

Proof: max_freq invalid at
some point will not affect
our answer!

max_freq will only become greater
if $[i, r)$ has a char's $\text{freq} > \text{max_freq}$

$\begin{array}{ccccccc} & & i & & & & r \\ & & \downarrow & & & & \downarrow \\ a_0 & a_1 & a_2 & a_3 & \dots & & a_{n-1} \end{array}$

Case 1: for every $[i, r)$ where $\text{true_max_freq} \leq \text{max_freq}$

\Rightarrow Because their $\text{true_max_freq} \leq \text{max_freq}$
they will only expand at most $\text{len} = \text{max_freq} + k$
 \Rightarrow will not affect answer

Case 2:- for every $[i, r)$ where $\text{true-max-freq} > \text{max-freq}$

Because

max-freq will be updated in the while

loop, $\text{len} = \text{true-max-freq} + 1$