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Theme 5. Interpolation of Rotations

Linear Techniques

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Outline

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Rotations

Rotations description

- Rotation Matrix
- Principal Euler's vector/angle
- Euler's angles: yaw, pitch and roll
- Rotation vector
- Quaternions

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Definition

The interpolation curve between two points can be defined as follows:

Interpolation between two points (rotations)

Given an arbitrary set $\mathcal{M}$ we interpolate between two points $\boldsymbol{x}_0$, $\boldsymbol{x}_1 \in \mathcal{M}$ parametrised by $h \in [0, 1]$. The resulting interpolation curve

$$\gamma : \mathcal{M} \times \mathcal{M} \times [0, 1] \longrightarrow \mathcal{M}$$

$$\gamma(\boldsymbol{x}_0, \boldsymbol{x}_1, h) = \boldsymbol{x}_h \in \mathcal{M}$$

satisfy the constraints:

$$\begin{aligned} \gamma(\boldsymbol{x}_0, \boldsymbol{x}_1, 0) &= \boldsymbol{x}_0 \\ \gamma(\boldsymbol{x}_0, \boldsymbol{x}_1, 1) &= \boldsymbol{x}_1 \end{aligned}$$

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Some examples

Some examples

- Several techniques can be used like in this Example.
- In this Example translation has been added for visualization.
- ... and two more examples: Example 1 and Example 2.

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Techniques
Linear

The most obvious method is simply linear interpolation between two tuples of Euler angles.

Linear Euler Interpolation, *LinEuler*

Interpolation between $\boldsymbol{x}_0 = (\psi_0, \theta_0, \phi_0) \in \mathbb{R}^3$ and $\boldsymbol{x}_1 = (\psi_1, \theta_1, \phi_1) \in \mathbb{R}^3$ using $h \in [0, 1]$ is defined as:

$$LinEuler(\boldsymbol{x}_0, \boldsymbol{x}_1, h) = \boldsymbol{x}_0(1-h) + \boldsymbol{x}_1 h$$

An example of Result for motion capture

Techniques
Linear

An alternative simple attempt is linear interpolation between rotation matrices – meaning linear interpolation of every single matrix element independently of the others.

Linear Matrix Interpolation, *LinMat*

Interpolation between $\mathbf{M}_0 \in \mathcal{M}_{3\times 3}$ and $\mathbf{M}_1 \in \mathcal{M}_{3\times 3}$ using $h \in [0,1]$ is defined as:

$$LinMat(\mathbf{M}_0, \mathbf{M}_1, h) = \mathbf{M}_0(1-h) + \mathbf{M}_1 h$$

An example of Result for motion capture

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Another obvious method is linear interpolation between quaternions.

Linear Quaternion Interpolation, *Lerp*

Interpolation between $\mathring{q}_0 \in \mathbb{H}$ and $\mathring{q}_1 \in \mathbb{H}$ using $h \in [0, 1]$ is defined as:

$$Lerp(\mathring{q}_0, \mathring{q}_1, h) = \mathring{q}_0(1 - h) + \mathring{q}_0 h$$

The interpolation curve for linear interpolation between quaternions gives a straight line in quaternion space. The curve therefore dips below the surface of the unit sphere.

An example of Result for motion capture

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Visualization

Applied / Sphere / Angular velocity

Most usual form of rotations is through quaternions. However, they are not easily visualized.

How to visualize interpolation of rotations using quaternions?

1. Visualization of rotations as applied on an object, like in motion capture.
2. Visualization of rotations on the unit sphere, considering unit quaternions, $\|\mathring{q}\| = 1$.
3. Visualization of the smoothness of interpolation curves using angular velocity,

$$\underset{\sim}{V}(\mathring{q}_h) = \frac{\|\mathring{q}_h - \mathring{q}_{h-1}\| + \|\mathring{q}_h - \mathring{q}_{h+1}\|}{2}$$

Linear Euler Interpolation, *LinEuler*

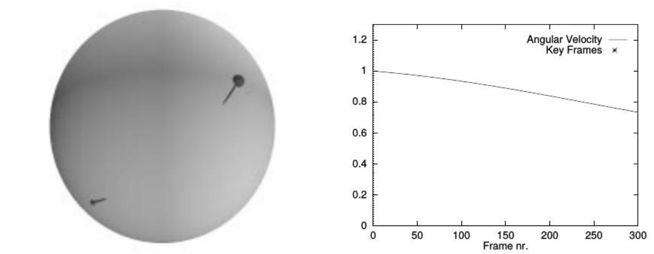

Figure: The curve disappears from the surface of the unit sphere. Angular velocity will gradually slow down.

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Linear Matrix Interpolation, *LinMat*

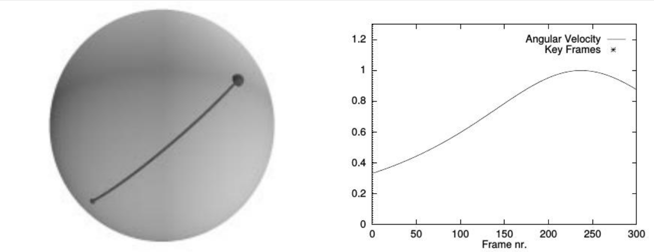

Figure: The curve for linear matrix interpolation does not in general lie on the unit sphere, since linear interpolation between orthonormal matrices will not in general produce orthonormal matrices (It cannot be appreciated here).

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Linear Quaternion Interpolation, *Lerp*

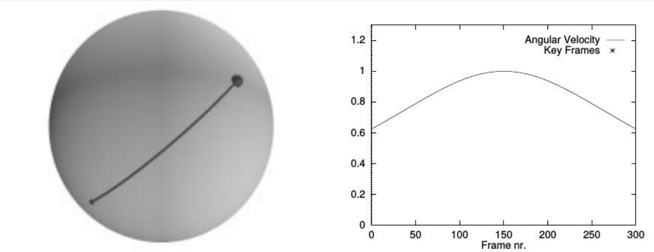

Figure: The interpolation curve for Lerp gives a straight line in quaternion space. The curve therefore dips below the surface of the unit sphere. Since all quaternions on a line through the origin give the same rotation, the curve can be projected on to the unit sphere without changing the corresponding rotations.

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Homeworks

Homeworks

Using your Matlab's functions to convert rotation matrices and Euler angles into quaternions, check the visualization results.