General Sir John Kotelawala Defence University
ET3122 Antennas and Propagation

High Frequency Propagation

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Outline

Introduction

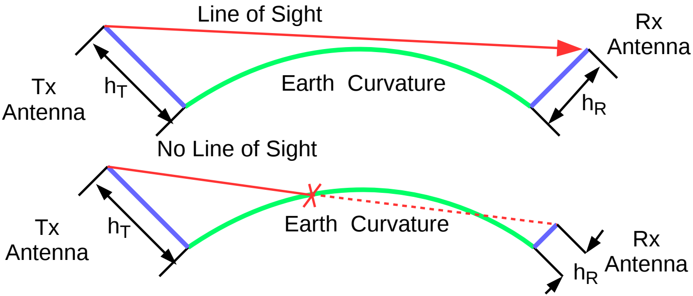
Microwave Propagation

- Microwaves are space waves
  - Ideally travel along a straight line in a vacuum
  - Require line of sight due to the earth curvature
- Tropospheric conditions alter propagation
  - Cause bending of the waves due to the refractive index gradient
  - Also absorption due to the atmosphere and rainfall
  - Anomalous propagation
- Effects due to obstacles and the ground

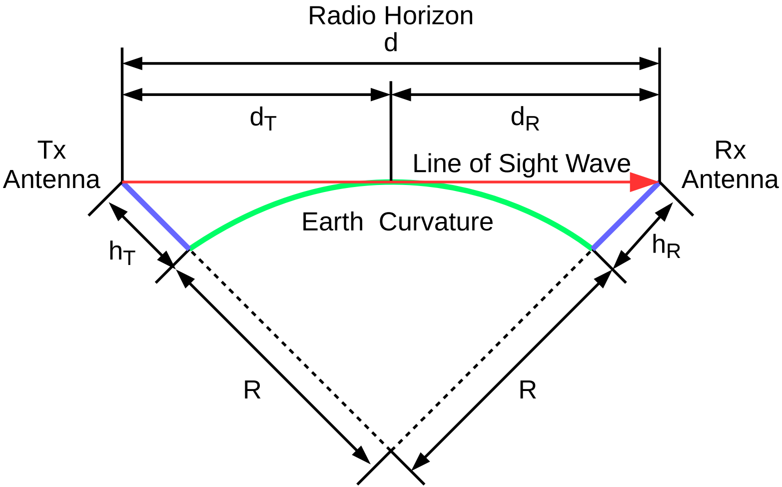
Radio Horizon

Line of Sight

- Microwave propagation requires line of sight
  - How high must the link antennas be?



Radio Horizon



Radio Horizon (Contd..)

- The formula that gives the relationship between the heights of the antennas and line of sight
  - The result is the *critical distance* for the given antenna heights where line of sight is about to be lost
- Let
  - R is the radius of the earth
  - h_T and h_R are the heights of the transmitting and receiving antennas respectively

$$d_T = \sqrt{(R+h_T)^2 - R^2} = \sqrt{2Rh_T + h_T^2} = \sqrt{h_T(2R+h_T)} \approx \sqrt{2Rh_T}$$

Radio Horizon (Contd..)

- Similarly

$$\begin{aligned} d_R &\approx \sqrt{2Rh_r} \\ d &= d_T + d_R \approx \sqrt{2R}(\sqrt{h_T} + \sqrt{h_R}) \end{aligned}$$

- Taking $R = 6378$ km

$$\begin{aligned} d &\approx \sqrt{2 \times 6378 \times 10^3}(\sqrt{h_T \times 10^{-3}} + \sqrt{h_R \times 10^{-3}}) \\ &\approx 3.6\left(\sqrt{h_T} + \sqrt{h_R}\right) \text{ (d in km and h_T,h_R in m)} \end{aligned}$$

- As an inequality of distance between antennas

$$d < 3.6\left(\sqrt{h_T} + \sqrt{h_R}\right) \tag{1}$$

Exercises

1. Find the radio horizon of a transmitting antenna 2 m above mean sea level
2. Assuming a curvature at mean sea level, what is the maximum feasible distance between two antennas elevated 20 m and 14 m above mean sea level?
3. What is the minimum antenna height required for a 20 km link?

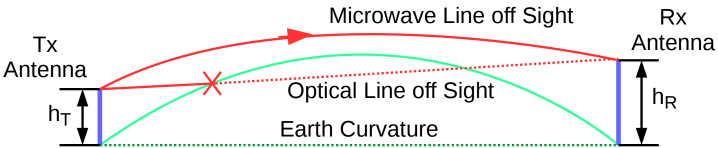
Answers

1. Take $h_R = 0$, $d = 3.6\left(\sqrt{2} + 0\right) = 5.09$ km
2. $d < 3.6\left(\sqrt{20} + \sqrt{14}\right) = 29.57$ km
3. Take $h_T = h_R = h$, $h = (d/(2 \times 3.6))^2 = (20/7.2)^2 = 7.7$ m

