Homework 6

Due: Sunday, Dec. 20
at 11:59 pm EST

Please give complete, well-written solutions to the following exercises and submit via Canvas.

1. (15 points) Consider the maximization problem

$$\begin{aligned} \max\ & x_1^2 + 2x_1x_2 + 2x_2^2 - 3x_1 + x_2 \\ s.t.\ & x_1 + x_2 = 1 \\ & x_1, x_2 \geq 0. \end{aligned}$$

   (i). Is the problem convex?

   (ii). Find all the KKT points of the problem.

   (iii). Find the optimal solution of the problem.

2. (15 points) Use the KKT conditions to solve the problem

$$\begin{aligned} \min\ & x_1^2 + x_2^2 \\ s.t.\ & -2x_1 - x_2 + 10 \leq 0 \\ & x_2 \geq 0. \end{aligned}$$

3. (20 points) Consider the optimization problem

$$\begin{aligned} \min\ & x_1 - 4x_2 + x_3 \\ (\mathrm{P})\quad s.t.\ & x_1 + 2x_2 + 2x_3 = -2 \\ & x_1^2 + x_2^2 + x_3^2 \leq 1. \end{aligned}$$

   (i). Given a KKT point of problem (P), must it be an optimal solution?

   (iii). Find the optimal solution of the problem using the KKT conditions.

4. (20 points) Consider the optimization problem

$$\begin{aligned} \min\ & x_1^2 - x_2^2 - x_3^2 \\ s.t.\ & x_1^4 + x_2^4 + x_3^4 \leq 1 \end{aligned}$$

   (i). Is the problem convex?

   (ii). Find all the KKT points of the problem.

   (iii). Find the optimial solution of the problem.