Causal evaluation and stochastic revealed preferences with unobserved randomness: a structural simulation approach

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Abstract

Conventional approaches to causal inference analysis, such as the Difference-in-Differences method (DiD), are susceptible to suffer from bias due to unobserved measurement error in the data, thus biasing the inference of the effect from a given shock or intervention over a variable of interest. In this paper, consumption data is simulated to follow a situation in which a subset of a population is randomly assigned an exogenous shock to their demand for a given good in their consumption bundle at some moment of time, while suffering from measurement error. The methodology applied, recovered from Aguiar and Kashaev (2020) provides a framework for testing the causal effect of any such situation in the households' bundle of consumption, leading, given the structural nature of this methodology, to testing the change in the structure of the households' preferences and identifying an Average Treatment Effect on the Treated (ATT) set. In contrast to the bias attained by a conventional DiD reduced-form estimation, the proposed structural estimation of this effect accounts for existing unobserved measurement error in the data.

Keywords: Microeconomics, Revealed Preferences, Structural Estimation, Causal Inference, Set-Identification, Difference-in-Differences, Econometric Methodology.

1 Introduction

What is the causal effect of an exogenous shock hitting a population's demand for a given good on their consumption bundle composition? While some works in the development economics literature have tackled the effect of such kind of situations, existent approaches have been mainly based on exploding reduced-form specifications based on conventional causal inference methods (Angelucci & De Giorgi, 2009; Cunha, De Giorgi, & Jayachandran, 2019; O. Attanasio et al., 2010; Oosterbeek, Ponce, & Schady, 2008).

One standard approach consists in calculating the change in the demand share of a given good in the consumption bundle of a population with respect to full bundle expenditure, via a reduced form specification. These conventional approaches to causal testing in consumption data are vulnerable to suffering from unobserved measurement error, thus biasing the estimated effect and potentially distorting the inferred magnitude of any such effect. This paper presents a framework for estimating causal Average Treatment effects on the Treated (ATT) on a population for the specific case of the change in the consumption bundle of a population facing exogenous shocks. This work introduces the structural revealed preference framework, in the fashion of Aguiar and Kashaev (2020), to a parametric Cobb-Douglas utility function. Importantly, this approach, dislike a reduced form estimations allows to account for unobserved randomness in the data (e.g., measurement error) while making results interpretable under the basis of a change in the structure of preferences of a population.

A data generating process (DGP) is simulated to portray a population in the following situation: Decision-Makers (DM) maximize their individual utility subject to a budget constraint at each period of time for T periods. At a given moment, a positive random exogenous shock hits the demand of a random subset of this population. This effect on the observed consumption shares for each good of the DM's bundle can be causally estimated by a reduced-form Difference-in-Differences (DiD) approach. However, the DGP is induced to contain unobserved randomness, thus portraying a phenomenon observed in real data (Bound, Brown, & Mathiowetz, 2001). This issue is solved by structurally estimating this causal effect via a system of moment conditions on the structural model that accounts for unobserved randomness in the data. In this case, this takes the form of measurement error, for which a single, centering assumption is considered. It is later shown that, while the reduced-form DiD estimation is biased, the structural model allows for the inference of an identified set that asymptotically matches the effect yielded by the reduced-form ATT in the "unobserved" real data without measurement error. In other words, by properly modeling the centered measurement error's moment condition in our model, we can infer an asymptotically unbiased causal estimator despite data suffering from measurement error.

This strategy explodes recent methodological developments in the Revealed Preferences (RP) literature. It aims, not only to causally test the change of the observed consumption bundle of a population in the face of an exogenous shock but to statistically test the preference structure evolution of the treatment group over multiple time periods and for a given menu of consumption goods. Our approach is parametric and provides a structural formulation to estimate the interest parameter, using the moment conditions approach and the robustness of the method provided by Aguiar and Kashaev (2020). This testing scheme is made operational via an algorithm designed to deal with multivariate conditional distributions over latent vectors. The latter is implemented by the Entropic Latent Variable Integration via Simulation (ELVIS) method developed by Schennach (2014) over the space of moment conditions.

This paper's structure is the following. Section 2 presents the model environment. Section 3 presents the causal inference procedure provided by a conventional Difference-in-Differences method and discusses the introduction of measurement error in the data. Section 4 builds on the elements presented in Section 2 to provide an alternative causal estimation that accounts for measurement error using a structural framework. Section 5 provides simulation results. Finally, section 6 discusses the final remarks.

1.1 Relation to the literature

By combining causal intervention evaluation and stochastic revealed preferences in the prescience of measurement error this study contributes to the economics literature by providing a method to disentangle causal effects that will otherwise be biased. With regards to causal intervention evaluation, this paper relates to the evaluation of either policy interventions or exogenous shocks on the consumption bundle composition of households in a given population. Some contributions in this direction include Angelucci and De Giorgi (2009), Cunha et al. (2019), O. Attanasio et al. (2010), and Oosterbeek et al. (2008) who apply mostly reduced-form causal inference techniques.

The second area for which this paper is relevant is the set of potential applications provided by stochastic revealed preference (RP) analysis. Recent theoretical developments have embarked into disentangling forms of measuring the fitness of a given set of individual consumption decisions into the traditional Utility Maximization Theory (UMT) and its deviations. These approaches have used structural, non-parametric estimation methods to test rationalizability in the data. Consistency with the classical Exponential Discounting (ED) model for intertemporal consumption has been tested as well.

Intending to test rationalizability non-parametrically in an RP context, Kitamura and Stoye (2018) used a testing method built on a model based on a linear program over 'budget patches'. Others, such as Aguiar and Kashaev (2020) applied such testing in the context of Afriat inequalities, and Demuynck and Potoms (2022) applied a column-generation approach and tools from convex analysis. Kashaev, Gauthier, and Aguiar (2023) provide a further innovation in which a flexible, stochastic, and dynamic approach to RP can handle serial correlation and cross-section heterogeneity in preferences.

While most of these approaches have been non-parametric, we innovate in providing a practical application of such framework in a parametric environment that builds upon the empirical consumption bundle composition of households. This applied approach sums to relevant contributions due to O. P. Attanasio, Meghir, and Santiago (2012); Todd and Wolpin (2006). Other works in the literature, such as Deb, Kitamura, Quah, and Stoye (2023), analyze welfare consequences via preferences for prices. Adams (2020) alike develop a method to rationalize on consumption predictions, pushing the revealed preference literature to address the prediction of rational demands over a set of new budgets.

Our work builds upon the work of Aguiar and Kashaev (2020), particularly by formulating the testing procedure that takes moment conditions relying on a model-driven "first order conditions approach". Despite the specific form of the moment conditions, our method outperforms the Difference-in-Differences reduced form specification when attempting to find the causal effect of a policy program in the presence of measurement error because we are able to provide a narrower set that asymptotically approximates the true effect. As long as the model can be expressed in the form of moment conditions[1], this study provides a procedure to correctly evaluate policy in settings where the data suffers from measurement error and other forms of unobserved randomness. Further work will most likely apply this method to Conditional Cash Transfer data in Mexico and Ecuador, exploiting the work of O. Attanasio and Pastorino (2020); Schady, Araujo, Peña, and López-Calva (2008); Oosterbeek et al. (2008).

