$ ?

A.  $A \rightarrow B$

$A^+ = A$ , so  $A \rightarrow B$  does not hold

B.  $AB \rightarrow E$

$E$  is not in  $S$ , so  $AB \rightarrow E$  does not hold

C.  $AE \rightarrow B$

$E$  is not in  $S$ , so  $AE \rightarrow B$  does not hold

D.  $BCD \rightarrow A$

Yes.  $BCD^+ = ABCDE$ ; all in  $S$

E. None of the above

Note that we use all FDs for finding closures, so for  $D$  we use  $DC \rightarrow E$  Even though  $E$  is not present in  $S$ .

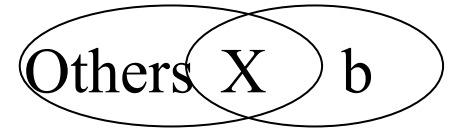
# Another BCNF Example

- $R(ABCDE)$

fd1:  $AB \rightarrow C$

fd2:  $D \rightarrow E$

$X \rightarrow b$



- Find closure of the following

- $AB^+ = ABC$

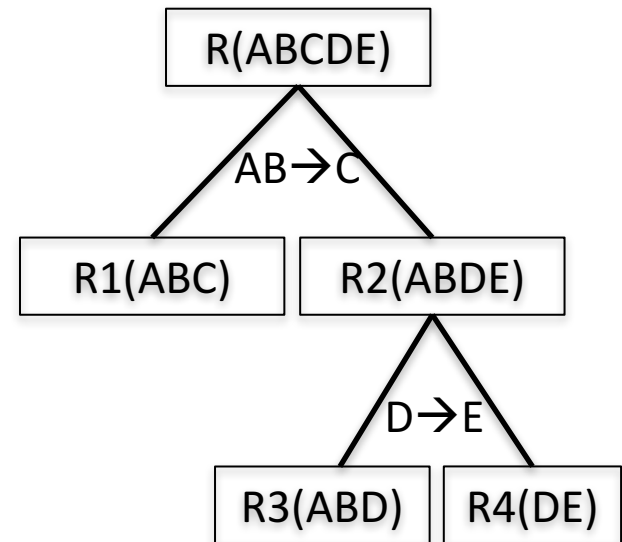
- $D^+ = DE$

- $AB \rightarrow C$  violates BCNF in  $R$

- $R1(ABC), R2(ABDE)$

- $D \rightarrow E$  violates BCNF in  $R2$

- $R3(ABD), R4(DE)$



Final answer:  $R1(ABC), R3(ABD), R4(DE)$

# Example: Implicit FDs matter

- $R(A,B,C,D,E,F)$

- $A^+ = ABC$

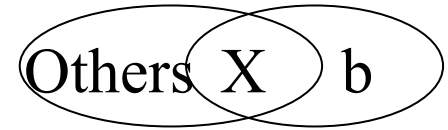
- $B^+ = BC$

- $DE^+ = DEF$

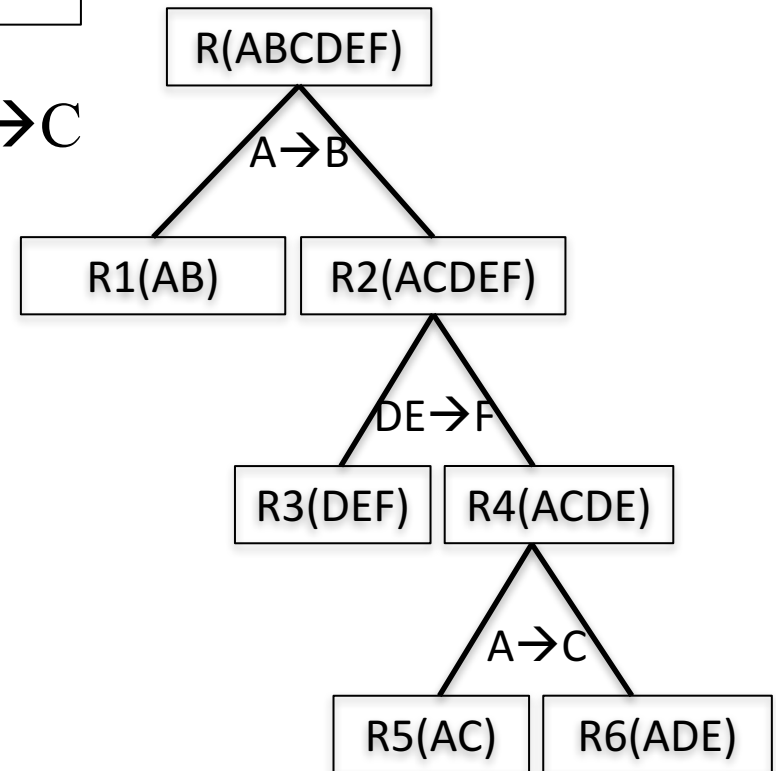
- $A^+$  contains C so an implicit FD  $A \rightarrow C$  holds. We add fd4 as  $A \rightarrow C$

