ET3213 Microwave Engineering
Noise in Microwave Systems

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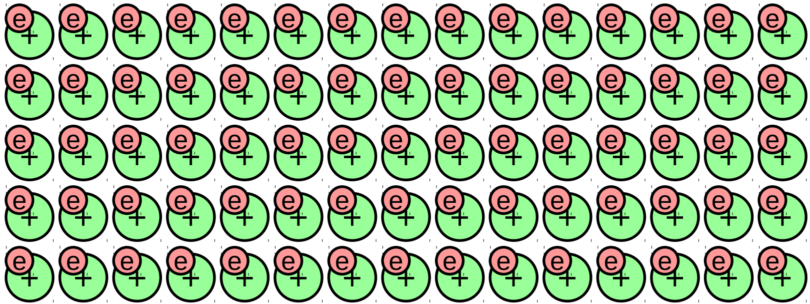
Introduction

- Noise is present in all communications systems
- Due to attenuation the receiver signal is significantly weaker than the transmitted signal
  - Effects of noise will be significant
- There are two main types of noise that affect communication systems
  1. Thermal (aka Johnson-Nyquist) noise (Gaussian) due to the thermal excitation of the conductor lattice and electrons
  2. Shot noise (Poisson) due to discrete electron arrivals

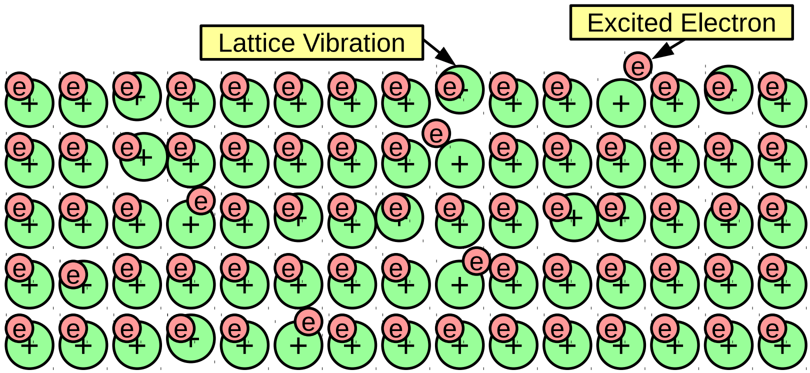
Thermal Noise

- At absolute zero all atoms and electrons are stationary
  - Electrons can move if excited by a potential difference
- Above absolute zero
  - Lattice atoms begin to vibrate
  - Electrons have higher mobility
- These phenomena interfere with the propagating signal

