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六 39、42、44、52、53
七 1、4、11、16、23

39. (1) $EX=\int_0^\infty e^{-\frac{x^2}{\theta}}dx=\sqrt{\theta}\cdot\int_0^\infty e^{-x^2}dx=\sqrt{\theta}\cdot\frac{\sqrt{\pi}}{2}=\frac{\sqrt{\pi\theta}}{2}$

(2) $EX^2=\int_0^\infty 2xe^{-\frac{x^2}{\theta}}dx=\int_0^\infty e^{-\frac{x^2}{\theta}}dx^2=\theta$

$(E|X|^p=\int_0^\infty pt^{p-1}P(|X|>t)dt)$

(2) $f=F'=\frac{2x}{\theta}e^{-\frac{x^2}{\theta}}$

$l=\sum_{i=1}^n \ln\frac{2x_i}{\theta}-\frac{x_i^2}{\theta}$

$\frac{\partial l}{\partial\theta}=\sum_{i=1}^n\frac{x_i^2-\theta}{\theta^2}=0 \quad \hat\theta=\frac{1}{n}\sum_{i=1}^n x_i^2$

(3) 由大数定律. $\hat\theta\xrightarrow{P}\theta$

42 $EX=\frac{1}{p}$. $VarX=\frac{1-p}{p^2}$

$EX+varX=\frac{1}{p^2}$

$\therefore$ $\bar X$ 为 $\frac{1}{p}$ 的无偏估计，$\bar X+S$ 为 $\frac{1}{p^2}$ 的无偏估计

44 (1) $EX=\int_\theta^\infty x\cdot\frac{1}{\sigma}e^{-\frac{x-\theta}{\sigma}}dx=\int_\theta^\infty\frac{x-\theta+\theta}{\sigma}e^{-\frac{x-\theta}{\sigma}}d\frac{x-\theta}{\sigma}\sigma$

$=\sigma\cdot\int_0^\infty(x+\frac{\theta}{\sigma})e^{-x}dx=\sigma+\theta$

$\hat\theta_1=\frac{1}{n}\sum_{i=1}^n X_i-\sigma$

$f=\prod_{i=1}^n\frac{1}{\sigma}e^{-\frac{x_i-\theta}{\sigma}}I_{x_i>0}\Rightarrow l=\sum_{i=1}^n -\ln\sigma-\frac{x_i-\theta}{\sigma}-\ln I_{x_i>\theta}$ 关于 $\theta\uparrow$

$\hat\theta_2=\min X_i$

(2) ~~由大数律~~ $E\hat\theta_1=\frac{1}{n}E(\sum_{i=1}^n X_i)-\sigma=\theta$ 无偏

$P(\hat\theta_2>x)=p(X_1>x,\cdots X_n>x)=(\int_t^\infty\frac{1}{\sigma}e^{-\frac{x-\theta}{\sigma}}dx)^n=e^{-n(\frac{x-\theta}{\sigma})}$

$E\hat\theta_2=\int_\theta^\infty e^{-n(\frac{x-\theta}{\sigma})}dx\ \cancel{=\frac{\sigma}{n}}+\int_0^\theta 1dx=\theta+\frac{\sigma}{n}$ 有偏

$\tilde{\theta}_2 = \hat{\theta}_2 - \frac{\sigma}{n}$.

(3) $\operatorname{var} X = \sigma^2$ (指数分布)

$\operatorname{var}(\hat{\theta}_1) = \operatorname{var}(\frac{1}{n}\sum X_i) = \frac{\sigma^2}{n}$

$E\hat{\theta}_2^2 = \int_0^\theta 2x\,dx + \int_\theta^\infty 2x e^{-n(\frac{x-\theta}{\sigma})} dx = \theta^2 + \frac{2\sigma^2}{n^2} + \frac{2\sigma\theta}{n}$

$\operatorname{var}\hat{\theta}_2 = E\hat{\theta}_2^2 - (E\hat{\theta}_2)^2 = \theta^2 + \frac{2\sigma^2}{n^2} + \frac{2\sigma\theta}{n} - (\theta + \frac{\sigma}{n})^2 = \frac{\sigma^2}{n^2}$

$\operatorname{Var}(\tilde{\theta}_2) = \operatorname{Var}(\hat{\theta}_2) = \frac{\sigma^2}{n^2} < \operatorname{Var}(\hat{\theta}_1) \Longrightarrow \tilde{\theta}_2$ 为优

52 该题为44题 $\sigma=1$ 的情形.

