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An Investigation of the Mechanical Properties of Materials at very High Rates of Loading

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ABSTRACT. A method of determining the stress–strain relation of materials when stresses are applied for times of the order of 20 microseconds is described. The apparatus employed was a modification of the Hopkinson pressure bar, and detonators were used to produce large transient stresses. Thin specimens of rubbers, plastics and metals were investigated and the compressions produced were as high as 20% with the softer materials. It was found that whilst Perspex recovered almost as soon as the stress was removed, rubbers and polythene showed delayed recovery, and copper and lead showed irrecoverable flow. The phenomenon of delayed recovery is discussed in terms of the theory of mechanical relaxation and *memory* effects in the material.

§1. INTRODUCTION

THE mechanical behaviour of materials at high rates of loading is of interest from two points of view. Firstly, the engineer, who wishes to use the materials under conditions where they may have to withstand sudden impacts, requires to know how their mechanical properties depend on the rate at which stresses are applied. Secondly, since the variation of the stress–strain diagram with loading rate is related to relaxation processes taking place on a microscopic scale in the material, the results are of interest to the physicist who is studying the relation between physical problems and molecular structure.

The present investigation was undertaken largely from this second point of view and the first experiments were carried out with specimens of several rubbers and plastics. Earlier work (Naunton and Waring 1938, Alexandrov and Lazurkin 1940) has shown that the mechanical behaviour of these materials is highly dependent on the rate of application of stress, and it was considered of interest to investigate their mechanical properties when they are being submitted to large stresses which are maintained for only a few microseconds. Some experiments on the behaviour of copper and lead under these conditions have also been carried out.

Two problems are encountered in such an experimental investigation of the mechanical behaviour of materials at very high rates of loading. These are associated with inertia effects in the apparatus and with the recording of transient stresses and strains.

As the rate of loading is increased, the acceleration of any moving parts of the straining apparatus begins to require forces comparable with those necessary to deform the specimen. It is often impossible to separate these inertia effects from the effects due to the physical properties of the material under investigation, and at the highest rates of loading the inertia of the specimen itself will result in a non-uniform distribution of stress along its length. In the work described here these difficulties were overcome by the use of very thin specimens and the stresses were applied by a simple mechanical system for which the inertia effects could be calculated in terms of the propagation of stress waves.

The natural period of ordinary stress gauges is far too long for them to respond to forces which are applied for only a few microseconds, and special techniques must be used for recording such transient stresses. Hopkinson (1914) was the first to devise a satisfactory method of doing this and the technique was later developed by Landon and Quinney (1923). More recently R. M. Davies (1948) has made a critical study of the method and has described a modified form of the apparatus in which the measurements are made electrically. This overcomes many of the disadvantages associated with the earlier experimental work.

Hopkinson's original apparatus, which has become known as the *Hopkinson pressure bar*, consisted of a cylindrical steel bar suspended ballistically. The pressure to be measured was applied to one end of the bar whilst at the other end a cylindrical pellet, known as the *time piece*, was wrung on. These pellets were of the same diameter as the bar and of the same type of steel. The end of the bar and one surface of the pellet were ground flat so that the pellet could be attached to the end of the bar with very little grease.

The principle of the method is that when a pulse of compression travels down the bar it is transmitted through the greased joint without change of form. When it reaches the other face of the pellet, which is a free boundary, it is reflected as a pulse of tension. Since the greased joint is unable to withstand any appreciable tensile stress, the pellet flies off as soon as the reflected pulse builds up any tension across the interface. The momentum trapped in the pellet corresponds to a section of pulse the length of which is twice that of the pellet. If the experiment is repeated with pellets of different length, and the momentum measured in each case by capturing the pellet in a ballistic pendulum, the nature of the pressure–time relation for the pulse can be investigated.

Although Hopkinson's original method has the advantage of simplicity and has been used with success by the Research Department, Woolwich (Robertson 1921), for measuring the pressures set up by the detonation of various explosives, it suffers from several serious limitations. Firstly, it gives only a series of pressure–time integrals for different sections of the pulse. From these the maximum amplitude of the pressure, and the time for which any value of the pressure was exceeded may be deduced, but the actual pressure–time curve is not determined. Secondly, the force necessary to break the greased joint introduces an unknown variable into the experiments, and this precludes the use of the apparatus for the measurement of pulses of small amplitude. Finally, the method assumes that the pulse is propagated down the bar without appreciable change of form: this is true only for pulses which are long in comparison with the diameter of the bar and within which there are no sudden changes in pressure.

In the Davies method (Davies 1948) the first two of these difficulties are overcome by eliminating the pellet and greased joint and obtaining a continuous record of the very small displacement of the free end of the bar. This is done by making the free end of the bar the earthed conductor of a parallel-plate condenser which operates as a condenser microphone. The amplified output from this microphone is fed into a cathode-ray oscillograph and a photographic record made. From this record the displacement–time curve may be obtained and if the applied pressures do not exceed the elastic limit of the steel the pressure–time curve can be deduced. Davies has also investigated the theory of the propagation of pulses down cylindrical bars and shown under what conditions the distortions become too severe for the method to be used satisfactorily.

In the work described here an apparatus based on that designed by Davies has been used for making stress–strain measurements. The specimens, which were in the form of thin discs, were placed between the flat faces of two cylindrical steel bars. The transient pressure was applied by firing a detonator at the end of one of the bars and the displacement at the free end of the other bar was measured with a parallel-plate condenser microphone. From this the pressure on the specimen could be calculated. A cylindrical condenser microphone was also fitted around the bar between the detonator and the specimen; this was used to measure the amplitude of the pressure pulse arriving at the specimen, and the deformation of the specimen could then be deduced.

§ 2 APPARATUS

(i) *General Description*

Figure 1 shows the general arrangement of the apparatus; this is similar to that described by Davies except that the bar is in two parts and a second condenser microphone and amplifier are introduced.

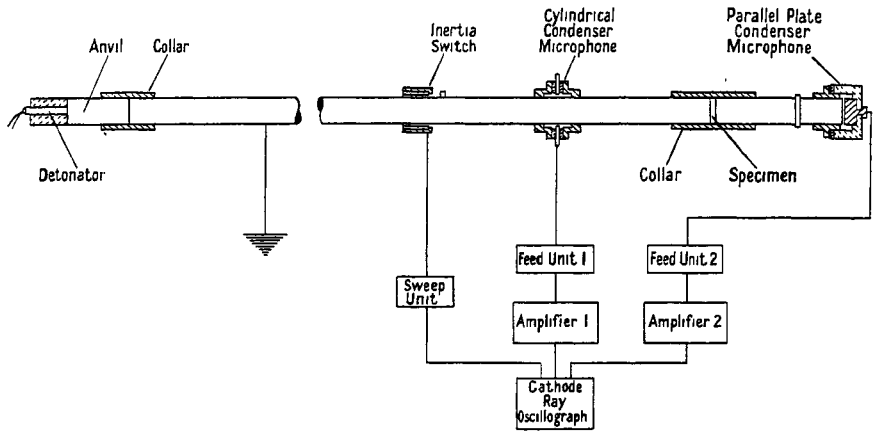

Figure 1. General arrangement of apparatus.

(ii) *The Pressure Bar*

The bar and extensions were of silver steel (yield point 40 tons/in^2) 1 in. in diameter. The bar was 6 feet in length and extensions 4 in., 6 in. or 8 in. in length could be attached with a brass collar, the specimens being lubricated and placed between the flat faces of the bar inside the collar. The ends of both the main bar and the extensions were carefully ground and the collar was made to be a good sliding fit over both bars. In order to protect the pressure end of the bar, the detonators were fired against hardened steel anvils of the same diameter as the bar. These anvils, which were replaced frequently, were cylinders $1\frac{1}{2}$ in. in length with their ends ground flat. A close-fitting steel collar held them against the end of the bar, a very small quantity of grease or heavy oil being used between the flat surfaces to ensure uniform contact.

The detonators, which could be fired electrically, were specially prepared by the Research Department of the Nobel Division of Imperial Chemical Industries Limited, and consisted of flat-based aluminium tubes 0·25 in. in diameter containing

0·5 gm. of tetryl loaded and pressed in increments of 0·1 gm., and primed with 0·35 gm. of a composition of lead azide and lead styphnate. The results obtained with these detonators were found to be consistent to one or two per cent.

