28、35、37、38、40、42

28. (1) $\cancel{C_n^n}$ $C_n^m p^m (1-p)^{n-m}$

(2)

$$p(X=n, Y=m) = p(Y=m \mid X=n)\, p(X=n) = C_n^m p^m (1-p)^{n-m} \frac{\lambda^n}{n!} e^{-\lambda}$$

35 (1) $$\int_0^\infty \int_0^\infty A e^{-(3x+4y)} dx\, dy = 1$$

$$\int_0^\infty \int_0^\infty e^{-3x} dx \cdot e^{-4y} dy = \int_0^\infty \frac{1}{3} e^{-4y} dy = \frac{1}{12}$$

$A = 12$

(2) $$f(x) = 12\int_0^\infty e^{-(3x+4y)} dy = 12 e^{-3x} \cdot \int_0^\infty e^{-4y} dy = 3e^{-3x}$$

$$f(y) = 12\int_0^\infty e^{-(3x+4y)} dx = 12 e^{-4y} \int_0^\infty e^{-3x} dx = 4e^{-4y}$$

$\Rightarrow f(x,y) = f(x) f(y)$ ∴独立

(3) $\cancel{f_Z(z) = \int_0^z \int_0^y 4e^{-4y} dy}$

$$\begin{aligned} f_Z(z) &= \int_0^z f_X(x) f_Y(z-x) dx \\ &= 12\int_0^z e^{-3x} e^{-4(z-x)} dx \\ &= 12\int_0^z e^{-4z+x} dx \\ &= 12 e^{-4z} \int_0^z e^x dx \\ &= 12 e^{-4z} (e^z - 1) \\ &= 12 e^{-3z} - 12 e^{-4z} \end{aligned}$$

(4) $$p(X>0.5 \mid X+Y=1) = \frac{p(X>0.5,\ X+Y=1)}{p(X+Y=1)} = \frac{p(X>0.5,\ 1-\varepsilon < X+Y < 1+\varepsilon)}{p(1-\varepsilon < X+Y \le 1+\varepsilon)}$$

$$= \frac{\int_{0.5}^1 f_X(x) f_Y(1-x) dx \cdot \Delta\varepsilon}{f_Z(1)\Delta\varepsilon} = \frac{12e^{-4}(e - e^{\frac{1}{2}})}{12e^{-4}(e-1)} = \frac{e-\sqrt{e}}{e-1}$$

37.

(1) $f_{Y|X=\frac{1}{2}}(y) = \dfrac{f(\frac{1}{2}, y)}{f_X(\frac{1}{2})}$

$f_X(x) = \int_{-1}^{1} \frac{1}{4}(1+xy)\,dy = \frac{1}{2} + \frac{1}{4}x\int_{-1}^{1} y\,dy = \frac{1}{2}$

$f_{Y|X=\frac{1}{2}}(y) = \dfrac{\frac{1}{4}(1+\frac{1}{2}y)}{\frac{1}{2}} I_{|y|<1} = \frac{1}{2}(1+\frac{1}{2}y) I_{|y|<1}$

(2) $f_X(|x|) = 1\, I_{|x|<1}$ $\quad \therefore P(X^2 \le t) = P(|x| \le \sqrt{t}) = \sqrt{t} = \int_0^t \frac{1}{2\sqrt{t}}\,dt$

$\Rightarrow f_{X^2}(t) = \frac{1}{2\sqrt{t}}$ 同理 $f_{Y^2}(t) = \frac{1}{2\sqrt{t}}$

$f_{|X|,|Y|}(x,y) = 1 + xy.$

$P(X^2 \le t_1, Y^2 \le t_2) = P(|X| \le \sqrt{t_1}, |Y| \le \sqrt{t_2}) = \cancel{\int_0^{\sqrt{t_1}}\int_0^{\sqrt{t_2}} (1+xy)\,dx\,dy}$

$\cancel{= \int_0^{\sqrt{t_1}} (\sqrt{t_2} + \frac{1}{2}t_2 y)\,dy = \sqrt{t_1 t_2} + \frac{1}{4}t_2 t_1}$

$= \int_{-\sqrt{t_1}}^{\sqrt{t_1}} \int_{-\sqrt{t_2}}^{\sqrt{t_2}} \frac{1}{4}(1+xy)\,dy\,dx$

$= \int_{-\sqrt{t_1}}^{\sqrt{t_1}} \frac{1}{2}\sqrt{t_2}\,dx = \sqrt{t_1 t_2} = P(X^2 \le t_1)\cdot P(Y^2 \le t_2) \Rightarrow$ 独立.

38.

(1)

Y \ X	0	1
-1	0	$\frac{1}{3}$
0	$\frac{1}{3}$	0
1	0	$\frac{1}{3}$

(2) $P(Z=0) = \frac{1}{3}$

$P(Z=-1) = \frac{1}{3}$

$P(Z=1) = \frac{1}{3}$

40.

(1) $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^x 3x\,dy = 3x^2$

$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_y^1 3x\,dx = \frac{3}{2}(1-y^2)$

(2) 否

(3) $P(X+Y \le 1) = \int_0^{\frac{1}{2}}\int_0^x 3x\,dy\,dx + \int_{\frac{1}{2}}^1 \int_0^{1-x} 3x\,dy\,dx$

$= \int_0^{\frac{1}{2}} 3x^2\,dx + \int_{\frac{1}{2}}^1 3x(1-x)\,dx$

$= \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

42.

$$f_X(x) = \int_0^{2\pi}\int_0^{2\pi} \frac{1}{8\pi^3}(1-\sin x\sin y\sin z)\,dy\,dz$$

$$= \frac{1}{2\pi} - \frac{1}{8\pi^3}\sin x \cdot \int_0^{2\pi}\int_0^{2\pi} \sin y\sin z\,dy\,dz$$

$$= \frac{1}{2\pi} = f_Y(y) = f_Z(z)$$

$$f_{Y,Z}(y,z) = \int_0^{2\pi} \frac{1}{8\pi^3}(1-\sin x\sin y\sin z)\,dx = \frac{1}{4\pi^2} = f_Y(y)f_Z(z).$$

但 $f_X(x)f_Y(y)f_Z(z) = \frac{1}{8\pi^3} \neq f(x,y,z)$