Problem #3.3

		Second Shot	
		$S_2 = 1$	$S_2 = 0$
First Shot	$S_1 = 1$	251	34
	$S_1 = 0$	48	5

$H_0 : S_1 \perp\!\!\!\perp S_2$ vs H_1 : not H_0

Data is not ordinal so a restricted alternative is not necessary.

Statistics	**Value**	**p-value**	**Conclusion**
X^2	0.2727	0.6015	Do not reject H_0, there is evidence that the first and second shot are independent
G^2	0.2858	0.5930	Do not reject H_0, there is evidence that the first and second shot are independent

Problem #3.9(a)

Table 1: Standardized Pearson Residuals

	Drugs	No Drugs
Schizophrenia	7.87	-7.87
Affective disorder	1.60	-1.60
Neurosis	-2.39	2.39
Personality disorder	-4.84	4.84
Special systems	-5.14	5.14

The Pearson residuals are quite large which indicates that there is a lack of fit. This is likely due to the different proportion of those who received drugs. For Schizophrenia and Affective disorder, there are *more people who are* assigned drugs than those who are not assigned drugs. Where as with Neurosis, Personality disorder and Special systems, there are *more people who are not* assigned drugs than those who are assigned drugs. This explains why the first two rows have standardized residuals with signs (+, -) and the last three rows have standardized residuals with signs (-, +).

Problem #3.12

Gamma, γ: 0.3873
95% CI: (0.3156, 0.4591)

Gamma is 0.3873 which indicates that when attitudes disagree (i.e. counts that are not on the diagonal), the proportion of concordant attitudes towards abortions ($\uparrow$ school = $\uparrow$ approval) is larger than the proportion of discordant attitudes. This means that there is greater approval of abortion when there is more schooling.

Problem #3.15

		Normalization	
		Yes	No
Group	Treatment	7	8
	Control	0	15

	V1	CI Lower	CI Uppder
A	Woolf (Wald)	NaN	Inf
B	Cornfield Exact	1.9784	Inf
C(1)	Profile Likelihood	0.0000	NA
C(2)	Profe Likelihood, counts+1	2.1242	286.7235

Having a cell with a value of 0 makes analysis difficult.

Problem #3.31

For a 2 ×2 table, consider $H_0 : \pi_{11} = \theta^2, \quad \pi_{12} = \pi_{21} = \theta(1-\theta), \quad \pi_{22} = (1-\theta)^2$.

Problem #3.31(a)

Show that the marginal distributions are identical and that independence holds.

$$
\begin{array}{|c|c|c}
\cline{1-2}
\pi_{11} & \pi_{12} & \pi_{1+} \\
\cline{1-2}
\pi_{21} & \pi_{22} & \pi_{2+} \\
\cline{1-2}
\multicolumn{1}{c}{\pi_{+1}} & \multicolumn{1}{c}{\pi_{+2}} &
\end{array}
\overset{H_0}{=}
\begin{array}{|c|c|c}
\cline{1-2}
\theta^2 & \theta(1-\theta) & \theta \\
\cline{1-2}
\theta(1-\theta) & (1-\theta)^2 & 1-\theta \\
\cline{1-2}
\multicolumn{1}{c}{\theta} & \multicolumn{1}{c}{1-\theta} &
\end{array}
$$

$$
\left.\begin{array}{l}
\pi_{+1} = \pi_{+1} = \theta \\
\pi_{+2} = \pi_{+2} = 1-\theta
\end{array}\right\} \text{marginal distributions are the same}
$$

$$
\left.\begin{array}{rcrcl}
\theta^2 & = & \pi_{11} = \pi_{1+}\pi_{+1} & = & \theta \cdot \theta = \theta^2 \\
\theta(1-\theta) & = & \pi_{12} = \pi_{1+}\pi_{+2} & = & \theta \cdot (1-\theta) \\
\theta(1-\theta) & = & \pi_{21} = \pi_{2+}\pi_{+1} & = & (1-\theta) \cdot \theta \\
(1-\theta)^2 & = & \pi_{22} = \pi_{2+}\pi_{+2} & = & (1-\theta) \cdot (1-\theta) = (1-\theta)^2
\end{array}\right\} \text{independence holds}
$$

Problem #3.31(b)

$$
\begin{aligned}
f &\propto \pi_{11}^{n_{11}} \pi_{12}^{n_{12}} \pi_{21}^{n_{21}} \pi_{22}^{n_{22}} \\
&\stackrel{H_0}{=} (\theta^2)^{n_{11}} \cdot (\theta(1-\theta))^{n_{12}+n_{21}} \cdot ((1-\theta)^2)^{n_{22}} \\
L &= n_{11} \log\left[\theta^2\right] + (n_{12}+n_{21}) \log\left[\theta(1-\theta)\right] + n_{22} \log\left[(1-\theta)^2\right] \\
\frac{\partial L}{\partial \theta} &= \frac{2n_{11}}{\theta} + \frac{n_{12}+n_{21}}{\theta} - \frac{n_{12}+n_{21}}{(1-\theta)} - \frac{2n_{22}}{(1-\theta)} \\
&= \frac{2n_{11}+n_{12}+n_{21}}{\theta} - \frac{2n_{22}+n_{12}+n_{21}}{1-\theta} \stackrel{set}{=} 0 \\
\implies \hat{\theta} &= \frac{2n_{11}+n_{12}+n_{21}}{2(n_{11}+n_{12}+n_{21}+n_{22})} \\
&= \frac{n_{1+}+n_{+1}}{2n} = \frac{1}{2}\left(\frac{n_{1+}}{n} \cdot \frac{n_{+1}}{n}\right) \\
\hat{\theta} &= \frac{p_{1+}+p_{+1}}{2}
\end{aligned}
$$

$$
\begin{aligned}
n &= n_{11}+n_{12}+n_{21}+n_{22} \\
n_{+1} &= n_{11}+n_{21} \\
n_{1+} &= n_{11}+n_{12}
\end{aligned}
$$

Problem #3.31(c)

Calculate the expected frequencies:

$\hat{\mu}_{11} = n \cdot \hat{\pi}_{11} = n \cdot \hat{\theta}^2$

$\hat{\mu}_{12} = n \cdot \hat{\pi}_{12} = n \cdot \hat{\theta}(1-\hat{\theta})$

$\hat{\mu}_{21} = n \cdot \hat{\pi}_{21} = n \cdot \hat{\theta}(1-\hat{\theta})$

$\hat{\mu}_{22} = n \cdot \hat{\pi}_{22} = n \cdot (1-\hat{\theta})^2$

Using the expected frequencies obtain Pearson's $X^2 = \sum \frac{(n_{ij}-\hat{\mu}_{ij})^2}{\hat{\mu}_{ij}}$ and compare to a χ^2 distribution with $df = 2$ ($df = \dim(H_0 \cup H_1) - \dim(H_0) = 3 - 1 = 2$). Note that when testing independence for a 2×2 table $df = 1$.

Problem #3.31(c)

H_0	X^2	df	**p-value**	**Conclusion**
indep.	0.27274	1	0.6015	Do not reject H_0, there is evidence that the first and second shot are independent
iid	0.27274	2	0.8725	Do not reject H_0, there is evidence that the first and second shot are iid

It is plausible that the free throws are independent and identically distributed.