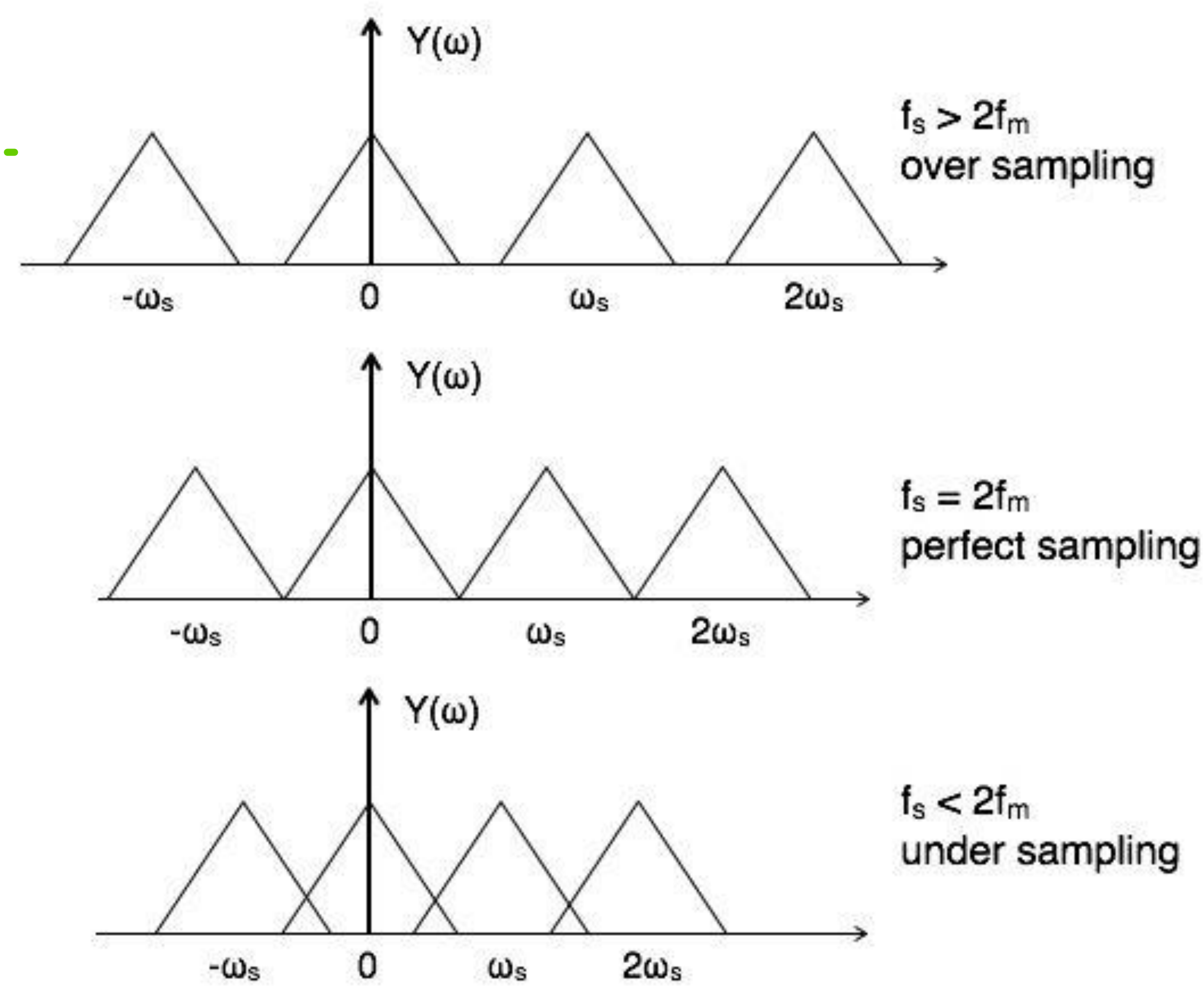
Possibility of sampled frequency spectrum with different conditions is given by the following diagrams:



In Over Sampling , the Original Signal can be extracted using practical law pass filter which having cut off frequency of fm.

In perfect Sampling , the Original Signal can be extracted using Ideal law pass filter which having cut off frequency of fm.

In Under Sampling, the aliasing effect will be occurred and a part of spectrum will be distorted.

Aliasing Effect

The overlapped region in case of **under sampling** represents **Aliasing effect**. It can be termed as "**the phenomenon of a high-frequency component in the spectrum of a signal, taking on the identity of a lower-frequency component in the spectrum of its sampled version.**

This effect can be removed by considering

(i) fs >2fm or

(ii) by using anti aliasing filters which are low pass filters and eliminate high frequency components

IN CLASS TASK

Suppose, if the msg signal is a sinusoidal signal, draw the frequency domain representation of sampled signal (Y(f)), {when fs>2.fm : Over Sampling}

{take msg signal , $x(t)=A_m.Cos(w_m t)=A_m.Cos(2\pi f_m t)$}

IDEAL OR IMPULSE SAMPLING

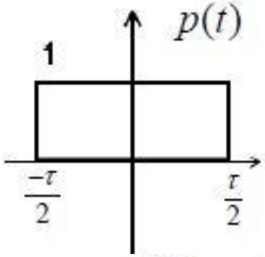
Obtained by multiplying input signal x(t) with impulse train of period 'Ts. Also called ideal sampling.