|     |                    |
|-----|--------------------|
| fd1 | $A \rightarrow B$  |
| fd2 | $DE \rightarrow F$ |
| fd3 | $B \rightarrow C$  |

$X \rightarrow b$



- $A \rightarrow B$  is violating BCNF in R
  - $R1(AB), R2(ACDEF)$
  - $R1$  is BCNF, but  $R2$  is not in BCNF
- $DE \rightarrow F$  is violating BCNF in  $R2$ 
  - $R3(DEF), R4(ACDE)$
  - $R3$  is in BCNF, is  $R4$  in BCNF?
- $A \rightarrow C$  is violating BCNF in  $R4$ 
  - $R5(AC), R6(ADE)$



Final answer  $R1(AB), R3(DEF), R5(AC), R6(ADE)$

# Clicker Exercise: More BCNF

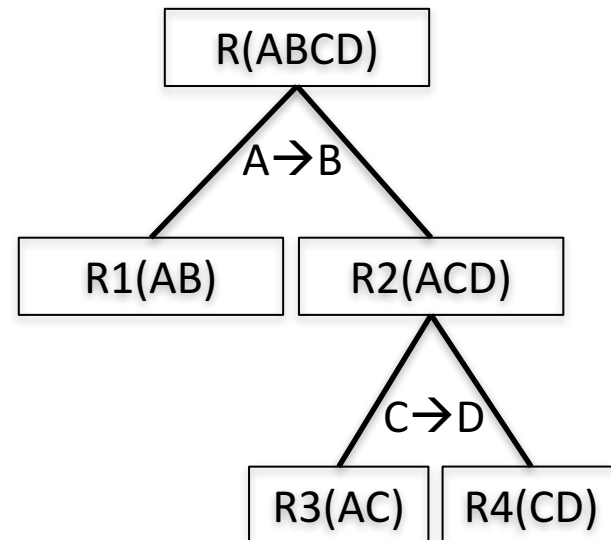
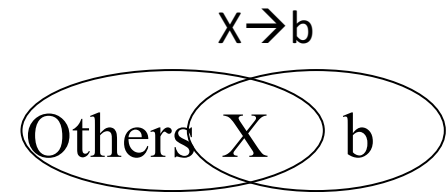
- Let  $R(ABCD)$  be a relation with functional dependencies  $A \rightarrow B$ ,  $C \rightarrow D$ ,  $AD \rightarrow C$ ,  $BC \rightarrow A$
- Decompose into BCNF. Which of the following is a lossless-join decomposition of  $R$  into Boyce-Codd Normal Form (BCNF)?

