Wind turbine efficiency

February, 2019

A. Drag coefficient variability

1. Imagine a large modern 3-blade turbine installed on a flatbed rolling on rails with negligible track friction.

2. Push the flatbed at a speed of 6.5 m/s (14.5 mph), which is the design speed for most large turbines.

3. Since the blades fill only 5% of the swept area and the drag coefficient is most probably less than 0.5 (the drag coefficient of a cone), the total equivalent drag coefficient over the whole swept area for a locked rotor would be less than C_D=2.5%.

4. Meanwhile, such a large turbine would exhibit a C_p equal to 50% when the rotor is released, C_P being the power coefficient, by analogy with the drag coefficient.

5. C_p is therefore at least 20 times greater than C_D.

6. The power required to push the flatbed at speed V with the rotor locked would be $P_B=1/2.C_D.\rho,S.V^3$ (ρ being the density of air, and S the surface of the swept area).

7. The power delivered by the locked turbine pushed on the flatbed at speed V is 0.

8. However, after unlocking the rotor, the power delivered by the turbine pushed on the flatbed at speed V would be $P_A=1/2.C_p.\rho,S.V^3$ (ρ being the density of air, and S the surface of the swept area).

9. $P_A > 20.P_B$

10. So what becomes of C_D when the rotor is unlocked? Could it be the same as when the rotor is locked? No, C_D would need to increase in order to be at least equal to C_p, since if C_D were less than C_p the power produced by the system would be greater than the power required to push it, which would be nice, but miraculous.

11. In the absence of friction, if the rotor were allowed to be freewheeling, both C_p and C_D would be equal to zero.

12. When the rotor is released in a wind of speed V while a resisting torque is applied, C_D increases more than 20-fold. Static C_D is less than 1/20th of dynamic C_D. Dynamic C_D must be at least equal to C_P.

B. Power as a function of blade number

Below is a set of representative formulas devised by the author of this paper:

$$P_n = \frac{1}{2} \cdot C_{Pn} \cdot S \cdot V_i^3$$

$$C_{Pn} = \frac{1}{2} \cdot \left(1 + \beta^n\right) \cdot \left(1 - \beta^{2n}\right)$$

$$P_n = \frac{1}{4} \cdot \rho \cdot S \cdot \left(1 + \beta^n\right) \cdot \left(1 - \beta^{2 \cdot n}\right) \cdot V_i^3$$

where β is the ratio V_e/V_i, V_i being the speed of air near the intake and V_e the speed of air near the exhaust, ρ is the air density, S is the system surface area also called the swept area), and n is the number of blades, or of any other gizmo designed to make the system produce power.

P_n is the power and C_{Pn} the power coefficient, by analogy with the drag coefficient.

Depending on how the blades are designed for a certain wind air speed, different behaviors will be observed, and, in addition, turbines have to be designed to withstand winds much stronger than at benign design wind speeds.

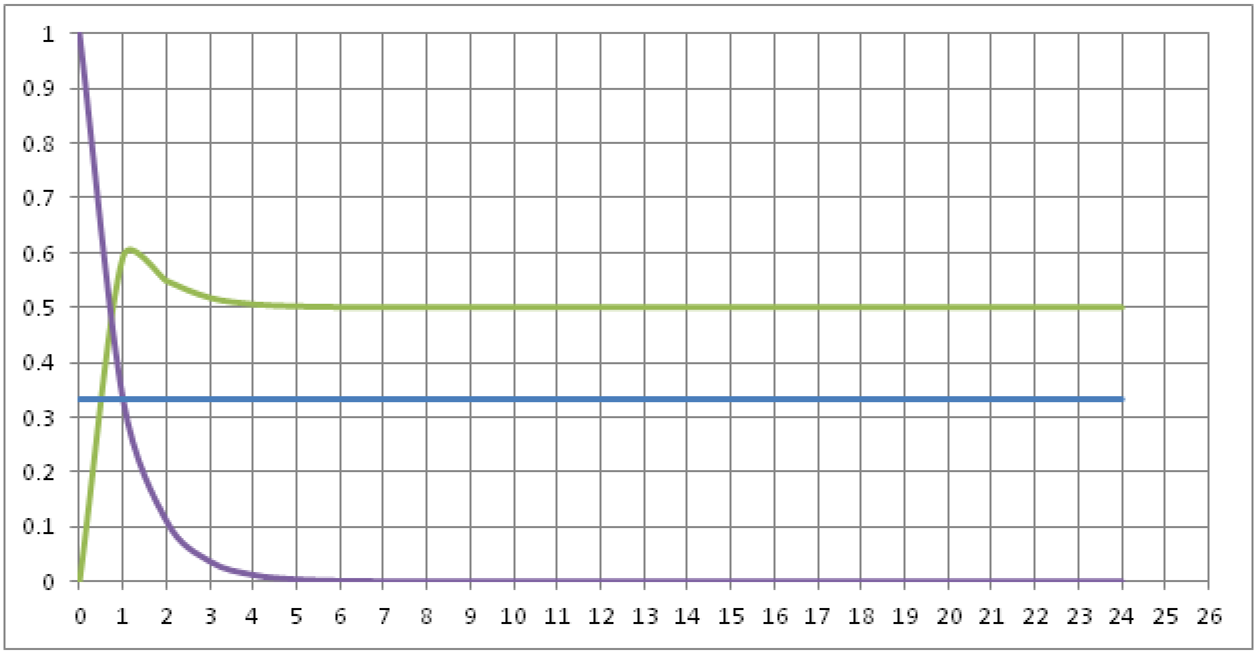
Cases 1 to 3 below assume a 6.5 m/s, 14.5 mph wind speed, while cases 4 to 6 assume a 10 m/s, 22 mph wind speed.

