Attributes of color represented by a spherical model

Article *in* Journal of Electronic Imaging · October 2013

DOI: 10.1117/1.JEI.22.4.043032

3 authors, including:

Tieling Chen
University of South Carolina Aiken

Journal of
Electronic Imaging

SPIEDigitalLibrary.org/jei

Attributes of color represented by a spherical model

Tieling Chen
Jun Ma
Zhongmin Deng

Attributes of color represented by a spherical model

Tieling Chen
Wuhan Textile University, China
School of Mathematics and Computer Science
1 Textile Road, Wuhan
Hubei 430073, China
and
University of South Carolina Aiken
Department of Mathematical Sciences
471 University Parkway
Aiken, South Carolina 29801
E-mail: tielingc@usca.edu

Jun Ma
Wuhan Textile University, China
School of Mathematics and Computer Science
1 Textile Road, Wuhan
Hubei 430073, China

Zhongmin Deng
Wuhan Textile University, China
School of Textile and Material
1 Textile Road, Wuhan
Hubei 430073, China

Abstract. *The paper introduces a spherical coordinate system-based color model and studies the color change in the model. A circular cone with a spherical top tightly circumscribing the RGB color cube is equipped with a properly rotated spherical coordinate system. Similar to the commonly used color models with a hue component such as the HSV model, the spherical model specifies color by describing the color attributes recognized by human vision, using the components of the spherical coordinate system. The formulas of conversions between the spherical model and the RGB color model are provided, which are mathematically simpler and more intuitively understandable than those for commonly used models with a hue component. Most importantly, color changes are perceptually smoother in the spherical model. Comparisons between the spherical model and the HSV model on color changes are made in the paper.* © *2013 SPIE and IS&T [DOI: 10.1117/1.JEI.22.4.043032]*

1 Introduction

A color model is an abstract mathematical model with two aspects, one is the geometric shape of a subset in the three-dimensional space and another one is the rule that maps colors to the points in the subset. For example, the RGB model describes color with a tuple of three numbers representing the amount of the three primary colors red, green, and blue, which is perfectly represented by the unit cube in the three-dimensional Euclidean space. The RGB model is also used as a theoretical platform to study other color models, mainly because of the identity correspondence between the geometric shape of the cube and the set of colors. To convert a color model to the RGB color model, one just needs to provide the transformation to convert the space subset representing the model to the unit cube, such that a point in the space subset is mapped to the point in the cube determined by its color. In other words, the conversion is a transformation of the shape of the subset into the cube, subject to the constraint that color does not change. The RGB model is well used in electronic systems such as computers. Unfortunately, the RGB model does not provide the human vision a natural way of understanding colors.

Human vision perceives color by recognizing the attributes of color such as hue, saturation, and brightness. A color model that specifies color by describing color attributes is more convenient than the RGB model in applications where color manipulation is a main concern. There are several such well-used color models, and among them the HSV color model is a very important one. The model uses three components to describe the hue H, saturation S, and the brightness value V of a color. The HSV model was originally introduced as a hexagonal cone model named the hexcone model;[1] then it evolved to a cylindrical model that uses the center angle, the radius of a cylindrical shell, and the height of a horizontal disc to measure the H, S, and V components. In computer graphics applications, other commonly used color models describing color with color attributes include the hue, saturation, and lightness (HSL) model[1–3] and the

Paper 13272P received May 12, 2013; revised manuscript received Oct. 8, 2013; accepted for publication Oct. 30, 2013; published online Dec. 18, 2013.

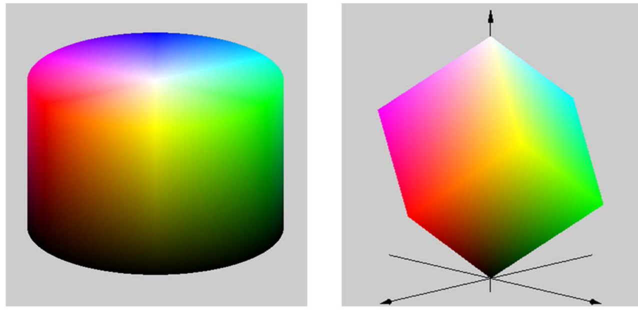
hue, saturation, and intensity (HSI) model.[4] A common characteristic of these color models is that they all use the hue as one component in their systems. Because the HSL model and the HSI model are very similar in concept to the hue, saturation, and brightness (HSV) model, and also because the HSV model and the spherical color model proposed in the paper have a similar geometrical structure, the paper uses the HSV model as the representative of the commonly used models with a hue component in the comparison.

