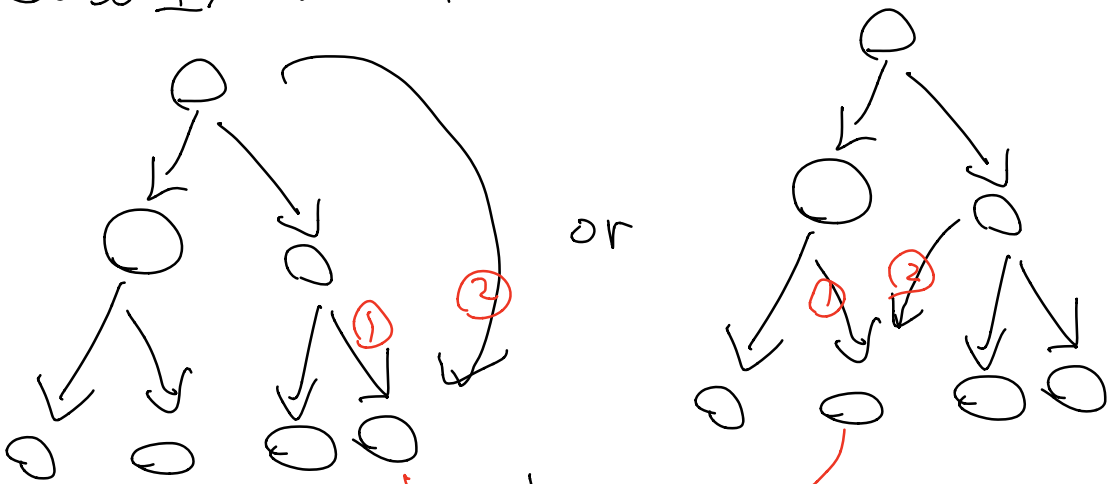
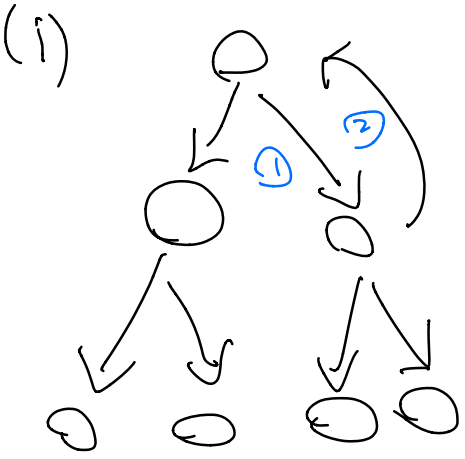
Case 1: ↓ : parent points to a descendant



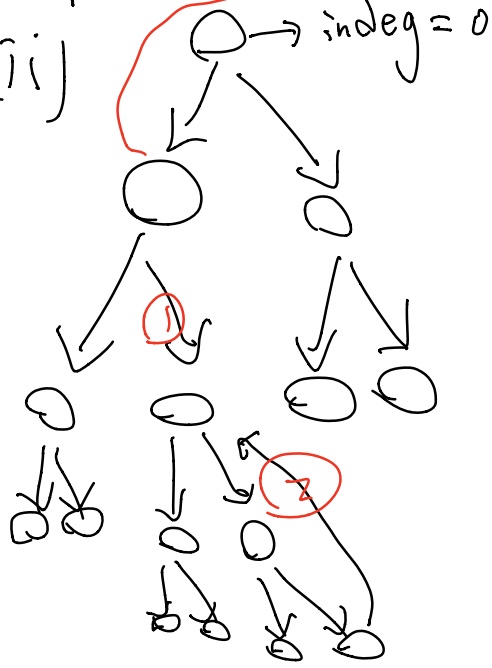
Case 2: ↑ a descendant points to a parent

(i)

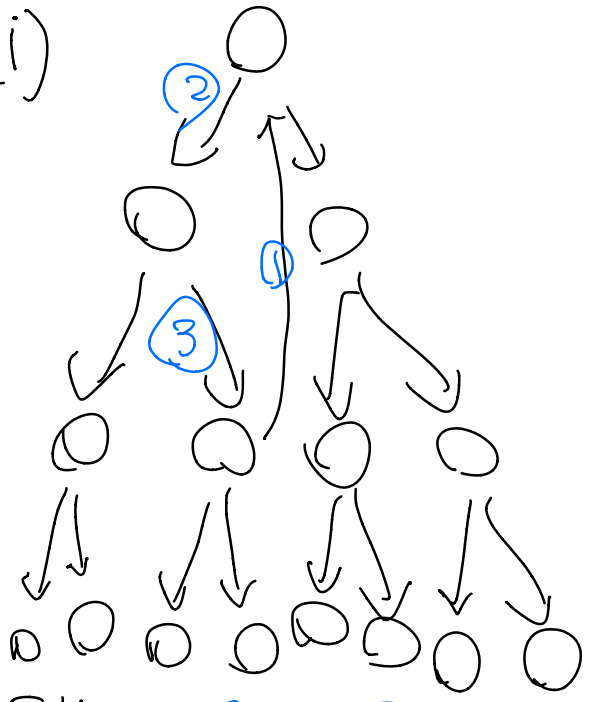


Either (1) or (2) can be removed

(ii)



(i)



Either (1), (2), (3) can be removed

Algorithm:

For Case 1, Case 2(ii),
we only have (1), (2) to choose.
And because Case 2(ii), we don't know which edge should we remove.
⇒ Just try removing (1) and do an union find,
if we don't see a loop ⇒ remove (1)
else ⇒ remove (2)

For case 2 (i),
every edge on the loop can be removed

$\Rightarrow$ Do an union find

$\Rightarrow$ return the edge that causes a loop

#