Case 2#

Optimizing network design to minimize costs

Applied Machine Learning and Optimization
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1. Executive summary

This report is motivated by two main questions regarding the network design for the shipment of coffee beans from Brazil and Colombia: (i) is the current dock selection –namely using Rotterdam and Amsterdam– going to be optimal moving forward given the information currently available on prices, or should Beanie Limited expand its commercial relations to further options across Europe? and (ii) is the current allocation criteria optimal, or is there room for cost reduction were Beanie Limited to change the strategy regarding how many containers to ship to each dock in use?

In reality, both these questions aim to tackle the objective of minimizing costs related to coffee shipment activities. As it becomes patent during the report, the answer to both questions is affirmative:

(i) Given an amount X of containers to ship and the prices provided in the table (appendix 1), we can obtain a reduction in overall costs by expanding our dock selection in the scenario at hand because both Rotterdam and Amsterdam are dominated –that is, there's a cheaper alternative– by at least one other dock in terms of price per container (appendix 1), even in those cases where there are fixed costs; and

(ii) since the space of possible alternatives is so big (for illustration purposes, even if we were to use only three docks, there would be over 288 million possible combinations to ship exactly 24,000 containers) the probability of obtaining the optimal allocation from selecting the docks following an arbitrary rule –as is the case with the current policy, everything is routed through Rotterdam while possible– is extremely slim, which means that developing a mathematical model to optimize the costs will yield a more cost efficient policy than the one currently in use, all other things being equal. As an example, if you were to ship 24,000 containers (as you did last year) using prices for next year, the total costs would be 11,952,000€, but with our policy Beanie Limited they would come down to 6,840,000€, so changing exclusively the strategy would mean a **total saveup of 6,860,000€.**

One important disclaimer, however, is that we can guarantee that the shipping strategy will be optimal, but we cannot guarantee that it will always produce a lower total cost nor a lower cost per container than the 13.7 million euros (570.83€/container) spent last year. That is because there are four factors that alter the costs: price per container shipped (variable cost), sign up fee (fixed costs), amount of containers to be sent, and the allocation. We can only optimize for the allocation of containers, but have no control over the other three factors. In practice, this means that the cost will be the lowest *it can be* given the amount of containers to be sent, the fixed and variable costs, but we can't guarantee it will be necessarily lower than the 570.83€/container of last year. Indeed, this is the situation we find ourselves in, we cannot lower the total costs (although in this case we are in fact lowering the *per container* cost) because the prices per container shipped in each dock differ from last year, and so does the amount of containers to be sent, but the strategy we propose is optimal regardless.

Moving on to the results, next year's cost to ship 50,506 containers in the event there are no sign up fees and no restrictions on the amount of docks (level 2), will be 16,392,820€. This is an increase of just 2,692,820€ from last year for more than twice the quantity of containers, which translates into a cost decrease of 43%, or 246.26€ (from 570.83€ to 324.57€) in terms of cost per container.

In the second scenario (level 3), where sign up fees are present and only up to three docks can be selected, the cost of shipping 50,506 containers will be 18,977,820€. This is again above last year's cost of 13.7 million euros –which means a total increase of 5,277,820€– but a save up of 34.17% in terms of price per container, or 195.08€ (from 570.83€ to 375.75€).

Details on exactly where and how much to ship are available in table format in their respective levels in the methodology section, but as a highlight, in all three situations considered during the report we are using Algeciras to its maximum capacity (20,000) so on a strategic level, fostering a good relationship with the dock may prove to be quite beneficial no matter the scenario to (1) potentially get a reduction in costs, which even if minimal could translate into a few thousand euros, and (2) improving certainty about lead times whenever the dock goes through periods of high activity and reducing the chance that the stock gets held off were a bottleneck to happen.