The concept of color models with a hue component is easy to understand, but the conversions to the RGB model for display are generally complicated and not easy to interpret. Although the conversions between the RGB model and the commonly used models with a hue component such as the HSV model can be implemented in a few lines in code, the principles of the conversions are not revealed by the conversion formulas. In other words, the conversions are not intuitively simple and are hard to understand to nonspecialists. Furthermore, the commonly used color models with a hue component have a more unfavorable aspect: color does not change perceptually smoothly in their color spaces. For example, in the HSV model, apparent brilliant star-like rays appear on color discs with constant values, making color change perceptually unsmooth near these rays. It looks like colors condense along the rays. In addition, lines near which colors condense also occur on the color surfaces with a constant amount of saturations. These phenomena cause inconveniences in color picking and inaccuracies in color comparisons.

The motivation of the paper is to establish a color model with a hue component that meets the following two criteria. First, the conversions between the model and the RGB model are mathematically simple and intuitively understandable, and second, color changes perceptually smoothly throughout the system. We discovered that a properly rotated spherical system meets the criteria very well, and the three components of a color point in the system perfectly represent the attributes of color. Similar to the HSV model, the spherical model specifies color by describing the color attributes recognized by human vision and has the hue as a component. Mathematical formulas to convert between the spherical color model and the RGB model are obtained in the paper. The conversion formulas are much mathematically neater than those of the HSV model, and the interpretation of the model is more intuitive too. Besides, the model is expandable because it can specify more colors than the RGB color cube. Most importantly, the spherical color model does not have the color condensation phenomenon that occurs in the HSV model and other existing color models with a hue component, and therefore color changes more perceptually smoothly.

2 Mathematical View of the HSV Model

In this section, we examine the HSV model from a mathematical point view to understand how the color attributes are described and why the color condensation phenomenon occurs in the system. The HSV color model was first introduced in Ref. 1 in 1978 as a hexagonal cone model. The commonly used cylindrical HSV model is a variation of the original hexcone model. It is a cylindrical system that measures the hue component H with the center angle, the saturation component S with the radius of the cylindrical shell, and the brightness value component V with the height of the disc above the base. Figure 1(a) shows a cylinder in the model space. Colors on the top disc have the same value in the V component and colors on the side surface have the same value in the S component. All the hues can be found on the top disc, while the bottom disc contains only the black color. It is apparent that color does not change perceptually smoothly on the top disc and on the side surface either. A star-like ray phenomenon occurs on the top disc and some vertical color ridges occur on the side surface.

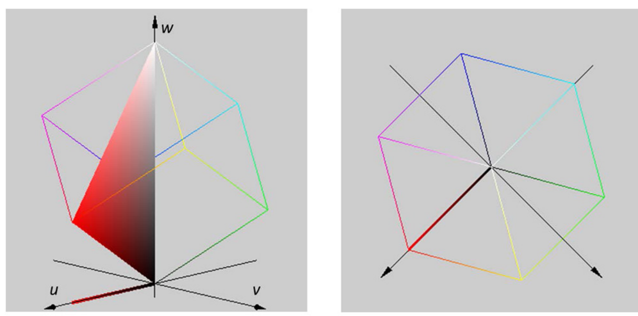


(a) (b)

Fig. 1 (a) A cylinder of the cylindrical HSV color model. Color does not change perceptually smoothly. A star-like ray phenomenon occurs on the top disc and vertical color ridges occur on the side cylinder surface. (b) The RGB color cube is repositioned such that the main diagonal is vertical. The shape of the subset of the cylindrical model is transformed from the repositioned cube.

Because the cylindrical HSV model is derived from its original version, named hexcone model, we mainly check the original model and look into the relations with the RGB model to explain the two unfavorable aspects of the HSV model that are mentioned in the Introduction; one is the obscurity in the formulas of the conversion transformations between HSV and RGB and the other one is the unsmooth change of color in the model. The hexcone HSV model can be visualized in the following way. Reposition the color cube in the three-dimensional space such that the black vertex is on the origin and the main diagonal connecting the black vertex and the white vertex is on the vertical axis, see Fig. 1(b).

A semiplain bounded by the main diagonal crosses the cube and gives a color triangle. All the colors on the color triangle have the same hue. As shown in Fig. 2(a), we set up a right-handed coordinate system (u, v, w) in such a way that the main diagonal of the color cube is on the positive side of



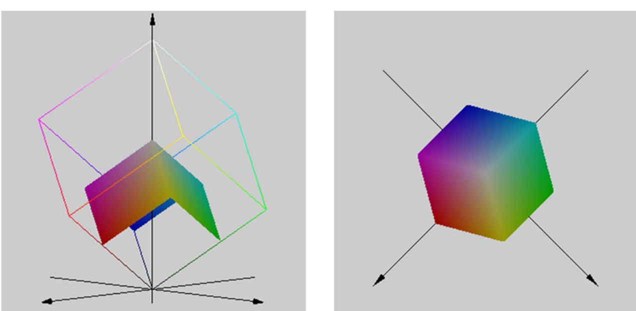
(a) (b)

