Linear Quadratic Gaussian (LQG) Control Design for Position and Trajectory Tracking of the Ball and Plate System

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***Abstract*—In this paper, the position and trajectory tracking control scheme for the ball and plate system using the double feedback loop structure (a loop within a loop) has been presented. The inner loop was designed using linear algebraic method by solving a set of Diophantine equations. The outer loop was designed using linear quadratic Gaussian (LQG) controller. Simulation was done in MATLAB/SIMULINK environment, and the simulation results showed that the plate was stabilized at 0.3546 seconds, and the ball was able to settle at 1.5075 seconds, when given a circular trajectory of radius 0.4m, with an angular frequency of 1.33 rad/sec, with a trajectory tracking error of 0.0112m, which shows that the controller have strong robustness, adaptability and high control performance for the ball and plate system.**

Keywords—Ball and plate system, linear algebraic method, Diophantine equation, linear quadratic Gaussian (LQG) controller.)

I. INTRODUCTION

Balancing systems is one of the most common and challenging test platforms for the control engineers. Example of such systems include ball and beam, traditional cart-pole system (inverted pendulum) and double and multiple inverted pendulums [1]. The ball and plate system (BPS) consist of a ball that can roll easily on a horizontal flat plate. The ball is required to track a desired path by adjusting the tilt of the plate with respect to two mutually perpendicular directions [2]. The motion of the ball in the BPS has four degree of freedom (DOF), but it is controlled by two actuating inputs, which implies under-actuation [3], and it is an open loop unstable system as the position of the ball becomes unbounded when the plate is tilted around either its x-axis or y-axis [4], which pose a challenge for modelling and control [2]. The system is generalization of the ball and beam benchmark system [5]. However, the system is more complicated than the traditional ball and beam system due to its coupling of multivariable. This under-actuated system has two actuators, and it is stabilized by the two control inputs [6]. The ball and plate system finds application in areas like humanoid robot, satellite control, unmanned aerial vehicle (UAV) and rocket system [7] in the field of path planning, trajectory tracking and friction compensation [8].

A servo system which consist of motor controller card and two servo motors is used for tilting the plate. Intelligent vision system is used for the measurement of the ball from a CCD camera [9]. Motion control of the ball and plate system is to control the position of a ball on a plate for both static positions and desired path trajectory tracking. The slope of the plate can be controlled in two perpendicular directions, so that the tilting of the plate will make the ball move on the plate [9].

However, various control techniques have been used in the recent years for the ball and plate system. A controller design for two electro-mechanical ball and plate system based on classical and modern theories was presented by [10]. In [11], back-stepping control design was proposed based on Lyapunov stability theory. The system was constructed using two magnetic suspension actuators for two degree of freedom control. Various conditions of dynamic operation including oscillatory stabilization and circular trajectory tracking were tested to prove the system performance and capability. [12] designed a controller which confirmed the BPS stability by using a loop shaping method based on Normalized Right Coprime Factos (NRCF) perturbation technique, in which the familiar lead-lag series compensation design methods were innovatively designed to find appropriate pre and post compensators as the weighting functions to guarantee the BPS time domain performance requirements. [13] proposed and examines the application of sliding mode control (SMC) to the ball and plate control problem. The linear full-state feedback controller was compared with the SMC, and their respective performance was experimentally validated on a physical test bed that was designed and constructed for the purpose of the research.

However, a unique motion controller, based on evolved lookup tables have been developed by [14] to move a ball on a set-point on a typical ball and plate system. Also, in order to overcome the problem of instability, nonlinearity and under-actuation, which is attributed to the BPS. [15] proposed the design of cascaded SMC for position control of a ball in a BPS. The efficiency of the proposed controller was tested through simulation analysis, by making the ball to follow a circular and square trajectories, in which the chattering effect was found to be within the acceptable limit.

One of the most effective control scheme for the ball and plate system is the double feedback loop structure, i.e. a loop within a loop. The inner loop works as dc motor servo position controller, while the outer loop control's the position of the ball [16].

In this paper, the ball and plate is considered as a double feedback loop structure for position and trajectory control. The inner feedback loop will be designed based on linear algebraic method, by solving a set of Diophantine equations, while the outer loop will be designed using linear quadratic Gaussian (LQG) controller, which is one of the robust controller.

The rest of the paper is organized as follows. Section 2 introduces the mathematical modelling of the BPS, section 3 discusses the design of the controllers. Section 4 shows the trajectory tracking simulation results and finally section 5 presents the conclusion.

