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**AQUI EL TÍTOL DEL TREBALL**

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The main goal of this work is to prove the existence of.....

**Notation:** We will work over an algebraically closed field  $K$  of characteristic zero.

# Chapter 1

## títol del capítol 1

### 1.1 Sub apartat del capítol

**Definition 1.1.** Given a projective scheme  $X \subseteq \mathbb{P}^n$  and a coherent sheaf  $\mathcal{E}$  on  $X$ , we say that  $\mathcal{E}$  is an *Ulrich sheaf* if  $\mathcal{E}$  is an ACM sheaf and  $h^0(\mathcal{E}_{init}) = \deg(X)\text{rank}(\mathcal{E})$ , i.e. the minimal number of generators of Ulrich sheaves is as large as possible.

Notice that in Definition 1.1 we do not require.....

**Theorem 1.2. (Borel-Bott-Weil)** Assume that all entries of  $\alpha + \rho$  are distinct. Let  $\sigma \in S_{n+1}$  be the unique permutation such that  $\sigma(\alpha + \rho)$  is strictly decreasing. Then

$$H^m(F_c, L_\alpha) = \begin{cases} \Sigma^{\sigma(\alpha+\rho)-\rho} V^* & \text{if } m = l(\sigma) \\ 0 & \text{otherwise.} \end{cases}$$

If at least two entries of  $\alpha + \rho$  coincide then

$$H^i(F_c, L_\alpha) = 0 \quad \text{for all integer } i.$$

*Proof.* See [3]; Theorem 2. □

**Theorem 1.3. (Borel-Bott-Weil)** Let  $\beta = (\beta_1, \dots, \beta_{p+1}) \in \mathbb{Z}^{p+1}$ ,  $\alpha = (\alpha_1, \dots, \alpha_{q-p}) \in \mathbb{Z}^{q-p}$  and  $\gamma = (\gamma_1, \dots, \gamma_{n-q}) \in \mathbb{Z}^{n-q}$  be non-increasing integer sequences and let  $\mu = (\beta, \alpha, \gamma) \in \mathbb{Z}^{n+1}$  be their concatenation.

(a) Assume that all entries of  $\mu + \rho$  are distinct and let  $\sigma \in S_{n+1}$  be the unique permutation such that  $\sigma(\mu + \rho)$  is strictly decreasing. Then

$$H^m(F(p, q, n), \Sigma^\beta \mathcal{P} \otimes \Sigma^\alpha \mathcal{S}_2 \otimes \Sigma^\gamma \mathcal{S}_1) = \begin{cases} \Sigma^{\sigma(\mu+\rho)-\rho} V^* & \text{if } m = l(\sigma) \\ 0 & \text{otherwise.} \end{cases}$$

(b) *If at least two entries of  $\mu + \rho$  coincide then*

$$H^i(F(p, q, n), \Sigma^\beta \mathcal{P} \otimes \Sigma^\alpha \mathcal{S}_2 \otimes \Sigma^\gamma \mathcal{S}_1) = 0 \quad \text{for all interger } i.$$

*Proof.* It follows from Theorem 1.2

□

## **Chapter 2**

### **títol del capítol 2**





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