12, 13, 18, 20, 24, 30. 36, 40. 48.

12. $P(Y=y)=\sum_{y}^{\infty} C_n^y p^y(1-p)^{n-y}\cdot\frac{\lambda^n}{n!}e^{-\lambda}$ （全概率公式）

$=\sum_{y}^{\infty}\frac{\cancel{n!}}{(n-y)!\,y!}p^y(1-p)^{n-y}\frac{\lambda^n}{\cancel{n!}}e^{-\lambda}$

$=\sum_{y}^{\infty}\frac{p^y(1-p)^{n-y}}{y!\,(n-y)!}\lambda^n e^{-\lambda}$

$=\sum_{y}^{\infty}\frac{[\lambda(1-p)]^{n-y}(\lambda p)^y}{(n-y)!\quad y!}e^{-\lambda}$

$=\frac{(\lambda p)^y}{y!}\cdot\sum_{y}^{\infty}\frac{[\lambda(1-p)]^{n-y}}{(n-y)!}e^{-\lambda}$

$=\frac{(\lambda p)^y}{y!}e^{-\lambda p}\sim P(\lambda)$ 同理. $Z\sim P(1-\lambda)$

$P(Y=y,Z=z)=C_{y+z}^{y}p^y(1-p)^{\cancel{n-y}\,z}\frac{\lambda^{y+z}}{(y+z)!}e^{-\lambda}$

$=\frac{(y+z)!}{y!\;z!}p^y(1-p)^z\frac{\lambda^{y+z}}{(y+z)!}e^{-\lambda}$

$=\frac{(\lambda p)^y}{y!}e^{-\lambda p}\cdot\frac{[\lambda(1-p)]^z}{z!}e^{-\lambda(1-p)}=P(Y=y)P(Z=z)$

∴ 独立

13. $X_{poi}\sim B(1000,0.001)$ $\lambda=1000\times0.001=1.$

$B(1000,0.001)\sim P(1)$ $P(X_{poi}=0)\approx\frac{1}{e}$

~~14.~~ 18. (1) $P(X=1)=P(X\le1)-P(X<1)=\frac{1}{2}+\frac{1-1}{4}-\frac{1}{4}=\frac{1}{4}$

$P(X=2)=P(X\le2)-P(X<2)=\frac{5}{6}-(\frac{1}{2}+\frac{2-1}{4})=\frac{5}{6}-\frac{3}{4}=\frac{10-9}{12}=\frac{1}{12}$

$P(X=3)=P(X\le3)-P(X<3)=1-\frac{5}{6}=\frac{1}{6}$

(2) $P(\frac{1}{2}<X<\frac{3}{2})=P(X<\frac{3}{2})-P(X\le\frac{1}{2})=P(X\le\frac{3}{2})-P(X\le\frac{1}{2})$

$=\frac{1}{2}+\frac{\frac{3}{2}-1}{4}-\frac{\frac{1}{2}}{4}=\frac{1}{2}+\frac{1}{8}-\frac{1}{8}=\frac{1}{2}.$

20. $\int_1^2 ax\,dx = \int_2^3 b\,dx$ $\quad \frac{1}{2}a(4-1) = b \quad b=\frac{3}{2}a$

$\int_1^2 ax\,dx + \int_2^3 b\,dx = 1 \Rightarrow b=\frac{1}{2} \Rightarrow a=\frac{1}{3},\ b=\frac{1}{2}.$

24. $\Delta = X^2-4 \geq 0 \Rightarrow \cancel{-2\le X\le 2}\ |X|\geq 2 \Rightarrow p=\frac{6}{10}=\frac{3}{5}$

30. (1) 必要性. $p(X\le t+x \mid X>t) = \dfrac{p(t<X\le t+x)}{p(X>t)} = \dfrac{p(X\le t+x)-p(X\le t)}{1-p(X\le t)}$

$= \dfrac{1-e^{-\lambda(t+x)}-1+e^{-\lambda t}}{e^{-\lambda t}} = 1-e^{-\lambda x} = p(X\le x)$

充分性：设 $p(X>t)=G(t)$. $\quad \dfrac{G(t)-G(t+x)}{G(t)} = 1-G(x)$

$\Rightarrow G(t)G(x)=G(t+x) \Rightarrow G(t)=e^{-\lambda t}$.

(2) 必要性：$p(X\le m+n \mid X>n) = \dfrac{p(n<X\le m+n)}{p(X>n)} = \dfrac{\sum_{n+1}^{m+n} p(1-p)^{1-i}}{\sum_{n+1}^{\infty} p(1-p)^{1-i}} =$

$\dfrac{\dfrac{p(1-p)^{-n}-p(1-p)^{1-n-m}}{1-(1-p)}}{\dfrac{p(1-p)^{-n}}{1-(1-p)}} = 1-(1-p)^{-m} = \dfrac{p-p(1-p)^{-m}}{p} = p(X\le m)$

充分性：设 $p(X=i)=p_i$ $\quad \dfrac{\sum_{n+1}^{m+n} p_i}{\sum_{n+1}^{\infty} p_i} = \sum_1^m p_i \quad \sum_{n+1}^{m+n} p_i = (1-\sum_1^n p_i)\sum_1^m p_i$

取 $m=n=1$ 得 $p_2=p_1(1-p_1)$

$m=1$. $\quad p_{n+1}=p_1(1-\sum_1^n p_i)$ 由归纳法易得 $p_i=p_1(1-p_1)^{i-1}$

36. (1)

3	1	-1	-3
0.2	0.3	0.1	0.4

(2)

0	1	2
0.3	0.3	0.4

(3)

4	1	0
0.2	0.7	0.1

40 (1) $f_{Y_1}(y)=\frac{1}{y}I_{[1,e]}$ (2) $f_{Y_2}(y)=\frac{1}{y^2}I_{(1,\infty)}$ (3) $x=e^{-\lambda y}$. $f_{Y_3}(y)=\lambda e^{-\lambda y}\cdot I_{(0,\infty)}$

48. (1) $\int_0^3 \frac{1}{a}x^2\,dx = \frac{1}{3a}\cdot 27 = \frac{9}{a} = 1 \Rightarrow a=9$

$p(X\le 1) = \int_0^1 \frac{1}{9}x^2\,dx = \frac{1}{27}$ $\quad p(X>2)=\int_2^3 \frac{1}{a}x^2\,dx = \frac{1}{27}(27-8) = \frac{19}{27}$

$1<y<2$ 时 $\cancel{p(Y\le y)}$ $p(1<Y\le y) = \int_1^y \frac{1}{9}y^2\,dy = \frac{1}{27}(y^3-1)$

$$F(y)=\begin{cases} 0 & y<1 \\ \frac{19}{27} & y=1 \\ \frac{y^3}{27}+\frac{2}{3} & 1\le y<2 \\ 1 & y\geq 2. \end{cases}$$

(2) $p(X\le Y) = p(X<2) = 1-\frac{19}{27} = \frac{8}{27}.$