We have already discussed this topic.

This is called ideal sampling or impulse sampling. You cannot use this practically because pulse width cannot be zero and the generation of impulse train is not possible practically.

NATURAL SAMPLING

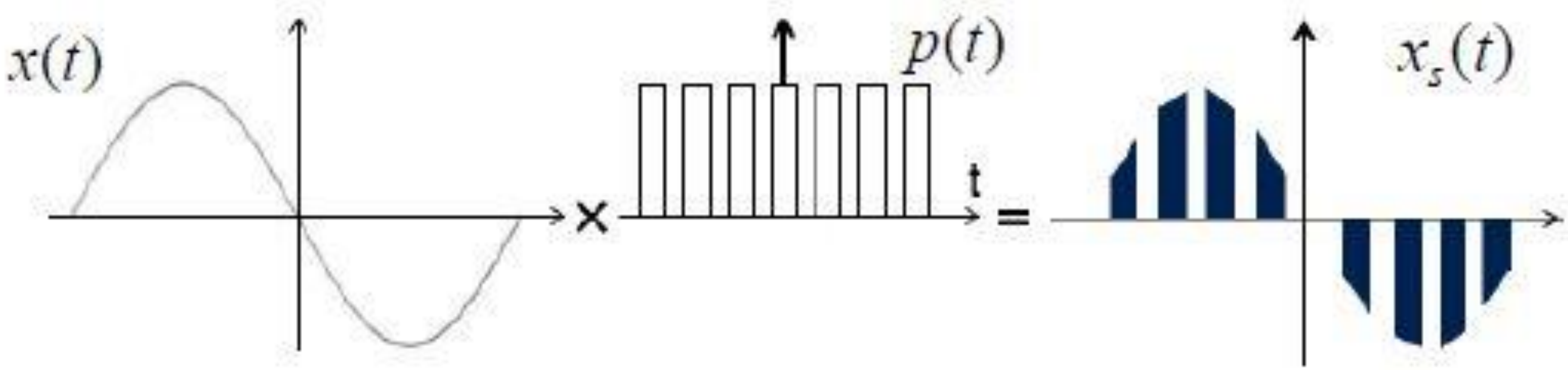
Natural sampling is similar to impulse sampling, except the impulse train is replaced by pulse train of period Ts. i.e.



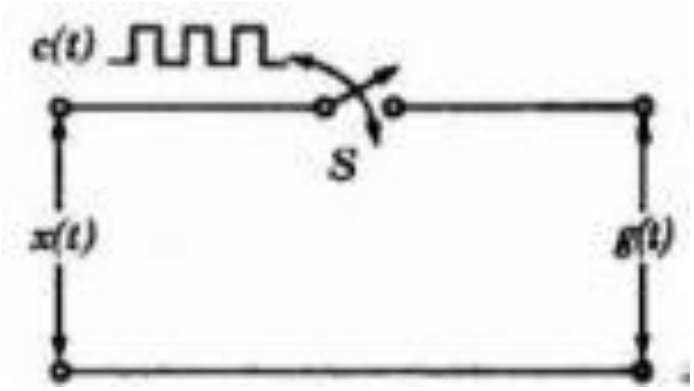
The pulse train equation is being given as:

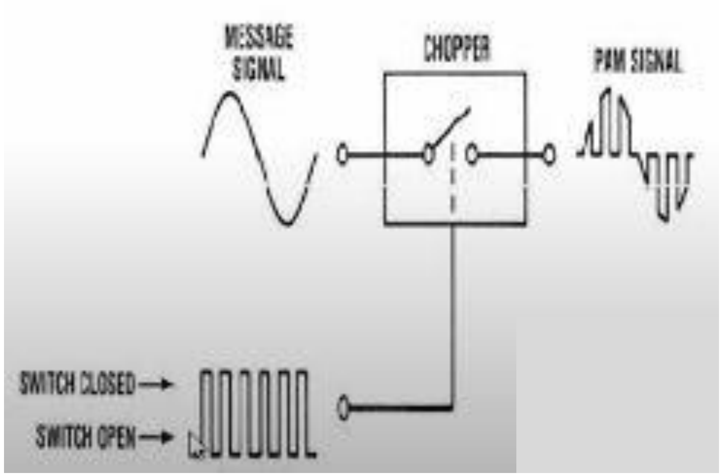
$$p(t) = \sum_{n=-\infty}^{+\infty} p(t - nT_s)$$

With the help of functional diagram of a Natural sampler, a sampled signal $x_s(t)$ is obtained by multiplication of Pulse train P(t) and the input signal x(t) :



Functional Diagram of Natural Sampler:





When Pulse Train is in high state the switch will be closed.

