

第一章

$$37. P(A) = P(A|C)P(C) + P(A|\bar{C})P(\bar{C}) = 0.9 \times 0.5 + 0.2 \times 0.5 = 0.55$$

$$P(B) = P(B|C)P(C) + P(B|\bar{C})P(\bar{C}) = 0.9 \times 0.5 + 0.1 \times 0.5 = 0.5$$

$$P(AB) = P(AB|C)P(C) + P(AB|\bar{C})P(\bar{C}) = P(A|C)P(B|C)P(C) + P(A|\bar{C})P(B|\bar{C})P(\bar{C}) = 0.9 \times 0.9 \times 0.5 + 0.2 \times 0.1 \times 0.5 = 0.415$$

$$P(A)P(B) = 0.275 \neq P(AB), \text{ 故 } A \text{ 与 } B \text{ 不相互独立.}$$

38. (1): 设 $A = \{\text{第一次命中}\}$, $B = \{\text{第二次命中}\}$, $C = \{\text{第三次命中}\}$

$$\text{则 } P(\text{恰有一次命中}) = P(AB\bar{C}) + P(A\bar{B}\bar{C}) + P(\bar{A}B\bar{C}) = 0.5 \times 0.4 \times 0.2 + 0.5 \times 0.6 \times 0.2 + 0.5 \times 0.4 \times 0.8 = 0.26$$

$$(2): P(\text{至少命中一次}) = 1 - P(\text{一次都没命中}) = 1 - 0.5 \times 0.4 \times 0.2 = 0.96$$

$$39. (4): P(\text{正常工作}) = P(D_1) \cdot P(A \cup B \cup C) \cdot P(D_2) = P_D^2 (1 - (1 - P_A)(1 - P_B)(1 - P_C))$$

$$(5): P(\text{正常工作}) = P(\text{工作}|C)P(C) + P(\text{工作}|\bar{C})P(\bar{C})$$

$$= P_C [1 - (1 - P_A)^2] [1 - (1 - P_B)^2] + (1 - P_C) [1 - (1 - P_A P_B)^2]$$

第二章

2. 设 X_k 为 k 次投篮命中次数

$$\text{由题意 } P(X_1=1) = \frac{1}{2} = P(X_2=2)$$

$$\text{设 } P(X_k=i) = \frac{1}{k-1}, \quad 1 \leq i \leq k-1$$

$$\text{则 } P(X_{k+1}=i) = P(X_k=i) \cdot P(\text{第 } k+1 \text{ 次未命中}) + P(X_k=i-1) \cdot P(\text{第 } k+1 \text{ 次命中})$$

$$= \frac{1}{k-1} \times \frac{k-i}{k} + \frac{1}{k-1} \times \frac{i-1}{k} = \frac{1}{k}, \quad 1 \leq i \leq k$$

由数学归纳法知假设成立.

$$\therefore P(X_{100}=i) = \frac{1}{99}, \quad 1 \leq i \leq 99$$

$$3. \begin{array}{c|c|c|c|c} X & -60 & 50 & 80 & 100 \\ \hline P & 0.1 & 0.1 & 0.2 & 0.6 \end{array}$$

$$10. P_4 = \left(\frac{3}{5}\right)^4 = \frac{81}{625}$$

$$P_5 = C_4^3 \left(\frac{3}{5}\right)^4 \times \frac{2}{5} = \frac{648}{3125}$$

$$P_6 = C_5^3 \cdot \left(\frac{3}{5}\right)^4 \times \left(\frac{2}{5}\right)^2 = \frac{648}{3125}$$

$$P_7 = C_6^3 \cdot \left(\frac{3}{5}\right)^4 \times \left(\frac{2}{5}\right)^3 = \frac{2592}{15625}$$

$$\text{甲胜率 } P_{\text{甲}} = P_4 + P_5 + P_6 + P_7 = \frac{11097}{15625}$$

$$\text{三局两胜制甲胜率 } P'_{\text{甲}} = \left(\frac{3}{5}\right)^3 + C_2^1 \times \left(\frac{3}{5}\right)^2 \times \frac{2}{5} = \frac{81}{125} < P_{\text{甲}}$$

故三局两胜制对乙更有利.

$$11. P(X=-1) = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$$

$$P(X=1) = C_3^1 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^2 = \frac{25}{72}$$

$$P(X=2) = C_3^2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right) = \frac{5}{72}$$

$$P(X=3) = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$$