第二章：随机信号与系统：

1. 随机信号（序列）通过线性时不变系统：时域频域的分析方法的关系

$$y(t)=x(t)*h(t)=\int_{-\infty}^{\infty}h(\tau)x(t-\tau)d\tau，Y(\omega)=H(j\omega)X(\omega)$$

$$S_y(\omega)=\int_{-\infty}^{\infty}R_y(\tau)e^{-j\omega\tau}d\tau=\left|H(j\omega)\right|^2 S_x(\omega)$$

2. 平稳随机序列的参数模型：ARMA 模型，AR 模型，MA 模型：自相关函数，功率谱密度函数，三种模型之间的联系，模型的建立

$$y(n)=\sum_{k=0}^{q}b_k w(n-k)-\sum_{k=1}^{p}a_k y(n-k) \qquad \text{滑动平均分量+自回归分量}$$

第三章：信号检测：判断是否存在信号或者存在哪个信号问题：假设检验问题处理

1. 几种准则下的判决规则都具有如下似然比检验形式：$\lambda(x)=\dfrac{f(x|H_1)}{f(x|H_0)}\underset{H_0}{\overset{H_1}{\gtrless}}th$

最大后验概率准则：$P(H_1\,|\,x)\underset{H_0}{\overset{H_1}{\gtrless}}P(H_0\,|\,x)$，$th=\dfrac{P(H_0)}{P(H_1)}$

最小错误概率准则：$\bar{P}_e=P(H_0)P(D_1|H_0)+P(H_1)P(D_0|H_1)\xrightarrow{R0,R1}\min$，$th=\dfrac{P(H_0)}{P(H_1)}$

$$P(D_1|H_0)=\int_{R_1}f(x|H_0)dx$$

贝叶斯平均风险最小准则：

$$\bar{C}=P(H_0)\left[C_{00}P(D_0|H_0)+C_{10}P(D_1|H_0)\right]+P(H_1)\left[C_{01}P(D_0|H_1)+C_{11}P(D_1|H_1)\right]\to\min$$

$$th=\frac{P(H_0)(C_{10}-C_{00})}{P(H_1)(C_{01}-C_{11})}$$

极小极大准则：

$$\bar{C}(p,p_1)=C_{00}\,p+C_{11}(1-p)+(C_{10}-C_{00})\alpha(p_1)p+(C_{01}-C_{11})\beta(p_1)(1-p)\ge\bar{C}(p,p)=\bar{C}_{\min}(p)$$

$$p_1=\arg\max_{p}\bar{C}_{\min}(p), \qquad th=\frac{p_1(C_{10}-C_{00})}{(1-p_1)(C_{01}-C_{11})}$$

纽曼-皮尔逊准则： $\min P(D_0\,|\,H_1)\quad s.t.\ P(D_1\,|\,H_0)=\alpha$，$th$ 由给定的虚警概率确定。

2. 对于 M 种假设的假设检验问题：

$$\bar{C}=\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}C_{ij}P(D_i|H_j)P(H_j)\xrightarrow{R_0,R_1,\cdots,R_{M-1}}\min$$

当$C_{ii}=0, C_{ij}=1, i\neq j$：似然比检验形式$\lambda\left(x\right)=\dfrac{f\left(x\middle|H_i\right)}{f\left(x\middle|H_j\right)}\overset{H_i}{\geq}\dfrac{P\left(H_j\right)}{P\left(H_i\right)}$，$j=0,\cdots,M-1,\neq i$

3. 多样本的假设检验问题：

$$\lambda(\mathbf{x})=\frac{f(\mathbf{x}|H_1)}{f(\mathbf{x}|H_0)}=\frac{f\left(x_1,x_2,\cdots x_N\middle|H_1\right)}{f\left(x_1,x_2,\cdots x_N\middle|H_0\right)}\underset{H_0}{\overset{H_1}{\gtrless}}\frac{P(H_0)(C_{10}-C_{00})}{P(H_1)(C_{01}-C_{11})}=th$$

4. 判决性能的计算

根据判决规则，确定检验统计量，原则是检验统计量的分布特性好分析（即两种假设下的概率密度函数）。如$P\left(D_1\middle|H_1\right)=\int_{th}^{+\infty}f\left(\lambda\middle|H_1\right)d\lambda$

