Mehul Pant | BSC(Hons)CS | 20211473| Practical- 8

Plot the integral surface of a given first order PDE with the initial data

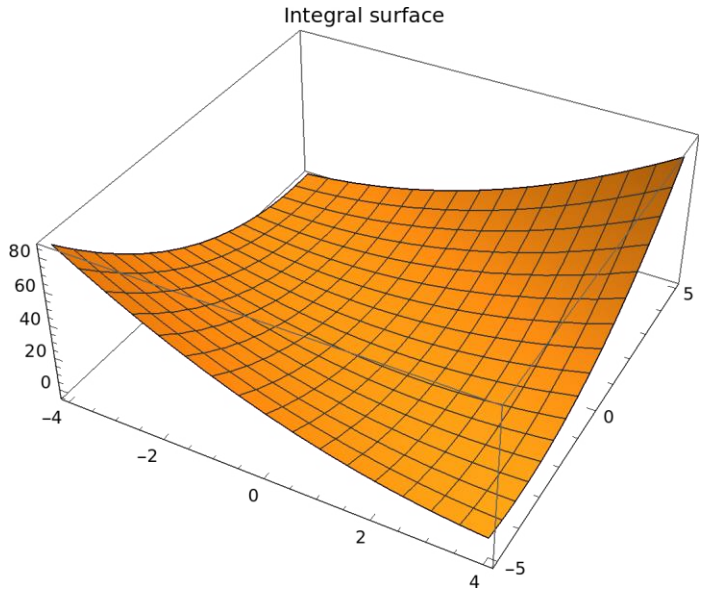
Problem 1: Obtain the solution of the linear equation u[(x,y),x]-u[(x,y),y]=1 with the Cauchy data u(x,0) = x*x.
Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.
Solution :

In[12]:=
```
pde = D[u[x, y], x] - D[u[x, y], y]==  1
DSolve[{D[u[x, y], x] - D[u[x, y], y]==  1, u[x, 0]==(  x * x)}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5},
PlotLabel → "Integral surface "]
```

Out[12]= $-u^{[0,1]}[x, y] + u^{[1,0]}[x, y] == 1$

Out[13]= $\{\{u[x, y] \to x^2 - y + 2\,x\,y + y^2\}\}$

Out[14]=



Problem 2: Find the solution of the equation

y*u[(x,y),x]-2*x*y*u[(x,y),y]=2*x*u[x,y] with the Cauchy data u(0,y) = y*y*y. Plot the integral surface with in the range {x,-7,7} and {y,-5,5}.

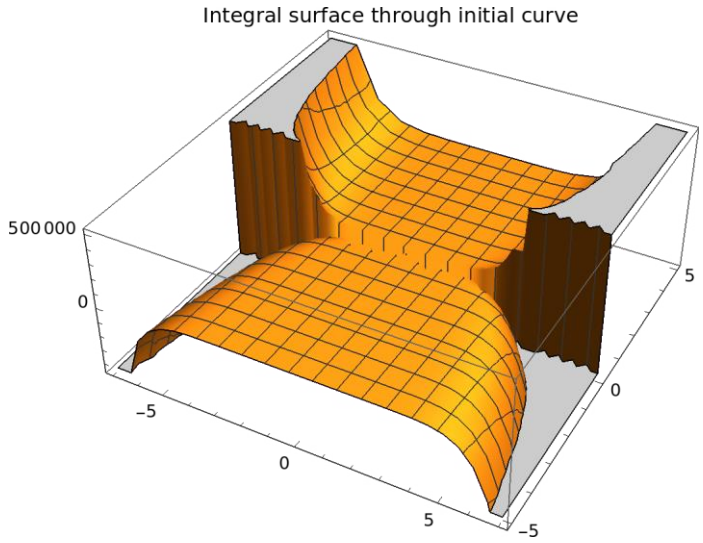
Solution :

In[15]:=
```
pde = y * D[u[x, y], x] - 2 * x * y * D[u[x, y], y]==  2 * x * u[x, y]
sol3 = DSolve[{pde, u[0, y]==  y * y * y}, u[x, y], {x, y}]
Plot3D[u[x, y]/. sol3, {x, -7, 7}, {y, -5, 5},
PlotLabel → "Integral surface through initial curve"]
```

