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2025/04/02

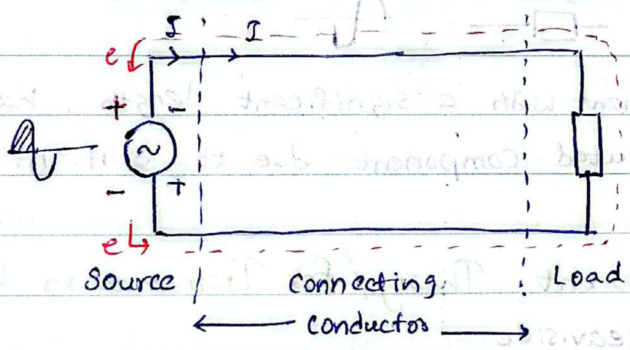
Microwave Engineering

1. Transmission lines.
2. Microwave antennas
3. S - parameters
   Switch chart
   Circuit basis
   } Network analizer

4. Microwave tube
5. Semiconductor devices.

legacy Technologies → Wave guide tubes

Transmission Line



$\omega \rightarrow f \rightarrow \lambda$

$f \propto \frac{1}{\lambda}$ so, if $f >>$, $\lambda <<$ so Short distance

if $f <<$, $\lambda >>$ so long distance

(condition) C = Circuit Theory → for every half cycle an electron, that leaves the Source from (-) returns to the (+)

1. $L << \lambda$ at low frequencies
2. at high frequiencies electron will not have enough time to complete the entire circuit,

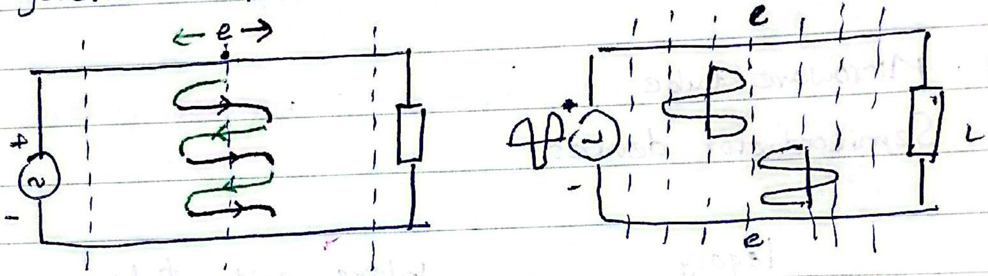
$L = \lambda$ → Condition C is not satisfied.

So, Circuit theory is no longer Valied.

No :

if we have electron at a given position, during positive half cycle, it attract to (+) terminal by attractive force so, becose of this electron move for little distance and then repelled away from the post (+) terminal, then again attract (+) terminal and repelled away from (+) termanil.

So, it will generate Simple Harmonic motion.

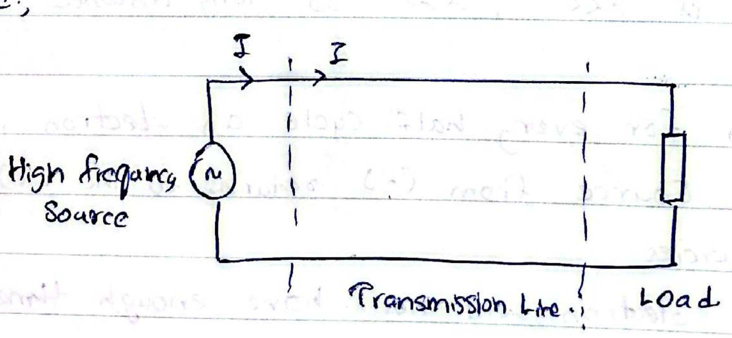


At high frequencies, e undergo Simple Harmonic Motion. (S.H.M)
- The entire S.H.M may not take place within a lumped component.

- every component with a significant length, has to be treated as a distributed component due to S.H.M.

Distributed Component Theory / Transmission Line theory
by Oliver Heaviside

In here we assume all electron undergo S.H.M and, in here,



Transmission Line has 2 conductors (at least two conductors) and there are transmission lines can have more than two conductors

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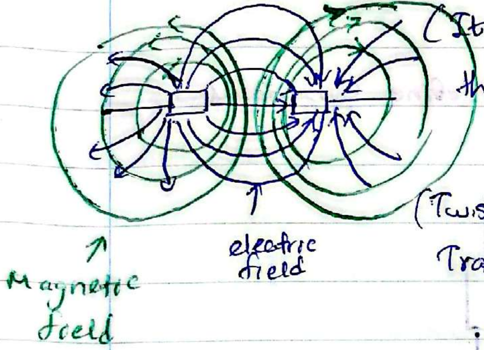
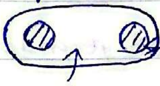
There are two basic forms of 2 conductors in transmission Line

① Twin wire (also called slotted line in Microwave)



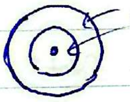
(It has two parallel conductors and a dielectric in between them)



(Twisted pair also part of this but it is used for lower frequency Transmission)

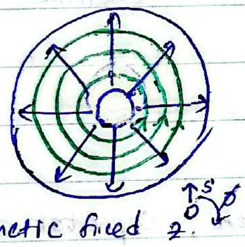
② Coaxial form.

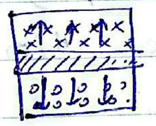
(It contained two concentric conductors, Two circular conductors in same center.)

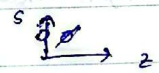


* Lower freqency Coaxial cable } very flexible and use for TV cable
* High frequency Coaxial cable } Rigit (not flexible)

- Transverse field -







Green → magnetic field

Blue → Electric field.

-longitudinal field-

* compared to twin wires, coaxial cables are better.
* In coaxial cable, bothe electric and Magnetic fields are contained inside the cable, due to both field is contained the E and M fields. co its better than twinwire.
* But in coaxial cable field can ~~be~~ have lickage ~~of~~ in little amount, and becouse of this outer cover is made to thick layer. So flexible ~~are~~ Coaxial cables ~~are~~ have leacage than the regid cables.

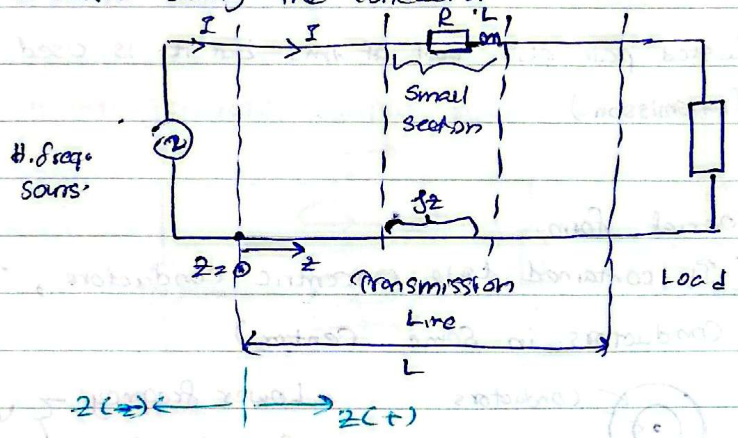
* So Coaxial cables have very low radiation loss.

Transmission Line Parameters

We are looking at it as distributed component (tt means transmission line)

We get a small section of it and, need to define coordinates.

* z is define along the conductor



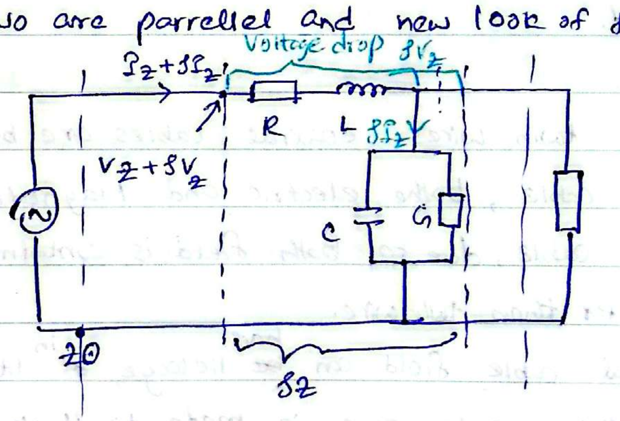
Small section has series small Resistance and small inductance

R - per length Resistors (Ω/m)

L - per length inductance (H/m)

and Small section has parrellel component, capacitor and conductors.

These two are parrellel and new look of the circuit



δV_z - Voltage drop across the small section

δI_z - leakage current within the small section

- Current entering to small section δz is $I_z + \delta I_z$
- Current leave the small section is I_z
- because there is voltage drop between δz and there is leakage current in capacitor and conductor

V_z and I_z can take any value, so we don't worry about voltage drop and leackage current between small section, and we only consider the fact that, if the leaekage current or Voltage drop can be handle by the transmission line.

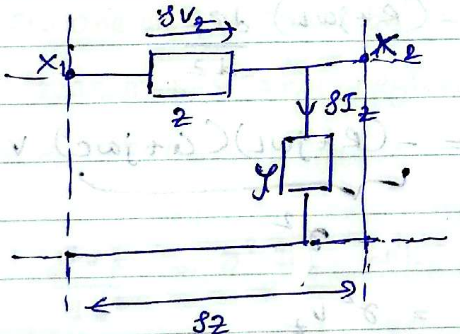
We can fed any current and any Voltage to the Transmission line in idealy. But the Voltage will be limited by the dielectric ~~bra~~ breakdown Used.

The curren ~~a~~ will be limited by the maximum current can be handled by the Current.

* any Conductor can act as faes when it exeed the maximum current can be handled by conductor

** R, L, C, G are Uniform across the transmission Line

We get small Section again as combine component, Series Impedance Z and Shunt admittance Y



$Z = R\delta z + j\omega L \delta z$

$Y = G\delta z + j\omega C \delta z$

** We can apply Ohms low for small section but not ~~for~~ for the entire component at once

Voltage drop $\rightarrow$ $\delta V_z = Z(I_z + \delta I_z)$

$$= (R\delta z + j\omega L\delta z)(I_z + \delta I_z)$$

$$= (R + j\omega L)(I_z + \delta I_z)\delta z$$

Admittance $\rightarrow$ Voltage at x_2 is V_z.

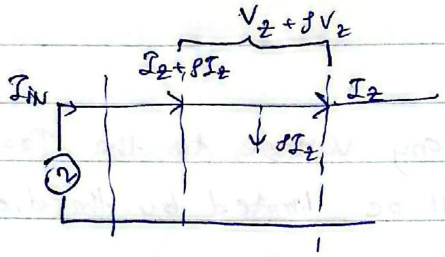
Leaekage current comming out of $x_2 = \delta I_z$

$$\delta I_z = Y V_z$$

$$= V_z(G\delta z + j\omega C\delta z)$$

$$= V_z(G + j\omega C)\delta z$$

There fore the rate of change in Voltage with respective to z,



$\delta V_z > 0 \quad \delta I_z > 0$

① — $V_z \downarrow \; z \uparrow$ $\quad \delta V_z$ (−) w.r.t δ_z

② — $I_z \downarrow \; z \uparrow$ $\quad \delta I_z$ (−) w.r.t. δ_z

$$I_z + \delta I_z \simeq I_z$$

$$\delta I_z \simeq 0$$

$$\delta V_z = (R + j\omega L)\underbrace{(I_z + \delta I_z)}_{I_z}\delta z$$

$$\frac{\delta V_z}{\delta_z} = -(R + j\omega L)\, I_z \qquad \left(\lim_{\delta z \to 0} \frac{\delta V_z}{\delta z}\right) = \frac{dV_z}{dz}$$

$$\frac{dV_z}{dz} = -(R + j\omega L)\, I_z$$

$$\frac{d^2 V_z}{dz^2} = -(R + j\omega L)\frac{dI_z}{dz}$$

$$\frac{d^2 V_z}{dz^2} = \underbrace{(R + j\omega L)(G + j\omega C)}_{\gamma^2} V_z$$

$$\frac{d^2 V_z}{dz^2} = \gamma^2 V_z$$

for current w.r.t δz

$$\frac{\delta I_z}{\delta z} = -(G + j\omega C) V_z$$

$$\frac{dI_z}{\delta z} = -(G + j\omega C) V_z$$

$$\frac{d^2 I_z}{dz^2} = -(G + j\omega C)\frac{dV_z}{dz}$$

$$= \underbrace{(G + j\omega C)(R + j\omega L)}_{\gamma^2} I_z$$

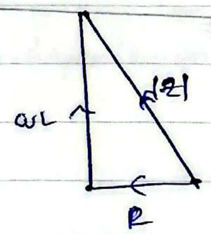
$$\frac{d^2 I_z}{dz^2} = \gamma^2 I_z$$

Similarly, $\dfrac{d^2 I_z}{dz^2} = \gamma^2 I_z$

$V_z \downarrow \; z \uparrow$

$I_z \downarrow \; z \uparrow$

V_z and I_z must dicreese with increse in z



at very high frequences,

$R <<< \omega L$

So, $Z = R + j\omega L$, $\because (R <<< \omega L)$

$Z \simeq j\omega L$

So, in $\gamma = \sqrt{(G + j\omega C)(R + j\omega L)}$

$\gamma = \sqrt{j^2\omega^2 CL} = \omega\sqrt{CL}$

if $\gamma = \alpha + j\beta$, when ω is very high.

$\gamma = j\beta$

but it will ~~be~~ always have small α

* at $V_z = Ae^{-\gamma z} + Be^{\gamma z}$

$A, B > 0$. and A, B constat (non-negative)

and one of A, B should be zero

* What ever the reason, we cannot neglect R because transmission line was made out of metel.

* Solution for $\dfrac{d^2 V_z}{dz^2} = \gamma^2 V_z$ will take the form $V_z = Ae^{-\gamma z}$

reason of this is, $V_z \downarrow$ $z \uparrow$

$I_z \downarrow$ $z \uparrow$.

