Privacy Preserving Group Ranking

Marc Ilunga Tshibumbu Mukendi

School of Computer and Communication Sciences

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Responsible
Prof. Serge Vaudenay
EPFL / LASEC

Supervisor
Handan Kilinç
EPFL / LASEC

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Chapter 1

Motivation

Group ranking is a process used to find the best candidates from a group. The selected candidates are the ones who satisfy certain criteria. As an example, online dating uses such process. Indeed, people have some preferences about the sleeked mate (age,profession, hobbies, etc..) and will be matched to people satisfying those preferences. As another example, consider a pharmaceutical company promoting a new drug for sleep disorder want to give a free trial of the drug to people and show that it actually works. To maximize the marketing effect people chosen for the trial are expected to be representatives of the targeted population. To ensure this, the company typically sets up a questionnaire for the people willing to participate to the trial. The questions might be about age, blood type, profession, etc. Once the questionnaire is filled up and returned to the company, it will be used to choose best candidates for the trial.

However, group ranking is more and more used in virtual environment in today's society and as a consequence privacy concerns emerge. A naive implementation of group ranking (e.g: for the drug company) would be a simple questionnaire on a web page or on a smart-phone application with questions about age, blood type, etc... The participant will then answer the question and send them to the company. Given that the questionnaire can contain questions about some sensitive information (e.g other diseases, annual income, etc..), such implementation of group ranking poses a privacy problems. Especially for a participant who will not be selected anyway.

Given the necessity of group ranking in a wide range of real world's application and the need foro preserving candidates' privacy, we searched for protocols promising privacy. Among them [1, 2, 3, 5, 6, 6, 7, 8, 9], we concentrate on the protocol proposed by Li et al. [1]. This protocol gives the privacy requirement that we need in the semi-honest adversary setting. In this project, we analyse and implement it.

Chapter 2

Definitions

In this section we define useful terms used later on.

2.1 Framework

The protocol is executed cooperatively by $n+1$ parties, an initiatorP_0 and n participants $P_1, P_2...P_n$. The protocol assumes that the participants are willing to accept the initiator's invitation and willing to submit their private information if selected eventually.

The questionnaire given by the initiator is represented as a m-dimensional name vector(i.e m questions). The initiator holds a m-dimensional vector v_0 indicating the preferred values of each the question in the questionnaire, another m-dimensional vector w represents the weight associated to each question. Furthermore, the questionnaire comprises: "Equal" questions meaning that for those the initiator is looking for an answer which has specific value. "Greater than" questions, meaning that the initiator is looking for values exceeding some threshold, and the more the value exceeds the threshold, the better. We assume without loss of generality that the first t questions are"greater than" questions and the rest are "Equal" questions. Finally the answer of P_j is also represented by a m-dimensional vector v_j. The participants are then ranked in a non-increasing order based on their gains. A participant's gain is defined as follows:

Definition 1 (Gain) *Given a criterion vector* $v_0 = [v_0^1, v_0^2, ..., v_0^m]^T$ *and the weight vector* $w = [w_1, w_2, ..., w_m]^T$*, the gain value of* P_j *is*
$g_j = \sum_{k=t+1}^{m} w_k(v_k^j - v_k^0) - \sum_{k=1}^{t} w_k(v_k^j - v_k^0)^2$.
As we can see partial gain value of P_j*, defined as* $p_j = \sum_{k=t+1}^{m} w_k v_k^j - \sum_{k=1}^{t}(w_k(v_k^j)^2 - 2w_k v_k^j v_k^0)$ *is sufficient for group ranking and hides part of the criterion vector.*

The partial gain of P_j can be represented by dot products, $wg \cdot vg_j$- $we \cdot (ve_j * ve_j) + 2(we * ve_0) \cdot ve_j$. Here, "" is the element-wise multiplication of two vectors, i.e $a * b = [a_1b_1, ..., a_mb_m]$. ve_0 is the sub-vector with "equals" elements of the criterion vector and vg_0 is the sub-vector with "greater than" elements of the criterion vector. we and wg are defined in the same fashion with respect to the weight vector w. Finally ve_j and vg_j are also defined in the same fashion with respect to v_j. In order, compute securely and efficiently P_J's gain, the protocol invokes a secure two-party protocol proposed by Ioannidis et al.[10].*

2.2 Two-Party Dot Product Protocol

This protocol allows two parties Alice and Bob to securely compute $w \cdot v$. where w and v are two vectors with the same dimensions held by two parties separately.

