GENERAL SIR JOHN KOTELAWALA DEFENCE UNIVERSITY

Faculty of Engineering

Department of Electrical, Electronic and Telecommunication Engineering

B.Sc. Engineering Degree

Semester 5 Examination – May 2023

(Intake 38 – ET/MC)

ET 3142– DIGITAL SIGNAL PROCESSING

Time allowed: 3 Hours 17th May 2023

ADDITIONAL MATERIAL PROVIDED

Pages 5 to 8 ANNEXURE

INSTRUCTIONS TO CANDIDATES

This paper contains 5 questions on 4 pages .

Answer ALL questions

This is a closed book examination

This examination accounts for 70% of the module assessment. The marks assigned for each question and parts thereof are indicated in square brackets

If you have any doubt as to the interpretation of the wordings of a question, make your own decision, but clearly state it on the script

Assume reasonable values for any data not given in or provided with the question paper, clearly make such assumptions made in the script

All examinations are conducted under the rules and regulations of the KDU

DETAILS OF ASSESSMENT

Learning Outcome (LO)	Questions that assess LO	Marks allocated (Total 70%)
LO1	Q2	14
LO2	Q4, Q5	28
LO3	Q1	14
LO4	Q3	14

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Question 1

Determine the system function H(z) of the lowest order Chebyshev IIR digital filter with the following specifications:

4 dB ripple in passband
30 dB attenuation in stopband $0.5\pi \leq \omega \leq \pi$
Use bilinear transformation with T = 1 [20]

Question 2

(a) Write the Fourier transform matrix F_4 for $k,n = 0,1,2,3$ where the complex number $W_N^{nk} = e^{-j\frac{2\pi nk}{N}}$. [06]

(b) Plot the derived values for W_N^{nk} in part (a) in a complex . [06]

(c) By using Fourier transform matrix F_4 in part (a) calculate the four-point DFT for sequence $x[n] = [0,1,2,3]$ [08]

Question 3

Design a low-pass Butterworth filter using the bilinear transformation method for satisfying the following specifications:

Passband: 0 - 400 Hz
Passband ripple: 2 dB
Sampling frequency: 10 kHz
Stopband: 2.1 – 4 kHz
Stopband attenuation : 20 dB [20]

Question 4

(a) Compare Analog and Digital Filters by including two advantages and two disadvantages for each. [08]

(b) Construct a digital band-pass filter for input x[n] to derive output y[n] using cascading and single stage methods. Use $h_1[n]$ and $h_2[n]$ as the impulse responses of low-pass and high-pass filters respectively. [04]

(c) Brief explain the main functionality of the following filters. [04]

i. Recursive Filter
ii. Moving average filter

(d) If you are designing a filter in order to filter the additive noise in a transferring signal, 'good performance in the time domain results in poor performance in the frequency domain, and vice versa'. Discuss this statement with two appropriate examples. [04]

Question 5

(a) A continuous time speech signal $x_a(t)$ is sampled at a rate of 8 kHz and the samples are subsequentially grouped in blocks each of size N. The DFT of each block is to be computed in real time using the radix-2 decimation in frequency FFT algorithm.
Assume that the processor performs all operations sequentially and takes 20μs for computing each complex multiplication (including multiplications by 1 and -1) and the time required for addition/subtraction is negligible.

i. Assume N points radix-2 FFT requires m stages. Derive an expression for m using N. [02]

ii. By using the derived statement in part ii state the number of multiplications for each block using N point FFT. [02]

iii. By approximating the number of samples to 8192 samples per second, state the number of required blocks. [02]

iv. Give a statement for total time taken for FFT of 8192 samples. [02]

v. Compute the maximum value of N. [02]

(b) Find 4-point DFT of a sequence $x(n) = [1,1,1,0]$ using radix -2 DIT-FFT algorithm. [10]

End of question paper

ANNEXURE

USEFUL FORMULAE

Fourier Transform Theorems

Sequence $x[n]$ $y[n]$	Fourier Transform $X(e^{j\omega})$ $Y(e^{j\omega})$
1. $ax[n]+by[n]$	$aX(e^{j\omega})+bY(e^{j\omega})$
2. $x[n-n_d]$ (n_d an integer)	$e^{-j\omega n_d}X(e^{j\omega})$
3. $e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
4. $x[-n]$	$X(e^{-j\omega})$ $X^*(e^{j\omega})$ if $x[n]$ real.
5. $nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
6. $x[n]*y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
7. $x[n]y[n]$	$\dfrac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$

Discrete Fourier Transform

DFT of $x[n]$,

$$X[k]=\sum_{n=0}^{N-1}x[n]e^{-j\Omega_0 kn}, 0\le k\le N-1, \Omega_0=\frac{2\pi}{N}$$

DFT of ideal frequency response

$$H_d\left(e^{j\omega}\right)=\sum_{-\infty}^{\infty}h_d[n]e^{-j\omega n}$$

Inverse Discrete Fourier transform (IDFT) of ideal frequency response

$$h_d[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d\left(e^{j\omega}\right)e^{j\omega n}d\omega$$