The detonators were fitted to the anvils by means of cylindrical polythene holders down the axes of which holes of suitable diameter had been drilled. The holders were attached to the anvils with adhesive tape so that the flat base of the detonator was firmly in contact with the face of the anvil. The bar was suspended ballistically and the momentum communicated on firing the detonator was measured with a light metal pointer which gave the maximum amplitude of swing. The various electrical connections were made with thin flexible wire so that they did not hinder the motion of the bar.

(iii) *The Condenser Microphones*

Figures 2 and 3 show the construction of the two condenser microphones used with the apparatus. The cylindrical microphone shown in Figure 2 consists

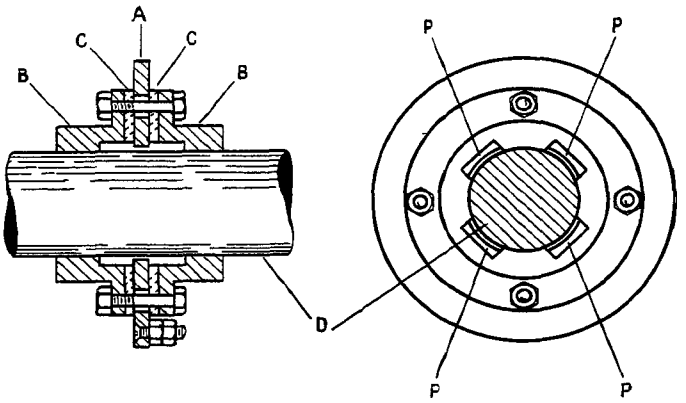

Figure 2. Cylindrical condenser microphone.

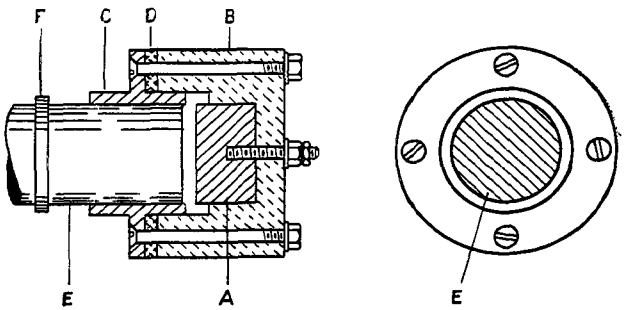

Figure 3. Parallel-plate condenser microphone.

of a circular disc of mild steel A $\frac{1}{8}$ in. thick, through which a concentric hole 1·020 in. in diameter has been drilled; this disc is held between two mild steel collars B by four bolts. Two Perspex washers C were used to insulate the disc

from the collars as this material was found to give good electrical insulation and mechanical rigidity. The collars were machined to slide easily over the bar D, the bore being slotted at four places P to allow feeler gauges to be inserted. The microphone was placed on the bar, the insulated ring was set concentrically by means of feeler gauges and the bolts were then tightened. This left an annular gap of 0·010 in. between the bar and the ring.

This condenser microphone was normally placed 7 in. from the end of the main bar and the insulated disc was charged to two or three hundred volts through a high resistance. The condenser feed-unit is shown in Figure 4. Whilst the compression pulse from the detonator was passing through the microphone, a lateral expansion of the bar took place, which caused a change in the capacity of the cylindrical condenser consisting of the bar and ring. This resulted in a small change in voltage across the condenser which was amplified and fed to one beam of a double-beam oscillograph as shown in Figure 1.

The parallel-plate condenser microphone shown in Figure 3 is similar to the one described by Davies (1948), except that provision has been made to set the insulated plate accurately by adjusting the bolts. The insulated metal plate

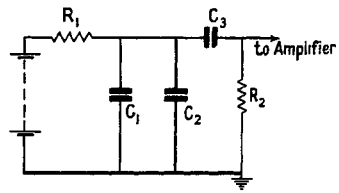

Figure 4. Condenser feed unit.

Typical values.

R_1 50 MΩ.
R_2 2 MΩ.
C_1 (condenser microphone) 0·00004 μF.
C_2 0·001 μF.
C_3 0·1 μF.

A is held in the cylindrical block of polythene B which is bolted to a mild steel collar C, a soft rubber disc D being placed between the polythene and the collar. The collar was made to slide easily over the ends of either the extension bar E or the main bar. The separation between the plate and the bar was adjusted by inserting ring gauges between the collar C and the small ridge F. The microphone was connected through a feed unit similar to that shown in Figure 4. The output was then amplified and fed to the other beam of the oscillograph (see Figure 1).

For both types of microphone the movements in the bar are converted into voltages which are proportional to the corresponding displacements so long as these displacements are small compared with the distance between the plates of the condenser units. For larger displacements the relation ceases to be linear.

As can be seen in Figure 4, the condenser microphone C_1 is connected across a much larger capacity C_2 and these are both charged through a high resistance R_1, C_3 being a large isolating condenser between the feed unit and the amplifier. The value of R_1 is sufficiently large for the charge across the two condensers in parallel to remain constant whilst the value of C_1 is being changed by the pressure pulse. Thus, if we write C for C_1+C_2, and c_1 is the change in capacity of C_1,

V the original potential difference across the condensers and v the change in potential difference, we have

$$Q = VC = (V - v)(C + c_1), \qquad \ldots\ldots(1)$$

and if c_1 is small compared with C (as it was with both condenser units), we may write

$$v = c_1 V/C. \qquad \ldots\ldots(2)$$

For the parallel-plate condenser unit the capacity C_1 is given by

$$C_1 = A(1 + \phi)/4\pi D, \qquad \ldots\ldots(3)$$

where A is the area of the plates, D the initial distance between them and ϕ a correction term to allow for edge effects. This term is small when D is small compared with the radius of the plates. If we take $C_1 = B/D$ where $B = A(1 + \phi)/4\pi$, B will remain appreciably constant for small changes in D.

If the end face of the bar moves a distance x to produce the change in capacity c_1, we have $C_1 + c_1 = B/(D - x)$, so that

$$c_1 = Bx/(D - x)D, \qquad \ldots\ldots(4)$$

or substituting in (2)

$$v = VBx/(D - x)CD = VC_1 x/(D - x)C. \qquad \ldots\ldots(5)$$

If x is very small compared with the separation D

$$v = VC_1 x/CD, \qquad \ldots\ldots(6)$$

so that the voltage is proportional to the displacement.

Since the annular gap in the cylindrical microphone is small compared with the radius of the bar r, the capacity of the condenser is here again inversely proportional to the width of this gap and equation (5) may be used. D is in this case the width of the gap, x the change in radius of the bar and, if edge effects are neglected, $B = rl/2$ where l is the thickness of the insulated disc.

(iv) *Amplifiers*

The output obtained from both types of condenser microphone described above will be proportional to the voltage applied across them, but even if this is increased to 1,000 v. the output resulting from the small movements of the bar produced by the pressure pulse will still be only a small fraction of a volt. In order to use this to produce a measurable deflection of the beam of a cathode-ray oscillograph a high-gain amplifier is therefore necessary. The design of such an amplifier which will give undistorted amplification of short transient pulses is a matter of some difficulty since stray capacities result in poor high-frequency response and attempts to compensate for this by using partially inductive loads in the anode circuits may easily lead to the setting up of parasitic oscillations. Fortunately, satisfactory wide-band amplifiers of high gain have been designed recently (Valley and Wallman 1948) and one of these, the No. 234 Pulse Amplifier, developed by the Telecommunications Research Establishment, was used in the present investigation. This amplifier gives an even voltage amplification from audio frequencies up to frequencies of several megacycles per second and has a maximum gain of about 30,000.

The amplifier is designed to work only with pulses of one sign but this did not matter with the parallel-plate microphone as the displacement of the bar resulting from a pressure pulse is always in one direction. With the cylindrical unit,

oscillations in the displacement are found to occur and in order to observe these a wide-band amplifier of smaller gain, which did not suffer from this limitation, was employed.

The connections between the feed units and the amplifiers were made with low-capacity coaxial cable and this was also used for connecting the outputs of the amplifiers to the Y plates of the double-beam oscillograph.