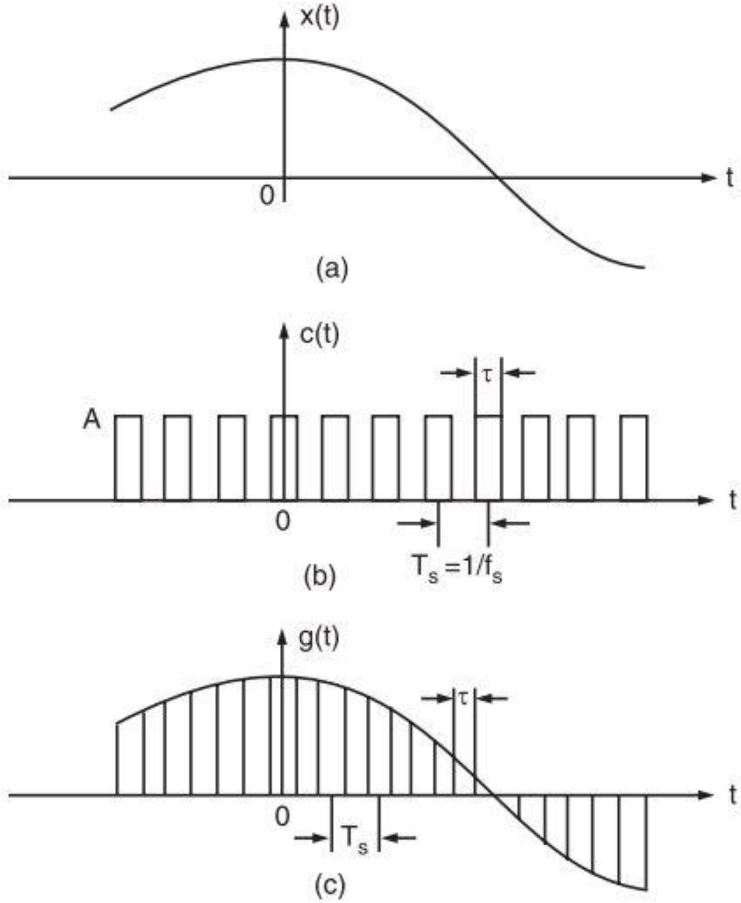
When Pulse Train is in Law state the switch will be Open.

Pulse train can be viewed as an opening and closing switch.

Practically it is impossible to obtain sampling function with pulse width approaching to Zero.

In natural Sampling a pulse, with finite width is used as a sampling function and Top of the each pulse of the Sampled Signal retain the shape of its corresponding analog segment during the pulse interval.

The waveforms of signals x(t), c(t) and g(t) have been illustrated in fig.3(a), (b) and (c) respectively.



(a) Continuous time signal x(t), (b) Sampling function waveform i.e., periodic pulse train, (c) Naturally sampled signal waveform g(t).

Now, the sampled signal g(t) may also be described mathematically as

g(t) = c(t) . x(t)

We can obtain g(t) value as:

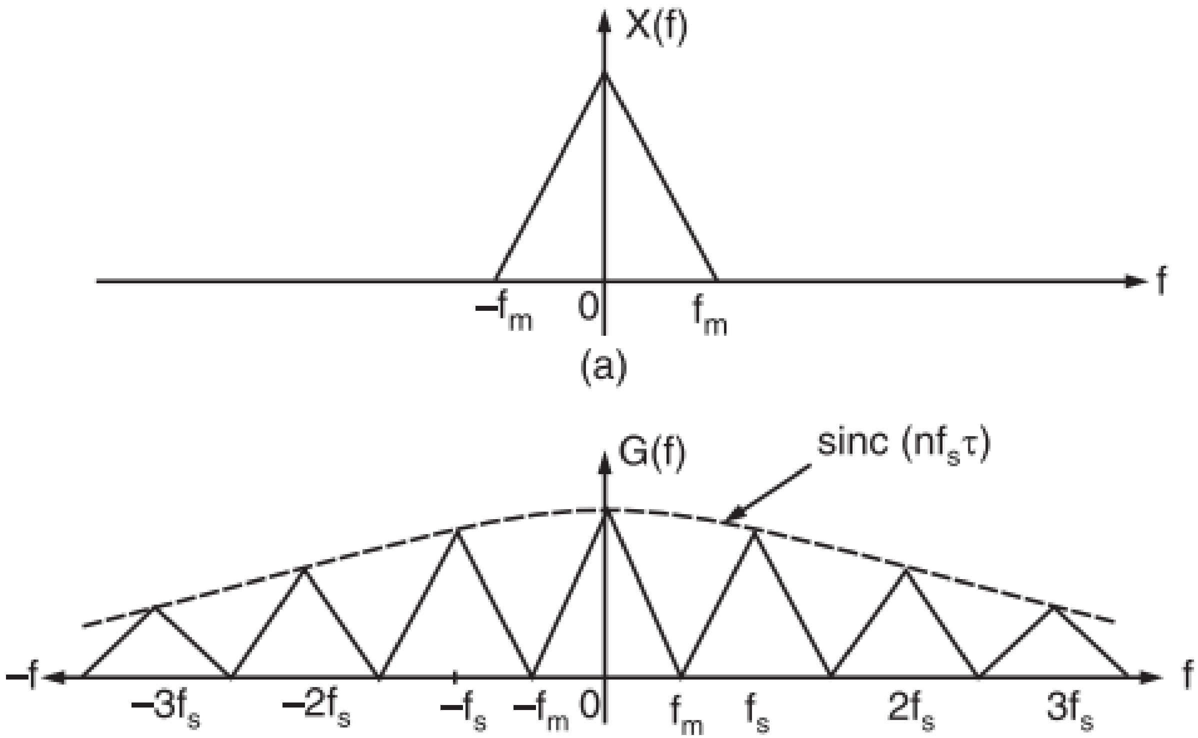
$$g(t) = \frac{\tau A}{T_s} \cdot \sum_{n=-\infty}^{\infty} \sin c(f_{n.} \tau).\ e^{j2\pi f_s nt} .x(t)$$

