Iqra University

Islamabad Campus

Quiz No. 3 (Open Book)

Course Code: CSC-103	Course Title: MULTIVARIATE CALCULUS
Teacher's Name: Imama Ejaz	Total Marks: 20 Time: 3hrs

Question No. 1 (3+4+3)

a. Find the value of $\partial x / \partial z$ at the point $(1, -1, -3)$ if the equation

$$xz + y \ln x - x^2 + 4 = 0$$

defines x as a function of the two independent variables y and z and the partial derivative exists.

b. **The heat equation** An important partial differential equation that describes the distribution of heat in a region at time t can be represented by the *one-dimensional heat equation*

$$\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2}.$$

Show that $u(x, t) = \sin(\alpha x) \cdot e^{-\beta t}$ satisfies the heat equation for constants α and β. What is the relationship between α and β for this function to be a solution?

c. Find and sketch the domain of f. Then find an equation for the level curve or surface of the function passing through the given point:

$$f(x, y) = \int_x^y \frac{d\theta}{\sqrt{1 - \theta^2}}, \quad (0, 1)$$

Question No. 3   (3+3+4)

a. Find an equation for the parabola with focus (4, 0) and directrix $x = 3$. Sketch the parabola together with its vertex, focus, and directrix.

b. The vertices of an ellipse of eccentricity 0.5 lie at the points $(0, \pm 2)$. Where do the foci lie?

c. Show that the function

$$f(x, y) = \frac{2x^2y}{x^4 + y^2}$$

has no limit as (x, y) approaches $(0, 0)$.