$V_z = Ae^{-\gamma z} + Be^{\gamma z} \Rightarrow V_z = Ae^{-\gamma z}$ $(\because B = 0)$ පුවකලොය

② — $\dfrac{dV_z}{dz} = -(R + j\omega L) I_z$ $\dfrac{dV_z}{dz} = -\gamma A e^{-\gamma z}$ — ①

① = ②, $-(R + j\omega L) I_z = -\gamma A e^{-\gamma z}$

$I_z = \dfrac{\cancel{-}\gamma A e^{-\gamma z}}{\cancel{-}(R + j\omega L)} = \dfrac{\gamma A e^{-\gamma z}}{(R + j\omega L)}$

$$I_z = \frac{\gamma A e^{-\gamma z}}{(R+j\omega L)} = \frac{\sqrt{(R+j\omega L)(G+j\omega C)}\; A e^{-\gamma z}}{(R+j\omega L)}$$

$$I_z = \sqrt{\frac{G+j\omega C}{R+j\omega L}}\; A e^{-\gamma z}$$

$$V_z = A e^{-\gamma z}$$

- the Voltage of the Transmission line at position z, V_z

$$I_z = \sqrt{\frac{G+j\omega C}{R+j\omega L}}\; A e^{-\gamma z}$$

- The Current of the transmission line at position z, I_z

- The impedance of the transmission Line at the position z,

$$\frac{V_z}{I_z} = \frac{A e^{-\gamma z}}{\sqrt{\left(\frac{G+j\omega C}{R+j\omega L}\right)} \times A e^{-\gamma z}}$$

$$= \sqrt{\frac{\boxed{R}+j\omega \boxed{L}}{\boxed{G}+j\omega \boxed{C}}}$$

R, L, G, and C are constant for the given transmission line and ω is independent on the z

- So, the value of impedance is independendent of the position within the transmission Line.
- So we call it Carasteristic impedance of transmission Line and we have special symbol - Z_0

$$\boxed{\text{Carastoristic Impedance} = Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$

Only depends on physical parameters, and when frequency is high

$\omega \uparrow \quad R <<< \omega L$

$G <<< \omega C$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \simeq \sqrt{\frac{\omega L}{\omega C}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$ ← we mostly use this equation since, this is for High frequency.

ex:

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A transmission line has a per unit inductance of 400 nH/m and per unit capacitance of 70 pF/m. Determine the characteristic impedance of the transmission line at High frequencies

$R \simeq 0$ and $G \simeq 0$ at High frequencies,

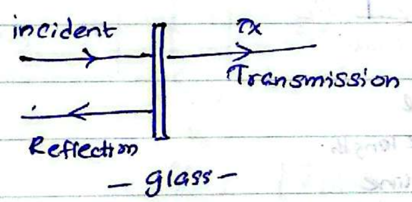
$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \simeq \sqrt{\frac{L}{C}} = \sqrt{\frac{400 \times 10^{-9}}{70 \times 10^{-12}}} = \sqrt{\frac{4}{7} 10^{4}} = 75.5\ \Omega$$

$\simeq 75\ \Omega$

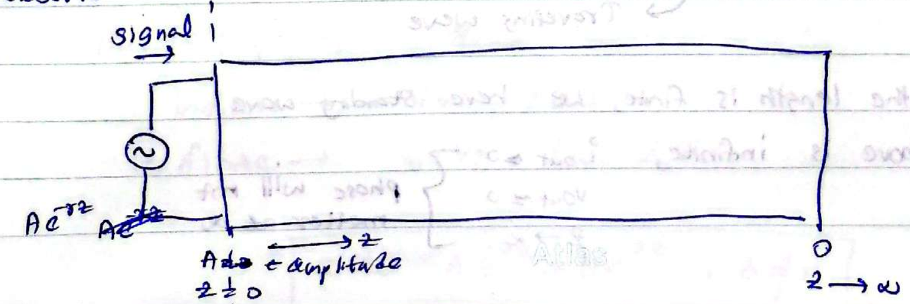
TV coaxial Cable ↗

Reflection

→ tipical of EM waves



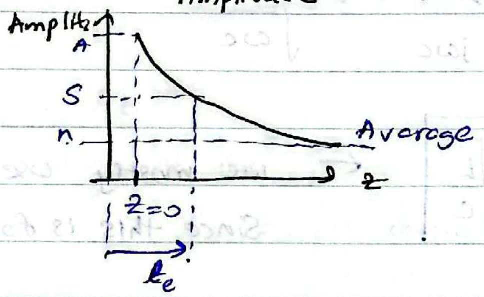
* Same thing happen for the electrical signal as well and we cannot directly observe



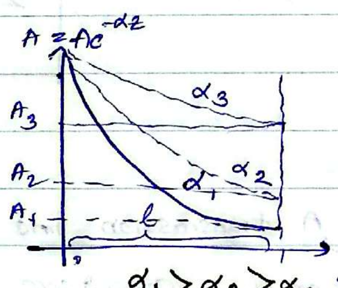
at the beginning amplitude is maximum and Amplitude will become zero when impedance become infinity.

$$\gamma = \alpha + j\beta$$

$$Ae^{-\gamma z} = \underbrace{Ae^{-\alpha z}}_{\text{Amplitude}}\ \underbrace{e^{-j\beta z}}_{\text{phase}}$$



* We have effective length of the cable ~~at the~~ with the average noise



$\alpha_1 > \alpha_2 > \alpha_3$

~~$A_3 > A$~~ $A_1 < A_2 < A_3$

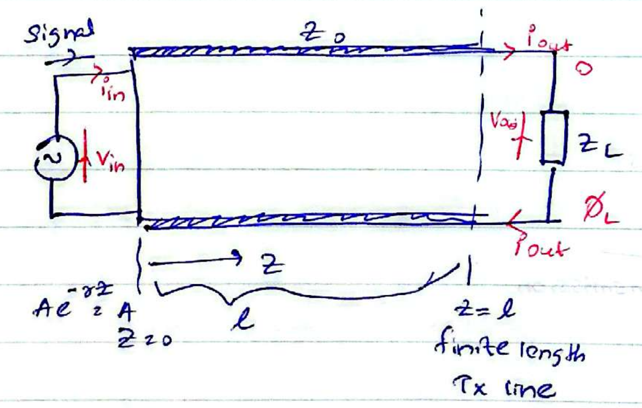
Signal to noice ratio $= \frac{S}{n}$

$$\frac{S}{n} = \frac{2n}{n} = 2$$

* Usually α is very low per meater, so for very ~~at~~ little affaction for transmission line, if α is very large effective length will be shorter

Loss of the transmission Line is highter for larger α

Lets get finite transmission ~~to~~ line,



$$Ae^{-\gamma z} + Be^{+\gamma z},$$

$$x(z) = Ae^{-\gamma z}$$ → Traveling wave

~~z~~ $x \downarrow$ $z \uparrow$

- When the length is finite, we here standing wave
- if above is infinite, $i_{out} \to 0$, $V_{out} \to 0$, $z \to \infty$ } phase will not matter at ∞

at the finite length,

Same i_{out} will enter again in to Transmission Line and phase shift can be happen.

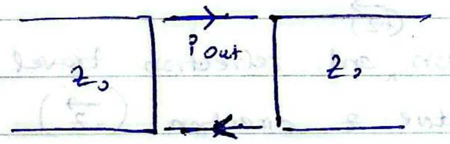
When transmission line finite in length and terminated by a load of impedance Z_L, where,

$$Z_L = |Z_L| \angle \phi_L$$

- Therefore, the current through the load will have a magnitude as well as phase ϕ_i,
- Fed back in to the Tx line, (Transmission line)

[illegible]

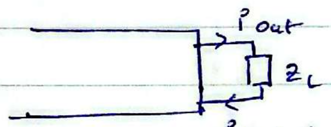
<u>Small</u> <u>section</u> of a transmission line,



If this were an infinite Tx line There is no phase change and if it is a finite Tx line, there is ~~no~~ phase change

$Z_0 \rightarrow \angle 0$

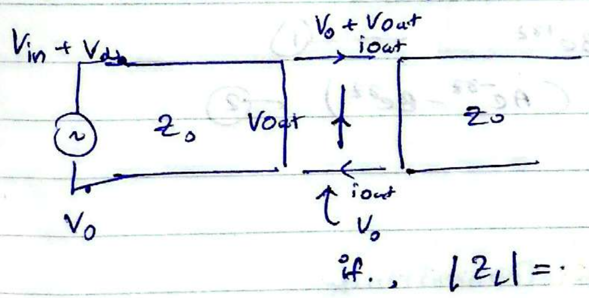
$Z_L \rightarrow \angle \phi_L$



z i_{out} + phase chare

and this is called effect of standing weve

- If the current is out, there is a voltage out as well,



if., $|Z_L| = Z_0$

$V_{out} = V_L + V_i$

$V_{out} + V_0 = V_L + V_i + V_0$

But $|Z_L| \neq Z_0$

infinite $\rightarrow$ V_0 i_{out}

finite $\rightarrow$ $V_0 + V_i$ $i_{out} \angle \phi_i$

$$\boxed{x(z) = Ae^{-\gamma z} + Be^{\gamma z}, \quad B \neq 0}$$

if $z\uparrow$ x has to decres. ($z\uparrow$ $x\downarrow$)

when $z\uparrow$ and $x\downarrow$.

$Ae^{-\gamma z}$ will decrese and $Be^{\gamma z}$ will increse and for this there is a solution,

V_{out} V_i

I_{out} $I_{out} \angle \phi_i$

$\boxed{\text{incident Signal}}$ $\boxed{\text{reflected signal}}$

$$x(z) = \underbrace{Ae^{-\gamma z}}_{\text{incident}} + \underbrace{Be^{\gamma z}}_{\text{reflection.}}$$

when $z \uparrow$, $Ae^{-\gamma z}\downarrow$

$z\uparrow$, $Be^{\gamma z}\uparrow$ ×

incident travel to positive z direction $(\overrightarrow{+z})$ and, reflection travel from Tx to load so its in negative z direction. $(\overrightarrow{-z})$

So, if we replace z with negative $\mathcal{Z}$,

$$x(z) = Ae^{-\gamma z} + Be^{\gamma \boxed{z}}$$

$$z = -\mathcal{Z}$$

$$x(z) = Ae^{-\gamma z} + Be^{-\gamma \mathcal{Z}}$$

so the both terms will decrese when z inerese

Solution for the finite Transmission Line

$$V_z = Ae^{-\gamma z} + Be^{+\gamma z} \quad \text{—— (1)}$$

$$I_z = \frac{\gamma}{(R + j\omega L)} (Ae^{-\gamma z} - Be^{\gamma z}) \quad \text{—— (2)}$$

The solution for the infinite Transmission line

$$V_z = Ae^{-\gamma z} \quad \text{—— (3)}$$

$$I_z = \frac{\gamma Ae^{-\gamma z}}{(R + j\omega L)} \quad \text{—— (4)}$$

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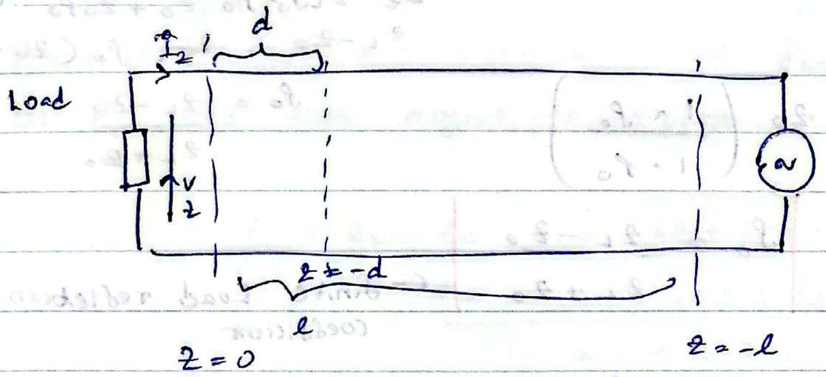


$$\gamma = \sqrt{R + j\omega L}\sqrt{G + j\omega C}$$

$$\frac{\gamma}{R + j\omega L} = \frac{\sqrt{(R + j\omega L)(G + j\omega C)}}{(R + j\omega L)}$$

$$= \sqrt{\frac{G + j\omega C}{R + j\omega L}}$$

$$= \frac{1}{Z_0}$$

from (2),

$$I_z = \frac{Ae^{-\gamma z}}{Z_0} - \frac{Be^{\gamma z}}{Z_0} \quad \text{--- (2)}'$$

$$V_z = Ae^{-\gamma z} + Be^{\gamma z} \quad \text{--- (1)}$$

$$I_z = \frac{Ae^{-\gamma z}}{Z_0} - \frac{Be^{\gamma z}}{Z_0} \quad \text{--- (2)}'$$

considering the load,

$$\left.\frac{V_z}{I_z}\right|_{z=0} = Z_L = \frac{Ae^{0} + Be^{0}}{\frac{Ae^{0}}{Z_0} - \frac{Be^{0}}{Z_0}} \qquad \because e^0 = 1$$

$$= \frac{A + B}{\frac{A}{Z_0} - \frac{B}{Z_0}} = \frac{Z_0(A+B)}{(A-B)}$$

$$\left.\frac{V_z}{I_z}\right|_{z=0} = Z_L = Z_0\left[\frac{A+B}{A-B}\right]$$

$$\rho_0 = \left.\frac{B}{A}\right|_{z=0}$$ ← Amplitude of the reflects wave.
← Amplitude of the incident wave

$B = \rho_0 A$

$$Z_L = Z_0\left[\frac{A + \rho_0 A}{A - \rho_0 A}\right] = \boxed{Z_0\left(\frac{1+\rho_0}{1-\rho_0}\right) = Z_L = \frac{V_z}{I_z}}$$

$Z_L(1-\rho_0) = Z_0(1+\rho_0)$

$Z_L - Z_L\rho_0 = Z_0 + Z_0\rho_0$

$Z_L - Z_0 = \rho_0(Z_0 + Z_L)$

$\rho_0 = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

No :

$$Z_L = Z_0\left(\frac{1+\rho_0}{1-\rho_0}\right)$$

so,
$$\boxed{\rho_0 = \frac{Z_L - Z_0}{Z_L + Z_0}}$$ ← finite Load reflection coefficient

When $Z_L = Z_0$, we called it as matched condition where no reflection $(B=0)$ so,

$\rho_0 = 0$

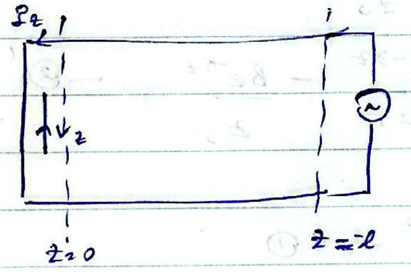
Load can be,

→ finite

→ open Circuit

→ short Circuit.

for short circuit,



, $Z_L = 0$

$\rho_0 = \dfrac{0 - Z_0}{0 + Z_0}$

$\rho = -1$

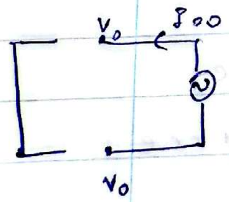
In here, no voltage drop and reflection Amplitude will be equal to incident Amplitude and just only 180° phase shift for the coefficient

