第14次作业.

第7章 27. (1) 记 $p=\frac{t}{n}$, $\hat{p}=\frac{t}{5}$, 可求得置信区间 $\frac{t}{5}\pm\mu_{\frac{\alpha}{2}}\sqrt{\frac{t/5(1-t/5)}{5}}$

(2) 由(1), $\frac{t}{n}\in\left[\frac{t}{5}-\mu_{\alpha/2}\sqrt{\frac{t/5(1-t/5)}{5}},\ \frac{t}{5}+\mu_{\alpha/2}\sqrt{\frac{t/5(1-t/5)}{5}}\right]$

$\Rightarrow N\in\left[r/\left(\frac{t}{5}+\mu_{\alpha/2}\sqrt{\frac{t/5(1-t/5)}{5}}\right),\ r/\left(\frac{t}{5}-\mu_{\alpha/2}\sqrt{\frac{t/5(1-t/5)}{5}}\right)\right]$

29. $\frac{\sqrt{n}(\bar{X}-\mu)}{\sigma}\sim N(0,1)$ $P(\bar{X}>75)=99\%\Leftrightarrow P\left(\frac{\sqrt{n}(\bar{X}-\mu)}{\sigma}>-\sqrt{n}\right)>0.99$. 即 $\mu(-\sqrt{n})<0.01$

$\sqrt{n}=2.33$ (查表) $n=5.43$ 向上取整, $n=6$

33. $\bar{X}=42086.33$, $S=1514.562$, $\frac{\sqrt{n}(\bar{X}-\mu)}{S}\sim t_{n-1}\Rightarrow \bar{X}-\frac{S}{\sqrt{n}}t_8(0.05)=41147.56$

34. (1). $\frac{\sqrt{n}(\bar{X}-\mu)}{S}\sim t_{n-1}$, $\bar{X}-\frac{S}{\sqrt{n}}t_{15}(0.05)=1593.426$

(2) $\frac{(n-1)S^2}{\sigma^2}\sim\chi^2_{n-1}$, $\frac{(n-1)S^2}{\chi^2_{n-1}(0.15)}=464.812$

第8章 1. (1) $P(\bar{X}>c)=0.05\Leftrightarrow P\left(\frac{(\bar{X}-\mu_0)}{\sigma}\sqrt{n}>\frac{(c-\mu_0)\sqrt{n}}{\sigma}\right)=0.05\Rightarrow \bar{X}>0.6645$.

(2) $P(\bar{X}\le 0.6645)=P\left(\frac{\sqrt{n}(\bar{X}-\mu)}{\sigma}\le\frac{\sqrt{n}(0.6645-0.65)}{\sigma}\right)=P(Z\le 0.145)=0.5576$

2. α 取大些. 减少次品混入正品的可能性为减少第二类错误概率.

3. (1). $4(\bar{X}-\mu_0)=4\bar{X}\sim N(0,1)$. $P(V_1)=P(4|\bar{X}|\ge\mu_{0.05})=2P(\bar{X}\ge\mu_{0.05})=0.1$

$P(V_2)=P(4|\bar{X}|\le\mu_{0.45})=P(-\mu_{0.45}\le 4\bar{X}\le\mu_{0.45})=0.1$

$P(V_3)=P(4\bar{X}\ge\mu_{0.1})=0.1$ $P(V_4)=P(4\bar{X}\le-\mu_{0.1})=0.1$

(2) H_1 成立时, $4(\bar{X}-1)\sim N(0,1)$ $\alpha_2(V_1)=P(-\mu_{0.05}-4\le 4(\bar{X}-1)\le\mu_{0.05}-4)=0.0093$

$\alpha_2(V_2)=P(4(\bar{X}-1)>\mu_{0.45}-4)+P(4(\bar{X}-1)<-\mu_{0.45}-4)=0.999$.

$\alpha_2(V_3)=P(4(\bar{X}-1)<\mu_{0.1}-4)=0.003$ $\alpha_2(V_4)=P(4(\bar{X}-1)>-\mu_{0.1}-4)=0.999$. , V_3 ✓

4. $\alpha_1=P(X>\frac{1}{2}|H_0)=\frac{63}{64}$, $\alpha_2=P(X\le\frac{1}{2}|H_1)=\frac{1}{16}$, $\beta=P(X>\frac{1}{2}|\theta=2)=\frac{7}{8}$

5. (1). $\lambda=0.25$: $\beta=0.287$, $\lambda=0.5$: $\beta=0.0458$. $\lambda=1$: $\beta=0.304$, $\alpha=0.0458$

(2) $\alpha_1=0.0458$. $\alpha_2=P(D^c|\lambda=0.25)=0.713$, $\alpha_2=P(D^c|\lambda=0.75)=0.916$.

把他城记为 D.

6. (1) $F_{X_{(n)}}(x)=\left(\frac{x}{3}\right)^n$, $\alpha=P(D|\theta=3)=\left(\frac{2.5}{3}\right)^n$, $\beta(\theta)=\left(\frac{2.5}{\theta}\right)^n$.

(2). $\alpha=0.05\Rightarrow n=\frac{\log(0.05)}{\log(2.5/3)}=16.43\approx 17$.