Fig. 2 (a) The color triangle passes the red vertex, and the hues of all the colors in the triangle are the same, given by $H = 0$, the hue of red. (b) The positions of the hues of red, yellow, green, cyan, blue, and magenta on the uv-plane.

the w axis and the projection of the color triangle containing the red color is on the positive side of the u axis. The v axis can be found such that its unit vector is the cross-product of the unit vectors of the other two axes. The projection of a color triangle onto the uv-plane is a line segment, and its position can be determined by the angle θ formed by the segment and the positive half of the u axis. Therefore, the hue component H of the HSV model is given by $H = \theta$. By revolving the semiplain with a full round about the w axis, we obtain all the hues of the HSV model. Figure 2(b) displays the positions of the hues of red, yellow, green, cyan, blue, and magenta on the uv-plane. Using the center angle θ to represent the hue attribute of a color is common to many color models such as HSV, HSL, and HSI. However, if θ itself is not a coordinate component in a system, then finding the value of θ is mathematically complicated. Take the hexcone HSV model as an example. The projection of the color cube to the uv-plane is a right hexagon. The value of H is determined by which sector of the hexagon the color point is projected to and what position the projected point is in the sector. Since θ is not directly used, H is determined by checking the subtle relations among the R, G, and B components of the color point, which have many different cases to consider.

The value of the V component of the HSV model can be obtained as follows. Every color point in the color cube determines a subcube with the origin as its black vertex and with its main diagonal on the w axis, such that the color point is on the shell consisting of the three top faces. Figure 3(a) shows such a shell. All the color points on the shell have the same value of the V component, which is determined by the side length of the faces of the shell. The V component measures how far the color point is from the black vertex.

In the hexcone HSV model, V is given by the maximum of the R, G, and B components of the color point. However, the maximum function is not a smooth function. Although all the color points on a shell have the same value of V, their Euclidean distances from the origin are not the same. Projecting a shell with constant value onto the uv-plane, we get a right hexagon, see Fig. 3(b). The projection of the three ridges of the shell is apparently displayed as three brilliant rays, showing that color has abrupt perceptual change when crossing these rays, although the V value of the color remains the same.



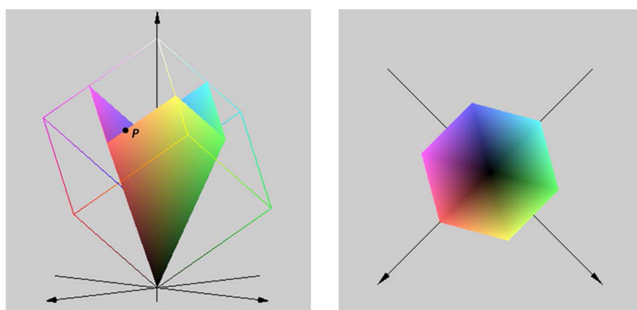
(a) (b)

Fig. 3 (a) A shell with constant value on the V component of the HSV model. The value of the V component measures the departure of a color from black. (b) The projection of a shell with constant value to the uv-plane. The resultant region is a right hexagon. Three brilliant rays occur in the region.

The saturation component S is determined by a hexagonal cone with the w axis as its center. For a color point in the cube, we can find the hexagonal cone that contains the point as follows. Suppose the line passing through the origin and the color point intersects one of the three top faces of the cube at point P, as shown in Fig. 4(a). If the color point lies on the main diagonal, then P is the white vertex at the very top. Color points on the main diagonal have zero saturation. On the face where P lies, find the square with the white vertex as one of its vertices and with point P on one of its sides. Find the square of the same size on each of the other two top faces. Looking from the top, the six outer sides of these three squares form a hexagon. Connect the origin with each point on the six outer sides to obtain a hexagonal cone. The saturation component measures how far a color point is from the main diagonal, where the gray colors are. Figure 4(a) shows a cone with constant saturation. All the color points on the side surface of the cone have the same saturation. The measure of saturation is determined by the opening of the hexagonal cone, which can be determined by the measure of the side of the three squares enclosed by the cone on the three top faces of the cube. In the hexcone HSV model, the computation for S involves the minimum of the R, G, and B components of the color point. The minimum function is not a smooth function.

Looking from the white vertex, the inner surface of a cone with constant saturation forms a right hexagon, see Fig. 4(b). All the colors have the same saturation, but apparently the color does not change perceptually smoothly when it is close to the edges joining two side faces of the cone.

