UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Centre de la Imatge i la Tecnologia Multimèdia

Row and Column pictures

A geometric view of 2D and 3D systems of equations

Julen Cayero, Cecilio Angulo

Bachelor's Degree in Video Game Design and Development

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Centre de la Imatge i la Tecnologia Multimèdia

Outline

Outline

Returning to linear equations

Understanding 2D LSE

Solving a linear system of equations $\mathbf{A}x = b$ means that $\mathbf{A}$ and $\mathbf{b}$ are known, so we might be focusing on finding x
However since the 2D and 3D systems of equations will be arising too often and they admit graphic representation it is interesting to give those linear systems a geometric meaning

Row picture

Understanding 2D LSE

$$\left.\begin{aligned} a_{11}x + a_{12}y &= b_1 \\ a_{21}x + a_{22}y &= b_2 \end{aligned}\right\} \tag{1}$$

Each equation in Eq. (1) represents a line in a plane and we are looking for the points that both lines have in common.

In the plane there are 3 possible solutions:

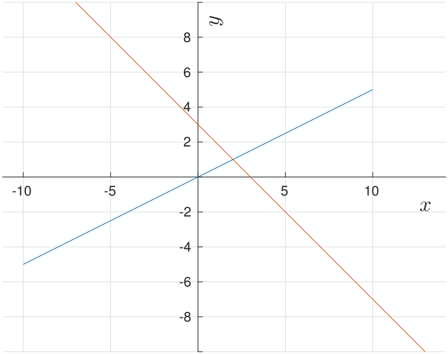
- Both lines are the same, and therefore the solution is achieved for all $x, y \in \mathbb{R}$
- Both lines are parallel and don't share any point. Therefore it does not exist the solution that we want
- The common case, both lines intersect on a unique point which is the solution.

Example
Understanding 2D LSE

$$\left.\begin{array}{rcl} x - 2y & = & 0 \\ x + y & = & 3 \end{array}\right\} \qquad (2)$$

Line 1:

x	y
0	0
2	1



Line 2:

x	y
0	3
3	0

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Centre de la Imatge i la Tecnologia Multimèdia

Column picture
Understanding 2D LSE

The column picture is based on the vector representation. The past system in Eq. (2) can we then written as:

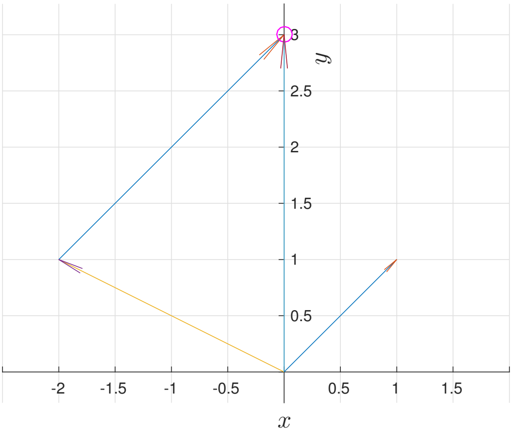
$$x\begin{pmatrix}1\\1\end{pmatrix}+y\begin{pmatrix}-2\\1\end{pmatrix}=\begin{pmatrix}0\\3\end{pmatrix}$$

So our purpose now is to find the scalars x and y such that the sum of the result vectors gives the right hand side term.

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Centre de la Imatge i la Tecnologia Multimèdia

Example
Understanding 2D LSE

By selecting $x = 2$ and $y = 1$



UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Centre de la Imatge i la Tecnologia Multimèdia

Outline

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Centre de la Imatge i la Tecnologia Multimèdia

Row picture
Understanding 3D LSE

In the 3D case we are dealing with linear systems of equations of the form:

$$\left.\begin{array}{l} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{array}\right\}$$

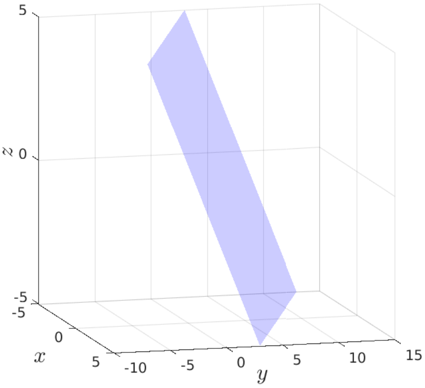
Now we are seeking for the points x, y, z contained on the past three equations. And every linear equation in 3D represents a plane