In the graphs below,

- The abscissa represents the number of blades
- The horizontal blue line is the single-blade β
- The purple curve represents the overall β for n blades
- The green curve is the overall C_P.

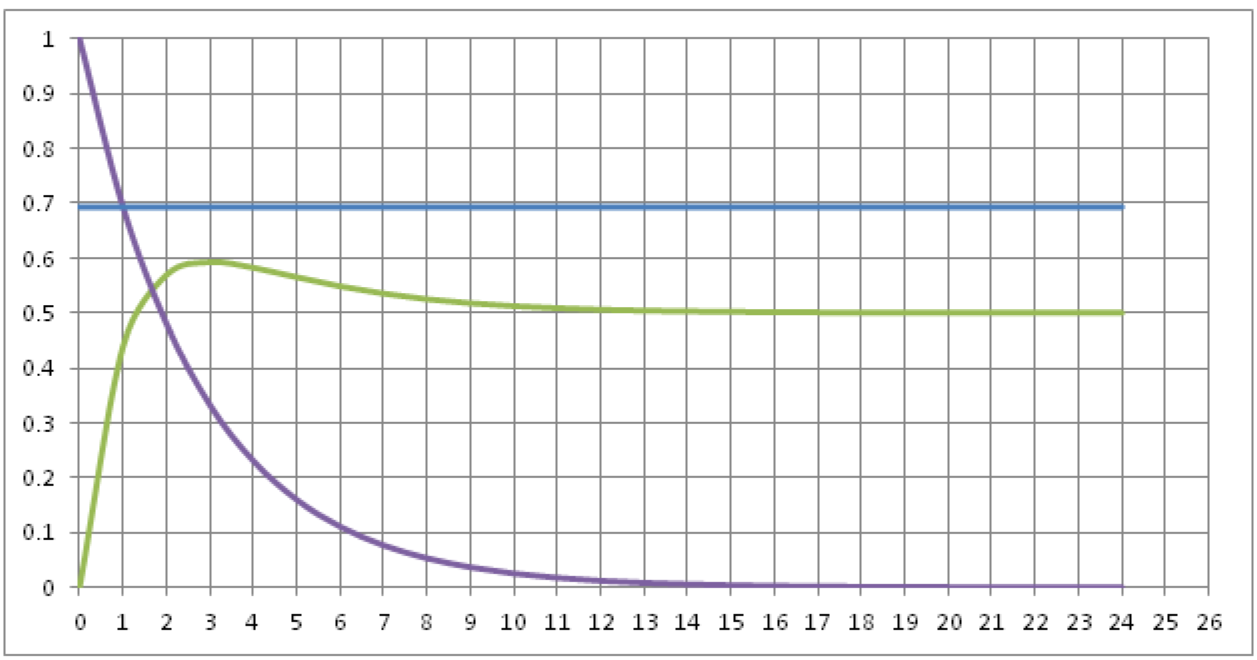
1. **A very efficient theoretical blade, with a single-blade β equal to 1/3**

   Maximum power is reached for 1 blade.