(v) *Cathode-ray Oscillograph*

A Cossor oscillograph Unit Model 3402, employing a 3259J double-beam tube, was used in these experiments. This tube can be run at a maximum anode voltage of 3,000 v. and was found to give an adequate writing speed for the experiments when fast photographic plates and a large aperture camera lens were employed.

The output from each amplifier was connected to the Y deflector plates of one of the beams, the other deflector plate in each case being at earth potential. Provision was made to substitute an oscillator of accurately known frequency on to one of the Y plates, and this was used as a time calibration for the records.

The X plates were connected across a balanced time-sweep unit which, when triggered, resulted in a single traverse of the two beams across the screen. At the same time it applied a transient positive potential to the modulating grid of the cathode-ray tube, which brought the beam to maximum brightness during the sweep.

The camera lens used had an aperture of f/2·9 and was of 8 in. focal length, a magnification of 0·5 was generally employed. In order to ensure that no distortion was being introduced by the camera, a graduated grid was placed in the plane of the cathode-ray tube screen and photographed; the plate was then measured with a travelling microscope. Ilford H.P.3 quarter plates were used and these were found to give a considerably higher writing speed than other plates with the J screen tube. The plates were measured with a Beck travelling microscope which gave traverses in two directions at right angles.

In carrying out the experiments the brilliancy control of the cathode-ray oscillograph was adjusted so that the spots just disappeared, the room was then placed in darkness and the camera shutter opened. When the detonator was fired it actuated an inertia switch on the bar which triggered the sweep unit giving a single traverse of the two beams.

(vi) *Sweep Unit*

The circuit of the sweep unit, together with the values of the resistances and condensers normally used, is shown in Figure 5. The sweep unit is designed to give a large transient voltage between X1 and X2 when a positive pulse is applied to terminal T. To prevent defocusing of the beam this voltage is balanced about earth potential. As soon as the unit is triggered it also applies a positive voltage to the modulating grid of the cathode-ray tube from terminal M, and this voltage is maintained positive for the duration of the sweep; it then falls back through zero to a small negative value and then asymptotically back to zero again. V1 is a rectifying valve and V2 a gas-filled or mercury vapour relay valve. For sweep voltages up to 500 v. a GT1C Osram valve with indirectly heated cathode was used. To obtain larger sweeps a BT19 mercury vapour thyratron working at 1,000 v. was employed.

The potentiometer R6 could be set so that the sweep unit triggered itself and gave a repeating stroke. It was used in this position whilst the other controls were being adjusted.

In order to apply a triggering voltage to terminal T some microseconds before the pressure pulse reached the cylindrical microphone, an inertia switch was fitted to the bar 6 in. from the microphone (see Figure 1). Several types of inertia switch were tried in attempts to effect triggering at a reproducible time before the head of the pressure pulse reached the specimen. With all mechanical types an uncertainty of 2 or 3 microseconds was found to occur and a simple *ring switch* similar to that described by Davies was finally employed. This is shown diagrammatically in Figure 1 and consists of a metal tube pressed round a plastic insulating collar which slides easily on the bar.

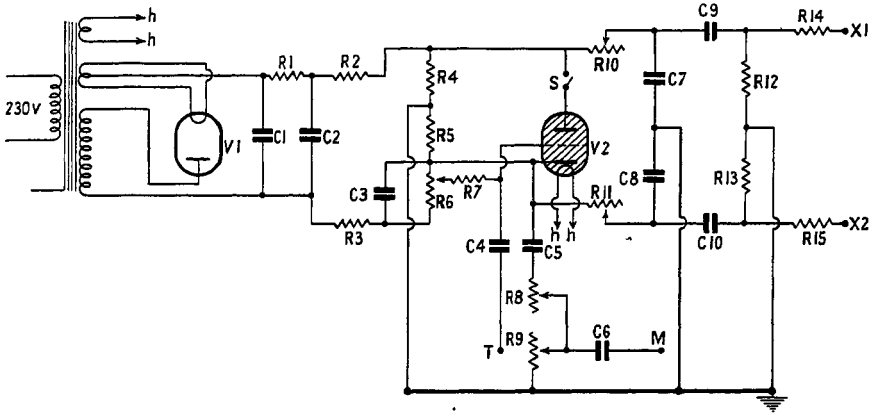

Figure 5. Sweep unit.

R1, 14, 15	30,000 Ω.	C1	0·5 μF.
R2, 3	0·5 MΩ.	C2	10 μF.
R4, 5	2 MΩ.	C3	50 μF.
R6, 7, 8, 9	100,000 Ω.	C4, 6, 7, 8, 9, 10	0·01 μF.
R10, 11	20,000 Ω.	C5	0·001 μF.
R12, 13	1 MΩ.		

It was found that the flash from the explosion of the detonator could be used to trigger the sweep unit satisfactorily by letting the light fall on a photoelectric cell with a one-valve amplifier. By this means results reproducible to within one microsecond were obtainable and this method was employed for some of the experiments.

Figure 6 shows a photograph of the traces obtained in an experiment when the bar and extension were wrung together with a thin layer of grease between them. The top and middle traces are the amplified outputs from the cylindrical and parallel-plate microphones respectively. The bottom trace is a timing wave of 5·30 μsec. period.

(vii) *Static Stress–Strain Apparatus*

Figure 7 shows a diagrammatic representation of the apparatus used to obtain stress–strain curves for the materials under static loading conditions. These were carried out for comparison with the results obtained for the Hopkinson bar apparatus. The specimens were placed in the hydraulic ram between the flat

end of the piston which was 1 in. in diameter and a stop of the same diameter, the faces of the specimen being lubricated to allow free lateral movement. The stress was measured with the pressure gauge which gave the oil pressure on the piston. The change in thickness of the specimen was measured with a travelling microscope which read directly to 0·01 mm. The pressure was built up with the pump, the booster being used as a fine control.

§ 3. THEORY OF THE METHOD

If a plane compression pulse is propagated along a cylindrical bar and the pressure does not exceed the elastic limit of the material, the velocity of propagation will be c where $c=\sqrt{(E/\rho)}$, E being Young's modulus for the material and ρ its

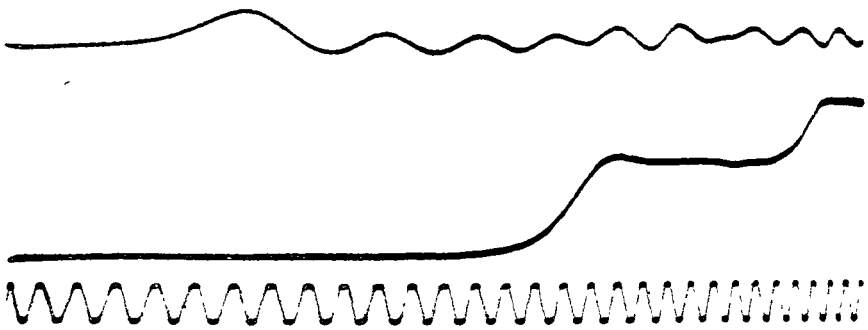

Figure 6. Cathode-ray oscillograph traces showing outputs from the two condenser microphones and 5·3μsec timing wave.

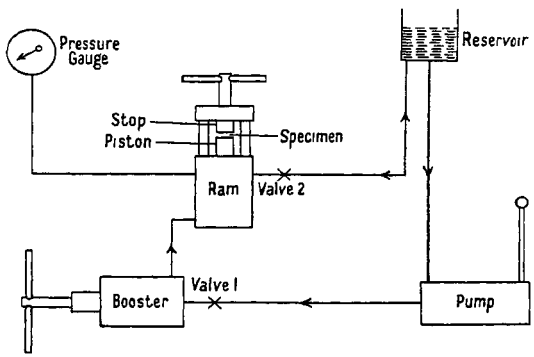

Figure 7. Static stress–strain apparatus.

density. Also if the particle velocity at any point is u and the amplitude of the pressure is p, we have $p=\rho cu$. (These relations assume that the pulse is propagated without distortion, the pressure being uniform over any given cross section; this is true only for pulses which are long in duration compared with the time taken for the pulse to travel a distance equal to the radius of the bar.)