2. Methodology

a. Level 1

Once we have examined the data provided we have been able to analyze your costs and strategy. Last year's policy was entering 80% of Beanie Limited containers through Rotterdam and the remaining 20% through Amsterdam. The total volume was around 24,000 containers. Your strategy led to a price of 570.83€ per container; about 10,960,000€ was spent in Rotterdam and 2,740,000€ in Amsterdam.

What would happen would you have applied another strategy? Now, we will explain to you an alternative strategy that optimizes your costs. We have maintained the volume of containers but we changed the allocation of the docks (i.e. a different allocation strategy), assuming we could use the full range of docks. We propose you to ship 20,000 containers through Algeciras, this leads to a cost of 5,600,000€ and 4,000 containers through Valencia, this leads to a cost of 1,240,000€. So, the total cost is 6,840,000€. If we compare this strategy to your previous one, you are saving 6,860,000€.

To examine if you could reduce costs during next year, we have fixed your strategy and changed the prices to the expected ones. The expected price to ship a container in Rotterdam is 470€ and 610€ in Amsterdam. So, the total cost leads to an amount of 9,024,000€ in Rotterdam and 2,928,000€ in Amsterdam. We have maintained the volume of containers shipped, this is 4,800 in Amsterdam and 19,200 in Rotterdam. The total cost if we maintain the strategy but we change the prices is 11,952,000€. Next year cost would be lower than this current year. So, we can conclude that next year prices for Rotterdam and Amsterdam will be lower –even if COVID has bumped up prices of transport– and you would save 1,748,000€ maintain the same strategy and same number of shipped containers.

The last thing that we have studied is aggregating all factors that factor into the total cost. What will happen next year with an optimal strategy? Here we will clear some doubts. In level 2, we have seen that the cost per container reaches 324.57€, saving 246,26€ per container. In level 3, we have brought to a cost of 375.75€ per container, saving 195,08€ per container. In level 4, we have an interval of some possible prices, these are 375.75€ as the best possible situation and 391.30€ as the worst possible situation. This led to an expected value of 377,48€ per container, saving 193,35€ per container. All these prices are below your current price (570.83€). So, applying any of these scenarios you are reducing costs.

Regarding the two questions that were laid down during the meetings,

The two questions laid down during the meetings were whether (i) there is a chance to reduce costs and whether costs will be lower next year. The latter question has been answered in the last paragraph, as well as the executive summary. Regarding the former, as stated in the executive report, we can't categorically say there will be a reduction in prices (neither overall nor per container prices). The reduction in prices is uncertain because the allocation strategy is just one part of what composes the overall price, but other factors that are not under out control, such as total containers shipped fixed and variable costs can make it so only overall costs or both overall and per container costs increase. In normal circumstances, however, because the strategy used up until now was far from optimal, we can expect (but not guarantee) that per container prices will indeed be lowered.

b. Level 2

In order to reach the optimal distribution for next year that minimizes costs in the event we have eight docks to choose from and no sign up fees, Beanie Limited has to minimize the target function to optimize the use of docks by the company.
The target function for the problem consists in the number of containers used in each dock multiplied by their respective price, mathematically:

$$\arg\min_{x_i} = \sum_{i=1}^{I} p_i \cdot x_i$$

$where:$
$p_i: price\ per\ container\ in\ dock\ i$
$x_i: units\ allocated\ to\ dock\ i$

Dock i	Price per container in dock i	max_capacity of dock i
Rotterdam	470	33000
Antwerp	470	25000
Hamburg	480	44000
Amsterdam	610	11000
Marseille	380	9000
Algeciras	280	20000
Valencia	310	11000
Genoa	340	7500

About the constraints used, must be taken into account different considerations:

- (1) The sum of all the containers used in each port should be equal or greater than the volume that is expected to arrive (although a *greater than* inequality is provided, minimizing the objective function implies that the optimum allocation will equate *S* = 50,506).
- (2) It must also be taken into account that there is a maximum capacity that each port is able to handle and the number of containers we send to each dock should be less than their actual capacity.

