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QUIZ: 03 (CAL-II)"

Q No: 1:-

a) Find the value $\partial x/\partial z$ at the point $(1,-1,-3)$ if the equation
$$xz + y\ln x - x^2 + 4 = 0$$

Sol.

operate $\frac{\partial}{\partial z}$ on each side of the implicit function

$$xz + y\ln x - x^2 + 4 = 0$$

$$\Rightarrow \frac{\partial}{\partial z}(xz + y\ln x - x^2 + 4) = \frac{\partial}{\partial z}(0)$$

$$\Rightarrow \frac{\partial}{\partial z}(xz) + \frac{\partial}{\partial z}(y\ln x) - \frac{\partial}{\partial z}(x^2) + \frac{\partial}{\partial z}(4) = 0$$

$$\Rightarrow x\frac{\partial}{\partial z}(z) + z\frac{\partial}{\partial z}x + y\frac{\partial}{\partial z}(\ln x)\frac{\partial}{\partial z} \cancel{\ln x}$$
$$- x - 2x\frac{\partial x}{\partial z} + 0 = 0$$

$$\Rightarrow x + z\frac{\partial x}{\partial z} + \frac{y}{x}\frac{\partial x}{\partial z} - 2x\frac{\partial x}{\partial z} = 0$$

Solve the obtained equation.

$$\Rightarrow x + z\frac{\partial x}{\partial z} + \frac{y}{x}\frac{\partial x}{\partial z} - 2x\frac{\partial x}{\partial z} = 0 \text{ for } \frac{\partial x}{\partial z}$$

$$\Rightarrow z\frac{\partial x}{\partial z} + \frac{y}{x}\frac{\partial x}{\partial z} - 2x\frac{\partial x}{\partial z} = -x$$

$$\Rightarrow \frac{\partial x}{\partial z}\left(z + \frac{y}{x} - 2x\right) = -x$$

$$\Rightarrow \frac{\partial x}{\partial z}\left(\frac{xz + y - 2x^2}{x}\right) = -x$$

$$\Rightarrow \frac{\partial x}{\partial z} = \frac{-x^2}{xz + y - 2x^2}$$

$\Rightarrow$ Now, From limits

$$\frac{dx}{dz}(1, -1, -3)$$

$\Rightarrow$ Apply limit

$$\Rightarrow \frac{-(1)^2}{(-3)(1) + (-1)(-2)(1)}$$

$$\Rightarrow \frac{1}{6}$$

b) Heat equation:-

$$\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2}$$

Show that $u(x, t) = \sin(ax) \cdot e^{-\beta t}$,

Sol.

We start with $\partial_t u$

$\Rightarrow \quad \partial_t u(x,t) = \partial_t \left(\sin(ax) \cdot e^{-\beta t}\right) =$

$\sin(ax)\, \partial_t (e^{-\beta t}) = -\beta \sin(ax) \cdot e^{-\beta t}$

$\Rightarrow$ Now for the $\partial_x u$ and $\partial_{xx} u$

$\Rightarrow \partial_x u(x,t) = \partial_x \left(\sin(ax) \cdot e^{-\beta t}\right) = \partial_x \left(\sin(ax)\right) \cdot e^{-\beta t} =$

$= a\cos(ax) \cdot e^{-\beta t}$

$\Rightarrow \partial_{xx} u(x,t) = \partial_x \left(a \cos(ax)\right) \cdot e^{-\beta t} = -a^2 \sin(ax) e^{-\beta t}$

Now we have,

$$\partial_t u(x,t) = \partial_{xx} u(x,t)$$

$$-\beta \sin(ax) \cdot e^{-\beta t} = -a^2 \sin(ax) \cdot e^{-\beta t}$$

Conclude from the solution that

$$\alpha^2 = \beta$$

~~α^2~~

c) Find and sketch the domain of f. then find an equation for the level curve or surface of the function passing through the given input.

$$f(x,y) = \int_x^y \frac{d\theta}{\sqrt{1-\theta^2}}, (0,1)$$

Sol.

Integerating the

$$\int_x^y \frac{d\theta}{\sqrt{1-\theta^2}} = \sin^{-1} x$$

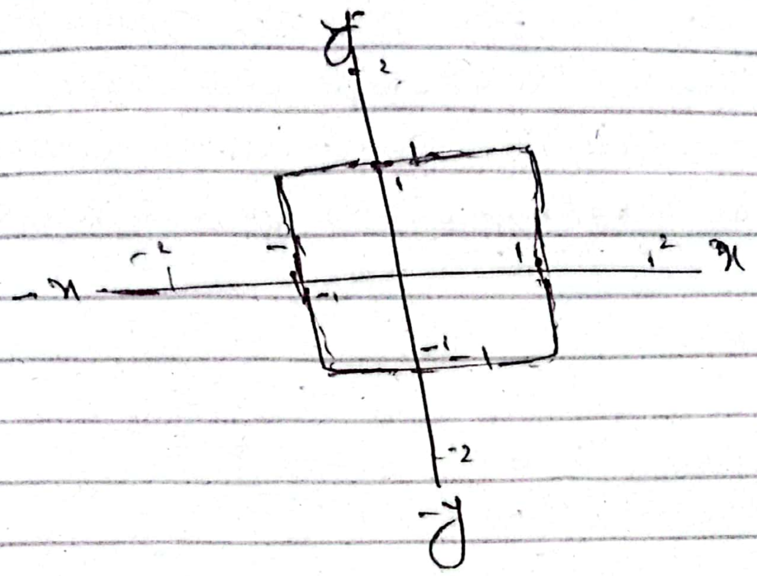
there fore

$$\Rightarrow f(x,y) = \int_x^y \frac{d\theta}{\sqrt{1-\theta^2}} = \left[\sin^{-1}\theta\right]_x^y = \sin^{-1} y - \sin^{-1} x$$

