IN4393 – Computer Vision



# Image matching

• What feature can we use to establish correspondences between images?



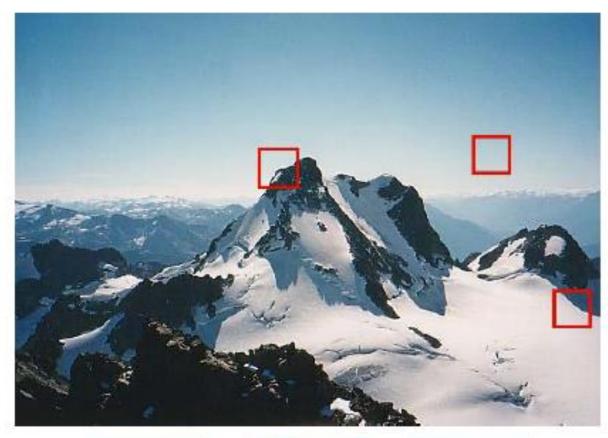


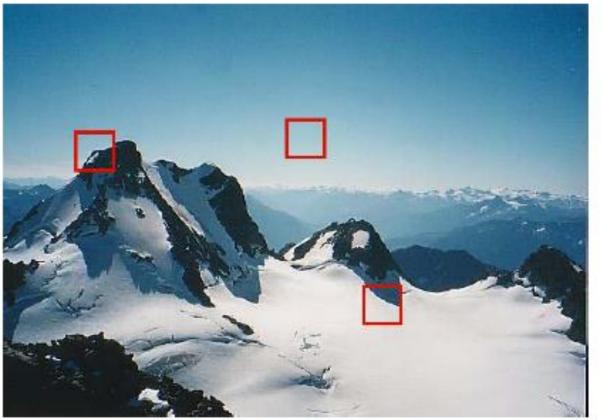




- The goal is to find locations that are stable under *image transformations*
- Feature points are frequently used in, among others:
  - Stereo matching
  - Image stitching
  - Video stabilization
  - Instance or object recognition

• Some feature points can be matched more accurately than others:









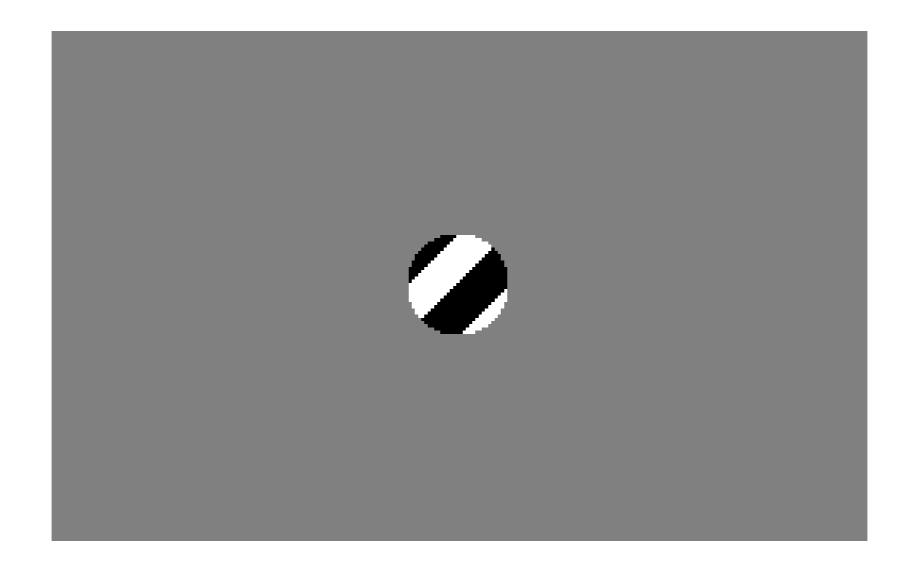




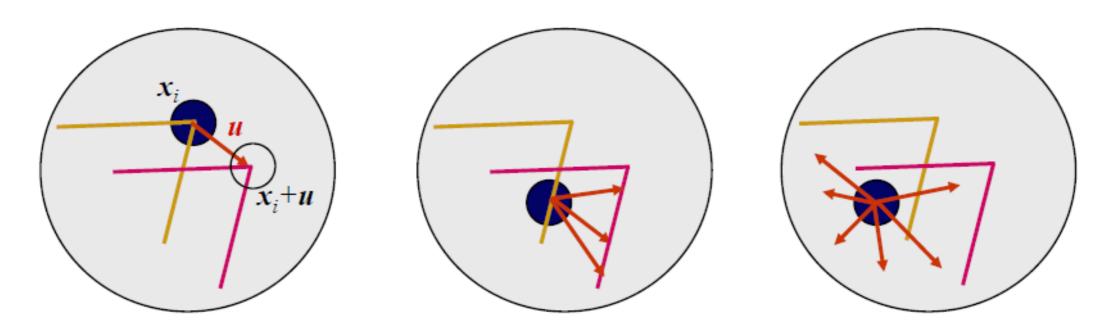




- Patches with large contrast changes are easier to localize
- However, straight lines with a single orientation suffer from aperture problem:



The most reliable points for matching are "corner"-like points:

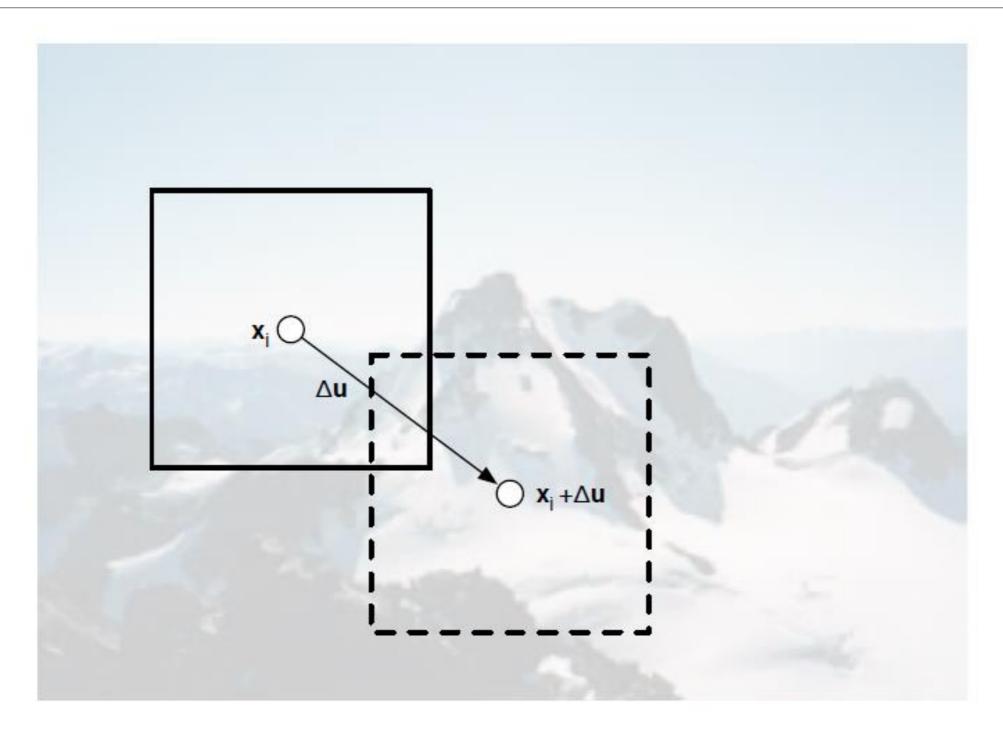


• We can formalize these intuitions by looking at the *autocorrelation function*:

$$E_{AC}(\Delta \mathbf{u}) = \sum_{i} w(\mathbf{x}_{i})[I_{0}(\mathbf{x}_{i} + \Delta \mathbf{u}) - I_{0}(\mathbf{x}_{i})]^{2}$$

Local weighting function: the summation over the window

## Autocorrelation function

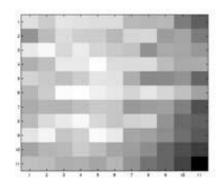


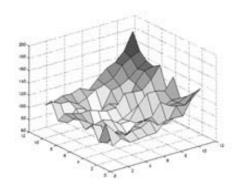
 $E_{AC}(\Delta \mathbf{u}) = weighted \ SSE(\square, \square)$ 