Out[15]=
$$-2\,x\,y\,u^{[0,1]}[x,y]+y\,u^{[1,0]}[x,y]==2\,x\,u[x,y]$$

Out[16]=
$$\{\{u[x,y]\to \frac{|x^2+y|^4}{y}\}\}$$

Out[17]=



Problem 3 : Determine the integral surfaces of the equation u[(x,y),x]+u[(x,y),y]=u[x,y]*u[x,y] , with the data x+y=0,u=1. Plot the integral surface with in the range {x,10,10} and {y,-10,10}.

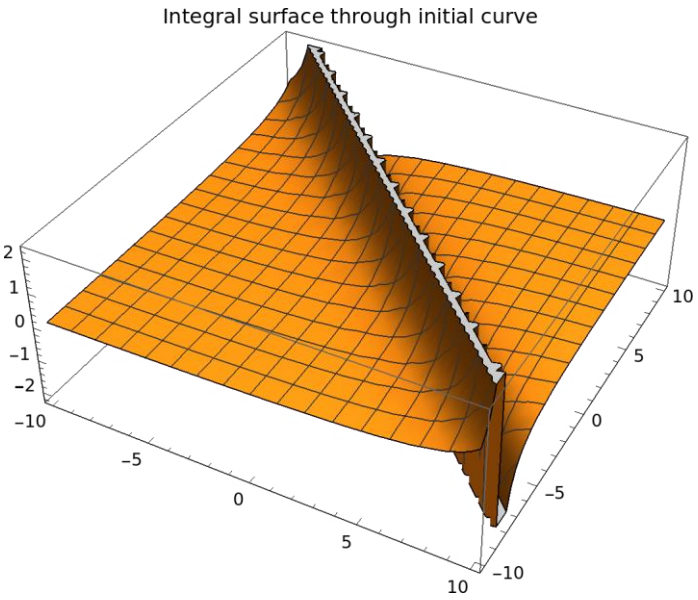
Solution :

In[18]:=
```
Eqn = D[u[x, y], x] + D[u[x, y], y]==  u[x, y] * u[x, y]
DSolve[{D[u[x, y], x] + D[u[x, y], y]==  u[x, y] * u[x, y], u[x, -x]==  1}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -10, 10}, {y, -10, 10},
PlotLabel → "Integral surface through initial curve "]
```

Out[18]= $u^{[0,1]}[x, y] + u^{[1,0]}[x, y] == u[x, y]^2$

Out[19]= $\{\{u[x, y] \to -\frac{2}{-2 + x + y}\}\}$

Out[20]=



Problem 4 : Obtain the solution of the linear equation u[(x,y),x]+u[(x,y),y]=1 with the Cauchy data u(x,2x)=x*x*x.

Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.

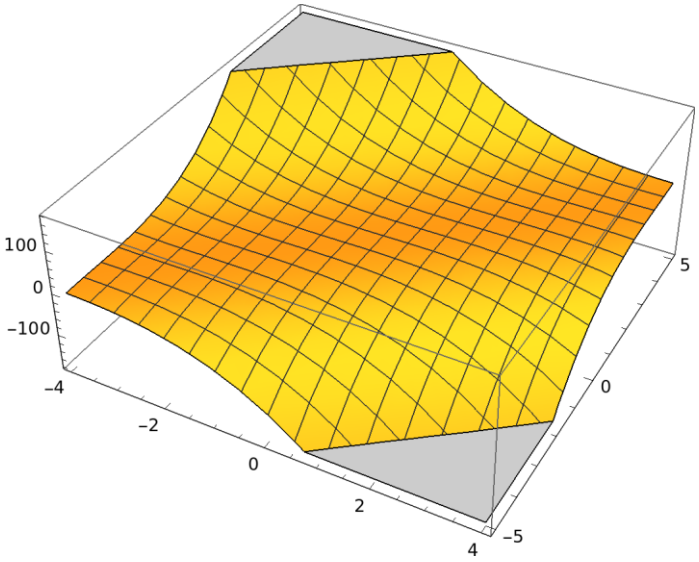
Solution :

```
In[21]:= D[u[x, y], x] + D[u[x, y], y]== 1
         DSolve[{D[u[x, y], x] + D[u[x, y], y]== 1, u[x, 2 x]== x * x * x}, u[x, y], {x, y}]
         Plot3D[u[x, y] /. %, {x, - 4, 4}, {y, - 5, 5}]
```