5. 复合的假设检验问题

$$\lambda\left(\mathbf{x}\right)=\frac{f(\mathbf{x}|H_1)}{f(\mathbf{x}|H_0)}=\frac{\int_{(\boldsymbol{\Theta})}f\left(\mathbf{x}\middle|\boldsymbol{\Theta},H_1\right)f_1\left(\boldsymbol{\Theta}\right)d\boldsymbol{\Theta}}{\int_{(\boldsymbol{\Phi})}f\left(\mathbf{x}\middle|\boldsymbol{\Phi},H_0\right)f_0\left(\boldsymbol{\Phi}\right)d\boldsymbol{\Phi}}\underset{H_0}{\overset{H_1}{\gtrless}}\frac{\left(C_{10}-C_{00}\right)P\left(H_0\right)}{\left(C_{01}-C_{11}\right)P\left(H_1\right)}=th$$

6．高斯白噪声中已知信号的检测： 根据似然比判决设计最佳最佳接收机，计算系统性能。

$$f\left(x\left(t\right)\middle|H_i\right)=\lim_{N\to\infty}f\left(x_1,x_2,\cdots,x_N\middle|H_i\right)=F\exp\left\{-\frac{1}{N_0}\int_0^T\left[x\left(t\right)-s_i\left(t\right)\right]^2dt\right\}$$

$$\lambda\left(x\left(t\right)\right)=\underset{H_0}{\overset{H_1}{\gtrless}}th\Rightarrow\int_0^T\left[s_1\left(t\right)-s_0\left(t\right)\right]x\left(t\right)dt\underset{H_0}{\overset{H_1}{\gtrless}}\frac{N_0}{2}\ln th+\frac{1}{2}\int_0^T\left[s_1^2\left(t\right)-s_0^2\left(t\right)\right]dt$$

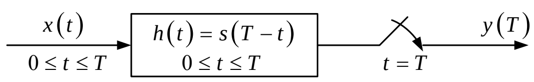
通信接收机（最小错误概率准则）：

$$P_e=1-\Phi\left(\sqrt{\left(1-\rho\right)E/N_0}\right),\quad \rho=\frac{1}{E}\int_0^T s_0\left(t\right)s_1\left(t\right)dt,\quad E=\frac{1}{2}\left(E_0+E_1\right)$$

雷达接收机（NP 准则）：$P_D=1-\Phi\left(\Phi^{-1}\left(1-\alpha\right)-\sqrt{2E_1/N_0}\right)$

7. 匹配滤波器

$t=t_0$时刻的瞬时输出信噪比$SNR_o=\dfrac{\left|s_o\left(t_0\right)\right|^2}{E\left\{n_o^2\left(t\right)\right\}}\leq\dfrac{E}{N_0/2}$，$H\left(j\omega\right)=CS^*\left(\omega\right)e^{-j\omega t_o}$等号成立。此时$s_o\left(t_0\right)=\int_{-\infty}^{\infty}\left|S\left(\omega\right)\right|^2d\omega\Big/2\pi$达到峰值。$t_0\geq T$。$y\left(T\right)=\int_0^T x\left(t\right)s\left(t\right)dt$

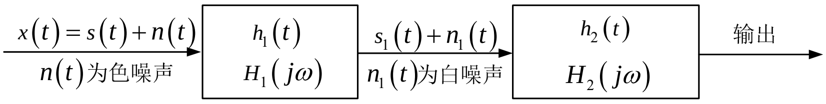


8. 信号的分集接收：设计最佳最佳接收机，计算系统性能

$$\lambda\left(\mathbf{x}(t)\right)=\frac{f\left(x_1(t),\cdots,x_M(t)\middle|H_1\right)}{f\left(x_1(t),\cdots,x_M(t)\middle|H_0\right)}=\prod_{i=1}^{M}\frac{f\left(x_i(t)\middle|H_1\right)}{f\left(x_i(t)\middle|H_0\right)}=\prod_{i=1}^{M}\lambda\left(x_i(t)\right)\underset{H_0}{\overset{H_1}{\gtrless}}th$$

9. 高斯色噪声中已知信号的检测

预白化方法：



$$H(j\omega)=H_1(j\omega)H_2(j\omega)=\frac{1}{S_n^+(\omega)}\frac{S^*(\omega)}{S_n^-(\omega)}e^{-j\omega T}=\frac{S^*(\omega)}{S_n(\omega)}e^{-j\omega T}$$