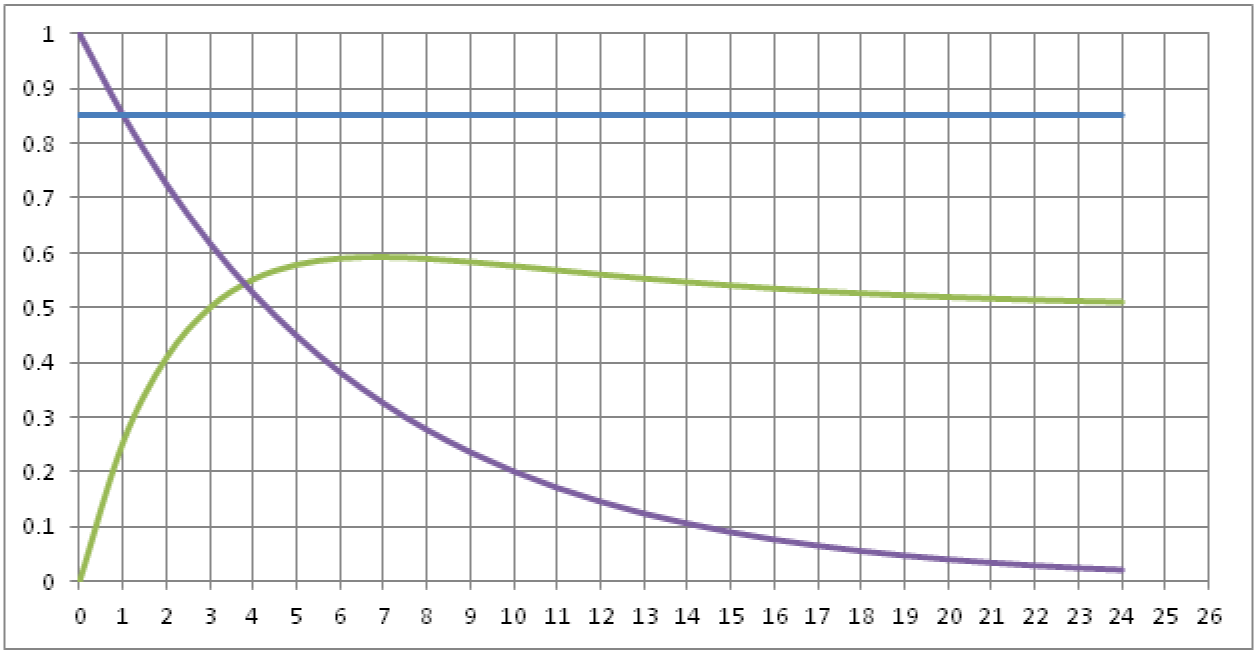
2. **A theoretical blade with a single-blade β equal to close to 0.7**

   Maximum power is reached for 3 blades.



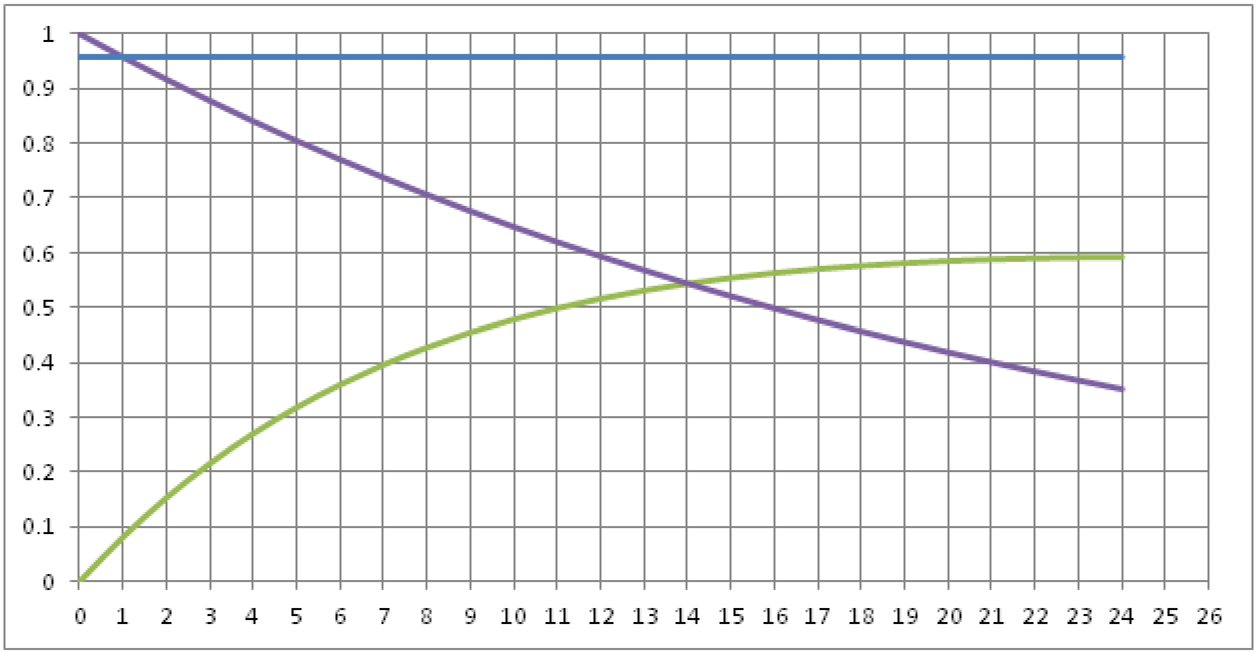
3. **A blade common in large tri-blade rotors (460' diameter, 140 m), with a single-blade β equal to close to 0.85**

