Rhea Agarwal | BA(Hons) Economics | 20202948 | Practical- 2

Plotting of second order solution family of differential equation

Question 1: Solve Second order Differential Equation y'' + y = 0
Solution :

In[60]:=
```
DSolve[y''[x] + y[x] == 0, y[x], x]
```

Out[60]=
$$\{\{y[x] \to c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}\}$$

Question 2: Solve Second order Differential Equation y'' + y' - 6y = 0
Solution :

In[64]:=
```
DSolve[y''[x] + y'[x] - 6 y[x] == 0, y[x], x]
```

Out[64]=
$$\{\{y[x] \to e^{-3x} c_1 + e^{2x} c_2\}\}$$

Question 3: Solve Second order Differential Equation 4y'' + 12y' - 6y = 0
Solution :

In[67]:=
```
DSolve[4 y''[x] + 12 y'[x] - 6 y[x] == 0, y[x], x]
```

Out[67]=
$$\left\{\left\{y[x] \to e^{\left(-\frac{3}{2}-\frac{\sqrt{15}}{2}\right)x} c_1 + e^{\left(-\frac{3}{2}+\frac{\sqrt{15}}{2}\right)x} c_2\right\}\right\}$$

Question 4: Solve Second order Differential Equation y'' - 6y' + 13y = 0
Solution :

In[69]:=
```
DSolve[y''[x] - 6 y'[x] + 13 y[x] == 0, y[x], x]
```

Out[69]=
$$\{\{y[x] \to e^{3x} c_2 \text{Cos}[2x] + e^{3x} c_1 \text{Sin}[2x]\}\}$$

Question 5: Solve Second order Differential Equation y'' - 2y' + y = 0
Solution:

In[71]:=
```
DSolve[y''[x] - 2 y'[x] + y[x] == 0, y[x], x]
```

Out[71]= $\{\{y[x] \to e^x c_1 + e^x x c_2\}\}$

Plotting Of Solution Of Second order Differential Equations

Question 1: Solve Second order Differential Equation y'' + y = 0 and Plot its three Solutions.
Solution :

In[77]:=
```
Sol = DSolve[y''[x] + y[x] == 0, y[x], x]
Sol1 = y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 2}
Sol2 = y[x] /. Sol[[1]] /. {C[1] → 1 / 2, C[2] → 5}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → -4}
Plot[{Sol1, Sol2, Sol3}, {x, - 20, 20},
PlotStyle → {{Red, Thickness[0.01]}, {Green , Thick}, {Purple , Thickness[0.02]}},
PlotLegends → {Sol1, Sol2, Sol3}]
```

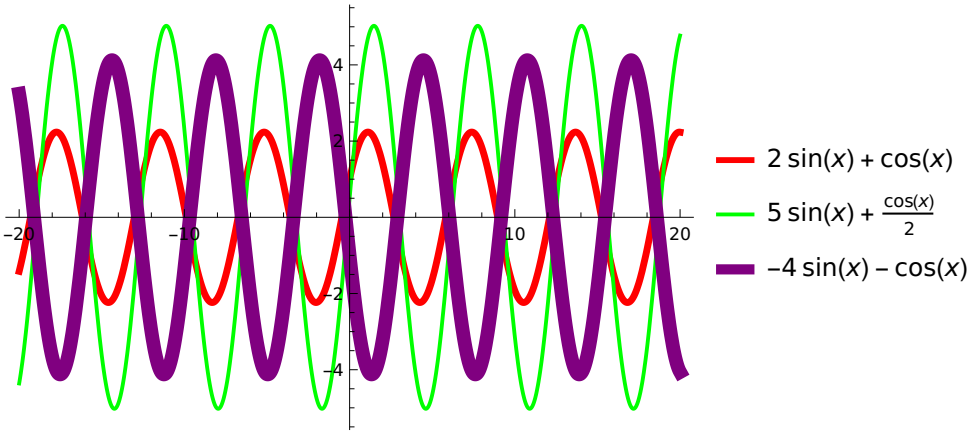
Out[77]= $\{\{y[x] \to c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}\}$

Out[78]= $\text{Cos}[x] + 2\ \text{Sin}[x]$

Out[79]= $\frac{\text{Cos}[x]}{2} + 5\ \text{Sin}[x]$

Out[80]= $-\text{Cos}[x] - 4\ \text{Sin}[x]$

Out[81]=



Question 2: Solve Second order Differential Equation y'' + y' - 6y = 0 and Plot its three Solutions.

