Problem #2.3

	Statistic	Value
1	Difference in Proportion	0.0085
2	Relative Risk	7.8970
3	Odds Ratio (OR)	7.9649

Interpretations:

- *Difference in Proportion:* The proportion for fatal injuries when a seat belt is not used is 0.0085 more than the proportion of fatal injuries when a seat belt is used.
- *Relative Risk:* The proportion of fatal injuries when a seat belt is not used is 7.8970 times that of the proportion of fatal injuries when a seat belt is used.
- *Odds Ratio (OR):* The odds of fatal injury when a seat belt is not used is 7.9649 times that of the odds of fatal injury when a seat belt is used.

The OR and the Relative Risk are similar to each other which makes sense because the difference in proportions is small and the event is rare.

Problem #2.8

Problem #2.8a

Correct interpretation of OR: The odds of survival for females is 11.4 times that of the odds of survival for males.

The interpretation "The probability of survival for females was 11.4 times that for males" would be approximately correct if the difference between proportions was small, i.e. the OR would approximately equal the relative risk.

Problem #2.8b

	Gender	Proportion Survived
1	Female	0.7436
2	Male	0.2028

Problem #2.12

	Department	Conditional OR
1	A	0.3492
2	B	0.8025
3	C	1.1331
4	D	0.9213
5	E	1.2216
6	F	0.8279

	Marginal OR
1	1.8411

The Conditional OR for each department is less than the marginal OR. This is due to an association between Gender and Department (i.e. Department is an effect modifier). This can be shown by calculating the corner OR using Female and Department F as the baseline, shown in the table to the right.

	Department	Corner OR
1	A	6.9835
2	B	20.4783
3	C	0.5010
4	D	1.0166
5	E	0.4443

Problem #2.19

	Statistic	Value	95% CI
1	Gamma, γ	0.3604	(0.1401, 0.5806)
2	Kappa, κ	0.1293	(-0.0051, 0.2638)

Since gamma is 0.3604, this indicates that of the couples who disagree, the proportion of concordant couples is larger than those of discordant couples. This means that the wives rating is usually high when the husband's rating is high.

If there is more agreement, Kappa will get closer to one. However, if Kappa negative that indicates that agreement is weaker than agreement by chance (the two disagree more). Since Kappa is fairly small (0.1293) and the 95% CI dips into the negative numbers - this shows that there isn't strong agreement between ratings for husbands and wives.

Problem #2.39

$$\lambda = \frac{V(Y) - \mathbb{E}\left[V(Y \mid X)\right]}{V(Y)} \quad \text{where} \quad V(Y) = 1 - \max(\pi_{+j}) \quad \text{and} \quad V(Y \mid X = i) = 1 - \max_j(\pi_{ij})$$

<u>Show: $\perp\!\!\!\perp = 0 \implies \lambda = 0$</u>

Note: $\lambda = 0 \iff V(Y) - \mathbb{E}\left[V(Y \mid X)\right] = 0$

$$\begin{aligned}
V(Y) - \mathbb{E}\left[V(Y \mid X)\right] &= \left(1 - \max_j \{\pi_{+j}\}\right) - \mathbb{E}\left[1 - \max_j \{\pi_{j|X=i}\}\right] \\
&= \mathbb{E}\left[\max_j \{\pi_{j|X=i}\}\right] - \max_j \{\pi_{+j}\} \\
&= \sum_i \pi_{i+} \max_j \{\pi_{j|i}\} - \max_j \{\pi_{+j}\} \\
&= \sum_i \pi_{i+} \max_j \left\{\frac{\pi_{ij}}{\pi_{i+}}\right\} - \max_j \{\pi_{+j}\} \\
&= \sum_i \max_j \{\pi_{ij}\} - \max_j \{\pi_{+j}\} \qquad (*)
\end{aligned}$$

$$X \perp\!\!\!\perp Y \implies \pi_{ij} = \pi_{i+} \cdot \pi_{j+} \qquad \text{(plug this into } (*))$$

$$\begin{aligned}
V(Y) - \mathbb{E}\left[V(Y \mid X)\right] &= \sum_i \max_j \{\pi_{i+} \cdot \pi_{j+}\} - \max_j \{\pi_{+j}\} \\
&= \sum_i \pi_{i+} \max_j \{\pi_{+j}\} - \max_j \{\pi_{+j}\} \qquad \sum_i \pi_{i+} = 1 \\
&= \max_j \{\pi_{+j}\} - \max_j \{\pi_{+j}\} = 0 \\
&\implies \lambda = 0
\end{aligned}$$

<u>Show $\lambda = 0 \not\Longrightarrow \perp\!\!\!\perp$</u>

$\lambda = 0 \iff \sum_i \max_j \{\pi_{ij}\} - \max_j \{\pi_{+j}\} \not\Longrightarrow X \perp\!\!\!\perp Y$

$\sum_i \max_j \{\pi_{ij}\} = 0.4 + 0.2 = 0.6$

$\max_j \{\pi_{+j}\} = \max(0.6, 0.2, 0.2) = 0.6$

$\pi_{1+} = 0.6 \quad \text{and} \quad \pi_{+1} = 0.6$

$\pi_{11} = 0.4 \neq 0.36 = \pi_{1+}\pi_{+1} \implies X \not\perp\!\!\!\perp Y$

		Y			
		$j=1$	$j=2$	$j=3$	
X	$i=1$	0.4	0.1	0.1	0.6
	$i=2$	0.2	0.1	0.1	0.4
		0.6	0.2	0.2	