At the free end of the bar the compression pulse will be reflected as a pulse of tension. This is of the same form as the outgoing pressure pulse, and the velocity of the end of the bar is therefore twice the corresponding particle velocity associated

with the pulse travelling along the bar. Thus, if we denote the displacement of the end of the bar, at time t, by ξ we have

$$p = \tfrac{1}{2}\rho c \frac{d\xi}{dt}. \qquad \ldots\ldots (7)$$

The output from the parallel-plate microphone gives the relation between ξ and t and by differentiating this curve the pressure–time relation for the pulse may be derived.

The radial displacement of the bar ζ is given by

$$\zeta = \sigma r p/E, \qquad \ldots\ldots (8)$$

where σ is Poisson's ratio and r is the radius of the bar. Thus the cylindrical condenser microphone gives the pressure–time curve directly, when the value of $\sigma r/E$ for the bar is known.

Davies (1948) has considered the two ways in which distortion appears when the pulse is of a length comparable with the cross section of the bar. He has done this in terms of the elastic equations given by Pochhammer (1876), Chree (1889) and Love (1927) for the propagation of longitudinal vibrations of a circular cylinder. This treatment shows that for such short pulses considerable dispersion takes place, the Fourier components of short wavelength travelling with lower velocities than those of longer wavelength. This results in a lengthening of the pulse as it travels down the bar. The other form that the distortion takes is in a non-uniformity of the distribution of the pressure and displacement across any section of the bar. When measurements are made with the parallel-plate microphone this results in an amplification of the high-frequency components.

Davies estimates that when a bar 2 ft. long and 1 in. in diameter is used for measuring pulses of 20 μsec. duration the distortion causes errors of between 2 and 3% in the values obtained. He also describes experiments which confirm that this is the order of the error observed. The distortion results in larger errors when a cylindrical condenser microphone is used and Davies suggests that this method can only be used profitably with pulses which are very long compared with the diameter of the bar.

In the method described here for obtaining stress–strain measurements, any distortion introduced by the main bar does not affect the validity of the results, but only changes the form of the stress cycle to which the specimen is subjected.

If a sufficiently thin specimen is inserted in the split Hopkinson bar, described in the previous section, the pressure is effectively the same throughout the specimen during the passage of the compression pulse, and this will be true so long as the thickness of the specimen is small compared with the wavelength of the shortest operative wave in the Fourier spectrum of the pulse. This pressure is communicated along the extension bar to the parallel-plate condenser microphone. On differentiating the displacement–time curve, which is obtained from this microphone, the stress–time relation for the specimen may be derived using equation (7).

If the displacement–time relation for the initial pulse travelling down the main bar is also known, the strain–time relation may be determined and the stress–strain curve can then be derived. Let the pressure of the incident pulse at any time t be p_I, of the transmitted pressure pulse which acts on the specimen

p_T, and of the reflected pulse, which is a pulse of tension travelling back along the bar $-p_R$, then

$$p_I = p_R + p_T. \qquad \ldots\ldots(9)$$

If we denote the displacement of the face of the main bar in contact with the specimen at time t by ξ_1 we have

$$\xi_1 = \frac{1}{\rho c}\int_0^t (p_I + p_R)dt, \qquad \ldots\ldots(10)$$

and if the displacement of the face of the extension bar in contact with the specimen is ξ_2,

$$\xi_2 = \frac{1}{\rho c}\int_0^t p_T dt. \qquad \ldots\ldots(11)$$

Now if the thickness of the specimen is z, the fractional strain s is given by $(\xi_1 - \xi_2)/z$, hence from (9), (10) and (11) we have

$$s = \frac{2}{\rho c z}\int_0^t (p_I - p_T)dt. \qquad \ldots\ldots(12)$$

Now

$$\frac{2}{\rho c}\int_0^t p_T dt = \xi_T$$

is the displacement measured by the parallel-plate microphone with the specimen between the bar and the extension, and

$$\frac{2}{\rho c}\int_0^t p_I dt = \xi_I,$$

the displacement produced by the pulse when no specimen is present, the two bars being in contact. Thus

$$s = (\xi_I - \xi_T)/z, \qquad \ldots\ldots(13)$$

so that if we know the displacement–time relations for ξ_I and ξ_T we can obtain the strain–time curve and, since the gradient of the ξ_T curve gives us the stress–time curve, we can derive the stress–strain relation for the material under the conditions of the experiment.

As mentioned above, the distortion introduced by the bar causes the values of the pressure amplitude obtained with the cylindrical microphone to be rather unreliable. Nevertheless, the experimental results showed that the pressure–time integral, which is all that is required to determine the strain in the specimen, agreed closely with the values obtained directly with the parallel-plate microphone.

In the stress–strain determinations described here, however, it was generally found most convenient to determine ξ_I and ξ_T in separate experiments with similar detonators, the two bars being wrung together with a little grease in the first case, and the specimen lubricated and inserted between the bars in the second case. The cylindrical-condenser microphone, did, however, serve two useful purposes. Firstly, it gave a convenient method of fixing the same zero in time for the two experiments, since measurements along the time scale could be made from the first intercept of the cylindrical microphone trace on the horizontal axis (Figure 6, top trace). This could be done with considerable accuracy whereas, since the displacement begins rising very slowly (Figure 6, middle trace), the arrival of the

pressure pulse cannot be fixed at all accurately. Secondly, the trace from the cylindrical microphone gave an accurate check that the pressure pulses obtained with different detonators were of the same shape and amplitude.

The forward acceleration of the specimen is proportional to the difference in pressure between the two faces and, so long as the specimen is sufficiently thin, the inertia of the specimen may be neglected from this point of view in determining its stress–strain behaviour. The radial motion, however, depends on the time rate of change of the longitudinal strain and a correction must be applied for the radial kinetic energy however thin the specimen may be.

If we take p_s as the stress required to produce a fractional strain s when no kinetic energy is given to the specimen, p_m to be the measured pressure with the specimen in the split Hopkinson bar apparatus, and G to be the radial kinetic energy, we have

$$Az\int_0^s (p_m - p_s)ds = G, \qquad \ldots\ldots(14)$$

where A is the cross-sectional area and z is the thickness of the specimen.

Differentiating with respect to s we obtain

$$p_m - p_s = \frac{dG}{ds}\Big/ Az. \qquad \ldots\ldots(15)$$

Now if the specimen is expanding uniformly and friction between it and the bars is neglected, the radial kinetic energy may be obtained by integration and is given by

$$G = \tfrac{1}{4}A\rho_s z V_r^2, \qquad \ldots\ldots(16)$$

where ρ_s is the density of the specimen and V_r is the radial velocity of a point on the circumference.

If σ_s is Poisson's ratio for the specimen and r its radius, we have

$$V_r = \sigma_s r \frac{ds}{dt},$$

and substituting in (16) and differentiating we obtain

$$\frac{dG}{ds} = \tfrac{1}{2}\sigma_s^2 r^2 A\rho_s z \frac{d^2s}{dt^2}; \qquad \ldots\ldots(17)$$

hence from (15)

$$p_m - p_s = \tfrac{1}{2}\sigma_s^2 r^2 \rho_s \frac{d^2s}{dt^2}. \qquad \ldots\ldots(18)$$

This correction was found to be not more than a few per cent in most of the experiments and is only important when the rate of straining is changing rapidly, which, in these experiments, was at the beginning of the stress cycle. Nevertheless, having to use this correction does introduce a certain amount of error into the results, in that the value of σ_s at these very high rates of loading is not known accurately, and the assumption that the expansion is uniform with no frictional effects is not necessarily justified. The double differentiation of the strain–time curve may also result in appreciable errors when the correction term is comparable with the actual applied stress.

The value of the correction term is proportional to the square of the radius of the specimen and if the method is to be used at higher rates of straining, smaller specimens would have to be employed.

§4. CALIBRATION OF THE APPARATUS

The foregoing treatment to the experimental method assumes that the apparatus fulfils the following conditions: (*a*) the extension bar does not introduce any further distortion to the stress pulses transmitted through it; (*b*) the pulse is transmitted across a greased joint between the bars without distortion; (*c*) the brass collar connecting the two bars does not transmit any appreciable stress-wave from the main bar to the extension bar.