卡亨南-洛维展开：$\int_0^T R_n(t_1-t_2)f_j(t_2)dt_2=\lambda_j f_j(t_1)$，$0\le t_1\le T$

实信号下，$x(t)=\sum_k x_k f_k(t)$，$x_k=\int_0^T x(t)f_k(t)dt$，$x_k$间互不相关。

$$\lambda\left(x(t)\right)=\lim_{N\to\infty}\lambda\left(x_1,x_2,\cdots,x_N\right)\underset{H_0}{\overset{H_1}{\gtrless}}th$$

10. 高斯白噪声中随机相位信号的检测

$$\lambda\left(x(t)\right)=\frac{f\left(x(t)\middle|H_1\right)}{f\left(x(t)\middle|H_0\right)}=\frac{\int_0^{2\pi}f\left(x(t)\middle|\theta,H_1\right)f(\theta)d\theta}{f\left(x(t)\middle|H_0\right)}=\exp\left(-\frac{A^2T}{2N_0}\right)I_0\left(\frac{2Aq}{N_0}\right)\underset{H_0}{\overset{H_1}{\gtrless}}th$$

判决规则：$q=\sqrt{a^2+b^2}\underset{H_0}{\overset{H_1}{\gtrless}}th'$ ------门限th'由α确定（NP 准则）

$$a=q\sin\theta_0=\int_0^T x(t)\cos\omega_c t dt$$
$$b=q\cos\theta_0=\int_0^T x(t)\sin\omega_c t dt$$

正交接收机

计算性能：$f\left(a,b\middle|\theta,H_i\right)\to f\left(q,\theta_0\middle|\theta,H_i\right)\to f\left(q\middle|\theta,H_i\right)\to f\left(q\middle|H_i\right)$

第五章：参量估计

1. 几种估计准则

最大后验概率估计：$\hat{\theta}_{MAP}=\underset{\theta}{\arg\max}\, f(\theta\,|\,\mathbf{x})=\underset{\theta}{\arg\max}\ln f(\theta\,|\,\mathbf{x})$

最大似然估计：$\hat{\theta}_{ML}=\underset{\theta}{\arg\max}\, f(\mathbf{x}\,|\,\theta)=\underset{\theta}{\arg\max}\ln f(\mathbf{x}\,|\,\theta)$

最小均方误差估计：$E\left\{e^2\left(\hat{\theta}\right)\right\}=\int\limits_{(\theta)}\int\limits_{(\mathbf{x})}\left(\theta-\hat{\theta}\right)^2 f\left(\theta,\mathbf{x}\right)d\theta d\mathbf{x}\to\min$，$\hat{\theta}_{MS}=\int\limits_{(\theta)}\theta f\left(\theta\,|\,\mathbf{x}\right)\mathrm{d}\theta$

线性最小均方误差估计：$E\left\{\left[\theta-\left(a+\sum_{k=1}^{N}b_k x_k\right)\right]^2\right\}\to\min$

$$\hat{\theta}_{LMS}=a+\mathbf{b}^T\mathbf{x}=E\left\{\theta\right\}+Cov\left\{\theta,\mathbf{x}\right\}Cov^{-1}\left\{\mathbf{x},\mathbf{x}\right\}\left[\mathbf{x}-E\left\{\mathbf{x}\right\}\right]$$

正交条件（充要）：$E\left\{\left(\theta-\hat{\theta}_{LMS}\right)\mathbf{x}^T\right\}=0$

最小平均绝对误差估计：$E\left\{\left|\mathrm{e}\left(\hat{\theta}\right)\right|^2\right\}\to\min$，$\int_{-\infty}^{\hat{\theta}_{ABS}}f\left(\theta\middle|\mathbf{x}\right)d\theta=\int_{\hat{\theta}_{ABS}}^{\infty}f\left(\theta\middle|\mathbf{x}\right)d\theta$

贝叶斯估计：$E\left\{c\left(\hat{\theta}\right)\right\}\to\min$

最小二乘估计：线性观测模型下$\hat{\theta}_{LS}=\underset{\hat{\theta}}{\arg\min}\left[\mathbf{x}-\mathbf{H}\hat{\boldsymbol{\theta}}\right]^T\left[\mathbf{x}-\mathbf{H}\hat{\boldsymbol{\theta}}\right]$，

$$\hat{\boldsymbol{\theta}}_{LS}=\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{x}=\mathbf{H}^{\#}\mathbf{x}$$