Out[21]= $u^{[0,1]}[x, y] + u^{[1,0]}[x, y] == 1$

Out[22]= $\{\{u[x, y] \to 2\,x - x^3 - y + 3\,x^2\,y - 3\,x\,y^2 + y^3\}\}$

Out[23]=



Problem 5 : Obtain the solution of the linear equation u(x+y)*u[(x,y),x]+u(x-y)*u[(x,y),y]=x*x+y*y with the Cauchy data u(x,2x)=0.
Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.
Solution :

In[24]:=
```
eqn = u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y]==  x * x + y * y
DSolve[{u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y]==(  x * x) + (y * y),
  u[x, 2 x]==  0}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, - 4, 4}, {y, - 5, 5}]
```

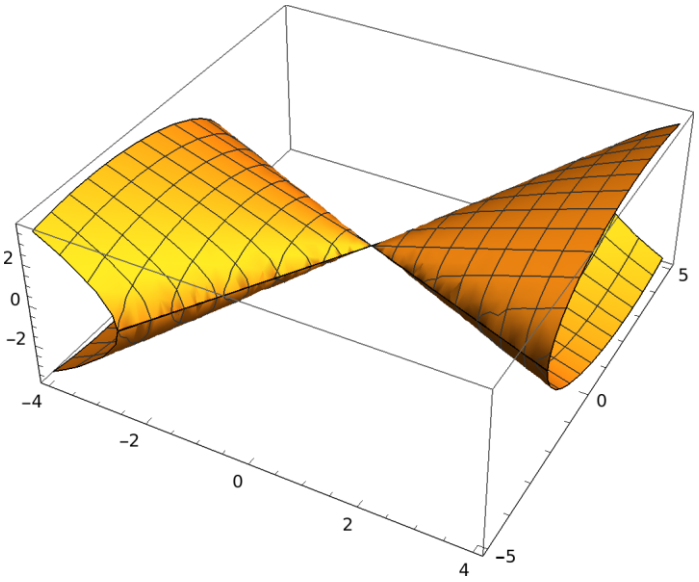
Out[24]= $(x - y)\,u[x, y]\,u^{[0,1]}[x, y] + (x + y)\,u[x, y]\,u^{[1,0]}[x, y] == x^2 + y^2$

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[25]=
$$\left\{\left\{u[x, y] \to -\sqrt{\frac{2}{7}}\,\sqrt{2\,x^2 + 3\,x\,y - 2\,y^2}\,\right\},\ \left\{u[x, y] \to \sqrt{\frac{2}{7}}\,\sqrt{2\,x^2 + 3\,x\,y - 2\,y^2}\,\right\}\right\}$$

$$\left\{\left\{u[x, y] \to -\sqrt{\frac{2}{7}}\,\sqrt{2\,x^2 + 3\,x\,y - 2\,y^2}\,\right\},\ \left\{u[x, y] \to \sqrt{\frac{2}{7}}\,\sqrt{2\,x^2 + 3\,x\,y - 2\,y^2}\,\right\}\right\}$$

Out[26]=



Problem 6 : Obtain the solution of the linear equation u[(x,y),x]+u[x,y]*u[(x,y),y]=1 with the Cauchy data u(0,y)=4*y.

Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.