- A.  $\{AB, AC, BD\}$
- B.  $\{AB, AC, CD\}$
- C.  $\{AB, AC, BCD\}$
- D. All of the above
- E. None of the above

# Clicker Exercise: More BCNF

- Let  $R(ABCD)$
- Decompose into BCNF.
  - $A^+ = AB$
  - $C^+ = CD$
  - $AD^+ = ADBC$
  - $BC^+ = BCAD$
- $A \rightarrow B$  is violating BCNF
  - $R_1(AB), R_2(ACD)$
- $C \rightarrow D$  is violating BCNF
  - $R_3(AC), R_4(CD)$

|     |                    |
|-----|--------------------|
| fd1 | $A \rightarrow B$  |
| fd2 | $C \rightarrow D$  |
| fd3 | $AD \rightarrow C$ |
| fd4 | $BC \rightarrow A$ |



Final answer  $R_1(AB), R_3(AC), R_4(CD)$



# Clicker Exercise: More BCNF

- Let  $R(ABCD)$  be a relation with functional dependencies  $A \rightarrow B$ ,  $C \rightarrow D$ ,  $AD \rightarrow C$ ,  $BC \rightarrow A$
- Decompose into BCNF. Which of the following is a lossless-join decomposition of  $R$  into Boyce-Codd Normal Form (BCNF)?

A.  $\{AB, AC, BD\}$

Let's see what happens if you randomly decompose

B.  $\{AB, AC, CD\}$

$R_1(\underline{A}B)$ ,  $R_3(\underline{A}C)$ ,  $R_4(\underline{C}D)$

C.  $\{AB, AC, BCD\}$

$C^+ = CD$ , so  $C$  is not key for  $(BCD)$

D. All of the above

E. None of the above

# Clicker Exercise: Option 'A' exposed

- $R(ABCD)$  and  $A \rightarrow B, C \rightarrow D, AD \rightarrow C, BC \rightarrow A$   
A. Is  $\{AB, AC, BD\}$  a lossless join?

Imagine tuples:

| A | B | C | D |
|---|---|---|---|
| 1 | 2 | 5 | 6 |
| 1 | 2 | 3 | 7 |
| 8 | 2 | 9 | 4 |

decompose

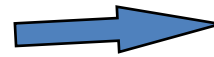


| A | B |
|---|---|
| 1 | 2 |
| 8 | 2 |

| A | C |
|---|---|
| 1 | 5 |
| 1 | 3 |
| 8 | 9 |

| B | D |
|---|---|
| 2 | 6 |
| 2 | 7 |
| 2 | 4 |

join



| A | B | C | D |
|---|---|---|---|
| 1 | 2 | 5 | 6 |
| 1 | 2 | 3 | 7 |
| 8 | 2 | 9 | 4 |
| 1 | 2 | 3 | 4 |

# BCNF is great, but...

- Guaranteed that there will be no redundancy of data
- Easy to understand (just look for superkeys)
- Easy to do.
- So what is the main problem with BCNF?
  - For one thing, BCNF may not preserve all dependencies

# Dependency Preservation

A functional dependency  $X \rightarrow Y$  is preserved in a relation R, if R contains all of the attributes of X and Y.

Example:

- $R(A,B,C)$       fd1  $A \rightarrow B$  ,      fd2  $A \rightarrow D$

fd1 is preserved in R and fd2 is not preserved in R

# An Illustrative BCNF example

| Unit | Company | Product |
|------|---------|---------|
|      |         |         |

Unit  $\rightarrow$  Company  
Company, Product  $\rightarrow$  Unit

Is unit a key?

| Unit | Company |
|------|---------|
|      |         |

Unit  $\rightarrow$  Company

| Unit | Product |
|------|---------|
|      |         |

The second functional dependency is no longer preserved.

# So What's the Problem?

| <u>Unit</u> | Company |
|-------------|---------|
| SKYWill     | ABC     |
| Team Meat   | ABC     |

| Unit      | Product   |
|-----------|-----------|
| SKYWill   | Databases |
| Team Meat | Databases |

Unit → Company

No problem so far. All *local* FD's are satisfied.  
Let's join the relations back into a single table again:

| Unit      | Company | Product   |
|-----------|---------|-----------|
| SKYWill   | ABC     | Databases |
| Team Meat | ABC     | Databases |

Violates the FD: Company, Product → Unit

# Third Normal Form (3NF)

A relation R is in 3NF if for all non-trivial dependencies in R:

If  $X \rightarrow b$  then

X is a superkey for R OR

b is a prime attribute of R

- A dependency is trivial if the LHS contains the RHS, e.g., City, Province  $\rightarrow$  City is a trivial dependency

Example: R(Unit, Company, Product)

- Unit  $\rightarrow$  Company
- Company, Product  $\rightarrow$  Unit  
Keys: {Company, Product}, {Unit, Product}
- R is not in BCNF but is in 3NF.