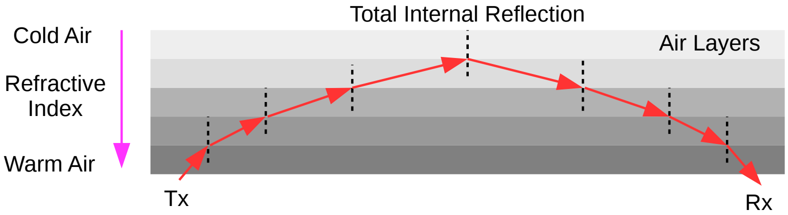
Effect of Refraction

Wave Bending

- Caused by the refractive index gradient
- Extends the range of the link beyond the optical LOS
  - Requires the radio horizon to be re-defined

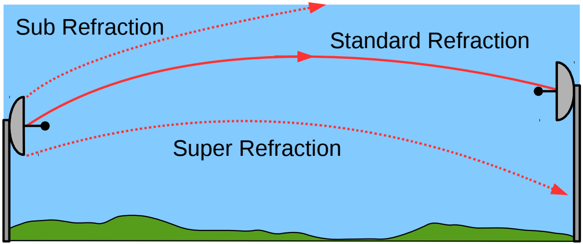


Wave Bending (Contd..)



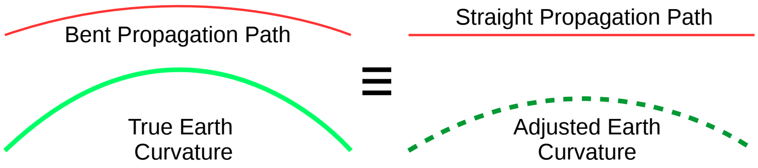
- The bending radius is an important parameter
  - Why?

Wave Bending (Contd..)



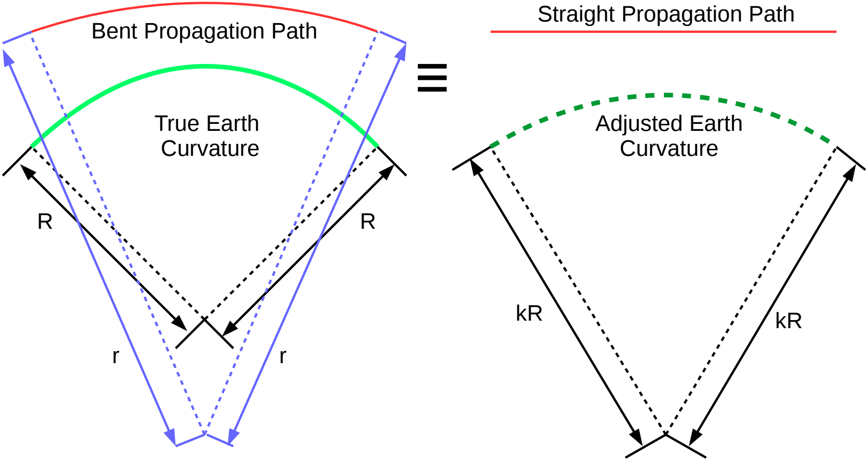
- Both the sub and super refracted waves miss the receiver
  - All have different refraction gradients and radii
  - How can the radius of bending be calculated?

Earth Radius Adjustment



- The curvature of the earth is reduced
  - Effective radius of the earth is increased

Earth Radius Adjustment (Contd..)



Earth Radius Adjustment (Contd..)

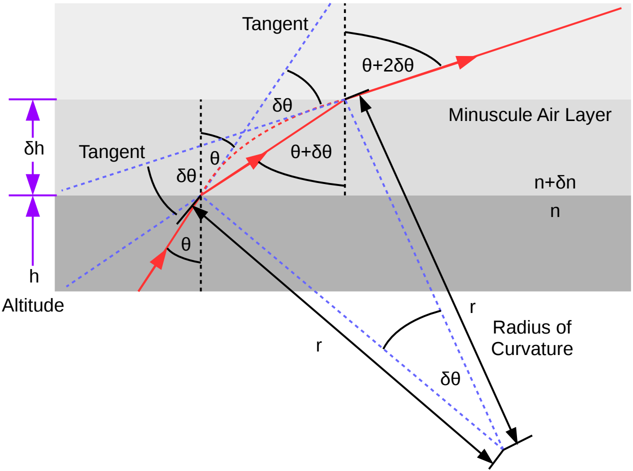
- Effective radius calculation
- Take the radius of the LOS wave as infinity

$$\frac{1}{R} - \frac{1}{r} = \frac{1}{kR} - \frac{1}{\infty}$$

- This results in

$$k = \frac{1}{1 - \left(\frac{R}{r}\right)} \tag{2}$$

Bending Radius



Bending Radius (Contd..)

