Graphs

Department of Computer Science

HCMC University of Technology, Viet Nam

05, 2020

Outline

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Introduction

Terminology

Representations of graphs

Graph Traversal

Graph Problems

Introduction

Graph

- Each node may have *multiple predecessors* as well as *multiple successors*.

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- Each node may have *multiple predecessors* as well as *multiple successors*.
- Graphs are used to represent *complex networks* and solve related problems.

Applications

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- Representing a map as a set of locations with distances between locations

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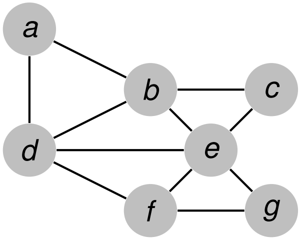
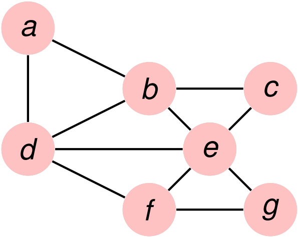


Terminology

- Vertex (vertices)



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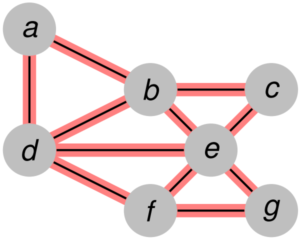

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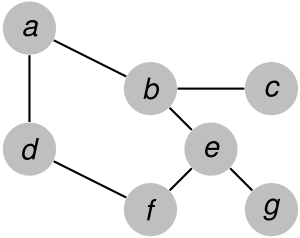
- Sparse - Dense - Complete



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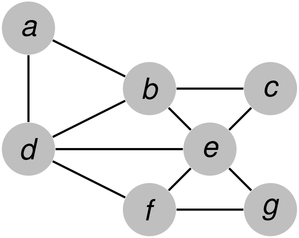

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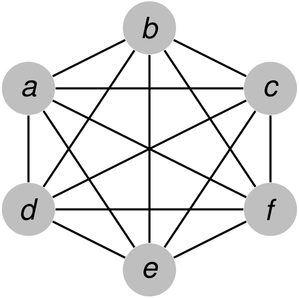

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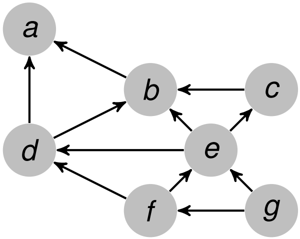

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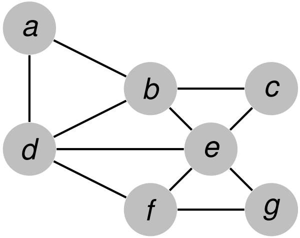

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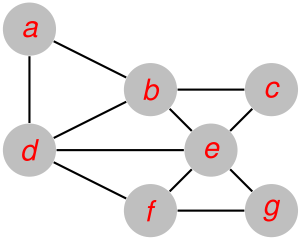

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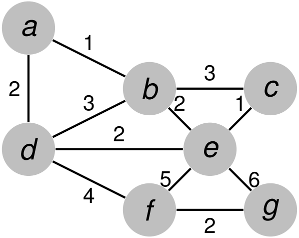

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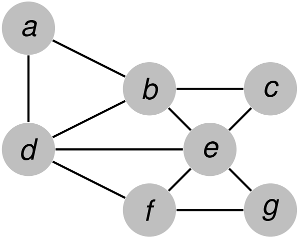

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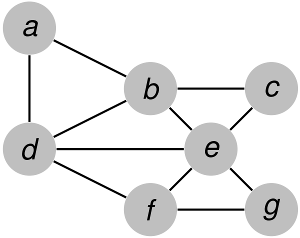

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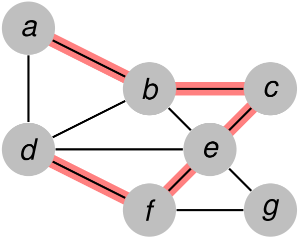

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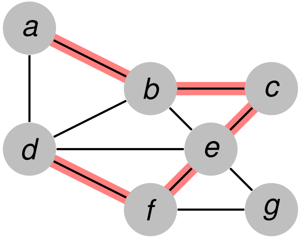

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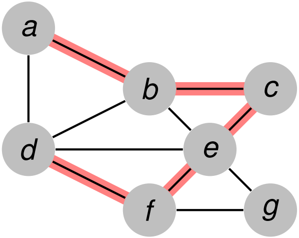

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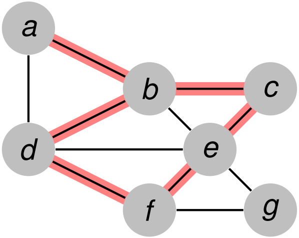

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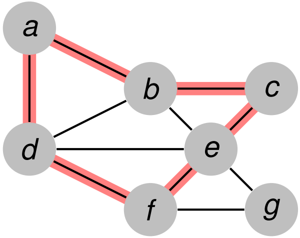

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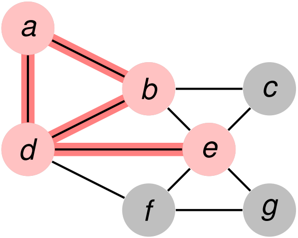

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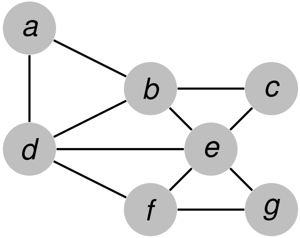

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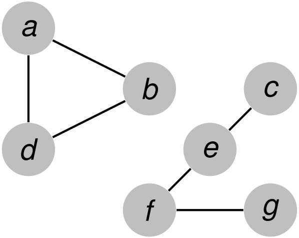

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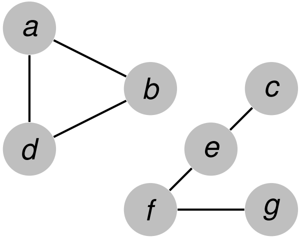

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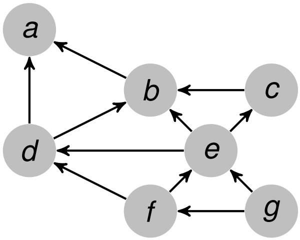

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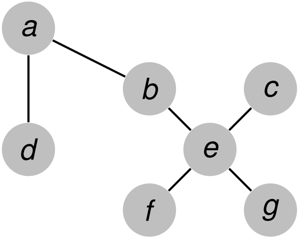

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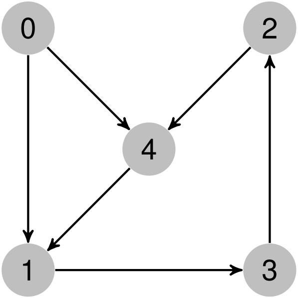

	0	1	2	3	4
0		1			1
1				1	
2					1
3			1		
4		1			

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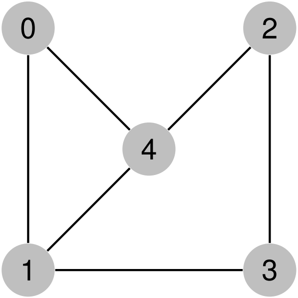

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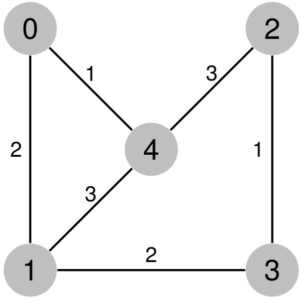

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0		2			1
1	2			2	3
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3		2	1		
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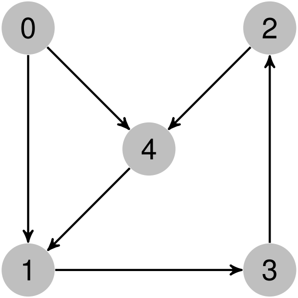
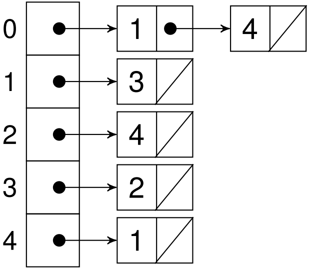
Adjacency List Representation of Graph





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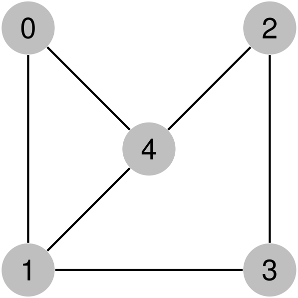