II. MATHEMATICAL MODEL OF THE BALL AND PLATE SYSTEM

Fig. 1 shows a typical laboratory model of the ball and plate system by HUMUSOFT.

Fig. 1. The Ball and Plate System by HUMUSOFT [17]

Fig. 2. Shows the mathematical model of the ball and plate system.

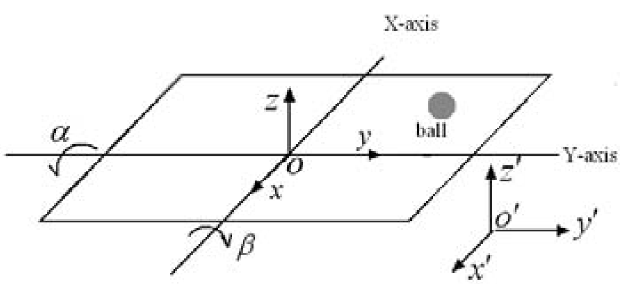

Fig. 2. The Mathematical model of the Ball and Plate System

The plate rotates about the x and y-axis in two perpendicular directions. The kinematic differential equations of the ball and plate system are derived using the Euler-Lagrange equation, given as follows [18]:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \tag{1}$$

Whrere q_i represents the i -direction coordinate; T is the kinetic energy of the system; V the potential energy, and Q is the composite force.

Ball and plate system can be simplified into a particle system made by two rigid bodies; the plate has a geometry limits in translation along x-axis, y-axis and z-axis. It has a geometry limit in rotation about the z-axis. The plate has two degree of freedom in rotation about the x-axis and y-axis. The ball has a geometry limit in translation along the z-axis. It has two degree of freedom along the x-axis and y-axis. The BPS system model has four degree of freedom (DOF), in which the generalized coordinates are [19]:

$q_1 = x$, $q_2 = y$, $q_3 = \alpha$, $q_4 = \beta$.

The total kinetic energy of the system is given as:

$$T = T_{ball} + T_{plate} \tag{2}$$

$$T_{ball} = \frac{1}{2}\left[\left(m_b + \frac{J_b}{{R_b}^2}\right)\left(\dot{x}^2 + \dot{y}^2\right) + J_b\left(\dot{\alpha}^2 + \dot{\beta}^2\right) + m_b\left(x\dot{\alpha} + y\dot{\beta}\right)^2\right] \tag{3}$$

$$T_{plate} = \frac{1}{2}\left[\begin{array}{c}\left(J_{Px}\dot{\alpha}^2 + J_{py}\dot{\beta}^2\right) + \left(m_b + \frac{J_b}{{R_b}^2}\right)\left(\dot{x}^2 + \dot{y}^2\right) \\ + J_b\left(\dot{\alpha}^2 + \dot{\beta}^2\right) + m_b\left(x\dot{\alpha} + y\dot{\beta}\right)^2\end{array}\right] \tag{4}$$

The potential energies of the system along the x-axis and y-axis is given as:

$$V_x = m_b g x \sin\alpha \tag{5}$$

$$V_y = m_b g y \sin\beta \tag{6}$$

And the mathematical equation of the ball and plate system is given as:

$$\left(m_b + \frac{J_b}{{R_b}^2}\right)\ddot{x} - m_b x\left(\dot{\alpha}\right)^2 - m_b y\dot{\alpha}\dot{\beta} + m_b g \sin\alpha = 0 \tag{7}$$

$$\left(m_b + \frac{J_b}{{R_b}^2}\right)\ddot{y} - m_b y\left(\dot{\beta}\right)^2 - m_b x\dot{\alpha}\dot{\beta} + m_b g \sin\alpha = 0 \tag{8}$$

$$\begin{array}{c}\left(m_b x^2 + J_b + J_{Px}\right)\ddot{\alpha} + 2m_b x\dot{x}\dot{\alpha} + m_b xy\ddot{\beta} \\ + m_b\left(\dot{x}y + x\dot{y}\right)\dot{\beta} + m_b g x\cos\alpha = \tau_x\end{array} \tag{9}$$

$$\begin{array}{c}\left(m_b y^2 + J_b + J_{Py}\right)\ddot{\beta} + 2m_b y\dot{y}\dot{\beta} + m_b xy\ddot{\alpha} \\ + m_b\left(\dot{x}y + x\dot{y}\right)\dot{\alpha} + m_b g y\cos\beta = \tau_y\end{array} \tag{10}$$