- Infinite solutions
  - The intersection between three planes is a plane or a line
- No solutions
  - The planes do not intersect or they intersect one-to-one
- The general case. The planes intersection results in one point

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Centre de la Imatge i la Tecnologia Multimèdia

Example
Understanding 3D LSE

$$\left.\begin{array}{rcl} x+y+z & = & 3 \\ x-y & = & 0 \\ x+z & = & 1 \end{array}\right\}$$

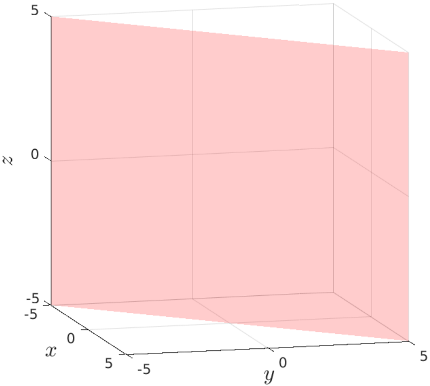


Plane $x+y+z=3$

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Centre de la Imatge i la Tecnologia Multimèdia

Example
Understanding 3D LSE

$$\left.\begin{array}{rcl} x+y+z & = & 3 \\ x-y & = & 0 \\ x+z & = & 1 \end{array}\right\}$$

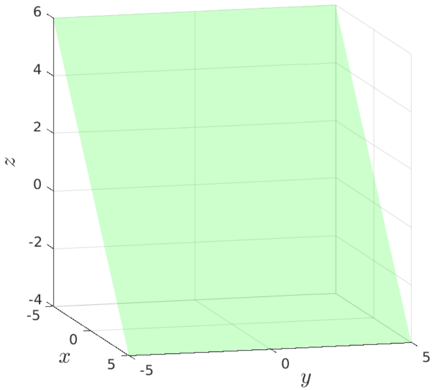


Plane $x - y = 0$

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Centre de la Imatge i la Tecnologia Multimèdia

Example
Understanding 3D LSE

$$\left.\begin{array}{rcl} x+y+z & = & 3 \\ x-y & = & 0 \\ x+z & = & 1 \end{array}\right\}$$

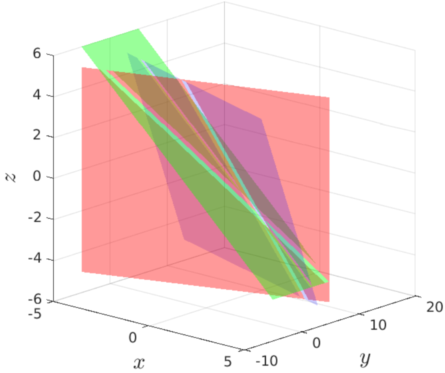


Plane $x+z=1$

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Centre de la Imatge i la Tecnologia Multimèdia

Example
Understanding 3D LSE

$$\left.\begin{aligned} x+y+z&=3\\ x-y&=0\\ x+z&=1 \end{aligned}\right\}$$



The three planes intersect at point:
$x=2,\ y=2,\ z=-1$

Column picture

Understanding 3D LSE

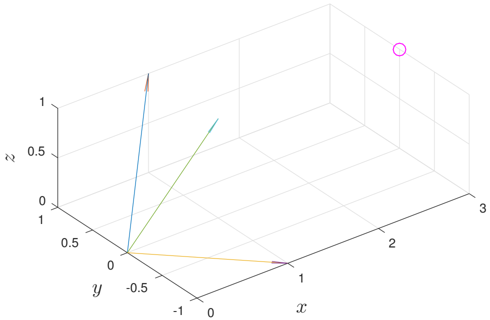
The column picture by contrast wants to find the scalars x, y and z that fulfill the equation

$$x \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + y \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + z \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Centre de la Imatge i la Tecnologia Multimèdia

Example
Understanding 3D LSE

$$x\begin{pmatrix}1\\1\\1\end{pmatrix}+y\begin{pmatrix}1\\-1\\0\end{pmatrix}+z\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}3\\0\\1\end{pmatrix}$$



UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Centre de la Imatge i la Tecnologia Multimèdia

Example
Understanding 3D LSE

$$2\begin{pmatrix}1\\1\\1\end{pmatrix}+2\begin{pmatrix}1\\-1\\0\end{pmatrix}-1\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}3\\0\\1\end{pmatrix}$$