To decompose into 3NF we rely on the *minimal cover*

# Minimal Cover for a Set of FDs

- Goal: Transform FDs to be as small as possible.

Minimal cover G for a set of FDs F:

1. Closure of F = closure of G (i.e., imply the same FDs)
2. Right hand side of each FD in G is a single attribute
3. If we delete an FD in G or delete attributes from an FD in G, the closure changes

- **Informally:** Every FD in G is needed, and is “*as small as possible*” in order to get the same closure as F

Example:

- $A \rightarrow B$ ,  $ABCD \rightarrow E$ ,  $EF \rightarrow GH$ ,  $ACDF \rightarrow EG$  has the following minimal cover:
  - $A \rightarrow B$ ,  $ACD \rightarrow E$ ,  $EF \rightarrow G$  and  $EF \rightarrow H$



# Finding Minimal Covers of FDs

1. Put FDs in standard form (have only one attribute on RHS)
2. Minimize LHS of each FD
3. Delete Redundant FDs

Example:

$A \rightarrow B$ ,  $ABCD \rightarrow E$ ,  $EF \rightarrow G$ ,  $EF \rightarrow H$ ,  $ACDF \rightarrow EG$

- Replace last rule with
  - $ACDF \rightarrow E$
  - $ACDF \rightarrow G$

# Finding Minimal Covers of FDs

1. Put FDs in standard form (have only one attribute on RHS)
2. Minimize LHS of each FD
3. Delete Redundant FDs

Example:

$A \rightarrow B$ ,  $ABCD \rightarrow E$ ,  $EF \rightarrow G$ ,  $EF \rightarrow H$ ,  $ACDF \rightarrow E$ ,  $ACDF \rightarrow G$

- Can we take anything away from the LHS?
  - $ABCD^+ = ABCDE$
  - $ACD^+ = ABCDE$ , so remove B from the FD

# Finding Minimal Covers of FDs

1. Put FDs in standard form (have only one attribute on RHS)
2. Minimize LHS of each FD
3. Delete Redundant FDs

Example:

$A \rightarrow B$ ,  $ACD \rightarrow E$ ,  $EF \rightarrow G$ ,  $EF \rightarrow H$ ,  $ACDF \rightarrow E$ ,  $ACDF \rightarrow G$

- Let's find  $ACDF^+$  without considering the highlighted FDs
  - $ACDF^+ = ACDFEBGH$ , so I can remove the highlighted rules
- Final answer:  $A \rightarrow B$ ,  $ACD \rightarrow E$ ,  $EF \rightarrow G$ ,  $EF \rightarrow H$

# Minimal Example Cover

- Consider the relation  $R(CSJDPQV)$  with FDs  
 $C \rightarrow SJDPQV$ ,  $JP \rightarrow C$ ,  $SD \rightarrow P$ ,  $J \rightarrow S$

Find a minimal cover

# Another Minimal Cover Example

- Consider the relation  $R(CSJDPQV)$  with FDs  
 $C \rightarrow SJDPQV$ ,  $JP \rightarrow C$ ,  $SD \rightarrow P$ ,  $J \rightarrow S$

Find a minimal cover

–  $C \rightarrow S$ ,  $C \rightarrow J$ ,  $C \rightarrow D$ ,  $C \rightarrow P$ ,  $C \rightarrow Q$ ,  $C \rightarrow V$ ,  $JP \rightarrow C$ ,  $SD \rightarrow P$ ,  $J \rightarrow S$

Can we shorten any FDs?

