Proofs with Sample Mean and Variance Equations

Givens: Let $x_1, x_2, \ldots, x_n$ be n observations of a variable of interest. Given that the sample mean $\bar{x}_n$ and sample variance s_n^2 are expressed as:

$$\bar{x}_n = \frac{1}{n}\sum_{k=1}^{n} x_k \quad \text{and} \quad s_n^2 = \frac{1}{n-1}\sum_{k=1}^{n}(x_k - \bar{x}_n)^2$$

where here the subscript n's indicate the number of observations in the sample.

Proof: I will show, algebraically, that the following relation holds between the mean of the first $n-1$ observations and the mean of all n observations:

$$\bar{x}_n = \bar{x}_{n-1} + \frac{x_n - \bar{x}_{n-1}}{n}$$

$$\bar{x}_n = \frac{1}{n}\sum_{k=1}^{n} x_k$$

$$(n)\bar{x}_n = \frac{1}{n}\sum_{k=1}^{n} x_k(n)$$

$$(n)\bar{x}_n = \sum_{k=1}^{n} x_k$$

$$(n)\bar{x}_n = \sum_{k=1}^{n-1} x_k + x_n$$

$$(\frac{1}{n-1})(n)\bar{x}_n = \sum_{k=1}^{n-1} x_k + x_n(\frac{1}{n-1})$$

$$\frac{n}{n-1}\bar{x}_n = \bar{x}_{n-1} + x_n\frac{1}{n-1}$$

$$(n-1)\frac{n}{n-1}\bar{x}_n = \bar{x}_{n-1} + x_n\frac{1}{n-1}(n-1)$$

$$(n)\bar{x}_n = \bar{x}_{n-1}(n-1) + x_n$$

$$(\frac{1}{n})(n)\bar{x}_n = \bar{x}_{n-1}(n-1) + x_n(\frac{1}{n})$$

$$\bar{x}_n = \bar{x}_{n-1} + \frac{x_n - \bar{x}_{n-1}}{n}$$