四 42.. 44 45 46 48 52

五 4 8 16 17

42. $P(|X_n-X|>\varepsilon)\to 0$ $P(|Y_n-Y|>\varepsilon)\to 0$

$P(|X_n+Y_n-X-Y|>\varepsilon)\le P(|X_n-X|<\frac{\varepsilon}{2})+P(|Y_n-Y|<\frac{\varepsilon}{2})$

$0\le \lim_{n\to\infty} P(|X_n+Y_n-X-Y|>\varepsilon)\le \lim_{n\to\infty} P(|X_n-X|<\frac{\varepsilon}{2})+\lim_{n\to\infty} P(|Y_n-Y|<\frac{\varepsilon}{2})$

$=0$

$\Rightarrow X_n+Y_n \xrightarrow{P} X+Y$.

44. (1) $P(X_n+Y_n\le t)=P(X_n+c+Y_n-c\le t)$

$=P(X_n+c+Y_n-c\le t, |Y_n-c|>\varepsilon)$

$+P(X_n+c+Y_n-c\le t, |Y_n-c|\le\varepsilon)$

$P(X_n+c+Y_n-c\le t, |Y_n-c|>\varepsilon)\le P(|Y_n-c|>\varepsilon)\to 0$

$P(X_n+c\le t-\varepsilon)\le P(X_n+c\le t-(Y_n-c), |Y_n-c|\le\varepsilon)\le P(X_n+c\le t+\varepsilon)$

$\therefore \lim_{n\to\infty} P(X_n+c\le t-\varepsilon)\le\lim_{n\to\infty} P(X_n+Y_n\le t)\le\lim_{n\to\infty} P(X_n+c\le t+\varepsilon)$

令 $\varepsilon\to 0$. 有 $P(X_n+Y_n\le t)\to P(X+c\le t)$

(2) $P(X_nY_n\le t)=P(X_nY_n\le t, |c-Y_n|>\varepsilon)+P(X_nY_n\le t, |c-Y_n|\le\varepsilon)$

$P(X_nY_n\le t, |c-Y_n|>\varepsilon)\le P(|c-Y_n|>\varepsilon)\to 0$.

$P((\varepsilon+c)X_n\le t)\le P(X_nY_n\le t, |c-Y_n|\le\varepsilon)\le P((c-\varepsilon)X_n\le t)$

$\therefore \lim_{n\to\infty} P((\varepsilon+c)X_n\le t)\le\lim_{n\to\infty} P(X_nY_n\le t)\le\lim_{n\to\infty} P((c-\varepsilon)X_n\le t)$

令 $\varepsilon\to 0$. $P(X_nY_n\le t)\to P(cX\le t)$

(3) $P(\frac{X_n}{Y_n}\le t) = P(\frac{X_n}{Y_n}\le t, |c-Y_n|>\varepsilon) + P(\frac{X_n}{Y_n}\le t, |c-Y_n|\le\varepsilon)$

$P(\frac{X_n}{Y_n}\le t, |c-Y_n|>\varepsilon) \le P(|c-Y_n|>\varepsilon) \to 0.$

$\cancel{P(\frac{X_n}{Y_n}\le t, |c}$

$P(\frac{X_n}{c}\le \frac{t(c-\varepsilon)}{c}) \le P(\frac{X_n}{Y_n}\le t, |c-Y_n|\le\varepsilon) \le P(\frac{X_n}{c}\le\frac{t(c+\varepsilon)}{c})$

$\therefore \lim_{n\to\infty} P(\frac{X_n}{c}\le\frac{t(c-\varepsilon)}{c}) \le \lim_{n\to\infty} P(\frac{X_n}{Y_n}\le t) \le \lim_{n\to\infty} P(\frac{X_n}{c}\le\frac{t(c+\varepsilon)}{c})$

令 $\varepsilon\to 0$. $P(\frac{X_n}{Y_n}\le t) \to P(\frac{X}{c}\le t)$

45.

① 切比雪夫 $Var(S_n) = np(1-p) = 80$

$P(|S_n-100|\ge 20) \le \frac{Var(S_n)}{20^2} = \frac{80}{400} = \frac{1}{5}$.