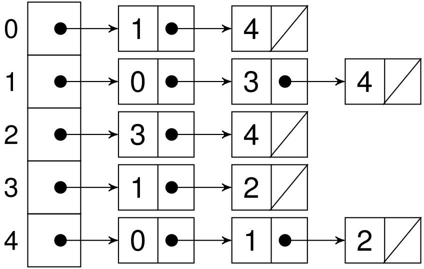

Outline

Graph Traversal

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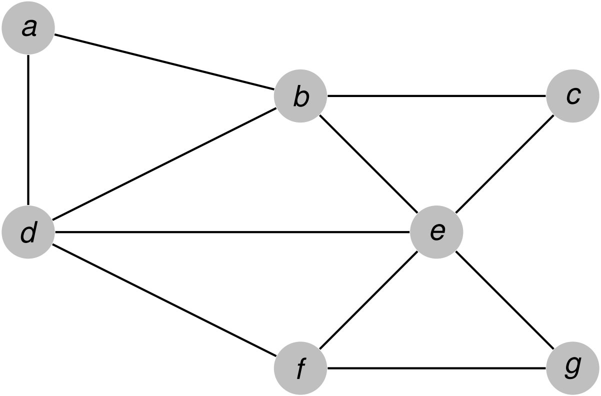
- Breadth-first search (BFS): all **adjacent** vertices of a vertex are processed **before** the **descendents** of the vertex

Depth-first search



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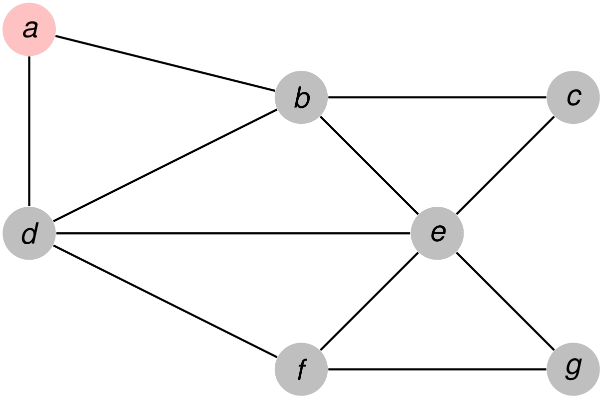

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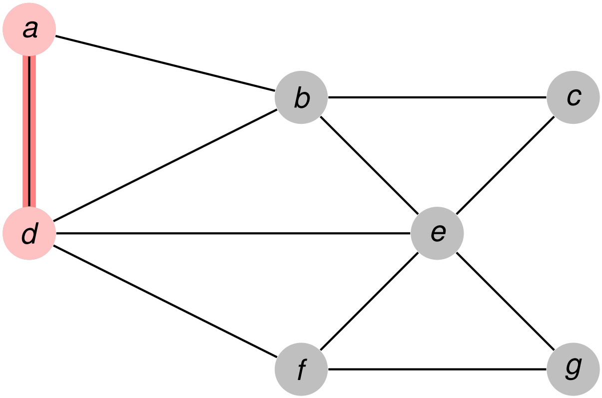

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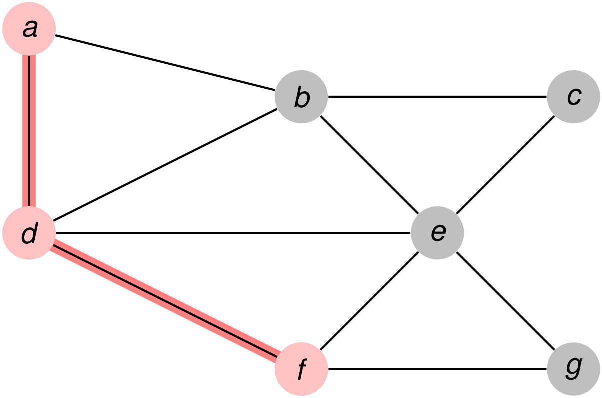

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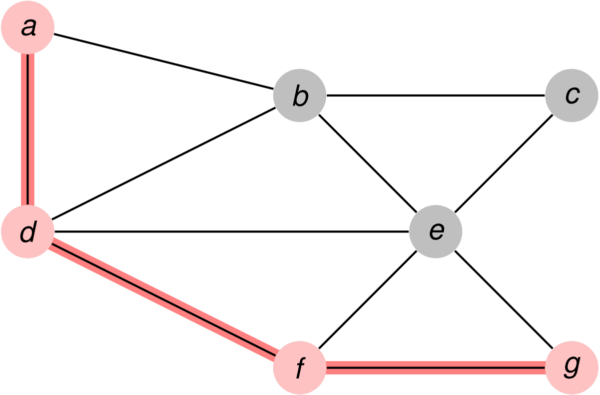

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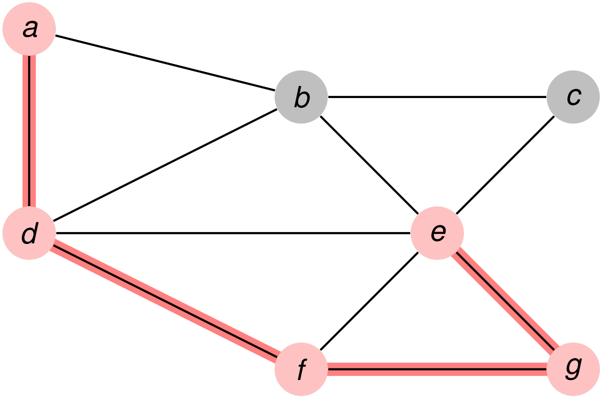

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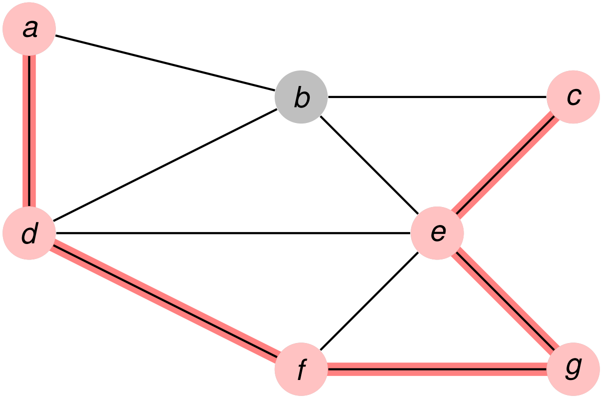

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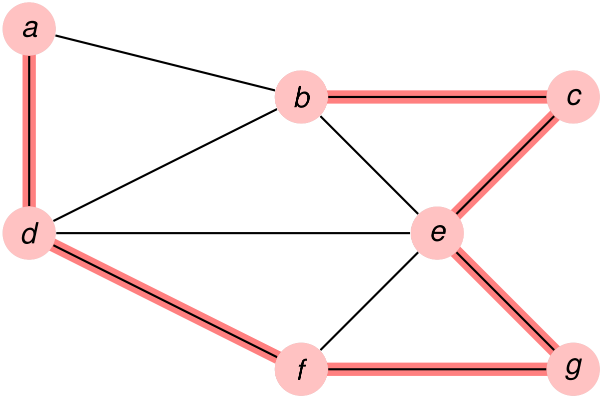
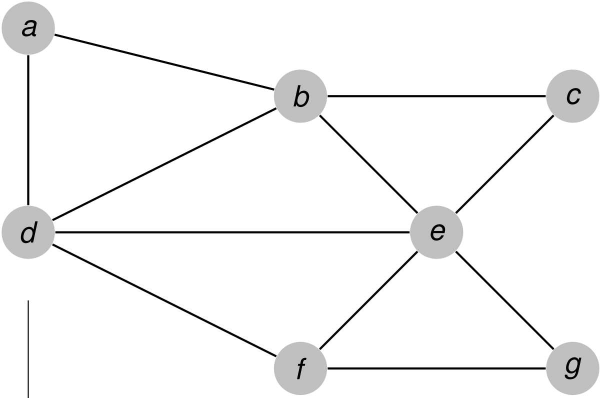
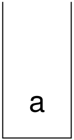


Depth-first search implementation





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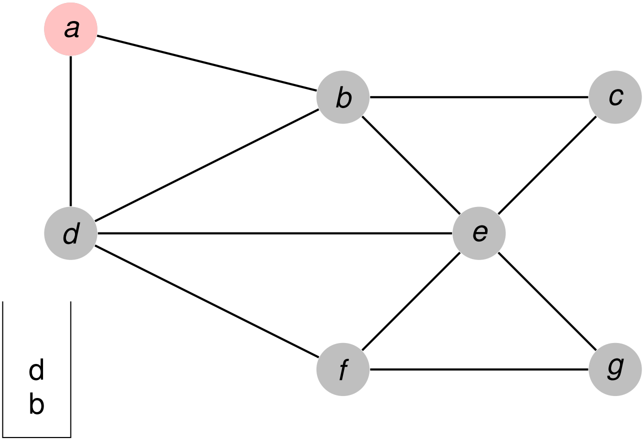

