"NAME: MAQSOOD AHMED

ID: 38186

DEPT: BSCS

CAL-II: ASSIGNMENT #02"

Apply nth-divergence test and find which of the following series converge, diverge or series inconclusive:

1) $\sum_{n=1}^{\infty} \frac{n(n+1)}{(n+2)(n+3)}$

Sol:-

Applying the Ratio Test

$\Rightarrow \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

$\Rightarrow \lim_{n\to\infty} \frac{(n+1)(n+2)}{(n+3)(n+4)} \cdot \frac{(n+2)(n+3)}{n(n+1)}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{n+4} = 0$

$\Rightarrow$ Since the limit is less than 1, the series converges.

) $\sum_{n=1}^{\infty} \ln\left(\frac{1}{n}\right)$

Applying the Ratio Test:

$\Rightarrow \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

$\Rightarrow \lim_{n\to\infty} \frac{\ln\left(\frac{1}{n+1}\right)}{\ln\left(\frac{1}{n}\right)}$

$\lim_{n\to\infty} \ln\left(\frac{n}{n+1}\right)$

$\Rightarrow \ln(1) = 0$

$\Rightarrow$ Since, the limit is less than 1 the series converges.

3) $\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n}$ (Geometric Series, if converge find sum)

This is a geometric series with the common ratio $r = \frac{2}{5}$. The general form of a geometric series is given by:

$$s = \frac{a}{1-r}$$

where s is the sum, a is the first term and r is the common ratio.

The first term a is given by

$$\Rightarrow a = \frac{2^{0+1}}{5} = \frac{1}{5}$$

$$\Rightarrow r = \frac{2}{5}$$

$$\Rightarrow s = \frac{a}{1-r} = \frac{1/5}{1-2/5} = \frac{1}{5} \cdot \frac{5}{3} = \frac{1}{3}$$

Apply Ratio or Root test to find which of the following series Converge, diverge or series inconclusive:

1) $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{1.2^n}$

Applying the Ratio Test

$\Rightarrow \lim_{n\to\infty} \sqrt[n]{\left|\frac{2+(-1)^n}{1.2^n}\right|}$

$= \lim_{n\to\infty} \sqrt[n]{\left|\frac{2}{1.2}\right|}$

$\Rightarrow \frac{1}{1.2} < 1$

Converge !

2) $\sum_{n=1}^{\infty} (-1)^n \frac{n+2}{3^n}$

Applying the Root Test

$\lim_{n\to\infty} \sqrt[n]{\left|(-1)^n \cdot \frac{n+2}{3^n}\right|}$

$\Rightarrow \lim_{n\to\infty} \frac{n+2}{3} \sqrt[n]{1/3}$

$$\Rightarrow \lim_{n \to \infty} \sqrt[n]{\frac{1}{3}} = \frac{1}{3}$$

$$\Rightarrow \lim_{n \to \infty} \frac{n+2}{3} = \infty$$ the series diverges.

3) $\sum_{n=1}^{\infty} \frac{(-1)^n (3n)!}{n!\,(n+1)!\,(n+2)!}$ Ratio Test.

Applying the Ratio Test

$$\Rightarrow \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$\Rightarrow \lim_{n \to \infty} \frac{(-1)^{n+1} (3n+3)!\, n!\, (n+1)!\, (n+2)!}{(-1)^n (3n)!\, (n+1)!\, (n+2)!\, (n+3)!}$$

$$\Rightarrow \lim_{n \to \infty} \frac{1}{8(n+3)} = 0$$

$\Rightarrow$ Since the limit is less than 1 the series converges.

4) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$ Ratio Test

Applying the Ratio Test.

$$\Rightarrow \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$\Rightarrow \lim_{n\to\infty} \frac{\ln(n+1)}{(n+1)^3} \cdot \frac{n^3}{\ln(n)}$$

$$\Rightarrow \lim_{n\to\infty} \frac{\ln(n+1)}{\ln(n)} \cdot \frac{n^3}{(n+1)^3} = 1$$

$\Rightarrow$ Since the limit is equal to 1 the ratio test is inconclusive. However, the series converges absolutely.

so $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$ is convergent.