$m_b\left(kg\right)$ represents the mass of the ball, $J_b\left(kgm^2\right)$ gives the rotational moment of inertia of the ball, $J_{P_x}\left(kgm^2\right)$ and $R_b\left(m\right)$ is the rotational moment of inertia of the plate and the radius of the ball; $x\left(m\right)$ and $y\left(m\right)$ is the position of the ball along the x-axis and y-axis; $\dot{x}\left(m/s\right)$ and $\ddot{x}\left(m/s^2\right)$ are the velocity and acceleration along the axis; $\dot{y}\left(m/s\right)$ and $\ddot{y}\left(m/s^2\right)$ are the velocity and acceleration along the y-axis; $\alpha\left(rad\right)$ and $\dot{\alpha}\left(rad/\sec\right)$ are the plate deflection angle, and angular velocity about x-axis; $\beta\left(rad\right)$ and $\dot{\beta}\left(rad/\sec\right)$ are the plate deflection angle and angular velocity about y-axis; $\tau_x\left(Nm\right)$ and $\tau_y\left(Nm\right)$ are the torques on the plate in the x-axis and y-axis.

Equation (7) and (8) describe the movement of the ball on the plate; it shows how the effect of the ball acceleration relies on the plate deflection angle and its angular velocity; also, equation (9) and (10) describe how the dynamics of the plate deflection rely on the external driving forces, and the ball position [20].

Considering the state variable assignment [21]:

$$X = \left(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\right)^T = \left(x, \dot{x}, \alpha, \dot{\alpha}, y, \dot{y}, \beta, \dot{\beta}\right)^T \tag{11}$$

And the state space equation of the ball and plate system is as follows [21]:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} x_2 \\ B\left(x_1 x_4^2 + x_4 x_5 x_8 - g\sin x_3\right) \\ x_4 \\ 0 \\ x_6 \\ B\left(x_5 x_8^2 + x_1 x_4 x_8 - g\sin x_7\right) \\ x_8 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix} \tag{12}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \left[X\right] \tag{13}$$

$$B = \frac{m_b}{\left(m_b + \frac{J_b}{R_b^2}\right)} \tag{14}$$

In steady state, the plate should in the horizontal position where both the inclination angles are equal to zero, if the inclination angle does not have much change, i.e. $\pm 5^0$, the sine function can be replaced by its argument. Then, the mathematical model of the BPS can be simplified and decomposed into x and y-axis as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ B\left(x_1 x_4^2 - g\sin x_3\right) \\ x_4 \\ 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \left[u_x\right] \tag{15}$$

$$\begin{bmatrix} \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} x_6 \\ B\left(x_5 x_8^2 - g\sin x_7\right) \\ x_8 \\ 0 \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \left[u_y\right] \tag{16}$$

III. CONTROLLER DESIGN

A. Determination of the Actuator Parameters

The actuator with a permanent DC motor is considered in the design of the inner loop. The relationship between θ_L and e_a is given as [22]:

$$\frac{\theta_L}{e_a} = \frac{K_t \frac{N_1}{N_2}}{\left[J_{eq} L_a s^3 + \left(J_{eq} R_a + D_{eq} L_a\right) s^2 + \left(D_{eq} R_a + K_t K_e\right) s\right]} = T_L \tag{17}$$

The parameters of the DC motors are derived based on the requirements of the load torque, moment of inertia, and the speed of the motor. This is given in Table I.

TABLE I. PARAMETERS OF THE BALL AND PLATE SYSTEM [23]

S/N	Description	Symbol	Unit	Value
1	Mass of the ball	m	kg	0.11
2	Radius of the ball	R	m	0.02
3	Dimension of the plate (square)	lxb	m^2	0.16
4	Mass moment of inertia of the plate	$J_{Px,y}$	kgm^2	0.5
5	Mass moment of inertia of the ball	J_b	kgm^2	1.76e-5
6	Maximum velocity of the ball	V	m/s	0.04

Equation (17) is given as:

$$\frac{\theta_L}{e_a} = \frac{0.105}{\left[0.47005s^2 + 421.113s\right]} \tag{18}$$

$$= \frac{0.2234}{s\left(s + 895.89\right)} \approx \frac{2.49x10^{-4}}{s} \tag{19}$$

B. Two-port Parameter Configuration

The two-port parameter configuration was used in the design of the inner loop. In order to find the value of ω_o, a step response is required, which could settle in 0.4 seconds. From the Humusoft ball and plate system manual, through simulation, $\omega_o = 20\,rad/\text{secs}$ was found to give the response. The ITAE optimal overall optimal overall transfer function with zero position error of the system $G_0(s)$ is [24]:

$$G_0(s) = \frac{\omega_0^2}{s^2 + 1.4\omega_0 s + \omega_0^2} \tag{20}$$

Additional gain of 494 was provided to limit the step response using a preamplifier. $G_0(s)$ is implemented as shown in Fig.3 using the two-port configuration.

