NAME: Mehul Pant
# ROLL NO: 20202948
COURSE: Bsc(HONS) CS

SOLUTION OF CAUCHY PROBLEM FOR FIRST ORDER PDE

QUESTION 1: Obtain the solution of the linear equation u[(x,y),x]- u[(x,y), y] = 1 with the Cauchy data u(x,0) = x*x

In[1]:= `pde = D[u[x, y], x] - D[u[x, y], y] == 1`

Out[1]= $-u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$

In[2]:= `sol = DSolve[{pde, u[x, 0] == x * x}, u[x, y], {x, y}]`

Out[2]= $\{\{u[x, y] \rightarrow x^2 - y + 2 x y + y^2\}\}$

In[3]:= `Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -5, 5},`
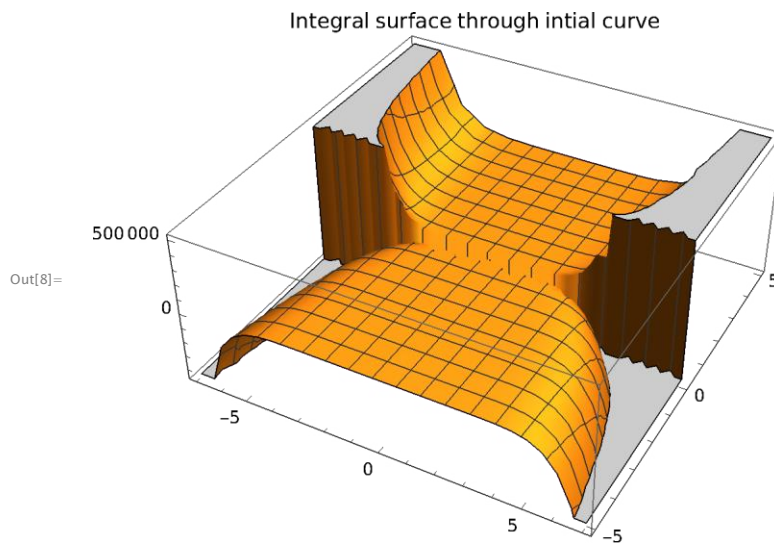   `PlotLabel → "Integral surface through intial curve"]`

Out[3]=

Integral surface through intial curve
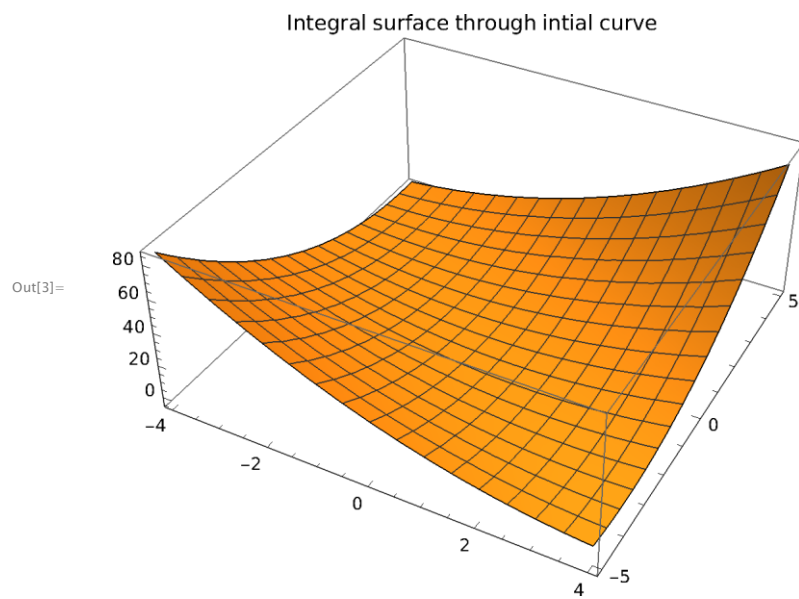
QUESTION 2: Obtain the solution of the linear equation y*u[(x,y),x]-2*x*u[x,y] with the Cauchy data u(0,y)=y*y*y
SOLUTION:

In[6]:= `pde = y * D[u[x, y], x] - 2 * x * y * D[u[x, y], y] == 2 * x * u[x, y]`

`sol3 = DSolve[{pde, u[0, y] == y * y * y}, u[x, y], {x, y}]`

`Plot3D[u[x, y] /. sol3, {x, -7, 7}, {y, -5, 5},`

` PlotLabel → "Integral  surface  through  intial  curve"]`

Out[6]= $-2 \, x \, y \, u^{(0,1)}[x, y] + y \, u^{(1,0)}[x, y] == 2 \, x \, u[x, y]$