$\rho_0 = -1 \rightarrow \angle\pi$

$B = A$

for open circuit,

$$\rho_0 = \frac{1 - \frac{Z_0}{Z_L} \rightarrow 0}{1 + \frac{Z_0}{Z_L} \rightarrow 0} \qquad Z_L \rightarrow \infty$$



$\rho_0 = 1$

in open circuit, there is no current flow and source will induce same voltage over the points we consider

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ex: $Z_0 = 75\,\Omega$, $Z_L = 60 + j20\,\Omega$ and find the reflection coefficient of ρ_0, and both magnitude and phase.

$$\rho_0 = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60\Omega + j20\Omega - 75\Omega}{60\Omega + j20\Omega + 75\Omega}$$

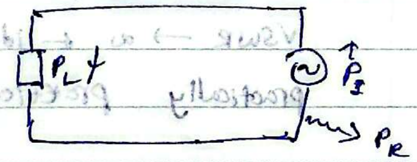
$$= \frac{-15\,\Omega + j20\,\Omega}{135 + j20\,\Omega}$$

$$= -0.087 + j0.16\,\Omega$$

$$= 0.178\,\angle 118°\,\Omega$$

(Power) $P \propto V^2$

reflected power → $P \propto |\rho_0|^2$



$\rho_0 = \frac{B}{A}$ B, A are amplitudes of voltage signals

$$P \propto V^2 \propto B^2, A^2$$

$$P \propto |\rho_0|^2$$

$$P_I = P_L + P_R$$

Insident powes ↑ — P_I; Powes dissipated by the load ↑ — P_L; ← reflected power — P_R

$$P_R = (\rho_0)^2 P_I$$

* Reflected Power (P_R) dissipated by source, it will be wasted. So the usefull power is the one taken by the load. So the

Usfull power (P_L) = $(1 - |\rho_0|^2) P_I$

Usefull power of previous example;

$$P_L = (1 - |\rho_0|^2) P_I$$

$$\frac{P_L}{P_I} = 1 - |(-0.087 + j0.16\,\Omega)|^2$$

$$= 1 - |0.087)^2 \ \%$$

Usful power precentage $= \frac{P_L}{P_I} = 96.8\%$, wasted presentage $= 3.2\%$

No : Date: / /

Voltage Standing Wave Ratio (VSWR)

* for a given load to achive minimum power wastage $|\rho_0| \downarrow$

$$VSWR = \frac{1 + |\rho_0|}{1 - |\rho_0|}$$

→ can be measured and from this we can find $|\rho|$

- Best VSWR value $= 1$ $(|\rho_0| = 0)$
- if we have open circuit or short circuit $|\rho_0| = 0$ and VSWR $\to \infty$ ← ideally

practically very large readings for VSWR

Attenuation of a Transmission Line

* The losses caused by the resistance and leakage of the Transmission line (R, G)

$$\gamma = \alpha + j\beta$$

$$V_z = Ae^{-\gamma z}$$ ← assuming it is either matched or infinite so $B = 0$

$$V_z = Ae^{-(\alpha + j\beta)z}$$

$$= Ae^{-\alpha z} e^{-j\beta z}$$

$$|V_z| = Ae^{-\alpha z}$$

$$\boxed{\alpha \approx \frac{R}{2Z_0} + \frac{GZ_0}{2}}$$

Exercise, $L = 190$ nH/m, $C = 75$ pF/m, $R = 2.6$ mΩ/m, $G = 67$ S/m Find Z_0 and α

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{190 \times 10^{-9}}{75 \times 10^{-12}}} = \sqrt{\frac{190 \times 10^{3}}{75}} = 50.3\,\Omega$$

$$\alpha = \frac{R}{2Z_0} + \frac{GZ_0}{2} = \frac{2.6 \times 10^{-3}}{2 \times 50.3} + \frac{67 \times 50.3}{2}$$

$$= 1675\ \text{m}^{-1}$$

No: coaxial cables, we can start from either (A) or (B) equation

Date: 2025/04/23

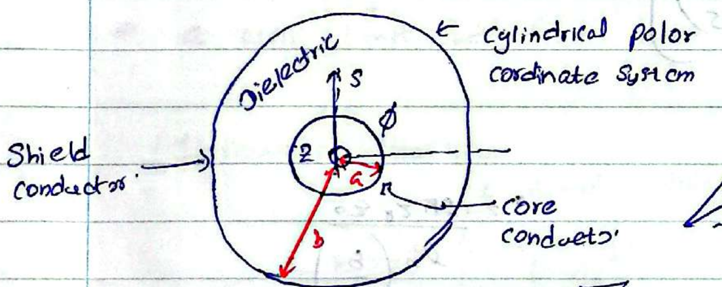
Transmission Lines used at microwave frequency

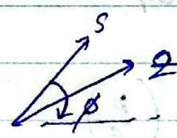
1. coaxial transmission lines
2. Substrate transmission line — embeded on ~~transmission~~ the substrate of the P.C.B.
   - ~~Sol~~. Slotlines
   - Microstrips
   - Striplines

* ~~Analog~~ Many adoptations necessary for high frequencies

* for Analog signals → All Voltage levels

Coaxial Cable





Per unit inductance and capacitance,

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$C = \frac{2\pi \varepsilon_r \varepsilon_0}{\ln(b/a)}$$

μ_0 = permeability of a vaccume

ε_r = relative permitivity

ε_0 = permitivity of a vaccume

$$Z_0 = \sqrt{\frac{L}{C}}$$

b - innermost radius of the outer conductor

a - radius of the inner conductor

$$Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} \ln\left(\frac{b}{a}\right) \quad \text{(A)}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$ - intrinsic impedance of free space

$$\eta_0 = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} \approx 120\pi$$

So from (A)

$$Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{b}{a}\right) \quad \text{(B)}$$

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ex) A coaxial transmission line has core and shielding radii of 0.5mm and 2.3 mm respectively it has a Characteristic impedance of 50 Ω

1) find the required value of ε_r
2) Find the per unit L and C

1)
$$Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{b}{a}\right)$$

$$50 = \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{2.3}{0.5}\right)$$

$$\sqrt{\varepsilon_r} = \frac{60}{50} \ln\left(\frac{2.3}{0.5}\right)$$

$$\varepsilon_r = \left[\frac{6}{5} \ln\left(\frac{2.3}{0.5}\right)\right]^2$$

$$\varepsilon_r = 3.354$$

2)
$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \ln\left(\frac{2.3}{0.5}\right)$$

$$= 3.052 \times 10^{-7} \ H/m$$

$$\simeq 305 \ nH/m$$

$$C = \frac{2\pi \varepsilon_r \varepsilon_0}{\ln\left(\frac{b}{a}\right)}$$

$$= \frac{2\pi \times 3.354 \times 8.85 \times 10^{-12}}{\ln\left(\frac{2.3}{0.5}\right)}$$

$$= 1.222 \times 10^{-10} \ F/m$$

$$= 122 \ pF/m$$

---

u = Velocity of the Signal

u = U_p - phase Velocity

U_p Should be less than c

$$U_p = \frac{1}{\sqrt{LC}} < c \quad \leftarrow \text{speed of light}$$

$$U_p = \frac{1}{\sqrt{305 \times 10^{-9} \times 122 \times 10^{-12}}}$$

$$= 1.6 \times 10^8 \ m/s$$

So, $U_p \cong 1.6 \times 10^8 < c$

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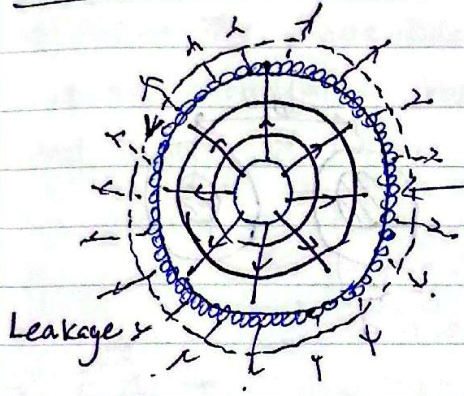
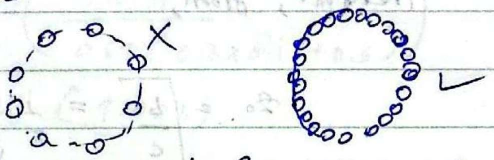
Coaxial Cable fields



- Core is a copper core

wire mesh provide flexibility for conductor

to minimize leakage from the mesh of the outer conductor has to be tightly spaced.



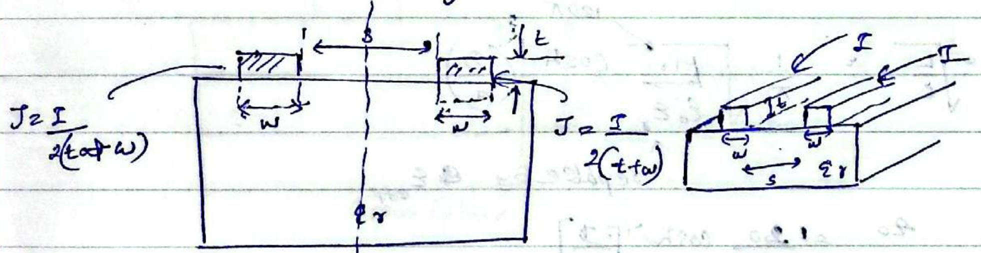
* N connector and SMA connector was used for microwave frequencies will limit the value of b

So there is a maximum b that we cannot go beyond that

* a will limit from inner strength of the conductor

Microwave Slotline

i.e symetric transmission line.



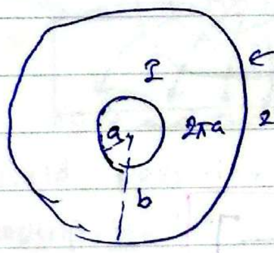
separation between 2 conductors - s

thickness of conductor strip - t

width of each conductor line - w

relative permittivity - ε_r

for cable



$J_a = \frac{I}{2\pi a}$

$J_b = \frac{I}{2\pi b}$

$J_a \neq J_b$

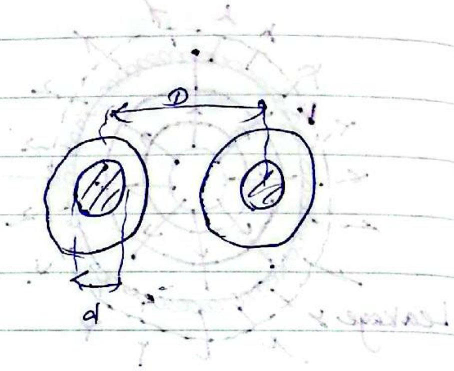
this is called a symetric T.L.

skin depth $J = \frac{I}{2tw}$

Twin wire Model

per unit inductance and capacitance,

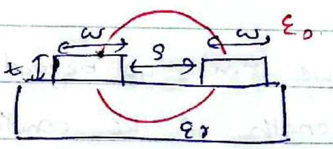
$$L = \frac{\mu_0}{\pi}\cosh^{-1}\left(\frac{D}{d}\right)$$

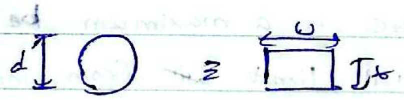


$$C = \frac{\cancel{\pi\varepsilon}\,\varepsilon_r\varepsilon_0}{\cosh^{-1}\left(\frac{D}{d}\right)}$$

Therefore, from,

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{\pi}\sqrt{\frac{\mu_0}{\varepsilon_r\varepsilon_0}}\cosh^{-1}\left(\frac{D}{d}\right)$$





$$\varepsilon_{eff} = \frac{1}{2}(1+\varepsilon_r)$$

effective relative permittivity approximation →

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{\pi}\sqrt{\frac{\mu_0}{\varepsilon_0'\varepsilon_r}}\cosh^{-1}\left(\frac{D}{d}\right)$$

120π

replace by ε_{eff}

$$Z_0 = \frac{120}{\sqrt{\varepsilon_{eff}}}\cosh^{-1}\left[\frac{D}{d}\right]$$

$$Z_0 = \frac{120}{\sqrt{\frac{1}{2}(1+\varepsilon_r)}}\cosh^{-1}\left(\frac{D}{d}\right)$$

$D = S + w$

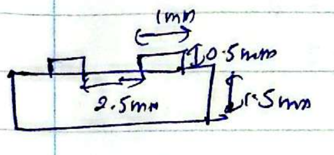
$D = S + 2 \times \frac{w}{2}$

$$= \frac{120\sqrt{2}}{\sqrt{1+\varepsilon_r}}\cosh^{-1}\left(\frac{D}{d}\right)$$

$$= \frac{120\sqrt{2}}{\sqrt{1+\varepsilon_r}}\cosh^{-1}\left(\frac{D}{d}\right)$$

$$\boxed{Z_0 = \frac{120\sqrt{2}}{\sqrt{1+\varepsilon_r}}\cosh^{-1}\left[\frac{\cancel{D}\; S+w}{0.67(0.8w+t)}\right]}$$

Ex: The substrate of a PCB has a relative permittivity of 4.2 and thickness of 1.5 mm. The single sided layer of copper has a thickness of 0.5 mm. Find the characteristic impedance of a ~~sd~~ slotline with a width of 1mm and spacing of 2.5 mm



$$Z_0 = \frac{120\sqrt{2}}{\sqrt{1+\varepsilon_r}} \cosh^{-1}\left(\frac{s+w}{0.67(0.8w+t)}\right)$$

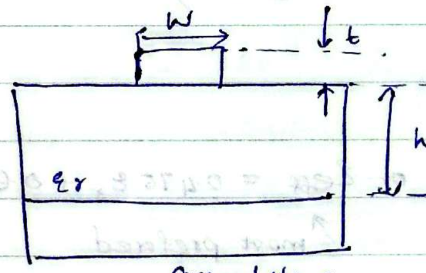
$$= \frac{120\sqrt{2}}{\sqrt{1+4.2}} \cosh^{-1}\left(\frac{2.5+1}{0.67(0.8\times 0.1+0.5)}\right)$$

$$= 74.421 \cosh^{-1}(9.006)$$

$$\approx 153.9\,\Omega \simeq 154$$

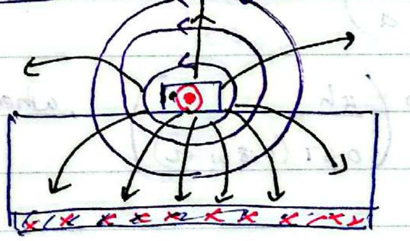
$$= 150\,\Omega$$

Microstrip Transmission line



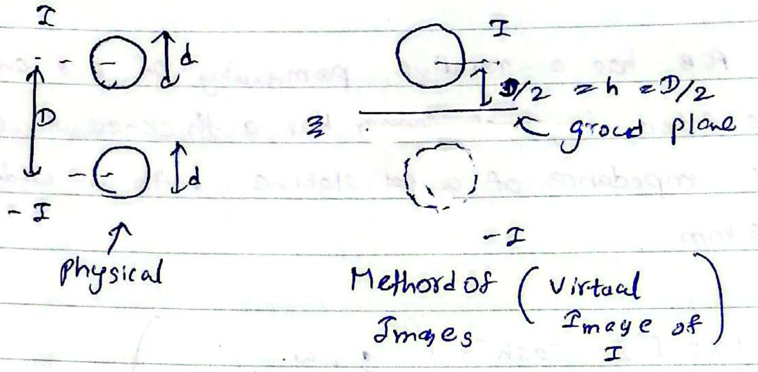
Separation between 2 conductors - h

Microstrip field - A symmetric transmission line



- No field pass from Ground plane
- Microstrip has less leackage compaired to a slotline

