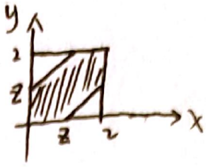
20: $f(x,y)=\frac{1}{4}$, $0\le x\le 2$, $0\le y\le 2$

$F(z)=P(|X-Y|\le z)$



当 $z<0$ 时, $F(z)=0$

当 $z\geq 2$ 时, $F(z)=1$

当 $0\le z<2$ 时, $F(z)=\frac{1}{4}\left(4-2\cdot\frac{1}{2}\cdot(2-z)^2\right)=z-\frac{1}{4}z^2$

于是 $f(z)=F'(z)=1-\frac{1}{2}z$, $0\le z<2$

23.

$V \backslash U$	0	1
0	$\frac{1}{4}$	$\frac{1}{4}$
1	0	$\frac{1}{2}$

25: (1). $P(X_1=m_1, X_2=m_2, \cdots, X_n=m_n)=C_m^{m_1}\cdot p_1^{m_1}\cdot C_{m-m_1}^{m_2}\cdot p_2^{m_2}\cdots\cdots C_{m-m_1-\cdots-m_{n-1}}^{m_n}p_n^{m_n}=\frac{m!}{m_1!m_2!\cdots m_n!}p_1^{m_1}p_2^{m_2}\cdots p_n^{m_n}$

(2): 易知 $X_k\sim B(m,p_k)$, 即 $P(X_k=m_k)=C_m^{m_k}p_k^{m_k}(1-p_k)^{m-m_k}$

(3): $P(X_1=m_1, X_2=m_2)=\sum\limits_{m_3+\cdots+m_n=m-m_1-m_2}\frac{m!}{m_1!m_2!\cdots m_n!}p_1^{m_1}p_2^{m_2}\cdots p_n^{m_n}$

$$=\frac{m!}{m_1!m_2!(m-m_1-m_2)!}p_1^{m_1}p_2^{m_2}(1-p_1-p_2)^{m-m_1-m_2}$$

(4): $P(X_2=m_2,\cdots,X_n=m_n \mid X_1=m_1)=\frac{P(X_1=m_1, X_2=m_2,\cdots, X_n=m_n)}{P(X_1=m_1)}$

$$=\frac{\frac{m!}{m_1!m_2!\cdots m_n!}p_1^{m_1}p_2^{m_2}\cdots p_n^{m_n}}{\frac{m!}{m_1!(m-m_1)!}p_1^{m_1}(1-p_1)^{m-m_1}}$$

$$=\frac{(m-m_1)!}{m_2!\cdots m_n!}\left(\frac{p_2}{1-p_1}\right)^{m_2}\cdots\left(\frac{p_n}{1-p_1}\right)^{m_n}$$

(RK: 此题错误率较高, 通过(1)问得出联合分布老老实实代入公式即可得出(2)(3)(4)问.
形如(1)的n维离散型分布称为多项分布, 记作 $M_n(m,p_1,p_2,\cdots,p_n)$;
二项分布就是 $M_2(m,p,1-p)$; 多项分布的k维边缘分布为 $n=k+1$ 维多项分布.)

31: $f(x,y,z)=\frac{1}{\frac{4}{3}\pi}=\frac{3}{4\pi}$, $x^2+y^2+z^2\le 1$

$f(x)=\iint\limits_{y^2+z^2\le 1-x^2}\frac{3}{4\pi}dydz=\frac{3}{4}(1-x^2)$, $-1\le x\le 1$

Poisson分布再生性证明：

$$P(Z=k)=\sum_{i=0}^{k}P(X=i)P(Y=k-i)$$

$$=\sum_{i=0}^{k}e^{-\lambda}\cdot\frac{\lambda^i}{i!}\cdot e^{-\mu}\frac{\mu^{k-i}}{(k-i)!}$$

$$=e^{-(\lambda+\mu)}\cdot\frac{1}{k!}\sum_{i=0}^{k}C_k^i\lambda^i\mu^{k-i}$$

$$=e^{-(\lambda+\mu)}\frac{(\lambda+\mu)^n}{k!}$$

即 $Z\sim Poi(\lambda+\mu)$