Bob, holding a $(d-1)$-dimensional vector w, chooses a random matrix Q of $s \times s$ dimensions, where s is a random integer. He generates another $(s \times d)$-dimensional matrix X, for which the r-th row is vector $[w^T, 1]^T$ and the rest of the numbers in the matrix are chosen randomly. Here, r is a random integer in $\{1, 2.., s\}$. Bob then calculates $b = \sum_{i=1}^{s} Qir$ and $c = \sum_{i=1, i\neq r}^{s}(x_i^T \cdot \sum_{j=1}^{s} Q_{ji})$, where x_i^T is the i-th row of the matrix X. Choosing a random d-dimensional vector f and three random numbers R_1, R_2, R_3, Bob sends QX, $c' = c + R_1 \cdot R_2 \cdot f^T$ and $g = R_1 \cdot R_3 \cdot f$ to the other party, Alice.
Alice generates a vector $v' = [v^T, \alpha]^T$, where α is a random number and v is a $(d-1)$-dimensional vector. Upon receiving data from Bob, Alice computes $y = QXv'$, $z = \sum_{i=1}^{s} y_i$, $a = z - c' \cdot v'$, $h = g^T \cdot v'$ and sends a, h to Bob.
Bob computes $\beta = \dfrac{(a + h \cdot \dfrac{R2}{R3})}{b}$, which is $w \cdot v + \alpha$.
They finally exchange α and β. The desired dot product is $\beta - \alpha$

2.3 ElGamal Cryptosystem

Let G_q be a prime order multiplicative group for which the DDH problem is hard and g is a generator in G_q.
Key generation: a private key is a random element x in $\mathbb{Z}_q$ and the corresponding public key is $y = g^x$.
Encryption: a ciphertext of a message M is of form $E(M) = (My^r, g^r)$, where r is a random element in $\mathbb{Z}_q$.
Decryption: the decryption of a ciphertext (c, c') is done by $M = \dfrac{c}{c^x}$.

The group ranking protocol uses a modified version of Eglamal cryptposystem where encryption is $E(M) = (g^M y^r, g^r)$. As a consequence ElGamal encryption turns to be an additive homomorphic encryption because $E(M_1) \circ E(M_2) = (c_1 \cdot c_2, c_1' \cdot c_2') = E(M1 + M2)$.

Given one cipher $E(M_1) = (g^{M_1} y^r, g^r)$ and a message M_2, the encryption can be seen as multiplicative homomorphic since $E(M_1)^{M_2} = (g^{M_1 M_2} y^{M_2}, g^{M_2}) = E(M_1 M_2)$
Decryption in this case is difficult or impossible because recovering M from g^M is hard in group G_q. However, this does not negatively affect the protocol since it only need to verify if $M = 0$, i.e., $g^M = 1$.

