中国科学技术大学
University of Science and Technology of China
地址：中国 安徽 合肥市金寨路96号 邮编：230026
电话：0551-63602184 传真：0551-63631760 Http://www.ustc.edu.cn

第十次作业.

第4章.

5. $\begin{cases} \int_0^1 f(x)dx=1 \\ \int_0^1 xf(x)dx=0.5 \\ \int_0^1 x^2 f(x)dx=0.4 \end{cases} \Rightarrow \begin{cases} a=12 \\ b=-12 \\ c=3. \end{cases}$

16. (1). $E\,sgn(X)=\int_{-\infty}^{+\infty} sgn(x)f(x)dx=\int_{-\infty}^{+\infty}[I(x>0)-I(x\le 0)]\cdot\frac{1}{3}\cdot I(-2\le x\le 1)\,dx=-\frac{1}{3}$

$E\,sgn(X)^2=\int_{-\infty}^{+\infty} sgn(x)^2\cdot f(x)dx=1$

$\Rightarrow Var(sgn(X))=E\,sgn(X)^2-(E\,sgn(X))^2=\frac{8}{9}.$

(2) $E[sgn(X)\cdot X]=\int_{-\infty}^{+\infty} x\,sgn(x)\cdot f(x)\,dx=\int_0^{+\infty}xf(x)dx-\int_{-\infty}^{0}xf(x)dx=2\int_0^{+\infty}xf(x)dx=\sqrt{\frac{2}{\pi}}$

20. (1). $X_2|X_1=k \sim B(m-k,\frac{p_2}{1-p_1}) \Rightarrow E(X_2|X_1=k)=\frac{(m-k)p_2}{1-p_1}$, $Var(X_2|X_1=k)=\frac{(m-k)p_2(1-p_1-p_2)}{(1-p_1)^2}$

(2). $X_i\sim B(m,p_i)$, $E(X_1+X_2)=m(p_1+p_2)$

$X_1+\cdots+X_k\sim B(m,\sum_{i=1}^{k}p_i)$, $Var(X_1+\cdots+X_k)=m\sum_{i=1}^{k}p_i(1-\sum_{i=1}^{k}p_i)$

22. (1). $f(y)=\sum_{i=0}^{2}f(y|X=i)p(X=i)=\frac{1}{3}(N(0,1)+N(1,1)+N(2,1))$ (不规范). , $EY=1$

(2). $F_{X+Y}(z)=P(X+Y\le z)=\sum_{i=0}^{2}P(Y\le z-i|X=i)\cdot P(X=i)=\frac{1}{3}(F(z)+F(z-2)+F(z-4))$

(3). $Cov(X,Y)=EXY-EXEY=E[XE(Y|X)]-1=EX^2-1=\frac{2}{3}$

29. (1). $P(Y\le z)=P(\min\{X_1,X_2\}\le z)=1-(1-F_{X_1}(z))(1-F_{X_2}(z))$

$f(z)=f_{X_1}(z)+f_{X_2}(z)-f_{X_1}(z)F_{X_2}(z)-f_{X_2}(z)F_{X_1}(z)=\frac{3}{2}e^{-\frac{3}{2}z}$, $EY=\frac{2}{3}$.

$P(Z\le z)=P(\max\{X_1,X_2\}\le z)=F_{X_1}(z)\cdot F_{X_2}(z)$, $f_Z(z)=e^{-z}+\frac{1}{2}e^{-\frac{1}{2}z}-\frac{3}{2}e^{-\frac{3}{2}z}$, $EZ=\frac{7}{3}$

(2). $Var(Y)=\frac{4}{9}$, $Var(Z)=EZ^2-(EZ)^2=\frac{82}{9}-(\frac{7}{3})^2=\frac{11}{3}$.

34. (1). $f(x,y)=\frac{1}{2}$, $EXY=\iint xy\cdot\frac{1}{2}\,dxdy=0 \Rightarrow Cov(X,Y)=0$

(2) $f(x)=\int_{|y|\le 1-|x|}f(x,y)dy=\begin{cases}1-x, & 0\le x\le 1\\ 1+x, & -1\le x<0\end{cases}$, $f(y)$ 同理. 不独立.

40. (1) $EN(T)=E[E(N(T)|T)]=E(\lambda T)=a\lambda$. $E(TN(T))=E[E(TN(T)|T)]=E(\lambda\cdot T^2)=\lambda(b+a^2)$

$Cov(T,N(T))=E(T\cdot N(T))-ET\cdot EN(T)=\lambda b.$

(2). $Var(N(T))=EN(T)^2-(EN(T))^2$

$EN(T)^2=E[E(N(T)^2|T)]=E(\lambda T+(\lambda T)^2)=a\lambda+(b+a^2)\lambda$

$\Rightarrow Var(N(T))=a\lambda+b\lambda^2$