Let's consider shortening  $JP \rightarrow C$

$$JP^+ = CSJDPQV$$

$$J^+ = JS$$

$$P^+ = P$$

Let's consider shortening  $SD \rightarrow P$

$$SD^+ = SDP$$

$$S^+ = S \text{ and } D^+ = D$$

# Another Minimal Cover Example

- Consider the relation  $R(CSJDPQV)$  with FDs  $C \rightarrow SJDPQV$ ,  $JP \rightarrow C$ ,  $SD \rightarrow P$ ,  $J \rightarrow S$

Find a minimal cover

–  $C \rightarrow S$ ,  $C \rightarrow J$ ,  $C \rightarrow D$ ,  $C \rightarrow P$ ,  $C \rightarrow Q$ ,  $C \rightarrow V$ ,  $JP \rightarrow C$ ,  $SD \rightarrow P$ ,  $J \rightarrow S$

Can we shorten any FDs? No

Can we remove any FDs?

- Let's consider  $C \rightarrow S$  and find  $C^+$  without considering this rule
  - $C^+ = SJDPQV$ , so we can delete this FD
- Let's consider  $C \rightarrow P$  and find  $C^+$  without considering this rule
  - $C^+ = SJDQVP$ , so we can delete this FD

# Minimize LHS before deleting redundant FDs

- If step 3 is done prior to step 2, the final set of FDs could still contain redundant FDs

- $ABCD \rightarrow E, E \rightarrow D, A \rightarrow B, AC \rightarrow D$

- Let's delete redundant FDs.
- None of the FDs are redundant.

1. One attribute on RHS
2. Delete Redundant FDs
3. Minimize LHS of each FD

Does not work

- $ABCD \rightarrow E, E \rightarrow D, A \rightarrow B, AC \rightarrow D$

- Now let's shorten FDs
- $ABCD \rightarrow E$  can be replaced by  $AC \rightarrow E$

- However the current set of FDs are not minimal

- $AC \rightarrow E, E \rightarrow D, A \rightarrow B, AC \rightarrow D$
- The highlighted FD can be deleted

# Decomposition into 3NF Using Minimal Cover

- Decomposition into 3NF:
  1. Given the FDs  $F$ , compute  $F'$ : the *minimal cover for  $F$*
  2. Decompose using  $F'$  if violating 3NF similar to how it was done for BCNF
  3. After each decomposition identify the set of dependencies  $N$  in  $F'$  that are not preserved by the decomposition.
  4. For each  $X \rightarrow a$  in  $N$  create a relation  $R_n(X \cup a)$  and add it to the decomposition



# 3NF Example

- Example:  $R(ABCDE)$

- Cover already minimal

- »  $ABE^+ = ABCDE$  **only key**

- $AB \rightarrow C$  is violating 3NF

- $R_1(ABC), R_2(ABDE)$

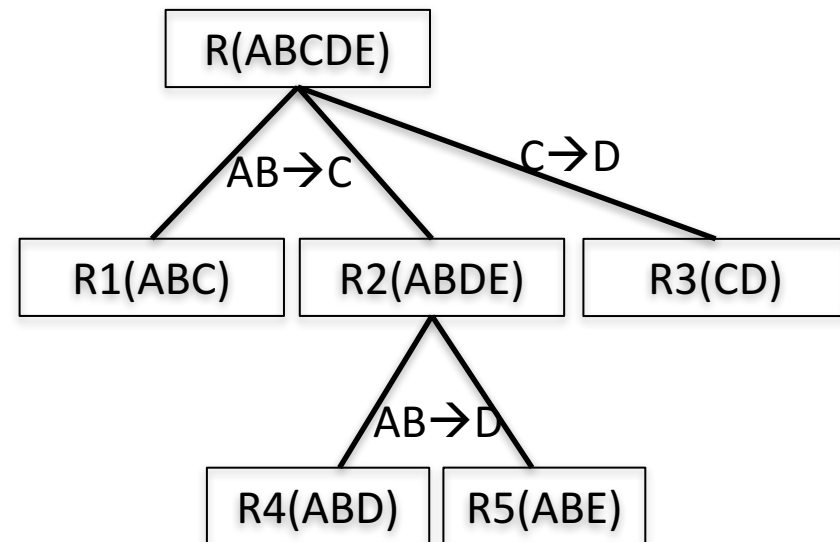
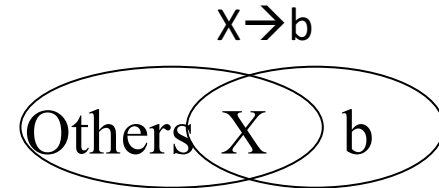
- Are all FDs preserved?

