$$
\begin{aligned}
&\text{Exact} \\
p_E(\theta \mid Y) &\propto \text{Beta}(x+a,\ n-x+b) \\
\mu_E &= \frac{x+a}{a+b+n}
\end{aligned}
$$

$$
\begin{aligned}
&\text{2nd Laplace Approx} \\
\mu_L &= \frac{\sigma^*}{\sigma^\dagger} \cdot \frac{\exp\{-nh^*(\theta^*)\}}{\exp\{-nh^\dagger(\theta^\dagger)\}}
\end{aligned}
$$

$$
\begin{aligned}
-nh^\dagger(\theta) &= \ell(\theta \mid Y) + \ln(p(\theta)) \\
&\qquad \ell(\theta \mid Y) = x\ln(\theta) + (n-x)\ln(\theta) \\
&\qquad \ln(p(\theta)) = (a-1)\ln(\theta) + (b-1)\ln(1-\theta) \\
&= \alpha_\dagger \ln(\theta) + \beta_\dagger \ln(1-\theta) \qquad \begin{aligned} \alpha_\dagger &= x+a-1 \\ \beta_\dagger &= n-x+b-1 \end{aligned}
\end{aligned}
$$

$$
\begin{aligned}
-nh^*(\theta) &= \ell(\theta \mid Y) + \ln(p(\theta)) + ln(g(\theta)) \\
&\qquad \begin{array}{rcl} \ell(\theta \mid Y) &=& x\ln(\theta) + (n-x)\ln(\theta) \\ \ln(p(\theta)) &=& (a-1)\ln(\theta) + (b-1)\ln(1-\theta) \\ ln(g(\theta)) &=& \ln(\theta) \end{array} \\
&= \alpha_* \ln(\theta) + \beta_* \ln(1-\theta) \qquad \begin{aligned} \alpha_* &= x+a \\ \beta_* &= n-x+b-1 \end{aligned}
\end{aligned}
$$

$$
\begin{aligned}
\theta^{(\cdot)} &= \frac{\alpha_{(\cdot)}}{\alpha_{(\cdot)} + \beta_{(\cdot)}} \\
&\qquad \arg\max_\theta(-nh^{(\cdot)}(\theta)) = \tfrac{\partial - nh^{(\cdot)}(\theta)}{\partial\theta} = \tfrac{\alpha_{(\cdot)}}{\theta} - \tfrac{\beta_{(\cdot)}}{1-\theta} \overset{\text{set}}{=} 0 \\
\sigma^{(\cdot)} &= \left[\frac{1}{n}\left(\frac{\alpha_{(\cdot)}}{\theta^{(\cdot)}} + \frac{\beta_{(\cdot)}}{(1-\theta^{(\cdot)})^2}\right)\right]^{-1/2} \\
&\qquad \left[\tfrac{\partial^2 h^{(\cdot)}(\theta)}{\partial\theta^2}\Big|_{\theta^{(\cdot)}}\right]^{-1/2}
\end{aligned}
$$