## Autocorrelation surfaces



• For homogeneous regions, there is no clear minimum

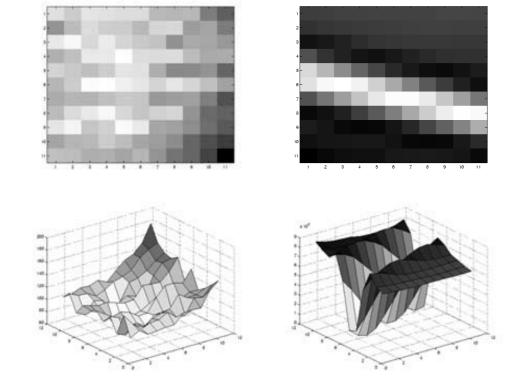




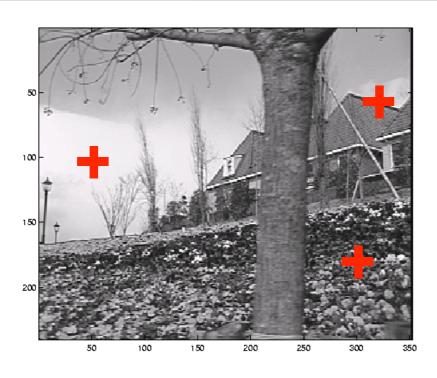
## Autocorrelation surfaces



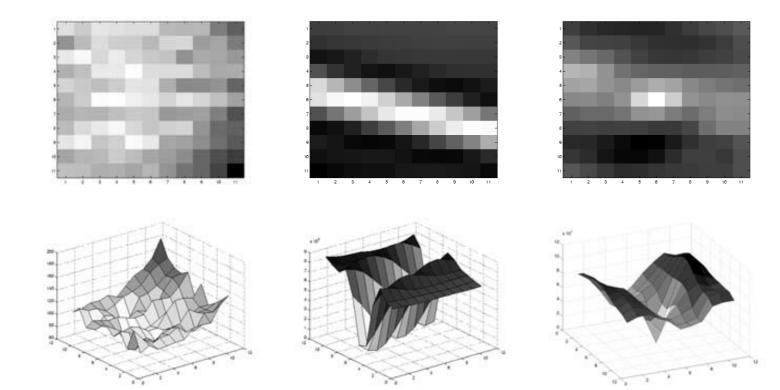
- For homogeneous regions, there is no clear minimum
- For lines, there is an ambiguity



#### Autocorrelation surfaces



- For homogeneous regions, there is no clear minimum
- For lines, there is an ambiguity
- The flower does have a clear minimum (= easy to localize!)



### Harris corner detector

We can approximate the autocorrelation via a Taylor expansion\* of the image:

$$\begin{split} E_{AC}(\Delta \mathbf{u}) &= \sum_{i} w(\mathbf{x}_{i}) [I_{0}(\mathbf{x}_{i} + \Delta \mathbf{u}) - I_{0}(\mathbf{x}_{i})]^{2} \\ &\approx \sum_{i} w(\mathbf{x}_{i}) [I_{0}(\mathbf{x}_{i}) + \nabla I_{0}(\mathbf{x}_{i}) \cdot \Delta \mathbf{u} - I_{0}(\mathbf{x}_{i})]^{2} \\ &= \sum_{i} w(\mathbf{x}_{i}) [\nabla I_{0}(\mathbf{x}_{i}) \cdot \Delta \mathbf{u}]^{2} \\ &= \Delta \mathbf{u}^{T} \mathbf{A} \Delta \mathbf{u} \end{split}$$

• Here, we define the autocorrelation matrix (aka second-moment matrix or structure tensor):

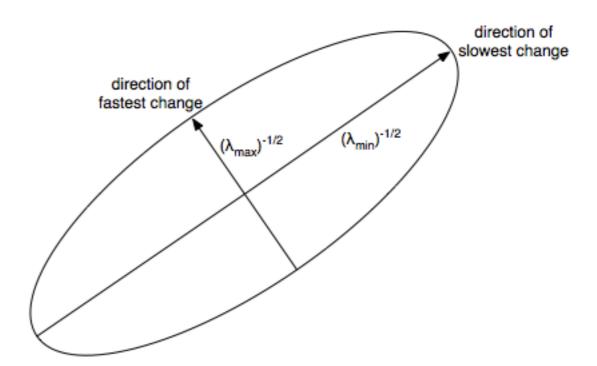
$$\mathbf{A} = w imes egin{bmatrix} I_x^2 & I_x I_y \ I_x I_y & I_y^2 \end{bmatrix}$$

\* A Taylor expansion of f(x) around a is given by:  $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$ 

