

10. As shown in Figure 1.13, Freddy Flintstone wishes to drive his car with square wheels along a strange road. How should you design the road so that his ride is perfectly smooth, i.e., so that the center of his wheel travels in a horizontal line? (Hints: Start with a square with vertices at  $(\pm 1, \pm 1)$ , with center

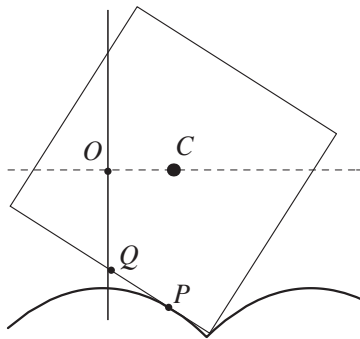


FIGURE 1.13

$C$  at the origin. If  $\alpha(s) = (x(s), y(s))$  is an arclength parametrization of the road, starting at  $(0, -1)$ , consider the vector  $\overrightarrow{OC} = \overrightarrow{OP} + \overrightarrow{PQ} + \overrightarrow{QC}$ , where  $P = \alpha(s)$  is the point of contact and  $Q$  is the midpoint of the edge of the square. Use  $\overrightarrow{QP} = s\alpha'(s)$  and the fact that  $\overrightarrow{QC}$  is a unit vector orthogonal to

$\overrightarrow{QP}$ . Express the fact that  $C$  moves horizontally to show that  $s = -\frac{y'(s)}{x'(s)}$ ; you will need to differentiate