- (3) Non-negativity: If we do not consider these constraints we would not obtain an optimal result since the variables could take minus infinitive values, which is clearly not realistic.
- (4) Integer values: Since containers are used in whole units and they must be paid completely even if you do not occupy the entire container the optimal dock's usage should be calculated with an integer number of containers in each dock.

Constraints

$$\sum_{i=1}^{I} x_i \geq S \tag{1}$$

$$x_i \leq c_i \, \forall i \tag{2}$$

$$x_i \geq 0 \, \forall i \tag{3}$$

$$x_i \text{ integer } \forall i \tag{4}$$

where:

S: Total number of containers to ship = $\frac{1500000 \cdot 1000}{66 \cdot 450} = 50{,}506$

c_i: *capacity in dock i*

After solving the problem with the target function and the constraints detailed above we get that the optimal solution is to use:

- Algeciras 20.000 containers (full capacity)
- Antwerp/Rotterdam 3.006 containers
- Genoa 7.500 containers (full capacity)
- Marseille 9.000 (full capacity)
- Valencia 11.000 (full capacity)

We are indifferent either to using the dock of Antwerp or Rotterdam since they both have the same price per container so both options can be optimal, so the minimum cost can still be achieved anyway if there's a strategic preference to either one of them. There is also another possibility which is probably more expensive due to greater complexity, but in case there are no costs for using one more dock, any combination that sums between Rotterdam and Antwerp 3006 containers would be optimal. This means that, although perhaps not practical, just one container in Antwerp and 3,005 containers in Rotterdam or 1503 containers in Antwerp and 1503 in Rotterdam would be two examples of optimal combinations that would lead to an efficient cost to the company.

The quantity of the other docks that were not mentioned before is zero since they are more expensive and we can fulfill the volume needed with the cheaper ones.

The total cost is thus 16,392,820€, it results from multiplying the optimal quantities by the price per container for each dock. The price per container in this case is 324.57€. As we can see, our total cost increased this year but the reason behind it is the increase in volume required (which more than doubled), not the price per container since it's significantly lower this year.

Recap: Optimal allocation

Dock	Optimal quantity
Algeciras	20,000
Amsterdam	0
Antwerp + Rotterdam	3,006
Genoa	7,500
Hamburg	0
Marseille	9,000
Valencia	11,000
Total cost	**16,392,820€**

c. Level 3

You asked us if we could solve the same problem but with new constraints. The first new restriction is that you only want to use a maximum of three different docks. Moreover, some docks have a sign-up fee, this is an only paid fee and independent of the container volume. The fees are 800,000€ to Algeciras, 500,000€ to Marseille and 1,000,000€ to Antwerp.

With this data, we present you the target function:

$$\arg\min_{x_i,\theta_j} = \sum_{i=1}^{I} p_i \cdot x_i + f_i\theta_i$$

$where$:

p_i: *price per container in dock i*
x_i: *units allocated to dock i*
$\theta_i: \begin{cases} 1 \text{ if dock } i \text{ is in use} \\ 0 \text{ otherwise} \end{cases}$
f_i: *sign up fee in port i*

Dock i	Price per container in dock i	max_capacity of dock i
Rotterdam	470	33000
Antwerp	470	25000
Hamburg	480	44000
Amsterdam	610	11000
Marseille	380	9000
Algeciras	280	20000
Valencia	310	11000
Genoa	340	7500

Dock j	Sign-up fee in dock j
Rotterdam	-
Antwerp	1,000,000
Hamburg	-
Amsterdam	-
Marseille	500,000
Algeciras	800,000
Valencia	-
Genoa	-

We want to specify the new variables; these are the binary ones. We have used the binary variables because there are two possible options: 1 if we use this dock and 0 otherwise. These variables have only been made to take into account the sign-up fee. For this reason, we only used the variables with docks that have a sign-up fee.