- For the δh layer the ray length is given by

$$\begin{aligned} \delta s &= \delta h \text{cosec}(\theta + \delta\theta) \approx r\delta\theta \text{ (length of arc)} \\ \delta h &= r\cos(\theta + \delta\theta)\delta\theta \\ \lim_{\delta\theta \to 0} \frac{\delta h}{\delta\theta} &\Rightarrow \frac{dh}{d\theta} = r\cos(\theta) \end{aligned} \tag{3}$$

- From Snell's law

$$\frac{n + \delta n}{n} = \frac{\sin(\theta)}{\sin(\theta + \delta\theta)} = \frac{\sin(\theta)}{\sin(\theta)\cos(\delta\theta) + \cos(\theta)\sin(\delta\theta)}$$

Bending Radius (Contd..)

- From the small angle approximations

$$
\begin{aligned}
\frac{n+\delta n}{n} &= \frac{\sin(\theta)}{\sin(\theta)+\cos(\theta)\delta\theta} \\
\Rightarrow (n+\delta n)(\sin(\theta)+\cos(\theta)\delta\theta) &= n\sin(\theta) \\
\Rightarrow \frac{\delta n}{\delta\theta} &= -(n+\delta n)\cot(\theta)\delta\theta \\
\lim_{\delta\theta\to 0}\frac{\delta n}{\delta\theta} \Rightarrow \frac{dn}{d\theta} &= -n\cot(\theta) \qquad (4)
\end{aligned}
$$

Bending Radius (Contd..)

- By applying the chain rule to (3) and (4)

$$\frac{dn}{dh} = -\frac{n}{r}\sin(\theta) \approx -\frac{n}{r} \text{ (Since } \theta \approx \tfrac{\pi}{2}) \Rightarrow r = \frac{-n}{\left(\frac{dn}{dh}\right)}$$

- Typically, $n = 1.0003$ and $\frac{dn}{dh}$ - $3.9 \times 10^{-8} m^{-1}$

$$r = 2.565 \times 10^{7} \text{ m}$$

- Furthermore, from (2) for standard atmospheric conditions

$$k_S = \frac{1}{1 + \left(\frac{R}{n}\right)\left(\frac{dn}{dh}\right)} = 1.33$$

Modified Radio Horizon

- By replacing R with kR,

$$d < 3.6\sqrt{k}\left(\sqrt{h_T} + \sqrt{h_R}\right) \tag{5}$$

- This gives the new radio horizon adjusted for tropospheric bending
  - For super refraction $k < k_S$
  - For sub refraction $k_S < k$

Exercises

Repeat the following questions assuming *standard atmospheric refraction*,

1. Find the radio horizon of a transmitting antenna 2 m above mean sea level
2. Assuming a curvature at mean sea level, what is the maximum feasible distance between two antennas elevated 20 m and 14 m above mean sea level?
3. What is the minimum antenna height required for a 20 km link?

Answers

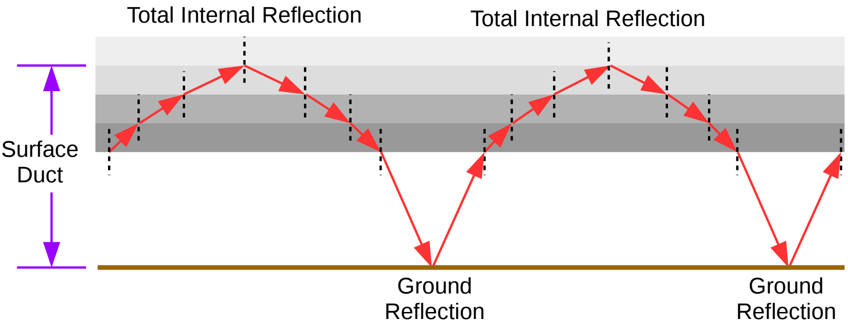
Take $k = k_S = 1.33$

1. Take $h_R = 0$, $d = 3.6\sqrt{1.33}\left(\sqrt{2} + 0\right) = 5.87$ km
2. $d < 3.6\sqrt{1.33}\left(\sqrt{20} + \sqrt{14}\right) = 34.1$ km
3. Take $h_T = h_R = h$,
   $h = (d/(2 \times 3.6 \times \sqrt{k}))^2 = (20/(7.2\sqrt{1.33}))^2 = 6.68$ m

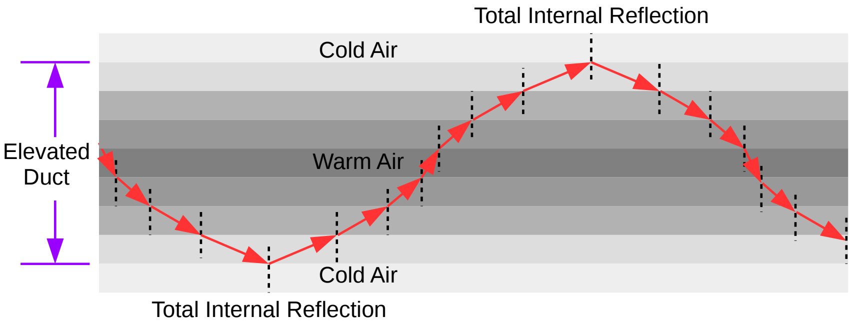
Ducting

- Ducting is an anomalous situation where multiple total internal and/or ground reflections make a wave propagate well beyond the expected range
  - A rare event but can seriously interfere with frequency planning
- Can be of two types
  - Surface ducts occur between a boundary and the ground
  - Elevated ducts occur between two boundaries