A series of experiments were carried out to test the extent to which these conditions were fulfilled by the apparatus. To investigate conditions (*a*) and (*b*) the collar and extension bar were removed and the parallel-plate microphone was set up at the end of the main bar. Records were then taken of the pulses from detonators, and the results compared with those obtained when the microphone was at the end of an extension bar wrung on to the main bar with a little grease. It was found that the results obtained with 4 in., 6 in. and 8 in. extension bars all gave the same displacement–time curve as that found when the microphone was at the end of the main bar.

Thus conditions (*a*) and (*b*) were fulfilled for the pressure pulse which had arrived at the end of the main bar. The method also assumes, however, that the modified pulse, produced when a specimen of some other material is inserted between the bars, travels down the extension bar without distortion. In the experiments it was found that these pulses were longer than the original pulse from the detonator and that the changes in pressure were generally more gradual. From Davies' treatment one would therefore expect that these pulses would travel down the extension bars with even less distortion than the pulse from the detonator. This could not be tested directly, but it was found that here again identical results were obtained whichever length of extension bar was employed.

It is perhaps worth mentioning that the results obtained when the microphone was used with extension bars differed in two minor respects from those obtained with the microphone at the end of the main bar. The middle trace of Figure 6 shows the trace from the parallel-plate microphone when it was placed at the end of a 6 in. extension wrung on to the main bar. It may be seen that the displacement rises to an approximately constant value at which it stays for about 35 μsec., after which it begins to rise again.

This second rise results from the reflected pulse travelling back along the extension bar, being reflected at the greased joint as a pulse of compression, and finally arriving at the condenser microphone as a pulse similar in form to the initial one. (The whole of the second pulse was not obtained in the experiment shown in Figure 6, as the amplifier was overloaded before the maximum displacement had been reached.)

When the microphone was placed at the end of the main bar no such second rise could be observed since it would arrive after the pulse had twice travelled the full 6 ft. length of the bar; it would thus be well outside the duration of the trace. The other difference between the traces obtained when the microphone was at the end of the main bar, and when an extension bar was used, is that in the former case instead of the trace being approximately flat after the main pulse had passed, it showed a series of oscillations. Such oscillations have been observed by Davies and shown by him to be due to the distortion introduced by the bar; this distortion results in a lagging behind of the high-frequency components of the pulse.

These oscillations may also be seen in the top trace of Figure 6 which corresponds to the radial displacements of the surface of the main bar when the pulse is passing. The theoretical treatment shows that these will be more marked than the corresponding longitudinal vibrations in the bar and it can be seen from the Figure that in this case the vibrations are comparable with the main pulse.

The reason these oscillations do not appear when an extension bar is used is that the greased joint does not transmit tensile stresses. The bars consequently separate slightly as soon as the first oscillation in the stress pulse builds up an appreciable tension across the greased joint; this joint thus acts as a *mechanical rectifier.*

To test whether condition (*c*) was fulfilled, the two bars were separated in the collar by an air gap of 0·020 inches (this gap being several times larger than the calculated maximum displacement of the end of the bar produced by the pulse). The amplifier of the parallel-plate microphone was then set at maximum gain and a polarizing voltage of 360 v. was used across the microphone. The sensitivity of the microphone was then about 30 times as great as it was in the stress–strain determinations. A detonator was fired at the end of the bar and the cathode-ray oscillograph traces photographed. The trace from the parallel-plate condenser microphone was perfectly flat, showing that no appreciable motion had been communicated to the extension bar through the brass collar.

Before the apparatus could be used for stress–strain measurements the relation between the values of the deflections on the cathode-ray oscillograph records and the corresponding displacements of the end of the bar had to be determined. In addition, as may be seen from equation (7), in order to calculate the pressure from the gradient of the displacement–time curve, the value of ρc for the material of the bar had to be ascertained.

The relation between the voltage output of the condenser microphone system and the displacement at the end of the bar is given in equation (5) and this equation may be rewritten

$$x = \frac{Dv}{C_1V/C + v}. \qquad \ldots\ldots(19)$$

If unit deflection on the photographic plate corresponds to an output of N volts from the condenser microphone and feed unit, we have

$$x = \frac{y}{C_1V/CND + y/D} = \frac{y}{K + y/D}, \qquad \ldots\ldots(20)$$

where y is the deflection on the photographic plate corresponding to a displacement x at the end of the pressure bar. Now N may be determined if the gain of the amplifier, the sensitivity of the cathode-ray oscillograph and the photographic reduction are known. If the capacity of the parallel-plate condenser unit, C_1, can also be determined for a known value of D, either by calculation or with a capacity bridge, equation (20) may be used to calculate the displacements from the measurements of the cathode-ray oscillograph records.

As this method involves the accurate determination of all the above constants, it was considered both simpler and more accurate to determine the quantity $C_1V/CND = K$, experimentally. From this value of K, and D as measured with gauges, equation (20) could be used to convert the values of y into displacements.

If the value of y for one known displacement x can be measured the value of

K may be derived. Now, by integrating equation (7) it can be seen that when a pressure pulse is reflected at the end of the bar the maximum displacement $\bar{x}$ is given by

$$\bar{x} = \frac{2}{\rho c} \int p dt. \qquad \ldots\ldots (21)$$

But if M is the momentum associated with the pulse, and A is the cross-sectional area of the bar, we have

$$M = A \int p dt. \qquad \ldots\ldots (22)$$

nus from (21) and (22)

$$\bar{x} = 2M/A\rho c. \qquad \ldots\ldots (23)$$

Hence, if the momentum M associated with a pressure pulse is known, the maximum displacement of the end of the bar when the pulse is reflected may be calculated.

Relation (23) can thus be used to determine the value of $\bar{x}$ for one known value of y, and K can hence be determined. To do this a detonator was fired at the end of the bar, the maximum amplitude of swing was determined, the cathode-ray oscillograph traces were photographed and the gain of the amplifiers measured. From the amplitude of swing, the mass of the bar and its period of oscillation the value of M was calculated. Hence the value of $\bar{x}$ was derived using the values of ρ and c, which, as described later in this section, had to be determined for the pressure measurements.

Substituting $\bar{x}$ and the corresponding value of y (measured on the cathode-ray oscillograph record) in equation (20), the value of K was determined. Since K is proportional to the gain of the amplifier, which may vary with time, in the later work the gain was measured in each experiment and the value of K changed accordingly.

To ensure that the momentum was entirely that of the short pressure pulse and that the amplitude of swing of the bar had not been increased by the general rush of gas following the explosion, the following experiments were carried out. The condenser microphone and the extension bar were removed and time pieces of different lengths, similar to the ones used by Hopkinson (1914) were wrung on to the bar. Detonators were then fired at the other end of the bar and the amplitude of swing measured. It was found that with a time piece $2\frac{1}{2}$ in. long there was no measurable deflection of the bar, the entire momentum having been trapped in the time piece.

It may be seen from equation (20) that in order to convert the values of y into displacements, the value of D is required as well as the value of K. D was determined with mechanical gauges and there may have been an inaccuracy of one or two per cent in the value taken. y/D was, however, always small compared with K and the error introduced into the values of x by this factor was consequently negligible.

As was mentioned earlier, the value of ρc of the material of the bar is required to convert the displacement measurements into pressures. The density ρ of the steel was measured by weighing a section of known dimensions and was found to be 7·89 gm/cm^3. The velocity of propagation c of the pressure pulses was measured by comparing the interval of time between the arrival of the pulse at the end of

the main bar, and at the ends of extension bars of different lengths. Records were taken with the parallel-plate condenser microphone first at the end of the main bar and then at the end of an extension bar, the cylindrical microphone being kept at the same position for all the experiments. The velocity of propagation c could then be calculated from the difference in separation between the pulses from the two microphones with and without the extension bar (cf. Figure 6). The mean value obtained for the velocity was 5,320 metres/sec. This value was checked by setting a length of magnetized bar in longitudinal resonance by means of an oscillator and energizing coil, and measuring the frequency, and it was found that the velocity of sound measured in this way agreed closely with the velocity of the pulses.