- We lost  $C \rightarrow D$  so add  **$R_3(CD)$**

- $AB \rightarrow D$  (transitivity) is violating 3NF

- $R_4(ABD), R_5(ABE)$

|     |                    |
|-----|--------------------|
| fd1 | $AB \rightarrow C$ |
| fd2 | $C \rightarrow D$  |

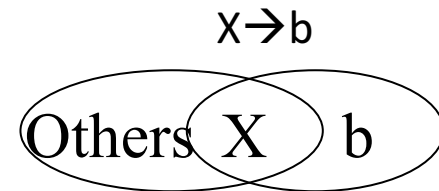


Final answer:  $R_1(ABC), R_3(CD), R_4(ABD), R_5(ABE)$

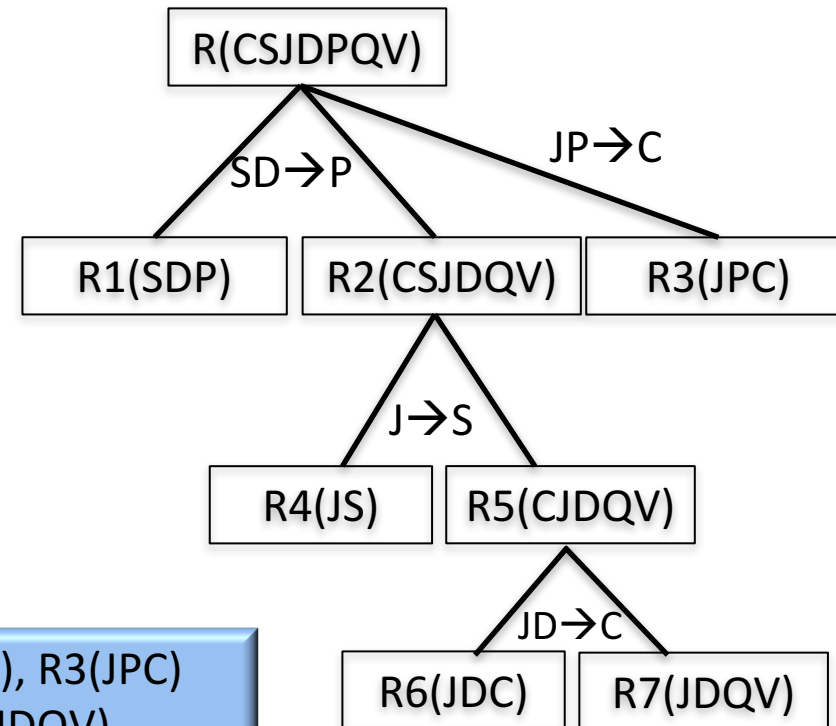
# Another 3NF Example

- Let  $R(CSJDPQV)$  be a relation with the following FDs. Decompose  $R$  into 3NF
  - Already in minimal cover
  - $SD^+ = SDP$
  - $JP^+ = JPSC$
  - $J^+ = JS$
  - $JD^+ = JDSPC$
  - $JDQV^+ = CSJDPQV$  **Key**

|     |                    |
|-----|--------------------|
| fd1 | $SD \rightarrow P$ |
| fd2 | $JP \rightarrow C$ |
| fd3 | $J \rightarrow S$  |



- $SD \rightarrow P$  violates 3NF in  $R$ 
  - $R_1(SDP)$ ,  $R_2(CSJ DQV)$
- Are all FDs preserved?
  - We lost  $JP \rightarrow C$  so add  **$R_3(JPC)$**
- $J \rightarrow S$  violates 3NF in  $R_2$ 
  - $R_4(JS)$ ,  $R_5(CJ DQV)$
- $JD \rightarrow C$  (implicit)
  - $R_6(JDC)$ ,  $R_7(JDQV)$