Solution:

In[87]:=
```
Sol = DSolve[y''[x] + y'[x] - 6 y[x] == 0, y[x], x]
Sol1 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 2.5}
Sol2 = y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 5}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → -1/2, C[2] → 5}
Plot[{Sol1, Sol2, Sol3}, {x, -2, 2},
PlotStyle → {{Pink, Thickness[0.01]}, {Green , Thick}, {Orange , Thickness[0.02]}},
PlotLegends → {Sol1, Sol2, Sol3}]
```

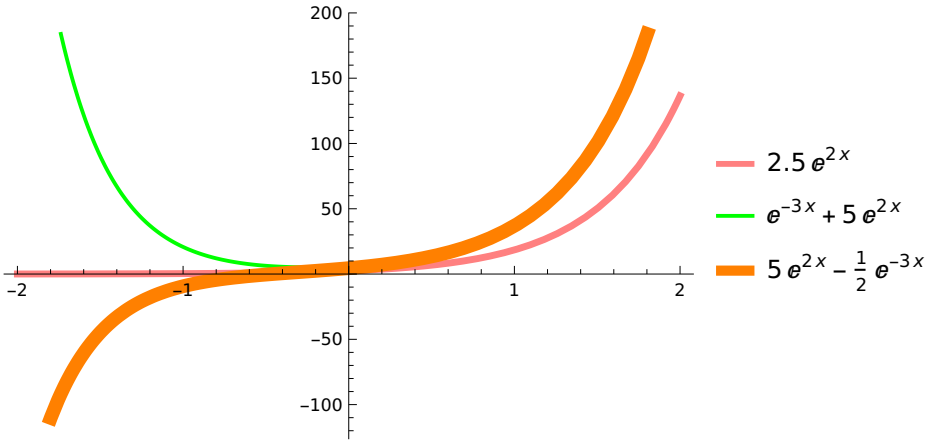
Out[87]= $\{\{y[x] \to e^{-3x} C_1 + e^{2x} C_2\}\}$

Out[88]= $2.5\, e^{2x}$

Out[89]= $e^{-3x} + 5\, e^{2x}$

Out[90]= $-\frac{1}{2} e^{-3x} + 5\, e^{2x}$

Out[91]=



Question 3: Solve Second order Differential Equation 4y'' + 12y' + 9y = 0 and Plot its four Solutions for

(i) C[1] = -1 , C[2] = 4

(ii) C[1] = -3 , C[2] = 6

(iii) C[1] = -10 , C[2] = 7

(iv) C[1] = -1.5 , C[2] = -5

Solution :

In[132]:=
```
Sol = DSolve[4 y ''[x] + 12 y '[x] + 9 y[x] == 0, y[x], x]
Sol1 = y[x] /. Sol[[1]] /. {C[1] → - 1, C[2] → 4}
Sol2 = y[x] /. Sol[[1]] /. {C[1] → 3, C[2] → 6}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → - 10, C[2] → 7}
Sol4 = y[x] /. Sol[[1]] /. {C[1] → - 1.5, C[2] → - 5}
Plot[{Sol1, Sol2, Sol3, Sol4}, {x, - 2, 2},
PlotStyle → {{Red, Thickness[0.01]}, {Green , Thick},
   {Purple , Thickness[0.02]}, {Yellow , Thickness[0.03]}},
PlotLegends → {Sol1, Sol2, Sol3, Sol4}]
```

Out[132]=

$$\{\{y[x] \to e^{-3\,x/2}\,c_1 + e^{-3\,x/2}\,x\,c_2\}\}$$

Out[133]=

$$-e^{-3\,x/2} + 4\,e^{-3\,x/2}\,x$$

Out[134]=

$$3\,e^{-3\,x/2} + 6\,e^{-3\,x/2}\,x$$

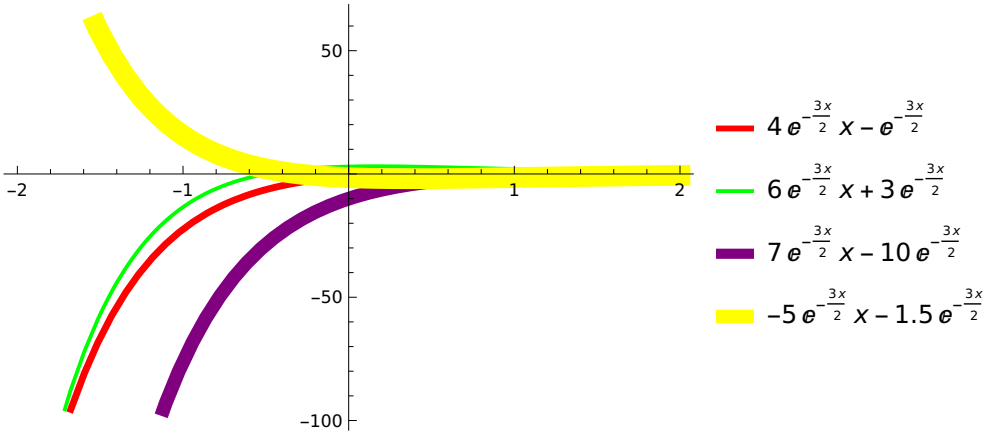
Out[135]=

$$-10\,e^{-3\,x/2} + 7\,e^{-3\,x/2}\,x$$

Out[136]=

$$-1.5\,e^{-3\,x/2} - 5\,e^{-3\,x/2}\,x$$

Out[137]=



Question 4: Solve Second order Differential Equation 4y'' - 6y' +13y = 0 and Plot

its any three Solutions.

