☆ 几乎没人对…

学会分类讨论

标注个表示引起待提升

1.7 ① 设 $a+b>1$，则

$P(AB)=P(A)+P(B)-P(A\cup B)\geq a+b-1$

② $a+b\leq 1$ 则 $P(AB)\geq 0$

再由 $P(AB)\leq P(A)$，$P(AB)\leq P(B)$

得 $P(AB)\in[(a+b-1)^+,\ a\wedge b]$

1.12. 五局三胜利甲：$A:=\{$甲在五局三胜中胜$\}$ $B:=\{$甲在三局两胜中胜$\}$

$$P(A)=\binom{5}{3}p^3(1-p)^2+\binom{5}{4}p^4(1-p)+\binom{5}{5}p^5=p^3(6p^2-15p+10)$$

$$P(B)=\binom{3}{2}p^2(1-p)+\binom{3}{3}p^3=p^2(3-2p)$$

$$P(A)-P(B)=6p^5-15p^4+10p^3-3p^2+2p^3=3p^2(p-1)^2(2p-1)\geq 0$$

用猜根因式分解

1.13 乙有利：

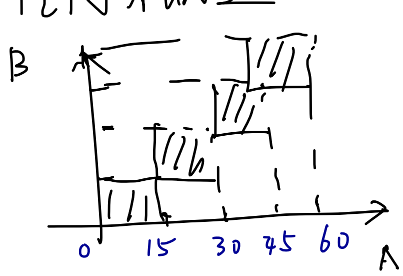
$A_k:=\{k\text{th}$ 盒摸白$\}$ $B_k:=\{k\text{th}$ 掷正$\}$ $k\geq 1$

$$P(\text{甲胜})=\sum_{k=1}^{\infty}P(A_k)=\sum_{k=1}^{\infty}\underbrace{P(\bar{B}_1\cdots\bar{B}_{k-1}B_k)}_{P(k\text{th 盒})}\underbrace{\frac{1}{k+1}}_{P(\text{摸白}\mid k\text{th 盒})}=\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^k\frac{1}{k+1}$$

$$<\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^k\frac{1}{2}=\frac{1}{2}$$

1.17 ☆ 几何概型

引起！



$$\frac{4}{16}=\frac{1}{4}$$

2.24

(1). $A:=\{$首抽一等$\}$，则

$$P(A)=\frac{1}{2}\times\frac{10}{50}+\frac{1}{2}\times\frac{18}{30}=\frac{2}{5}$$

(2). $B:=\{$2nd 抽一等$\}$ 则

$$P(B\mid A)=\frac{P(AB)}{P(A)}=\frac{\frac{1}{2}\times\frac{10}{50}\times\frac{9}{49}+\frac{1}{2}\times\frac{18}{30}\times\frac{17}{29}}{\frac{2}{5}}=0.4856$$

或 $\frac{690}{1421}$

2.25

$A_k := \{k\text{th 红}\}$ $B_k := \{k\text{th 黑}\}$ $k=1,2,3,4$

$$P(A_1A_2A_3A_4) = P(A_1)P(A_2|A_1)P(A_3|A_2A_1)P(A_4|A_3A_2A_1)$$

$$= \frac{r}{r+b}\cdot\frac{r+a}{r+a+b}\cdot\frac{b}{r+2a+b}\cdot\frac{a+b}{r+3a+b}$$

2.29 ☆ 错很多！大家要使用合适方法！观点 matters！

$A_0 := \{\text{前5全摸黑}\}$ $A_k := \{\text{前5摸}k\text{红}\}$ $k=0,1,2,\cdots,5$ $B := \{\text{无放回摸第6球红}\}$

则 $P(A_0|B) = \dfrac{P(A_0B)}{P(B)}$ $P(A_1|B) = \dfrac{P(A_1B)}{P(B)}$

$P(B) = \dfrac{20}{25} = \dfrac{4}{5}$ 每个位置都有5黑20红竞争

$P(A_0B) = \dfrac{1}{C_{25}^5} = \dfrac{1}{53130}$ 黑球排前5；选5个放黑

$P(A_1B) = \dfrac{C_5^4 C_{19}^1}{C_{25}^5} = \dfrac{95}{53130}$ 一红排前5，一红排第6；选5个放黑

$P(A_0|B) = \dfrac{1}{42504}$ $P(A_1|B) = \dfrac{95}{42504}$

从而 $P(A_0\cup A_1|B) = \dfrac{96}{42504} = \dfrac{12}{5313} = \dfrac{4}{1771}$

无交并

故 $P(A_2\cup A_3\cup A_4\cup A_5|B) = \dfrac{1767}{1771}$

2.33 (1) $A_k := \{\text{甲摸}k\text{个白}\}$ $k=0,1,2$

$B := \{\text{(换球后)乙摸白}\}$

$P(B) = \sum_{k=0}^{2} P(B|A_k)P(A_k)$

代入 $P(B|A_0) = \frac{4}{11}$

$P(B|A_1) = \frac{5}{11}$

$P(B|A_2) = \frac{6}{11}$

$P(A_2) = \dfrac{\binom{5}{2}}{\binom{7}{2}} = \dfrac{10}{21}$

$P(A_1) = \dfrac{\binom{5}{1}\binom{2}{1}}{\binom{7}{2}} = \dfrac{10}{21}$ 验证加和为1

$P(A_0) = \dfrac{\binom{2}{2}}{\binom{7}{2}} = \dfrac{1}{21}$

从而 $P(B) = \dfrac{114}{231} = \dfrac{38}{77}$

(2) $P(A_0|B) = \dfrac{P(A_0B)}{P(B)} = \dfrac{P(B|A_0)P(A_0)}{P(B)} = \dfrac{2}{57}$

从而 $P(\overline{A_0}|B) = \dfrac{55}{57}$