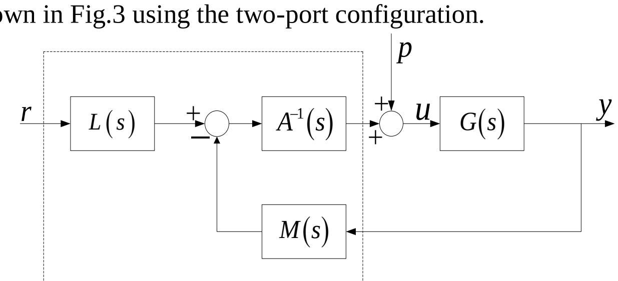

Fig. 3. Two-Port Parameter Configuration

Where $L(s)$, $M(s)$ and $A(s)$ are the polynomials that describes the compensator, p is the input disturbance. Solving the Diophantine equation, the compensator and the DC motor actuator has the following values.

$$A(s) = 28 + s \tag{20}$$

$$M(s) = 3252 + s \tag{21}$$

$$L(s) = 3252 \tag{22}$$

C. Linear Quadratic Gaussian (LQG) Control

This is considered a robust control method because the state output equations is explicitly considered. However, quantitative information about the noise is taking into consideration in the controller design [25]. Given the state space equation of the plant [25]:

$$\dot{x}(t) = Ax(t) + Bu(t) + \Gamma\xi(t) \tag{23}$$

$$y(t) = Cx(t) + \theta(t) \tag{24}$$

$\xi(t)$ and $\theta(t)$ are random noises in the state equation and output measurements. Assuming that $\xi(t)$ and $\theta(t)$ as the mean zero Gaussian random process with covariance matrices, which is given by [25]:

$$E\left[\xi(t)\xi^T(t)\right] = \Xi \geq 0,\ E\left[\theta(t)\theta^T(t)\right] = \Theta > 0 \tag{25}$$

$E[x]$ is the mean value of x, and $E\left[xx^T\right]$, the covariance matrix of the zero mean Gaussian signal x. However, the random signals $\xi(t)$and $\theta(t)$ are also assumed to be mutually independent [25]:

$$E\left[\xi(t)\xi^T(t)\right] = 0 \tag{26}$$

The states can be approximated optimally when a Kalman filter, rather than an observer, is used. An optimal state estimation $\hat{x}(t)$, which minimizes the covariance $E\left[(x-\hat{x})(x-\hat{x})^T\right]$, and the estimated signal, $\hat{x}(t)$ is used to replace the actual state variables in such a way that the problem can be modified to an LQ optimal control problem. The Kalman filter gain matrix can be written as [25]:

$$K_f = P_f C^T \Theta^{-1} \tag{27}$$

Where P_f satisfies the algebraic Riccatti equation (ARE) [25]:

$$P_f A^T + AP_f - P_f C^T \Theta^{-1} C P_f + \Gamma \Xi \Gamma^T = 0 \tag{28}$$

P_f is a symmetrical semi-positive-definite matrix $P_f = P_f^{\,T} \geq 0$. Obtaining the optimal filter signal $\hat{x}(t)$, the block diagram of the LQG compensator can be constructed as shown in Fig. 4.

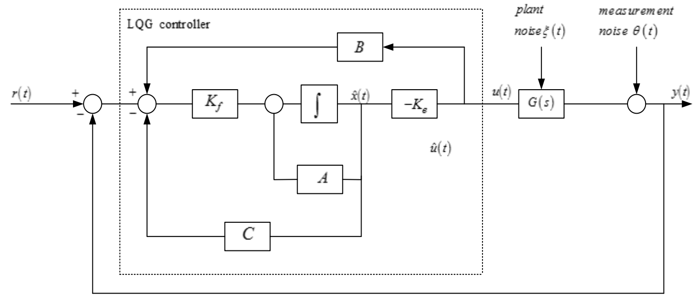

Fig. 4. LQG Control Structure

The optimal control $u^*(t)$ is given as:

$$u^*(t) = -K_c \hat{x}(t) \tag{29}$$

Also, the optimal state feedback matrix K_c is given as:

$$K_c = R^{-1} B^T P_c \tag{30}$$

The symmetrical semi-positive-definite matrix should satisfy the (ARE), which is given in equation (31).

$$A^T P_c + P_c A - P_c B R^{-1} B^T P_c + M^T Q M = 0 \tag{31}$$

Observing that in the LQG optimal control problem, the optimal estimation and optimal control problem can be solved separately, which is the well-known separation principle. In order to design the LQG controller, the state estimator could be designed first, then use the estimated state as if the states were exactly measurable to design the LQR state feedback controller.