Spectrum of naturally sampled signal

$$G(f) = \frac{\tau A}{T_s} \cdot \sum_{n=-\infty}^{\infty} \sin c(nf_{s\,.}\ \tau)\ X(f - nf_s)$$

This equation shows that the spectra of x(t) i.e., X(f) are periodic in f_s and are weighed by the sinc function.

Figure illustrates some arbitrary spectra for x(t) and corresponding spectrum G(f).



MATHEMATICAL REPRESENTATION

- The sampled signal is given by,

$$s(t) = c(t)g(t) \tag{1}$$

- In natural sampling the top of each pulse retains the shape of the analog segment at that pulse interval.

- The sampling function $c(t)$ can be represented by the complex Fourier series as;

$$c(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi f_s nt} \tag{2}$$

- where, the coefficient C_n can be calculated using the equation,

$$C_n = \frac{1}{T_s}\int_{-T/2}^{T/2} c(t)e^{-j2\pi f_s nt}dt$$

- Note that the amplitude of $c(t)$=A.

$$C_n = \frac{1}{T_s}\int_{-T/2}^{T/2} Ae^{-j2\pi f_s nt}dt$$

$$= Af_s\left[\frac{e^{-j2\pi f_s nt}}{-j2\pi f_s n}\right]_{-T/2}^{T/2}$$

$$= Af_s\left[\frac{e^{-j2\pi f_s nT/2} - e^{j2\pi f_s nT/2}}{-j2\pi f_s n}\right]$$

$$= Af_s\left[\frac{e^{-j\pi f_s nT} - e^{j\pi f_s nT}}{-j2\pi f_s n}\right]$$

$$
\begin{aligned}
&= \frac{Af_s}{\pi f_s n}\left[\frac{e^{-j\pi f_s nT} - e^{j\pi f_s nT}}{-2j}\right] \\
&= \frac{A}{\pi n}\left[\frac{e^{j\pi f_s nT} - e^{-j\pi f_s nT}}{2j}\right] \\
&= \frac{A}{\pi n} sin(\pi f_s nT) \\
&= A\frac{sin(\pi f_s nT)}{\pi n} \\
&= A\frac{sin(\pi f_s nT)}{\frac{\pi f_s nT}{f_s T}} \\
&= Af_s T\frac{sin(\pi f_s nT)}{\pi f_s nT}
\end{aligned}
$$

$$
C_n = Af_s T sinc(f_s nT) \tag{3}
$$

- Substituting Eq. (3) into Eq. (2)

$$c(t) = \sum_{n=-\infty}^{\infty} Af_sTsinc(f_snT)e^{j2\pi f_snt} \tag{4}$$

- Substituting Eq. (4) in Eq. (1) we get,

$$s(t) = g(t) \sum_{n=-\infty}^{\infty} Af_sTsinc(f_snT)e^{j2\pi f_snt}$$

$$s(t) = Af_sT \sum_{n=-\infty}^{\infty} sinc(f_snT)g(t)e^{j2\pi f_snt} \tag{5}$$

- Taking Fourier transform on both sides of Eq. (5) we get;

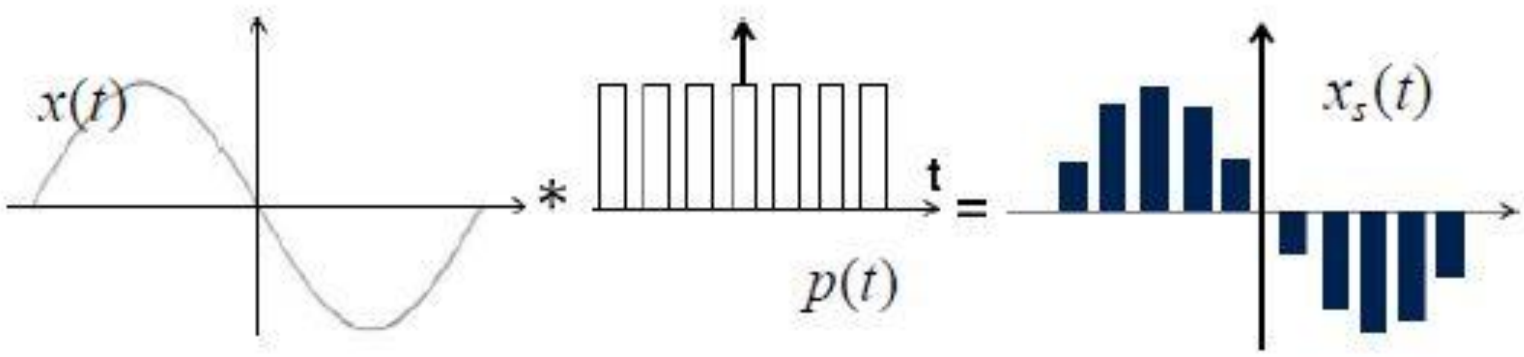
$$S(f) = Af_sT \sum_{n=-\infty}^{\infty} sinc(f_snT)G(f - nf_s) \tag{6}$$