§ 5. EXPERIMENTAL RESULTS

(i) *General Procedure*

The specimens used for the experiments with the pressure bar were in the form of circular discs of uniform thickness. In order to allow for the lateral expansion of the specimens when they were being compressed, their initial diameters were made slightly less than that of the bar, the maximum permissible diameter in each case being determined by preliminary experiments. To reduce friction between the specimens and the bars both surfaces of each specimen were lubricated before it was placed in the apparatus. A thin layer of grease was generally used except with the rubber specimens, when glycerol was used. A number of different oils and greases were first tried in order to ensure that the viscosity of the lubricant did not materially influence the results. Similar results were obtained with all the lubricants used; when, however, no lubricant was used, it was found that larger stresses were required to obtain a given strain, which indicated that there was then a considerable amount of friction between the specimen and the bars.

For each group of experiments the gains of the amplifiers were measured, and in addition one or two experiments were included in which the main bar and the extension bar were wrung together with no specimen between them. The traces obtained under these conditions were useful as a calibration of the microphones and as a check that the apparatus as a whole was operating satisfactorily.

The results of the ballistic calibration of the pressure bar, described in the previous section, showed that when the pressure pulse from one of the detonators used was reflected at the free end of the main bar, this end surface moved forward a distance of 0·12 mm. In the limiting case of a very soft specimen the movement of the end face would be very nearly as much as for a free end. The other face of the specimen in contact with the extension bar would remain almost fixed, and the maximum fractional strain would then be given by the ratio of 0·12 mm. to the initial thickness of the specimen (cf. equation (13)).

With harder materials both surfaces of the specimen move and the strains are consequently smaller; nevertheless, by using specimens of different thicknesses, the rate of straining and the maximum fractional strain can be varied within certain limits. These limits are set on the one hand by the fact that, for very thin specimens, there is not sufficient difference between the cathode-ray oscillograph traces obtained when the specimen is present and when the bars are wrung together for accurate values of the strain to be determined. On the other hand, if the specimens are too thick, the assumption that the pressures on both sides of the specimen are sensibly the same is no longer valid.

The method of determining the stress–strain curve from the photographic records is described in the section dealing with the theory of the method. The procedure after the plates had been measured was first to convert the measurements into displacement–time curves, using equation (20). The displacement–time curve obtained from the experiment when the specimen was present was then differentiated numerically, and this gave the pressure–time curve, as may be seen from equation (7). The strain–time curve was derived from the difference between the corresponding values of the two displacement–time curves, using equation (13). All measurements of time were made from the same point on the trace from the cylindrical microphone; as this microphone was in the same position along the bar in both experiments, this ensured that the values of stress and strain corresponded to the same instants of time.

Before deriving the stress–strain curve from these results, the stresses had to be corrected for the radial kinetic energy effect, as described in an earlier section, and where the strains were large the change in the cross-sectional area of the specimen during straining had to be allowed for to obtain the true values of the stress.

The experiments were all carried out at room temperature and calculation showed that the heating produced by the compression of the specimens never amounted to more than 1·5° C. in these experiments.

(ii) *Polythene*

Figure 8 shows the stress–strain curves obtained with four different thicknesses of polythene (polymerized ethylene) of a type known as grade 20. The specimens were prepared from cast sheet which was carefully annealed to ensure that there was no preferred molecular orientation. The points on each curve correspond to 2 μsec. intervals in the stress–strain cycle, so that the rates of stressing and straining may be derived from the curves. This same convention has been used in the dynamic stress–strain curves for other materials shown in the later figures. To avoid confusion around the origin in Figure 8 the first two or three points on each curve have been omitted. The broken curve on this figure shows the results obtained for the static stress- strain relation, using the apparatus shown in Figure 7.

It may be seen that the effective elastic modulus for the dynamic tests is very much higher than that observed statically, and that in all four experiments the strain continues to increase after the stress has passed its maximum and has decreased to a fraction of its maximum value. Since there was no permanent plastic flow in the material, the specimens being of exactly the same dimensions after the experiment, this increase of strain with decreasing stress may appear at first sight rather surprising. The interpretation of this effect in terms of a memory function of the type first suggested by Boltzmann is discussed in detail in the next section.

(iii) *Rubber*

Experiments were carried out with specimens of natural and synthetic rubbers, and the results obtained were somewhat similar to those observed with polythene. The delayed recovery effect with these materials was even more pronounced than for polythene, the strain continuing to increase rapidly after the stress had passed its maximum value.

The specimens of the three synthetic rubbers investigated, neoprene GN, Perbunan and GR-S were closely similar in their static mechanical behaviour to

the specimen of natural rubber used, and it was found that their dynamic behaviour was also very similar to that of the natural rubber. For all these materials no permanent deformation could be observed after the dynamic tests.

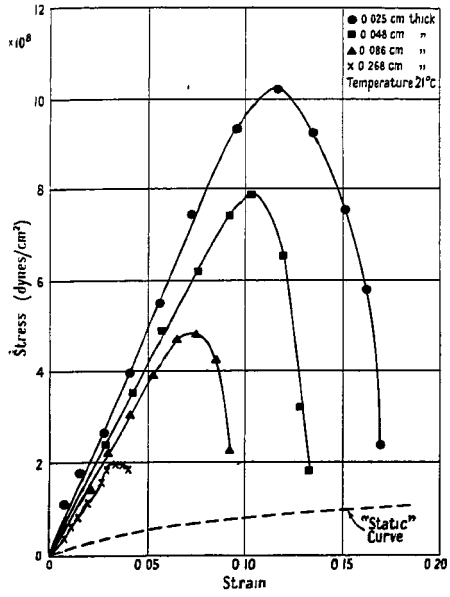

Figure 8. Dynamic stress–strain curves for polythene.

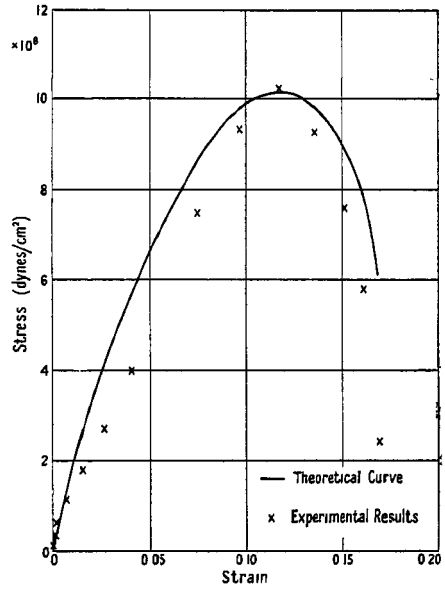

Figure 9. Theoretical curve for polythene.

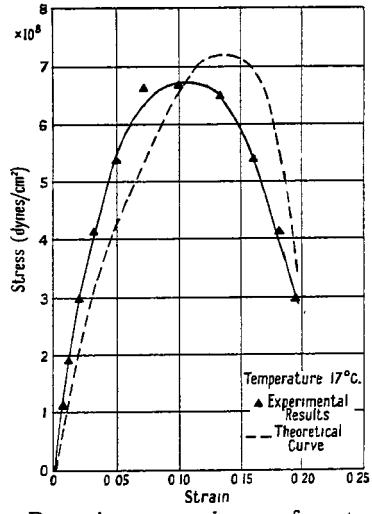

Figure 10. Dynamic stress–strain curve for natural rubber.

Figure 10 shows a typical dynamic stress–strain curve obtained with natural rubber. The specimen used for this experiment was initially 0·03 cm. in thickness,

and the maximum strain observed was about 20%. The broken curve shown on the same figure was one derived theoretically in terms of a *memory* function, and is discussed in the next section. No static stress–strain curve is shown, since on the scale used it would be indistinguishable from the strain axis, the static value of Young's modulus for the material being about 2×10^7 dynes/cm^2. The difference between the static and dynamic behaviour of rubber is thus very much more marked than it is for polythene.

(iv) *Polymethyl-methacrylate*

This is a hard glass-like plastic of which Perspex is a plasticized form. Figure 11 shows the experimental results given at 2 μsec. intervals for a stress–strain cycle, together with the static stress–strain curve obtained with a similar specimen in the hydraulic ram apparatus. The continuous curve in this case is the one derived theoretically and is discussed in the next section.

It may be seen that the dynamic elastic modulus for this material is very much higher than that found with rubber and polythene, and that as the stress is removed the strain decreases, although the material does show considerable hysteresis at these rates of loading.