2. 多参量估计

如：$\left[\dfrac{\partial}{\partial\theta_i}\ln f\left(\boldsymbol{\theta}\,|\,\mathbf{x}\right)\right]\Bigg|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_{MAP}}=0$，$\left[\dfrac{\partial}{\partial\theta_i}\ln f\left(\mathbf{x}\,|\,\boldsymbol{\theta}\right)\right]\Bigg|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_{ML}}=0$，$i=1,2,\cdots,M$

$$\hat{\boldsymbol{\theta}}_{LS}=\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{x}=\mathbf{H}^{\#}\mathbf{x}$$

3. 计算估计性能：$E\left\{\hat{\theta}\right\},E\left\{\left(\theta-E\{\hat{\theta}\}\right)^2\right\},E\left\{(\theta-\hat{\theta})^2\right\}$

无偏性：$E\left\{\hat{\boldsymbol{\theta}}\right\}=\boldsymbol{\theta}$或$E\left\{\hat{\boldsymbol{\theta}}\right\}=E\left\{\boldsymbol{\theta}\right\}$

有效性：无偏估计量的均方误差达到最小。对于确定单参量无偏估计量$\hat{\theta}$，此时有

$$E\left\{\left(\hat{\theta}-\theta\right)^2\right\}=\left\{E\left\{\left[\frac{\partial}{\partial\theta}\ln f\left(\mathbf{x}\,|\,\theta\right)\right]^2\right\}\right\}^{-1}=-\left\{E\left\{\frac{\partial^2}{\partial\theta^2}\ln f\left(\mathbf{x}\,|\,\theta\right)\right\}\right\}^{-1}$$

第六章：波形估计

1. 连续维纳滤波器

$$E\left\{e^2\left(t\right)\right\}=E\left\{\left[g\left(t\right)-y\left(t\right)\right]^2\right\}=E\left\{\left[g\left(t\right)-\int_{-\infty}^{\infty}h\left(\tau\right)x\left(t-\tau\right)d\tau\right]^2\right\}\xrightarrow{h(t)}\min$$

线性最小均方误差估计的正交条件：$E\left\{e(t)x(\tau')\right\}=0,\ \begin{cases}-\infty<\tau'<\infty & \text{非因果}\\ -\infty<\tau'\le t & \text{因果}\end{cases}$

维纳—霍夫方程：$R_{gx}(\eta)=\int_{-\infty}^{\infty}h(\lambda)R_x(\eta-\lambda)d\lambda,\ \begin{cases}-\infty<\eta<\infty & \text{非因果}\\ 0\le\eta<\infty & \text{因果}\end{cases}$

求解：$H(j\omega)=\dfrac{S_{gx}(\omega)}{S_x(\omega)}$ -------物理不可实现

$$H(s)=\frac{1}{S_x^+(s)}\left[\frac{S_{gx}(s)}{S_x^-(s)}\right]^+ \text{------ 物理可实现}$$

$$E\left\{e^2(t)\right\}_{\min}=E\left\{e(t)g(t)\right\}=R_g(0)-\int_{-\infty}^{\infty}h(\lambda)R_{gx}(\lambda)d\lambda$$

2. 离散维纳滤波器

$$E\left\{e^2(k)\right\}=E\left\{\left[g(k)-y(k)\right]^2\right\}=E\left\{\left[g(k)-\sum_{i=-\infty}^{\infty}h(i)x(k-i)\right]^2\right\}\to\min$$

线性最小均方误差估计的正交条件：$E\left\{e(k)x(j)\right\}=0,\ \begin{cases}-\infty<j<\infty & \text{非因果}\\ -\infty<j\le k & \text{因果}\end{cases}$

维纳—霍夫方程：$\sum_{i=-\infty}^{+\infty}h(i)R_x(l-i)=R_{gx}(l),\begin{cases}-\infty<l<\infty & \text{非因果}\\ 0\le l<\infty & \text{因果}\end{cases}$

求解：$H(z)=\dfrac{S_{gx}(z)}{S_x(z)}$ -------物理不可实现

$$H(z)=\frac{1}{S_x^+(z)}\left[\frac{S_{gx}(z)}{S_x^-(z)}\right]^+ \text{------ 物理可实现}$$

对应有限样本的维纳滤波器：$\sum_{i=0}^{N-1}h(i)R_x(l-i)=R_{gx}(l),l=0,1,\cdots,N-1$

$$\mathbf{R}_x\mathbf{h}=\mathbf{r}_{gx},\quad \mathbf{h}=\mathbf{R}_x^{-1}\mathbf{r}_{gx}$$