Melodic Sounds of Oddly Shaped Drums

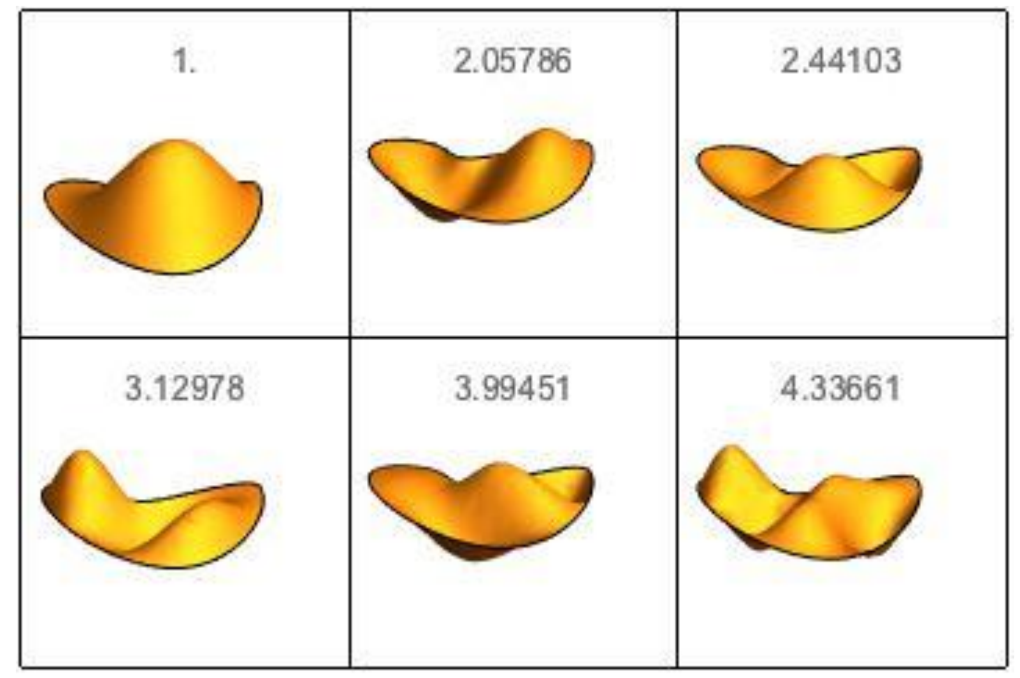
Alex Campbell: Century College
Tom Eichlersmith, Mohamed Mohamed, Ashwin Sridhar: Hamline University

Introduction

Most instruments, when hit, will naturally vibrate at a series of distinct frequencies known as normal modes (Figure 1). The lowest normal mode is known as the fundamental frequency while the higher frequencies are called overtones. Overtones that are integer multiples of the fundamental frequency are known as harmonic overtones. Some instruments, such as string and wind instruments produce such overtones, resulting in consonance. However, other instruments such as drums do not produce frequencies that are multiples of the fundamental frequency; therefore, the sound a drum makes will often be similar to noise to the human ear.

Our main objective was to find a surface to act as the drumhead such that when hit, will produce harmonic overtones. We approached this in both a computational and experimental method, the former using Mathematica to explore the mathematics behind different test shapes and the latter physically implementing these shapes into actual drums using clamps.

Fig 1: Images of Eigenshapes for a non-circular border



Computational Methods

- Represented different borders with their 2D parametrization
- Able to calculate normal modes and overtones easily and quickly using built in commands in Mathematica (e.g Figure 1)
- Required an objective rating system that would decide whether a set of frequencies would sound good or bad (without having to listen to every one)
- Decided upon using a neural network for this rating system over other more mathematical systems because it allowed for the most flexibility
  - Obtained a training set by rating randomly generated frequencies manually and adding sets of harmonic frequencies rated perfectly.

Complications

Self-Cancellation

- Some normal modes are so highly symmetric that they effectively cancel themselves out when attempting to create a sound wave in air (Figure 2)

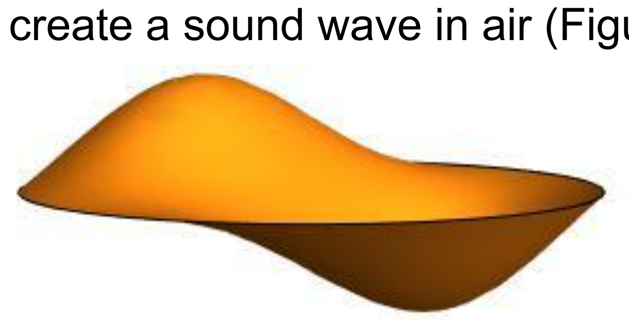


Fig 2: 3D Plot of 2nd Normal Mode for a circular border. Symmetric enough to produce self-cancellation.