Geometrically, the unsmooth shape of the hexagonal cone with constant value in S and the unsmooth shape of the shell with constant value in V are the main reasons of the unsmooth change of color in the HSV model. Analytically, the unsmooth functions involved in the computations of S and V cause the unsmooth change of color in the model. The hexcone HSV model normalizes the domains of the three components H, S, and V to the interval [0, 1]. The math expressions of the conversions between the RGB model and the HSV model are given as piecewise functions, and the conversion algorithms also consider many subtle cases. The mathematical relations between the two models are not clearly revealed in the algorithms. The relations are even more complicated when the HSV model is transformed to



(a) (b)

Fig. 4 (a) The hexagonal cone determined by a point P on the top faces of the color cube. The saturations of all the colors on the surface of the cone are the same. (b) The projection of a cone with constant saturation to the uv-plane. The resultant region is a right hexagon. The star-like rays are caused by the edges on the side surface of the cone.

the cylindrical model. Each hexagon projected from a shell with constant value in V is expanded to a circular disc and the radius is uniformly normalized. All the circular discs are integrated into the cylindrical model. In the transformation, color triangles with constant hues in H still keep the same hues, but hexagonal cones with constant saturations in S are transformed to cylindrical shells, and the shells with constant values in V are transformed to circular discs. Just like in the original hexcone model, in the cylindrical model, color does not change perceptually smoothly on both the cylindrical shells and the circular discs, as shown in Fig. 1. In fact, the situation for the cylindrical HSV color model is even worse because the transformation converting a hexagon to a circle does not stretch the points of the hexagon uniformly. There are six color rays that occur on each color disc. Vertical color ridges also occur on the side cylinder surface.

3 Spherical Color Model

We introduce a spherical color model in this section. The idea of using a spherical space in color modeling is actually not new, and examples can be found in Refs. 5, 6, and 7. However, the relationship between the components of the mathematical spherical coordinate system and the color attributes used in color description has not been well studied. We impose the mathematical spherical coordinate system over a properly rotated cube and then use the three coordinate components of the spherical system to describe the color attributes. Mathematical formulas of the conversions between the spherical model and the RGB model are obtained. Compared with the HSV model, the spherical model is mathematically simpler and color changes are perceptually smoother over the system.

We first build the spherical model starting from the RGB model. Suppose the coordinate system (r, g, b) represents the space of the RGB model. With a rotation, the (r, g, b) coordinate system is transformed to the (u, v, w) coordinate system as shown in Fig. 2. To find the transformation from the (r, g, b) system to the (u, v, w) system, we need to find the expressions of the basis of the (u, v, w) system in the (r, g, b) system. Since the w axis coincides with the main diagonal of the color cube, the unit vector of the w axis is $(\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3)$ in the (r, g, b) system. The v axis is perpendicular to the plane determined by the main diagonal and the r axis, so its direction is determined by the cross-product of the vector $(\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3)$ and the vector (1,0,0). After normalization, we get the unit vector of the v axis, $(0, \sqrt{2}, -\sqrt{2})$. The unit vector of the u axis is the cross-product of $(0, \sqrt{2}, -\sqrt{2})$ and $(\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3)$, which gives $(\sqrt{6}/3, -\sqrt{6}/6, -\sqrt{6}/6)$. Therefore, the transformation from (r, g, b) to (u, v, w) is given by

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \sqrt{6}/3 & -\sqrt{6}/6 & -\sqrt{6}/6 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{3}/3 & \sqrt{3}/3 & \sqrt{3}/3 \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}. \quad (1)$$

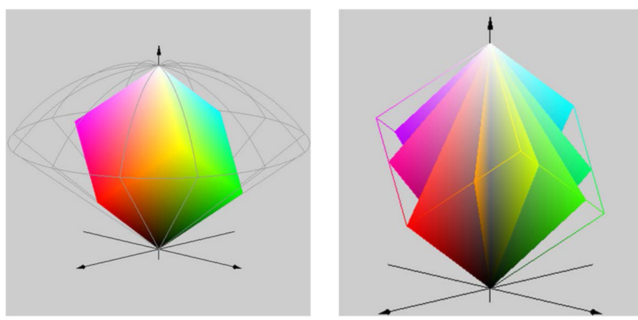
After the transformation, the main diagonal of the RGB cube is on the vertical w axis and the white point is at (0, 0, $\sqrt{3}$) in the new system, as shown in Fig. 2. As an example, the red vertex in the RGB space $(r, g, b) = (1, 0, 0)$ has a new coordinate $(u, v, w) = (\sqrt{6}/3, 0, \sqrt{3}/3)$.

