Mehul Pant | BSC(Hons)CS | 20211473| Practical 7

Find the Characteristics for the first order PDE and Plotting them

Example 1: Find the Characteristics of the equation (u-y)u[(x,y),x] +y*u[(x,y),y] = x+y and plot them.

Solution :

The characteristics system is

dx/(u-y) = dy/y = du/(x+y)

using (i)+(ii)+(iii), we have v = (u+x)/y = c1, is a first integral.

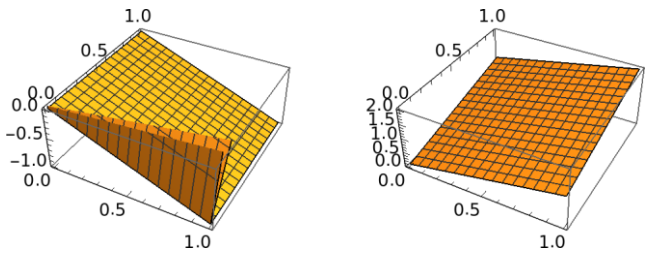
using (i)+(ii) = (iii), we have w = (x+y)^2 - u*u = c2,

is a second first integral

In[235]:=
```
f0 = Plot3D[- x, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
f1 = Plot3D[5 y - x, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
f2 = Plot3D[10 y - x, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
g1 = Show[f0, f1, f2];
h0 = Plot3D[x + y, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
h1 = Plot3D[Sqrt[(x + y)^ 2 + 5], {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
h2 = Plot3D[Sqrt[(x + y)^ 2 + 10], {x, 0, 1}, {y, 0, 1}, PlotPoints → 10];
g2 = Show[h0, h1, h2];
Show[GraphicsArray [{g1, g2}]]
```

GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.

Out[243]=



Example 2 : The solution of the equation u[(x,y),y] + u[x,y]*u[(x,y),x] = 0, can be

interpreted as a vector field on the x-axis varying with time y. Find the integral satisfying the initial condition u(s,0)=h(s), where h is a given function.

Solution :

We plot the curves

{Ct: x= s+t(s^3 - 3s^2 +4),

u= s^3 - 3s^2 +4}

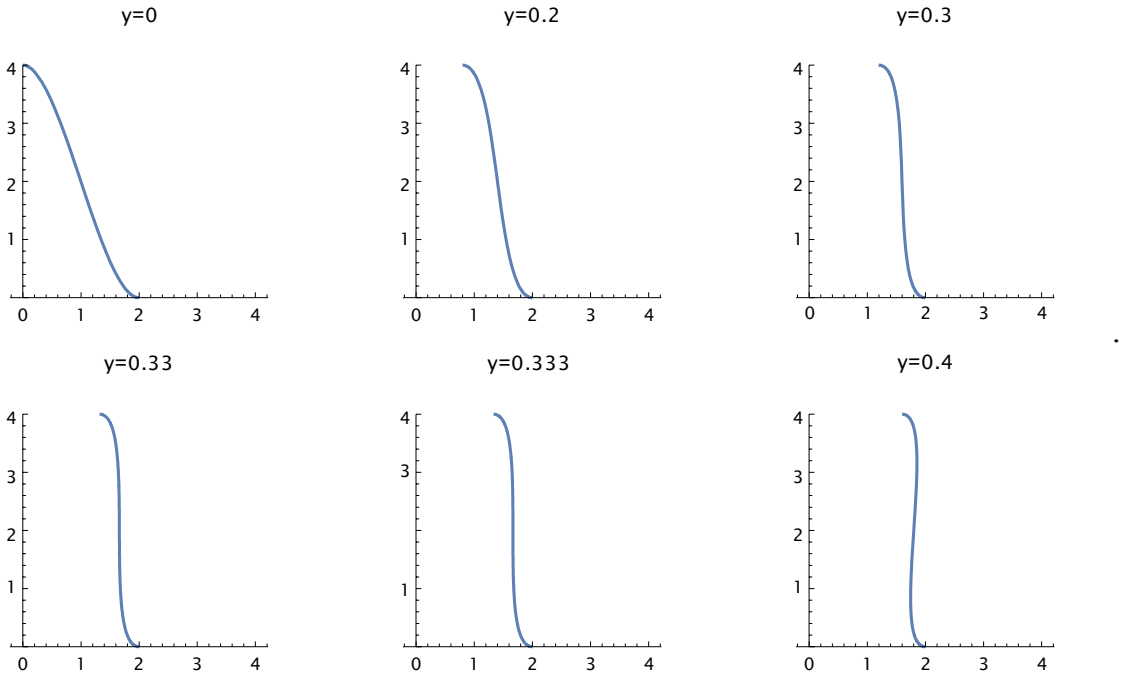
In[244]:=
```
u[s_] := s ^ 3 - 3 s ^ 2 + 4;
x[s_, t_] := s + t * u[s];
h0 =
  ParametricPlot[{x[s, 0], u[s]}, {s, 0, 2}, PlotRange →{ 0, 4}, PlotLabel → "y=0"];
h1 = ParametricPlot[{x[s, 0.2], u[s]},
   {s, 0, 2}, PlotRange →{ 0, 4}, PlotLabel → "y=0.2"];
h2 = ParametricPlot[{x[s, 0.3], u[s]},
   {s, 0, 2}, PlotRange →{ 0, 4}, PlotLabel → "y=0.3"];
h3 = ParametricPlot[{x[s, 0.33], u[s]},
   {s, 0, 2}, PlotRange →{ 0, 4}, PlotLabel → "y=0.33"];
h4 = ParametricPlot[{x[s, 0.333], u[s]},
   {s, 0, 2}, PlotRange →{ 0, 4}, PlotLabel → "y=0.333 "];
h5 = ParametricPlot[{x[s, 0.4], u[s]},
   {s, 0, 2}, PlotRange →{ 0, 4}, PlotLabel → "y=0.4"];
Show[GraphicsArray[{{h0, h1, h2}, {h3, h4, h5}}].FrameTicks → None, Frame → False]
```

GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.

Show: No graphical objects to show.

Out[252]=

Show



FrameTicks → None, Frame → False