[1] With a centering assumption

2 Model environment

Consider a given number of households determined by $\mathcal{J} = \{1, \ldots, J\}$. For a finite set of goods $\mathcal{L} = \{1, \ldots, L\}$, each household $j \in \mathcal{J}$ consumes over a space $X \subseteq \mathbb{R}^L_{++}$ for a number of periods given by $\mathcal{T} = \{1, \ldots, T\}$. Let the household's preferences be defined by a utility function $u_{j,t}(c_{j,t})$ where, for each $j \in \mathcal{J}$, $u_t(\cdot)$ is continuous, concave, and locally non-satiated for all $t \in \mathcal{T}$, and $c_t \in X$ is the vector of consumption for each household j across all periods $t \in \mathcal{T}$. In fact, consider the case where the preferences of each household can be represented by Cobb-Douglas (CD) preferences:

$$u_t(c_t) = \prod_{l=1}^{L} c_{l,t}^{\alpha_{l,t}} \tag{1}$$

with the property of $\sum_l \alpha_{l,t} = 1$ at each $t \in \mathcal{T}$, such that the elements of $c_t \in X$ are multiplicatively considered within $u_t(\cdot)$ with each one contributing a share $\alpha_{l,t}$.

Since we are modeling this as a "repeated cross-section", DM's hold no savings and do not discount future income flows. DM's are assumed to have perfect foresight over their exogenous income. By writing the DM's utility maximization problem, we define our first relevant testable condition of interest at the individual level:

Definition 1 (CD-rationalizability). *A deterministic array $(p_t, c_t)_{t\in\mathcal{T}}$ is CD-rationalizable if, for Cobb-Douglas preferences as in equation 1, $(c_t)_{t\in\mathcal{T}}$ solves:*

$$\max_{c_t \in X} \prod_{l=1}^{L} c_{l,t}^{\alpha_{l,t}} \tag{2}$$

s.t.

$$p_t' c_t = y_t \tag{3}$$

As implied by the parametric assumption on preferences, $u(\cdot)$ is in fact concave, and since equation (3) is convex, there must exist a unique solution to the utility maximization problem in definition 1. With that in mind, the following serves as a preamble to showing the testing conditions of our model.

Theorem 1. *For a Cobb-Douglas model of preferences, the following are equivalent:*

1. *The deterministic array $(p_t, c_t)_{t\in\mathcal{T}}$ is CD-rationalizable.*
2. *There exists a matrix $(\alpha_{l,t})_{l\in\mathcal{L},t\in\mathcal{T}}$ such that $\sum_l \alpha_{l,t} = 1$, and*

$$c_{l,t} = \frac{\alpha_{l,t} y_t}{p_{l,t}}$$

for all $t \in \mathcal{T}$.

For now the model setup does not consider any perturbation in the data, the next subsection introduces measurement error to the model.

2.1 Introducing Measurement Error.

Assume income y_t is normalized to one for each DM at each period and denote a random variable as $\mathbf{x}$. Additionally, denote $\mathbf{x}^*$ as a random variable, data set or array that does not suffer from any perturbation i.e., measurement error.

As argued by Aguiar and Kashaev (2020), conventional RP testing methodologies, even in fully non-parametric environments, tend to overreject rationalizability when unobserved randomness (e.g., measurement error in the data) is not accounted for. A similar argument could be made for the case of conventional, reduced-form causal estimations. Before defining measurement error, CD-rationalizability must be defined in a fully stochastic context. For that matter, the following definition is neccesary:

Definition 2 (**s/CD-rationalizability**). *A random array $(\mathbf{p}^*_t, \mathbf{c}^*_t)_{t\in\mathcal{T}}$ is said to be statistically CD-rationalizable for a given structure of Cobb-Douglas preferences, if there exists a random matrix $(\mathbf{a}_{l,t})_{l\in\mathcal{L},t\in\mathcal{T}}$*[2] *such that:*

1. *Each entry of $(\mathbf{a}_{l,t})_{l\in\mathcal{L},t\in\mathcal{T}}$ is a random variable supported on or inside a known set $\mathbf{A}\subseteq[0,1]^{L\times T}$ and $\sum_l^L \mathbf{a}_{l,t}=1$ for every $l\in\mathcal{L}$ and every $t\in\mathcal{T}$;*
2. *$\mathbf{c}^*_{l,t}=\frac{\mathbf{a}_{l,t}}{\mathbf{p}^*_{l,t}}$ a.s. for all $l\in\mathcal{L}$ and for all $t\in\mathcal{T}$;*
3. *For every $l\in\mathcal{L}$, and $t\in\mathcal{T}$, it must be the case that*

$$\mathbb{P}\left(\mathbf{c}^*_{l,t}>0,\ \mathbf{c}^*_{l,t}=\frac{\mathbf{a}_{l,t}}{\mathbf{p}^*_{l,t}}\right)=1$$

In fact, for a given collection of random sequences $(\mathbf{p}^*_t, \mathbf{c}^*_t)_{t\in\mathcal{T}}$ to be s/CD-rationalizable is equivalent, given some random positive matrix $(\mathbf{a}_{l,t})_{l\in\mathcal{L},t\in\mathcal{T}}$ supported on or inside some $\mathbf{A}\subseteq[0,1]^{L\times T}$, to be such that $\mathbf{c}^*_{l,t}=\frac{\mathbf{a}_{l,t}}{\mathbf{p}^*_{l,t}}$, a.s. for all $t\in\mathcal{T}$. This is proven in a general, non-parametric manner, by Aguiar and Kashaev (2020) who also show that this condition does not guarantee, for any implementable test, that there would be a conclusive telling, for a data set being either "almost" s/ED-Rationalizable or "exact" s/ED-Rationalizable. This makes the following necessary:

Definition 3 (**Approximate s/CD-rationalizability**). *We say that $(\mathbf{p}^*_t, \mathbf{c}^*_t)_{t\in\mathcal{T}}$ is approximately consistent with s/CD-rationalizability if there exists a positive matrix $(\mathbf{a}_{t,l,r})_{t\in\mathcal{T},l\in\mathcal{L}}$ supported on or inside $\mathbf{A}\subseteq[0.1]^{L\times T}$, such that*

$$\mathbb{P}\left(\mathbb{I}\Big(\boldsymbol{c}^*_{l,t}=\frac{\mathbf{a}_{t,l,r}}{\mathbf{p}^*_{l,t}}\Big)=1\right)\longrightarrow_{r\to+\infty}1 \tag{4}$$

for all $t\in\mathcal{T}$

Consider any non-systematic measurement error on observed data that arises from misrecording issues, e.g., trembling hand errors. As in the case of Aguiar and Kashaev (2020), we *define* this error, for which the

[2] In definition 2 $(\mathbf{a}_{l,t})_{l\in\mathcal{L},t\in\mathcal{T}}$ stands for the stochastic counterpart of the deterministic matrix $(\alpha_{l,t})_{l\in\mathcal{L},t\in\mathcal{T}}$ stated in theorem 1

framework used is consistent with any measurement error type with these characteristics. As in the fashion of Demuynck and Potoms (2022), we may as well conceive this as "unobserved randomness".

