Problem #1.a

Restricted:

$$
\begin{aligned}
p(\alpha \mid \boldsymbol{y}) &= y_1^{\alpha} y_2^{\alpha} y_3^{1-2\alpha} \\
L(\alpha \mid \boldsymbol{y}) &= (y_1 + y_2)\log\alpha + y_3 \log(1-2\alpha) \\
s(\alpha \mid \boldsymbol{y}) &= \frac{y_1+y_2}{\alpha} - \frac{2y_3}{1-2\alpha} \\
B(\alpha) &= \frac{2n}{\alpha(1-2\alpha)} \\
\alpha_0 &= {}^1\!/_3 \\
s(\alpha_0 \mid \boldsymbol{y}) &= \frac{y_1+y_2}{\alpha_0} - \frac{2y_3}{1-2\alpha_0} \\
&= \frac{y_1+y_2-2\left(n-y_1-y_2\right)}{{}^1\!/_3} \\
&= \frac{y_1+y_2-2n+2y_1+2y_2}{{}^1\!/_3} \\
&= \frac{3\left(y_1+y_2-\frac{2}{3}n\right)}{{}^1\!/_3} \\
&= 9\cdot\left(y_1+y_2-\tfrac{2}{3}n\right) \\
B(\alpha_0) &= \frac{2n}{\alpha_0(1-2\alpha_0)} \\
&= \frac{2n}{\frac{1}{3}\cdot\frac{1}{3}} = \frac{2n}{{}^1\!/_9} = 9\cdot 2n \\
&= 18n \\
S_R^2 &= \frac{s(\alpha_0 \mid \boldsymbol{y})^2}{B(\alpha_0)} \\
&= \frac{\left(9\cdot\left(y_1+y_2-\frac{2}{3}n\right)\right)^2}{18n} \\
&= \frac{9\cdot 9\cdot\left(y_1+y_2-\frac{2}{3}n\right)^2}{9\cdot 2\cdot n} \\
&= \frac{\left[y_1+y_2-({}^2\!/_3)\,n\right]^2}{({}^2\!/_9)\,n}
\end{aligned}
$$

Unrestricted:

$$
\begin{aligned}
\hat{m}_{0,i} &= n\cdot {}^1\!/_3 = {}^n\!/_3 \\
S_U^2 &= \sum_{i=1}^{3}\frac{\left(y_i-\hat{m}_{0,i}\right)^2}{\hat{m}_{0,i}} \\
&= \sum_{i=1}^{3}\frac{\left(y_i-{}^n\!/_3\right)^2}{{}^n\!/_3} \\
&= \frac{\left(y_1-{}^n\!/_3\right)^2}{{}^n\!/_3} + \frac{\left(y_2-{}^n\!/_3\right)^2}{{}^n\!/_3} + \frac{\left(y_3-{}^n\!/_3\right)^2}{{}^n\!/_3}
\end{aligned}
$$

Problem #1.b

	Sample.Size	pi.T1	pi.T2	pi.T3	P.R	aP.R	P.U	aP.U
1	75	0.3333333	0.3333333	0.3333333	0.0373	0.0500	0.0508	0.0500
2	75	0.2500000	0.2500000	0.5000000	0.8242	0.8647	0.7795	0.7884
3	75	0.1666667	0.5000000	0.3333333	0.0378	0.0500	0.9216	0.8962
4	75	0.2000000	0.3000000	0.5000000	0.8238	0.8647	0.8397	0.8349
5	250	0.3333333	0.3333333	0.3333333	0.0519	0.0500	0.0467	0.0500
6	250	0.3000000	0.3000000	0.4000000	0.6256	0.6088	0.4902	0.5037
7	250	0.2200000	0.4467000	0.3333000	0.0542	0.0500	0.9868	0.9819
8	250	0.2500000	0.3000000	0.4500000	0.9727	0.9746	0.9543	0.9594
9	250	0.2200000	0.4000000	0.3800000	0.3721	0.3467	0.9598	0.9381

The power tends to increase when the true probabilities align with the alternative and when there is an increasing in sample size.

Problem #1.c

	pi.T1	pi.T2	pi.T3	n.R	n.U
1	0.3333333	0.3333333	0.3333333	Inf	Inf
2	0.2500000	0.2500000	0.5000000	63	78
3	0.1666667	0.5000000	0.3333333	Inf	58
4	0.2000000	0.3000000	0.5000000	63	69
5	0.3000000	0.3000000	0.4000000	393	482
6	0.2200000	0.4467000	0.3333000	1569772001	125
7	0.2500000	0.3000000	0.4500000	129	149
8	0.2200000	0.4000000	0.3800000	801	165

It makes sense that when the true probabilities are equal that is no sample possible to get 80% power to detect the differences. It also makes sense that when the power calculated in part b is lower than 80% a larger sample than in part b is needed to achieve that power. When the calculated power is greater than 80%, then a smaller sample is needed.