The optimization criterion is given as [25]:

$$J = \lim_{t_f \to \infty} E\left\{ \int_0^{t_f} \left[x^T\ u^T \right] \begin{bmatrix} Q & N_c \\ N_c^{\,T} & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} dt \right\} \tag{32}$$

N_c can normally be chosen as a zero matrix, Q and R are the weighted matrices; Q is semi-positive-definite, and R is positive-definite [26]. The state feedback matrix K_c, and Kalman filter gain matrix K_f have been obtained from the separation principle. The Kalman filter dynamic equation is given as [25]:

$$\hat{x} = A\hat{x} + Bu + K_f\left(y - C\hat{x} - Du\right) \tag{33}$$

The observer based LQG controller can be concisely given as [25]:

$$G_c(s) = \left[\begin{array}{c:c} A - K_f C - BK_c + K_f D K_c & K_f \\ \hdashline K_c & 0 \end{array} \right] \tag{34}$$

IV. RESULTS OF THE SIMULATION

The following simulation results were obtained from MATLAB 2016a software.

A. Inner Loop Design

The step response of the actuator is shown in Fig. 5.

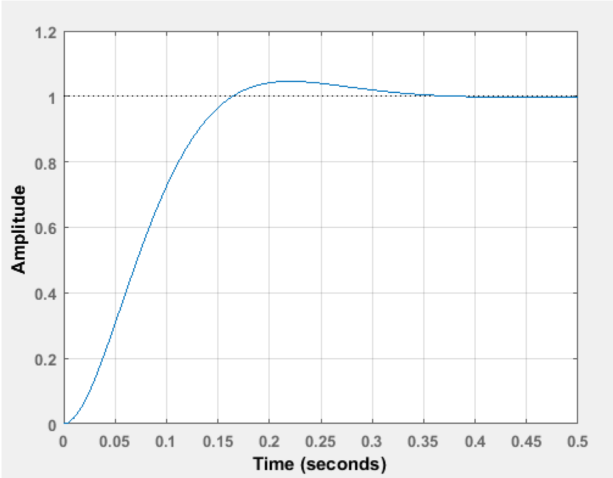

Fig. 5. Step Response of the Actuator

Table II shows the properties of the actuator

Table II. Properties of the Actuator

System Response	Value
Settling Time (sec)	0.2989
Overshoot (%)	4.5989

From Table II, the settling time of the actuator is 0.2989 seconds, which shows that the plate will settle before 0.4 seconds that was set for it. However, the actuator $U(s)$ due to a step input should not exceed threated DC motor voltage, which is 75V. Fig. 6 shows the step response of the actuator with open parameters (rated power and voltage) of the DC motor.

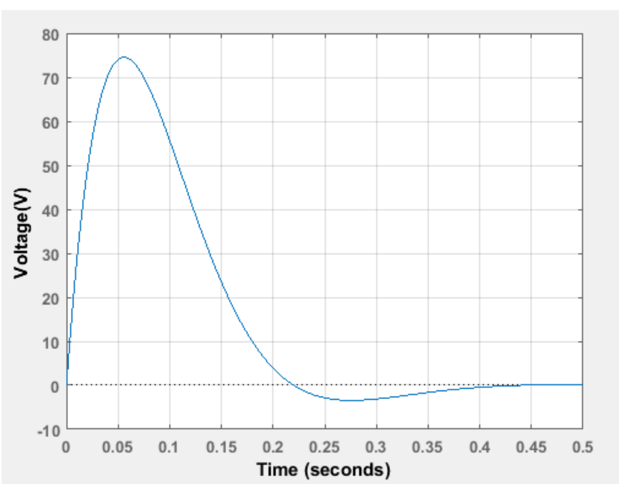

Fig. 6. Step Response of the Actautor with Open Parameters

From Fig. 6, the plate stabilized at 0.3546 seconds. Also, the peak voltage is 74.56V, which is closer to the rated voltage of 75V. from this, it shows that a proper inner loop design of the DC motor actuator has the following properties, which is given in Table III

Table III. Properties of the Actuator with Open Parameters

Actuator System Response	Value
Settling Time (sec)	0.3546
Peak Voltage (V)	74.5631

B. Outer Loop Design

The parameters of the LQG controller are:

$$Q = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R = 0.1$

$\Xi = 7e-03$

$\Theta = 1e-04$

The step response of the LQG controller is given in Fig. 7.