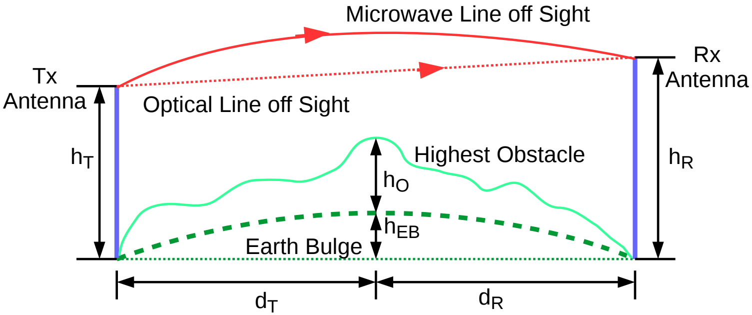
Surface Duct



Elevated Duct



Earth Bulge


Microwave Line off Sight
Rx
Antenna
Tx
Antenna
Optical Line off Sight
h_T
Highest Obstacle
h_R
h_O
h_{EB}
Earth Bulge
d_T
d_R

Earth Bulge (Contd..)

- The additional height of an obstacle due to the earth bulge
- If the obstacle is a distance d_T from the transmitter,

$$d_T = \sqrt{(kR)^2 - (kR - h_{EB})^2} = \sqrt{(2kR - h_{EB})h_{EB}} \approx \sqrt{2kRh_{EB}}$$

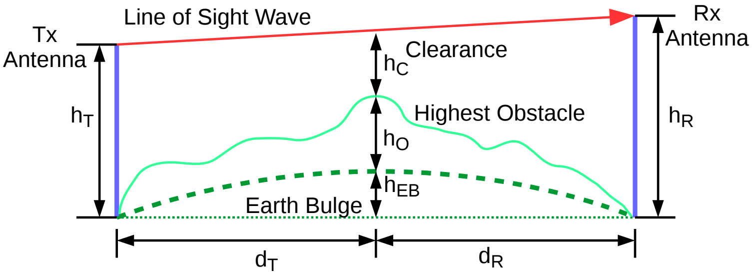
- Also for the receiver

$$d - d_T = d_R \approx \sqrt{2kRh_{EB}}$$

- Therefore, the earth bulge is given by

$$d_T d_R \approx 2kRh_{EB} \Rightarrow h_{EB} = \frac{d_T d_R}{2kR} = \frac{d_T(d - d_T)}{2kR}$$

Obstacle Clearance



- The earth bulge can be used to determine the link feasiblity
- At the critical point for the given k, $h_C = 0$

Exercise

The transmitting antenna of a microwave link is 10 m above MSL and the receiving antenna is 7.2 m above MSL. The two antennas are 6.5 km apart. Recently two building construction projects A and B have began on the flat ground between the two antennas. The distances from the transmitter to sites A and B are 1 km and 4 km respectively.

1. Calculate the earth bulge at each location assuming normal atmospheric conditions (i.e., $k = 1.33$).
2. Calculate the critical MSL height of each building to ensure that the link will not fail under normal atmospheric conditions.

Answers

For site A, $d_T = 1$ km

$$h_{EB} = \frac{1 \times 10^3 \times 5.5 \times 10^3}{2 \times 1.33 \times 6378 \times 10^3} = 0.32 \text{ m}$$

From the tangent ratio,

$$\frac{h_T - h_R}{d} = \frac{h_T - (h_O + h_{EB})}{d_T} \Rightarrow h_O = -\left(\frac{h_T - h_R}{d}\right) d_T + h_T - h_{EB}$$

$$h_O = -\frac{(10 - 7.2)}{6.5 \times 10^3}(1 \times 10^3) + 10 - 0.32 = 9.25 \text{ m}$$

Answers (Contd..)