Finally, the ElGamal encryption can be done in distributed way by letting each party choose x_i at random in $\mathbb{Z}_q$ and publish $y_i = g^{x_i}$. The joint public key then becomes $y = \prod_{i=1}^n y_i$. A ciphertext (c, c') encrypted by the joint public key can be decrypted by M $= \dfrac{c}{\prod_{i=1}^n c^{x_i}}$

2.4 Zero-Knowledge Proof

In order to be sure that the y_i's used to compute the common key in the previous paragraph come from the participants and not from someone trying to impersonate a participant, the protocol requires that every participant P_j proves to other participants that he is the one who delivered y_j. The proof is executed by two parties, a prover and a verifier. At the end of the proof the verifier will eventually be convinced that the prover knows some knowledge without learning any information about the knowledge. We are interested in the zero-knowledge proof of the discrete logarithm of y_j in G_q which is x_i the private key of P_j .This is known as the Schnorr identification scheme [11]. The proof works as follows:

The prover chooses a random number r in $\mathbb{Z}_q$ and sends $h = g^r$ to the verifier.
The verifier chooses a random number c in $\mathbb{Z}_q$ and sends back to the prover. The prover calculates $z = (r + xc \bmod q)$ and sends z to the verifier. The verifier verifies $g^z = hy^c$.

Chapter 3

Detailed explanation of the protocol

Now we have all necessary tool to explain the protocol in details.

3.1 Initialization phase

In this phase every P_j, $0 \le j \le n$ sets up the data needed to begin the protocol.
P_0 generates a group G_q and a generator g and publishes them along with a vector of attribute names and an integer k, $1 \le k \le n$. k is the number of high ranking participant who will be selected. P_0 keeps as private input v_0 and w. The participants P_j keep v_j, $1 \le j \le n$.

3.2 Secure gain computation

In this phase, every participant P_j, $1 \le j \le n$ securely computes his/her gain. Since the gain can be computed as the dot product wg$\cdot vg_j - we \cdot (ve_j * ve_j) + 2(we * ve_0) \cdot ve_j$, every participant P_j invokes the secure dot product with P_0, the secure dot product protocol is described in Sec 2.3

step 1) P_0 generates a random h-bits integer ρ

step 2) Every participant P_j generates $w'_j = [vg_j^T, (ve_j * ve_j)^T, ve_j^T, 1]$. vg_j, ve_j are describe in section 2.2. Next P_j computes Q_jX_j, c'_j, g_j and sends them to P_0.

step 3) Once Q_jX_j, c'_j, g_j are received from a participant P_j, P_0 chooses randomly ρ_j from $\{0, 1, \cdots, \rho\}$ and constructs v'_j =

$[\rho w g^T, -\rho w e^t, 2\rho(we * ve_0)^T, \rho_j]$. P_0 then computes $a_j = z_j - c'_j \cdot v'_j$, $h_j = g_j^T \cdot v'_j$ and sends them back to P_j.

step 4) Upon receiving (a_j, h_j) from P_0, P_j computes $\beta_j = \dfrac{(a_j + h_j \cdot \dfrac{R_2}{R_3})}{b_j}$. This is a masked partial gain. $\beta_j = \rho p_j + \rho_j$. and convert it to an unsigned integer. We show in the appendix that the mask does not affect badly the comparison.(i.e if $\beta_j > \beta_i$ then masked$(\beta_j) >$ masked(β_i))

3.3 Unlinkeable gain comparison

In this phase, every participants compares his/her gain value β_j to other β values. However, the comparison are done on encrypted values, the details are given below.

step 5) First every participant P_j generates a privates key x_j taken randomly from from $\mathbb{Z}_q$, publishes $y_j = g^{x_j}$. Then P_j proves the knowledge of x_j to other participants via the the zero-knowledge proof protocol presented in Sec 2.5.

step 6) Next, each P_j convert his/her β_j to an array of bits.(i.e the array is equivalent to the binary representation). $[\beta_j]_B = [\beta_j^l, \beta_j^{l-1}, ..., \beta_j^1]$, encrypts them using the common public $y = \prod_{i=1}^n y_j$. The result of the encryption is $[E(\beta_j)]_B = [E(\beta_j^l), E(\beta_j^{l-1}), ..., E(\beta_j^1)]$, P_j then sends this to the other participants.

step 7) Upon receiving $[E(\beta_i)]_B$ from other participants, P_j applies the algorithm below to compare each $[E(\beta_i)]_B$. The algorithm and it's validity is explained in the appendix section.