- Hence, from Eq. (6) we note that spectrum of $S(f)$ is a periodic signal weighted by a sinc function.

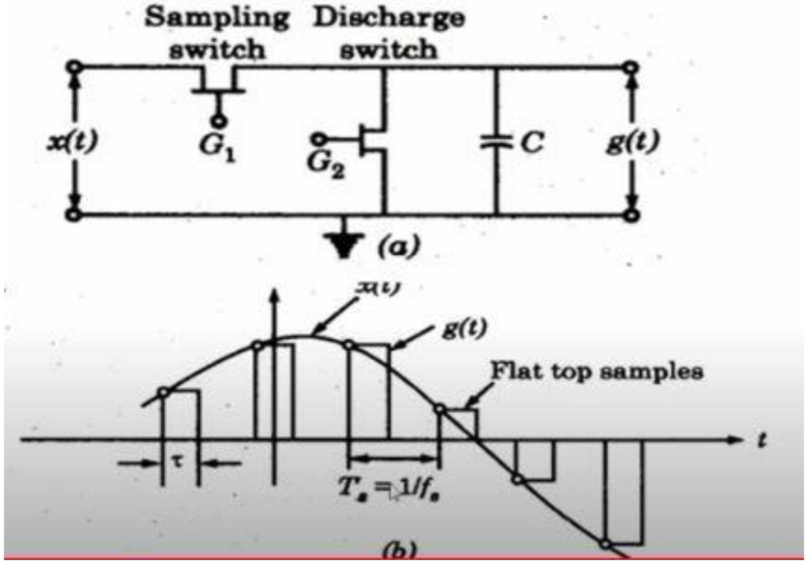
FLAT TOP SAMPLING

During transmission, noise is introduced at top of the transmission pulse which can be easily removed if the pulse is in the form of flat top.

Here, the top of the samples are flat i.e. they have constant amplitude. Hence, it is called as flat top sampling or practical sampling.



Sample and hold circuit is used for the generation of the sampled signal to attain flat top sampling, which is shown in the Fig below.

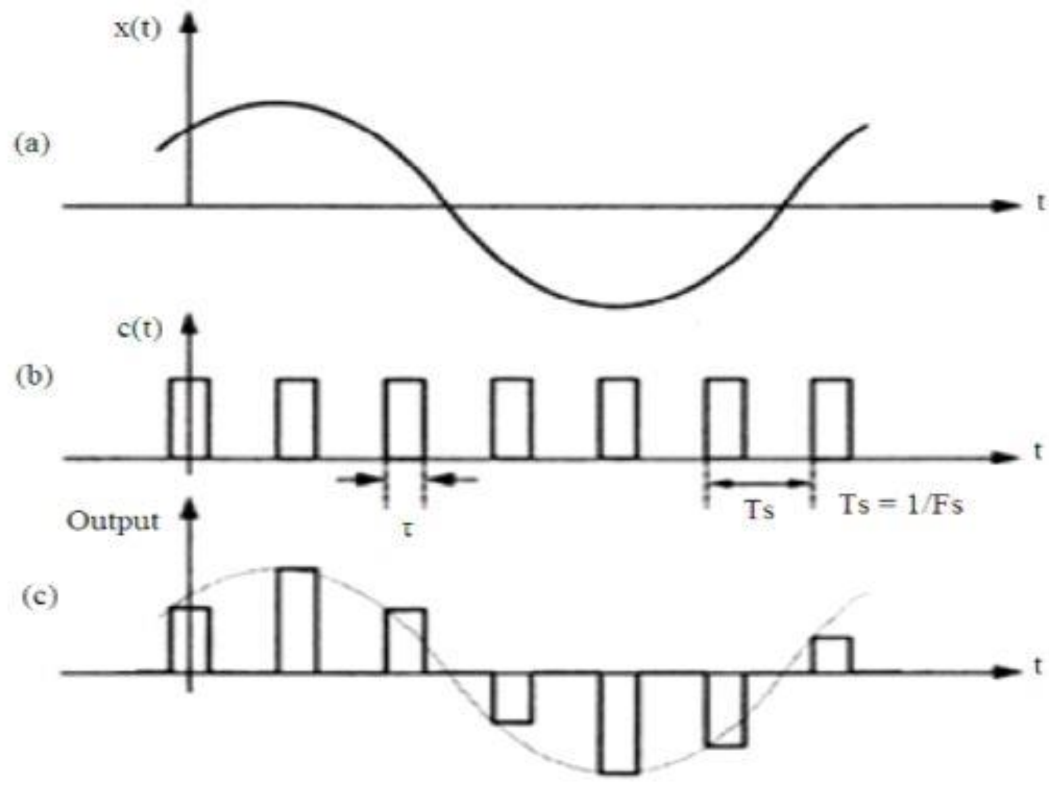


Above Sample and hold circuit to generate flat top samples

The switch G1 closes at each sampling instant to sample the modulating signal.