Now we set up the standard mathematical spherical coordinate system over the (u, v, w) system. Let ρ, θ, and φ be the three coordinate components of the spherical system. Given a point in the spherical system, the component ρ measures the distance between the point and the origin, which is the black vertex, the component θ measures the azimuthal angle between the plane of $v = 0$ and the plane determined by the w axis and the point, and the component φ measures the polar angle of the circular cone containing the point, with the origin as its vertex and the w axis as its center axis.

In a standard mathematical spherical system, ρ is non-negative, θ is in the interval $[0, 2\pi]$, and φ is in the interval $[0, \pi]$. However, we only need a subspace that is a circular cone with a spherical top tightly circumscribing the RGB color cube, see Fig. 5(a). The component θ is still in the interval $[0, 2\pi]$. Since the largest value of ρ is reached at the white vertex, ρ is in the interval $[0, \sqrt{3}]$. The largest value of φ is reached at the red, green, and blue vertices. At the red vertex, we have $\tan\varphi = (\sqrt{6}/3)/(\sqrt{3}/3) = \sqrt{2}$, so $\varphi = \arctan\sqrt{2}$. Therefore, the value of φ is in the interval $[0, \arctan\sqrt{2}]$. Thus, the subspace in the spherical system is limited by $0 \le \rho \le \sqrt{3}$, $0 \le \theta < 2\pi$, and $0 \le \varphi \le \arctan\sqrt{2}$. These measures can be normalized to the range [0,1], but we still keep their original ranges for the convenience of system transformations. The subspace contains more points than the RGB color cube, which means it can describe colors that cannot be properly described by the RGB model. In this paper, we focus on the colors that are within the RGB color cube.

The three components θ, φ, and ρ describe the color attributes hue, saturation, and brightness very well. Let us look at the component θ first. Obviously, it is essentially the same as the hue component in the HSV color model. Therefore, it describes the hue attribute of a color. We limit the spherical color model within the RGB color cube. Figure 5(b) shows some color triangles within the color cube, with constant hues represented by different values of the angle θ.

Then we look at the component φ. It measures the opening of a circular cone with the vertical axis as its center line. It is comparable to the saturation component in the hexcone HSV model because it measures how far the colors on the cone are from the gray color diagonal. Hence, it describes the saturation attribute of a color. Figure 6(a) shows a circular cone within the color cube with a constant value of φ. Compared with the hexagonal cone of the HSV model in Fig. 4(a), the circular cone of the spherical model has a



(a) (b)

Fig. 5 (a) The circular cone with a spherical top that tightly circumscribe the RGB color cube. (b) A set of color triangles with constant hues for colors within the color cube.

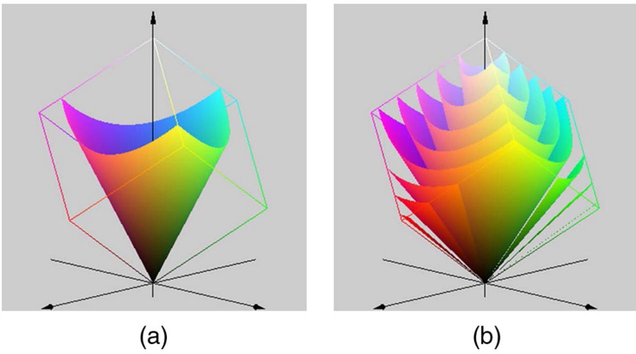


Fig. 6 (a) A circular cone with constant saturation within the color cube. (b) A set of circular cones with constant saturation within the color cube.

smoother side surface. Figure 6(b) displays some circular cones within the color cube with different φ values.

Finally, we look at component ρ, which is the distance between a color point and the origin. It measures how far a color is from the black color, so it describes the brightness attribute of a color. In concept, the component is similar to the V component in the HSV model, but it is more geometrically meaningful and the distance function is smooth. Figure 7(a) shows one spherical surface within the color cube. The surface is smoother than the shell with constant V value shown in Fig. 3(a). Some spherical surfaces within the color cube with different values of ρ are shown in Fig. 7(b).

From the above analysis, we can see that the three components of the spherical system describe the color attributes very well. In concept, it is very close to the HSV color model, but it is more intuitive. The mathematical transformation from the spherical coordinate system to the Cartesian coordinate system of (u, v, w) is given by

$$\begin{cases} u = \rho \sin \varphi \cos \theta, \\ v = \rho \sin \varphi \sin \theta, \\ w = \rho \cos \varphi. \end{cases} \tag{2}$$

From Eq. (1) we get the inverse transformation from the (u, v, w) system to the (r, g, b) system.