Algorithm: $\forall i, 1 \leq i \leq n \; i \neq j$ and $\forall t, 1 \leq t \leq l$:

P_j computes $E(\gamma_i^t) = E(\beta_j^t + \beta_i^t + 2\beta_j^t\beta_i^t)$. Here $\gamma_i^t = \beta_j^t \oplus \beta_i^t$. The encrypted additions are possible thanks to the additive homomorphic properties of ElGamal cryptosystem and the term $E(2\beta_j^t\beta_i^t)$ is obtained due to the multiplicative homomorphism of the cryptosystem and the fact that P_j knows β_j^t.

P_j computes $E(\omega_i^t)$ where $\omega_i^t = (l - t + 1)(1 - \gamma_i^t) + \sum_{v=t+1}^l \gamma_i^v$.

Lastly, P_j computes $E(\tau_i^t)$ where $\tau_i^t = \omega_i^t + \beta_i^t$. Again all operations on encrypted data is possible through the homomorphic properties of the cryptosystem.

To sum up, for each $\forall i, 1 \leq i \leq n$, $i \neq j$, P_j generates $[E(\tau_i)] =$

$[E(\tau_i^l), E(\tau_i^{(l-1)}, ..., E(\tau_i^1)]$ and sends them all P_1 as $\varepsilon_j = \left\{[E(\tau_i^l)]\right\}$, where $1 \leq i \leq n$, and $i \neq j$

3.4 Chained Comparison Decryption

In this section, all the participants decrypt the comparisons in chained fashion. Indeed, the decryption needs each participant's private key.

step 8) After receiving all the ε_j, P_1 assembles them as a vector $V = [\varepsilon_n, \varepsilon_{n-1}, ..., \varepsilon_1]$. Starting at P_1, every participant does the following:
For every $\varepsilon_i, i \neq j$,in V and for every cipher (c, c') in ε_i, P_j picks r randomly from $\mathbb{Z}_q$ and computes $\tilde{c} = c/c'^{x_j}$ and updates the cipher by $((\tilde{c})^r, (c')^r)$.
Finally P_j permutes the ciphers in every ε_i and send them to P_{j+1}. if $j = n$, P_n sends the partially decrypted ε_j to P_j.
To sum up, every participants partially decrypt the encrypted comparison vectors (i.e the $\tau' s$), the decryption in this case needs $x = \prod_{i=1}^{n} x_j$ and P_j only knows his/her x_j. The random r and the permutation of the ciphers ensure that no information is leaked.

3.5 Ranking submission

In this last phase, every P_j decrypts every cipher (c, c') in ε_j by using $g^m = c/c'^{x_j}$ and checks if $g^m = 1$. P_j counts the number of zeros in the decrypted messages. Let d be this number, the ranking of P_j is then $d_j = d + 1$. If $d_j \leq k$, P_j submits v_j and d_j to P_0.

Chapter 4

Simulation and results

In this section we describe how the protocol have been implemented using Java programming language. The execution of the protocol and the code flow will be presented here as a UML diagram. Here is a list and short description of classes we used to implement the protocol.

