Rhea Agarwal | BA (Hons) Economics | 20202948 | Practical- 4

Method of Variation of Parameters

QUESTION 1 : Solve second order differential equation y''[x]+y[x]=tan[x] by method of variation of parameter
Solution:

Step -1: Find complementary function

In[169]:=
```
eqn := y''[x] + y[x];
f[x_] := Tan[x];
P = DSolve[eqn == 0, y[x], x]
```
Out[171]=
$\{\{y[x] \to c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}\}$

Step -2 Consider fundamental solution function u(x) and v(x)

In[172]:=
```
u[x_] := Cos[x];
v[x_] := Sin[x];
```

Step -3 Find Wronskian W = ({ u[x] , v[x] } , { u'[x] , v'[x] })

In[174]:=
```
w = Simplify[Det[{{u[x], v[x]}, {u'[x], v'[x]}}]]
```
Out[174]=
1

Step -4 Find g[x]= (-v[x]f[x])/w and h[x] = (u[x] f[x])/w

In[175]:=
```
g[x_] := (-v[x] × f[x]) / w
h[x] := (u[x] × f[x]) / w
```

Step -5 Find G = Integrate[g[x],x] and H=Integrate[h[x],x]

In[178]:=
```
G = Integrate[g[x], x]
H = Simplify[Integrate[h[x], x]]
```

Out[178]=

$$\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Sin}[x]$$

Out[179]=

$$-\text{Cos}[x]$$

Step -6 Find PI=u[x]G+v[x]H

In[180]:=
```
PI = u[x] G + v[x] H
```

Out[180]=

$$-\text{Cos}[x]\times\text{Sin}[x]+\text{Cos}[x]\left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Sin}[x]\right)$$

QUESTION 2 : Solve second order differential equation y''[x]-2y'[x]=e^x Sin[x] by method of variation of parameter

Step -1: Find complementary function

In[181]:=
```
eqn := y''[x] - 2 y'[x];
f[x_] := e ^ x * Sin[x];
P = DSolve[eqn == 0, y[x], x]
```

Out[183]=

$$\left\{\left\{y[x]\to\frac{1}{2}e^{2x}c_1+c_2\right\}\right\}$$

Step -2 Consider fundamental solution function u(x) and v(x)

In[192]:=
```
u[x_] := 1/2 Exp[2 x]
v[x_] := 1
```

Step -3 Find Wronskian W = ({ u[x] , v[x] } , { u'[x] , v'[x] })

In[194]:=
```
w = Simplify[Det[{{u[x], v[x]}, {u'[x], v'[x]}}]]
```

Out[194]=

$$-e^{2x}$$

Step -4 Find g[x]= (-v[x]f[x])/w and h[x] = (u[x] f[x])/w

In[195]:=
```
g[x_] := (- v[x] × f[x]) / w
h[x_] := (u[x] × f[x]) / w
```

Step -5 Find G = Integrate[g[x],x] and H=Integrate[h[x],x]

In[197]:=
```
G = Integrate[g[x], x]
H = Simplify[Integrate[h[x], x]]
```

Out[197]=

$$\frac{e^{x}\,e^{-2x}\,(-\mathrm{Cos}[x]+(-2+\mathrm{Log}[e])\,\mathrm{Sin}[x])}{5-4\,\mathrm{Log}[e]+\mathrm{Log}[e]^{2}}$$

Out[198]=

$$\frac{e^{x}\,(\mathrm{Cos}[x]-\mathrm{Log}[e]\times\mathrm{Sin}[x])}{2\left(1+\mathrm{Log}[e]^{2}\right)}$$

Step -6 Find PI=u[x]G+v[x]H

In[199]:=
```
PI = u[x] G + v[x] H
```

Out[199]=

$$\frac{e^{x}\,(-\mathrm{Cos}[x]+(-2+\mathrm{Log}[e])\,\mathrm{Sin}[x])}{2\left(5-4\,\mathrm{Log}[e]+\mathrm{Log}[e]^{2}\right)}+\frac{e^{x}\,(\mathrm{Cos}[x]-\mathrm{Log}[e]\times\mathrm{Sin}[x])}{2\left(1+\mathrm{Log}[e]^{2}\right)}$$