

Problem #3.3

		Second Shot		$H_0 : S_1 \perp S_2$ vs $H_1 : \text{not } H_0$
		$S_2 = 1$	$S_2 = 0$	
First Shot	$S_1 = 1$	251	34	
	$S_1 = 0$	48	5	

Data is not ordinal so a restricted alternative is not necessary.

Statistics	Value	p-value	Conclusion
X^2	0.2727	0.6015	Do not reject H_0 , there is evidence that the first and second shot are independent
G^2	2.8060	0.2459	Do not reject H_0 , there is evidence that the first and second shot are independent

Problem #3.9(a)

Table 1: Standardized Pearson Residuals

	Drugs	No Drugs
Schizophrenia	7.87	-7.87
Affective disorder	1.60	-1.60
Neurosis	-2.39	2.39
Personality disorder	-4.84	4.84
Special systems	-5.14	5.14

The Pearson residuals are quite large which indicates that there is a lack of fit. This is likely due to the different proportion of those who received drugs. For Schizophrenia and Affective disorder, there are *more people who are* assigned drugs than those who are not assigned drugs. Where as with Neurosis, Personality disorder and Special systems, there are *more people who are not* assigned drugs than those who are as-

signed drugs. This explains why the first two rows have standardized residuals with signs (+, -) and the last three rows have standardized residuals with signs (-, +).

Problem #3.12

Gamma, γ : 0.3873

95% CI: (0.3156, 0.4591)

Gamma is 0.3873 which indicates that when attitudes disagree (i.e. counts that are not on the diagonal), the proportion of concordant attitudes towards abortions (\uparrow school = \uparrow approval) is larger than the proportion of discordant attitudes. This means that there is greater approval of abortion when there is more schooling.

Problem #3.15

Group	Treatment	Normalization		Name	95% CI
		Yes	No		
		7	8		
	Control	0	15	A Woolf (Wald)	(0.000, ∞)
				B Cornfield Exact	(1.9784, ∞)
				C Profile Likelihood	(5.117, ∞)

Having a cell with a value of 0 makes analysis difficult.

Problem #3.31

For a 2×2 table, consider $H_0 : \pi_{11} = \theta^2, \quad \pi_{12} = \pi_{21} = \theta(1 - \theta), \quad \pi_{22} = (1 - \theta)^2$.

Problem #3.31(a)

Show that the marginal distributions are identical and that independence holds.

π_{11}	π_{12}	π_{1+}	$\stackrel{H_0}{=}$	θ^2	$\theta(1 - \theta)$	θ
π_{21}	π_{22}	π_{2+}		$\theta(1 - \theta)$	$(1 - \theta)^2$	$1 - \theta$
π_{+1}	π_{+2}			θ	$1 - \theta$	

$$\left. \begin{array}{l} \pi_{+1} = \pi_{1+} = \theta \\ \pi_{+2} = \pi_{2+} = 1 - \theta \end{array} \right\} \text{marginal distributions are the same}$$

$$\left. \begin{array}{l} \theta^2 = \pi_{11} = \pi_{1+}\pi_{+1} = \theta \cdot \theta = \theta^2 \\ \theta(1 - \theta) = \pi_{12} = \pi_{1+}\pi_{+2} = \theta \cdot (1 - \theta) \\ \theta(1 - \theta) = \pi_{21} = \pi_{2+}\pi_{+1} = (1 - \theta) \cdot \theta \\ (1 - \theta)^2 = \pi_{22} = \pi_{2+}\pi_{+2} = (1 - \theta) \cdot (1 - \theta) = (1 - \theta)^2 \end{array} \right\} \text{independence holds}$$

Problem #3.31(b)

$$\begin{aligned}
 f &\propto \pi_{11}^{n_{11}} \pi_{12}^{n_{12}} \pi_{21}^{n_{21}} \pi_{22}^{n_{22}} \\
 &\stackrel{H_0}{=} (\theta^2)^{n_{11}} \cdot (\theta(1-\theta))^{n_{12}+n_{21}} \cdot ((1-\theta)^2)^{n_{22}} \\
 L &= n_{11} \log [\theta^2] + (n_{12} + n_{21}) \log [\theta(1-\theta)] + n_{22} \log [(1-\theta)^2] \\
 \frac{\partial L}{\partial \theta} &= \frac{2n_{11}}{\theta} + \frac{n_{12} + n_{21}}{\theta} - \frac{n_{12} + n_{21}}{(1-\theta)} - \frac{2n_{22}}{(1-\theta)} \\
 &= \frac{2n_{11} + n_{12} + n_{21}}{\theta} - \frac{2n_{22} + n_{12} + n_{21}}{1-\theta} \stackrel{set}{=} 0 \\
 \Rightarrow \hat{\theta} &= \frac{2n_{11} + n_{12} + n_{21}}{2(n_{11} + n_{12} + n_{21} + n_{22})} \quad \begin{array}{l} n = n_{11} + n_{12} + n_{21} + n_{22} \\ n_{+1} = n_{11} + n_{21} \\ n_{1+} = n_{11} + n_{12} \end{array} \\
 &= \frac{n_{1+} + n_{+1}}{2n} = \frac{1}{2} \left(\frac{n_{1+}}{n} \cdot \frac{n_{+1}}{n} \right) \\
 \hat{\theta} &= \frac{p_{1+} + p_{+1}}{2}
 \end{aligned}$$

Problem #3.31(c)

Calculate the expected frequencies:

$$\begin{aligned}
 \hat{\mu}_{11} &= n \cdot \hat{\pi}_{11} = n \cdot \hat{\theta}^2 \\
 \hat{\mu}_{12} &= n \cdot \hat{\pi}_{12} = n \cdot \hat{\theta}(1-\hat{\theta}) \\
 \hat{\mu}_{21} &= n \cdot \hat{\pi}_{21} = n \cdot \hat{\theta}(1-\hat{\theta}) \\
 \hat{\mu}_{22} &= n \cdot \hat{\pi}_{22} = n \cdot (1-\hat{\theta})^2
 \end{aligned}$$

Using the expected frequencies obtain Pearson's $X^2 = \sum \frac{(n_{ij} - \hat{\mu}_{ij})^2}{\hat{\mu}_{ij}}$ and compare to a χ^2 distribution with $df = 2$ ($df = \dim(H_0 \cup H_1) - \dim(H_0) = 3 - 1 = 2$). Note that when testing independence for a 2×2 table $df = 1$.

Problem #3.31(c)

H_0	X^2	df	p-value	Conclusion
indep.	0.27274	1	0.6015	Do not reject H_0 , there is evidence that the first and second shot are independent
iid	0.27274	2	0.8725	Do not reject H_0 , there is evidence that the first and second shot are iid

It is plausible that the free throws are independent and identically distributed.