For site B, $d_T = 4$ km

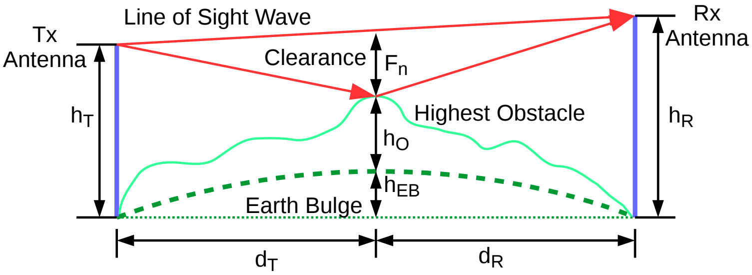
$$h_{EB} = \frac{4 \times 10^3 \times 2.5 \times 10^3}{2 \times 1.33 \times 6378 \times 10^3} = 0.59 \text{ m}$$

The critical height,

$$h_O = -\frac{(10 - 7.2)}{6.5 \times 10^3}(4 \times 10^3) + 10 - 0.59 = 7.69 \text{ m}$$

Fresnel Clearance

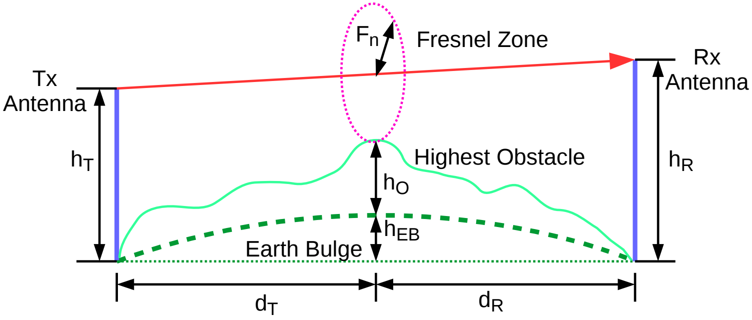
Fresnel Zones



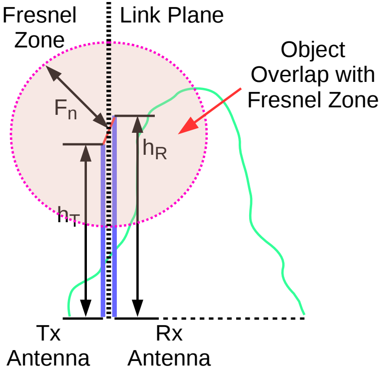
Fresnel Clearance (Contd..)

- So far it was only checked if the obstacle blocked the LOS
- Obstacles can also cause losses due to
  - Interference
  - Diffraction
- Therefore, an additional clearance is necessary
  - Known as the Fresnel clearance
- An obstacle can affect a link when within the *Fresnel zone* even if *not within the plane of the two antennas*
  - Multiple obstacles can be contribute to the interference

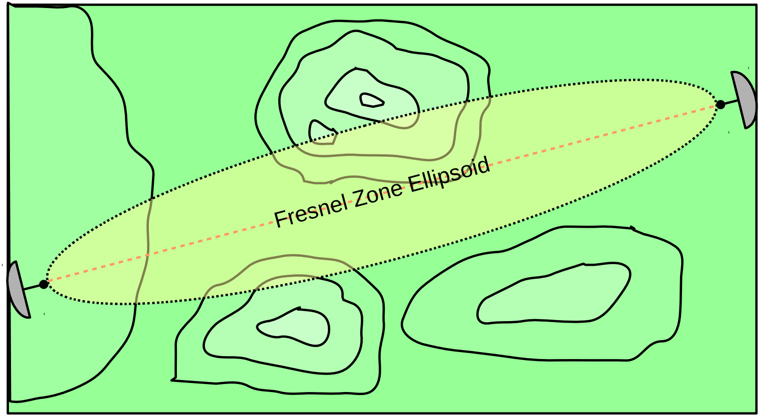
The Fresnel Zone



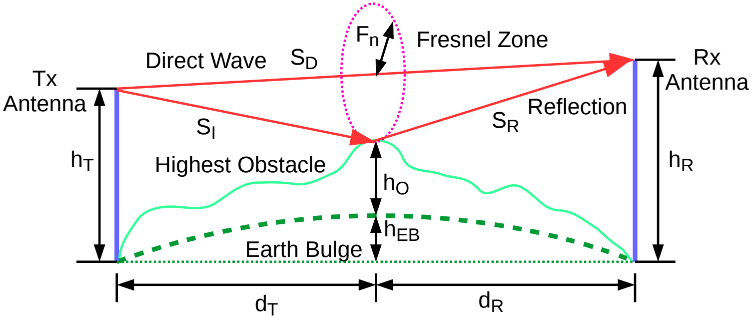
The Fresnel Zone (Off Plane Obstacle)



Fresnel Zone Ellipsoid Topographical Overlay



Fresnel Zone Interference



- Fresnel interference is caused by the direct wave (S_D) and the reflection (S_R) off the obstacle
  - It is assumed that there is no reflection loss (i.e., $S_I = S_R$)

Fresnel Zone Interference (Contd..)

