

Problem 2:

$$T(n) = T(n/2) + 3 \text{ if } n > 1 \text{ and } T(1) = 2$$

To solve this recurrence, I will assume that all division is integer division.

Now, suppose  $n$  is a power of 2, say,  $n = 2^m$ .

If so, what we will have is:

$$T(2^m) = T\left(\frac{2^m}{2}\right)$$

The above expression, however, can be re-written recursively as:

$$\begin{aligned} & T(2^{m-1}) + 3 \\ & T(2^{m-2}) + 3 + 3 \\ & T(2^{m-3}) + 3 + 3 + 3 \end{aligned}$$

So, for all  $k$ , we can claim that:

$$T(2^{m-k}) + 3k$$

And if we apply this reasoning to  $k = m$ , we get:

$$T(2^{m-m}) + 3m$$

And this results in:

$$2 + 3m$$

Now, since I have substituted  $2^m$  for  $n$ , we must take the log of both sides of the equation to get the true value of  $m$ :

$$n = 2^m$$

$$\log(n) = \log(2^m)$$

$$\log(n) = m$$

Therefore,

$$T(n) = 2 + 3\log_2 n$$

To determine the complexity, we can use the master theorem:

For this function, we have:

$$a = 1, b = 2, c = 0$$

$$\log_b a = \log_2 1 = 0$$

Therefore,

$$T(n) = \theta(n^0 \log n)$$

Or,

$$\theta(\log n)$$