APMA E4306: Applied Stochastic Analysis

Course Syllabus, Spring 2022

Course Description. This course aims at providing an elementary introduction to fundamental ideas in stochastic analysis for applied mathematics. The course has four main parts: (i) a quick review of elementary probability theory (including limit theorems), and a short introduction to discrete Markov chains and Monte Carlo methods; (ii) elementary theory of stochastic process (e.g. continuous Markov process and Wiener process), Itô's stochastic calculus and stochastic differential equations; (iii) introductions to probabilistic representation of elliptic partial differential equations (the Fokker-Planck equation theory); and (iv) stochastic approximation algorithms and asymptotic analysis of SDEs. The main objective is not to give thorough technical studies on stochastic analysis, but rather to provide an elementary introduction to the fundamental concepts and tools in the topic and connect them to various applications in applied mathematics.

Prerequisite. Good understanding of probability theory (with law of large numbers and the central limit theorem) (on the level of IEOR E3658 or STAT G4001) and elementary stochastic process (on the level of the first part of IEOR E4106, STAT G4264 or STAT W5207) are required. Knowledge on elementary analysis (on the level of MATH GU4601), numerical methods (on the level of APMA E4300), and basic programming skills (with Python, MATLAB or equivalent) are also necessary. *Email the instructor if you are not sure if you have the required background.*

Class Meetings. The class meets MW 10:10-11:25 AM @ 627 Seeley W. Mudd Building

Instructor. Kui Ren (215 S. W. Mudd; kr2002@columbia.edu; 212-854-4731).

Office Hours. MW 11:30AM-12:30PM + Appointments

Textbooks. The course is based on the following textbooks.

`Applied Stochastic Analysis (required)`
`Weinan E, Tiejun Li and Eric Vanden-Einden`

American Mathematical Society, 2019

Stochastic Differential Equations (Optional)
Second Edition
Bernt Oksendal
Springer-Verlag, New York, 1995

Stochastic Approximation and Recursive Algorithms
and Applications (Optional)
Second Edition
Harold J. Kushner G. George Yin
Springer-Verlag, Berlin, 1999

Class Attendance. Attendance is required at all class meetings.

Grading Policy. There will be 12 homework sets, each based on 100 points. There will be a take-home final exam and a final class project, both based on 100 points. The final grade will be weighted roughly as follows:

Homework 30%, Take-home Exam 30%, Final Project 40%

Course Webpage. All the homework will be posted on the university teaching tool, the `Courseworks` system:

https://courseworks.columbia.edu/

Important Dates.

- 01/19/2022, First day of class for E4306
- 03/14/2022, Spring Break; No Class
- 03/16/2022, Spring Break; No Class
- 05/02/2022, Last day of class for E4306

Academic Dishonesty. Discussions and team works among students are encouraged in general. However, the work a student submits for grading, including homework and exams, must be his/her own work. Students who violate university rules on academic dishonesty are subject to disciplinary penalties, including the possibility of failing the course and/or dismissal from the University. Detailed information on academic integrity at Columbia University is available here:

https://www.college.columbia.edu/academics/academicintegrity

Students with Disabilities. Columbia University makes every effort to accommodate students with disabilities. If you require disability accommodations to attend the classes or the exams, please contact Columbia Disability Services at 212-854-2388. For more information, please visit:

https://health.columbia.edu/content/disability-services

Students with disabilities may be eligible for accommodations related to the administration of examinations. Here are more details:

https://health.columbia.edu/services/testing-accommodations

Tentative Weekly Lecture Schedule. Here is a rough schedule for the lectures.

- Week # 01: Review of probabilistic tools
- Week # 02: Discrete Markov chains
- Week # 03: Continuous Markov chains
- Week # 04: Markov chain Monte Carlo methods (Metropolis algorithms and Gibbs sampling)
- Week # 05: Gaussian processes, Karhunen-Loeve expansion and applications
- Week # 06: Wiener Process, Polynomial Chaos Expansion and applications
- Week # 07: Stochastic integrals
- Week # 08: Stochastic differential equations
- Week # 09: Numerical analysis of SDEs
- Week # 10: Fokker-Planck theory

- Week # 11: Stochastic representation of elliptic PDEs
- Week # 12: Stochastic gradient and related methods
- Week # 13: Asymptotic expansions of SDEs

This schedule is only tentative. Changes of the schedule will be announced in class. Homework will be handed out on the dates indicated.