

First refinement:  
for each prime,

mark  $\boxed{p^2}$ ,  $(p+1)p$ ,  $\dots$

$\downarrow$   
because for  $\text{num} = \underline{x} \cdot p < p \cdot p$

assume every  $p' < p$  has  
mark all its multiples

$$\Rightarrow \text{num} = \underline{\tilde{x}} \cdot p$$

$\hookrightarrow$  does not contain  
 $p$  as factor

$$\therefore \tilde{x} < p$$

$\tilde{x}$  must be a composite number  
formed by smaller primes

Second refinement:

$2 \quad 3 \quad 4 \quad 5 \quad 6 \quad - - - \sqrt{n} \quad - - -$

Note that:

for each  $p$ ,  $p^2, (p+1)p^2, \dots$   
are what we mark

but when  $p > \sqrt{n}$ ,  $p \cdot q > n$

$\Rightarrow$  no need to mark any more!

bound =  $\lfloor \sqrt{n} \rfloor \rightarrow$  why not  $\lceil \sqrt{n} \rceil$

for  $i$  in  $[2, \text{bound}]$

mark  $i^2, (i+1)i, \dots$

because say

$$\sqrt{n} = 2 \dots$$

when  $p=2$  still have check

$p=3$  we don't need to check