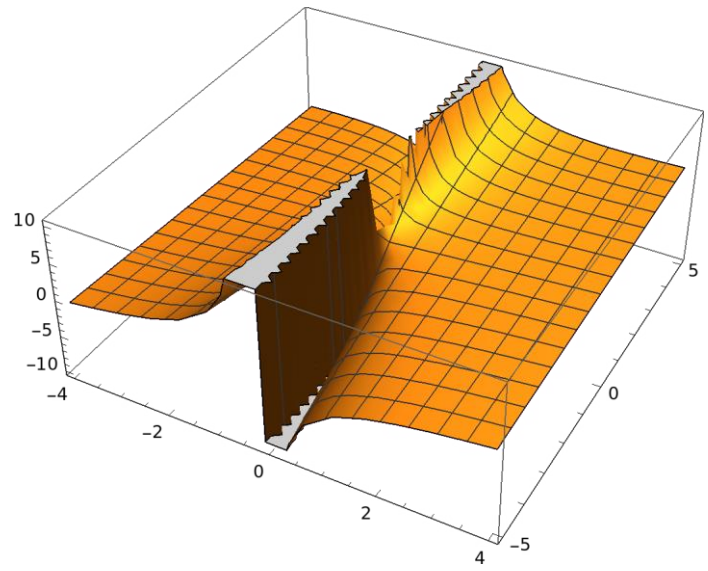
Solution :

In[27]:=
```
D[u[x, y], x] + u[x, y] * D[u[x, y], y]== 1
DSolve[{D[u[x, y], x] + u[x, y] * D[u[x, y], y]== 1, u[0, y]== 4 * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, - 4, 4}, {y, - 5, 5}]
```

Out[27]=
$$u[x, y]\, u^{[0,1]}[x, y] + u^{[1,0]}[x, y] == 1$$

Out[28]=
$$\left\{\left\{u[x, y] \to \frac{x + 2\, x^2 + 4\, y}{1 + 4\, x}\right\}\right\}$$

Out[29]=



Problem 7 : Obtain the solution of the linear equation u[(x,y),x]+y*u[(x,y),y]=0 with the Cauchy data u(0,y)=4*y.

Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.

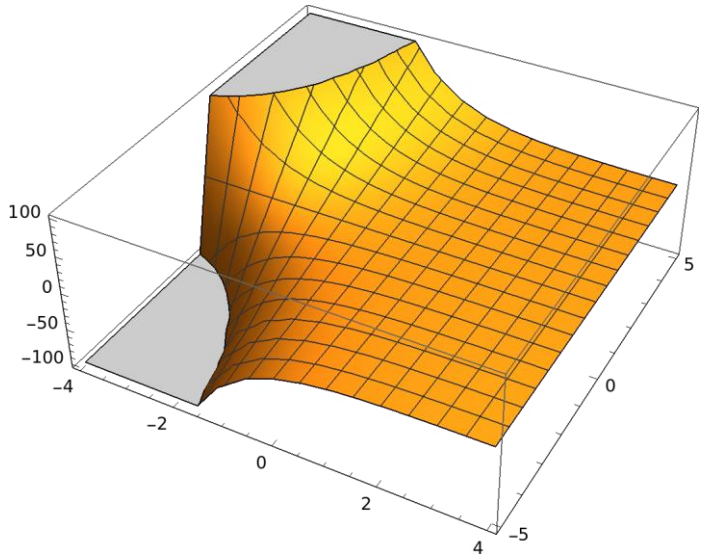
Solution :

In[30]:=
```
D[u[x, y], x] + y * D[u[x, y], y]== 0
DSolve[{D[u[x, y], x] + y * D[u[x, y], y]== 0, u[0, y]== 4 * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, - 4, 4}, {y, - 5, 5}]
```

Out[30]= $y\,u^{[0,1]}[x, y] + u^{[1,0]}[x, y] == 0$

Out[31]= $\{\{u[x, y] \to 4\,e^{-x}\,y\}\}$

Out[32]=



Problem 8 : Obtain the solution of the linear equation u[(x,y),x]+2*u[(x,y),y]=0 with the Cauchy data u(0,y)=Exp[-y*y].
Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.
Solution :

In[33]:=
```
D[u[x, y], x] + 2 * D[u[x, y], y]== 0
DSolve[{D[u[x, y], x] + 2 * D[u[x, y], y]== 0, u[0, y]== Exp[- y * y]}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, - 4, 4}, {y, - 5, 5}]
```

Out[33]= $2\,u^{[0,1]}[x, y] + u^{[1,0]}[x, y] == 0$

Out[34]= $\{\{u[x, y] \to e^{\frac{-1}{\ldots}\ldots^{2}}\}\}$

Out[35]=