#### Harris corner detector

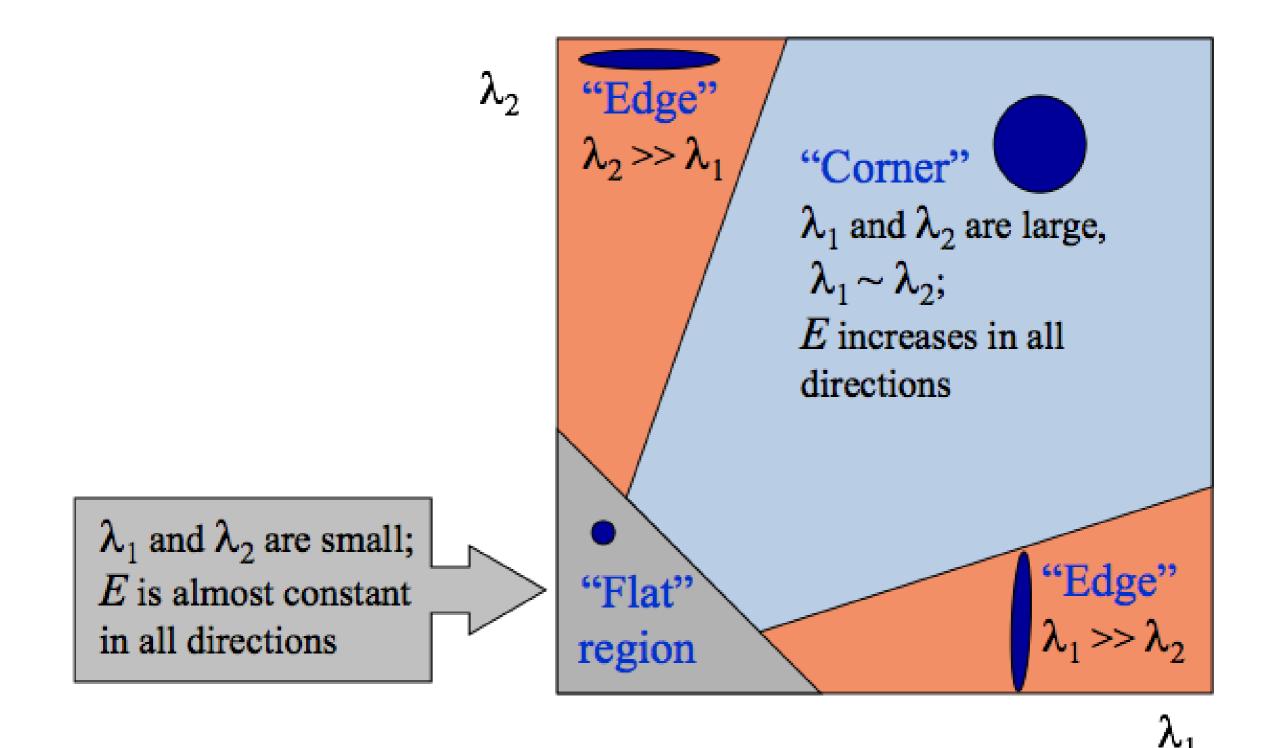
- ullet Corners correspond to large changes in  $E_{AC}$  in all directions
- Eigenanalysis of autocorrelation matrix reveals direction and speed of change:

$$\mathbf{A}\mathbf{v} = \lambda \mathbf{v}$$



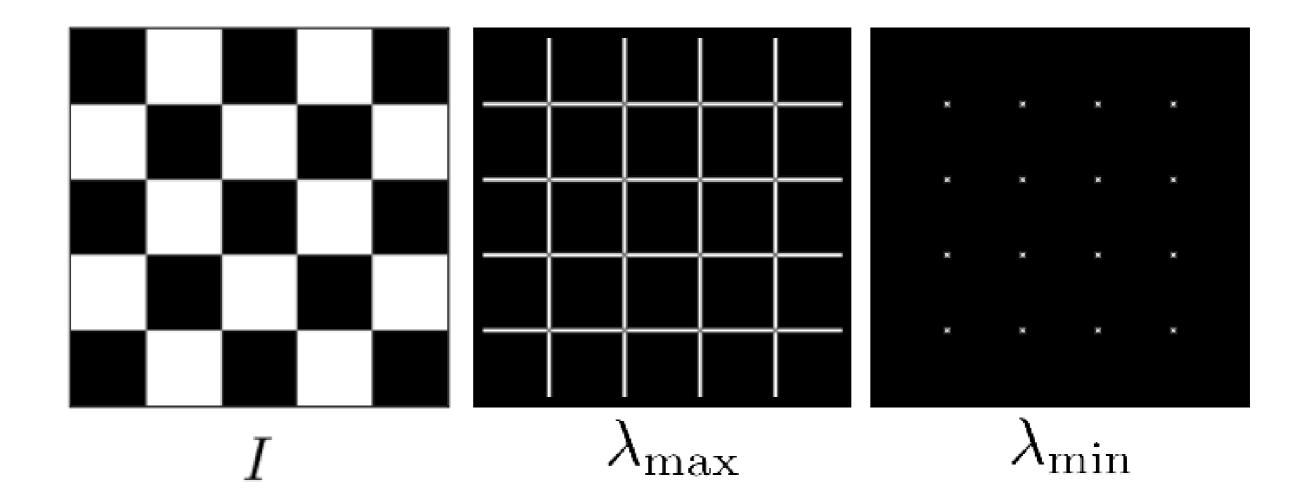
- Harris detector finds local maxima of:  $\det(\mathbf{A}) \alpha \operatorname{trace}(\mathbf{A})^2 = \lambda_0 \lambda_1 \alpha (\lambda_0 + \lambda_1)^2$
- Shi and Tomasi detector finds local maxima in the smallest eigenvalue  $\lambda_0$