Let the *measurement error* be an array $\mathbf{w} = (\mathbf{w}_t)_{t\in\mathcal{T}} \in W$ given as the difference between recorded and true data for consumption and prices:

$$\mathbf{w}_t = \begin{bmatrix} \mathbf{w}_t^c \\ \mathbf{w}_t^p \end{bmatrix} \tag{5}$$

with $\mathbf{w}_t^c = \mathbf{c}_t - \mathbf{c}_t^*$ and $\mathbf{w}_t^c = \mathbf{p}_t - \mathbf{p}_t^*$.

A random array $(\mathbf{p}_t, \mathbf{c}_t)_{t\in\mathcal{T}}$ is s/CD-rationalizable if and only if there exist and array $(\mathbf{a}_{l,t}, \mathbf{w}_t)_{t\in\mathcal{T}, l\in\mathcal{L}}$ such that

$$\mathbf{c}_{l,t} - \mathbf{w}_{l,t}^c = \frac{\mathbf{a}_{l,t}}{(\mathbf{p}_{l,t} - \mathbf{w}_{l,t}^p)}; \text{a.s.}, \forall t \in \mathcal{T} \tag{6}$$

In what follows, we assume measurement error hits consumption only, and not prices[3]. Let $\mathbf{e} = (\mathbf{a}', \mathbf{w}') \in E|X$ be the vector of unobserved, random latent variables of the model, supported on or inside the conditional support $E|X$, where $\mathbf{x} = (\mathbf{p}_t, \mathbf{c}_t) \in X$ is the observed data. Define $g_M : X \times E|X \longrightarrow \mathbb{R}^{d_M}$ as a *measurement error moment.* The only imposed assumption in our model is the following:

Assumption 1 (Centered measurement error). *(i) The random vector of latent variables* $\mathbf{e}$ *is supported on or inside* $E|X$*, and (ii) There exists a known mapping* $g_M : X \times E|X \longrightarrow \mathbb{R}^{d_M}$ *such that*

$$\mathbb{E}[g_M(\mathbf{x}, \mathbf{e})] = 0 \tag{7}$$

3 The Difference-in-Differences approach

Difference-in-Differences (DiD) is a statistical method to identify causal effects in quasi-experimental frameworks. In its simplest form, the method exploits the random assignment of individuals into treated and non-treated groups to compare mean outcomes among groups. The simple DiD method needs two assumptions to identify causal effects, the parallel trend and the no-anticipation assumptions. The former implies that in the absence of the treatment, the average outcomes of the treated and control groups would follow a similar trend over time. The second assumption states that the treatment has no causal effect prior to its implementation, i.e., no previous behavior by individuals (Roth, Sant'Anna, Bilinski, & Poe, 2023).

In the context of an intervention, the econometrician observes an outcome variable Y_{jt} for each and the group assignation $D_j \in (0,1)$ for each household in a data panel structure. Note that contrary to c_t representation in equation (1) Y_{jt} refers to the household j. Assuming that both parallel trends and no-anticipation assumption hold let the Average Treatment effect on the Treated (ATT) be:

$$ATT = \mathbb{E}[Y_{j,t^*} - Y_{j,t^*-1} | D_j = 1] - \mathbb{E}[Y_{j,t^*} - Y_{j,t^*-1} | D_j = 0] \tag{8}$$

[3] This is consistent with Appendix B

where $D_j = 1$ denotes that household j belongs to the treatment group and $D_j = 0$ to the control group. Additionally, t^* represents a post-treatment period, and $t^* - 1$ a pre-treatment period.

Given the Cobb Douglas structure described in section 2, the econometrician is able to ask whether an intervention changes the share of consumption for a given good l in period t for the group of intervened households. First, the econometrician transforms the consumption data into shares (see equation (9)). The effect of interest is described by equation (10) where $j \in \tau \equiv D_j = 1$ and $j \in \mathcal{C} \equiv D_j = 0$ describe treatment and control assignation[4].

$$\mathbf{a}_{j,l,t} = \frac{\mathbf{c}^*_{j,l,t}}{\sum_{k=1}^{L} \mathbf{c}^*_{j,k,t}\mathbf{p}_{k,t}} \tag{9}$$

$$ATT = \beta = \left[\bar{\mathbf{a}}_{j\in\tau,\, t^*} - \bar{\mathbf{a}}_{j\in\tau,\, t^*-1}\right] - \left[\bar{\mathbf{a}}_{j\in\mathcal{C},\, t^*} - \bar{\mathbf{a}}_{j\in\mathcal{C},\, t^*-1}\right] \tag{10}$$

The classic method to estimate ATT in econometrics uses a two-way fixed effect reduced form model in which the parameter of interest captures the time and group interaction. The following definition introduces the method tailored to our model of consumption data:

Definition 4 (**Difference-in-Differences estimation**). *Assume that a subset of the population $\tau \subset \mathcal{J}$ receives an exogenous shock effective on their demand for the good $l \in \mathcal{L}$. The Difference-in-Differences (DiD) effect is given by estimating parameter β in the following regression equation:*

$$\mathbf{a}_{j,l,t} = \phi_0 + \phi_1 Time_{j,t} + \phi_2 Group_{j,t} + \beta(Time_{j,t} \times Group_{j,t}) + \epsilon_{j,t} \tag{11}$$

where $Time_{j,t}$ is a dummy variable indicating the post-treatment period t^*, and $Group_{j,t}$ is a dummy indicating that j belongs to the treatment group, and $\epsilon_{t,l,l}$ the residual term.

In presence of measurement error in $\mathbf{a}_{j,l,t}$[5] given by $\tilde{\mathbf{a}}_{j,l,t} = \frac{\mathbf{c}^*_{j,l,t} + \mathbf{w^c_{j,l,t}}}{\sum_{k=1}^{L}(\mathbf{c}^*_{j,k,t} + \mathbf{w}^c_{j,k,t})\mathbf{p}_{k,t}}$ with observed consumption $\mathbf{c}_{j,l,t} = \mathbf{c}^*_{j,l,t} + \mathbf{w}^c_{j,l,t}$ the econometrician will measure a different specification

$$\tilde{\mathbf{a}}_{j,l,t} = \frac{\mathbf{c}_{j,l,t}}{\sum_{k=1}^{L} \mathbf{c}_{j,k,t}\mathbf{p}_{k,t}} = \tilde{\phi}_0 + \tilde{\phi_1} Time_{j,t} + \tilde{\phi}_2 Group_{j,t} + \tilde{\beta}(Time_{j,t} \times Group_{j,t}) + \epsilon_{j,t} \tag{12}$$

Although the $\mathbb{E}[w^c_{j,l,t}] = 0$, if $\sigma_{w^c} > 0$ is high enough, estimates retrieved by standard DiD methodology could appear to be non significant. To the best of our knowledge, there are no statistical reasons to assume that classical measurement error in elicited consumption data is delimited by some arbitrary bounds. In empirical applications, such as surveys, this can cause the inability to reject the null hypothesis, therefore erroneously inferring that a an exogenous controlled intervention or a shock did not affect an outcome of interest when it actually did.