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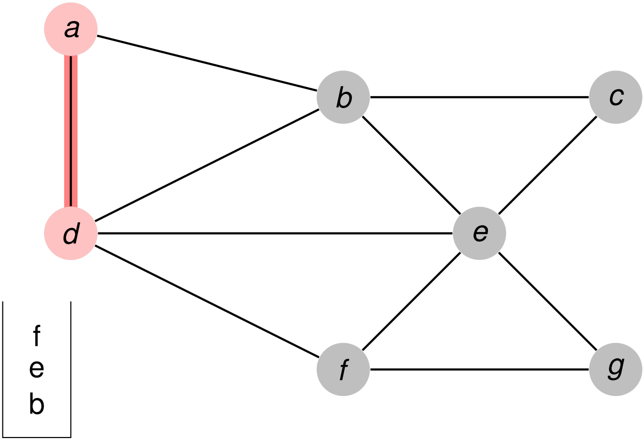

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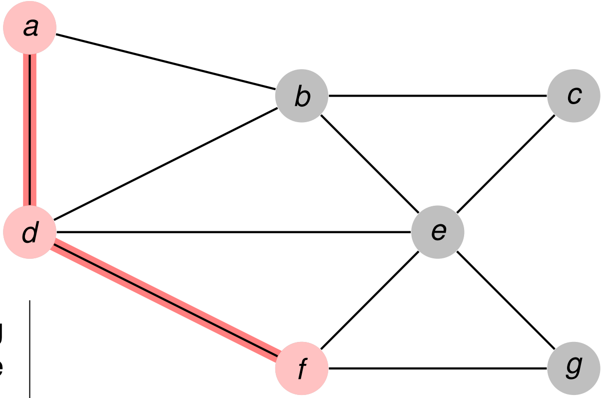

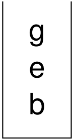

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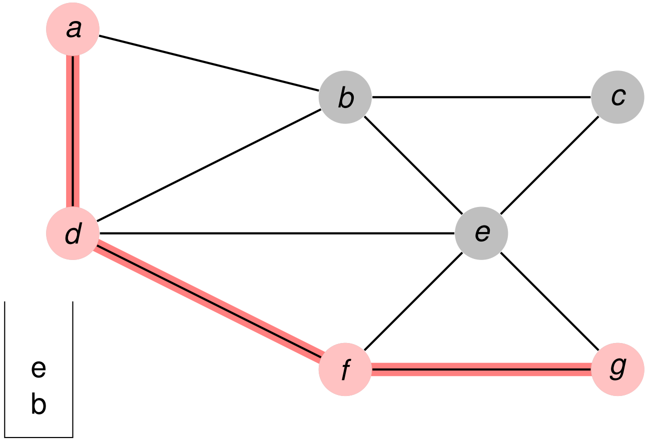

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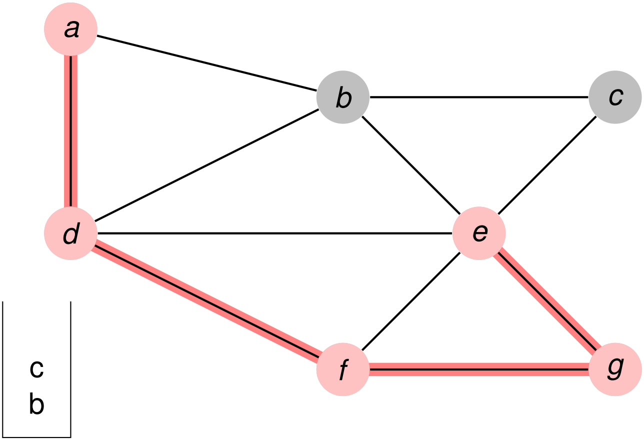
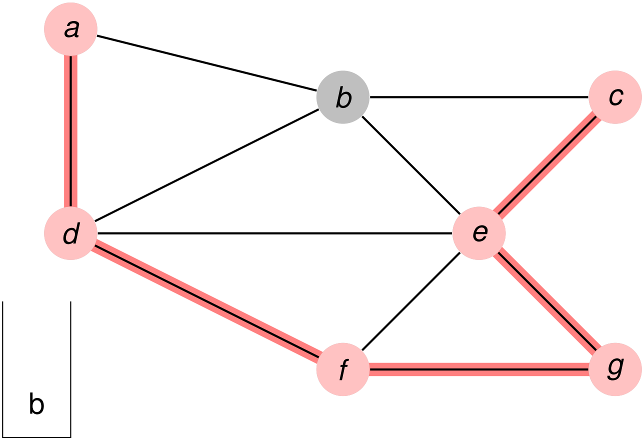
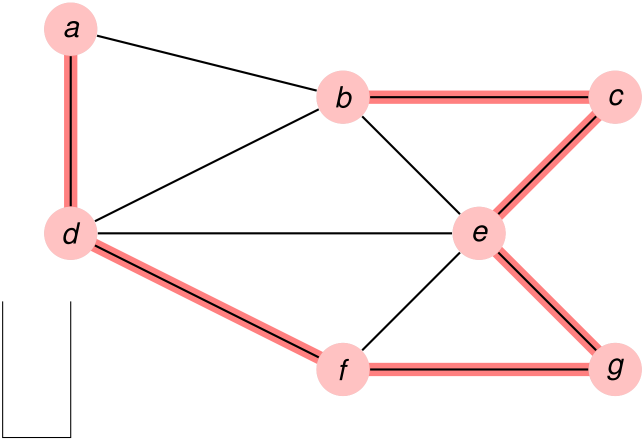


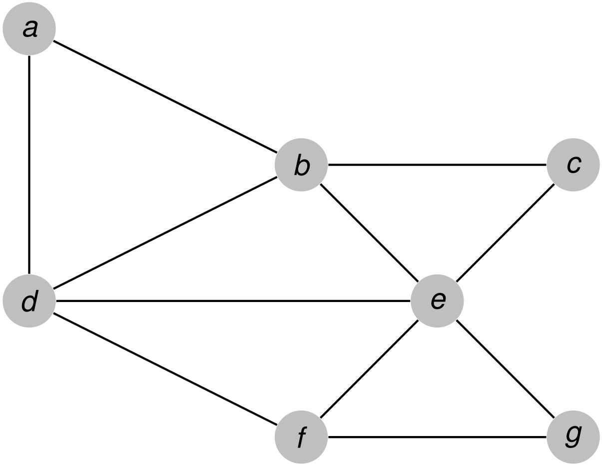
Depth-first search implementation



Depth-first search implementation



Breadth-first search



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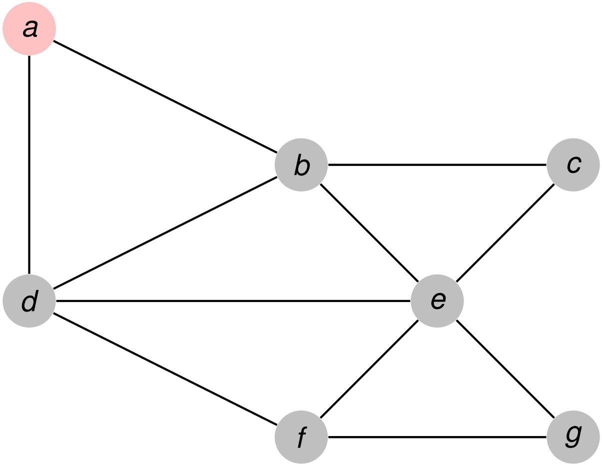

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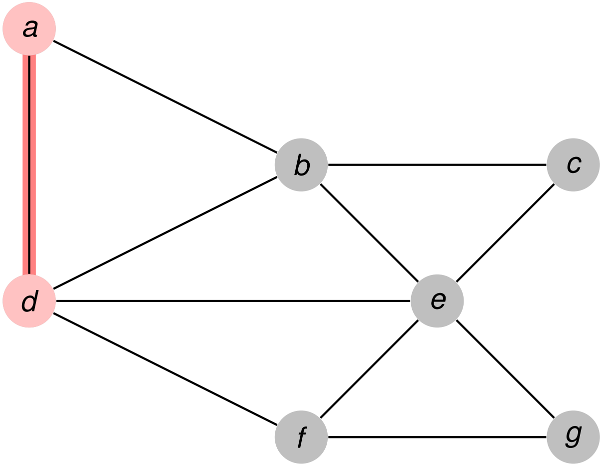

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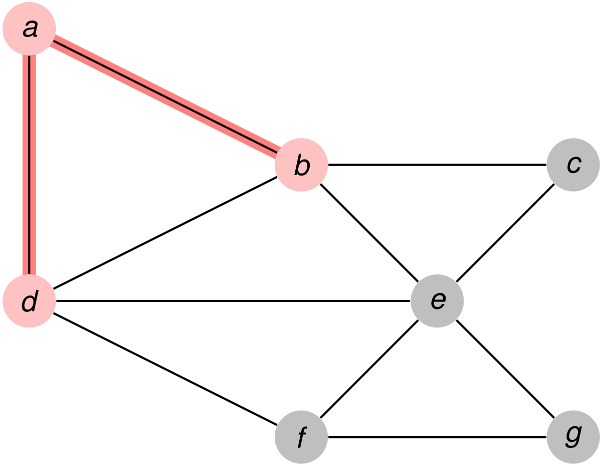