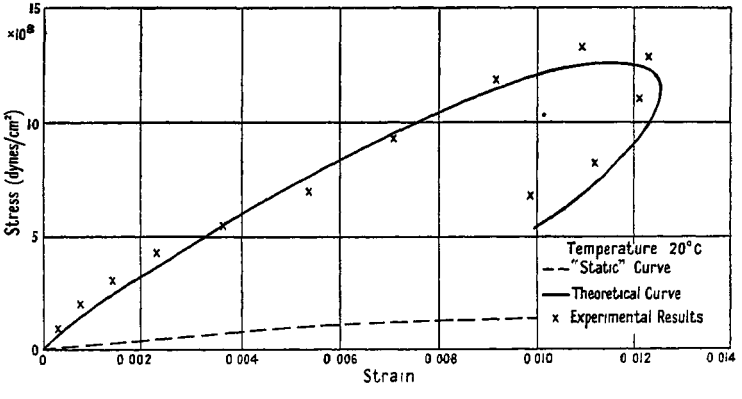

Figure 11. Dynamic stress–strain curve for Perspex.

When thick specimens of Perspex were inserted between the bars it was found that a pressure pulse was communicated through them from the main bar to the extension bar. The transmitted pulse was flatter in shape and of longer duration than the initial pulse, as well as being much smaller in amplitude. By using a series of specimens of up to 2 in. in thickness, it was found that the head of the pulse was propagated with a velocity of 2,400 metres/second; this corresponds to a value of Young's modulus of $6{\cdot}8 \times 10^{10}$ dynes/cm^2. In making this determination of the velocity, the separations between the head of the pulse and a fixed point on the trace from the cylindrical condenser microphone were measured on the cathode-ray oscillograph records.

When comparable thicknesses of either rubber or polythene were tried no well-defined pulse was transmitted through them. This is to be expected from the dynamic stress–strain curves shown in Figures 8 and 10. From these it can

be seen that the layers of material will not recover immediately the stress is removed, so that a continued lengthening of the pulse with a corresponding drop in the pressure amplitude will take place as the pulse travels through the material.

(v) *Copper*

Figure 12 shows the dynamic and static curves for specimens of pure annealed copper sheet 0·05 cm. in thickness. When the specimens were measured after the experiments it was found that they had decreased approximately 4% in thickness. The behaviour of the metal is thus very different from that found with polythene and rubber, although the appearance of the curves is rather similar. The material is here flowing under the influence of the applied stress, the only recoverable part of the deformation being the very small strain corresponding to the initial steep rise in the stress–strain curve. The difference between the static and dynamic behaviour for copper is not nearly so marked as it was found to be with the other materials investigated.

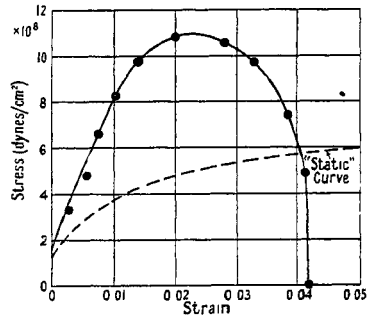

Figure 12. Dynamic stress–strain curve for copper.

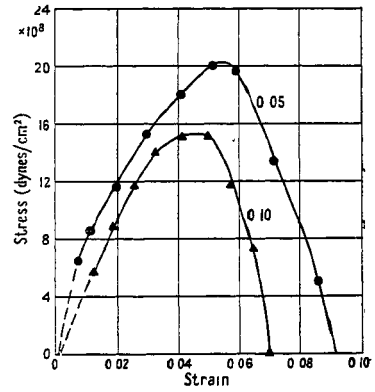

Figure 13. Dynamic stress–strain curve for lead.

(vi) *Lead*

Figure 13 shows the dynamic stress–strain curves obtained with lead specimens 0·05 and 0·10 cm. thick. As in the case of copper, it was found that after the tests the specimens were thinner by the fraction indicated by the curves. As a result of the comparatively small stress required to start deforming the specimens and the high density of lead, the radial kinetic energy correction was found to be large, and the values of the stress in the region near the origin were consequently rather uncertain. The results did, however, show that the yield point was very small compared with the order of magnitude of the maximum stress applied in these experiments.

It was found that for this material the values of the stress were approximately proportional to the rate of straining; this indicates that the lead is behaving like a viscous liquid under these dynamic conditions.

As in the case of rubber the stresses required to deform the material statically were found to be too small to be shown on the same scale as the dynamic results.

§6. DISCUSSION OF RESULTS

The experimental results described in the previous section have shown three types of mechanical behaviour that may occur when materials are submitted to rapidly changing stresses. Under these conditions, polythene and rubber only begin to recover after the stress has fallen back to zero, but the recovery is complete and no permanent deformation can be observed after the experiments. Polymethyl-methacrylate shows a hysteresis loop at these high rates of loading, but with this material a considerable fraction of the strain disappears within a few microseconds of the stress passing its maximum. Here again, after the experiments, there is no permanent deformation and, when the specimens were examined through crossed nicols, they showed no optical strain.

Copper and lead, unlike the plastics and rubbers, were permanently deformed and showed no recovery either immediately the stress began to fall or subsequently. The difference in behaviour between copper and lead results from the much higher yield point of the former metal, which has a definite elastic region in its stress–strain curve. The yield point of lead on the other hand is so low that in these experiments its behaviour is very much like that of a viscous fluid. The static stress–strain curve for copper shows that even after the yield point has been passed the material continues to offer increasing resistance to further deformation as a result of work-hardening.

In the interpretation of the results obtained with the plastic and rubber specimens it is necessary to assume that the relation between the stress and the strain is also a function of time. When a specimen of such a material is stressed, the strain does not reach its final value immediately but increases with time. Conversely, if it is deformed a fixed amount and the strain is then maintained at this value, the stress relaxes with time.

Boltzmann (1876) was among the first to consider this type of behaviour analytically, and more recently several surveys (Slonimsky 1940, Leaderman 1943, Alfrey and Doty 1945) of the subject have been published. Boltzmann introduced a new concept in his treatment of the problem, and suggested that the mechanical behaviour of a specimen was a function of its entire previous history. He further assumed that when a specimen had undergone a series of deformations, the effect of each deformation was independent of the others, and the behaviour of the specimen could be calculated by a simple addition of the effects that would occur when the deformations took place singly. This assumption has become known as the Principle of Superposition.

In considering the behaviour of a specimen in which both the stress and the strain are varying simultaneously, we may consider the strains produced by known stresses, and the problem is then one of *creep*. Alternatively we may calculate the stresses produced when the specimen is deformed a known amount and treat the problem in terms of the *relaxation of stress*. It has been shown (Gross 1947) that the two treatments are equivalent, and it is generally a matter of convenience which of them is followed.

In Boltzmann's original treatment he considered the problem in terms of the relaxation of stress and postulated that the stresses produced by dilatational and shear strains may relax in different ways. The case of uniaxial compression or tension involves both shear and dilatational deformations, and if two separate relaxation phenomena have to be taken into account the problem becomes rather complex. Boltzmann, however, suggested that it was unlikely that many materials

would show dilatational relaxation phenomena and in fact there is no experimental evidence to show that such phenomena occur. Recent work (Pinkerton 1947, Mason *et al.* 1948) on liquids has shown that when such relaxation does occur, the times involved are very much shorter than those with which we are concerned here.

Boltzmann considered the past strain history of the specimen as divided into a number of infinitesimal time elements. At time t the residual stress due to a strain $\epsilon(\tau)$ which took place at time τ and was maintained for an element of time $\delta\tau$ is expressed as $-\phi(t-\tau)\epsilon(\tau)$ where $\phi(t-\tau)$ is called a *memory* function. If at time t the value of the strain is $\epsilon(t)$ there will be a stress $\psi[\epsilon(t)]$ due to this, together with the residual stresses due to all the previous deformations. Thus, the resultant value of the stress $\sigma(t, \epsilon)$ will be given by

$$\sigma(t, \epsilon) = \psi[\epsilon(t)] - \int_{-\infty}^{t} \phi(t-\tau)\epsilon(\tau)d\tau. \qquad \ldots\ldots(24)$$

In interpreting the delayed recovery effects observed in the present work on plastic and rubber specimens, it has been found convenient to use a slightly different treatment from that suggested by Boltzmann. This treatment is similar to that used by G. I. Taylor and E. Volterra (Taylor 1946) in their work on the mechanical behaviour of polythene at lower rates of loading, and an analogous approach has been described by Leaderman (1943) in his treatment of creep phenomena.