4.1 Class list

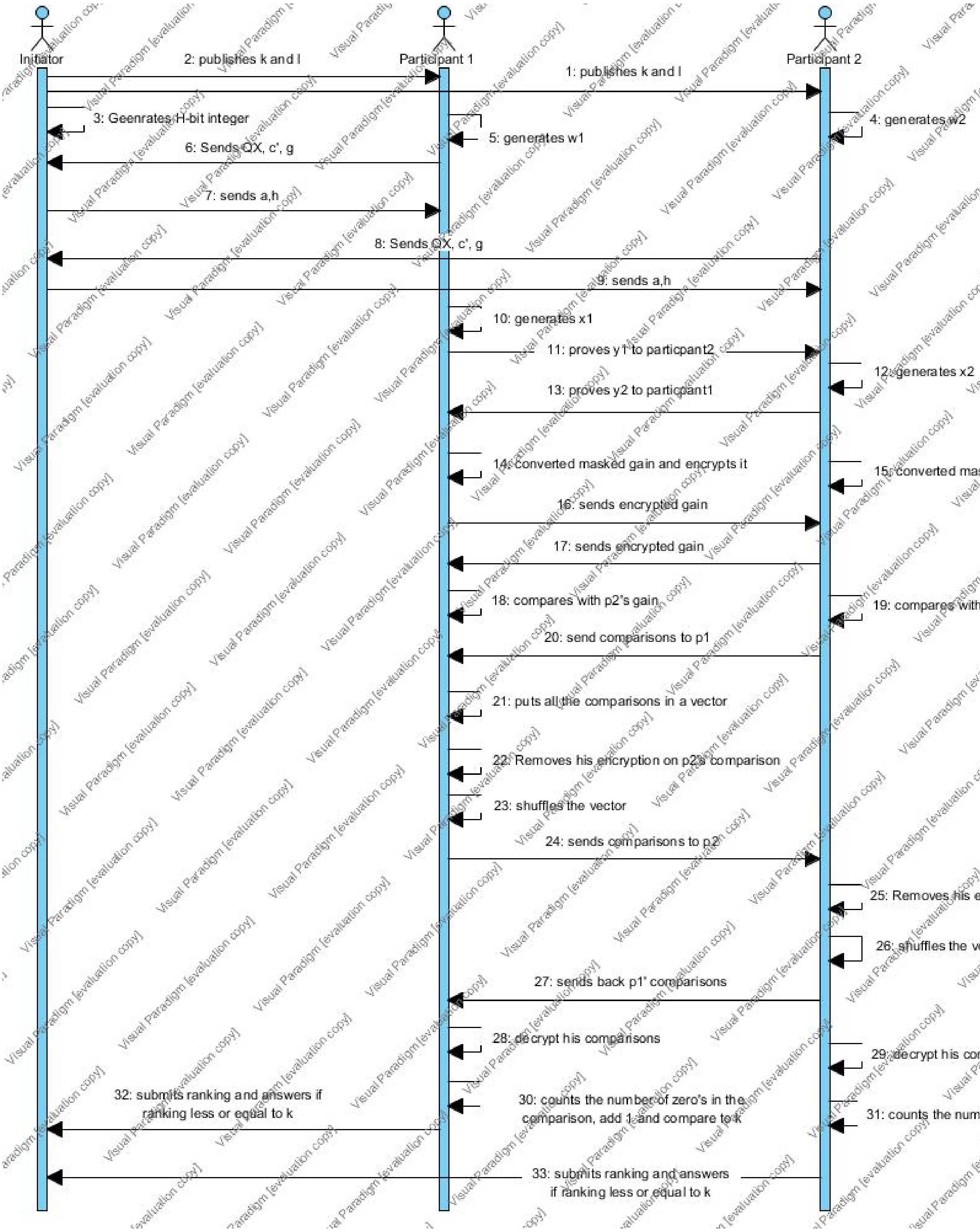
Owner.java This class is used to represent the initiator P_0.
Participant.java This class is used to represent one participant P_j, $1 \leq j \leq n$.
ZeroKnowledgeProver.java This class is an helper that represents the prover in the zero-knowledge proof protocol.
ZeroKnoledgeVerifier.java This class is an helper that represents the verifier in the zero-knowledge proof protocol.
SecuredotProductParty.java This class is an helper that represents a party in the secure dot product protocol.
ElGamal.java This class represents the ElGamal cryptosystem used in the protocol.
GroupGenerator.java This class is used to generate a group G_q. This is done by generating first a safe-prime order group P then we find a prime order sub-group G of P along with it's generator g. The complete algorithm is given at the end of the section.
Database.java This class mocks a server that would be used for an application using our code. The database contains app's users and lists of polls. The database also serves a data exchange interface. The data of concern are the one use for example in the secure dot product, in the zero-knowledge proof, etc...

