Manacher's algorithm

Idea: using Mirror property

Assume: for $a_0 a_1 a_2 \cdots a_{n-1}$

P[i] is the longest palindrome centered at i's diameter (i.e. $a_{i-P[i]} \cdots a_{i-1} a_i \cdots a_{i+P[i]}$)

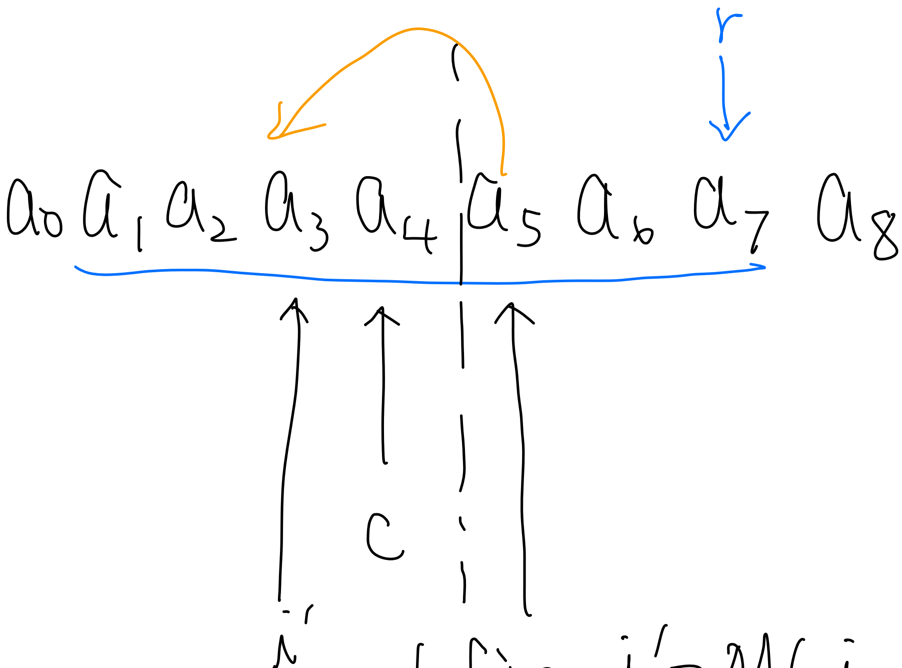
Assume P[0]~P[4] is known

r

$a_0 \underbrace{a_1 a_2 a_3 a_4 \mid a_5 a_6 a_7} a_8$

C

How to effectively use mirror property to compute P[5]~P[7]



define $i' = M(i, c)$ as the mirror index of i around c

$$i' = M(i, c) \Rightarrow c - i' = i - c$$
$$\Rightarrow i' = 2c - i$$

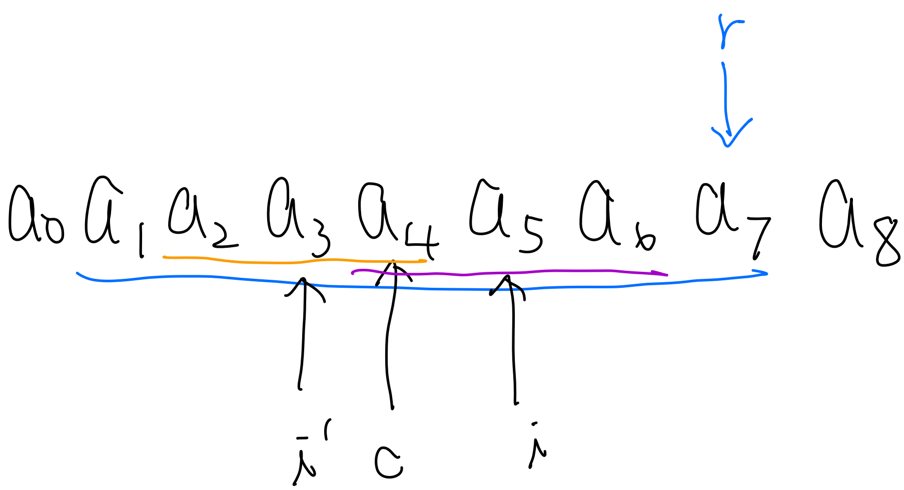
There are 3 cases for $P[i']$:

Case 1: $i + P[i'] < r$

Case 2: $i + P[i'] = r$

Case 3: $i + P[i'] > r$

Case 1: $i + P[i'] < r$



We know that apparently $P[i] \geq P[i']$,

Let's check if i can expand beyond $P[i']$

Assume $a_{i-P[i']-1} = a_{i+P[i']+1}$

let's mirror $\underline{i-P[i']-1}$ and $\underline{i+P[i']+1}$

$$\Rightarrow a_{M(i-P[i']-1,\, C)} = a_{2C-(i-P[i']-1)}$$

$$= a_{\underline{2C-i}+P[i']+1}$$

$$= a_{i'+P[i']+1}$$

$$\Rightarrow a_{M(i+P[i']+1,\, c)} = a_{\underline{2c-i}-P[i']-1}$$

$$= a_{i'-P[i']-1}$$

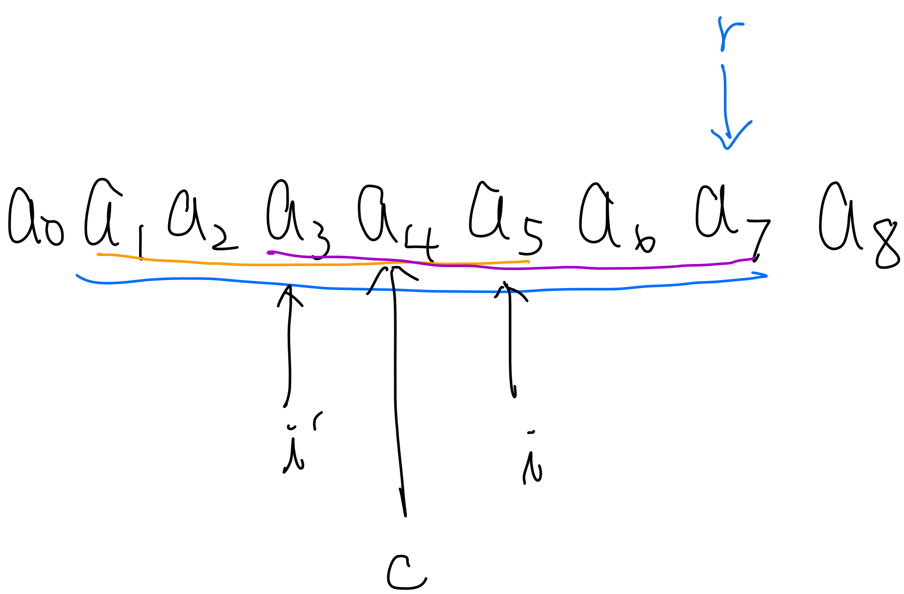
So $a_{i-P[i']-1} = a_{i+P[i']+1}$

$\Leftrightarrow a_{i'+P[i']+1} = a_{i'-P[i']-1}$

This tells us that i' can be further expanded, contradict our assumption that $P[i']$ is the longest palindrome diameter.

$\Rightarrow P[i] = P[i']$

Case 2: $i + P[i'] = r$



apparently, $P[i] \geq P[i']$, but we need to check if it is possible $P[i] > P[i']$

We want to check

$$a_{i+P[i']+1} \overset{?}{=} a_{i-P[i']-1}$$

let us mirror the above equation around C

$$a_{M(i+P[i']+1,\, C)} = a_{\underline{2C-i}-P[i']-1}$$
$$= a_{i'-P[i']-1}$$

$$a_{M(i-P[i']-1,\, C)} = a_{\underline{2C-i}+P[i']+1}$$
$$= a_{i'+P[i']+1}$$

Observe that

$a_{i+P[i']+1}$ and $a_{i'-P[i']+1}$

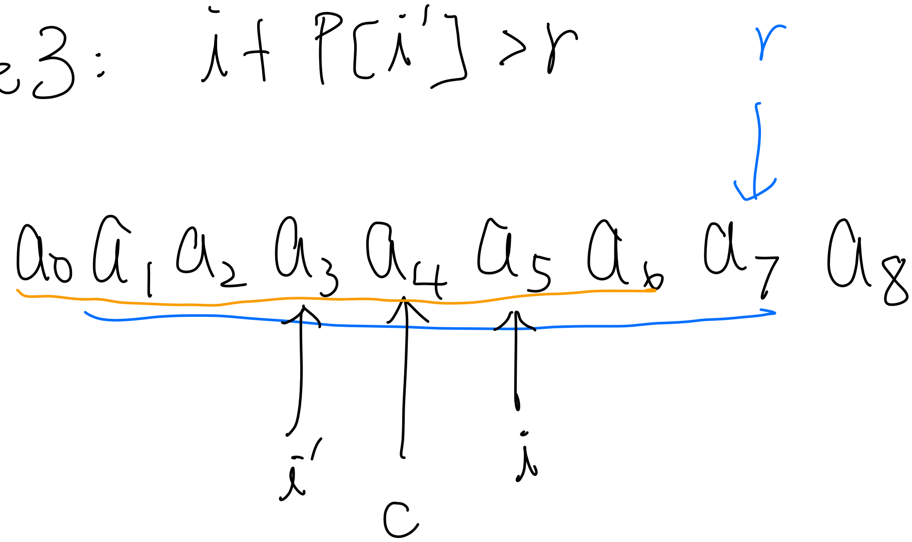
equality is unknown because

$i+P[i']+1 > r$ and $i'-P[i']+1 < M(r, C)$

$\Rightarrow$ We have no information to use by mirroring!

Therefore, in case 2, we have to expand r until we encounter an inequality!

Case 3: if $P[i'] > r$



Apparently, $P[i] \geq P[i']$, but we still need to check if $P[i] > P[i']$

Let's assume $a_{i+P[i']+1} = a_{i-P[i']-1}$

let us mirror it around c:

$$a_{M(i+P[i']+1,\, c)} = a_{2c-i-P[i']-1}$$
$$= a_{i'-P[i']-1}$$
$$a_{M(i-P[i']+1,\, c)} = a_{2c-i+P[i']-1}$$

$$= a_{i'} + P[i'] + 1$$

observe that

$$\underline{a_i + P[i'] + 1} = a_i - P[i'] - 1$$

and $a_i + P[i'] + 1 = \underline{a_{i'} - P[i'] - 1}$

and $a_i - P[i'] - 1 = a_{i'} + P[i'] + 1$

$$\Rightarrow \quad a_i + P[i'] + 1 = a_{i'} - P[i'] - 1$$

This tells us that $P[c]$ can be further expanded!

Contradicts that $P[c]$ already is the maximum

$$\Rightarrow P[i] = r - i$$