## Interpreting the eigenvalues

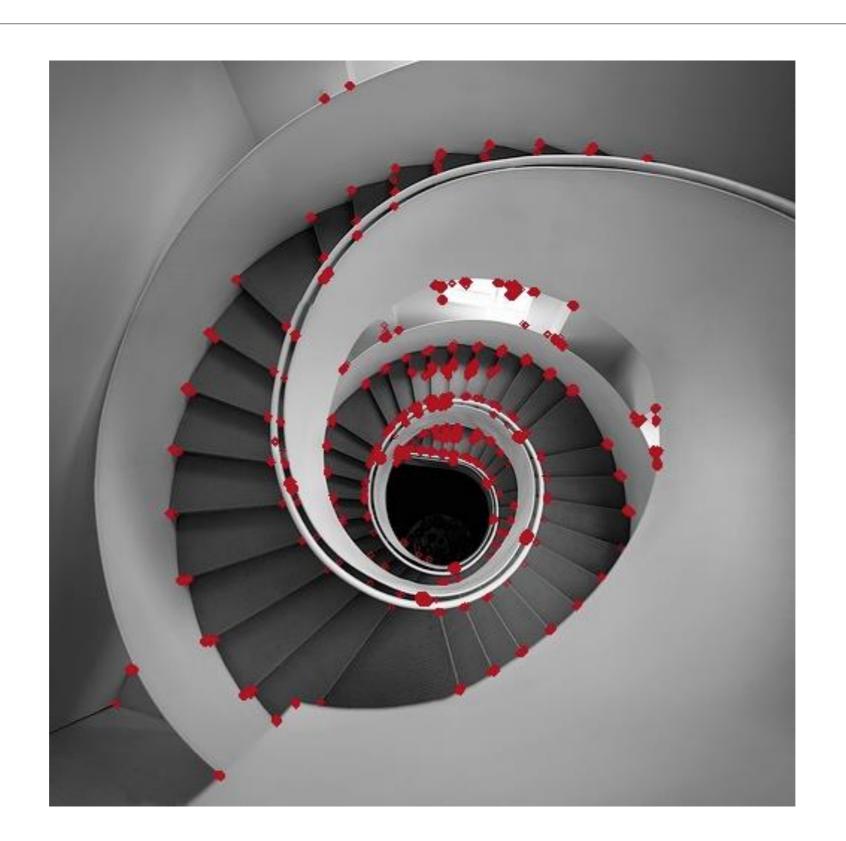


#### Harris corner detector

- Perform eigendecomposition of the second-moment matrix at each location
- Find local maxima in the field of the smallest eigenvalues:



# Example: Harris corner detector

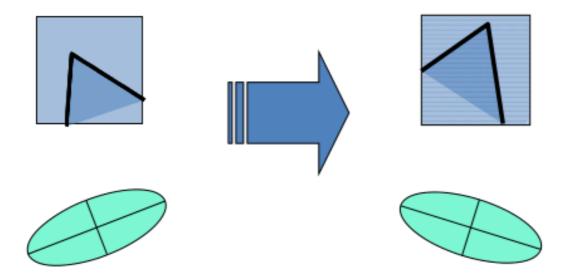


# Example: Harris corner detector

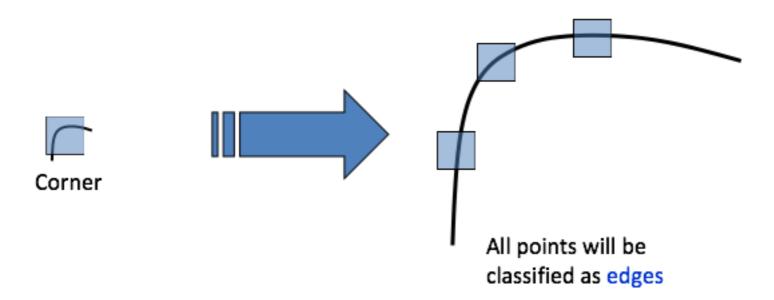


## Invariances of Harris detector

• The Harris detector is invariant to *rotations*:



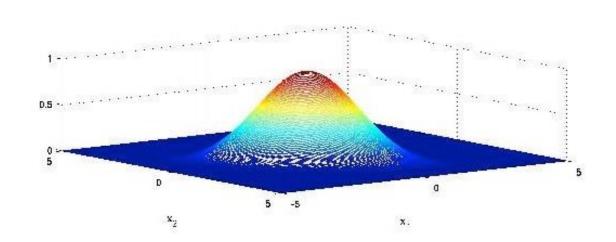
• The Harris detector is not invariant to scale changes:



• In contrast to Harris corners, SIFT features are stable in location and scale

- Overview of the SIFT feature point detector:
  - Perform band-pass filtering on a wide range of image scales
  - Non-maxima suppression to find candidate keypoints in location and scale
  - Remove candidate keypoints in low-contrast regions and keypoints on edges