This implementation rely on a third party called ***the system owner***.

He generates (P, G, g) and publishes them, these parameters will be used by all parties involved in the protocol.

4.2 Simulation of the protocol

Since a picture is worth a thousand words, in this section we will use UML diagrams to show interaction between classes throughout the running of the protocol.


Initiator
Participant 1
Participant 2
1: publishes k and l
2: publishes k and l
3: Geenrates H-bit integer
4: generates w2
5: generates w1
6: Sends QX, c', g
7: sends a,h
8: Sends QX, c', g
9: sends a,h
10: generates x1
11: proves y1 to particpant2
12: generates x2
13: proves y2 to particpant1
14: converted masked gain and encrypts it
15: converted mas
16: sends encrypted gain
17: sends encrypted gain
18: compares with p2's gain
19: compares with
20: send comparisons to p1
21: puts all the comparisons in a vector
22: Removes his encryption on p2's comparison
23: shuffles the vector
24: sends comparisons to p2
25: Removes his e
26: shuffles the v
27: sends back p1' comparisons
28: decrypt his comparisons
29: decrypt his co
30: counts the number of zero's in the comparison, add 1 and compare to k
31: counts the num
32: submits ranking and answers if ranking less or equal to k
33: submits ranking and answers if ranking less or equal to k

Chapter 5

Conclusion

Objective: In this project we have analysed and implemented a secure group ranking protocol using Java programming language. The protocol we have studied gives the required security in the HBC model, and protects an honest participant from passive inside attacks.

Implementation Status: We need to fix the chained decryption (i.e step 9). Furthermore, the code also needs re-factoring so that it can be used more generally as library for example.

Future work Since it has been shown that there exist no secure distributed dot product protocol[12]. Knowing whether a dishonest initiator could learn some knowledge about the participant is a question of great interest. Unfortunately, due to time constraint the question has not been investigated further.

challenges: Choosing an implementation project was at the same time a hard choice and an interesting one. Hard because I am more versed in theoretical matters, interesting because I believe that it is a good hands-on experience. In this project, the implementation has been tedious and has required being really careful.

Lessons learned: Implementation of mathematical concepts to create a cryptosystem, implementation of other various tools. Programming wise, I have come to develop more tools that helped me to be efficient and spot my mistakes faster than before. Knowledge wise, I have learned new cryptographic concepts. This includes how to securely generates a cyclic group used in cryposystems, homomorphic cryptosytems, HBC models, Zero-Knowledge proof and secure dot product protocol.
Finally, I would like to acknowledge Handan Kilinç, who supervised this project.

Chapter 6

Appendix

Proof of correctness of gain comparison. In this paragraph we show that the comparison algorithm used at step 7 is correct and gives the desired results.
Let a = $a_{l-1}...a_1a_0$ and b = $b_{l-1}...b_1b_0$ be two l bits unsigned numbers in binary form and we want to compare a to b. Let suppose W.L.O.G that the first k-1 most significant bits of a and b are the same, $0 \leq k-1 \leq l$. γ is computed computed as a bit-by-bit xor of a and b, i.e $\gamma_k = a_k \oplus b_b$. We notice here that γ_v is always 0 for $v > k$ and γ_k is always 1. Let ω be a n-bits numbers where $\omega_k = (l-t+1)(1-\gamma_t)$ we notice here that ω_k is always 0. Lastly τ is a n-bits number such that $\tau_t = \omega_t + a_t$

Case 1: a is smaller than b, so $a_k = 0$ and $b_k = 1$. This implies that $\gamma^k = 1$. This implies that $\tau_k = 0 + a_k = 0$.

Case 2: a is greater than b. In this case $a_k = 1$ and $b_k = 0$. This implies that that $\omega_k = 0$ and $\tau_t = 0 + a_k = 1$.