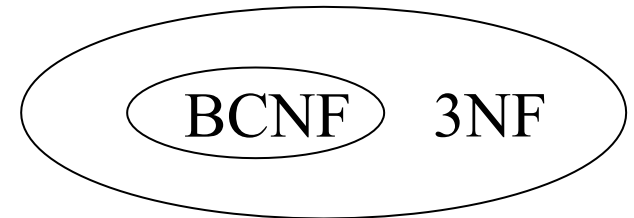
Final answer  $R_1(SDP)$ ,  $R_3(JPC)$   
 $R_4(JS)$ ,  $R_6(JDC)$ ,  $R_7(JDQV)$

# Notes

- Please use the form published under **Learning Resources --> Module 3** to provide feedback on your learning experience in this module.
- **Lecture:** Finishing Module 3 and then starting Module 4, which focuses on SQL.
- **Tutorial:** Reviewing Quiz 1
- **Practical:** Providing support for assignment 1. Make sure you use the provided template and the spacing provide.
- **RiPPLE:** Round 2 grades have been released on BB. Round 3 ends 28 September.

# BCNF & 3NF

- BCNF guarantees removal of all anomalies
- 3NF has some anomalies, but preserves all dependencies
- If a relation R is in BCNF it is in 3NF.
- A 3NF relation R may not be in BCNF.



- Most organizations go to BCNF or 3NF.

# Clicker Question: BCNF and 3NF

- Consider the following relation and functional dependencies:

R(ABCD) FD's:  $ACD \rightarrow B$  ;  $AC \rightarrow D$  ;  $D \rightarrow C$  ;  
 $AC \rightarrow B$

Which of the following is true:

- A. R is in neither BCNF nor 3NF
- B. R is in BCNF but not 3NF
- C. R is in 3NF but not in BCNF
- D. R is in both BCNF and 3NF

# Clicker Question: BCNF and 3NF

- Consider the following relation and functional dependencies:

R(ABCD) FD's:  $ACD \rightarrow B$  ;  $AC \rightarrow D$  ;  $D \rightarrow C$  ;  
 $AC \rightarrow B$

Which of the following is true:

- A. R is in neither BCNF nor 3NF
- B. R is in BCNF but not 3NF
- C. R is in 3NF but not in BCNF
- D. R is in both BCNF and 3NF

$ACD^+ = ABCD$

$AC^+ = ACDB$

$D^+ = DC$

$AD^+ = ADCB$

Keys: AC, AD

$D \rightarrow C$  (D not key so NOT BCNF)

C is part of a minimal key  
so R is in 3NF

# Denormalization

- Process of intentionally violating a normal form to gain performance improvements
- Performance improvements:
  1. Fewer joins
  2. Reduces number of foreign keys
- Useful in data analysis or if certain queries often require (joined) results, and the queries are frequent enough.

# Learning Outcomes

| Description  | Tag                     |
|--|-------------------------|
| Provide examples of modification, insertion, and deletion anomalies.   | Functional-dependencies |
| Explain the term functional dependency.  |                         |
| Given a set F of functional dependencies and a functional dependency fd, determine whether fd can be inferred from F using Armstrong's Axioms.   |                         |
| Given a set of functional dependencies that hold over a table, determine the keys.   |                         |
| Given a set of functional dependencies that hold over a table, determine the superkeys.  |                         |
| Given a set F of functional dependencies and X of attributes of a relation, compute the closure of X, which is X <sup>+</sup>                    |                         |
| Given a relation R, be able to correctly decompose it into two or more relations   | Decomposition           |
| Justify why lossless join decompositions are preferred decompositions.   |                         |
| Explain the benefits of Normalization.   | Normalization           |
| Given a relation schema R and a set of functional dependencies on it, show that R is/isn't in 1NF.   |                         |
| Explain what dependency preservation means.  |                         |
| Given a relation schema R and a set of functional dependencies on it, show that R is/isn't in 3NF.   |                         |
| Given a relation schema R and a set of functional dependencies on it, show that R is/isn't in BCNF.  |                         |
| Describe the process and benefits of Denormalization.  |                         |
| Reason with the logical foundation of the relational data model and understand the fundamental principles of correct relational database design. |                         |