By adding a fourth blade (+33%), power increases by 10% only. Going to 6 blades (+100%) increases power by 18% only. By taking one blade away (-33%) power decreases only by 20%, but 2 blades is impractical for reasons of balance. Taking away 2 blades (-67%) power decreases only by 50%, but 1 blade only is also impractical for reasons of stability.



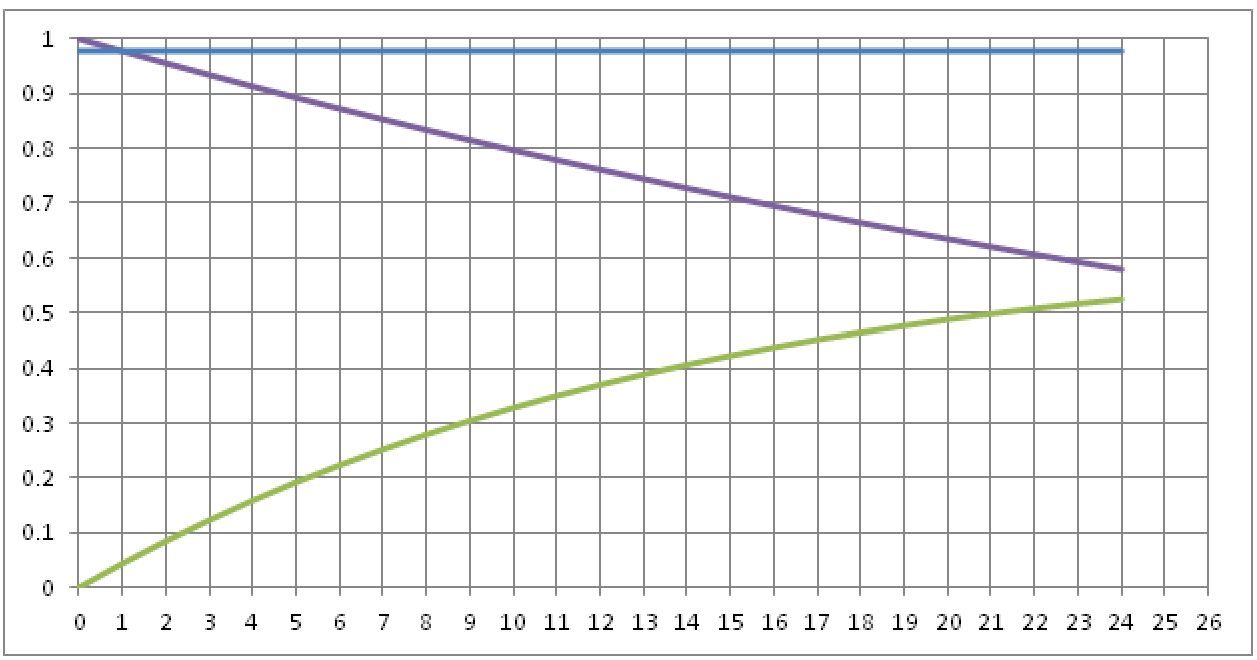
4. **A blade similar to those available in small 6-blade turbines (5' diameter, 1.50 m), with a single-blade β equal to close to close to 0.96**

   An additional 8 blades (+133%) would be required, for a total of 14, to increase power by 50%, if so many blades could fit.