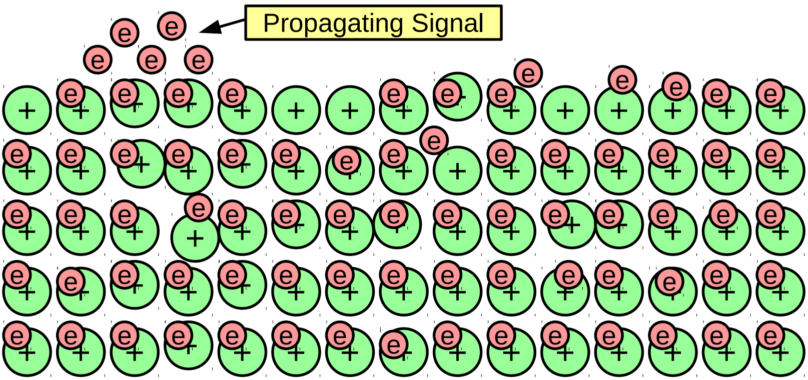
Metal Lattice at Absolute Zero



Thermally Excited Lattice



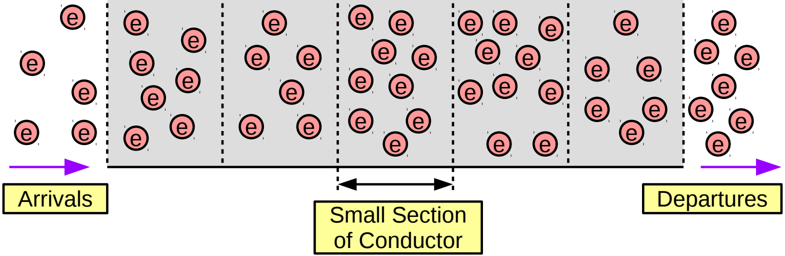
Thermal Noise



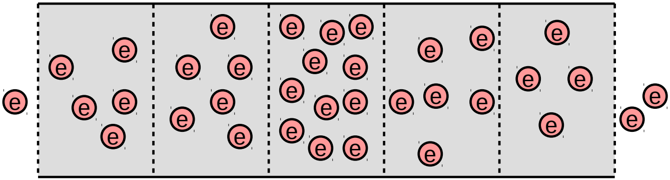
Shot Noise

- Electrons are discrete particles
- When the current is small and the frequency is high the discrete nature of a current becomes prominant
  - Electron arrival and departure will follow a Poisson distribution
- The number of electrons in a small section of the conductor will vary

Shot Noise



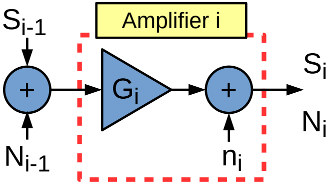
Modulation Shot Noise



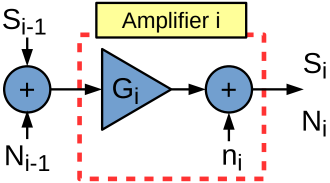
- The modulated signal will degrade due to shot noise

Friis Noise Formula

- Quantifies performance of an amplifier based upon the signal to noise ratio
- i.e., $\mathrm{SNR}_i = S_i/N_i$



Friis Noise Formula (Contd..)



The output signal and noise of a single stage are given by

$$
\begin{aligned}
S_i &= G_i S_{i-1} \\
N_i &= G_i N_{i-1} + n_i
\end{aligned}
$$

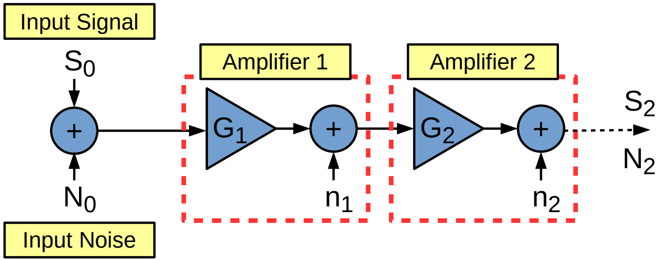
Always $\text{SNR}_i < \text{SNR}_{i-1}$ due to the addition of the internal noise n_i

Friis Noise Formula (Contd..)

The noise factor (F) of the amplifier stage

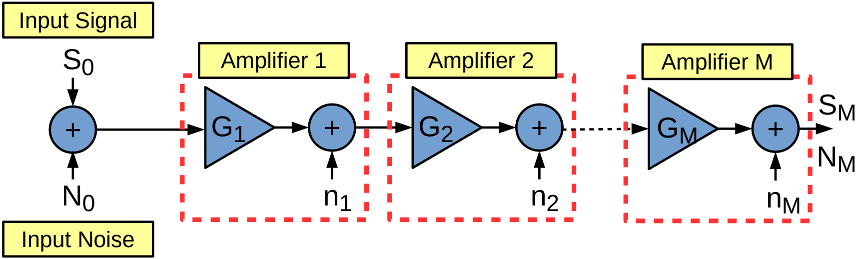
$$F_i = \frac{\mathsf{SNR}_{i-1}}{\mathsf{SNR}_i} = \frac{S_{i-1}/N_{i-1}}{S_i/N_i} = 1 + \frac{n_i}{GN_i}$$

Friis Noise Formula (Contd..)



Exercise: Determine the noise factor of a two stage cascade

Friis Noise Formula (Contd..)



The noise factor of the entire cascade is given by,

$$F_T = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \cdots + \frac{(F_M - 1)}{\prod_{i=1}^{M-1} G_i}$$

Friis Noise Formula (Contd..)