- This mode is known as a quasi TEM Mode
  (A small longitudinal E field exists but it is very small compared to the transverse fields.)



has the same Z_0 as the twin wire

$$Z_0 = \frac{1}{\pi}\sqrt{\frac{\mu_0}{\varepsilon_{eff}\,\varepsilon_0}}\cosh^{-1}\left(\frac{D}{d}\right)$$

$$= \frac{120\cancel{\pi}}{\cancel{\pi}\sqrt{\varepsilon_{eff}}}\cosh^{-1}\left(\frac{2h}{d}\right)$$

(1) → $$Z_0 = \frac{120}{\sqrt{\varepsilon_{eff}}}\cosh^{-1}\left(\frac{2h}{d}\right)$$

(A) → * $d = 0.67\,(0.8w + t)$

(B) → * $\varepsilon_{eff} = \frac{1}{2}(1 + \varepsilon_r)$ or $\varepsilon_{eff} = 0.475\,\varepsilon_r + 0.67$

↑ most prefered

we need to apply (A), (B) to (1)

[illegible]

* if the copper thickness is less than substrate thickness

we get $Z_0 = \frac{120}{\sqrt{\varepsilon_{eff}}}\ln\left(\frac{4h}{d}\right)$

narrow microstrips

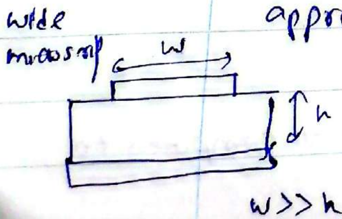
$$Z_0 = \frac{120}{\sqrt{\varepsilon_{eff}}}\ln\left(\frac{4h}{0.67\,(0.8w + t)}\right), \text{ when } 0.8w < h$$

$0.8w < h$

$\boxed{0.5\,w < h}$ ← Best

* for wide microstrips ($w \gg h$), the Assadourian and Rimai approximation can be used,

wide microstrip



$$Z_0 \simeq \frac{120\pi}{\sqrt{\varepsilon_r}}\,\frac{h}{w}$$

$w \gg h$

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ex: $t = 0.5$ mm, $\varepsilon_r = 4.2$, $h = 1.5$ mm

$Z_0 = 50$

$$W = \frac{120\pi}{\sqrt{\varepsilon_r}}\left(\frac{h}{Z_0}\right)$$

$$= \frac{120\pi}{\sqrt{4.2}}\left(\frac{1.5}{50}\right)$$

$W = 5.518$ mm $>$ $h = 1.5$ mm ✓

feasible (wide)

$Z_0 = 100$

$$W = \frac{120\pi}{\sqrt{\varepsilon_r}}\left(\frac{h}{Z_0}\right)$$

$$= \frac{120\pi}{\sqrt{4.2}}\left(\frac{1.5}{100}\right)$$

$W = \cancel{2.7\text{ mm}}$ 2.75 mm $> h = 1.5$ m ✓

wide fiesible)

$Z_0 = 300$ ~~0.92 mm~~

$$W = \frac{120\pi}{\sqrt{\varepsilon_r}}\left(\frac{h}{Z_0}\right)$$

$$= \frac{120\pi}{\sqrt{4.2}}\left(\frac{1.5}{300}\right)$$

$W = 0.92$ mm $<$ $h = 1.5$ mm ✗. we have to go for narrow approximation.

So, $$Z_0 = \frac{120}{\sqrt{\varepsilon_{eff}}}\ln\left(\frac{4h}{d}\right)$$

$$= \frac{120}{\sqrt{0.475\,\varepsilon_r + 0.67}}\ln\left[\frac{4h}{0.67\,W\left(0.8 + \frac{t}{W}\right)}\right]$$

$$\ln\left(\frac{4 \times 1.5}{0.67 \times 0.8\,W + 0.67 \times 0.5}\right) = \frac{300 \times \sqrt{0.475 \times 4.2 + 0.67}}{120}$$

$$\ln\left(\frac{6}{0.536W + 0.335}\right) = 4.08$$

$$e^{4.08} = \frac{6}{0.536W + 0.335}$$

$$0.536W = \frac{6}{e^{4.08}} - 0.335$$

$W = -0.435$

for this minus values were incorres there is some propblem with equation.

from every eqation it will give negative value.
So, 300 Ω is anyway infeasible

No :

Date: 02 / 05 / 2025

Example for Microwave Slotline

Ex: 8 The sustrate of a printed circuit board has a relative permittivity (ε_r) of 4.2 and thickness of 1.5 mm. The one sided copper has a thickness of 0.5 mm. Find the characteristic impedance of a slotline with a width of 1 mm and spacing of 2.5 mm

$\varepsilon_r = 4.2$

$h = 1.5\,mm \quad t = 0.5\,mm \quad w = 1\,mm \quad S = 2.5\,mm$

$$Z_0 = \sqrt{\frac{L}{C}} = \cancel{\frac{1}{\pi}\sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} \cosh^{-1} \& D} = \frac{1}{\pi}\sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} \cosh$$

$$= \frac{120}{\sqrt{\varepsilon_{eff}}} \cosh^{-1}\left(\frac{D}{d}\right)$$

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2}$$

$$D = S + 2\left(\frac{w}{2}\right)$$

$$= 2.5 + 2\left(\frac{1}{2}\right) = 3.5\,mm$$

$$= \frac{120\sqrt{2}}{\sqrt{1+\varepsilon_r}} \cosh^{-1}\left[\frac{S + D}{0.67(0.8w + t)}\right]$$

$$= \frac{120\sqrt{2}}{\sqrt{1+4.2}} \cosh^{-1}\left[\frac{2.5 + 1}{0.67(0.8 \times 1 + 0.5)}\right]$$

$$= 74.42 \times \cosh^{-1}(4.018)$$

$$Z_0 = 153.90\ \Omega$$

$$\simeq 150\ \Omega$$

Ex. 9 A printed circuit board has a relative permittivity (ε_r) of 4 and thickness of 1.4 mm which excludes the 0.6 mm copper layer on either side. Calculate the carastoristic impedance of the rel microstrip transmission line if the width is,

1) a narrow 0.8 mm $\longrightarrow$ 120 Ω

2) a wide 3.4 mm $\longrightarrow$ 75 Ω

$\varepsilon_r = 4.3 \quad h = 1.4 \quad t = 0.6$

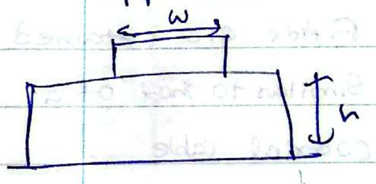
1) $$Z_0 = \frac{120\sqrt{2}}{\sqrt{1+\varepsilon_r}} \cosh^{-1}\left[S + \right.$$

$$w = \frac{0.8}{1.4} = 0.57 \simeq 0.6 \quad \cancel{\ge 2}$$

$$\cancel{\frac{w}{h} = 75\ \Omega}$$

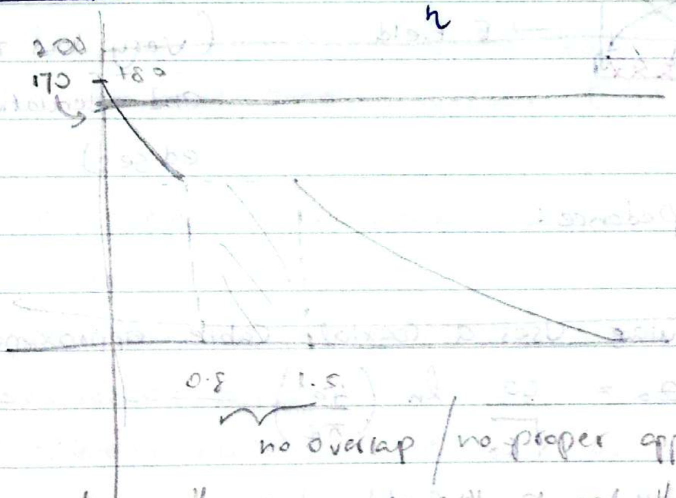
2). $$\frac{w}{h} > \frac{3.4}{1.4}$$

Ex ⑩ The printed circuit board of a microwave circuit has a thickness (h) of 2.5 mm, copper thickness (t) of 0.7 mm and relative permittivity (ε_r) of 4.2 . The circuit requires a microstrip transmission line to connect a power amplifier output to a printed antenna. The required impedance of the transmission line is 170 Ω. Using a suitable ~~approxim~~ approximation find the required width (ω) of the microstrip.



$$\left.\begin{array}{l} \frac{w}{h} < 0.8 \rightarrow \text{narrow} \\ 5 > \frac{w}{h} > 1.5 \rightarrow \text{wide} \end{array}\right\} \text{programe}$$

from this range There is no overlap and no proper approximation



If we change the range, we will not get the unique answer.
no intersection with wide approximation.

if 200 Ω, there is no intersection i.e. the in narrow and wide so 200 Ω is infeasable

3 types → 1) Slotline
2) Microstrip
3) Stripline

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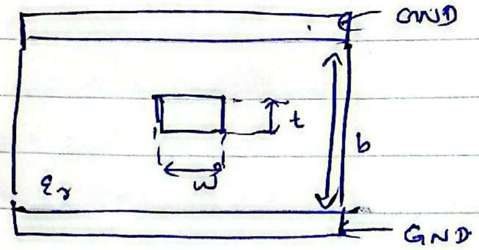
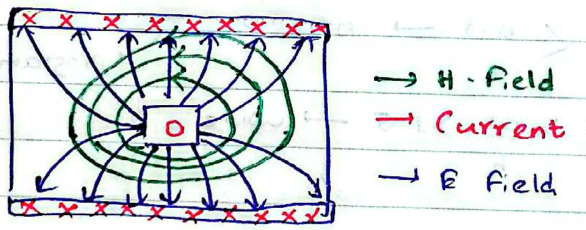
Microwave Striplines



* There are 2 ground plates

* A stripline has a conducting strip within substrate with relative permittivity ε_r enclosed by two ground planes
* benfit of having 2 ground plates: is the fact that it has minimal field leackage. becouse of that it has a very low attenuation.
* This is very similer to coaxial cable (in term of it struct.

Striplines Fields



* Fields are contained similar to that of a coaxial cable (very low radiation and attenuation - from edges).

Charasteristic Impedance:

~~Ch~~

Cohn's formulas uses a coaxial cable approximation

$$Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{4b}{\pi d}\right) \leftarrow \text{obserlete now}$$

(ප්‍රමාණවත් නැත)

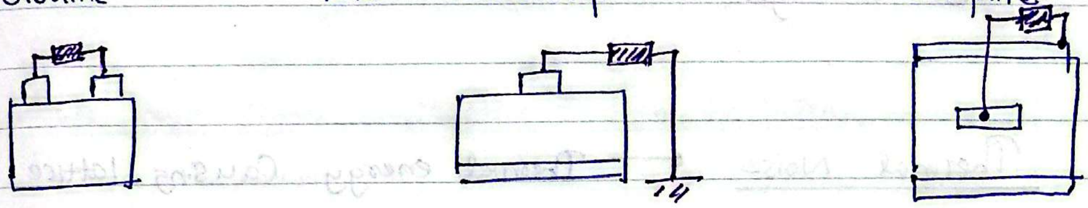
Where d. is the springfield diameater, subjected to the constrains.

$$\omega \leq 0.35\,(b-t) \quad \text{and} \quad 4t \leq b.$$

This is obserlete becouse ~~of~~, very hard to calculete Z_0 in this type of transmission line. So now adays microstrips are the most prefered.

Comparison between 3 types.

Slotline	Microstrip	Stripline



	Slotline	Microstrip	Stripline
	need Single layer PCB	need double layer PCB	need Multi layer PCB
1) Connecting components:	Most conveneant (becose its the matter of conneting 2 lines)	less convenient (we have to make sure that one line is conneted to microstrip and other must connect through PCB to other side ground in PCB)	least conveneat. (approximation is very inaccurate)
2) loss:	Highest loss	Moderate loss	least loss
3) Symmetry	Symmetric	Asymmetric	Asymmetric

Date 2025/05/07.

Noise in Microwave Systems

→ Basis of Microwave System design

→ Radar and satelite system. (hand held in present)

* Noise is present in all communication systems (We cannot get rid of them practically)
* Due to attenuation the receiver signal is significantly weaker than the transmitted signal
  → Effect of noise will be significant
* There are two main types of noise that affect communication system
  1) Thermal (aka Johnson-Nyquist) noise (Gaussian) due to the thermal excitation of the conductor lattice and electrons.
  2) Shot noise (Poisson) due to discrete electron arrivals

~~At~~

No : Date: / /

* In a free space communication link free space loss is very. So, when recive signal is weeker and weeker affect of noise is larger

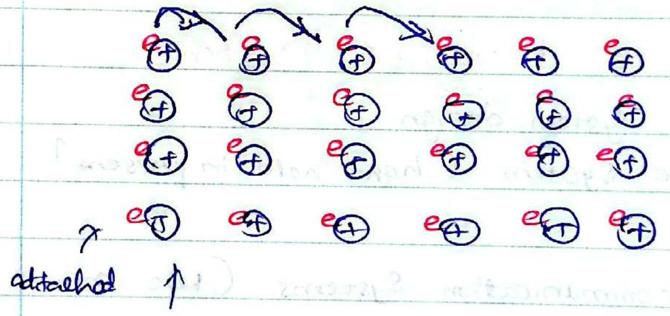
Thermal Noise ⟵ Thermal energy causing lattice vibra

* At absolute zero, all ~~et~~ atoms and electrons are stationary.
  → Electrons can move if excited by a potental difference

* Above absolute zero,
  → Lattice attoms begin to vibrate
  → Electrons have highter mobility

* These phenomena interfere with the propagating signal.

