第六次书面作业参考答案

1 习题 5

1(2). $\|A\|_1 = 2, \|A\|_2 = \sqrt{\rho(A^TA)} = \sqrt{\frac{3+\sqrt{5}}{2}}, \|A\|_\infty = 2$

2(1). $\rho(B) = 4, Cond_\infty(B) = \|B\|_\infty \|B^{-1}\|_\infty = 2$

4(2). 设迭代格式为 $x_{k+1} = Mx_k + g$, 则 $M = \begin{pmatrix} 0 & \frac{1}{5} & \frac{1}{5} \\ -\frac{1}{2} & 0 & -\frac{1}{3} \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}, g = \begin{pmatrix} -\frac{1}{5} \\ 0 \\ 2 \end{pmatrix}$

$$x^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, x^{(1)} = \begin{pmatrix} -\frac{1}{5} \\ 0 \\ 2 \end{pmatrix}, x^{(2)} = \begin{pmatrix} \frac{1}{5} \\ -\frac{17}{30} \\ \frac{21}{10} \end{pmatrix}$$

解为: $\begin{cases} x_1 = \frac{1}{5} \\ x_2 = -\frac{17}{30} \\ x_3 = \frac{21}{10} \end{cases}$

5(2). 设迭代格式为 $x_{k+1} = Mx_k + g$, 则 $M = \begin{pmatrix} 0 & \frac{1}{5} & \frac{1}{5} \\ 0 & -\frac{1}{10} & -\frac{13}{30} \\ 0 & -\frac{3}{20} & -\frac{19}{60} \end{pmatrix}, g = \begin{pmatrix} \frac{16}{5} \\ \frac{7}{30} \\ -\frac{149}{40} \end{pmatrix}$, 也可以用各分量去迭代。

迭代 11 步，解为：$\begin{cases} x_1 = 3.0002 \\ x_2 = 0.9997 \\ x_3 = -2.0002 \end{cases}$

6.(1)Jacobi 迭代矩阵：$M = \begin{pmatrix} 0 & -t \\ -\frac{1}{2}t & 0 \end{pmatrix}, \rho(M) = \frac{|t|}{\sqrt{2}} < 1 \iff |t| < \sqrt{2}$

(2)Gauss-Seidel 迭代矩阵：$M = \begin{pmatrix} 0 & -t \\ 0 & \frac{1}{2}t^2 \end{pmatrix}, \rho(M) = \frac{t^2}{2} < 1 \iff |t| < \sqrt{2}$

8(1).

$$A^{-1} = \begin{pmatrix} -0.9286 & -0.7857 & 0.5714 & 0.1429 \\ 1.1071 & 0.3214 & -0.3929 & -0.0357 \\ -0.0357 & -0.1071 & -0.0357 & 0.1786 \\ -0.2857 & 0.1429 & 0.2143 & -0.0714 \end{pmatrix}$$

10.(1) 设迭代矩阵为 M, 则 $M = \begin{pmatrix} 1-3\alpha & -2\alpha \\ -\alpha & 1-2\alpha \end{pmatrix}$

(2) 渐进收敛速度：$R_\infty = -ln(\rho(M))$,$\rho(M) = max\{|4\alpha - 1|, |\alpha - 1|\}$,$\alpha = \frac{2}{5}$ 时，$\rho(M)$ 最小，迭代收敛速度最快。(关于收敛速度，可参考群文件参考教材《数值线性代数》4.3 节的内容)

2 习题 8

1(2). 按模最大特征值 $\lambda \approx 3.8281$, 特征向量 $v \approx (0.7070, 1)^T$

2(2). 按模最小特征值 $\lambda \approx 1.9938$, 特征向量 $v \approx (-0.9953, 1)^T$