Out[7]= $\left\{\left\{u[x, y] \rightarrow \dfrac{(x^2 + y)^4}{y}\right\}\right\}$

Out[8]=

Integral surface through intial curve

QUESTION 3 : Determine the integral surfaces of the equation u[(x,y),x]+u[(x,y),y]=u[x,y]*u[x,y] , (a) with the data x+y=0,u=1. (b) with the data u(x,0)=tanh(x).
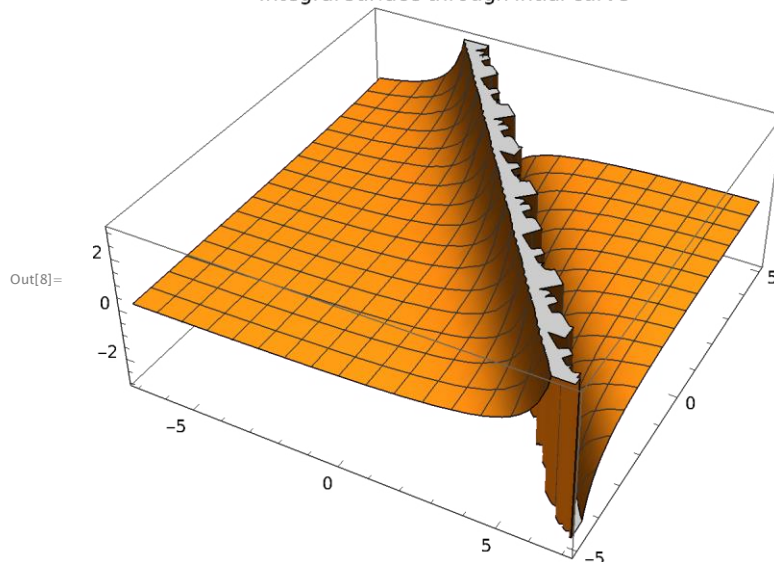
In[6]:= Eqn = D[u[x, y], x] + D[u[x, y], y]== u[x, y]*u[x, y]

sol4 =
 DSolve[{D[u[x, y], x] + D[u[x, y], y]== u[x, y] * u[x, y], u[x, - x]== 1}, u[x, y], {x, y}]
Plot3D[u[x, y] /. sol4, {x, -7, 7}, {y, -5, 5},
PlotLabel → "Integral surface through intial curve"]

Out[6]= $u^{(0,1)}[x, y] + u^{(1,0)}[x, y]== u[x, y]^2$

Out[7]= $\left\{\left\{ u[x, y] \to - \frac{2}{-2 + x + y}\right\}\right\}$

Integral surface through intial curve

Out[8]=

(b)

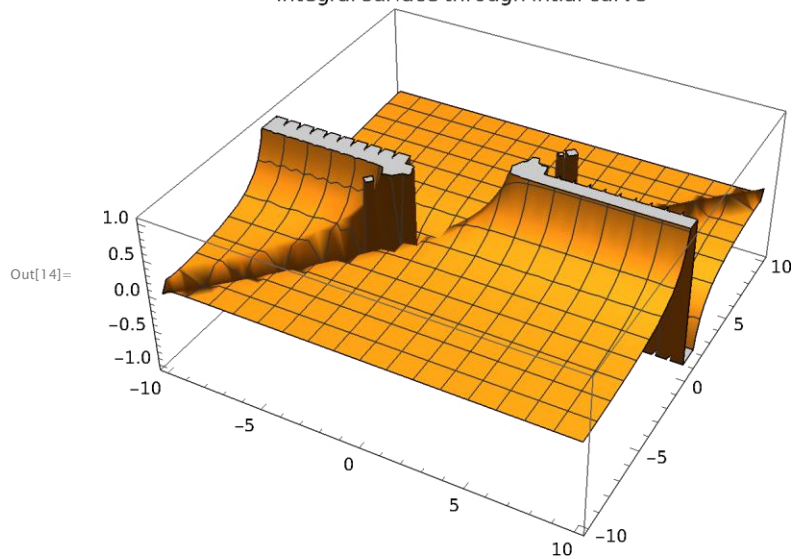In[9]:= D[u[x, y], x] + D[u[x, y], y]== u[x, y] * u[x, y]