Metal lattice at absolute zero,



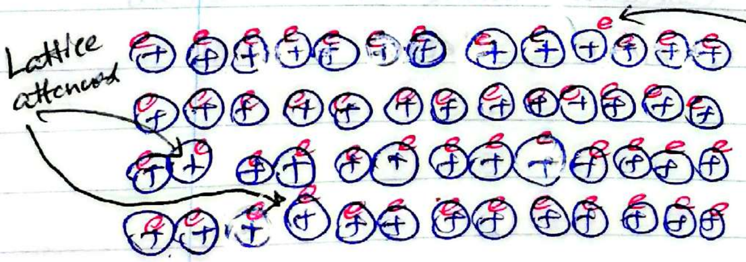
— highly unform manr and its supperconduc

⊕ - nucleas ⟵ stantionary at 0 K

all of this is lost when, the temperature becomes trastion 0 K, T > 0 K ⟵ absolute zero

So at this time we have, ① Lattice bibration (there are some displacements)

② Excited Electrons (when recere the energy, electro move to higher or from ground)

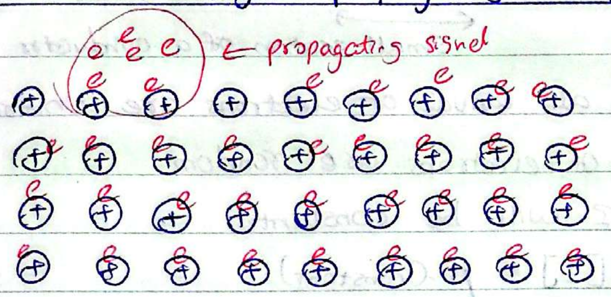


- When there is current flow it will become highly disorganized

If we take thermal Noise as "N";

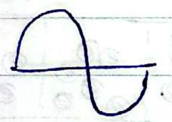
$N \propto T$ (absolute Temparature)

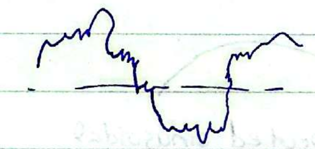
When we have Signal propageting through the lattice,



* at high freqancies ⟶ electrons propegate using simple harmonic Moto
* when displaced the attoms
* So, Dsount the flow of electron.

Propayating throgh conductor



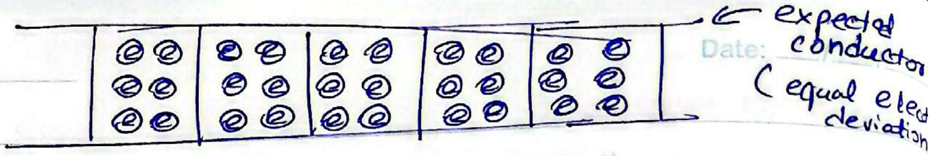


Shot Noise

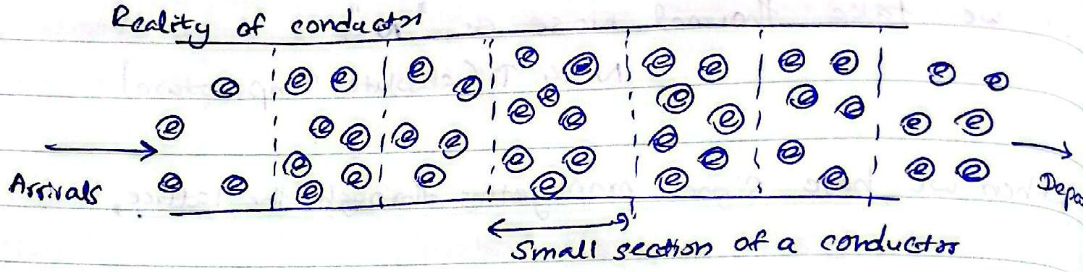
* Electrons are discrete pertkals
* When the current is Small and frequncy is high the discrete nature of a current becomes prominant.
  → Electrons arrival and depature will ~~flow~~ follow a poison distribu
* The number of electrons in a small section of the conductor will vary

Poison distribution ⟶ number of arrivals
(↑ Include Mean(μ) & Standard deriation(σ))
↳ What we have here is the randomness in no. of arrivals

Current I will be allways Average Current.



Reality of conductor

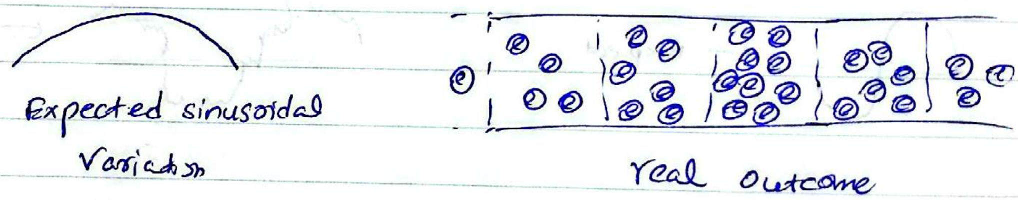


* We are assuming that all arival of electrons are Uniform
* But In reality arrival of electrons are random
* Expected Value of I will be constant.

$$E[I] = \rho \text{ (constant)}$$

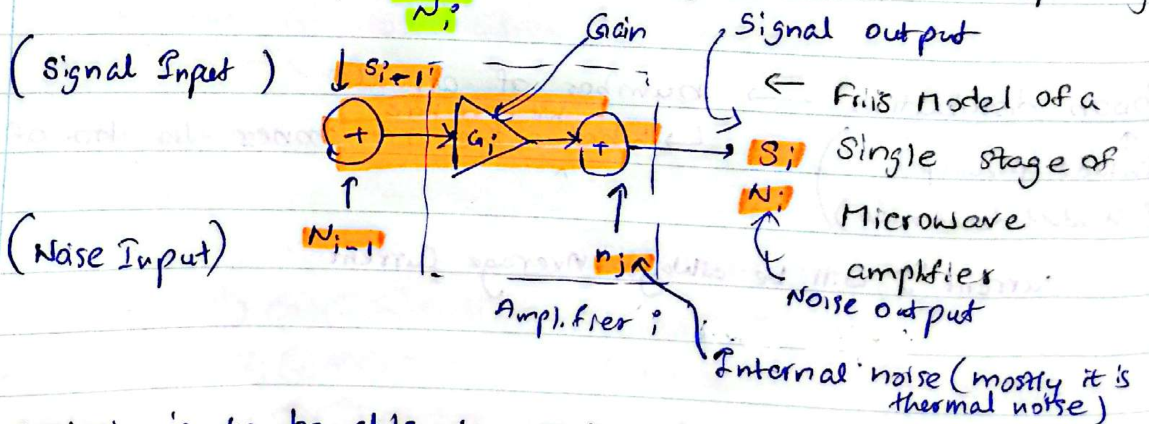
Modulation Shot Noise

* The modulated signal will 'degrade' due to shot Noise.
* If we have sinusoidal signal, we expect to see sinusoid variation. but we will recive different



Friis Noise formula

- use for analysis and design
- Quantifies performance of amplifier based upon the Signal to noise ratio.
- i.e, $SNR_i = \frac{S_i}{N_i}$ ← usually defines reciver capability



← Friis Model of a Single stage of Microwave amplifier.

anlysis is to be able to predict behavior and for that we have to design systems.

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o/p i/p

① — $S_i = G_i(S_{i-1})$
② — $N_i = G_i(N_{i-1}) + n_i$ } Any amplifier will add noise to the system

So, always
output → $SNR_i < SNR_{i-1}$ ← Input

* SNR will degrade when amplified, The reason is internal Noise (n_i)

Noise Factor (multiplier) (F_i) & Noise figure (dB) ← official IEEE unit to discribe the preformance of Amplifier

In data sheets we'll find the noise figure rather than Noise factor

$$F_i \text{ (Noise factor)} = \frac{S_{i-1}/N_{i-1}}{S_i/N_i} = \frac{SNR \text{ input}}{SNR \text{ output}}$$

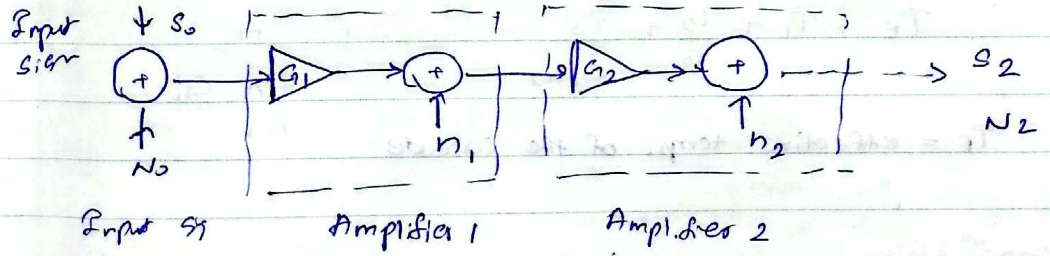
Since $SNR_{i-1} > SNR_i \rightarrow \boxed{F_i > 1}$ $F_i > 1$ (This is must)

from ① & ②,

$$F_i = \frac{S_{i-1}/N_{i-1}}{S_i/N_i} = \frac{S_{i-1}}{N_{i-1}} \times \frac{N_i}{S_i} = \frac{\cancel{S_{i-1}}}{N_{i-1}} \times \frac{G_i N_{i-1} + n_i}{G_i \cancel{S_{i-1}}}$$

$$F_i = 1 + \frac{n_i}{G_i N_{i-1}}$$

Ex: Determine the noise factor of ~~a~~ two stage cascade



We start with stage 2, $S_2 = G_1 S_1 = G_1[G_1 S_0] = G_2 G_1 S_0$

$N_2 = G_2 N_1 + n_2 = G_2[G_1 N_0 + n_1] + n_2 = G_2 G_1 N_0 + G_2 n_1 + n_2$

$N_1 = G_1 N_0 + n_1$

$$F_T = \frac{S_0/N_0}{S_2/N_2} = \frac{S_0}{N_0} \times \frac{N_2}{S_2} = \frac{S_0}{N_0} \times \frac{G_1 G_2 N_0 + G_2 n_1 + n_2}{G_2 S_1 S_0}$$

$$F_T = 1 + \frac{n_1}{G_1 N_0} + \frac{n_2}{G_2 G_1 N_0}$$ //

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$$F_T = 1 + \frac{n_1}{G_1 N_0} + \frac{n_2}{G_2 G_1 N_0} + \frac{n_3}{G_3 G_2 G_1 N_0} + \dots + \frac{n_M}{G_M G_{M-1} \dots G_1 N_0}$$

$$F_T = \frac{n_M}{\left(\prod_{i=1}^{n} G_i\right) N_0}$$

Σ - + sums

π - $\times$ products

$$F_i = 1 + \frac{n_i}{G_i N_{i-1}}$$

$$F_1 = 1 + \frac{n_1}{G_1 N_0} \qquad F_2 = 1 + \frac{n_2}{G_2 N_1}$$

$$F_2 - 1 = \frac{n_2}{G_2 N_1}$$

$$F_T = \underbrace{1 + \frac{n_1}{G_1 N_0}}_{F_1} + \underbrace{\frac{n_2}{G_2 G_1 N_0}}_{\frac{1}{G_1}(F_2 - 1)} + \underbrace{\frac{n_3}{G_3 G_2 G_1 N_0}}_{\frac{1}{G_1 G_2}(F_3 - 1)} + \dots \frac{(F_m - 1)}{\prod_{i=1}^{m-1} G_i}$$

* Alternative noise tem. formula

$$\boxed{F_i = 1 + \frac{T_i}{T_0}}$$

T_0 = ambient temperature

T_i = equivalent noise temp. of the i

$$F_1 = 1 + \frac{T_1}{T_0}$$

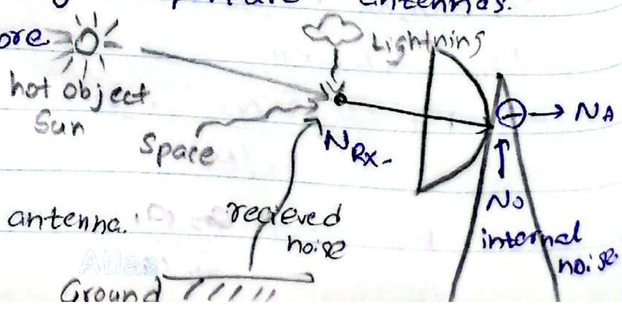
* Entire cascade,

$$T_E = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \dots + \frac{T_M}{\prod_{i=1}^{m-1} G_i}$$

T_E = effective temp. of the cascade

Antenna Noise

- It ~~is~~ the noise captured by the antenna
- Becomes a problem with large aperture antennas.
- Larger apeture capture more



N_A - noise of the antenna

$N_A \propto T_A \leftarrow$ Noise temp. of antenna.

* Antenna noise is given by Livingston's formula.

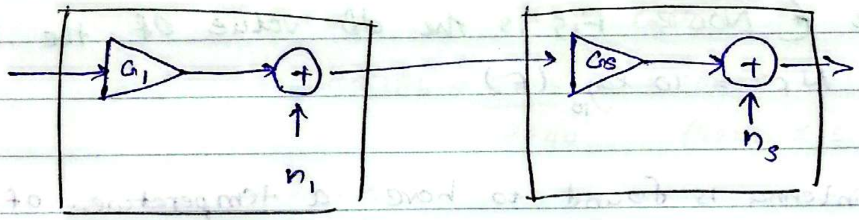
$$T_A = \sum_{i=1}^{N} \alpha_i T_i$$

α – coefficient.

* Finding α_i is difficult, have to relay on heuristic estimates.
* T_i is individual Temp. Source
* The overall temp.

$$T_S = T_A + T_E$$

Receiver Noise Analysis



First stage — All subsequent Stages

$$\boxed{F_T = F_1 + \frac{(F_s - 1)}{G_1}}$$

All subsequent stages

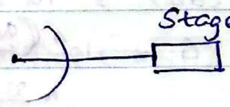
Gain of 1st stage

Noise factor of 1st stage

* By making G_1 large, the noise factor of all subsequent stages can be minimized.

$G\uparrow$ very large $\left[\frac{(F_s-1)}{G}\right] \simeq 0$

* However, the noise factor of the first stage will always dominate
* $F_1 \downarrow$ should be lower as possible

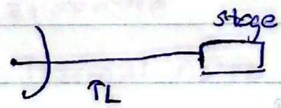


(LNA) Low noise Amp. $F_1 \downarrow$ $G\uparrow$

G_1 – attenuation ?