About the constraints used, must be taken into account different considerations:

- (1) The sum of all the containers used in each port should be equal or greater than the volume that is expected to arrive.
- (2) It must also be taken into account that there is a maximum capacity that each port is able to handle and the number of containers we send to each dock should be less than their actual capacity. In this level we multiply it by the binary that indicates whether we use the dock to link it with the objective function

- (3) Non-negativity: If we do not consider these constraints we would not obtain an optimal result since the variables could take minus infinitive values, which is clearly not realistic.
- (4) Integer values: Since containers are used in whole units and they must be paid completely even if you do not occupy the entire container the optimal dock's usage should be calculated with an integer number of containers in each dock.
- (5) Maximum number of docks: we have been told that we must use a maximum of three different docks.
- (6) Binary variables: this type of variable can only take two values, 0 or 1.

Constraints

$$\sum_{i=1}^{I} x_i \geq S \tag{1}$$

$$x_i \leq c_i\theta_i \ \forall i \tag{2}$$

$$x_i \geq 0 \ \forall i \tag{3}$$

$$x_i \ integer \ \forall i \tag{4}$$

$$\sum_{i=1}^{I} \theta_i \leq 3 \tag{5}$$

$$\theta_i \ binary \ \forall i \tag{6}$$

where:

S: Total number of containers to ship = $\frac{1500000 \cdot 1000}{66 \cdot 450} = 50{,}506$

$c_i: capacity\ in\ dock\ i$

After we solve the problem with the target function and the constraints detailed above we get that the optimal solution is to use:

- Algeciras: 20.000 containers (full capacity)
- Rotterdam: 19,506
- Valencia: 11.000 (full capacity)
- Total cost: 18,977,820€

To sum up, we want to show you the impact of this restriction in comparison with level 2. We can see that in level 2 (without maximum dock and without fees), you had a total cost of 16,392,820€. The impact of these new restrictions increased the cost to 18,977,820€. This represents an increase of **2,584,990€**. In level 2, we told you to use the full capacity of Algeciras, Genoa, Marseille and Valencia. These docks are the cheaper ones and some of them have little capacity. This is why, in this level you must prioritize docks with larger capacity when putting a maximum amount of docks restriction. For this reason, in this situation we told you to use the full capacity of Algeciras and Valencia and some capacity of the Rotterdam dock.

We can conclude that the price per container in level 2 is 324.57€ compared to level 3 that is 375.75€. So, on average, there is a difference of +51.18€ per container. So, it is more expensive in this situation than the previous one, but still cheaper than the baseline of last year by 34.17%, or 195.08€ (from 570.83€ to 375.75€.

Recap: Optimal allocation

Dock	Optimal quantity
Algeciras	20,000
Amsterdam	0
Antwerp	0
Genoa	0
Hamburg	0
Marseille	0
Rotterdam	19,506
Valencia	11,000
Total cost	**18,977,820€**

d. Level 4

In this section, we face uncertainty about the prices for Valencia and Algeciras. Your expert thinks that the price in Valencia can end up somewhere between today's price 310€/40ft and 390€/40ft. For Algeciras, your expert thinks the price can end up somewhere between today's price 280€/40ft and 330€/40ft. Both ranges follow a uniform distribution, so any combination of prices is equally as likely to happen.

Simulation methodology

To carry out the simulation, we will iterate through every possible price for Valencia and Algeciras (a total of 4,131 combinations) and compute the optimal solution and its cost for every one of them. Using this approach will allow us to carry out a sensitivity analysis to evaluate how optimality changes with prices for each dock, and see how the optimal costs and allocations are distributed. For the sake of simplicity, and because there's no need to simulate otherwise, we will assume prices are integer values, so no decimal places will be used throughout the simulation.