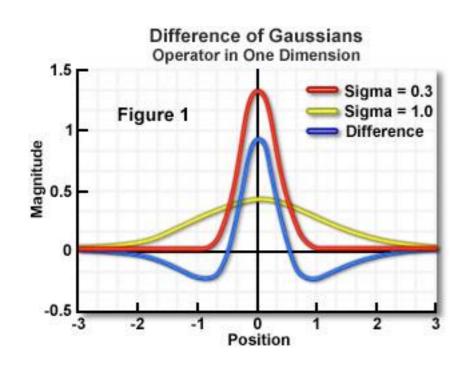
Gaussian scale space removes features at fine scales via repeated blurring:

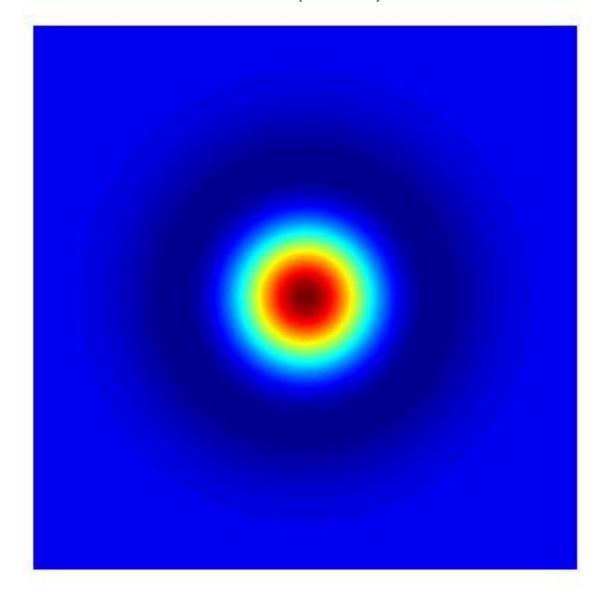




larger sigma / coarser scale  $\rightarrow$ 

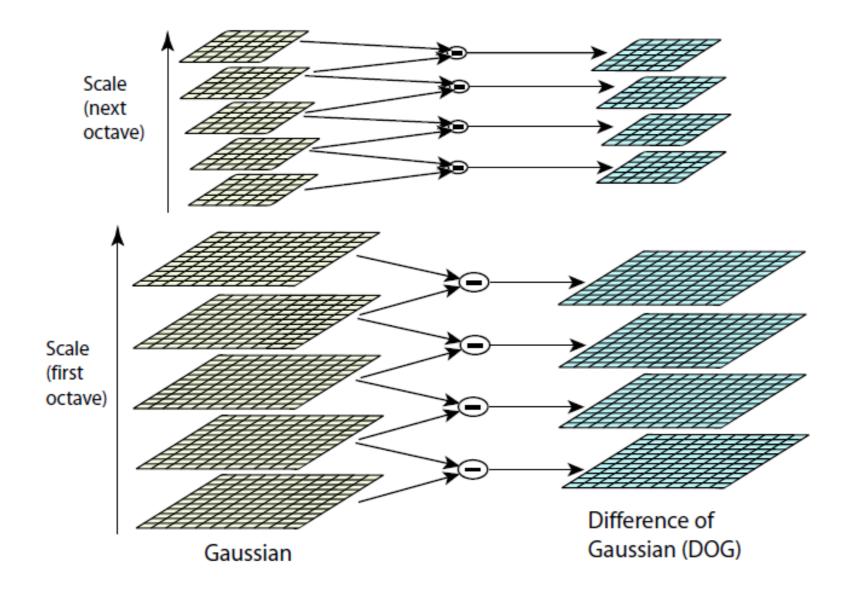
• SIFT performs filtering using difference-of-Gaussian (DoG) filters:





• This is a band-pass filter that only retains particular spatial frequencies

• Efficiently computing difference-of-Gaussian response images:



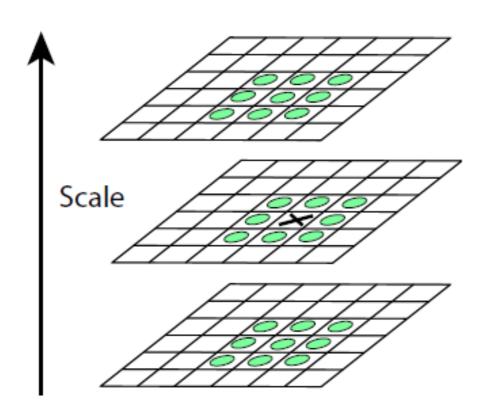


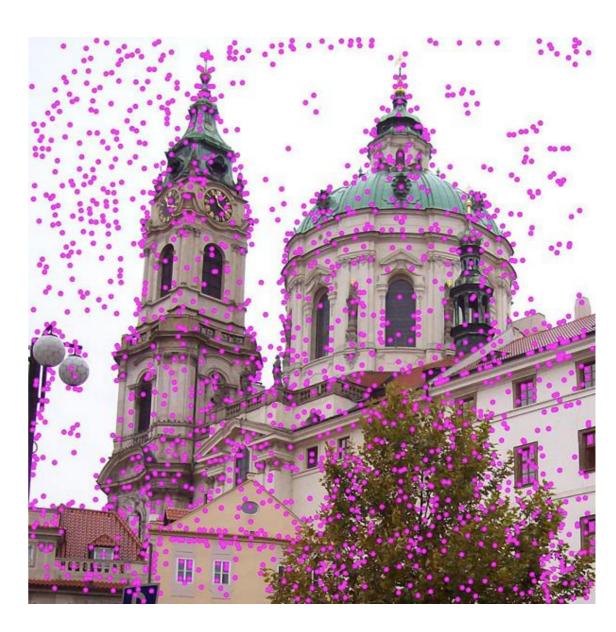




(o,s)=(-1,-1), sigma=0.800000	(o,s)=(-1,0), sigma=1.007937	(o,s)=(-1,1), sigma=1.269921	(o,s)=(-1,2), sigma=1.600000	(o,s)=(-1,3), sigma=2.015874	(o,s)=(0,-1), sigma=1.600000
(o,s)=(0,0), sigma=2.015874	(o,s)=(0,1), sigma=2.539842	(o,s)=(0,2), sigma=3.200000	(o,s)=(0,3), sigma=4.031747	(o,s)=(1,-1), sigma=3.200000	(o,s)=(1,0), sigma=4.031747
(o,s)=(1,1), sigma=5.079683	(o,s)=(1,2), sigma=6.400000	(o,s)=(1,3), sigma=8.063495	(o,s)=(2,-1), sigma=6.400000	(o,s)=(2,0), sigma=8.063495	(o,s)=(2,1), sigma=10.159367
(o,s)=(2,2), sigma=12.800000	(o,s)=(2,3), sigma=16.126989	(o,s)=(3,-1), sigma=12.800000	(o,s)=(3,0), sigma=16.126989	(o,s)=(3,1), sigma=20.318733	(o,s)=(3,2), sigma=25.600000
(o,s)=(3,3), sigma=32.253979	(o,s)=(4,-1), sigma=25.600000	(o,s)=(4,0), sigma=32.253979	(o,s)=(4,1), sigma=40.637467	(o,s)=(4,2), sigma=51.200000	(o,s)=(4,3), sigma=64.507958

- Keypoints are found as local maxima and minima of the DoG responses
- Search for maxima in eight-pixel spatial neighborhood across three scales:





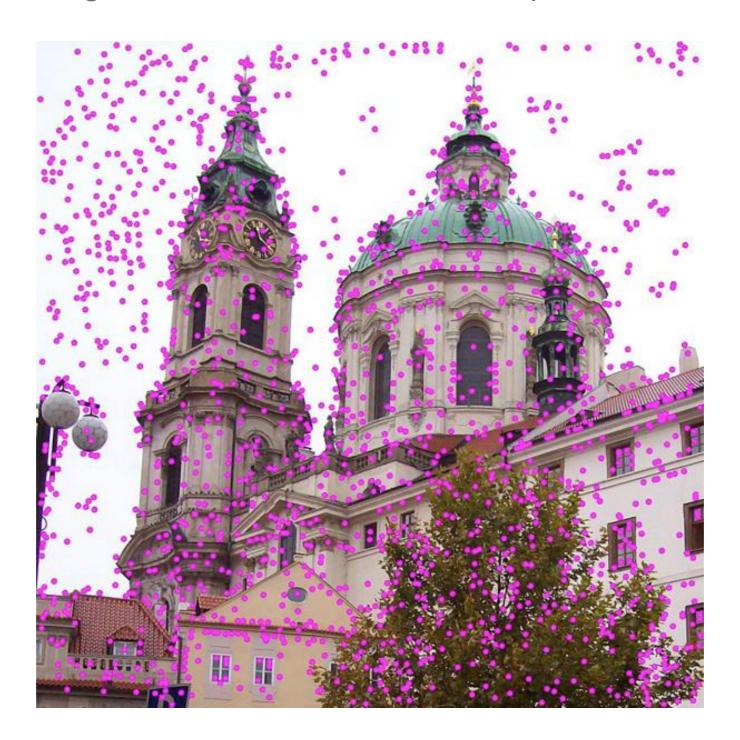
- Candidate keypoints with low contrast or that lie on edges are removed
- This is done based on the *local Hessian* of the response  $D(x,y,\sigma)$ :