If there's a transmission line, it will increse noise ($F_s \uparrow$)

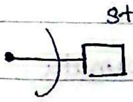
$(F_s - 1)$ will increse since $G_1 < 1$



noise factor $\uparrow$

No :

state 1



* as close as possible to the receiving antenna (Rx)

$F_T \approx F_1$

* It is essential that the 1st stage amplifier has,

1) Low F_1
2) high G_1 (gain)
3) is as close as possible to Rx

Rx, the point where EM wave is converted to an electrical signal

Noise Figure (Noise Fig is the dB value of the noise factor)

$$NF = 10 \log_{10} (F)$$

* A microwave antenna is found to have a temperature of 76 K. It is connected to an amp. cascade with noise figures & gains of the table given below with stage 1 being the LNA & the ambient temp. 300 K

Stage	1	2	3
Noise figure (dB)	2.1	1.1	1.1
Gain (dB)	33.5	12	12

Determine the system noise factor & noise temperature

Stage 1

$$NF = 10 \log_{10} (F)$$

$$F = 10^{\wedge}\left(\frac{NF}{10}\right)$$

$$= 10^{\wedge} \frac{2.1}{10}$$

$$F = 1.622$$

$$G\,(dB) = 10 \log G$$

$$G = 10^{\wedge}\left(\frac{G_{dB}}{10}\right)$$

$$= 10^{\wedge}\frac{33.5}{10}$$

$$= 2240$$

Stage 2 and 3

$$F = 10^{1.1/10}$$

$$= 1.28$$

$$G = 10^{12/10}$$

$$= 15.8$$

$$F_i = 1 + \frac{T_i}{T_o}$$

$$T_i = T_o (F_i - 1)$$

$$T_1 = T_o (F_1 - 1) = 300(1.622 - 1) = 186K$$

$$T_2 = T_3 = T_o (F_2 - 1) = 300(1.28 - 1) = 86.5K$$

$$T_S = T_A + T_E$$

$$= T_A + T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2}$$

$$= 76 + 186 + \frac{86.5}{2240} + \frac{86.5}{(2240 \times 15.8)}$$

$$= 262 K$$

$$F_S = 1 + \frac{T_S}{T_o}$$

$$F_S = 1 + \frac{262}{300} = 1.875$$

$$NF_S = 2.7 \text{ dB} \rightarrow (10 \log_{10} 1.875)$$

Receiver Noise Analysis Contd....

* How can the noise factor of the 1st be reduced?
  - → By minimizing the input noise
  - → By minimizing ~~inetied~~ internally generated noise (analog circuit design) + The end result is known as an Low noise Amplifier (LNA)

* Let input be $x(t)$ & signal $s(t)$

$$x(t) = [A_o + p(t)] s(t) + n(t)$$

A_o = Amplitude, $s(t)$ = Signal

$p(t)$ = Shot noise

$n(t)$ - Thermal noise

* Have to minimize $p(t)$ & ~~Ea~~ $n(t)$

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* $A_o >> P(t) \simeq A_o$ ⟶ not the case at a RX, $A_o \to 0$
signal is very weak

* $R \propto N$ → Thermal Noise ∝ Resistance

* To minimize Shot noise, $A_o >> P(t)$
  - Can be achieved by ~~com~~ capturing more of the radiated energy
    ie. incresing the gain of the antenna

* Thermal noise can be minimized by,
  - Minimizing the capture of external noise
    - Aperature ↓ reduans
    - Not be feasible.

  - Reducing the antenna resistance
    - May not be feasible
    - Resistance ↓
    - $R \propto N$ (thermal Noise)

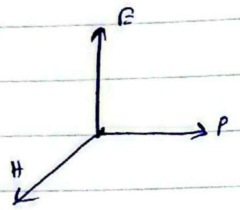
  - Cooling the antenna
    - Using Cryogenic fluid → Liquid N (77°K) or Liquid He (4°K)
    - very expensive

  - Having the LNA as close ~~as~~ to the antenna as possible
    - Most practical.

Date: 2025/05/14

Skin Depth - Transmission line

EM wave in free space



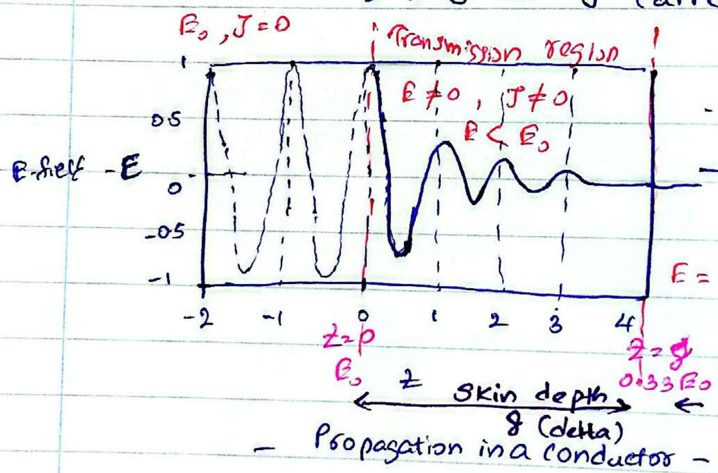
E - E-field

H - magnetic field

P - Power

* Power which is in perpendicular to the plain containing E field and magnetic field.
* EM field will come to metal and, both E, M fields will attenuate as their energy is ~~contain~~ converted ~~to~~ to current.
* Within metal there can be no Electric field. The energy in the electric field will convert int current density

$$E = 0 \rightarrow J \text{ current density.}$$



--- outside conductor

— within conductor.

E = 0 → neglegible E-field so, J = 0

← region between boundary and the dotted line

* Beyond the skin depth there is no significant current within the conductor.

$$E = E_0 e^{-\alpha z}$$

; α is the real

$$E = E_0 e^{-\gamma z}$$

$(\gamma = \alpha + \beta j)$

$$= E_0 e^{-(\alpha+\beta j)z}$$

attenuation.

$$|E| = E_0 e^{-\alpha z} = E_0 e^{-1} \quad (\because \alpha z = 1)$$

$\alpha z = 1$

$e^{-1} = 0.33$

$$|E| = 0.33 E_0$$

The skin depth or penetration depth (δ) is defined as the distance a wave travels before it is attenuated by a factor of 0.368 ($\approx e^{-1}$)

δ (skin depth),

$$z = \delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$z = \delta \rightarrow \alpha \delta = 1$$

$$\delta = \frac{1}{\alpha}$$

f - frequency

μ - permeability

σ - conductivity

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

- when, $f\uparrow$, $\delta\downarrow$
- when, $\mu\uparrow$, $\delta\downarrow$
- when, $\sigma\uparrow$, $\delta\downarrow$

Ex. 3

Find the skin depth of aluminum and iron at 50 Hz and 1 GHz

$\mu_r (Fe) = 500$ @ 50 Hz $\qquad \mu_r(Fe) = 10$ @ 1 GHz

$\mu_r (Al) = 1$

$\sigma_{Fe} = 1.03 \times 10^7 \ Sm^{-1}$

$\sigma_{Al} = 3.78 \times 10^7 \ Sm^{-1}$ $\qquad \mu = \mu_0 \mu_r$

50 Hz,

$$\delta_{Fe} = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \times 50 \times 500 \times 1.03 \times 10^7 \times \mu_0}} = 0.7 \text{ mm}$$

for low frequencies, So we use this conductor for power cable

$$\delta_{Al} = \frac{1}{\sqrt{\pi \times 50 \times 1 \times 3.78 \times 10^7 \times \mu_0}} = 1.16 \text{ cm}$$

1 GHz,

$$\delta_{Fe} = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \times 1 \times 10^9 \times 10 \times 1.03 \times 10^7 \times \mu_0}} = 0.175 \mu m$$

but at high frequencies these two doesn't have much difference and matters is durability for conductor

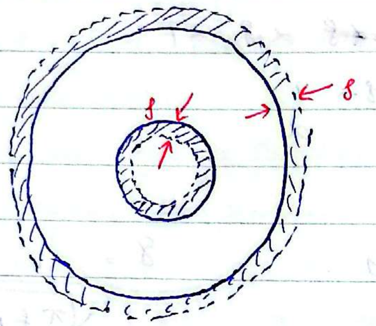
$$\delta_{Al} = \frac{1}{\sqrt{\pi \times 1 \times 10^9 \times 1 \times 3.07 \times 10^7 \times \mu_0}} = 2.584 \mu m$$

$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu_0 \mu_r \sigma}} = \frac{503.2921}{\sqrt{f \mu_r \sigma}}}$$

- In profecianaly enclosure will be made with Duralumin which were aloy of Al and other metals.

Attenuation of transmission line.

$$R = \frac{\rho l}{A} = \left(\frac{1}{\sigma}\right)\left(\frac{l}{A}\right)$$



colord area – crossectional area of the conductor (These should be polished as much as possible.)

– Skin depth of coaxial cable –

<u>Power attenuation</u>

Surface resistance : $R_s = \frac{1}{\sigma_c \delta} = \frac{1}{\sigma_c \frac{1}{\sqrt{\pi f \mu_r \mu_0 \sigma_c}}}$

$$= \sqrt{\frac{\pi f \mu_r \mu_0}{\sigma_c}}$$

$$= \sqrt{\frac{\omega \mu_c}{2\sigma_c}} \quad \because \omega = 2\pi f,\ \mu_c = \mu_r \mu_0$$

at microwave frequencies resistance of any conductor is proportional to the frequency; (either interms of ω or f)

$$R_s \propto \sqrt{\omega}$$

or

$$R_s \propto \sqrt{f}$$

Asymetric antenna should not be connected to Symmetric Load.

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Attenuation Coefficient.

$$\alpha = \frac{R_s}{2Z_0}$$

Assadourian and Rimai approximation for α of a microstrip

$$\alpha = \sqrt{\frac{\varepsilon_{eff}\varepsilon_0\omega}{2\sigma_c}}\left[\frac{1+\frac{h}{d}}{2\sqrt{h^2-d^2}\cosh^{-1}(h/d)}\right]$$

← where σ_c is the conductivity and μ_c is permeability of the microstrip, d is Springfield diameter.

* Microstrip (Substrate Transmission line) α_M

coaxial α_X

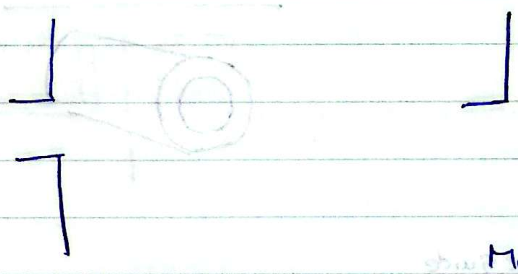
$$\alpha_M \simeq 100\,\alpha_X$$

Substrate T.L.

- used as minimally as possible

Long distance → Coaxial cable preferred

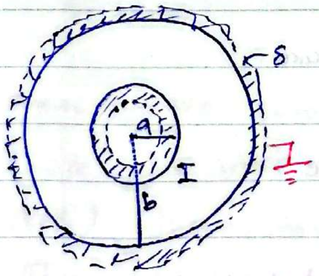
Transmission line / Load Symmetry.



Monopole → Asymetric

Dipole → Symetric

Coaxial cables' symmetry ← It is Asymetric



core = $\delta \times 2\pi a$

shild = $2\,\delta \times 2\pi b$

$$J_a = \frac{I}{\delta \times 2\pi a} \qquad J_b = \frac{I}{\delta \times 2\pi b}$$

$J_a \neq J_b$; Asymetric

$J_a << J_a$

J_b negligible

→ The outer conductor can be grounded

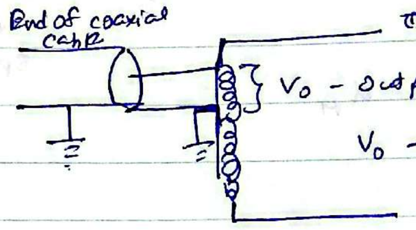
* To connect dipole twin wire is needed. [illegible] we cannot find them and using balun transformers coaxial cables were connected.

* Asymetric Load cannot connects to Symmetric antenna

* When ever it is necessary to connect an asymetric load or Tx line to a symetric Tx line or local

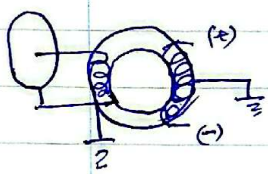
* a balun Transformer is essential.

Balance - Unbalance Transformer ≡ Balun Transformer



} V_0 - output of Tx line

$V_0 \rightarrow$ Symmetric output generated by the Balun

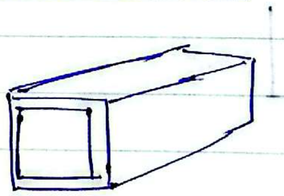


(-)

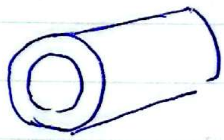
Mirror potential generated by the Balun.