Table 1 in section 5 reports the estimated coefficients for equations (11) and (12) , as well as their corresponding confidence intervals. These results are reported as the mean value on a 1000 repetitions simulation. A significance non-rejection rate is also presented in Table 1, indicating the share of repetitions

[4]In this notation, and $\bar{x}$ stands for the mean of a variable x_i

[5]Please recall that in this case l remains fixed

for which the statistical significance at the 95% level of these estimators is non-rejected. Similarly, a "mis-inference rate" is reported, indicating the share of repetitions for which $\hat{\hat{\beta}}$ is not included in the 95% confidence interval of $\hat{\beta}$, thus providing an argument for preferring the structural estimation presented in the next section.

4 A framework for structural causal inference via the methodology of Aguiar and Kashaev (2020)

Given the model and the first order conditions stated previously, we apply the testing frameworks provided by Aguiar and Kashaev (2020). For the established structure of preferences, the corresponding moment conditions, for each $l \in \mathcal{L}$ and all $t \in \mathcal{T}$, are given by:

$$g_{A,j,l,t}(\mathbf{x},\mathbf{e}) = \mathbb{I}\left[\mathbf{c}_{j,l,t} - \mathbf{w}^{c}_{j,l,t} = \frac{\mathbf{a}_{j,l,t}}{\mathbf{p}_{j,l,t}}\right] - 1 \tag{13}$$

$$g_{D,j,l=1}(\mathbf{e};\theta) = \left[\left[\mathbf{a}_{j\in\mathcal{T},\, t^*} - \mathbf{a}_{j\in\mathcal{T},\, t^*-1}\right] - \left[\mathbf{a}_{j\in\mathcal{C},\, t^*} - \mathbf{a}_{j\in\mathcal{C},\, t^*-1}\right]\right] - \theta \tag{14}$$

$$g_{M,j,l,t}(\mathbf{x},\mathbf{e}) = \mathbf{w}_{j,l,t} \tag{15}$$

Equations (13)-(15) constitute the moment conditions that are to be tested. In particular, equation (13) corresponds to the CD-rationalizability First Order Conditions and equation (15) is to be used as part of the centered measurement error assumption. On the other hand, equation (14) is part of a user-defined moment allowing for recovering effects from the model of interest, as shown by Aguiar and Kashaev (2020). In this case, the model corresponds to the conditions set by the DiD framework over the structural parameters $\mathbf{a}_{j,l,t}$ as set in previous section.

Note that since we have not simulated the DGP with measurement error in prices, but only in consumption, observed prices are actual prices and the measurement error vector. contains exclusively consumption errors. There are some important clarifications to make. In equation (14), since $\mathbf{a}_{l,t} \in [0,1]$, then, $\theta \in [-2,2]$. For that matter, the baseline slack variable's value for θ is taken to be the unbiased DiD estimator. For a sufficiently big testing interval, a set of values for θ can be found such that the model is not rejected. The moment conditions corresponding to equations 13-15 are the following:

$$\mathbb{E}[g_{A,j,l,t}(\mathbf{x},\mathbf{e})] = 0 \tag{16}$$

$$\mathbb{E}[g_{D,j,l=1}(\mathbf{e};\theta)] = 0 \tag{17}$$

$$\mathbb{E}[g_{M,j,l,t}(\mathbf{x},\mathbf{e})] = 0 \tag{18}$$

Before introducing the econometric framework for this structural estimation, consider the following synthesis of equations (16) and (17):

$$g_{I,j,l,t}(\mathbf{x},\mathbf{e};\theta) = (g_{A,j,l,t}(\mathbf{x},\mathbf{e})', g_{D,j,l=1}(\mathbf{e},\theta)') \tag{19}$$

Equation (20) synthesize the (13)-(15) system.

$$g_{j,l,t}(\mathbf{x},\mathbf{e};\theta) = (g_{A,j,l,t}(\mathbf{x},\mathbf{e})', g_{D,j,l=1}(\mathbf{e};\theta)',, g_{M,j,l,t}(\mathbf{x},\mathbf{e})') \tag{20}$$

Therefore, the conditions required, associated with moments equations (16)-(18) are the following:

$$\mathbb{E}[g_{j,l,t}(\mathbf{x},\mathbf{e};\theta)] = 0 \tag{21}$$

Where $\mathbf{x} = (\mathbf{c}_{j,l,t}, \mathbf{p}_{j,l,t})$ and $\mathbf{e} = (\mathbf{w}_{j,l,t},, \mathbf{a}_{j,l,t})$. For each household, there are k moment functions corresponding to equation (16) and an additional unique function taken from equation (17). Additionally, there are q moment functions corresponding to the centered measurement error condition in (18). Testing (21) will in fact, allow for consistent inference of the estimated parameter $\hat{\beta}$ while accounting for unobserved measurement error in the data.

Note that the left-hand side of equation (21) can be written in the following form:

$$\mathbb{E}_{\mu\times\pi}[g_{j,l,t}(\mathbf{x},\mathbf{e};\theta)] = \int_X \int_{E|X} g(x,e;\theta) d\mu d\pi \tag{22}$$

where $\mu \in \mathcal{P}_{E|X}$ and $\pi \in \mathcal{P}_X$. With this in mind, we can formulate the condition required by the model's moment. Aguiar and Kashaev (2020) prove that for a random array $(\mathbf{p}_t, \mathbf{c}_t)_{t\in\mathcal{T}}$ to be approximately s/CD-rationalizable such that Assumption 1 holds is equivalent to

$$\inf_{\mu\in\mathcal{P}_{E|X}} \|\mathbb{E}_{\mu\times\pi}[g(\mathbf{x},\mathbf{e};\theta)]\| = 0 \tag{23}$$

where $\pi_0 \in \mathcal{P}_X$ is the observed distribution of $\mathbf{x}$ and the subscripts of $g_{j,l,t}(\cdot)$ have been omitted to save on notation. This is a preliminary approach to our testing conditions, which sum up to testing, at a given significance level, if whether there exists an unknown distribution of latent variables conditional on observables such that the moment conditions are fulfilled.

