

Turbulence Monte Carlo

M. V. da Cunha Pereira

January 14, 2014

Abstract

This document describes some of the theory behind simulations performed in this project.

1 From Fourth to Second-Order Simulation

Performing simulations for quantities that are fourth-order in field is a formidable task when propagation under atmospheric turbulence is considered, so we try to reframe the model in terms of equivalent second-order beam propagation. This can be done for special cases detailed below, using, for the phase screens that emulate turbulent refractive index fields, a modified 'effective field'.

1.1 Delta Correlation in Near Field

We first consider a special case in which the fourth-order beam is delta correlated in the near-field, so that on the source-plane it has the form

$$E_0(\rho'_1, \rho'_2) = E_p \left(\frac{\rho'_1 \pm \rho'_2}{2} \right) V_a(\rho'_1 \mp \rho'_2) \quad (1)$$

, where $V_a(\rho) = \delta^2(\rho - a)$.

Top and bottom sign refer to the case where there is and isn't a coordinate inversion scheme in place. We consider the former for concreteness and as it

corresponds to the more interesting case. Hence its propagation reads:

$$\begin{aligned}
E_a(\rho_1, \rho_2) &= \iint E_p \left(\frac{\rho'_1 - \rho'_2}{2} \right) V_a(\rho'_1 + \rho'_2) h(\rho_1, \rho'_1) h(\rho_2, \rho'_2) d^2 \rho'_1 d^2 \rho'_2 \\
&= \int E_p \left(\rho'_1 + \frac{a}{2} \right) h(\rho_1, \rho'_1) h(\rho_2, -\rho'_1 + a) d^2 \rho'_1 \\
&= \int E_p(\rho') h \left(\rho_1, \rho' + \frac{a}{2} \right) h \left(\rho_2, -\rho' + \frac{a}{2} \right) d^2 \rho'. \tag{2}
\end{aligned}$$

The propagators $h(x, y)$ can be split in the following form:

$$h(x, y) = h_0(x, y) \exp[\psi(x, y)] \tag{3}$$

$$, \text{ where } h_0(x, y) \propto \exp \left[\frac{ik(x - y)^2}{2z} \right]. \tag{4}$$

The term h_0 is the usual vacuum propagator, while ψ describes the contribution from refractive index turbulence is written

$$\begin{aligned}
\psi(\rho, \rho') &\propto \int_0^L dz' \int d^2 \rho'' \frac{n_1(\rho'', z')}{z'(L - z')} \\
&\quad \times \exp \left\{ \frac{ik}{2} \left[\frac{(\rho - \rho'')^2}{L - z'} + \frac{(\rho'' - \rho')^2}{z'} - \frac{(\rho - \rho')^2}{L} \right] \right\}. \tag{5}
\end{aligned}$$

Close inspection reveals that

$$h_0(\rho_1, \rho' + a/2) = h_0(\rho_1 - a/2, \rho') \tag{6}$$

$$h_0(\rho_2, -\rho' + a/2) = h_0(-\rho_2 + a/2, \rho') \tag{7}$$