FROM NO-LINEAR PROBLEM TO CGA

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$$Y(t) = f(X(t), t) \tag{1}$$

$$\frac{\partial Y}{\partial t} = F(X,t), \frac{\partial Y}{\partial X} = G(X,t), \frac{\partial X}{\partial t} = M(X,t) \tag{2}$$

$$Y(X,t) - Y(X_0,t_0) = \int\limits_{t_0}^{t} F(X,t)dt \tag{3}$$

$$\delta Y(t) = F(X,t)|_{X=X_0,t=t_0}\delta t + G(X,t)|_{X=X_0,t=t_0}\delta X \tag{4}$$

$$\lim\nolimits_{\delta t\to 0} \delta Y(t) = G(X,t)|_{X=X_0,t=t_0}\delta X, \text{for quickly changing situation.} \tag{5}$$

So,many problems could be transformed into linear solve method, even it is no-linear system. For $\delta Y = G\delta X$, the solver is $\delta X = U_{m\times m}G^T V_{o\times o}\delta Y$,in which

$$\begin{cases} U = (GP^{-1}G^T + R^{-1})^{-1} \\ V = R^{-1} \end{cases} ,\text{or } \begin{cases} U = P \\ V = (GPG^T + R)^{-1} \end{cases} \tag{6}$$

.

Let's begin with

$$\begin{cases} \delta Y = GS\delta X \\ \delta Y = PG\delta X \end{cases} \to \begin{cases} P = R^{-1} = (G^T G)^{-1}G^T SG \\ S = GPG^T(GG^T)^{-1} \end{cases} \tag{7}$$

.

For $\begin{cases} S \to \alpha_1 I_{m\times m} \\ P \to \alpha_2 I_{o\times o} \end{cases}$, we could get

$$\begin{cases} GP^{-1}G^T \to \beta_1 I_{o\times o} \\ GPG^T \to \beta_2 I_{o\times o} \end{cases} \to \begin{cases} U \to \kappa_1 I_{o\times o} \\ V \to \kappa_2 I_{m\times m} \end{cases} \to \delta X = \lambda G^T \delta Y. \tag{8}$$

, which is Conjugate-Gradinet Algorithm.

From Eq 4,we could get