In order to deal with a test of an unknown distribution of latent vectors, we apply the Entropic Latent Variable Integration via Simulation (ELVIS) approach by Schennach (2014) and used in Aguiar and Kashaev (2020). This approach will let us write our testing conditions in terms of the observables' distributions only.

Following the notation by Aguiar and Kashaev (2020), the maximum-entropy (ME) moment of $g(x,e;\theta)$ can be written as:

$$h(x;\gamma,\theta) = \int_{e\in E|X} g(x,e;\theta) d\eta^*(e|x;\gamma,\theta) \tag{24}$$

where

$$\left\{ d\eta^*(\cdot|x;\gamma,\theta) = \frac{\exp(\gamma' g(x,\cdot)) d\eta(\cdot|x)}{\int_{e\in E|X} \exp(\gamma' g(x,e;\theta)) d\eta(e|x)}, \gamma \in \mathbb{R}^{k+q} \right\} \tag{25}$$

is a family of exponential conditional probability measures. By this way, we express the ME moment of $g(x,e;\theta)$ as:

$$h(x;\gamma,\theta) = \frac{\int_{e\in E|X} g(x,e;\theta) \exp(\gamma' g(x,e;\theta)) d\eta(e|x)}{\int_{e\in E|X} \exp(\gamma' g(x,e;\theta)) d\eta(e|x)} \tag{26}$$

where, in this case, $\gamma \in \mathbb{R}^{k+q}$ is a nuissance parameter and $\eta \in \mathcal{P}_{E|X}$ is an arbitrary user-inputed distribution supported on $E|X$ such that

$$\mathbb{E}_{\pi_0}[\log \mathbb{E}_\eta[\exp(\gamma' g(\mathbf{x}, \mathbf{e}; \theta))|\mathbf{x}]] \tag{27}$$

exists and is twice continuously differentiable in γ.

This implies that h is the marginal moment of g where the latent vector has been "integrated out" using some probability measure (e.g., some distribution) in the family (25).

An important implication of this approach emanates from what was already proved by Aguiar and Kashaev (2020). For a random array $(\mathbf{p}_t, \mathbf{c}_t)_{t \in \mathcal{T}}$ to be approximately s/CD-rationalizable such that Assumption 1 holds is equivalent to

$$\inf_{\gamma \in \mathbb{R}^{k+q}} \|\mathbb{E}_{\pi_0}[h(\mathbf{x}; \gamma, \theta)]\| = 0 \tag{28}$$

where $\pi_0 \in \mathcal{P}_X$ is the observed distribution of $\mathbf{x}$. Importantly, the choice of η does not affect (27), only the nuissance parameter γ.

This is both a necessary and sufficient condition for arguing the following: the existence of any set of distributions part of the family of probability measures on (25) such that the property for the ME moment in (26) holds (for this we rely on distributions on the observed data only), implies the existence of some set of distributions, conditional on unobserved latent variables, such that the condition for the original moment as in (23) is fulfilled. More importantly, this in turn implies that the data set $\mathbf{x}$ while being rationalized by the stated structure of preferences, fulfills Assumption 1 on (15) and our model of interest, portrayed by equation (14), holds for some nonempty subset of parameters $\theta \in \Theta$. We can now test this without relying on identifying distributions conditional on unobserved latent variables $\mathbf{e}$.

4.1 Implementation

Emanating from (28), the following condition applies: for a random array $(\mathbf{p}_t, \mathbf{c}_t)_{t \in \mathcal{T}}$ to be approximately consistent with s/CD-rationalizability such that Assumption 1 holds, is equal to the following:

$$\min_{\gamma_M \in \mathbb{R}^q} \|\mathbb{E}_{\pi_0}[h_M(\tilde{\mathbf{x}}; \gamma, \theta)]\| = 0 \tag{29}$$

with:

$$\tilde{h}_M(x; \gamma, \theta) = \frac{\int_{e \in E|X} g_M(x, e) \exp(\gamma' g_M(x, e)) \mathbb{I}(g_I(x, e; \theta) = 0) d\eta(e|x)}{\int_{e \in E|X} \exp(\gamma' g(x, e; \theta)) \mathbb{I}(g_I(x, e; \theta) = 0) d\eta(e|x)} \tag{30}$$

This is proved by Aguiar and Kashaev (2020), who also show, by terms of any divergent sequence $\{\gamma_{I,l}\}_{l=1}^{+\infty}$, that for a random array $(\mathbf{p}_t, \mathbf{c}_t)_{t \in \mathcal{T}}$ to be approximately consistent with s/CD-rationalizability equals to the sequence $\{\gamma_{M,l}\}$ of minimizers of

$$\lim_{l \to +\infty} \min_{\gamma_M \in \mathbb{R}^q} \|\mathbb{E}_0[h(\mathbf{x}; (\gamma'_{I,l}, \gamma'_M), \theta)]\| = 0 \tag{31}$$

being convergent to some finite γ_0 not depending on $\{\gamma_{I,l}\}_{l=1}^{+\infty}$. In other words, our optimization procedure will no longer depend on $\gamma \in \mathbb{R}^{k+q}$ and only on $\gamma_M \in \mathbb{R}^q$. Given the calibration of our DGP, we have that

$q = (T \times K \times \tau c) + 1 = 13$ (where τc is the number of interest groups: treatment and control groups). In practice, to compute (30), Markov Chain Monte Carlo methods are used. More specifically, a Metropolis-Hastings algorithm sampling from η is implemented.

Now, the testing procedure is introduced. For a given observed data set $\{\mathbf{x}_i\}_{i=1}^n = \{(\mathbf{p}_{t,i}, \mathbf{c}_{i,t})_{t\in\mathcal{T}}\}_{i=1}^n$ with sample size $n \in \mathbb{N}$, the finite-sample analogue of the ME moment condition in (30), and its corresponding variance are given by:

$$\hat{\tilde{h}}_M(\gamma,\theta) = \frac{1}{n}\sum_{i=1}^{n} \tilde{h}_M(\mathbf{x}_i;\gamma,\theta); \tag{32}$$

$$\hat{\tilde{\Omega}}(\gamma,\theta) = \frac{1}{n}\sum_{i=1}^{n} \tilde{h}_M(\mathbf{x}_i,\gamma,\theta)\tilde{h}_M(\mathbf{x}_i,\gamma,\theta)' - \tilde{h}_M(\gamma,\theta)\tilde{h}_M(\gamma,\theta)' \tag{33}$$

Therefore, as derived from Schennach (2014), the test statistic proposed by Aguiar and Kashaev (2020) is

$$TS_n = n \inf_{\gamma\in\mathbb{R}^q} \tilde{h}_M(\gamma,\theta)'\hat{\tilde{\Omega}}(\gamma,\theta)^{-1}\tilde{h}_M(\gamma,\theta) \tag{34}$$

where $\hat{\tilde{\Omega}}(\gamma,\theta)^{-1}$ is the inverse of the matrix in (33). This implies, as proved by Aguiar and Kashaev (2020), that under the null hypothesis that the data is approximately consistent with s/CD-rationalizability, it occurs that:

$$\lim_{n\to\infty} \mathbb{P}\Big(TS_n > \chi^2_{q,1-\alpha}\Big) \leq \alpha \tag{35}$$

