

Problem 3

$T(n)=T(n)+1$ if $n>2$ and $T(n)=0$ otherwise

$$T(n) = T(\sqrt{n}) + 1$$

$$T(n^{\frac{1}{2}}) = T(n^{\frac{1}{2}}) + 1$$

$$T(n^{\frac{1}{4}}) = T(n^{\frac{1}{4}}) + 1 + 1$$

$$T(n) = T(n^{\frac{1}{8}}) + 1 + 1 + 1$$

$$T(n) = T(n^{\frac{1}{2^k}}) + k$$

$$2 \geq n^{\frac{1}{2^k}}$$

$$\log(2) \geq \log(n^{\frac{1}{2^k}})$$

$$\log(2) \geq \frac{1}{2^k} \log(n)$$

$$\frac{\log(2)}{\log(n)} \geq \frac{1}{2^k}$$

$$\frac{\log(n)}{\log(2)} \leq 2^k$$

$$\log(n) \leq 2^k$$

$$\log(\log(n)) \leq k \log 2$$

$$\log(\log(n)) \leq k$$

Therefore,

$$T(n) = \theta(\log(\log(n)))$$