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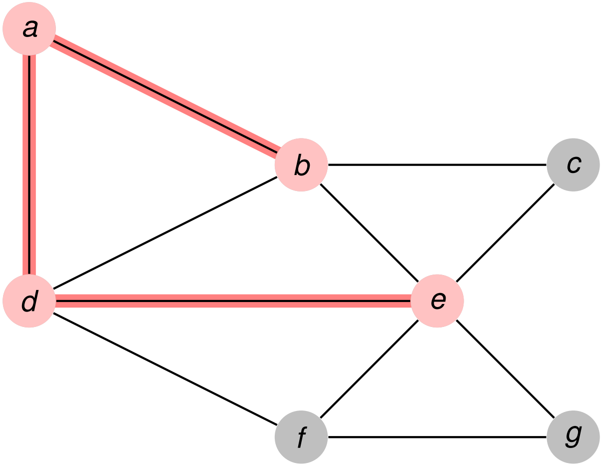

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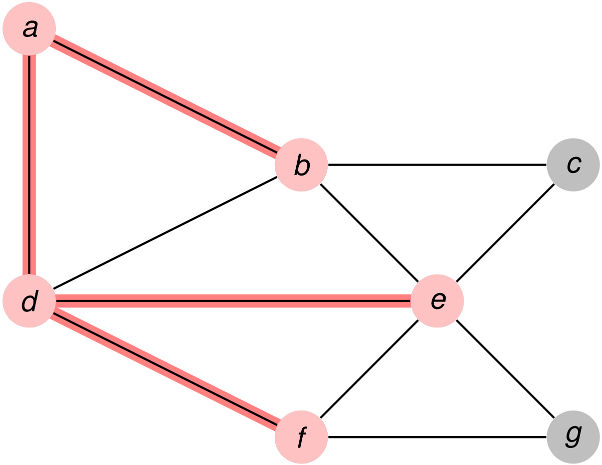

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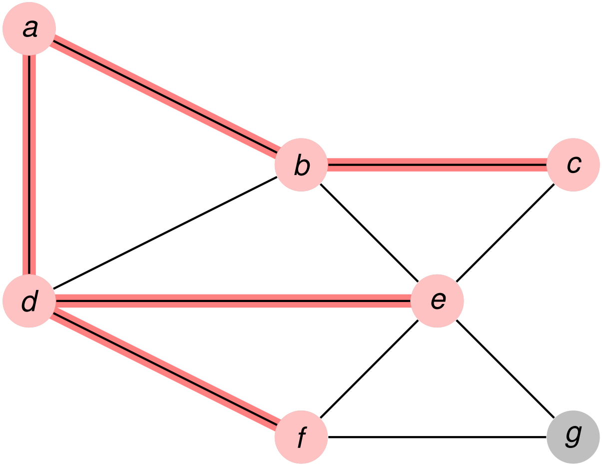

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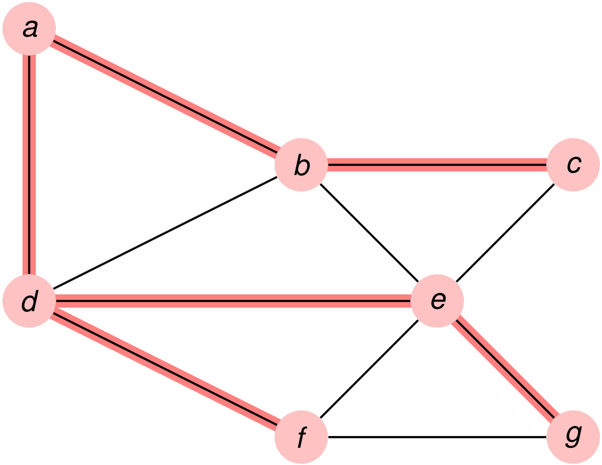
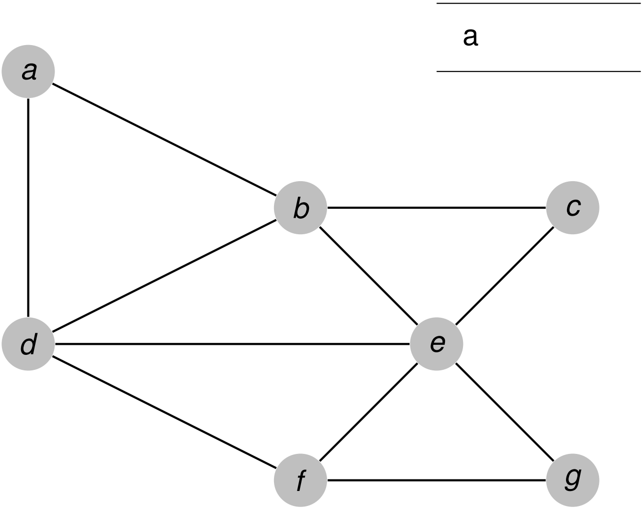
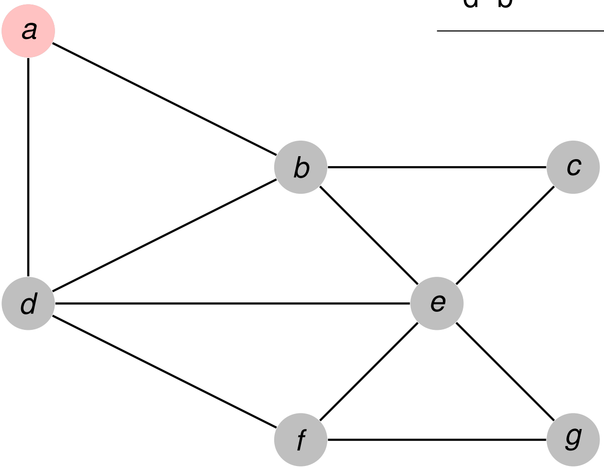


Breadth-first search implementation



Breadth-first search implementation

d b



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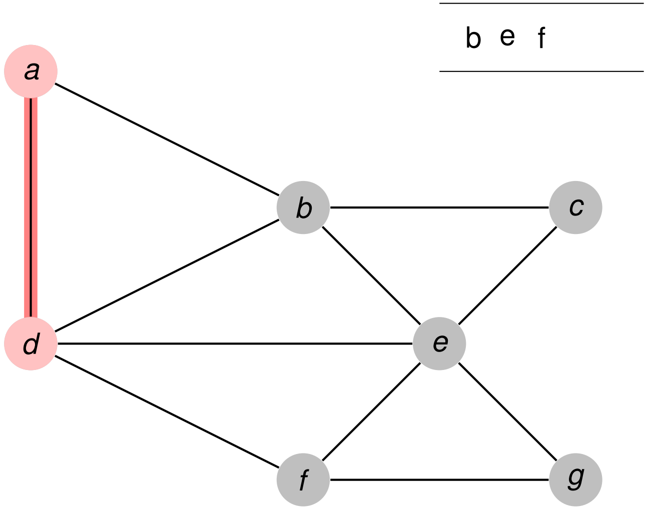

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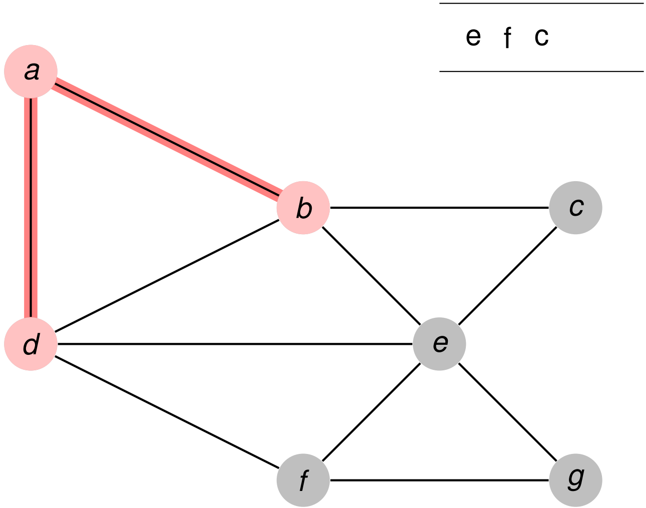

Breadth-first search implementation

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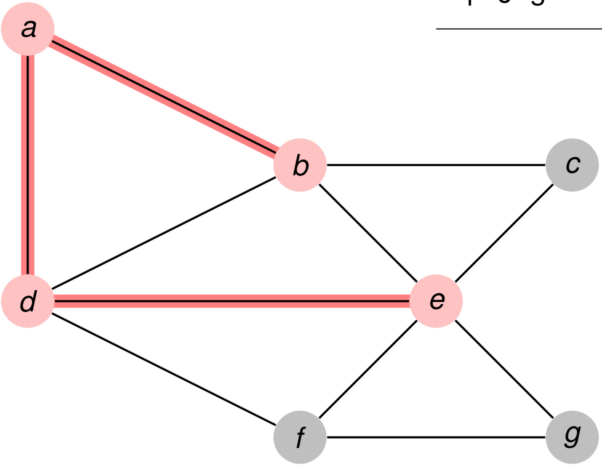

