四、27. (1). $M_X(s)=E[e^{sX}]=\sum_{k=0}^{n}e^{sk}C_n^k p^k(1-p)^{n-k}=\sum_{k=0}^{n}C_n^k(pe^s)^k(1-p)^{n-k}=(pe^s+1-p)^n$

(2). $M_X(s)=\sum_{k=0}^{+\infty}e^{sk}\cdot e^{-\lambda}\cdot\frac{\lambda^k}{k!}=e^{-\lambda}\sum_{k=0}^{+\infty}\frac{(\lambda e^s)^k}{k!}=e^{-\lambda}\cdot e^{\lambda e^s}=e^{\lambda(e^s-1)}$

(3). $M_X(s)=\int_0^{+\infty}e^{sx}\lambda e^{-\lambda x}dx=\frac{\lambda}{s-\lambda}e^{(s-\lambda)x}\Big|_0^{+\infty}=\frac{\lambda}{\lambda-s}$ , $|s|<\lambda$

(4). $M_X(s)=\int_{-\infty}^{+\infty}e^{sx}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx \xlongequal{x=\mu+\sigma z} e^{\mu s}\int_{-\infty}^{+\infty}e^{z\sigma s}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz=e^{\mu s+\frac{1}{2}\sigma^2s^2}$

41. $(X,Y)\sim N(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho)$，则 $X|Y\sim N(\mu_1+\rho\frac{\sigma_1}{\sigma_2}(y-\mu_2),\ (1-\rho^2)\sigma_1^2)$

则 $E[X|Y=2]=\mu_1+\rho\frac{\sigma_1}{\sigma_2}(2-\mu_2)=1$

$E[XY^2+Y|Y=1]=E[X+1|Y=1]=\mu_1+\rho\frac{\sigma_1}{\sigma_2}(1-\mu_2)+1=1.8$

43. $P(X_n=k)=(1-\frac{\lambda}{n})^{k-1}\frac{\lambda}{n}$，$k=1,2,\cdots$，$n=1,2,\cdots$，$P(Y_n=k)=P(X_n=nk)=(1-\frac{\lambda}{n})^{nk-1}\frac{\lambda}{n}$

$F_n(y)=P(Y_n\le y)=1-P(Y_n>y)=1-\sum_{k=y}^{+\infty}(1-\frac{\lambda}{n})^{nk-1}\frac{\lambda}{n}=1-(1-\frac{\lambda}{n})^{ny-1}$，$F(y)=1-e^{-\lambda y}$，$y>0$

$\lim_{n\to\infty}F_n(y)=\lim_{n\to\infty}(1-(1-\frac{\lambda}{n})^{ny-1})=1-e^{-\lambda y}=F(y)$

$\therefore Y_n\xrightarrow{L}Y$

45. X 记事件A发生次数，$X\sim B(500,0.2)$，$EX=100$，$Var(X)=80$

切比雪夫不等式：$P(|X-\mu|\ge\varepsilon)\le\frac{Var(X)}{\varepsilon^2}$，取 $\varepsilon=20$ 有 $P(80\le X\le 100)=P(|X-100|\le 20)\ge 1-\frac{80}{20^2}=0.8$

中心极限定理：$P(80\le X\le 120)\approx\phi(\frac{120-100}{\sqrt{80}})-\phi(\frac{80-100}{\sqrt{80}})=2\phi(\sqrt{5})-1\approx 0.97$

46. 令 $S_n=\sum_{i=1}^{n}X_i^2$，$ES_n=n\alpha_2$，$Var\,S_n=n\,Var(X_1^2)=n(\alpha_4-\alpha_2^2)$

由中心极限定理，$\frac{S_n-n\alpha_2}{\sqrt{n(\alpha_4-\alpha_2^2)}}\xrightarrow{L}N(0,1)$

渐近分布 $N(n\alpha_2,\ n(\alpha_4-\alpha_2^2))$

47. 设X为零件总质量，$EX=2500$，$VarX=50$

由中心极限定理，$P(X>2510)=1-P(X\le 2510)=1-\phi(\frac{2510-2500}{\sqrt{50}})=1-\phi(\sqrt{2})\approx 0.0787$

49. 设X为200次取款总额，$EX=1100$，$VarX=1650$

由中心极限定理，$P(X\le S)=\phi(\frac{S-1100}{\sqrt{1650}})\ge 0.95$，解得 $S\ge 1166.8$

该取款机应至少存入1167(百元)才能保证0.95概率不出现余额不足