

Byron's solution:

1. Given $[2, 5, 3, 4, 1]$

2. sort them and give each number

an index. 1 2 3 4 5
 \Rightarrow
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $[1] [2] [3] [4] [5]$
* # of rating on the left of i and $< \text{ratings}[i]$

3. compute $\text{less}[i]$ using Binary Indexed Tree

for i, r in ratings : and on the left
ratings $< r$

$\text{less}[i] = \text{query}(\text{map}[r], \text{bit})$

$\text{update}(\text{map}[r], \text{bit}, 1)$

4. compute $\text{triplet}[i]$ using BIT and $\text{less}[i]$

for i, r in ratings:

$\text{triplet}[i] = \text{query}(\text{map}[r], \text{bit}, 2)$

$\text{update}(\text{map}[r], \text{bit2}, \text{less}[i])$

for every r_j where $j < i$
and $r_j < r_i$

we sum over their
 $\text{less}[j]$

Intuition:

$r_0 r_1 r_2 r_3 r_4 r_5 r_6 r_7 r_8$
 $\text{less}: [0] [1] [2] [3] [4] [5] [6] [7] [8]$

When $i = 5$, Ex. triplet $[5] =$

we have only $\text{Sum}(\text{less}[j] \text{ for } j < i \text{ and}$
seen these numbers $r_j < r_i)$

$\hookrightarrow s = \text{query}(\text{map}[r_5], \text{bit2})$ Ensure $r_j < r_i$

Generalized to $k \geq 4$ soldiers:

Given: $[r_0 r_1 r_2 r_3 r_4 r_5 r_6 r_7 r_8]$

Compute triplet,

$$\text{triplet}[i] = \text{sum} \left(1 \text{ for } k < j < i \text{ and } r_k < r_j < r_i \right)$$

$$\text{quad}[i] = \text{sum} \left(\text{triplet}[j] \text{ for } j < i \text{ and } r_j < r_i \right)$$

Ex.



If only $r_2 < r_6$, $r_4 < r_6$:

then $\text{quad}[6] = \text{triplet}[2] + \text{triplet}[4]$