$P(80\le S_n\le 120) \ge \frac{1}{5}$

② CLT

$P \approx \Phi(\frac{120-100}{\sqrt{80}}) - \Phi(\frac{80-100}{\sqrt{80}}) = \Phi(\sqrt{5}) - \Phi(-\sqrt{5})$

$= 0.975$

46. $EX_i^2 = \alpha_2$ $Var(X_i^2) = EX_i^4 - (EX_i^2)^2 = \alpha_4 - \alpha_2$.

CLT $\frac{\sum_{i=1}^n X_i^2 - n\alpha_2}{\sqrt{n(\alpha_4-\alpha_2)}} \to N(0,1)$

$\sum_{i=1}^n X_i^2 \to N(n\alpha_2, n(\alpha_4-\alpha_2))$

48

(1) $p = 1 - \Phi\left(\frac{85-90}{\sqrt{100\times0.1\times0.9}}\right) = 1-\Phi(-\frac{5}{3}) = \Phi(\frac{5}{3}) = 0.95$

(2) $\Phi\left(\frac{0.8n-0.9n}{\sqrt{0.09n}}\right) < 0.05$

$\frac{0.9n-0.8n}{\sqrt{0.09n}} > 1.65 \quad \frac{\sqrt{n}}{3} > 1.65 \quad n \geq 25$

52.

单个车索赔金额 $Z_i = \sum_{j=1}^{X_i} Y_j \quad X_i \sim P(2) \quad Y_j \sim U[1000,5000]$

$EZ_i = E(E(\sum_{j=1}^{X_i} Y_j | X_i)) = E(EY_j \cdot X_i) = EY_j \cdot EX_i = 3000\times2$
$= 6000$

条件方差公式 $var(\sum_{j=1}^{X_i} Y_j) = var(E(\sum_{j=1}^{X_i} Y_j | X_i)) + E(var(\sum_{j=1}^{X_i} Y_j | X_i))$

$= var(3000 X_i) + E(X_i \cdot \frac{1}{12} \cdot 4000^2)$

$= 3000^2 \times 2 + \frac{1}{12}\times4000^2\times2$

$= 2.07\times10^7$

CLT $\frac{\sum_{i=1}^{2400} Z_i - EZ_i}{\sqrt{2400\times2.07\times10^7}} \to N(0,1)$

~~1200~~ $2400\times5000 = 12000000$ 盈利200万 $\Rightarrow \sum_{i=1}^{2400} Z_i \leq 1000$万

$P\left(\frac{\sum_{i=1}^{2400} Z_i - EZ_i}{\sqrt{2400\, var Z_i}} \leq \frac{-4.4\times10^6}{\sqrt{2400\, var Z_i}}\right) \approx \Phi\left(\frac{-4.4\times10^6}{2.2\times10^5}\right) \approx \Phi(-19.7)$
≈ 0

4. 总体分布 $B(1,p)$ （注意书上或PPT上的定义）

抽样分布 $f = \binom{10}{\sum x_i} p^{\sum x_i}(1-p)^{10-\sum x_i} \quad B(10,p)$

8. (1) $\Omega = \{0,1\}^5$ 抽样分布 $f = p^{\sum x_i}(1-p)^{5-\sum x_i}\binom{5}{\sum x_i} \quad B(5,p$

(2) 统计量：$X_1+X_2 \quad \min_{1\leq i\leq 5} X_i$. 其它不是因为包含未知参数.

16. 不妨设 $X_1, X_2, \cdots X_9 \sim i.i.d\ N(0,1)$

$Y_1 \sim N(0, \frac{1}{6})$ $Y_2 \sim N(0, \frac{1}{3})$ 且 Y_1 与 Y_2 独立

$Y_1 - Y_2 \sim N(0, \frac{1}{6} + \frac{1}{3} = \frac{1}{2})$

$\sqrt{2}(Y_1 - Y_2) \sim N(0,1)$

$S^2 \sim \chi^2_{(2)}$ 且与 Y_1、Y_2 独立 $\Longrightarrow Z = \frac{\sqrt{2}(Y_1 - Y_2)}{S} \sim t_{(2)}$

17. $$Y = \frac{1}{2} \frac{\frac{X_1^2 + X_2^2 + \cdots + X_{10}^2}{4}}{\frac{X_{11}^2 + X_{12}^2 + \cdots + X_{15}^2}{4}} = \frac{1}{2} \frac{\chi^2_{10}}{\chi^2_5} = \frac{\frac{\chi^2_{10}}{10}}{\frac{\chi^2_5}{5}} \sim F_{10,5}$$