Requirements for Optimization

- We are forced to represent each border by a set of points that can be freely changed by the optimization software.

Varying Intensities

- Calculating the intensity of each of the resonant frequencies sometimes reveals significant changes depending on strike location (for example Figure 3)

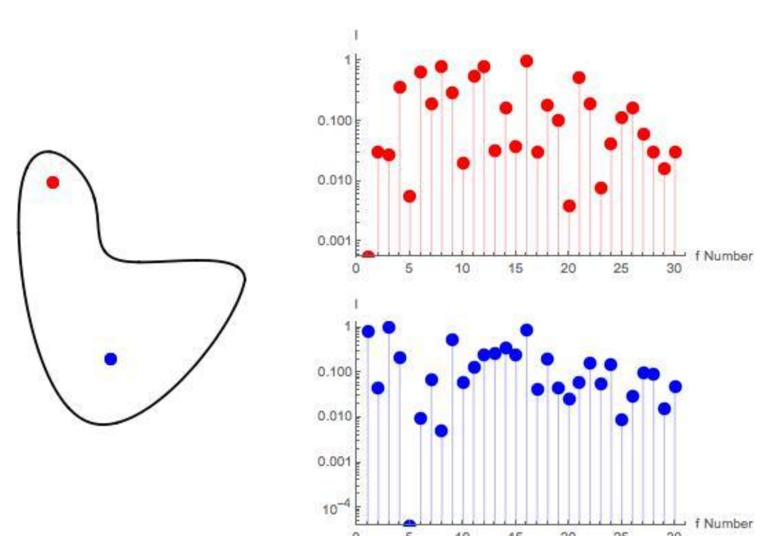


Fig 3: Depiction of different strike locations for a non-circular border. The graphs plot the relative intensity versus the frequency number (1st frequency is the fundamental).

Final Optimization

As a proof of concept we used our quickest and simplest methods:

- Did not take varying intensities or self-cancellation into account
- Simply connected control points with straight lines
- Rated sets of frequencies using a neural network

This led to a shape (Figure 4) rated a 3.2187 out of five that is an improvement on the traditional circle (rated 2.7208).

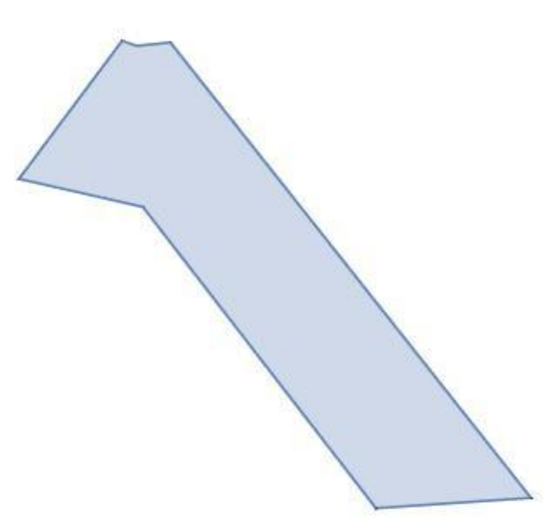


Fig 4: Optimized polygon with 7 control points.