In both cases, $\omega_j \geq 1$, for $j < k$. This comes form the fact that $(l-t+1)(1-\gamma_j) \geq 0$ and sum of γ's is at least 1 due to the fact that $\gamma_k = 1$. We can also see that $\omega_j \geq 1$, for $j > k$ since the ω_j are always 1. As a consequence, corresponding $\tau_j \geq 1$ for $j \neq k$. This shows that τ will have at most one zero and will contain a zero iff a < b.
Now it is easy to convince ourselves that the procedure applied here for a and b is the same as the one used by the protocol expect that in the protocol the numbers are encrypted and represented as an array.

ElGamal group generation. Pick a prime number q.
Take a random $p = aq + 1$ until it is prime.
Take a random number in $\mathbb{Z}_p^*$, raise it to the power a modulo p, and get g - if $g = 1$, try again (otherwise, it must be of order q in $\mathbb{Z}_p^*$).

Proof of correctness of masked gain. Let a, b, c be integers such that $a > b$ and $c > 0$. let $a' = ca + c_a$ and $b' = cb + c_b$, where c_a, c_b in

$\{0, 1, .., c - 1\}$. Is is the case that $ca - cb \geq c$. We also have that $c > c_b - c_a$. Thus $ca - cb > c_b - c_a$, finally we have $ca + c_a > cb + c_b$. This shows that the order of the partial gains are preserved by the masks

Bibliography

[1] Lingjun Li, Xinxin Zhao, Guoliang Xue, and Gabriel Silva. *Privacy Preserving Group Ranking.* 2012 32nd IEEE International Conference on Distributed Computing Systems

[2] K. Jonsson, G. Kreitz, and M. Uddin, *"Secure multi-party sorting and applications"*,2011, http://eprint.iacr.org/.

[3] M. Burkhart and X. Dimitropoulos, *"Fast privacy-preserving top-k queries using secret sharing",* in Proc. IEEE ICCCN10, Zrich, Switzerland, Aug. 2010.

[4] T. Nishide and K. Ohta, *"Multiparty computation for interval, equality, and comparison without bit-decomposition protocol"*, Public Key CryptographyPKC 2007, pp. 343360, 2007.

[5] I. Damgard, M. Fitzi, E. Kiltz, J. Nielsen, and T. Toft, *"Unconditionally secure constant-rounds multi-party computation for equality, comparison, bits and exponentiation"*, Theory of Cryptography, pp. 285304, 2006.

[6] S. Goldwasser, S. Micali, and C. Rackoff, *"The knowledge complexity of interactive proof-systems"*, in Proc. ACM STOC85. ACM, 1985, pp. 291304.

[7] I. Damgard, M. Geisler, and M. Kroigard, *"Homomorphic encryption and secure comparison"*, International Journal of Applied Cryptography, vol. 1, no. 1, pp. 2231, 2008.

[8] H. Lin and W. Tzeng, *"An efficient solution to the millionaires problem based on homomorphic encryption"*, in Proc. IACR ACNS. New York, USA: Springer, 2005, pp. 456466.

[9] P. Paillier, *"Public-key cryptosystems based on composite degree residuosity classes"*, in Proc. IACR EUROCRYPT99, Prague, Czech Republic, May 1999.

[10] I. Ioannidis, A. Grama, and M. Atallah, *"A secure protocol for computing dot-products in clustered and distributed environments"*, in Proc. IEEE ICPP02, Vancouver, British Columbia, Aug. 2002.

[11] C. Schnorr, *"Efficient identification and signatures for smart cards"*, in Proc. Crypto89. Springer, 1990, pp. 239252.

[12] Pedersen, Thomas Brochmann and Sava, Erkay (2009) *"Impossibility of unconditionally secure scalar products"*. Data and Knowledge Engineering, 68 (10). pp. 1059-1070. ISSN 0169-023X