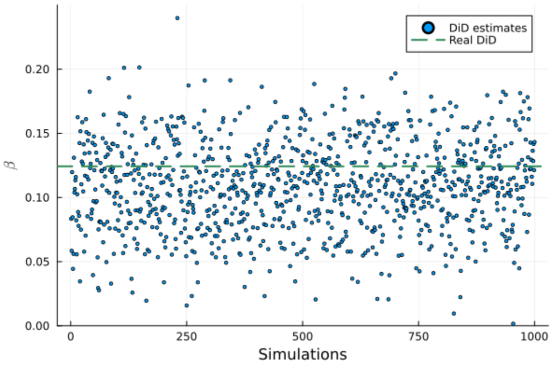
for any $\alpha \in (0,1)$. Moreover, the confidence set for θ_0 can be obtained by inverting $TS_n(\theta_0)$. Therefore, in our model's case, the confidence set for θ_0 is:

$$\Big\{\theta_0 \in \Theta : TS_n \leq \chi^2_{13,1-\alpha}\Big\} \tag{36}$$

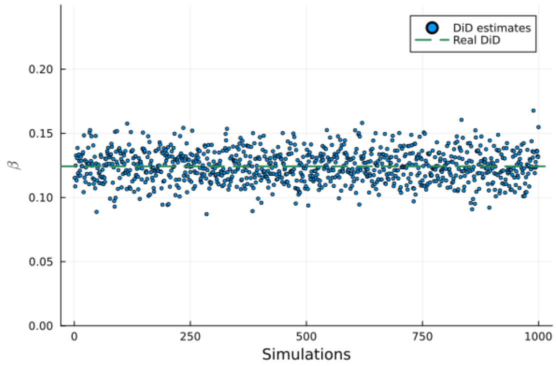
There is no need for rationalizability to be tested independently in (36) prior to our model's parameter's identified set. The rejection of rationalizability would imply an asymptotically empty $\theta_0 \in \Theta$. In our case, the confidence set of our structural DiD model in (36) is tested simultaneously to measurement error robustness (and Cobb-Douglas rationalizability, which is controlled by our DGP) .

5 Results

Table 1 summarizes the reduced-form DiD estimation model performance for two different settings in the simulated number of repetitions. In both cases the estimation with observed data yields a significance rejection rate above 11% as compared with a 0% when estimated with real data. Additionally, the misinference rate, the frequency of the estimated parameter falling outside the real data's estimated confidence interval is, in both cases, above 45%. Interestingly, in both cases, the estimated effect with real data yields a value below the estimated effect with real data. This difference is a bias due to measurement error and to the reduced-form DiD estimation not being able to account for it on observed data. Figures 1 and 2 illustrate some DiD estimator issues when exposed to measurement error, particularly regarding the direction and magnitude of the bias at the confidence interval level.

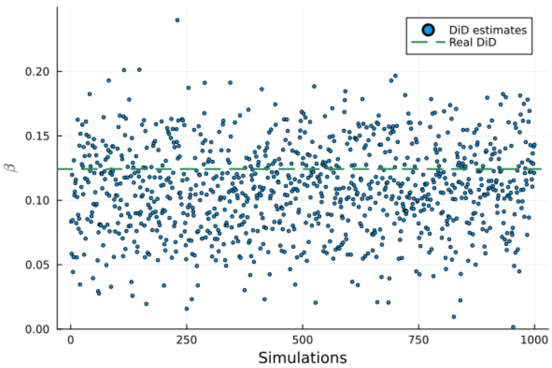


(a) DiD with measurement error

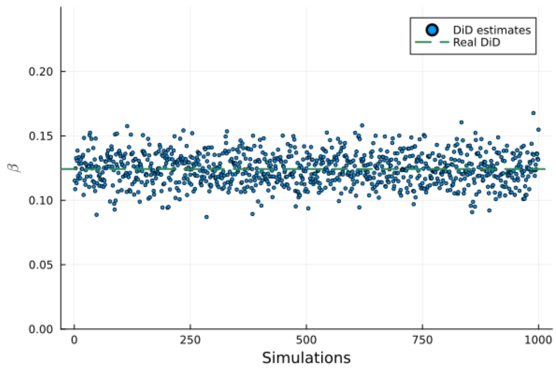


(b) DiD without measurement error

Figure 1: Difference-in-Differences estimates with 100 simulations



(a) DiD with measurement error



(b) DiD without measurement error

Figure 2: Difference-in-Differences estimates with 1000 simulations

	Observed Data	Real Data	Observed Data	Real Data
Estimated effect	0.1089	0.1242	0.1068	0.1247
Standard error	0.0344	0.0122	0.0349	0.0128
Confidence interval	(0.0424, 0.1753)	(0.1001, 0.1483)	(0.0407, 0.1729)	(0.1007, 0.1488)
Significance rejection rate	0.112	0.00	0.13	0.00
Significance level	95%		95%	
No. of repetitions	1,000		100	
Misinference rate	0.495		0.460	

Table 1: Reduced-form Difference-in-Differences (DiD) estimation for both observed data suffering from measurement error, and real data, according to our simulated DGP. Results are the average of a 1,000 and 100 simulation repetitions. The Misinference rate denotes the share of repetitions on which the observed data's estimated effect was not included in the real data's estimator's confidence interval.

Simulation 1: Extended Grid		Simulation 2: Reduced Grid	
θ	Rejection rate	θ	Rejection rate
0.0424	1.00	0.1162	0.30
0.0572	1.00	0.1195	0.18
0.0719	1.00	0.1228	0.13
0.0867	0.96	0.1261	0.12
0.1015	0.73	0.1294	0.10
0.1162	0.30	0.1326	0.10
0.1310	0.10	0.1359	0.09
0.1458	0.27	0.1392	0.12
0.1605	0.73	0.1425	0.18
0.1753	0.95	0.1458	0.27

Table 2: Structural DiD set-identified estimation of the slack parameter for the ATT with observed data, for two simulation tranches with 100 repetitions. Simulation 1 has as upper and lower bounds the bounds of the confidence interval thrown by the reduced form DiD estimation with observed data. The second simulation builds on a reduced grid taken from a subset of the grid used in Simulation 1 where convergence to high non-rejection was achieved. In both simulations, the average real effect was **ATT = 0.1237**.

Now, the results from the structural estimation. Table 2 reports two simulations on the structural estimation model described in section 4. In both cases, a size grid of 10 was taken over the ATT slack parameter θ. Simulation 1 was estimated taking the reduced-form confidence interval with the real data as bounds. While around a 87.0% - 88.8% of the times the reduced-form estimation with observed data will not be rejected (as shown in Table 1), the structural model, accounting for measurement error, yields a more precise non-rejection region on the estimator. The second simulation of the structural estimation model ran over a subset of the original grid where the test statistic achieved parabolic convergence to a deeper non-rejection region.