$$\mathbf{H} = egin{bmatrix} D_{xx} & D_{xy} \ D_{xy} & D_{yy} \end{bmatrix}$$

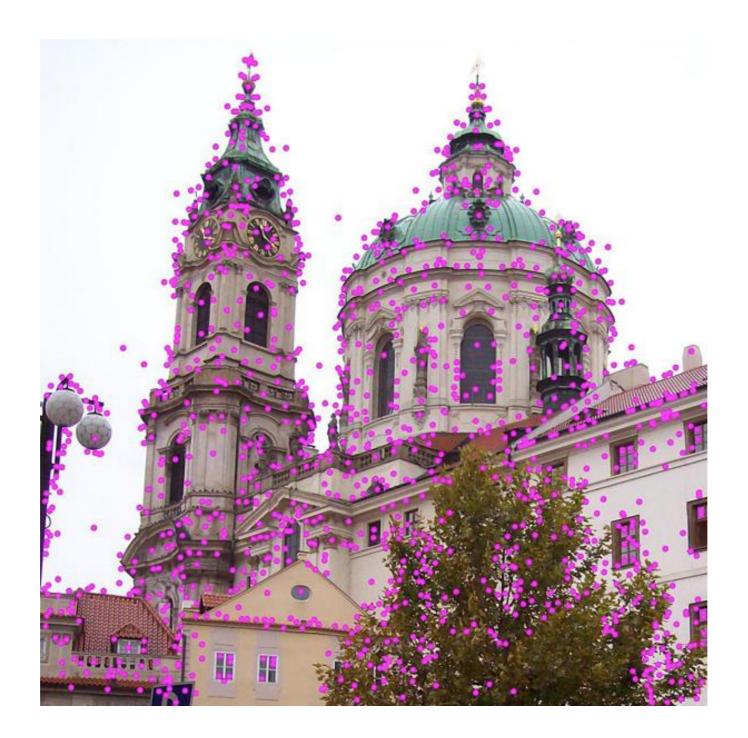
- Eigenvalues of the local Hessian are proportional to the principal curvature
- SIFT looks at the ratio of the two eigenvalues  $r=rac{\lambda_1}{\lambda_2}$  ; it rejects whenever:

$$\frac{(r+1)^2}{r} = \frac{\operatorname{trace}(\mathbf{H})^2}{\det(\mathbf{H})} > 10$$

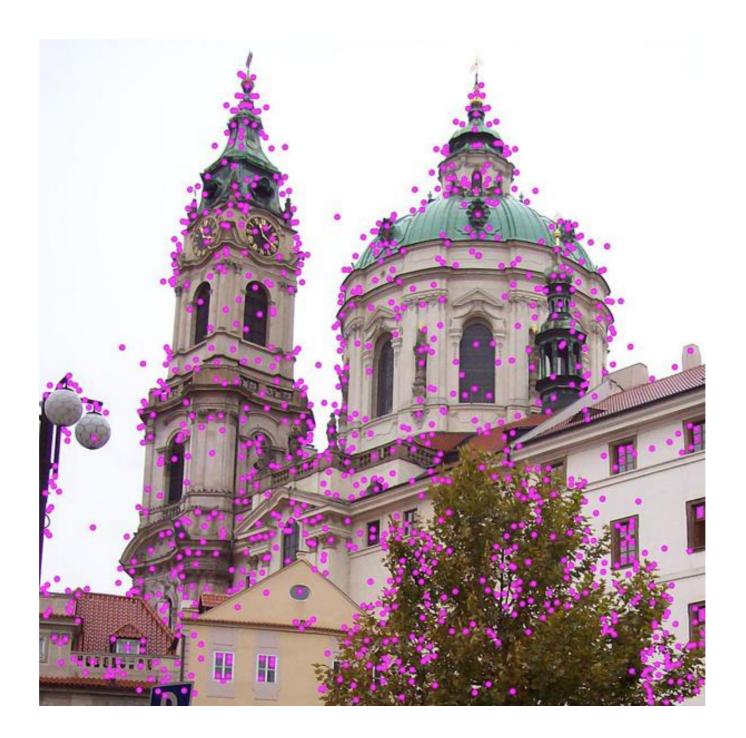
• Example of removing unstable candidate feature points:



• Feature points in low-contrast regions are removed:



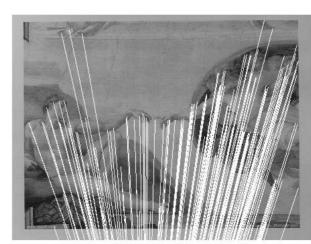
• Feature points that lie on edges are removed:



#### What have we learned in this lecture?

- Feature points are stable in location, rotation, and preferably also scale
- Harris performs spectral analysis of second-moment matrix / structure tensor
- By contrast, SIFT inspects the local Hessian of difference images

Next: Feature description and matching





Reading material: Section 3 and 4.1.1