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} \sqrt{6}/3 & -\sqrt{6}/6 & -\sqrt{6}/6 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{3}/3 & \sqrt{3}/3 & \sqrt{3}/3 \end{bmatrix}^{-1} \begin{bmatrix} u \\ v \\ w \end{bmatrix}. \tag{3}$$

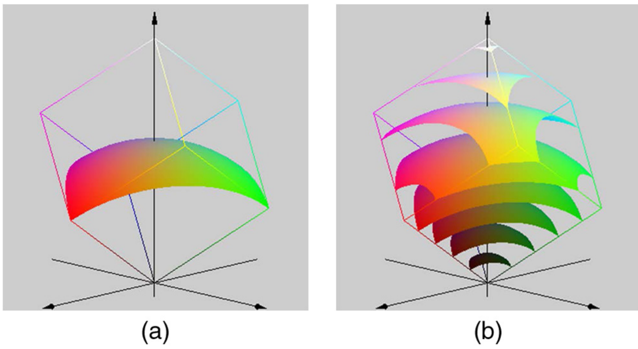


Fig. 7 (a) A spherical surface within the color cube. (b) A set of spherical surfaces with different distances from the black vertex within the color cube.

On composing the transformations in Eqs. (3) and (2), we obtain

$$\begin{cases} r = \sqrt{6}/3\rho \sin\varphi \cos\theta + \sqrt{3}/3\rho \cos\varphi, \\ g = -\sqrt{6}/6\rho \sin\varphi \cos\theta + \sqrt{2}/2\rho \sin\varphi \sin\theta + \sqrt{3}/3\rho\cos\varphi, \\ b = -\sqrt{6}/6\rho \sin\varphi \cos\theta - \sqrt{2}/2\rho \sin\varphi \sin\theta + \sqrt{3}/3\rho\cos\varphi. \end{cases} \tag{4}$$

Equation (4) is the transformation from the spherical color model to the RGB color model. We use the restricted domain of $\rho \in [0, \sqrt{3}]$, $\theta \in [0, 2\pi]$, and $\varphi \in [0, \arctan\sqrt{2}]$ for the transformation, which is the circular cone with a spherical top tightly circumscribing the RGB color cube shown in Fig. 5(a). The resulting set of the transformation contains more colors than the RGB color cube. This raises a technical problem that some of (ρ, θ, φ) in the circular cone are mapped outside the RGB color cube and the corresponding colors cannot be represented by the RGB model. In the actual application, gamut-mapping algorithms that keep the smooth change of color may be required to resolve the problem. In this paper, we only consider the color points that can be properly represented by the RGB model.

For the converse transformation from the RGB model to the spherical model, we first transform the system of (r, g, b) to the system of (u, v, w) using Eq. (1) and then transform the intermediate result to the spherical system (ρ, θ, φ), using the standard mathematical transformation,

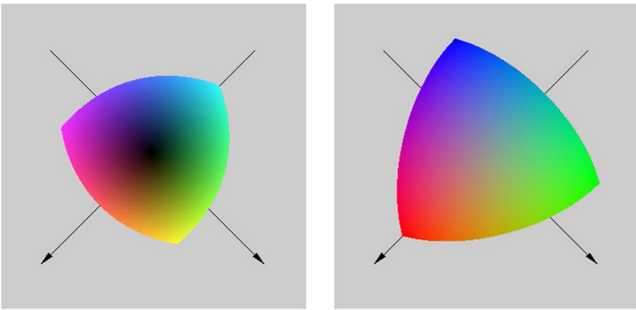
$$\begin{cases} \rho = \sqrt{u^2 + v^2 + w^2}, \\ \theta = \begin{cases} \arccos\frac{u}{\sqrt{u^2+v^2}}, & v \geq 0, \\ 2\pi - \arccos\frac{u}{\sqrt{u^2+v^2}}, & v < 0, \end{cases} \\ \varphi = \arccos\frac{w}{\sqrt{u^2+v^2+w^2}}. \end{cases} \tag{5}$$

On composing the transformations in Eqs. (5) and (1), we get the conversion formula from the RGB color model to the spherical color model, which is

$$\begin{cases} \rho = \sqrt{r^2 + g^2 + b^2}, \\ \theta = \begin{cases} \arccos\frac{2r-g-b}{2\sqrt{r^2+g^2+b^2-rg-rb-gb}}, & g \geq b, \\ 2\pi - \arccos\frac{2r-g-b}{2\sqrt{r^2+g^2+b^2-rg-rb-gb}}, & g < b, \end{cases} \\ \varphi = \arccos\frac{r+g+b}{\sqrt{3}\sqrt{r^2+g^2+b^2}}. \end{cases} \tag{6}$$

The domain of the transformation in Eq. (6) is limited to the color cube, with the values of r, g, and b in [0,1]. The formulas in Eqs. (5) and (6) are conversions between a rotated Cartesian system and the spherical system. The mathematical spherical system is a well-known coordinate system, so it is a perfect tool to explain the perceptual attributes of color. Compared to the hexcone HSV model, the conversion formulas are more straightforward and easier to comprehend. Because the transformations are smooth functions, color changes perceptually smoothly over the spherical system.