Waveguides and Resonant Cavities

Rectangular Wave guides ;



Circular wave guide

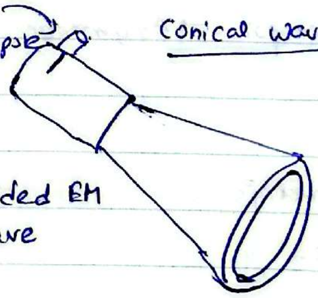


Feeder with monopole antenna

↓

Electrical Signal ⟺ Guided EM Wave

Conical wave guide

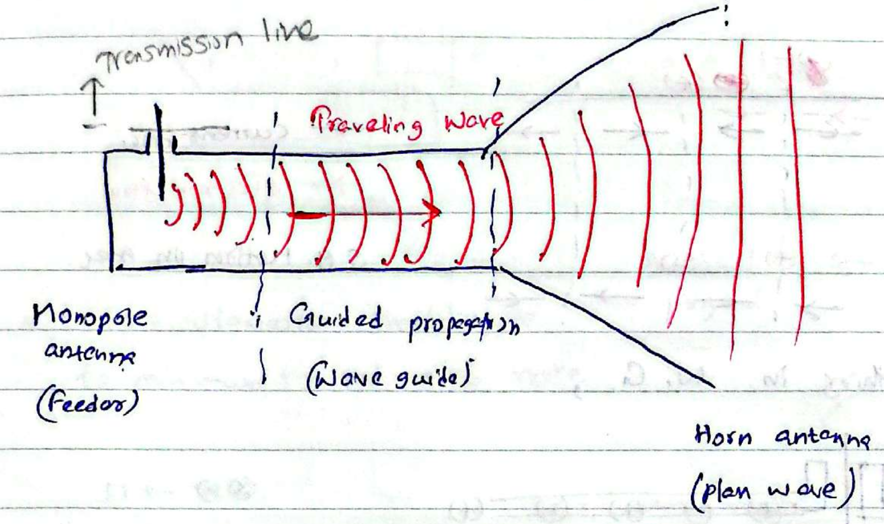


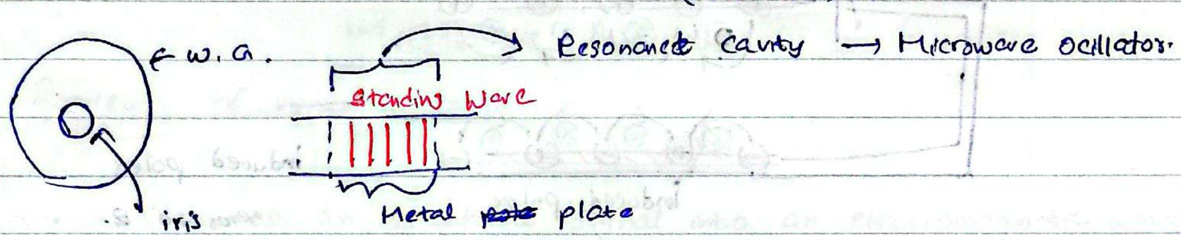
← conical Horn Antenna at the One end of wave guide

↓

Guided EM wave ⟺ Plane wave

* horn antenna can be Conical or Rectangular, but in the present Rectangular one is not common.

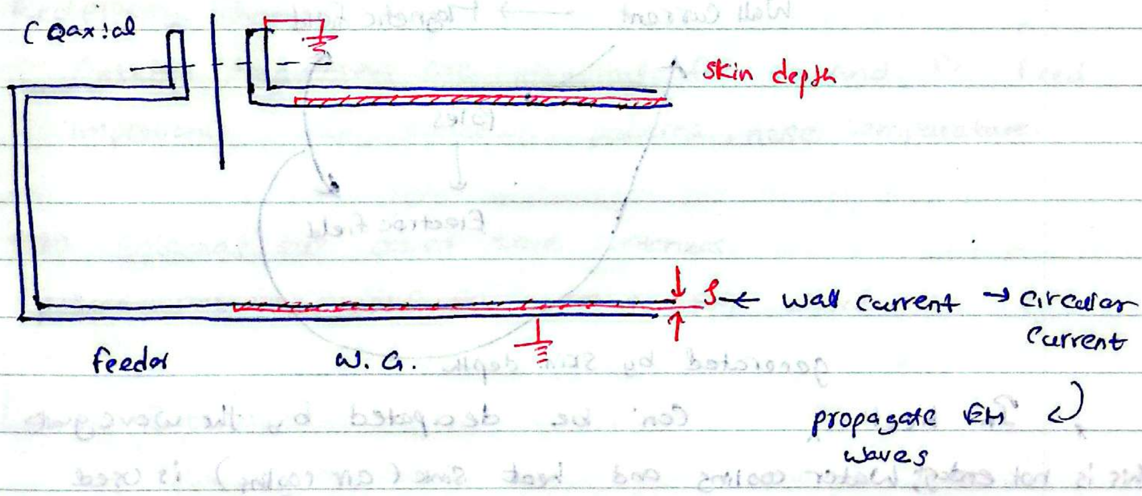




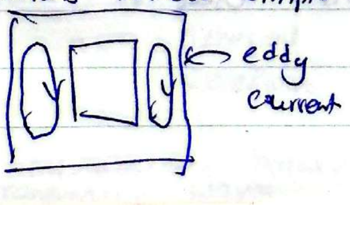
Small opennings at the centre

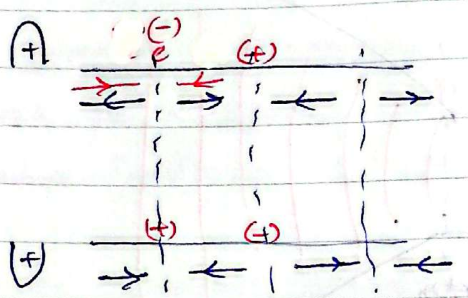
why we use guided Propagation over Transmission line ?

* Wave guide can handle very large power compaired to transmission line
* Single pice of Metal can do this
* The high power radars dosent have feeder.



* entire wave guide is grounded
* Inside the inner surface of the wave guide (within the distance of skin depth) there are wall current
* This wall current propagate electromagnetic waves
* electrons follow Simple Harmonic motion at the high frequencies

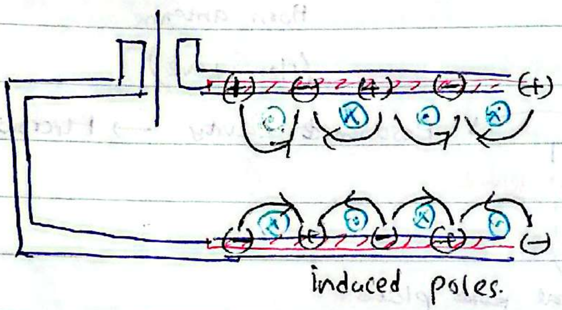




→ current flow
→ electron flow

∴ S.H. Motion in me

Similar thing in W.G.,



⊗⊙ → H

induced poles generate guided EM wave

Wall current → poles

Guided EM wave ← Initial

Initial Guided EM wave will be generated by feeder

Wall current ⟶ Magnetic field

Poles

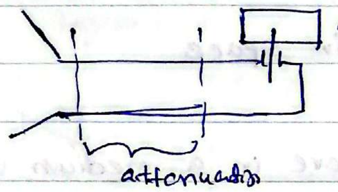
Electric field

generated by skin depth

* In heat can be decipated by the waveguide
If this is not enough, water cooling and heat sink (air cooling) is used

* Wave guide can be connected to the output of high power microwave vaccume tubes, for very high power Tx (eg: in radar)

* During Rx only a short section is often used



← Low Noise Amplifier → Down converter → IF → DC → Baseband signal

* in here attenuation should make to minimum to reduce the noise

Microwave Antennas

Date: 2025/05/28

Review of Antenna basis

Antenna: converts an electrical signal into an electromagnetic wave or vice versa.

Signal ⟷ EM wave

Two types → Wire antennas
→ ~~Appur~~ Aperture antennas

antenna → TX
RX ←

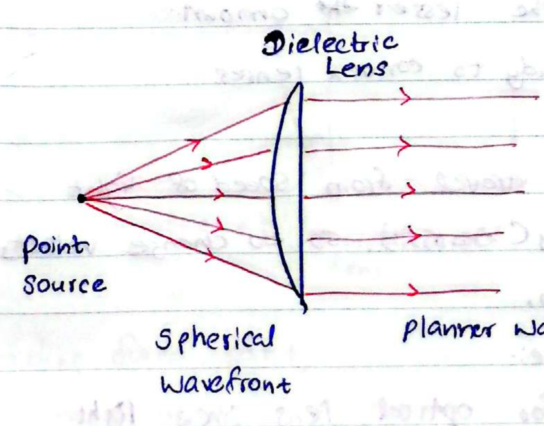
Reciprocity Theorem.

Antenna parameters are identical for tx and Rx. Feed impedance, gain, radiation paterns, noise temparature.

Wire antennas are called true antennas.

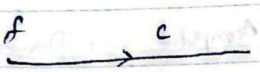
Apeture antennas modify an existing EM wave

Lens Antennas.

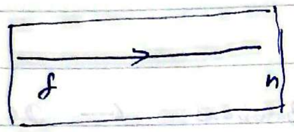


- Similar to an optical lens (same principle and analysis)
- analysis - geomatric ~~lens~~ Optics

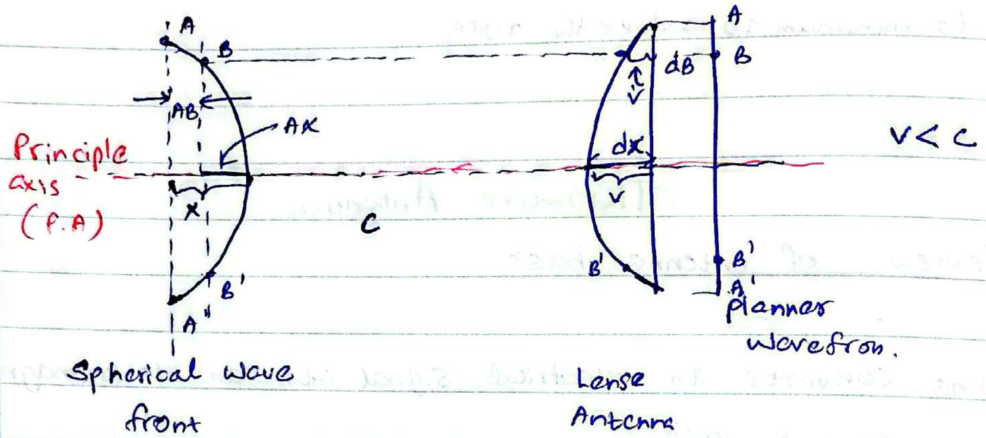
• no interfearance between sourse and transmitted waves

A 

EM wave in space

B 

EM wave in a medium with refractive index n



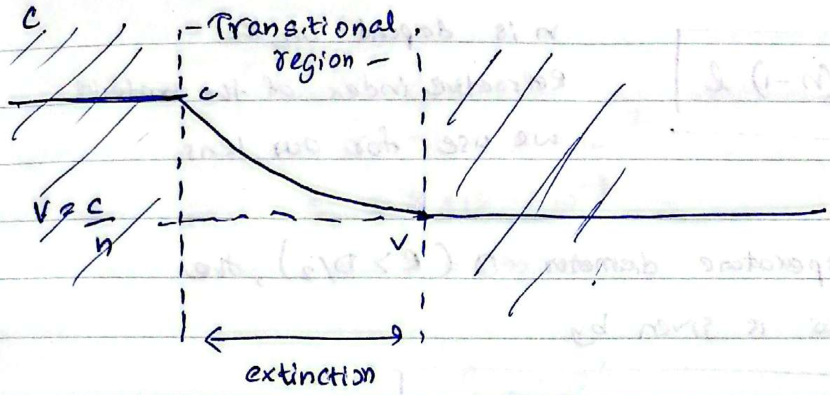
The dielectric will slove down the ~~wave~~ EM wave from C in the vaccum to $v = \frac{c}{n}$ in the dielectric medium.

* In order to convert ~~a~~ spherical wave front to a planner wave front, the points along the wave front are slow down acording to their position with respective to the principle axis
* Point X on the P.A has to be slowed down the most - so through the maximum distance through the lense
* points that are further away from principle axis has to slow down by the lesser ~~the~~ amount.
* We will limit our study to convex lenses.

EM wave which were travel from speed of light cannot be change instantly (suddenly), so to change velocity there is Transitional region.

Next page:-

- This is not problem for optical lens since light wavelength's in nano meters.



unless you except very precise result.

- This is not a very critical rule. (If we can do this it is better)

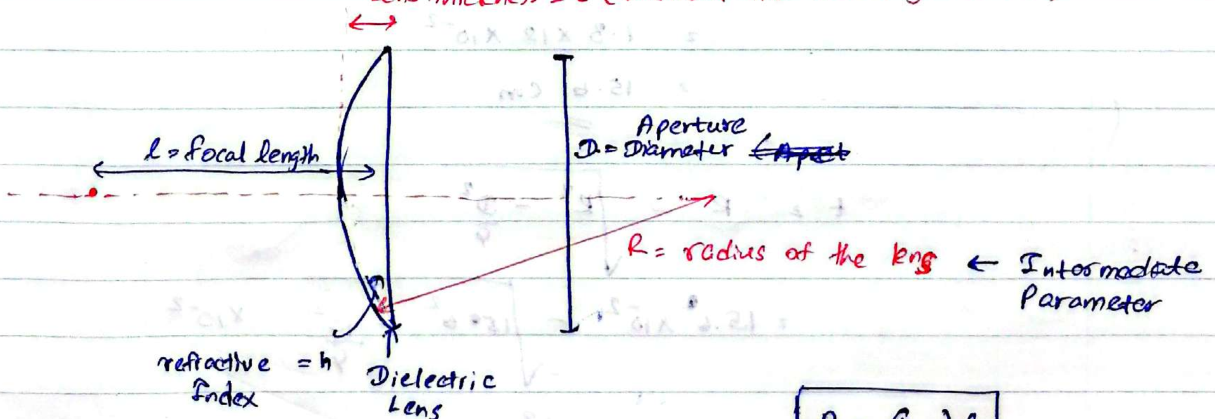
Jackson formula,

$$t_e = \frac{\lambda}{2\pi(n-1)}$$

the thickness of the lense > t_e at all point.

at microwave frequencies extinction lenth taken by the additional mounting section in the lense wich made out of same material as lens.

Antenna lens. Design (parameters)



$$\boxed{R = (n-1)l}$$

regarding focal point,

Sperical wave of a point sourse placed at the focal point will become a plane (perfect) wave.

