第六次作业

第三章　2. (1). $P(X=1 \mid Z=0) = \dfrac{P(X=1, Y=1, Z=0)}{P(X=2, Z=0) + P(Y=2, Z=0) + P(X=1, Y=1, Z=0)} = \dfrac{4}{9}$

(2). $f(x,y) = \dfrac{1}{4} C_2^x \left(\dfrac{1}{3}\right)^x \left(\dfrac{2}{3}\right)^y$, $x,y \in N$ 且 $0 \le x+y \le 2$.

4.

(X_1, X_2)	$(1,1)$	$(1,0)$	$(0,1)$	$(0,0)$
P	0	$\frac{4}{5}$	$\frac{1}{10}$	$\frac{1}{10}$

6. (1) $P(X=i, Y=j) = P^2(1-p)^{j-2}$, $j=2,\cdots$, $i=1,\cdots,j-1$.

(2). $P(Y=j) = \sum\limits_{x=1}^{j-1} P(X=x, Y=j) = (j-1)p^2(1-p)^{j-2}$

$P(X=i) = \sum\limits_{j=2}^{\infty} P(X=i, Y=j) = P(1-p)^{i-1}$

11. (1) $f_X(x) = xe^{-x}$, $x>0$ $\Rightarrow$ $f_{Y|X}(y|x) = \dfrac{1}{x}$, $0<y<x$.

(2) $P(X \le 1 \mid Y \le 1) = \dfrac{P(X \le 1, Y \le 1)}{P(Y \le 1)} = \dfrac{\int_0^1 \int_0^x e^{-x} dy\, dx}{\int_0^1 \int_y^{\infty} e^{-x} dx\, dy} = \dfrac{e-2}{e-1}$

17. (1) $f(x,y) = f_X(x) \cdot f_{Y|X}(y|x) = \begin{cases} \dfrac{9y^2}{x} , & 0<y<x<1 \\ 0 , & \text{其他}. \end{cases}$

(2) $f_Y(y) = \int_y^1 f(x,y)\, dx = -9y^2 \ln y$, $0<y<1$

22. (1) $\int_0^1 \int_0^x f(x,y)\, dy\, dx = 1 \Rightarrow f(x,y) = 2$, $0 \le x \le y \le 1$

(2). $f_1(x) = \int_x^1 f(x,y)\, dy = 2(1-x)$. $0 \le x \le 1$, $\qquad f_2(y) = \int_0^y f(x,y)\, dx = 2y$, $0 \le y \le 1$

(3). $f_{X|Y}(x|y) = \dfrac{f(x,y)}{f_2(y)} = \dfrac{1}{y}$, $0 \le x \le y$, $0 \le y \le 1$, $\qquad$ (4). $P(X \le 0.5 \mid Y=y) = \dfrac{\int_0^{0.5 \wedge y} f(x,y)\, dx}{f_2(y)} = \dfrac{y \wedge 0.5}{y}$

25. (1) $f(X_1=x_1, \cdots, X_n=x_n) = \dfrac{m!}{x_1! \cdots x_n!} \prod\limits_{k=1}^n P_k^{x_k}$, $\sum\limits_{k=1}^n x_k = m$

(2). $f(X_k=x_k) = \dfrac{m!}{x_k!(m-x_k)!} P_k^{x_k}(1-P_k)^{m-x_k}$

(3). $f(X_1=x_1, X_2=x_2) = \dfrac{m!}{x_1! x_2! (m-x_1-x_2)!} P_1^{x_1} P_2^{x_2}(1-P_1-P_2)^{m-x_1-x_2}$

(4). $f(X_2=x_2, \cdots, X_n=x_n \mid X_1=x_1) = \dfrac{(m-x_1)!}{x_2! \cdots x_n!} \cdot \dfrac{P_2^{x_2} \cdots P_n^{x_n}}{(1-P_1)^{m-x_1}}$