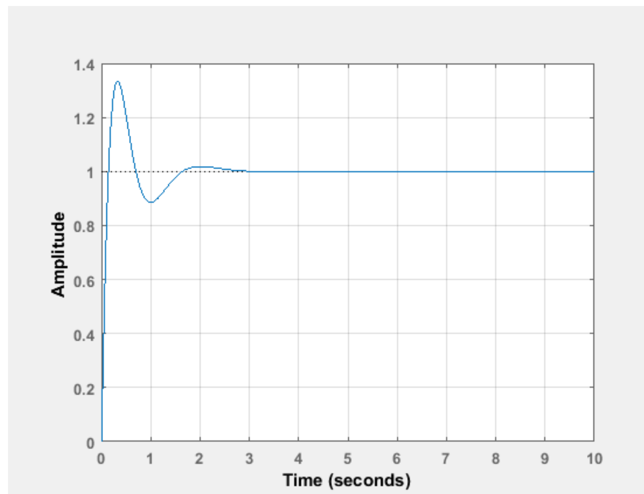

Fig. 7. Step Response of the LQG Controller

From Fig. 7, the properties of the LQg controller is given in Table IV.

Table IV. Properties of the LQG Controller

LQG Controller System Response	Value
Settling Time (sec)	1.5075
Overshoot (%)	33.4996

From Table IV, it shows that the LQG controller stabilized the ball at 1.5075 seconds, with an overshoot of 33.4996 %. This shows a good indication of tracking the ball on the desired path on the plate.

A circular trajectory of radius 0.4 m, and a sinusoidal reference input of $x = 0.4(1-\cos\omega t)$ and $y = 0.4(\sin\omega t)$ was taken into consideration, and was used to demonstrate the trajectory tracking performance of the ball. The angular frequency of the sinusoidal reference signal used was 1.33 rad/sec. this is shown in Fig. 8.

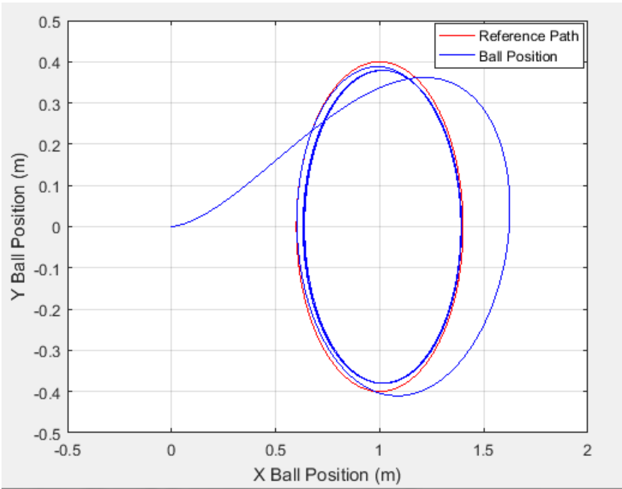

Fig. 8. Circular Trajectory Tracking using LQG Controller

The ball was allowed to track a circular trajectory of 0.46 rad/sec at a complete revolution of 13.6 seconds. When the speed was increased to 0.8 rad/sec, at a complete revolution of 8 seconds, the trajectory tracking error increased.

However, it was observed that using the LQG controller, the staedy state tracking error of the ball is 0.0112m, which shows that the ball was able to track the circular reference signal with a trajectory tracking error of 0.0112m.

V. CONCLUSION

The position and trajectory tracking control of the ball and plate system using a double feedback loop structure (a loop within a loop) has been proposed. The inner loop was designed using linear algebraic method by solving a set of Diophantine equation, while the outer loop was designed using LQG controller. The results of the simulation shows that the controllers has strong robustness, adaptability and high control performance for the ball and plate system. However, future research work will consider incorporating artificial intelligent techniques with the controllers for optimal path trajectory tracking of the ball on the plate.

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