Breadth-first search implementation

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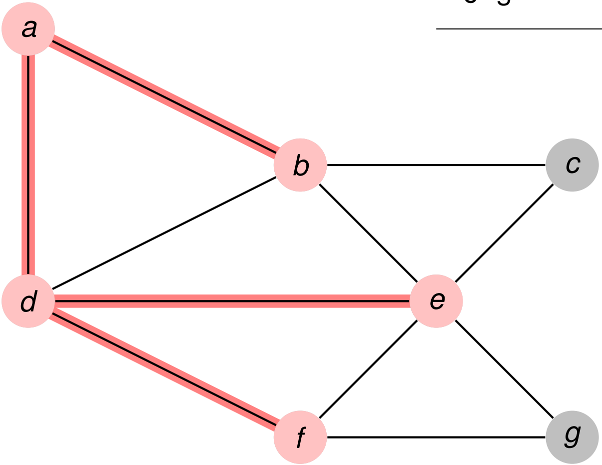

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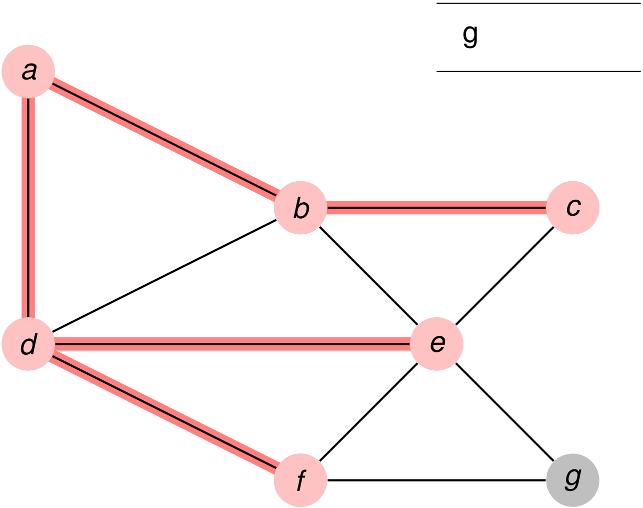

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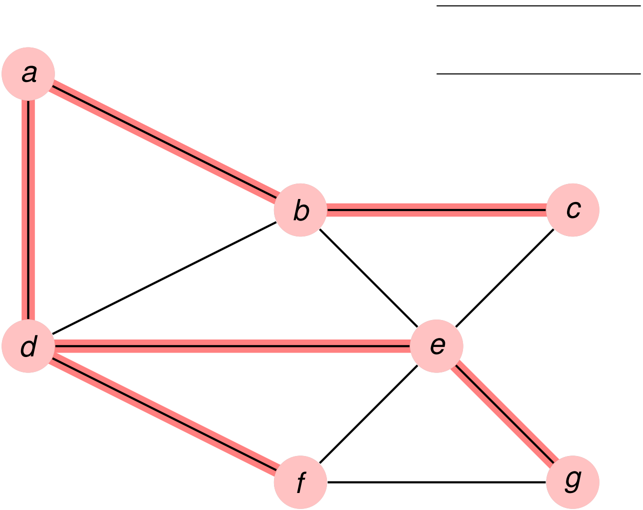
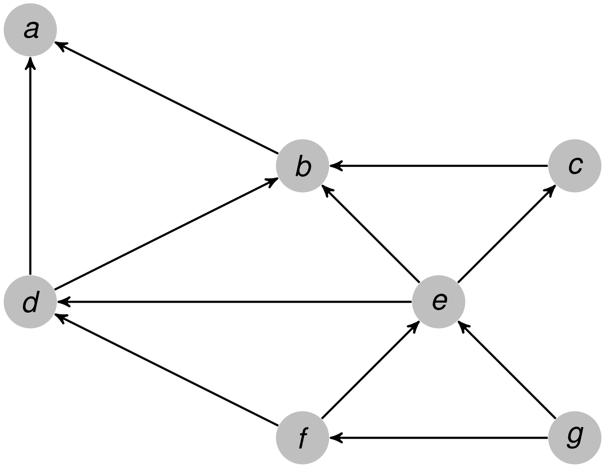


Topological Sort



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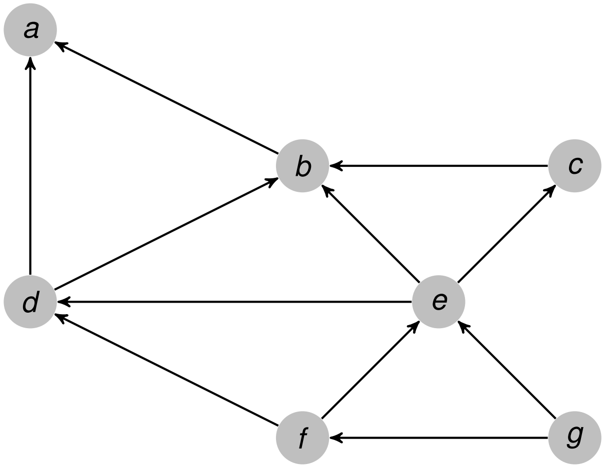

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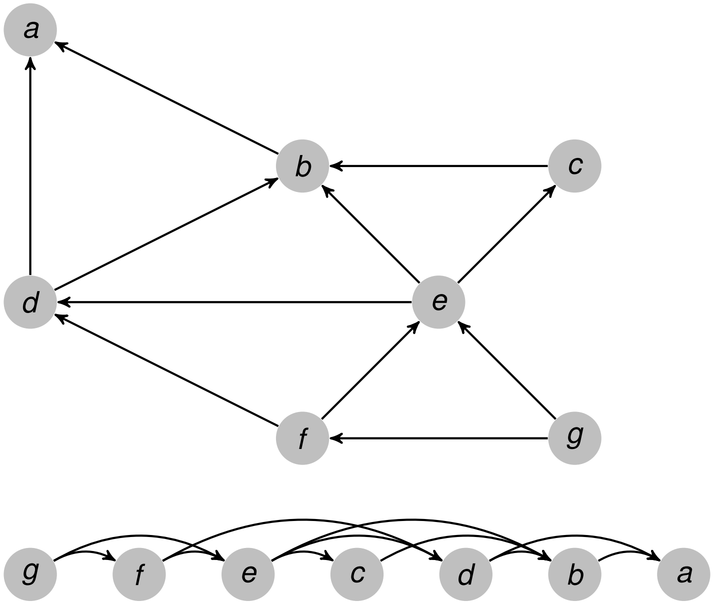

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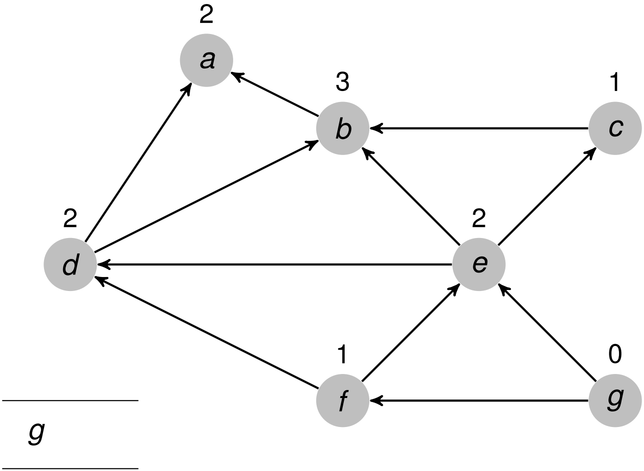

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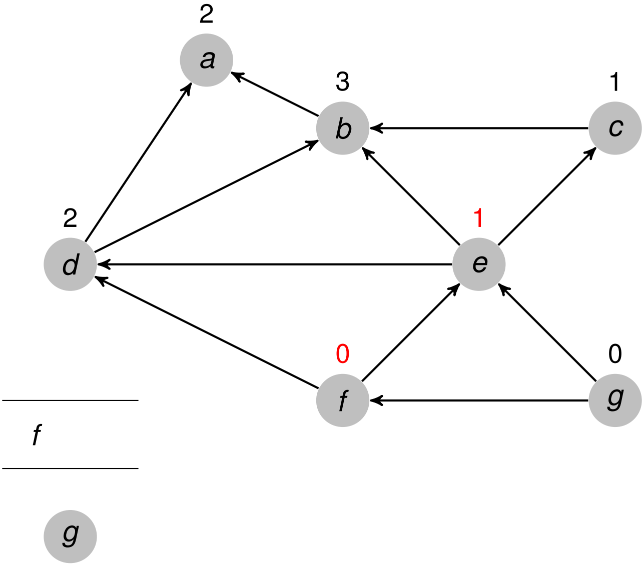