- Alternative noise temperature formula

$$F_i = 1 + \frac{T_i}{T_0}$$

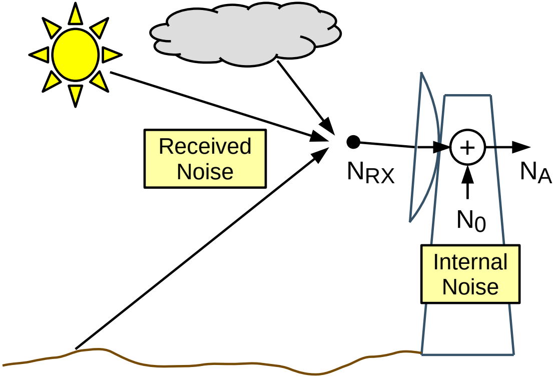
where T_0 is the ambient temperature and T_i is the *eqivalent* temperature of the stage

- For the entire cascade

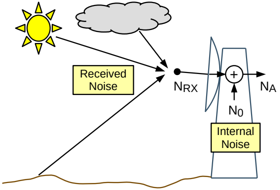
$$T_E = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots + \frac{T_M}{\prod_{i=1}^{M-1} G_i}$$

where T_E is the effective temperature of the cascade

Antenna Noise



Antenna Noise (Contd..)



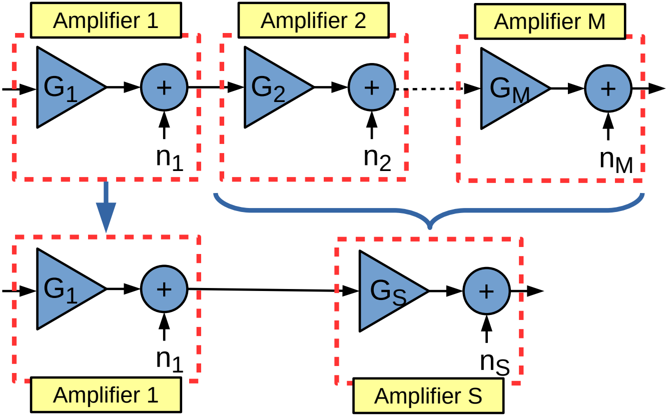
- Antenna noise is given by Livingston's formula

$$T_A = \sum_{i=1}^{N} \alpha_i \tau_i$$

where τ_i is the individual temperature source

- The overall temperature is given by $T_S = T_A + T_E$

Receiver Noise Analysis



Receiver Noise Analysis (Contd..)

- Keep the first stage independant and combine all other subsequent stages of the cascade into a single stage
- The resulting noise factor becomes

$$F_T = F_1 + \frac{(F_S - 1)}{G_1}$$

- By making G_1 large, the noise factor of all subsequent stages can be minimized
  - However, the noise factor of the first stage will always dominate

Receiver Noise Analysis (Contd..)

- How can the noise factor of the first stage be reduced?
  1. By reducing the input noise
  2. By minimizing internally generated noise (analog circuit design)
     - The end result is known as a Low Noise Amplifier (LNA)
- Let the input be $x(t)$ and signal $s(t)$

$$x(t) = [A_0 + p(t)]\, s(t) + n(t)$$

- Have to minimize
  - Shot noise $p(t)$
  - Thermal noise $n(t)$

Receiver Noise Analysis (Contd..)

- To minimize shot noise: $A_0 \gg p(t)$
  - Can be achieved by capturing more of the radiated energy i.e., increasing the gain of the antenna
- Thermal noise can be minimized by
  - Minimizing the capture of external noise
  - Reducing the antenna resistance
  - Cooling of the antenna
  - Having the LNA as close to the antenna as possible

Low Noise Circuit Design

- Consideration of the thermal noise of a resistor (R) and its contribution to the circuit

$$
\begin{aligned}
v_N &\propto \sqrt{R} \text{ (if as a voltage)} \\
i_N &\propto \frac{1}{\sqrt{R}} \text{ (if as a current)}
\end{aligned}
$$

  - Also consider the bandwidth for both v_N and i_N are proportional to $\sqrt{\text{BW}}$

- Consideration of transistor Noise Figure

$$\text{NF} = 10 \log_{10}(F)$$

Exercise

A microwave antenna is found to have a temperature of 76 K. It is connected to an amplifier cascade with the noise figures and gains of the table given below with Stage 1 being the LNA and the ambient temperature being 300 K.

Stage	1	2	3
Noise Figure (dB)	2.1	1.1	1.1
Gain (dB)	33.5	12	12

Determine the system noise factor and noise temperature.

Conclusion

- Noise is a significant issue in telecommunication systems
- It is caused by inherent properties of matter and conduction
- Can be analyzed using the Friis noise formula
- Can be reduced by cooling and careful system design