Requiring the partition function to be modular invariant, gives some properties to $\epsilon(\alpha,\beta)$.

▶ Under
$$\tau \rightarrow \tau + 1$$

$$\{\alpha,\beta\} \to e^{-i\pi n(\alpha)/8} \{\alpha,\bar{\alpha}\beta\}.$$

This will require $\epsilon(\alpha, \beta) = \varepsilon_{\alpha} \epsilon(\alpha, \bar{\alpha}\beta)$.

▶ Under $\tau \to -\frac{1}{\tau}$

$$\{\alpha,\beta\} \to e^{i\pi n(\alpha\cap\beta)/4}\{\beta,\alpha\}.$$

This will require
$$\epsilon(\alpha, \beta) = \delta_{\alpha} \delta_{\beta} \epsilon(\beta, \alpha) \varepsilon_{\alpha \cap \beta}^{-2}$$
.

In the equations above $\varepsilon_{\alpha} = e^{-i\pi n(\alpha)/8}$.