Take the path difference as ΔS,

$$
\begin{aligned}
\Delta S &= S_I + S_R - S_D \\
&= \sqrt{d_T^2 + F_n^2} + \sqrt{d_R^2 + F_n^2} - (d_T + d_R) \\
&= d_T \left[1 + \left(\frac{F_n}{d_T}\right)^2\right]^{\frac{1}{2}} + d_R \left[1 + \left(\frac{F_n}{d_R}\right)^2\right]^{\frac{1}{2}} - (d_T + d_R) \\
&= d_T \left[1 + \frac{1}{2}\left(\frac{F_n}{d_T}\right)^2\right] + d_R \left[1 + \frac{1}{2}\left(\frac{F_n}{d_R}\right)^2\right] - (d_T + d_R) \\
&= \frac{F_n^2}{2}\left[\frac{1}{d_T} + \frac{1}{d_R}\right] = \frac{F_n^2 d}{2 d_T d_R}
\end{aligned}
$$

Fresnel Zone Interference (Contd..)

The resulting phase difference is given by,

$$\Delta\theta = \frac{2\pi\Delta S}{\lambda}$$

At a Fresnel Zone, for destructive interference $\Delta\theta = n\pi$ where n is odd such that

$$E = E_D + E_R e^{j\Delta\theta} \Rightarrow E_{\min} = E_D - E_R$$

Therefore,

$$\Delta S = \frac{n\lambda}{2} = \frac{F_n^2 d}{2 d_R d_T} \Rightarrow F_n = \sqrt{\frac{n\lambda d_T d_R}{d}} \text{ (n odd)}$$

Fresnel Zone Interference (Contd..)

- The heuristic Fresnel clearance is
  1. $0.4F_1$ for rural areas
  2. $0.2F_1$ for urban areas plus gain compensation
- Where F_1 is the first Fresnel zone
- This includes the clearance from the ground for low lying links

Exercise

For the previous exercise, calculate the critical MSL height of each building if the link frequency is 10 GHz and an additional clearence equal to 40% of the first Fresnel zone is needed.

Answers

- $\lambda = 3$ cm therefore,
- F_1 at A,

$$F_1 = \sqrt{\frac{n\lambda d_T d_R}{d}} = \sqrt{\frac{1 \times 0.03 \times 1 \times 10^3 \times 5.5 \times 10^3}{6.5 \times 10^3}} = 5 \text{ m}$$

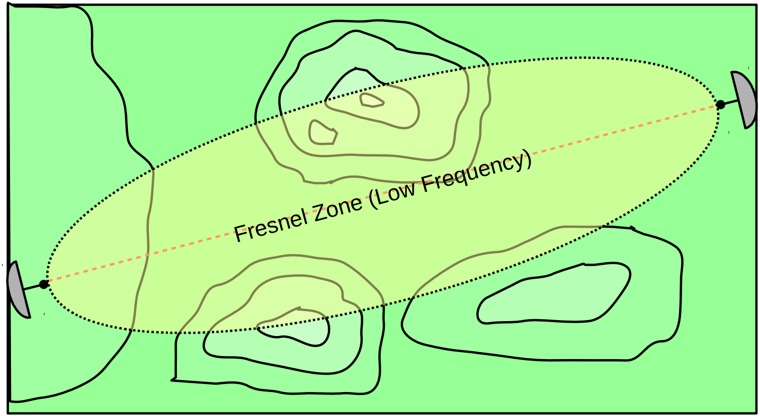
  Therefore, the height is reduced to $9.25 - 0.4 \times 5 = 7.25$ m

- F_1 at B,

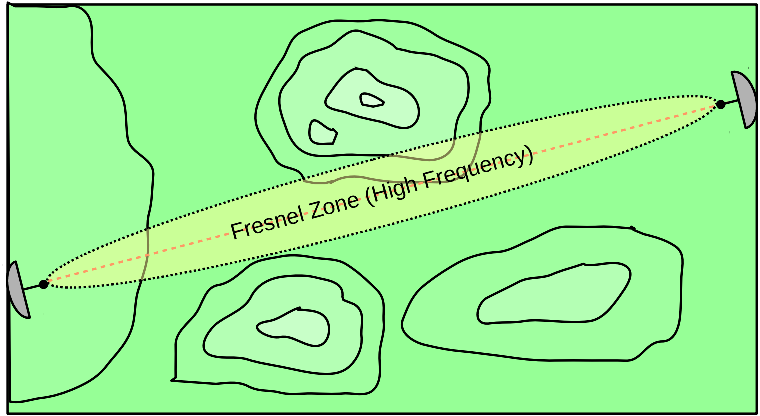
$$F_1 = \sqrt{\frac{1 \times 0.03 \times 4 \times 10^3 \times 1.5 \times 10^3}{6.5 \times 10^3}} = 6.8 \text{ m}$$

  Therefore, the new height is $7.7 - 0.4 \times 6.8 \approx 5$ m

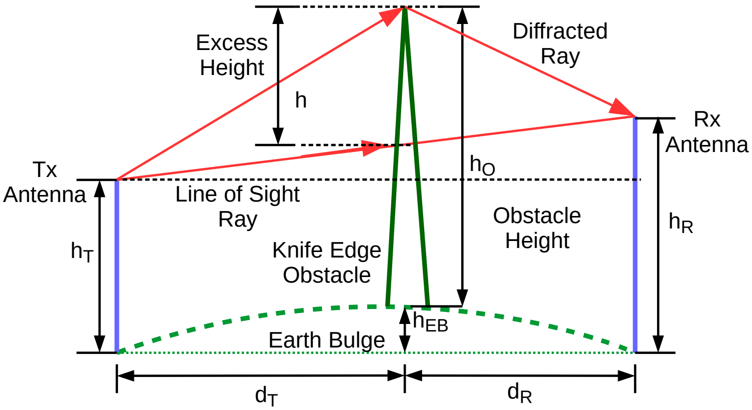
Fresnel Zone Ellipsoid (Low Frequency)



