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Centre de la Imatge i la Tecnologia Multimèdia

Theme 5. Interpolation of Rotations

SLERP

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Bachelor's Degree in Video Game Design and Development

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Outline

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Motivation

SLERP vs LERP

- Angular velocity should be constant when moving from a rotation to another one.
- Angular velocity behaviour for LERP is easily illustrate in this video.
- Quaternion linear interpolation on the unit sphere (SLERP) is preferable since angular velocity will be constant.

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Definition

The spherical linear interpolation curve between two quaternions can be defined as follows:

SLERP between two quaternions

Given two quaternions $\mathring{p}\,,\,\mathring{q} \in \mathbb{H}$ parametrised by $h \in [0, 1]$, the following five functions are equivalent expressions for spherical linear interpolation:

$$
\begin{aligned}
Slerp(\mathring{p}, \mathring{q}, h) &= \mathring{p}(\tilde{\mathring{p}}\mathring{q})^h \\
&= (\mathring{p}\tilde{\mathring{q}})^{1-h}\mathring{q} \\
&= (\mathring{q}\tilde{\mathring{p}})^h\mathring{p} \\
&= \mathring{q}(\tilde{\mathring{q}}\mathring{p})^{1-h} \\
&= \mathring{p}^{1-h}\mathring{q}^h
\end{aligned}
$$

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Properties

SLERP Properties

- Symmetry in definitions is due to

$$Slerp(\mathring{p}, \mathring{q}, h) = Slerp(\mathring{q}, \mathring{p}, 1 - h)$$

- It can be proven that the curvature equals one throughout the entire interpolation curve.
- Quaternions $\mathring{q}$ and $-\mathring{q}$ give the same rotation. Hence, there are two SLERP choices, pick the shortest.

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Practical use

Alternative SLERP Definition

Given two quaternions $\mathring{p}, \mathring{q} \in \mathbb{H}$ parametrised by $h \in [0, 1]$, the following expression is also valid for spherical linear interpolation:

$$Slerp(\mathring{p}, \mathring{q}, h) = \frac{\sin((1-h)\Omega)}{\sin(\Omega)}\mathring{p} + \frac{\sin(h\Omega)}{\sin(\Omega)}\mathring{q}$$

with

$$\cos(\Omega) = \mathring{p} \cdot \mathring{q} = p_0 q_0 + p_1 q_1 + p_2 q_2 + p_3 q_3$$

This formula does not produce a unit quaternion and must be normalized.

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SLERP quaternions interpolation of rotations

1. Visualization of rotations as applied on an object, like in motion capture. See video

SLERP quaternions interpolation of rotations

1. Visualization on the unit sphere.
2. Visualization of the smoothness of interpolation curves using angular velocity.

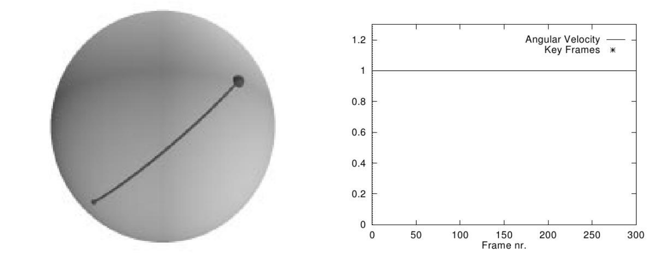

Figure: Interpolation curve and velocity graph for *Slerp*.

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Homeworks

Homeworks

For a quaternion $\mathring{r}$ in the unit sphere $\mathbb{S}$ representing a rotation of angle θ and axis $\boldsymbol{u}$, and a number h,

$$\mathring{r}^h = \begin{pmatrix} \cos\left(\dfrac{h\theta}{2}\right) \\ \\ \boldsymbol{u}\sin\left(\dfrac{h\theta}{2}\right) \end{pmatrix}$$

Calculate $Slerp(\mathring{p}, \mathring{q}, h)$ for $\mathring{p}, \mathring{q} \in \mathbb{S}$

Hint: For $\mathring{r} \in \mathbb{S}$, $\tilde{\mathring{r}} = \mathring{r}^{-1}$