Figure 8(a) shows a circular cone surface with constant φ projected on the perpendicular uv-plane. Perceptually, color changes smoothly on the surface. The star-like ray phenomenon shown in Fig. 4(b) does not occur on the surface, so we think φ is a better measure than S in the HSV model to

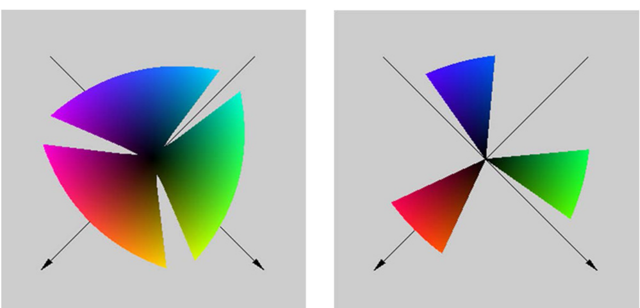


(a) (b)

Fig. 8 (a) A circular cone surface projected on the perpendicular *uv*-plane. The star-like ray phenomenon shown in Fig. 4(b) does not occur on the surface. (b) A spherical surface projected on the perpendicular *uv*-plane. The star-like ray phenomenon shown in Fig. 3(b) does not occur on the surface.

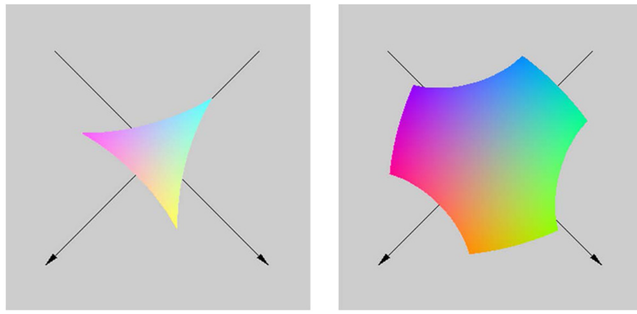
describe the saturation attribute of a color. Figure 8(b) shows a surface with constant ρ projected on the uv-plane. The brilliant star-like rays found in Fig. 3(b) are not shown on the surface. Since color changes perceptually smoothly on the surface, we think ρ is a better measure than V in the HSV model to describe the color attribute of brightness.

One thing that might not be appealing about the spherical color model is that when limited within the RGB color cube, the circular cone surface determined by φ and the spherical surface determined by ρ cannot be completely displayed. Let us look at the circular cone surface determined by φ first. Suppose φ increases from 0. When φ is small and the cone surface is above the vertices of yellow, cyan, and magenta, the projected region is convex as shown in Fig. 8(a). When φ is getting bigger and the cone surface is below the vertices of yellow, cyan, and magenta, the projected region starts splitting into three sectors, as shown in Fig. 9. Figure 9(a) is for a smaller φ and Fig. 9(b) is for a bigger φ. The value of φ when the projected region starts to split can be found as follows. The yellow vertex in the RGB model has coordinates $(r,g,b)=(1,1,0)$, and by Eq. (1) it has $(u,v,w)=(\sqrt{6}/6,\sqrt{2}/2,2\sqrt{3}/3)$. Then $\tan\varphi=\sqrt{u^2+v^2}/w=\sqrt{2}/2$, which gives $\varphi=\arctan(\sqrt{2}/2)$. When $\varphi\le\arctan(\sqrt{2}/2)$, the projected region is convex as shown in Fig. 8(a), otherwise it is concave as shown in Fig. 9.



(a) (b)

Fig. 9 Concave shapes of the projected regions of two circular cones that are limited within the RGB color cube. (a) The figure is for a smaller φ. (b) The figure is for a bigger φ.



(a) (b)

Fig. 10 Concave shapes of the projected regions of two spherical surfaces that are limited within the RGB color cube. (a) The figure is for a $\rho>\sqrt{2}$. (b) The figure is for a ρ between 1 and $\sqrt{2}$.

Similarly, when ρ changes, the projected region of the portion of the spherical surface within the RGB color cube changes its shape, too. This can be seen in Fig. 10, where Fig. 10(a) is the shape for $\rho\ge\sqrt{2}$ and Fig. 10(b) is the shape for $1<\rho<\sqrt{2}$. Both shapes are concave. When $\rho\le 1$, the shape of the spherical surface is like that shown in Fig. 8(b), which is convex. The shape in Fig. 10(a) is for the spherical surface higher than the vertices of yellow, cyan, and magenta, whose ρ values can be obtained by the first equation of Eq. (6), $\rho=\sqrt{2}$. The shape in Fig. 10(b) is for the spherical surfaces lower than the vertices of yellow, cyan, and magenta, but higher than the vertices of red, green, and blue. By the first equation of Eq. (6), the ρ values of the vertices of red, green, and blue are $\rho=1$.