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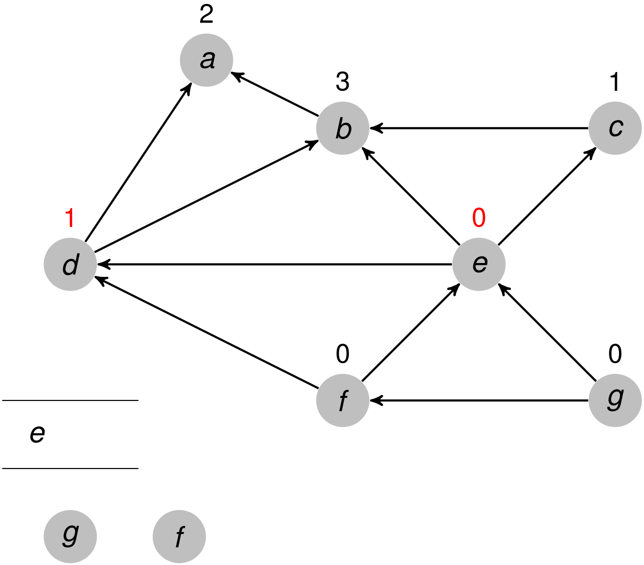

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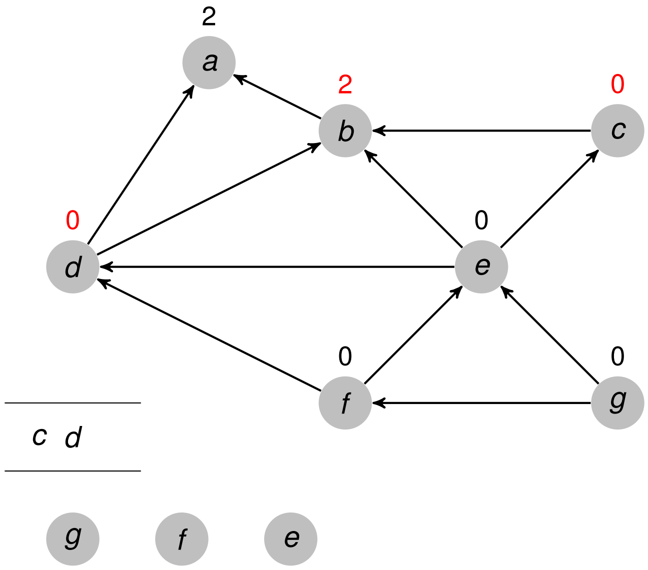

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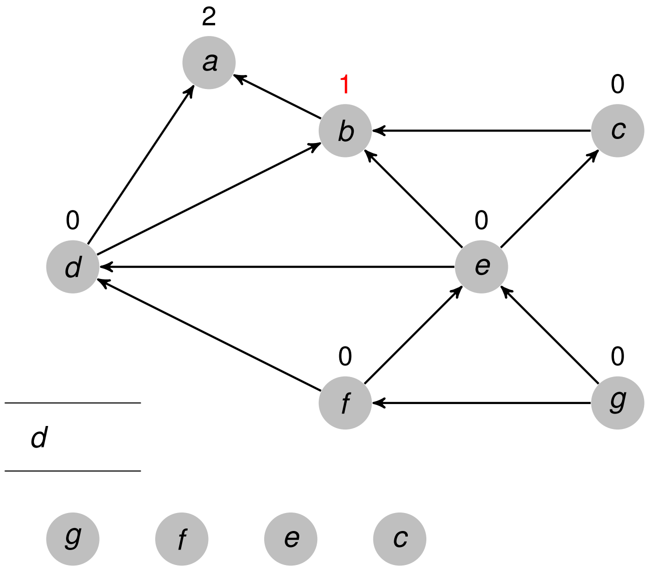
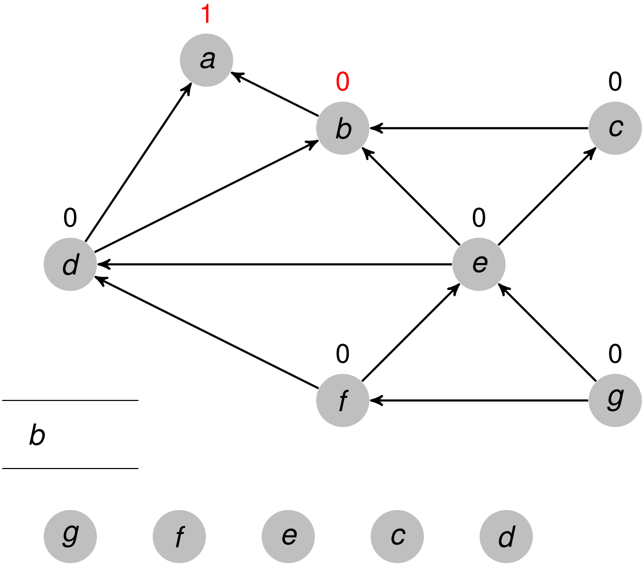
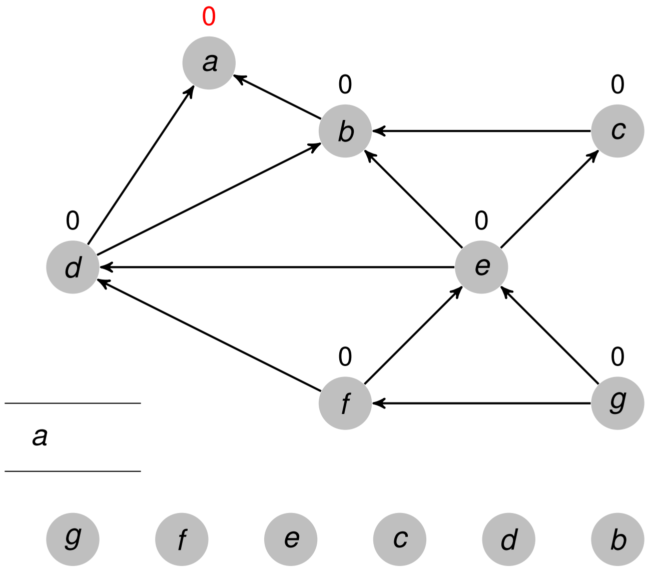


Topological Sort Implementation



Topological Sort Implementation



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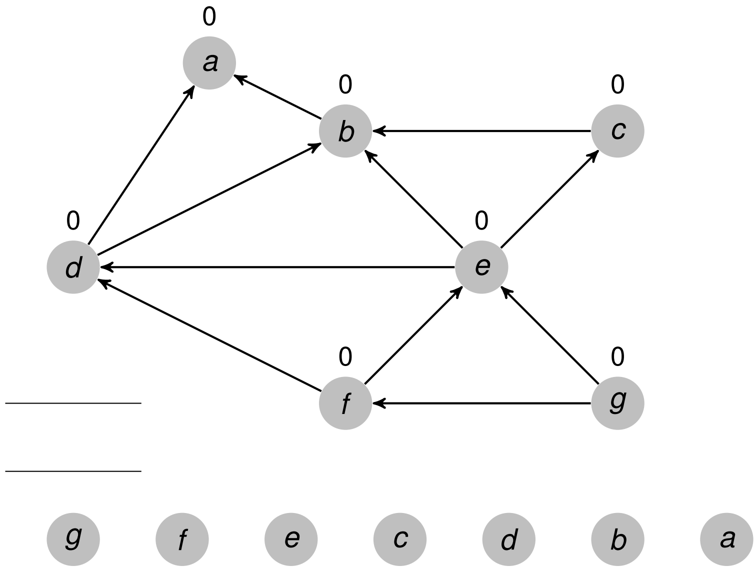

Outline

Single-source shortest path problem

What ?

Given Vertex S in Graph G, find a shortest path from S to every other vertex in G.

Why ?

Find the cheapest way for one computer to broadcast a message to all other computers on the computer network.

How ?

- Dijkstra’s algorithm

Dijkstra's algorithm

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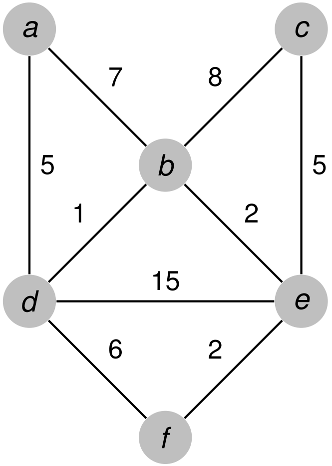
   3.1 Pick a vertex u $\notin$ sptSet and has minimum distance value.
   3.2 Include u to sptSet.
   3.3 For every adjacent vertex v of u, if sum of distance value of u (from source) and weight of edge u-v, is less than the distance value of v, then update the distance value of v.