The capacitor C holds the sampled voltage (Starting value of the massage Signal) for period Γ, at the end of which switch G2 is closed in order to discharge the capacitor.

Thus the signal generated as a result of sample and hold process is the flat top sampled signal. The spectrum of the generated flat top sampling signal along with the modulating signal and the sampling signal is shown below figure.



In above Fig (a) Modulating signal (b) sampling signal and (c) Flat top sampling spectrum

The starting edge of the pulse corresponds to the instantaneous value of the modulating signal x(t).

Flat top sampling is mostly used in digital transmission.

MATHEMATICAL REPRESENTATION

- Consider the situation where the analog signal $g(t)$ is sampled instantaneously at the rate $f_s = 1/T_s$, and that the duration of each sample is lengthened to T as illustrated in Fig. 1.
- Let $s(t)$ to denote the sequence of flat-top pulses generated in this way,

$$s(t) = \sum_{n=-\infty}^{\infty} g(nT_s)h(t - nT_s) \qquad (1)$$

- The $h(t)$ is a rectangular pulse of unit amplitude and duration T, defined as

$$h(t) = \begin{cases} 1 & 0 < t < T \\ 0 & t < 0, \text{ and } t > T \end{cases}$$

$$h(t) = rect\left(\frac{t}{T} - \frac{1}{2}\right) \qquad (2)$$

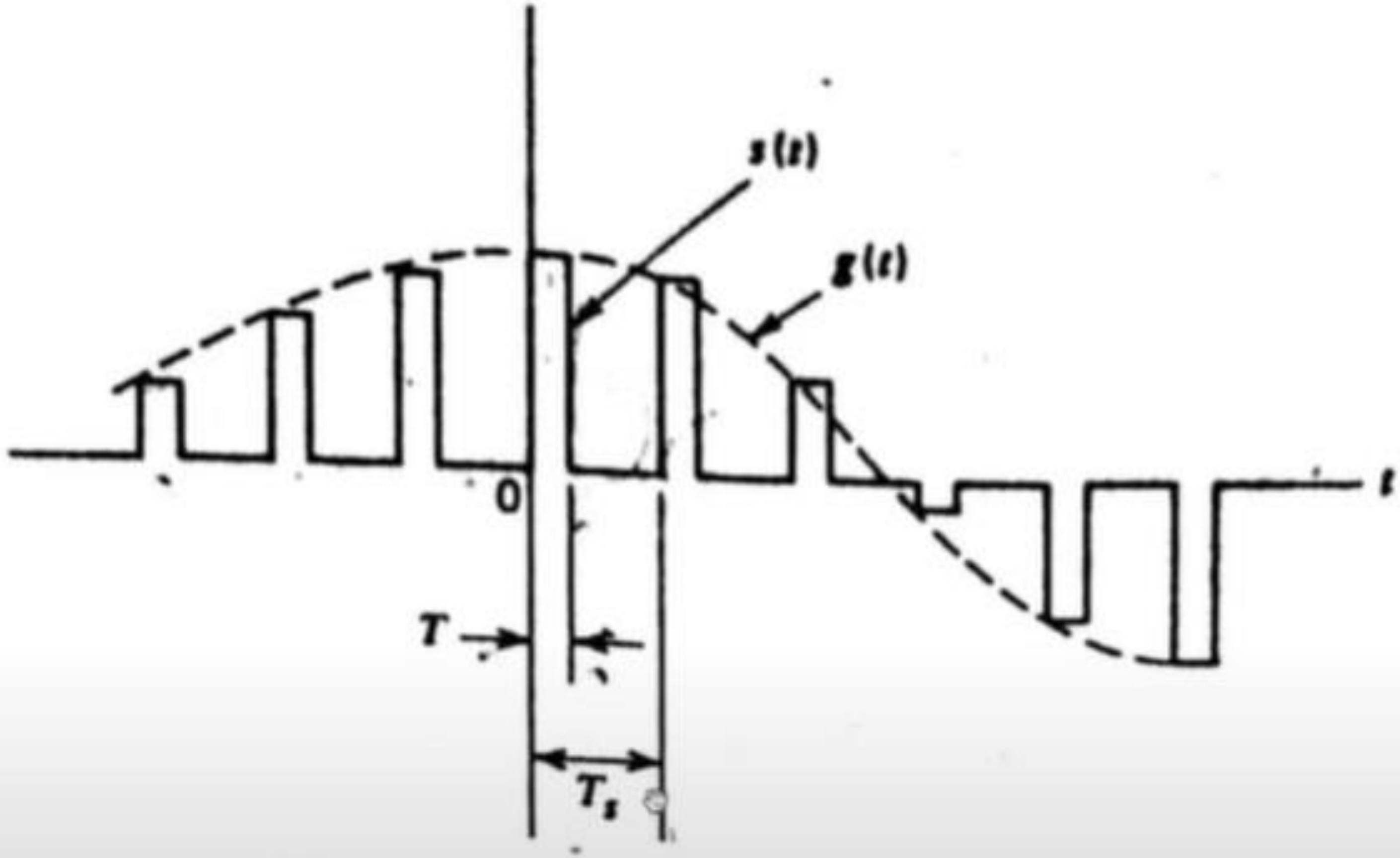


Figure 1: Flat-Top Samples

- From the ideal sampling theorem, the discrete-time signal $g_\delta(t)$, obtained by instantaneously sampling $g(t)$, is given by

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s) \tag{3}$$

- Convolving $g_\delta(t)$ with $h(t)$, we get;

$$\begin{aligned} g_\delta(t) \star h(t) &= \int_{-\infty}^{\infty} g_\delta(\tau)h(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g(nT_s)\delta(\tau - nT_s)h(t-\tau)d\tau \end{aligned}$$

$$g_\delta(t) \star h(t) = \sum_{n=-\infty}^{\infty} g(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s)h(t-\tau)d\tau \tag{4}$$

- The sifting property of the delta function is given by,

$$\int_{-\infty}^{\infty} \delta(\tau - nT_s)h(t-\tau)d\tau = h(t - nT_S) \tag{5}$$