Note that the phenomena demonstrated in Figs. 9 and 10 are not caused by the spherical model itself but by the limitation of color representation by the RGB color cube. In other words, the spherical model contains extra colors that are not contained in the RGB model. The spherical color model changes the way of color selection. Colors that can be represented by the RGB model must be selected from the restricted projected regions.

An important application of the spherical color model is color picking in computer graphics applications. Since color does not change abruptly in the model, it is easier to pick the desired color, compared with the HSV model and similar models. A user-friendly color selector could largely reduce the inconvenience in color picking caused by the restricted projected regions. For example, the user can select color from the restricted projected region of a surface with constant ρ component. By adjusting the value of ρ, the user can access all the colors in the RGB cube. On every projected region, color changes perceptually smoothly. In addition, color also changes perceptually smoothly when ρ changes. For the user's convenience, the color selector can be designed in such a way that the restricted projected region of each value of ρ is inscribed in a circle and all the circles are normalized to the same radius. Only those colors in the restricted regions can be selected. The user can adjust the value of ρ to select a color in a perceptually smooth environment. Because the existing color models including the HSV color model contain the same colors as RGB, the spherical color model is a much bigger model. Most importantly, color changes more perceptually smoothly in the spherical model. After all, the spherical color model is conceptually simple and perceptually harmonious.

4 Conclusion

The paper proposed a color model that uses the components of the mathematical spherical system to describe the color attributes hue, saturation, and brightness. The model is built on the RGB color cube after a proper rotation such that the main diagonal is on the vertical axis. The geometrical shape of the model is a circular cone with a spherical top that tightly circumscribes the rotated color cube. In the spherical system, each color point can be determined by the coordinates (ρ, θ, φ), where the radius ρ measures the brightness of the color, the azimuthal angle θ measures the hue of the color, and the polar angle φ measures the saturation of the color. Conceptually, the spherical model is similar to the existing color models with a hue component, including the commonly used HSV model, but it is more intuitive and easier to interpret. Mathematically, the conversion formulas between the spherical model and the RGB model are standard system transformations and are easy to understand to nonexperts. Most importantly, in the spherical color model, color changes more perceptually smoothly than in the exiting color models with a hue component. The phenomenon that colors condense along some rays or parallel lines that occurs in the existing color models with a hue component does not occur in the spherical model. Generally, with the spherical model, color specification is more convenient and color comparison is more perceptually accurate.

Acknowledgments

This work was partially supported by the National Natural Science Foundation of China under Grant No. 11071280.

References

1. A. R. Smith, "Color gamut transform pairs," *Comput. Graph.* **12**(3), 12–19 (1978).
2. G. H. Joblove and D. Greenberg, "Color spaces for computer graphics," *Comput. Graph.* **12**(3), 20–25 (1978).
3. A. M. Spalter, *The Computer in the Visual Arts*, Addison-Wesley, Boston, Massachusetts (1998).
4. R. C. Gonzales and R. E. Woods, *Digital Image Processing*, 3rd ed., Prentice Hall, Upper Saddle River, New Jersey (2008).
5. C. Izmailov, "Spherical model of discrimination of self-luminous and surface colors," in *Geometric Representations of Perceptual Phenomena*, R. D. Luce et al., Eds., pp. 153–168, Lawrence Erlbaum Associates Inc., Mahwah, New Jersey (1995).
6. A. D. Logvinenko, "An object-color space," *J. Vis.*, **9**(11), 1–23 (2009).
7. P. O. Runge, *Farben Kugel* (Color Sphere), translated by R. G. Kuehni, 2008, published 1810, http://www.iscc.org/pdf/RungeFarben-Kugel.pdf (16 July 2013).

Tieling Chen is an associate professor in the Department of Mathematical Sciences, University of South Carolina Aiken. His research interests include application of mathematical transformations to image processing and computer graphics. He is an adjunct professor of Wuhan Textile University under the Chu Tian Scholars Program of Hubei, China.

Jun Ma is a professor in the School of Mathematics and Computer Science, Wuhan Textile University, China. His research interests include wavelet transformation-based detection techniques at the yarn (i.e., single-strand) level, algorithm design, analysis of detection techniques, and mathematical transformations in image processing and computer graphics.

Zhongmin Deng is a professor in the School of Textile Science and Engineering and director of the Key Laboratory of Green Processing and Functional Textiles of New Textile Materials, Ministry of Education, China. His research interest includes integrating computer-aided design and manufacturing of textile products, image processing, and computer graphics.