In this treatment, instead of considering the total strains to be maintained for short elements of time, we consider the small changes in strain which have occurred in the past history of the specimen and sum the residual elements of stress produced by them. Thus, if between the times τ and $\tau+\delta\tau$ the strain changes from ϵ to $\epsilon+\delta\epsilon$, this will result in a change of stress $\delta\sigma$ which will relax with time. If we call the final value of the stress when relaxation is complete $F(\delta\epsilon)$, then at time t the residual value of the element of stress will be given by

$$\delta\sigma = F(\delta\epsilon) + f(t-\tau)\delta\epsilon, \qquad \ldots\ldots(25)$$

where $f(t-\tau)$ is a memory function which approaches zero as t increases.

The resultant value of the stress σ at time t when the strain is ϵ, is then given by

$$\sigma = F(\epsilon) + \int_{-\infty}^{t} f(t-\tau)\frac{d\epsilon}{d\tau}d\tau. \qquad \ldots\ldots(26)$$

A material whose mechanical behaviour is represented by (26) will not show any permanent strain but may show delayed recovery.

If when the strains are maintained for a sufficiently long time the material obeys Hooke's law, $F(\epsilon)$ will be given by $E\epsilon$ where E is the value of the appropriate elastic modulus. Taylor (1946) has indicated how the function $f(t-\tau)$ can be determined from experimental results, but it was found that when this method was applied to the results of the present experiments the scatter was too large for a satisfactory determination of the function to be made. Instead, it was assumed that $f(t-\tau)$ was of the form $A\exp[-(t-\tau)/\alpha]$ and the values of A and α which gave the best fit were then determined.

This assumption is equivalent to regarding the mechanical behaviour of the material as being similar to that of a system consisting of two springs in series, with a dashpot connected across one of the springs. The springs are assumed to obey Hooke's law and the dashpot is assumed to obey Newton's law of viscosity.

i.e. the viscous force is proportional to the velocity with which it moves. E is then the effective elastic modulus of the two springs in series, and $(E+A)$ is the effective modulus for the spring which has no dashpot across it. The time of relaxation of the system is governed by the ratio of the magnitude of the viscous and elastic elements of the dashpot and spring in parallel.

In deriving the theoretical curves which are shown in the figures the relation used has been

$$\sigma = E\epsilon + A\int_0^t \exp\left[-(t-\tau)/\alpha\right]\frac{d\epsilon}{d\tau}d\tau, \qquad \ldots\ldots(27)$$

the values of E, A and α having been chosen to give the best fit with the experimental results. It would appear possible to determine the value of E experimentally, since it should correspond to the value of Young's modulus for strains which have taken place very slowly. Unfortunately, it is found (Alexandrov and Lazurkin 1940, Hillier and Kolsky 1949) that the value obtained for E with many of these plastic materials depends on whether the forces are applied in times of the order of minutes, seconds or milliseconds, and it is consequently difficult to determine the appropriate value experimentally. In theory, the value to be taken should be that for strains which have been maintained for an infinitely long time, but since experimental evidence shows that irrecoverable plastic flow as well as relaxation phenomena of very much longer period take place, the exact experimental significance of E in equation (27) is not so well defined, and it must be considered as an average value of the modulus for times long compared with the duration of the *dynamic* experiments but short compared with those of the *static* tests.

The assumption that with these materials there is only a single relaxation time is a considerable over-simplification of the problem. The assumption that Hooke's Law is obeyed exactly with stresses which have been applied slowly is also not altogether justified. Nevertheless, fairly good agreement with the experimental results was obtained for polythene and polymethyl–methacrylate on these assumptions. With rubber the agreement was less satisfactory and it seems clear that the model assumed will give only qualitative agreement with the actual behaviour of rubber at these rates of loading.

Figure 9 shows a plot of equation (27) with $A = 4{\cdot}6 \times 10^{10}$, $E = 10^9$, in dynes/cm^2, and $\alpha = 2\,\mu$sec.; the values of ϵ and $d\epsilon/d\tau$ were obtained from the experimental results with polythene and the observed values of stress are also shown in the figure. Using a similar treatment for results obtained with polythene, the stresses having been applied for times of the order of ten milliseconds, Taylor (1946) obtained lower values of A and E and the value given for α is 1·7 milliseconds. Thus, it would appear that relaxation phenomena are exhibited over a range of times, but that in a given experiment the relaxation times of the order of the duration of the experiment are the only ones that are of importance.

Figure 10 gives the theoretical curve for rubber, with $A = 2{\cdot}4 \times 10^{10}$, $E = 10^8$, and α is again taken as $2\,\mu$sec. The value of E in this case is too small to affect the shape of the theoretical curve appreciably, and this value was chosen as one that corresponds to the modulus for stresses applied for times of the order of a few milliseconds. Although the values of A and α used were the ones which gave the best agreement with the experimental results, it can be seen that quantitatively the agreement is not very good and a more complicated model must be considered to account for the exact shape of the stress–strain curve.

Figure 11 gives the curve for $A = 14{\cdot}5 \times 10^{11}$, $E = 8 \times 10^{10}$ and $\alpha = 0{\cdot}5\,\mu$sec. The agreement with the experimental results obtained with polymethyl–methacrylate is here quite close and the difference in the behaviour of this material and that of polythene and rubber thus appears to be due to the shorter effective relaxation time. This results in the term $E\epsilon$ becoming very much more important in determining the instantaneous values of the stress than it was with either polythene or rubber.

§ 7. CONCLUSION

The experimental method described in this paper has enabled measurements to be made of the stress–strain behaviour of materials in times of the order of $20\,\mu$sec. The main disadvantage of the method lies in the fact that the specimens have to be compressed between two parallel surfaces, and that even when lubricants are used, there is some uncertainty whether there is free lateral movement of the specimen, and whether the effects of friction between the specimen and the bar are not introducing large errors. Since, however, the use of different lubricants did not affect the results, and where permanent deformations were obtained they were fairly uniform with no evidence of *barrelling*, friction effects would not appear to be large. In the experiments, it was attempted to maintain two thin lubricant layers between the surfaces of the specimen and the faces of the bars, and since there was not sufficient time for the lubricant to be squeezed out, the conditions are in this respect more favourable from the point of view of eliminating friction than in comparable static experiments where the lubricant layers may become very thin.

The interpretation of the results has shown that very marked relaxation phenomena occur in plastics and rubber in the microsecond region, and that their mechanical properties under these rapid conditions of loading are very different from those observed when forces are applied more slowly. Although the assumption of a single relaxation time has led to reasonable agreement with the experimental results, the complicated molecular structure of these materials would indicate that a large number of different relaxation phenomena take place simultaneously and a more complicated memory function is applicable. Work at lower rates of loading (Alexandrov and Lazurkin 1940, Nolle 1947) has indicated that the dynamic behaviour of materials composed of macromolecules is very sensitive to temperature, and it would therefore seem desirable to continue these experiments over a wide range of temperatures.

The theoretical interpretation of rubber-like elasticity in terms of molecular processes, which has been put forward by several workers (Guth and Mark 1934, Kuhn 1936), shows that the high extension and low elastic modulus of rubber-like materials may be accounted for in terms of the uncurling of molecular chains under the influence of the applied stress; this involves the rotation of segments of the chains around single carbon–carbon bonds. Two other molecular processes associated with very much higher elastic moduli may also occur in these materials; these are changes in the values of the angles between the bonds and changes in the distances between adjacent carbon atoms. Since the increase of interatomic distances will require very large forces, it seems reasonable to correlate the two components of the elastic behaviour given in equation (27) with the changing of bond angles and the uncurling of chains. E is then the modulus corresponding to the latter process and the integral involving the memory function corresponds

to the fact that the uncurling process requires times comparable with the time of application of the stress. Thus, when the stress is first applied, the initial deformation corresponds to only a change in the bond angles.

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