- Substituting Eq. (5) into Eq. (4) we get,

$$g_\delta(t) \star h(t) = \sum_{n=-\infty}^{\infty} g(nT_s)h(t - nT_S) \tag{6}$$

- Comparing Eq. (1) and Eq. (6) we find that their RHS are equal. Therefore, it follows that $s(t)$ is mathematically equivalent to the convolution of the instantaneously sampled signal, $g_\delta(t)$, and the pulse $h(t)$.

$$s(t) = g_\delta(t) \star h(t) \tag{7}$$

- Taking FT on both sides of Eq. (7) we get,

$$S(f) = G_\delta(f)H(f) \tag{8}$$

- From the ideal sampling theorem we know that;

$$G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G_(f - mf_s) \tag{9}$$

- Substituting Eq. (9) into Eq. (8) we get,

$$S(f) = f_s \sum_{m=-\infty}^{\infty} G_(f - mf_s)H(f) \tag{10}$$

- Lastly, suppose that $g(t)$ is strictly band-limited and that the sampling rate f_s is greater than the Nyquist rate.
- Then, passing s(t) through a low-pass reconstruction filter, we find that the spectrum of teh resulting filter output is equal to $G(f)H(f)$.
- This is equivalent to passing the original analog signal $g(t)$ through a low-pass signal of transfer function $H(f)$.
- The transfer function $H(f)$ is obtained by applying FT on Eq. (2);

$$H(f) = Tsinc(fT)\exp(-j\pi fT) \tag{11}$$

This is shown plotted in Fig. 2.

- Hence, we see that by using flat-top samples, we have introduced amplitude distortion as well as a delay of T/2.
- This effect is similar to the variation in transmission with frequency that is caused by the finite size of the scanning aperture in television and facsimile. Accordingly, the distortion caused by lengthening the samples is referred to as the aperture effect.
- This distortion may be corrected by connecting an equalizer in cascade with the low-pass reconstruction filter.
- The equalizer has the effect of decreasing the in-band loss of the reconstruction filter as the frequency increases in such a manner as to compensate for the aperture effect.
- Ideally, the amplitude response of the equalizer is given by

$$\frac{1}{|H(f)|} = \frac{1}{T sinc(fT)} = \frac{1}{T}\frac{\pi f T}{\sin(\pi f T)} \tag{12}$$