Linearity:

$$\begin{cases} \alpha x[n] \longleftrightarrow \alpha X[k] \\ x_1[n] + x_2[n] \longleftrightarrow X_1[k] + X_2[k] \end{cases}$$

Time Shift:

$$x\left[((n-n_0))_N\right] \longleftrightarrow e^{-j\Omega_0 n_0 k} X[k]$$

Frequency Shift:

$$e^{jin} x[n] \longleftrightarrow X\left[((k-i))_N\right]$$

Convolution:

$$x_1[n] * x_2[n] = \sum_{m=0}^{N-1} x_1\left[((n-m))_N\right] x_2[m]$$

$$x_1[n] * x_2[n] \longleftrightarrow X_1[k] X_2[k]$$

Multiplication:

$$X_1[k] * X_2[k] = \sum_{i=0}^{N-1} X_1\left[((k-i))_N\right] X_2[i]$$

$$x_1[n] x_2[n] \longleftrightarrow \frac{1}{N} X_1[k] * X_2[k]$$

Time Differencing:

$$x[n] - x\left[((n-1))_N\right] \longleftrightarrow \left(1 - e^{-j\Omega_0 k}\right) X[k]$$

Accumulation:

$$\sum_{m=0}^{n} x[m] \longleftrightarrow \frac{1}{1 - e^{-j\Omega_0 k}} X[k] \quad (\text{only for } X[0] = 0)$$

IIR Filter – Properties

A_1 = Gain at a passband frequency ω_1
A_2 = Gain at a passband frequency ω_2
Ω_1 = Analog frequency corresponding to ω_1
Ω_2 = Analog frequency corresponding to ω_2

$\Omega_1 = \dfrac{2}{T}\tan\dfrac{\omega_1}{2}$, $\Omega_2 = \dfrac{2}{T}\tan\dfrac{\omega_2}{2}$ where T is the time.

Low-Pass Digital Butterworth

The order N of the filter,

$$N \geq \frac{1}{2} \frac{\log\left\{\left[\dfrac{1}{A_2^2} - 1\right] \Big/ \left[\dfrac{1}{A_1^2} - 1\right]\right\}}{\log \dfrac{\Omega_2}{\Omega_1}}$$

Cutoff Frequency, $\Omega_c = \dfrac{\Omega_1}{\left[\dfrac{1}{A_1^2}-1\right]^{-1/2N}}$

Transfer Function,

when the order N is even for unity dc gain filter, $H_a(s)=\prod_{k=1}^{N/2}\dfrac{\Omega_c^2}{s^2+b_k\Omega_c s+\Omega_c^2}$.

when the order N is odd for unity dc gain filter, $H_a(s)=\dfrac{\Omega_c}{s+\Omega_c}\prod_{k=1}^{N-1/2}\dfrac{\Omega_c^2}{s^2+b_k\Omega_c s+\Omega_c^2}$.

The coefficient of b_k is given by $b_k = 2\sin\left[\dfrac{(2k-1)\pi}{2N}\right]$

Chebyshev Filter

Attenuation Constant $\varepsilon = \left[\dfrac{1}{A_1^2}-1\right]^{\frac{1}{2}}$

The order N of the filter,

$$N \geq \left\{\frac{\cosh^{-1}\left\{\dfrac{1}{\varepsilon}\left[\dfrac{1}{A_2^2}-1\right]^{1/2}\right\}}{\cosh^{-1}\left\{\dfrac{\Omega_2}{\Omega_1}\right\}}\right.$$

Cutoff Frequency, $\Omega_c = \dfrac{\Omega_1}{\left[\dfrac{1}{A_1^2}-1\right]^{-1/2N}}$

Transfer Function,

when the order N is even, $H_a(s)=\prod_{k=1}^{N/2}\dfrac{B_k\Omega_c^2}{s^2+b_k\Omega_c s+c_k\Omega_c^2}$.

when the order N is odd, $H_a(s)=\dfrac{B_0\Omega_c}{s+c_0\Omega_c}\prod_{k=1}^{N-1/2}\dfrac{B_k\Omega_c^2}{s^2+b_k\Omega_c s+c_k\Omega_c^2}$.

where

$$b_k = 2y_N \sin\left[\frac{(2k-1)\pi}{2N}\right]$$

$$c_k = y_N^2 \cos^2\left[\frac{(2k-1)\pi}{2N}\right]$$

$$c_0 = y_N$$

$$y_N = \frac{1}{2}\left\{\left[\left(\frac{1}{\varepsilon^2}+1\right)^{\frac{1}{2}}+\frac{1}{\varepsilon}\right]^{\frac{1}{N}} - \left[\left(\frac{1}{\varepsilon^2}+1\right)^{\frac{1}{2}}+\frac{1}{\varepsilon}\right]^{\frac{-1}{N}}\right\}$$