Example of Dijkstra's algorithm



	a	b	c	d	e	f
Initial	0	∞	∞	∞	∞	∞

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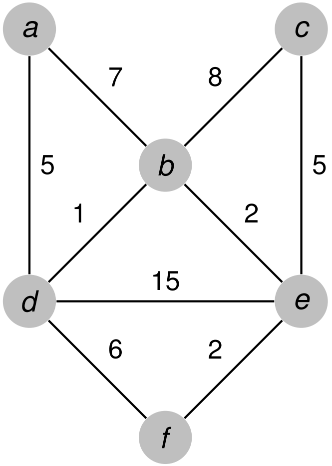

	a	b	c	d	e	f
Initial	0	∞	∞	∞	∞	∞
Proc.a	0	7	∞	5	∞	∞

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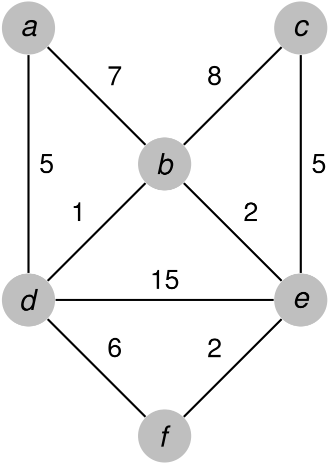

	a	b	c	d	e	f
Initial	0	∞	∞	∞	∞	∞
Proc.a	0	7	∞	5	∞	∞
Proc.d	0	6	∞	5	20	11

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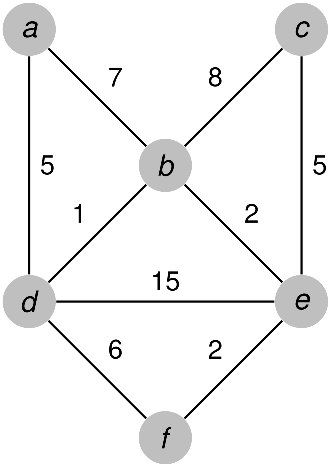

	a	b	*c*	d	e	*f*
Initial	0	∞	∞	∞	∞	∞
Proc.a	0	7	∞	5	∞	∞
Proc.d	0	6	∞	5	20	11
Proc.b	0	6	14	5	8	11

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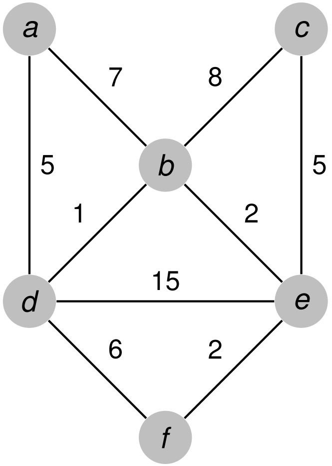

	a	b	c	d	e	f
Initial	0	∞	∞	∞	∞	∞
Proc.a	0	7	∞	5	∞	∞
Proc.d	0	6	∞	5	20	11
Proc.b	0	6	14	5	8	11
Proc.?	0	6		5		
Proc.?	0	6		5		
Proc.?	0	6		5		

Minimum Spanning Tree (MST)

What?

- Spanning tree: tree that contains all of the vertices in a connected graph.

Why?

How?

Minimum Spanning Tree (MST)

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- Minimum spanning tree: spanning tree such that the sum of its weights are minimal.

Why?

How?

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How?

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- Prim's algorithm

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- Spanning tree: tree that contains all of the vertices in a connected graph.
- Minimum spanning tree: spanning tree such that the sum of its weights are minimal.

Why?

- Communication
- Water supply
- Electricity Transmission

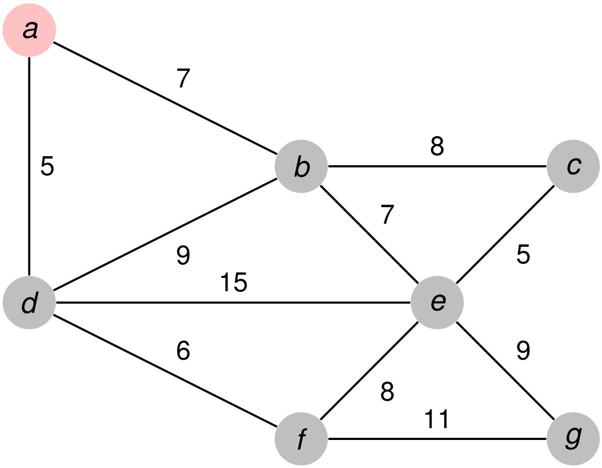
How?

- Prim's algorithm
- Kruskal's algorithm

Prim's Algorithm

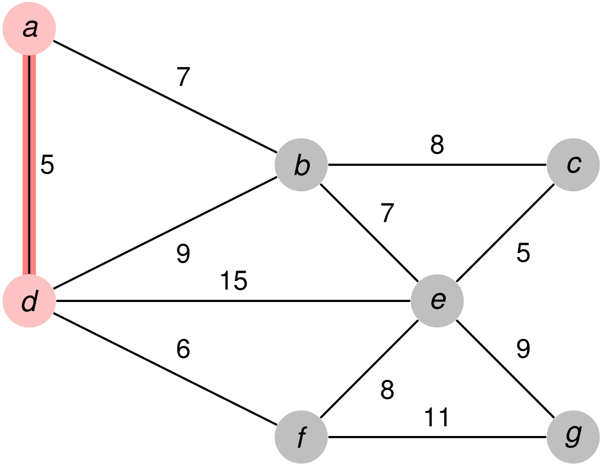
1. Create a set mstSet that keeps track of vertices already included in MST.
2. Assign a key value (0 for first, ∞ for others) to all vertices in the input graph.
3. While mstSet doesn't include all vertices
   3.1 Pick a vertex u $\notin$ mstSet and has minimum key value.
   3.2 Include u to mstSet.
   3.3 For every adjacent vertex v of u, if weight of edge u-v is less than the previous key value of v, update the key value as weight of u-v