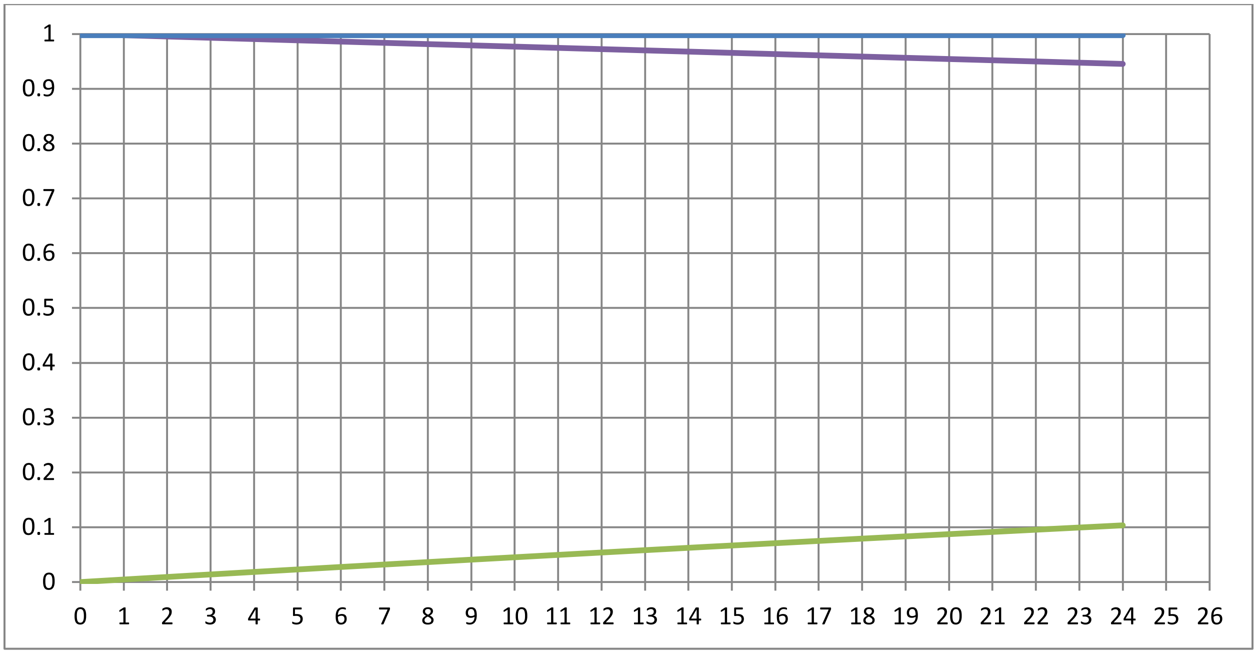
5. **A blade found on a small 11-blade rotor (5' diameter, 1.50 m), with a single-blade β equal to close to close to 0.98**

   Doubling the number of blades, if they could fit, would only increase power by 40%.



6. **A blade found on larger 18-blade farm wind mills (16' diameter, 4.90 m)), with a single-blade β equal to close to close to 0.99**

The green C_P curve is almost linear and power is almost proportional to the number of blades. This kind of wind mill is very inefficient power-wise. It is rather designed to continuously produce sufficient torque to lift water from deep wells, although with low instantaneous flows. The whole surface area of the rotor is almost covered with blades and no more than 20 could actually fit.



C. Scalability

The reasons for smaller blades not being as efficient as larger blades reside probably in (among others):

- Minute adjustments, which are relatively tiny at a larger scale, are not necessarily so at a smaller. The largest of the smaller diameter models available on the market are about 1% the linear dimension of the largest, and therefore lead to 10^{-4} the surface area and 10^{-6} the volume and mass. By comparison, realistic airplane models are built at a scale of between 1/10th and ½ and they still exhibit significantly different behaviors than real airplanes;

- The very high rotational speed required by smaller rotors to conserve a similar wing tip to wind speeds ratio, whereas large rotors revolve very slowly;

- The flexibility and dynamic deformation of the blades which must be significantly different in proportion;
- The local variability and turbulence of the wind, which affects a small area much more than a large;
- and, probably more importantly, the fact that air molecules and physics constants cannot be scaled down, particularly regarding inertia.

D. Adding blades

Adding blades to efficiently designed turbines is either uneconomical or counter-productive, while adding blades to an inefficient turbine does add to an otherwise limited power.

E. Overall futility of wind power

This being said, overall, wind turbines, whatever their efficiency, cannot be seriously thought of as a significant source of energy in either rich or poor countries. See papers about the subject at eastseas.com, particularly "*The electric car fever (October 2017)*". The same can be said of solar panels. The example of Denmark's energy policies hints that large scale wind power is a not too ingenuous (and probably short lived) swindle to divert wealth from the average consumer towards subsidized manufacturers.