Solution :

In[138]:=
```
Sol = DSolve[4 y ''[x] - 6 y '[x] + 13 y[x] == 0, y[x], x]
Sol1 = y[x] /. Sol[[1]] /. {C[1] → - 1, C[2] → 4}
Sol2 = y[x] /. Sol[[1]] /. {C[1] → 3, C[2] → 6}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → - 10, C[2] → 7}
Plot[{Sol1, Sol2, Sol3}, {x, - 5, 5},
PlotStyle → {{Red, Thickness[0.01]}, {Green , Thick}, {Purple , Thickness[0.02]}},
PlotLegends → {Sol1, Sol2, Sol3}]
```

Out[138]=

$$\left\{\left\{y[x] \to e^{3\,x/4}\,C_2\,\text{Cos}\left[\frac{\sqrt{43}\,x}{4}\right] + e^{3\,x/4}\,C_1\,\text{Sin}\left[\frac{\sqrt{43}\,x}{4}\right]\right\}\right\}$$

Out[139]=

$$4\,e^{3\,x/4}\,\text{Cos}\left[\frac{\sqrt{43}\,x}{4}\right] - e^{3\,x/4}\,\text{Sin}\left[\frac{\sqrt{43}\,x}{4}\right]$$

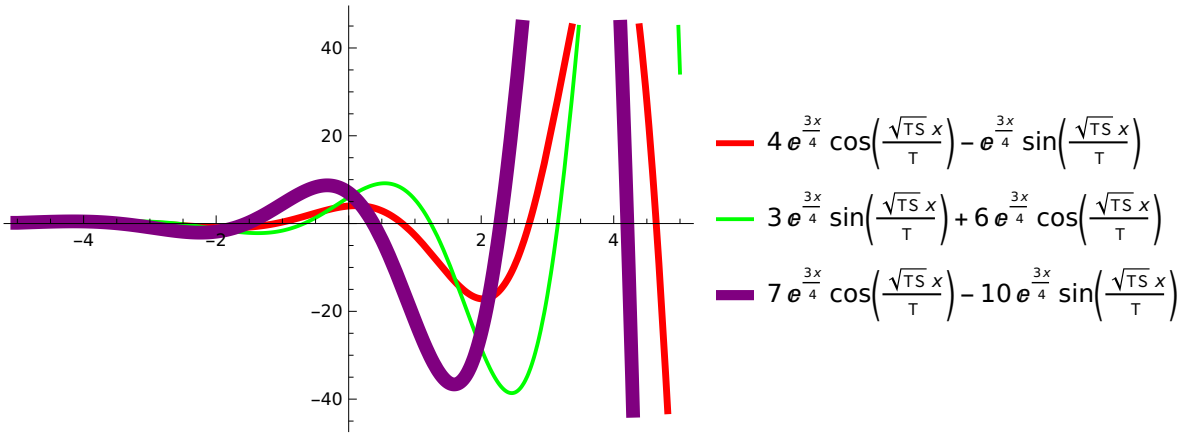
Out[140]=

$$6\,e^{3\,x/4}\,\text{Cos}\left[\frac{\sqrt{43}\,x}{4}\right] + 3\,e^{3\,x/4}\,\text{Sin}\left[\frac{\sqrt{43}\,x}{4}\right]$$

Out[141]=

$$7\,e^{3\,x/4}\,\text{Cos}\left[\frac{\sqrt{43}\,x}{4}\right] - 10\,e^{3\,x/4}\,\text{Sin}\left[\frac{\sqrt{43}\,x}{4}\right]$$

Out[142]=



Question 5: Solve Second order Differential Equation y'' - 2y' + y = 0 and Plot its five Solutions.

Solution :

In[143]:=
```
Sol = DSolve[y ''[x] - 2 y '[x] + y[x] == 0, y[x], x]
Sol1 = y[x] /. Sol[[1]] /. {C[1] → 0.5, C[2] → 3}
Sol2 = y[x] /. Sol[[1]] /. {C[1] → - 3, C[2] → - 2}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → - 1, C[2] → 7}
Sol4 = y[x] /. Sol[[1]] /. {C[1] → - 6, C[2] → 1}
Sol5 = y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 2 / 3}
Plot[{Sol1, Sol2, Sol3, Sol4, Sol5}, {x, - 2, 2},
PlotStyle → {{Red, Thickness[0.01]}, {Green , Thick}, {Purple , Thickness[0.02]},
   {Yellow , Thickness[0.03]}, {Orange , Thickness[0.04]}},
PlotLegends → {Sol1, Sol2, Sol3, Sol4, Sol5}]
```

Out[143]=
$$\{\{y[x] \to e^x c_1 + e^x x c_2\}\}$$

Out[144]=
$$0.5\, e^x + 3\, e^x x$$

Out[145]=
$$-3\, e^x - 2\, e^x x$$

Out[146]=
$$-e^x + 7\, e^x x$$

Out[147]=
$$-6\, e^x + e^x x$$

Out[148]=
$$e^x + \frac{2\, e^x x}{3}$$

Out[149]=