Fresnel Zone Ellipsoid (High Frequency)



Obstacle Diffraction



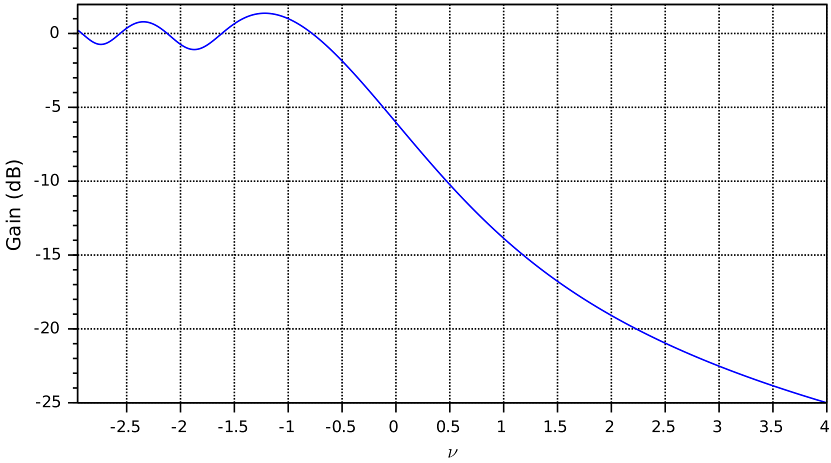
Obstacle Diffraction (Contd..)

- Dependant on the profile of the obstacle
  - Difficult to analytically model (using Heuygen's Principle)
  - Further complicated by the variation of h_{EB}
- Object is modeled as a knife-edge using the Fresnel-Kirchoff integral parameter ν given by

$$\nu = \frac{h}{\sqrt{2F_1}}$$

  - Where h is the *excess height*

Fresnel Gain of a Knife Edge Obstacle



Exercise

For the previous exercise, calculate the critical MSL height of each building for a frequency of 10 GHz and an additional clearence equal to 20% of the first Fresnel zone. What is the Fresnel gain in each case?

Answers

- $F_1 = 5$ at A. Therefore, the new height is $9.25 - 0.2 \times 5 = 8.25$ m
- $F_1 = 6.8$ at B. Therefore, the new height is $7.7 - 0.2 \times 6.8 \approx 6.3$ m
- For the loss, taking $h = -0.2F_1$

$$\nu = \frac{h}{\sqrt{2}F_1} = -\frac{0.2}{\sqrt{2}} \approx -0.15$$

which amounts to a loss of -5 dB

Foliage Losses

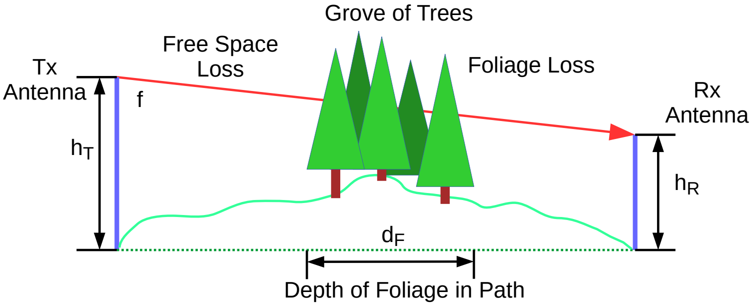
Foliage Loss

- The loss due to leaves (foliage) of vegetation
- Difficult to analytically model
- Empirical models are preferred where the loss in decibels is given by the generic equation

$$L = K_0 f^a (d_F)^b$$

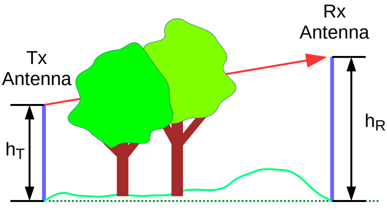
  where f is the frequency of the link, d_F is the depth of foliage and constants K_0, a and b are specific to the model
- The calulated loss added to the free space loss

Foliage Grove



- A typical grove of trees for which the loss is accounted for

Foliage State



a) Woodland In-Leaf b) Woodland Out-of-Leaf

- In addition some models could consider the foliage state
  - When leaves are shed during the dry season in the tropics
  - During the autumn and winter in temperate regions
- Typically the loss for out-of-leaf situations is less

Weissberger and ITR Models

- Valid for frequencies ranging from 230 MHz to 95 GHz
- The Weissberger model is given by

