FA2

Gaac

February 2026

Problem 3.49

$$\begin{aligned}\sum_{j=1}^{N}(X_j-1)^2 &= \sum_{j=1}^{N}(X_j^2-2X_j+1)\\ &= \sum_{j=1}^{N}X_j^2-\sum_{j=1}^{N}2X_j+\sum_{j=1}^{N}1\\ &= \sum_{j=1}^{N}X_j^2-2\sum_{j=1}^{N}X_j+N\end{aligned}$$

Problem 3.51

Given:
$U=\{3,-2,5\}$
$V=\{-4,-1,6\}$

1. **Calculate** $\sum UV$:

$$\begin{aligned}\sum_{i=1}^{3}U_iV_i &= (3)(-4)+(-2)(-1)+(5)(6)\\ &= -12+2+30\\ &= \mathbf{20}\end{aligned}$$

2. **Calculate** $\sum(U+3)(V-4)$:

$$\begin{aligned}\sum(U_i+3)(V_i-4) &= (3+3)(-4-4)+(-2+3)(-1-4)+(5+3)(6-4)\\ &= (6)(-8)+(1)(-5)+(8)(2)\\ &= -48-5+16\\ &= \mathbf{-37}\end{aligned}$$

3. **Calculate** $\sum V^2$:

$$\begin{aligned}\sum V_i^2 &= (-4)^2+(-1)^2+(6)^2\\ &= 16+1+36\\ &= \mathbf{53}\end{aligned}$$

4. **Calculate** $(\sum U)(\sum V)^2$:

$$\begin{aligned}\left(\sum U_i\right)\left(\sum V_i\right)^2 &= (3-2+5)(-4-1+6)^2\\ &= (6)(1)^2\\ &= \mathbf{6}\end{aligned}$$

5. **Calculate $\sum UV^2$:**

$$\begin{aligned}\sum U_i V_i^2 &= (3)(-4)^2 + (-2)(-1)^2 + (5)(6)^2 \\ &= 3(16) - 2(1) + 5(36) \\ &= 48 - 2 + 180 \\ &= \mathbf{226}\end{aligned}$$

6. **Calculate $\sum(U^2 - 2V^2 + 2)$:**

$$\begin{aligned}\sum(U_i^2 - 2V_i^2 + 2) &= \sum U_i^2 - 2\sum V_i^2 + \sum 2 \\ &= (38) - 2(53) + (3 \times 2) \\ &= 38 - 106 + 6 \\ &= \mathbf{-62}\end{aligned}$$

7. **Calculate $\sum(U/V)$:**

$$\begin{aligned}\sum \frac{U_i}{V_i} &= \frac{3}{-4} + \frac{-2}{-1} + \frac{5}{6} \\ &= -0.75 + 2 + 0.8333 \\ &= \mathbf{2.0833} \quad \text{or} \quad \frac{25}{12}\end{aligned}$$

Problem 3.90: Geometric Mean Calculation

The geometric mean G of a set of numbers $X_1, X_2, \ldots, X_n$ is given by:

$$G = \sqrt[n]{X_1 X_2 \ldots X_n}$$

(a) Set: {3, 5, 8, 3, 7, 2}

Here, $n = 6$.

$$\begin{aligned}G &= \sqrt[6]{3 \cdot 5 \cdot 8 \cdot 3 \cdot 7 \cdot 2} \\ &= \sqrt[6]{5040} \\ &\approx \mathbf{4.14}\end{aligned}$$

(b) Set: {28.5, 73.6, 47.2, 31.5, 64.8}

Here, $n = 5$.

$$\begin{aligned}G &= \sqrt[5]{28.5 \cdot 73.6 \cdot 47.2 \cdot 31.5 \cdot 64.8} \\ &= \sqrt[5]{202,092,516.9} \\ &\approx \mathbf{45.83}\end{aligned}$$