Recall that the model in Section 4 is by design built with a structural significance parameter of $\alpha = 0.05$. We provide Montecarlo evidence of this parameter being identified over a sharp set in (0.1294,0.1359) at the 0.1 level. Since we are interested in set identification rather than point identification, the fact that the later set resides slightly upward the causal effect point at the 0.1 level should not be of worry as it is explained due to the simulation running over a small sample. A potential gain is to be attained if we simulate 1000 times per each θ in the grid rather than 100 times in order to yield stronger asymptotic consistency. This was not implemented due to the time costs.

6 Discussion

It has been shown, under a controlled environment with simulated data, that observed data sets suffering from measurement error yield biased causal DiD estimations. For the DGP calibration proposed, this bias can be calculated in sign and magnitude as shown in Table 1. An alternative causal estimation method is proposed, making use of a structural solution provided by the stochastic revealed preferences' approach by Aguiar and Kashaev (2020) to account for measurement error. This methodology applies a Maximum-Entropy Moment Conditions' structure derived from the Entropic Latent Variable Integration via Simulation (ELVIS) methodology due to Schennach (2014). Montecarlo evidence is provided that this method provides set-identification that is robust to classical forms of measurement error on observed data when estimating the causal effect of an exogenous shock on the consumption bundles of a population. The identified set of the interest parameter on the structural model suggests that, when relying on observational data with unobserved measurement error, conventional reduced form DiD estimation techniques may yield biased estimators and broad confidence intervals for which a very reduced subset contains an identification region that actually accounts for unobserved randomness such as measurement error.

Some further important extensions are to be incorporated in further extensions to this testing approach. First and foremost, while the nature of this problem's formulation made it impossible to incur in parallel computing solutions, stronger computational capacity is definitely to be required, and the implementation mechanism optimized so that the Montecarlo estimation of the structural model yields asymptotic consistency at the 0.05 level for each θ in a grid. Additionally, the optimization methods applied in the algorithm could be further improved. Second, the identified set for which the structural estimated effect reaches non-rejection is potentially consistent to a given subset of distributions fulfilling parallel-trends conditions. However, this assumption should be explicitly modeled as an additional moment condition in equations (13) - (15) for such identified set to be sufficiently narrowed down to a space of pure causal effects.

Third, while CD preferences may be fairly inclusive of a vast range of stochastically diverse consumer behavior types, a robustness extension would consist of expanding the model to other flexible structure of preferences given, for example, by CES functions, making its parameters empirically interpretable in an RP context, specially considering that parametric rationalizability is implicitly tested in our moment conditions. Finally, while the use of classical forms of measurement error on this paper has been justified, it would be optimal to consider alternative types of unobserved measurement error. Other forms of non-classical systematic measurement error lead to more severe biases for which the applied methodology would still be robust. These are just some of the potential extensions that could be explored for this paper in the near future.

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Appendices

A Proof of Theorem 1

Proof. First, it will be shown that for a deterministic array $(p_t, c_t)_{t\in\mathcal{T}}$ to be CD-rationalizable is equal to the existence of some matrix $(\alpha_{l,t})_{l\in\mathcal{L},t\in\mathcal{T}}$ with $\sum_l \alpha_{j,l,t} = 1$ such that $c_{l,t} = \alpha_{l,t}y_t/p_{l,t}$ for all $l \in \mathcal{L}$ at each $t \in \mathcal{T}$.

Consider the convexity-preserving monotone transform of equation (1) given by $\ln u = \sum_l \alpha_l \ln c_l$. It will be shown that there exists some $(\alpha_{l,t})_{l\in\mathcal{L},t\in\mathcal{T}}$ such that

$$c_l = \arg\max_{c_l}\Big\{\sum_l \alpha_l \ln c_l \text{s.t.} p'c = y\Big\} \tag{37}$$

For this, solve for the first order conditions of the associated Lagrangian:

$$L = \sum_l \alpha_l \ln c_l + \lambda[y - p'c] \tag{38}$$

with yields the marginal condition

$$\alpha_l = \lambda p_l c_l \tag{39}$$

Since $\ln u$ is concave and the budget set $p'c = y$ is convex, this marginal condition is a perfectly identified system and there exists a unique interior solution for c_l. To see it, sum both sides of (39) over goods $k \in \mathcal{L}$, $\sum_k \alpha_k = \lambda \sum_k p_k c_k$ which implies that $\lambda = \sum_k \alpha_k / \sum_k p_k c_k$. Replacing this into (39) the following is obtained:

$$c_l = \frac{\alpha_l}{p_l \sum_k \alpha_k} \sum_k p_k c_k \tag{40}$$

By noting that $\sum_l p_l c_l \equiv p'c$ and realizing this is consistent for the case $\sum_l \alpha_{j,l,t} = 1$, the demand for good $l \in \mathcal{L}$ yields:

$$c_l = \frac{\alpha_l y}{p_l} \tag{41}$$

for each $t \in \mathcal{T}$ and any household $j \in \mathcal{J}$. Therefore, given $(c_t)_{t\in\mathcal{T}}$ solves (37) and so $(p_t, c_t)_{t\in\mathcal{T}}$ is CD-rationalizable, some $(\alpha_{l,t})_{l\in\mathcal{L},t\in\mathcal{T}}$ in fact exists such that $c_{l,t} = \alpha_{l,t}y_{l,t}/p_{l,t}$.

Now, it will be shown that existing a $(\alpha_{l,t})_{l\in\mathcal{L},t\in\mathcal{T}}$ for some $c_{l,t} = \alpha_{l,t}y_{l,t}/p_{l,t}$, the array $(p_t, c_t)_{t\in\mathcal{T}}$ is CD-rationalizable, that is, $(c_t)_{t\in\mathcal{T}}$ solves (37).

Since it has already been shown that 41 is in fact the maximum possible value of the CD utility function (1) given the budget constraint, this amounts to showing that the solution 41 is unique. Assume there is another level of demand, $\tilde{c}$ such that $\tilde{c} \sim c$ and both $u(\tilde{c})$ and $u(c)$ are maxima given $p'c = y$. This would imply that $u(\cdot)$ is not concave, which it is as per equation (1). Therefore, there is no other demand level yielding the same utility as 41 does.

Next, assume there is another level of consumption $\tilde{c} \succ c$ that solves (37), such that $u(\tilde{c}) > u(c)$ for $u(\cdot)$ concave, such that $\tilde{c} > c$. Then,

$$\arg\max_c \Big\{u(\tilde{c}) \text{s.t.} p'\tilde{c} > y\Big\} \succ \arg\max_c \Big\{u(c) \text{s.t.} p'c = y\Big\} \tag{42}$$

however, the left-hand side of (42) is impossible to hold as it is out of the bounds set by the convex constraint. Therefore, c as in 41 must be unique.