53. $f = \prod_{i=1}^n \frac{1}{\theta} I_{x_i \le \theta}$ 随 $\theta \downarrow$. $\hat{\theta}_n = \max_{1\le i\le n} X_i$

$P(\hat{\theta} \le x) = P(X_1 \le x, \cdots X_n \le x) = (\frac{x}{\theta})^n$ ~~$f_{\hat{\theta}_n}(x) = \frac{n x^{n-1}}{\theta^n}$~~

$P(|\hat{\theta} - \theta| > \varepsilon) = P(\hat{\theta} < \theta - \varepsilon) = (\frac{\theta-\varepsilon}{\theta})^n \longrightarrow 0$

1. 卡西欧计算器统计功能

$\bar{x} = 518.9 \quad \sigma x = 41.84 \quad s = \sqrt{41.84^2 \times 10 \div 9} = \cancel{44.38}\ 44.10$

$\bar{x} \pm \frac{s}{\sqrt{n}} t_{n-1}(0.05) = 518.9 \pm \frac{44.10}{\sqrt{10}} t_9(0.05)$

$= 518.9 \pm \cancel{13.23}\ \cancel{14.03}\ 13.95 \times 1.8331 = 518.9 \pm \cancel{24.25}\ 25.56$

$= \cancel{[494.65, 543.15]}\ \cancel{493.17}$

$= [493.33, 544.46]$

4. $\bar{x} = 2.13 \quad \sigma x = 0.0187 \quad s = 0.0187\sqrt{9 \div 8} = 0.0198$

(1) $\sigma = 0.01 \quad \bar{x} \pm \frac{\sigma}{\sqrt{n}} u_{\frac{\alpha}{2}} = 2.13 \pm \frac{0.01}{3} \times 1.645 = [2.125, 2.135]$

(2) $\bar{x} \pm \frac{s}{\sqrt{n}} t_{n-1}(0.05) = 2.13 \pm \frac{0.0198}{\sqrt{9}} t_8(0.05)$

$= 2.13 \pm 0.0066 \times 1.8595 = [2.118, 2.142]$

11. $\bar{x} = 91 \quad \sigma x = 28.342 \quad s = 28.342\sqrt{11 \div 10} = 29.275 \quad \bar{x} \pm \frac{s}{\sqrt{n}} t_{n-1}(0.025)$

(1) 前

$= 91 \pm \frac{29.275}{\sqrt{11}} \times 2.2281 = [71.33, 110.67]$

(2) 后 $\bar{x} = 93.59 \quad \sigma x = 30.330 \quad s = 30.330\sqrt{11 \div 10} = 31.810 \quad \bar{x} \pm \frac{s}{\sqrt{n}} t_{n-1}(0.025)$

$= 93.59 \pm \frac{31.810}{\sqrt{11}} \times 2.2281 = [72.22, 114.96]$

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(3) 后-前 3.2 -4.5 0.7 6.7 -6.1 -3.7 8.9 3.2 25.6 -17.3 11.8

$\bar{x}=2.59 \quad \sigma x=10.601 \quad sx=10.601\times\sqrt{11\div 10}=11.12$

$\bar{x}\pm\frac{s}{\sqrt{n}}t_{n-1}(0.025)=2.59\pm\frac{11.12}{\sqrt{11}}\times 2.2281=[-4.88, 10.06]$

16.

(1) $\mu=50mm \quad \hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{n}(x_i-50)^2=0.29 \sim \cancel{\frac{1}{10}\chi^2_{10}}$

$\frac{\sum_{i=1}^{n}(x_i-50)^2}{\sigma^2}\sim\chi^2_{10} \quad \left[\frac{\sum_{i=1}^{n}(x_i-50)^2}{\chi^2_{10}(0.025)}, \frac{\sum_{i=1}^{n}(x_i-50)^2}{\chi^2_{10}(1-0.025)}\right]$

$=\left[\frac{2.9}{20.483}, \frac{2.9}{3.247}\right]=[0.14, 0.89]$

(2) μ未知 $\quad \sigma^2 x=0.2836 \Rightarrow \sum_{i=1}^{n}(x_i-\bar{x})^2=2.836$

$\left[\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{\chi^2_9(0.025)}, \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{\chi^2_9(1-0.025)}\right]=\left[\frac{2.836}{19.023}, \frac{2.836}{2.7}\right]=[0.15, 1.05]$

23. $\frac{145}{200}=0.725 \quad \frac{172-145}{200}=0.135$

由书中P249公式(7.7) $p\in\frac{\hat{p}+\frac{u^2_{\alpha/2}}{2n}}{1+\frac{u^2_{\alpha/2}}{n}}\pm u_{\alpha/2}\frac{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}+\frac{u^2_{\alpha/2}}{4n^2}}}{1+\frac{u^2_{\alpha/2}}{n}}$

(1) $\frac{0.725+\frac{1.96^2}{400}}{1+\frac{1.96^2}{200}}\pm 1.96\frac{\sqrt{\frac{0.725(1-0.725)}{200}+\frac{1.96^2}{4\times 200^2}}}{1+\frac{1.96^2}{200}}$ $[0.66, 0.78]$

(2) $\frac{0.135+\frac{1.96^2}{400}}{1+\frac{1.96^2}{200}}\pm 1.96\frac{\sqrt{\frac{0.135(1-0.135)}{200}+\frac{1.96^2}{4\times 200^2}}}{1+\frac{1.96^2}{200}}$ $[0.09, 0.19]$