Example of Prim's Algorithm



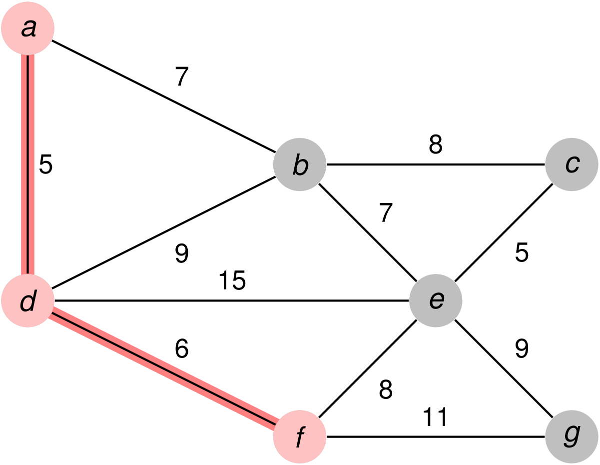
a	b	c	d	e	f	g
0	∞	∞	∞	∞	∞	∞

Example of Prim's Algorithm



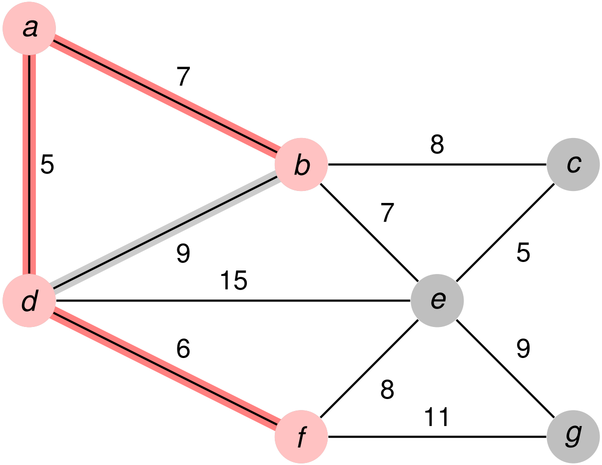
a	b	c	d	e	f	g
0	7	∞	5	∞	∞	∞

Example of Prim's Algorithm



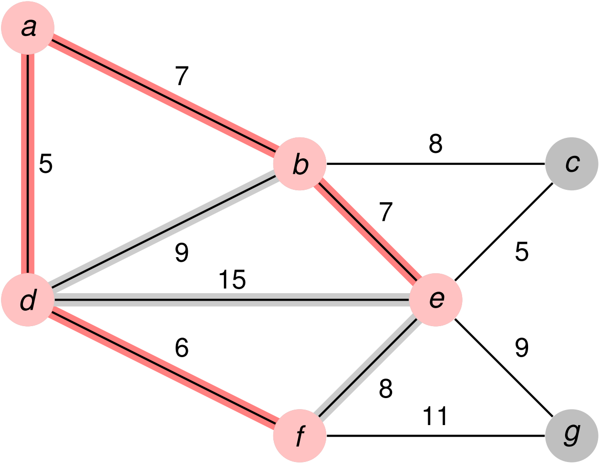
a	b	c	d	e	f	g
0	7	∞	5	15	6	∞

Example of Prim's Algorithm



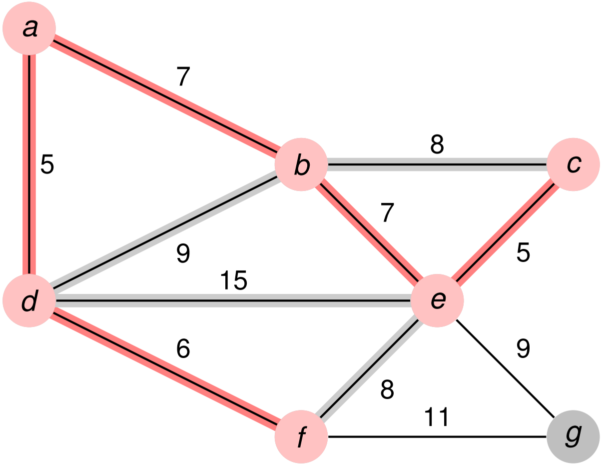
a	b	c	d	e	f	g
0	7	∞	5	8	6	11

Example of Prim's Algorithm



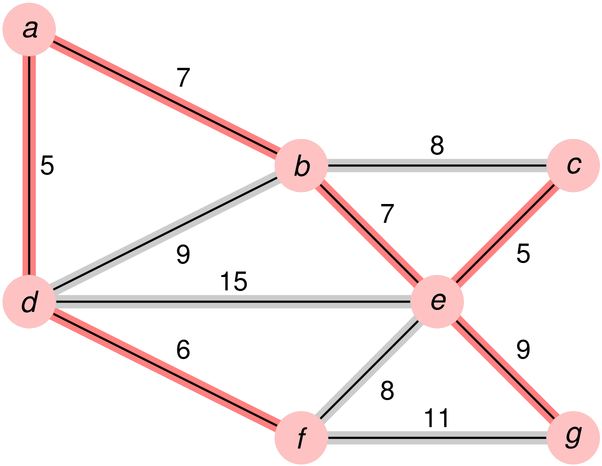
a	b	c	d	e	f	g
0	7	8	5	7	6	11

Example of Prim's Algorithm



a	b	c	d	e	f	g
0	7	5	5	7	6	9

Example of Prim's Algorithm



a	b	c	d	e	f	g
0	7	5	5	7	6	9

Kruskal's Algorithm

1. Sort all the edges in non-decreasing order of their weight.

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2. Pick the smallest edge. Check if it forms a cycle with the spanning tree formed so far. If cycle is not formed, include this edge. Else, discard it.

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1. Sort all the edges in non-decreasing order of their weight.
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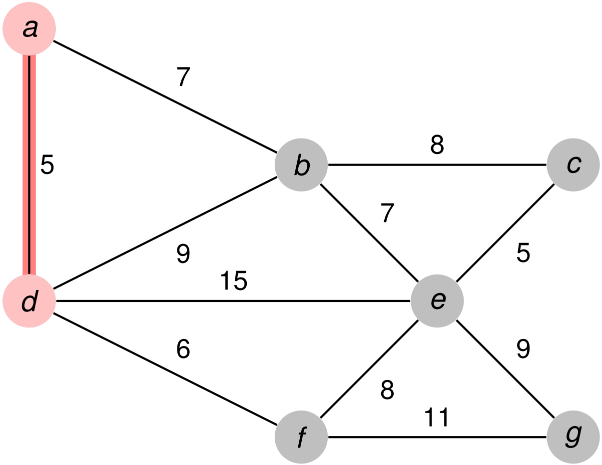
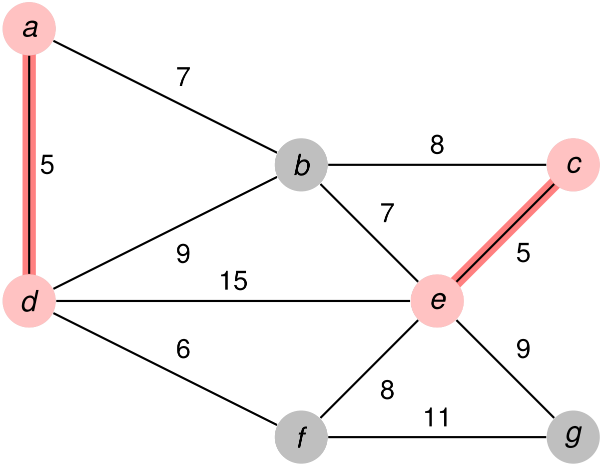
3. Repeat step #2 until there are (V-1) edges in the spanning tree.

Example of Kruskal’s Algorithm



Example of Kruskal's Algorithm



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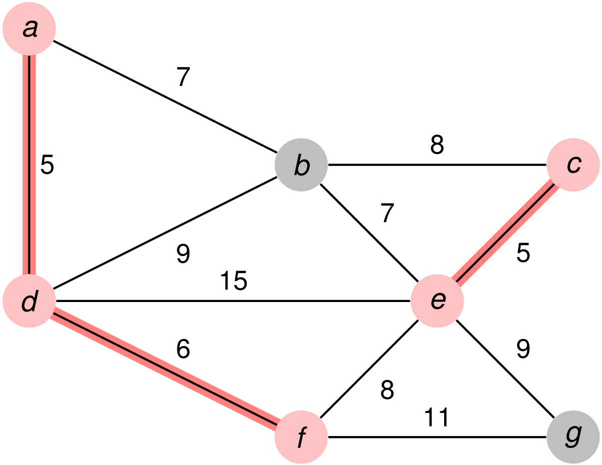

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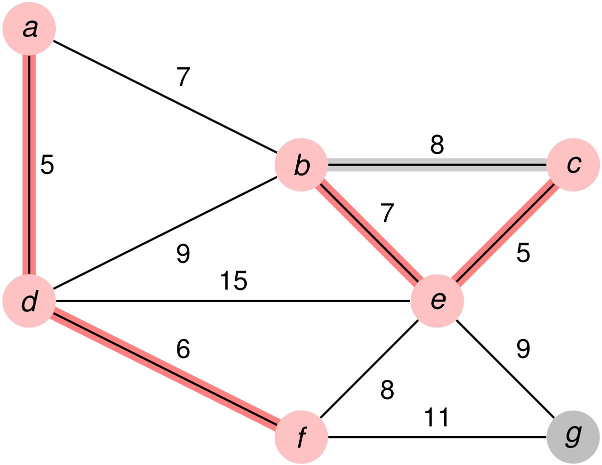

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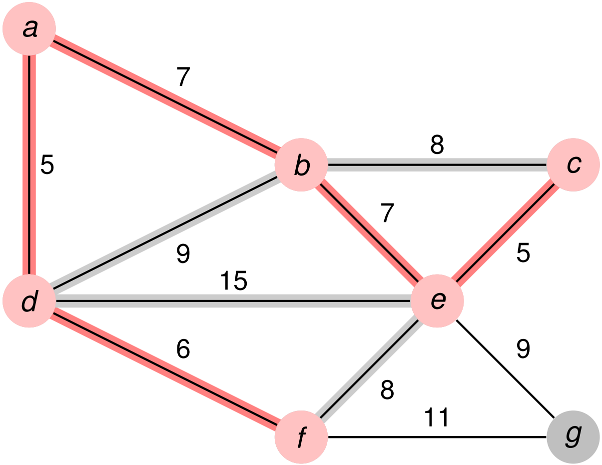

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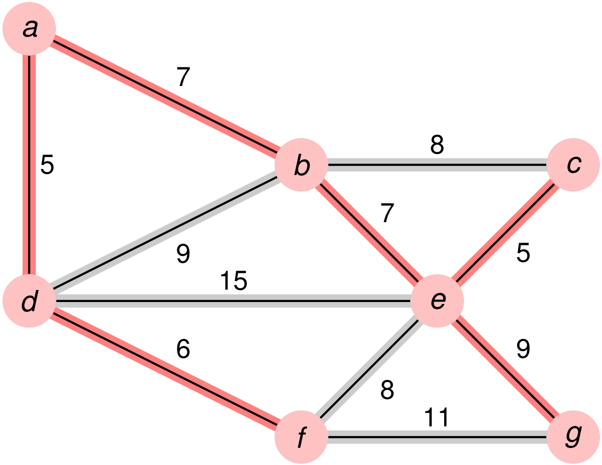

Summary

- Terminology
- Graph Representations: Adjacent Matrix, Adjacent List
- Graph Traversal: DFS, BFS
- Topological Sort
- Graph Problems: Shortest-Path, MST