Physical Implementation

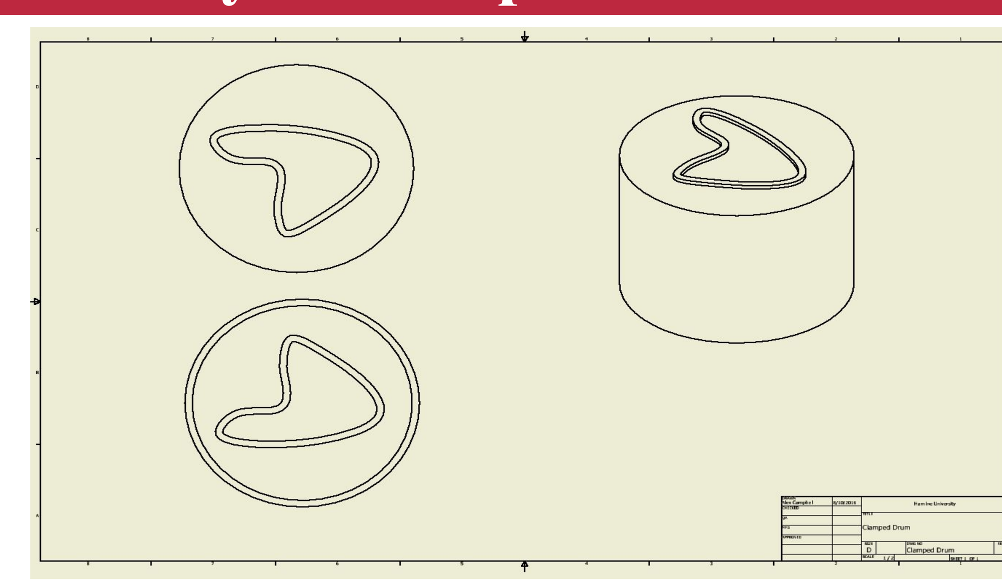


Fig. 8 (Above) Simplified drawing of the drum with the unclamped borders

Fig. 9 (Right) Picture of the drum with unclamped border

- Creating an oddly shaped drum from scratch has issues with maintaining constant tensions throughout the drumhead
- We quickly made a clamp system to hold 3D printed borders in and outside the drum in order to simulate a new drum head
  - Similar to frets on a guitar string
- Clamp system needed improvement as overtones weren't accurate
- New system had more points of pressure and more control over the location of these points
- Overtone ratios were better but not nearly as accurate as desired
  - Assumed to be due to vibrations continuing past the clamp.
  - Dampening helped but didn't solve the issue
- Created a fully 3D-printed drum to remove unwanted vibrations
  - We analyzed drum frequency due to a strike not due to resonant frequency. This creates a variable of the tension
  - A printed drum has no tension and cannot be used for our comparisons

Sound Analysis

- Calculated Eigenvalues for a circle, overlap with Periodograms from lab
- Scaled Eigenvalues to overlap with Periodogram peaks, majority overlap=success
- One of the earliest challenges we had was to figure out the constant that would be in front of our leplacian. The a value is our way of reading the tension of our system.
- Overlay predicted frequencies for unclamped and clamped drum head to their Periodograms
- Only the mathematical results of the unclamped drum lined up with the Periodogram
- This indicates that while our clamping system is working to a degree but not to the point where it can keep out noise and other sounds from bleeding into the node that the clamping system creates

Methods

- Record a sound file, Export it, Import it and get information from it using the "Data" command
- We then find the peaks of each thump and the round those peak values
- Take a mean of the peaks in each thump
- Separate the different thumps from each other
- Use the Periodogram command to do the FFT on our sound data
- Extract Periodogram data into a text file

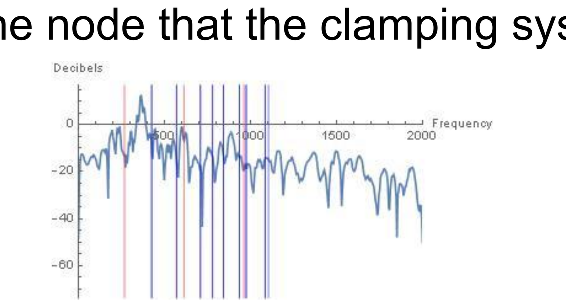


Fig. 5 Predicted frequencies overlaid onto a Periodogram of the clamped drum head.

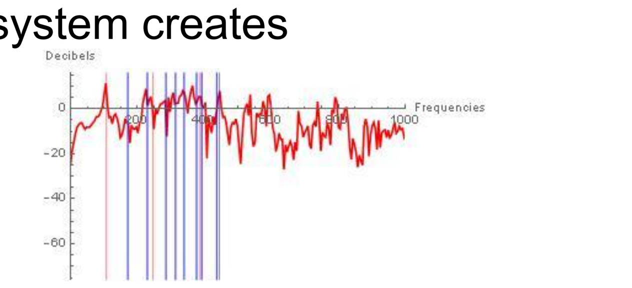
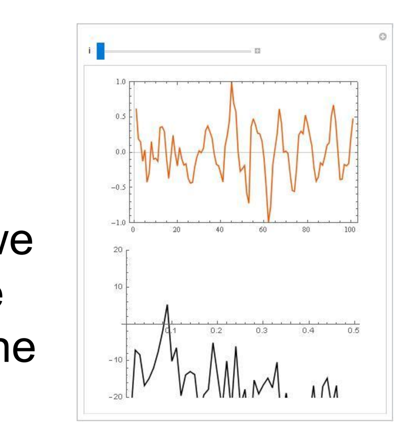


Fig. 6 Predicted frequencies overlaid onto a Periodogram of the unclamped drum head.

Windowing allows us to change the parts of the sound file we view so that we can select the section with the least noise.



Fig,7 The window command in Mathematica. The top graph is the sound file while the bottom one is the Periodogram.

Final Results

- We had our neural network generate an optimized shape for us with a given value of 3.2187 (on a scale of 1-5 where 1 is pure noise and 5 is pure tone)
- Once we had clamped the shape and ran it through our sound analysis code we were able to get data of all the peaks above -20 dBs.
- We used the first five frequencies above -20 dBs for the recording of thumps on the top, middle, and bottom of the shape.
- The results we got were 3.31598 for bottom and 2.91853 for middle and top.

Future Directions

- Investigate the role of internal forces versus the tension force
- Expand training set of neural network
- Include more aspects of a realistic drum in the model (for example: the frame and the air surrounding it)
- Develop a more reliable testing method

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