$\Rightarrow$ Domain of inverse function is $[1,-1]$

$\Rightarrow$ Domain of $f(x,y)$ is $D = -1 \leq x \leq 1,\ -1 \leq y \leq 1.$

Sketch the domain



$\Rightarrow$ To obtain a level curve for

$f(x,y) = \sin^{-1}y - \sin^{-1}(x)$

$\Rightarrow c = \sin^{-1}y - \sin^{-1}(x)$

Limit $(0,1)$

$\Rightarrow$ Putting limits

$\Rightarrow c = \sin^{-1}(1) - \sin^{-1}(0)$

$c = \frac{\pi}{2} - 0$

$c = \pi/2$,

the required level curve is $\sin^{-1}y - \sin^{-1}x = \frac{\pi}{2}$ &

Q No: 3

a) Find an equation for the parabola with focus $(4, 0)$ and directrix $x = 3$. Sketch the parabola together with its vertex, focus and directrix.

Soln.

Focus $(4, 0)$
Directrix $(x = 3)$

Here,

$$2p = 4 - 3$$
$$2p = 1$$
$$p = \frac{1}{2}$$

Now,

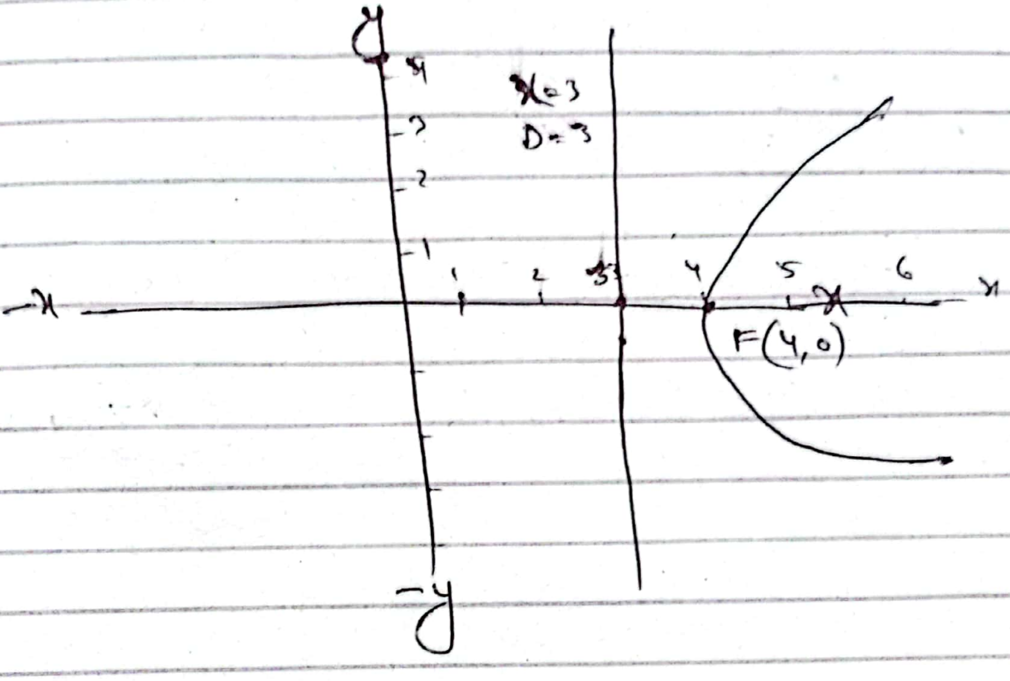
$$\Rightarrow \left(\frac{1}{2} + 3, \; 0\right)$$
$$\Rightarrow \left(\frac{1+6}{2}, \; 0\right)$$
$$\Rightarrow \left(\frac{7}{2}, \; 0\right)$$

then,

$$(y - 0)^2 = \left(4 \cdot \frac{1}{2}\right)\left(x - \frac{7}{2}\right)$$
$$y^2 = \frac{4}{2}\left(x - \frac{7}{2}\right)$$

$$y^2 = 2\left(x - \frac{7}{2}\right)$$

$$\frac{y^2}{2} = x - \frac{7}{2}$$



b) The vertices of an ellipse of eccentricity 0.5 lie at the points $(0, \pm 2)$, where do the foci lie?

Sol. :

Vertices are $(0, \pm 2) \Rightarrow a = 2$ and $e = 0.5$

Now,

$$e = \frac{c}{a}$$

$$0.5 = \frac{c}{2}$$

$$c = 1$$

Foci are $(0, \pm c) = (0, \pm 1)$

c) Show that the function

$$f(x,y) = \frac{2x^2y}{x^4+y^2}$$

has no limit as (x,y) approaches $(0,0)$

Soln:-

⇒ Approach along the x-axis $(y=0)$:

⇒ $\lim_{(x,0)\to(0,0)} \frac{2x^2 \cdot 0}{x^4+0}$ $= \frac{0}{0}$

$$\lim_{x\to 0} \frac{0}{(0)^4+0} = \frac{0}{0}$$

Applying L'hopital Rule:

$$\Rightarrow \lim_{(0,y)\to(0,0)} \frac{2 \cdot 0^2 y}{0^4+y^2} = 0$$

⇒ the function as $(x,y)\to(0,0)$ does not exist, from above eq we conclude that

$f(x,y) = \frac{2x^2y}{x^4+y^2}$ has no limit

as (x,y) approaches $(0,0)$.