This implies that for a demand $c_{l,t} = \alpha_{l,t} y_t / p_{l,t}$ to exist with its matrix $(\alpha_{l,t})_{l\in\mathcal{L},t\in\mathcal{T}}$ is equivalent to stating that $(c_t)_{t\in\mathcal{T}}$ solves (37) and therefore $(p_t, c_t)_{t\in\sqcup}$ is CD-rationalizable by CD preferences. □

B Data Generating Process via simulation

The data is generated by the following algorithm. The output is the observed data set of consumption and prices:

Step 0:

set $\mathcal{T} = 1,2$ and $\mathcal{L} = 1,2,3$ for $\mathcal{J} = 1,\ldots,200$ households.

set control and treatment groups in the population, $\mathcal{C},\tau \subset \mathcal{J}$ by randomly drawing an equal number of households such that $|\mathcal{C}| = |\tau| = 100$, with $\tau \cap \mathcal{C} = \emptyset$, $\tau \neq \emptyset$, $\mathcal{C} \neq \emptyset$ and $\tau \cup \mathcal{C} \equiv \mathcal{J}$.

set $(\mathbf{a}_{j,l,t})_{j\in\mathcal{J},l\in\mathcal{L},t\in\mathcal{T}} \sim \mathcal{N}(\mu_{\mathbf{a}} = 1/K, \sigma^2_{\mathbf{a}} = 0.05)$ truncated at [0.1,0.9] and randomly drawn accordingly for the simulated population.

set $\mathbf{a}_{j,1,t} \in (0,1)$ and $\mathbf{a}_{j,1,t} + \mathbf{a}_{j,2,t} + \mathbf{a}_{j,3,t} = 1$ for all $t \in \mathcal{T}$ and $j \in \mathcal{J}$.

set $\mathbf{a}_{j,l=1,t}$ as receiving a positive exogenous scaled shock derived from $\psi_{j\in\tau} \sim \text{Unif}(0,1)$ to simulate a random treatment effect assignment in the DGP.

set observed prices $(\mathbf{p}_{l,t})_{l\in\mathcal{L},t\in\mathcal{T}} \sim \text{Exp}(\lambda = 1)$

set income stream $(\mathbf{y}_{j,t})_{j\in\mathcal{J},t\in\mathcal{T}} = 1$ for all $t \in \mathcal{T}$ and $j \in \mathcal{J}$.

set demand $(\mathbf{c}_{j,l,t})_{j\in\mathcal{J},l\in\mathcal{L},t\in\mathcal{T}}$ as given by $\mathbf{c}_{j,l,t} = \mathbf{a}_{j,1,t}\mathbf{y}_{j,t}/\mathbf{p}_{l,t}$

set measurement error $\epsilon_{j,l,t} = (b-a) \times \zeta_{j,l,t} + a$ with $\zeta_{j,l,t} \sim \text{Unif}(0,1)$. Also, set $b = 1.9$ and $a = 0.1$

set observed demand $(\mathbf{c}^*_{j,l,t})_{j\in\mathcal{J},l\in\mathcal{L},t\in\mathcal{T}}$ as given by $\mathbf{c}^*_{j,l,t} = \mathbf{c}_{j,l,t} \times \epsilon_{j,l,t}$.

return observed data set $\mathbf{x} = \{(\mathbf{c}^*_{j,l,t}, \mathbf{p}_{l,t})\}_{j\in\mathcal{J},l\in\mathcal{L},t\in\mathcal{T}}$

C Algorithm for section 4

Step 0:

set a realization of $\{(\mathbf{c}^*_{j,l,t}, \mathbf{p}_{l,t})\}_{j\in\mathcal{J},l\in\mathcal{L},t\in\mathcal{T}}$

fix a user-defined grid $\Theta = \{\theta_k\}_{k=1}^{\Theta s}$

fix $g_I = (g'_A, g'_D)$ and g_M

fix $\eta \in \mathcal{P}_{E|X}$

fix $\mathbf{x} = (\mathbf{x}_i)_{i=1\ldots n}$ where $\mathbf{x}_i = \{(\mathbf{c}^*_{t,i}, \mathbf{p}_{t,i})\}_{t\in\mathcal{T}}$ is the i-th obseration of sample of size n.

end Step 0

Step 1 (Integration):

set $i = 1$

fix cl - MCMC length;

fix $\eta, \gamma, \mathbf{x}_i$, and the first element of the chain e_{-nburn} that satisfies the constraints.

set $r = -nburn + 1$ and $\hat{h}_M(x_i, \gamma) = 0$

while $r \leq nsims$ **do**

  draw $e_{jump} = \{(\alpha_{j,l,t})_{j \in \mathcal{J}, l \in \mathcal{L}, t \in \mathcal{T}}, w^c\}$ proportional to $\tilde{\eta}(\cdot|x_i, \theta) = \eta(\cdot|x_i)\mathbb{I}(g_I(x_i, \theta, \cdot) = 0)$

  draw α from Unif$(0,1)$

  set e_r equal to e_{jump} if $[g_M(x_i, e_{jump}) - g_M(x_i, e_{r-1}))]'\gamma > \log(\alpha)$

  if $r > 0$ **then**

    compute $\tilde{h}_M(x_i, \gamma, \theta) = h_M(x_i, \gamma, \theta) + g_M(x_i, e_r)/cl$

  end if

  set $r = r + 1$

end while

set $i = 1$

while $i \leq n$ **do**

  define the measure $\tilde{\eta}(\cdot|x_i, \theta) = \eta(\cdot|x_i)\mathbb{I}(g_I(x_i, \theta, \cdot) = 0)$

  integrate latent variables by $\tilde{\eta}(\cdot|x_i, \theta)$ to obtain $\tilde{h}_M(x_i, \gamma, \theta)$

  set $i = i + 1$

end while

compute

$$\hat{\tilde{h}}_M(\gamma, \theta) = \frac{1}{n}\sum_{i=1}^{n} \tilde{h}_M(\mathbf{x}_i, \gamma, \theta); \tag{43}$$

and

$$\hat{\tilde{\Omega}}(\gamma, \theta) = \frac{1}{n}\sum_{i=1}^{n} \tilde{h}_M(\mathbf{x}_i, \gamma, \theta)\tilde{h}_M(\mathbf{x}_i, \gamma, \theta)' - \hat{\tilde{h}}_M(\gamma, \theta)\hat{\tilde{h}}_M(\gamma, \theta)' \tag{44}$$

compute the objective function

$$\text{ObjFun}(\gamma, \theta) = n \times \hat{\tilde{h}}_M(\gamma, \theta)'\hat{\tilde{\Omega}}(\gamma.\theta)^{-}\hat{\tilde{h}}_M(\gamma, \theta) \tag{45}$$

end Step 1

Step 2: (Optimization)

compute

$$TS_n = \min_{\gamma} \text{ObjFun}(\gamma, \theta) \tag{46}$$

end Step 2