Out[9]= $u^{(0,1)}[x, y] + u^{(1,0)}[x, y]== u[x, y]^2$

In[10]:= sol5 = DSolve[
  {D[u[x, y], x] + D[u[x, y], y]== u[x, y] * u[x, y], u[x, 0]== Tanh[x]}, u[x, y], {x, y}]

Out[10]= $\left\{\left\{ u[x, y] \to \frac{1}{-y + Coth[x - y]}\right\}\right\}$

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In[14]:=  Plot3D[u[x, y] /. sol5, {x, -10, 10}, {y, -10, 10},
          PlotLabel → "Integral  surface  through  intial  curve"]
```

Integral surface through intial curve

Out[14]=

QUESTION 4 : Obtain the solution of the linear equation u[(x,y),x]+u[(x,y),y]=1 with the Cauchy data u(x,2x)=x*x*x

```
In[15]:=  D[u[x, y], x] + D[u[x, y], y]==  1
```

Out[15]= $u^{(0,1)}[x, y] + u^{(1,0)}[x, y]== 1$

```
In[16]:=  sol6 = DSolve[{D[u[x, y], x] + D[u[x, y], y]==  1, u[x, 2 x]==  x * x * x}, u[x, y], {x, y}]
```

Out[16]= $\{\{u[x, y] → 2 x - x^3 - y + 3 x^2 y - 3 x y^2 + y^3\}\}$

In[17]:= `Plot3D[u[x, y] /. sol6, {x, -4, 4}, {y, -5, 5},`
`PlotLabel → "Integral surface through intial curve"]`

Integral surface through intial curve

Out[17]=
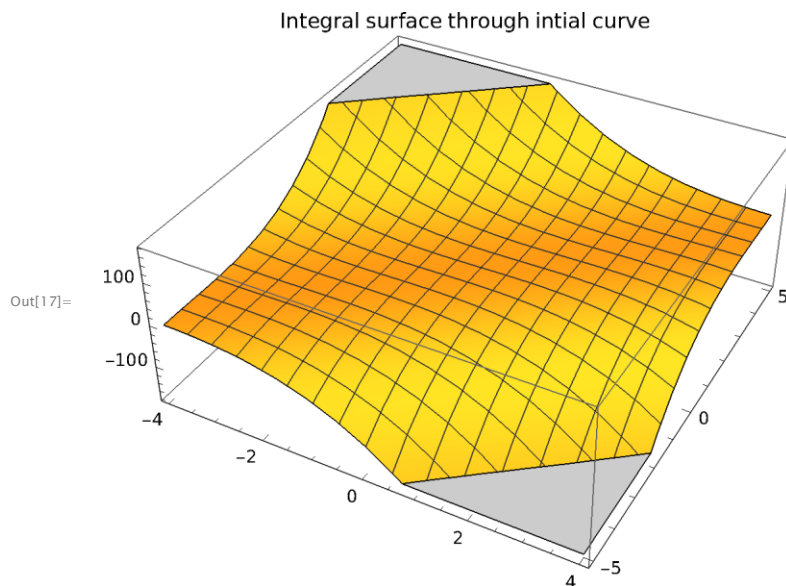
QUESTION 5 : Obtain the solution of the linear equation u(x+y)*u[(x,y),x]+u(x-y)*u[(x,y),y]=x*x+y*y with the Cauchy data u(x,2x)=0
SOLUTION :

In[18]:= `u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y]== x * x + y * y`

Out[18]= $(x - y) u[x, y] u^{(0,1)}[x, y] + (x + y) u[x, y] u^{(1,0)}[x, y]== x^2 + y^2$

In[19]:= `DSolve[{u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y]==( x * x) + (y * y),`
`u[x, 2 x]== 0}, u[x, y], {x, y}]`

Solve : Inverse functions are being used by Solve , so some solutions may not be found ; use Reduce for complete
solution information .

Out[19]= $\{\{u[x, y] \to - \sqrt{\frac{2}{7}} \sqrt{2 x^2 + 3 x y - 2 y^2}\}, \{u[x, y] \to \sqrt{\frac{2}{7}} \sqrt{2 x^2 + 3 x y - 2 y^2}\},$

$\{u[x, y] \to - \sqrt{\frac{2}{7}} \sqrt{2 x^2 + 3 x y - 2 y^2}\}, \{u[x, y] \to \sqrt{\frac{2}{7}} \sqrt{2 x^2 + 3 x y - 2 y^2}\}\}$

In[20]:= `Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5},`
`PlotLabel → "Integral  surface  through  intial  curve"]`

Out[20]=

Integral surface through intial curve