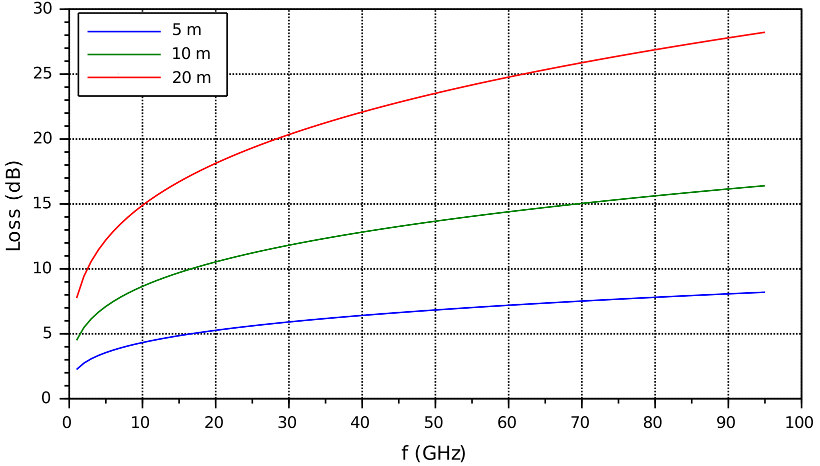
$$L = \begin{cases} 1.33 f^{0.284} d_F^{0.588} & 14 \text{ m} \le d_F \le 400 \text{ m} \\ 0.45 f^{0.284} d_F & 0 \text{ m} \le d_F < 14 \text{ m} \end{cases}$$

  where d_F is the depth of foliage in meters and f is in GHz
- The ITR model is given by

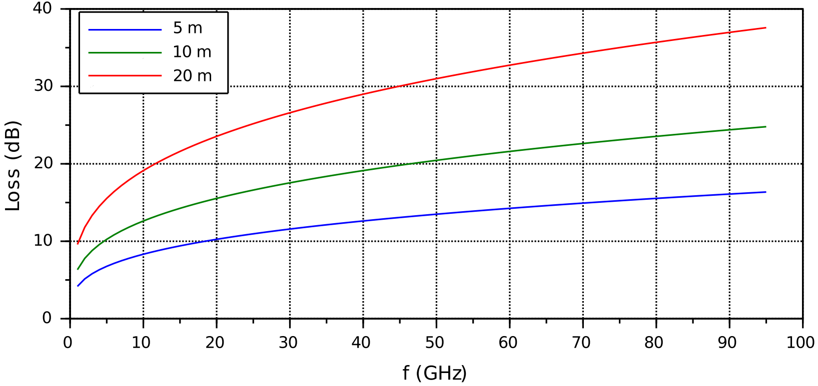
$$L = 0.2 f^{0.3} d_F^{0.6}$$

  where d_F is the depth of foliage in meters and f is in MHz

Weissberger Model Plot



ITR Model Plot



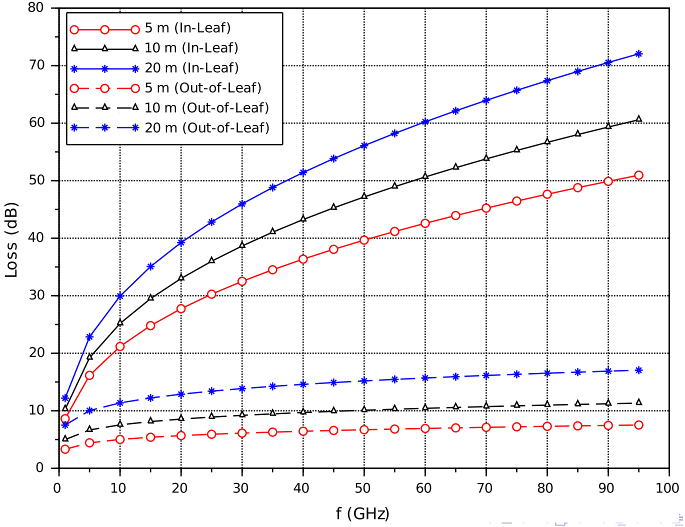
The Fitted-ITR Model

- An improvement of the ITR model that takes into account the foliage state
- Valid for frequencies ranging from 230 MHz to 95 GHz
- Given by

$$L = \begin{cases} 0.37 f^{0.18} d_F^{0.59} & \text{out-of-leaf} \\ 0.39 f^{0.39} d_F^{0.25} & \text{in-leaf} \end{cases}$$

where d_F is the depth of foliage in meters and f is in MHz

Fitted-ITR Model Plot



Conclusion

Conclusion

- Microwaves propagate as space waves
  - Require LOS
- Due to the curvature of the earth the LOS range of microwaves are limited to the radio horizon
- Tropospheric conditions (i.e., the refraction index gradient) can bend microwaves and increase there range
- Obstacles can cause Fresnel interference and diffraction
- Foliage losses account for the loss due to vegetation