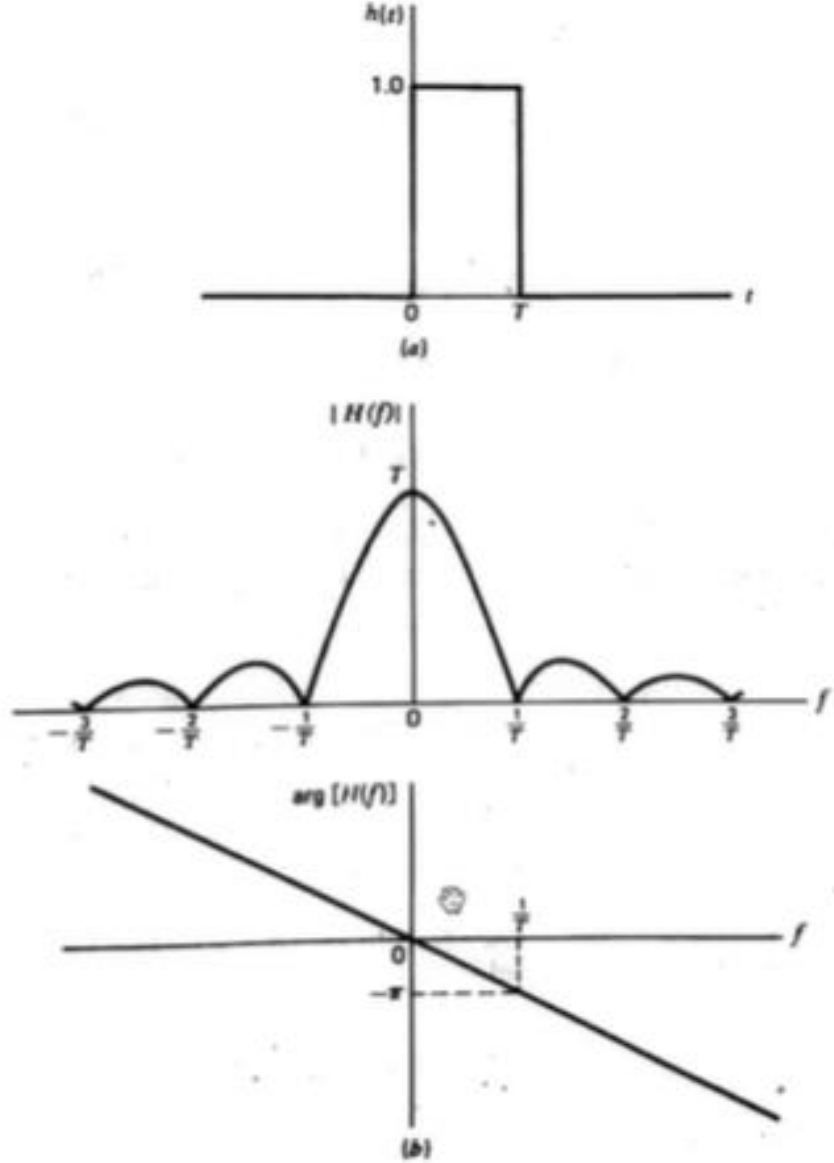
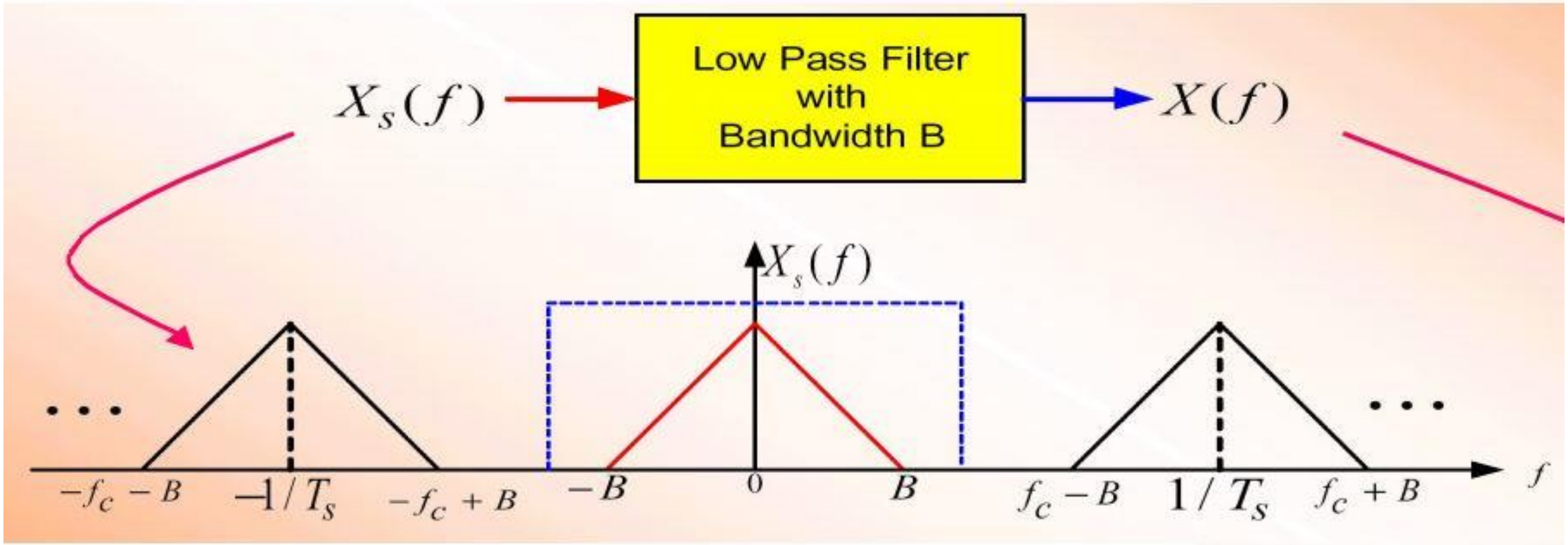


Figure 2: (a) Rectangular pulse $h(t)$, (b) spectrum $H(f)$

Recovering the Analog Signal One way of recovering the original signal from sampled signal Filter (LPF) as shown below



If fm>2B then we recover x(t) exactly.

Else we run into some problems and signal is not fully recovered.