Target function

In order to analyze this problem, we have carried out a simulation. The same target function from level 3 will be used, with the exception that we prices for Algeciras and Valencia will change. Note, however, that the target function will remain unchanged, but p_{Alg} and p_{Val} will change in each iteration.

$$\underset{x_i,\theta_j}{\arg min} = \sum_{i=1}^{I} p_i \cdot x_i + f_i\theta_i$$

$where:$

$$p_i: price\ per\ container\ in\ dock\ i$$
$$x_i: units\ allocated\ to\ dock\ i$$
$$\theta_i: \begin{cases} 1\ if\ dock\ i\ is\ in\ use \\ 0\ otherwise \end{cases}$$
$$f_i: sign\ up\ fee\ in\ port\ i$$

Dock i	Price per container in dock i	max_capacity of dock i
Rotterdam	470	33000
Antwerp	470	25000
Hamburg	480	44000
Amsterdam	610	11000
Marseille	380	9000
Algeciras	Range: [280, 330]	20000
Valencia	Range: [310, 390]	11000
Genoa	340	7500

Dock j	Sign-up fee in dock j
Rotterdam	-
Antwerp	1,000,000
Hamburg	-
Amsterdam	-
Marseille	500,000
Algeciras	800,000
Valencia	-
Genoa	-

$\boldsymbol{Constraints}$

Because the constrains are exactly the same as in level three, we will not be explaining them one but one, but the mathematical equations are show next for the sake of readability:

$$\sum_{i=1}^{I} x_i \geq S \tag{1}$$

$$x_i \leq c_i\theta_i \; \forall i \tag{2}$$

$$x_i \geq 0 \; \forall i \tag{3}$$

$$x_i \; integer \; \forall i \tag{4}$$

$$\sum_{i=1}^{I} \theta_i \leq 3 \tag{5}$$

$$\theta_i \; binary \; \forall i \tag{6}$$

$\boldsymbol{where}$:

S: Total number of containers to ship = $\frac{1500000 \cdot 1000}{66 \cdot 450} = 50{,}506$

$c_i: capacity \; in \; dock \; i$

Results

The results that we found are two possible scenarios; using Algeciras, Rotterdam, and either Valencia or using Genoa.

	Alg	***Ams***	***Ant***	***Gen***	***Ham***	***Mar***	***Rot***	***Val***	***Total cost***	***Prob***
Scenario 1	20,000	0	0	0	0	0	19,506	11,000	18,977,820€	88.89%
Scenario 2	20,000	0	0	7,500	0	0	23,006	0	19,762,820€	11.11%

You will carry out the first scenario when Valencia dock has a price between 310€ to 381€. In all these possible prices, you will have a total cost of 18,9M€. Otherwise, when the price of Valencia dock increases to a range between 382€ to 390€, you will move to the second scenario and your cost will increase to 19,7M€. Between these possible scenarios you could have a difference in cost of 785,000€.

Note that only the price for Valencia can change the optimal allocation, but changes in the price for Algeciras won't make us change the optimal allocation.

In scenario 1 the total cost will be 18,977,820€, for an increase of 5,277,820€ over the baseline of last year, or a per container save up of 179.53€. In scenario 2 the total cost will be 19,762,820€, which would increase prices by 6,062,820 from last year, or saving up 193.35€ per container

The expected value of these two scenarios is 19,064,170€.

$$E(X) = Total\ Costs_{Scenario\ 1} * 0.8889 + Total\ Costs_{Scenario\ 2} * 0.1111 = 19{,}064{,}170$$

This means an expected increase from last year of 5,364,170€.

The price per container in scenario 1 is 375.75€ and in scenario 2 is 391.30€. The expected value is 377.48€ per container.

$$E(X) = 375.75 * 0.8889 + 391.30 * 0.1111 = 377.48$$

In this situation, you are saving a range of 179.53€/container to 195.08€/container depending on which scenario you will end up to. The expected value of the savings goes to 193.35€/container.

Appendix 1

dock	40_ft_container_price_eur	max_capacity
Rotterdam	470	33000
Antwerp	470	25000
Hamburg	480	44000
Amsterdam	610	11000
Marseille	380	9000
Algeciras	280	20000
Valencia	310	11000
Genoa	340	7500