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$$\boxed{R = (n-1)\, l}$$

n is depend on the Refractive index of the material we use for our lens.

for given aperature diameter D $(R > D/2)$, the lens thickness is given by

$$\boxed{t = R - \sqrt{R^2 - \frac{D^2}{4}}}$$

Ex1 $D = 10\,cm$, $l = 12\,cm$, $n \Rightarrow$ (1) $\Rightarrow 1.8$
(2) $\rightarrow 2.3$
(3) $= 1.3$

1) $R = (n-1)\,l = (1.8-1) \times 12 \times 10^{-2}$
$= 0.8 \times 12 \times 10^{-2}$
$R = 9.6\ cm$

$$t = R - \sqrt{R^2 - \frac{D^2}{4}}$$

$$= 9.6 \times 10^{-2} - \sqrt{9.6^2 - \frac{10^2}{4}} \times 10^{-2}$$

$= 1.4 \times 10^{-2}\ m = 14\ mm$

2) $R = (n-1)\,l = (2.3-1) \times 12 \times 10^{-2}$
$= 1.3 \times 12 \times 10^{-2}$
$= 15.6\ cm$

$$t = R - \sqrt{R^2 - \frac{D^2}{4}}$$

$$= 15.6 \times 10^{-2} - \sqrt{15.6^2 - \frac{10^2}{4}} \times 10^{-2}$$

$= 0.82 \times 10^{-2}\ m$
$= 8.2\ mm$

3) $R = (n-1) l$

$= (1.3-1) \times 12 \times 10^{-2}$

$= 0.3 \times 12 \times 10^{-2}$

$= 3.6 \text{ cm}$

$$t = R - \sqrt{R^2 - \frac{D^2}{4}}$$

$$= 3.6 \times 10^{-2} - \sqrt{3.6^2 - \frac{10^2}{4}} \times 10^{-2} = 3.6 \times 10^{-2} - \sqrt{-12.0} \times 10^{-2}$$

= complex answer

= Anser is complex number

= so infeseble

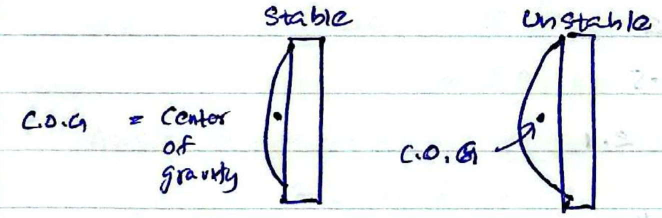
for certain value of n, l, D, realising value is infeseable

* Why is it benifitial to make t as small as possible? (at least 2 reasons)

→ to reduce the amount of Material

→ to reduce the loss (absorbtion loss) ← absorbtion loss will incree becouse of material amount

→ Large thickness results in Structural issues.



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Qu. A conical horn antenna has feed and Aperture diameters of 30 mm and 100 mm respectively. It has a length of 120mm. It is suggested to develop a lens antenna cover for the conical horn antenna to prevent dust and insects from entering it. it is to be used for a frequency of 15 GHz

a) If the effective aperture of the horn is 0.65 times physical aperture, calculate the gain of the conical horn.

b) If the lens increases the effective aperture of the horn ~~of~~ to 0.90 times the physical aperture but also absorbs 0.3 dB of radiated power, find the change in gain of the combined antenna.

c) From the ~~diameter~~ dimentions of the conical ~~ant~~ horn, Obtain the focal length of the lens.

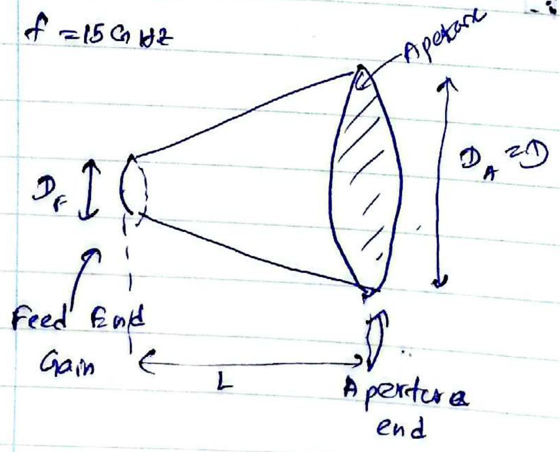
d) Determine the feasibility and necessary thickness of the lens if the materials with the following refractive index are to be used

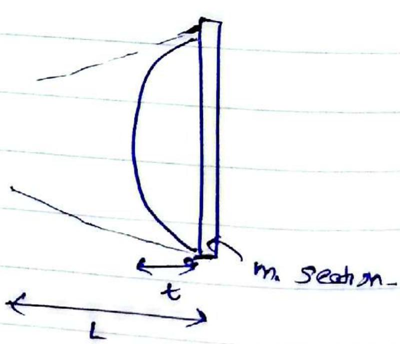
i) A with $n = 1.2$

ii) B with $n = 1.5$

iii) C with $n = 2.1$

$f = 15\ GHz$





Gain $= G \propto A$ (physical area of aperture)

$G \propto \eta$ (aperture efficiency)

$G \propto \frac{1}{\lambda^2}$

$c = f\lambda$ $\qquad$ $\lambda = \frac{c}{f} = \frac{3\times10^8}{15\times10^9} = \frac{1}{5}\times 0^{-1}$
$= 0.02\,m$

$\lambda = 2\,cm$

$\eta A \rightarrow$ effective Aperture

a)

$$G = \frac{4\pi\eta A}{\lambda^2} = 4\pi \times 0.65 \times \left[\pi \times \left(\frac{0.1}{2}\right)^2\right] \Big/ \lambda^2$$

$$= \cancel{4\pi\times0.65\,[\pi\times}$$

$$G_{dB} = 10\log_{10} G$$

$$= \cancel{0.064}\ 160.38$$

$$G\,dB = 10\log_{10} G = 10\log_{10} \cancel{0.064}\ 160.38$$

$$= 22.05\ dB.$$

b) $\eta = 0.65 \longrightarrow \eta = 0.9$

absorbs $\rightarrow$ 0.3 dB.

$$G = \frac{4\pi\eta A}{\lambda^2} = \frac{4\pi\times0.9\times\left[\pi\times\left(\frac{0.1}{2}\right)^2\right]}{0.02^2}$$

$$= 222.06$$

$$G\,dB = 10\log_{10} G$$

$$= 23.46\ dB$$

Absorb 0.3 = data

$$G_T = G_L - 0.3$$

$$= 23.46 - 0.3 = 23.16\ dB$$

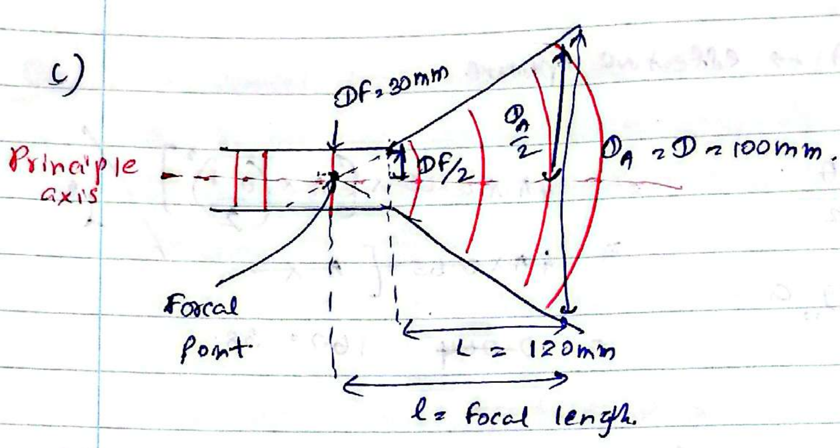
$$\Delta G = G_T - G_A$$

$$= 23.16 - 22.05\ dB$$

$$= 1.11\ dB$$ $\leftarrow$ This answer can be (−) value as well

~~c) $D_A = D = 100mm$~~

c)



$$\frac{D_A/2}{l} = \frac{D_A/2 - D_F/2}{L}$$

$$\frac{100\text{ mm}/2}{l} = \frac{100\text{ mm}/2 - 30/2\text{ mm}}{120\text{ mm}}$$

$$\cancel{l = \frac{30 \times 120}{100}\text{ mm}}$$

$$\cancel{l = 36\text{ mm}}$$

$$l = \frac{100 \times 120 \times 2}{2 \times 70}$$

$$= 171.42\text{ mm}$$

$n = 2.1$

d) iii) $t = R - \sqrt{R^2 - \frac{D^2}{4}}$

$R = (n-1)\,l$

$= (2.1 - 1) \times 171.42$

$= 1.1 \times 171.42$

$= 188.55$

$$\cancel{t = 188.55 \times 10^{-3}} - \sqrt{(188.55 \times 10^{-3})^2}$$

$$t = 188.55 \times 10^{-3} - \sqrt{(188.55 \times 10^{-3})^2 - \frac{(100 \times 10^{-3})^2}{4}}$$

$t = 6.75$ mm ← feasible

ii) $n = 1.5$

$R = (1.5-1)\,171.42$

$= 85.71$ mm

$t = 85.71\text{ mm} - \sqrt{(85.71)^2 - \frac{100^2}{4}}$ ~~m×10~~ mm

$= \underline{16.09}$ mm ← feasible

i) $n = 1.2$

$R = (1.2-1)\,171.42$

$= 0.2 \times 171.42$

$= 34.284$ mm

$t = 34.28\text{ mm} - \sqrt{\frac{34.28^2 - 100^2}{4}}$ mm

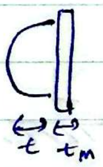
$= 34.28 - 36.39i$ ← complex number.

So . it is not feasible.

For Assignment,

4 ① Loss in here 0.3 dB

But for Assignment L & t_T based on cardan equato

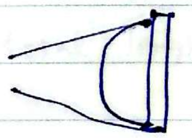
② 

$t_T = t + t_M$

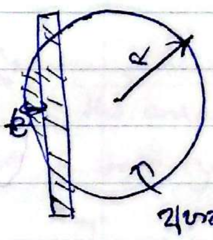
$t_M > t_e$

AutoCAD file and SCL.file

③ If thickness is very large it may not fit in cornical horn.



$R = (n-1)\lambda$



$\boxed{\text{Free CAD Software}}$

This has to be Solid Structure →

④ Autocad design.

• First define sphere $R = (n-1)\lambda$

• then get thickness and cut unwanted sphere and add the rectangular mounted thickness

• must have brackets to mount it

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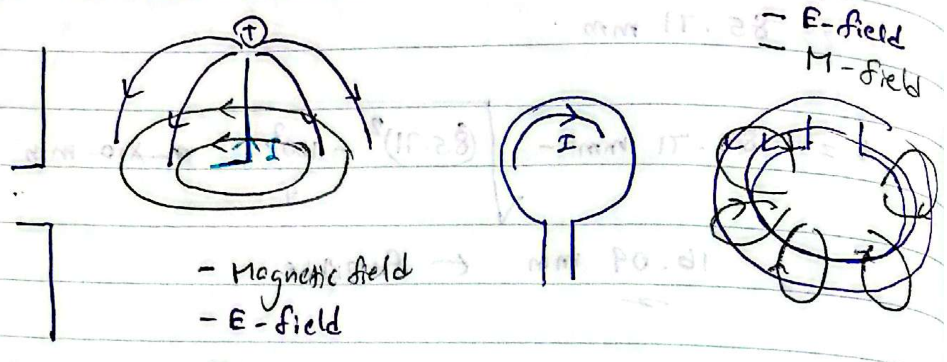
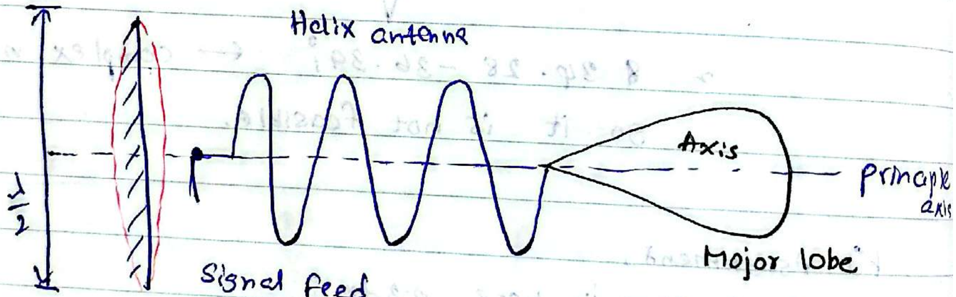
Helical Antennas

- These are wire antennas.



Dipole antenna   Monopol antenna   Loop antenna

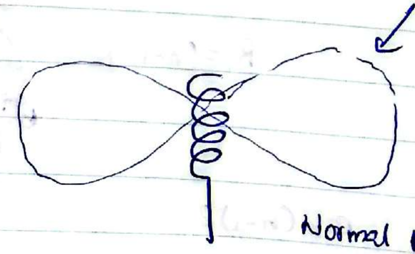
- all above antennas have 2 fields.
- Helical antenna is a single field antenna, and it prod[uces] circularly polaraized EM wave.



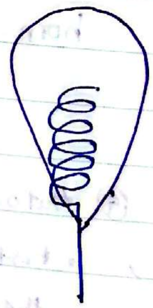
Circular Ground plane

In dipole  → Monopole ← missing plane is covered by the ground plane

Wehn λ >>> dimention of helix, low frequency



Normal Mode of helix



High frequency
λ ≏ dimention helix

- Unless we use low frequency this is absolutely outdated technology

- Axial Mode -

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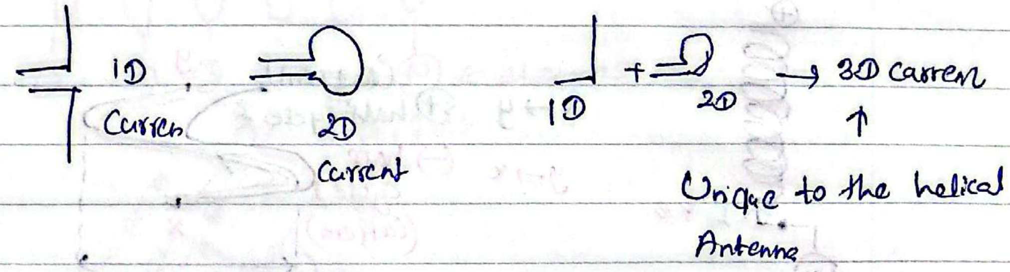
Overview of helix antenna

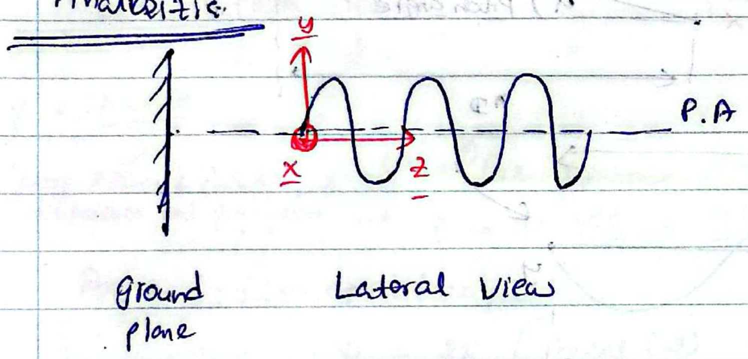
Analysis → 3D current

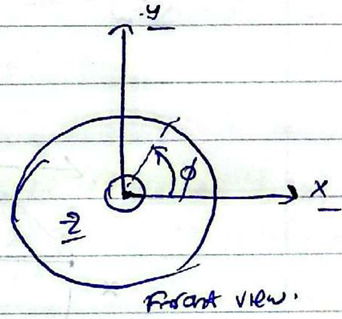


→ Elliptic or circularly polarized

- Two mode of operation
  1) ~~axo~~ normal Mode (infeasible in MW frequencies)
  2) axial mode

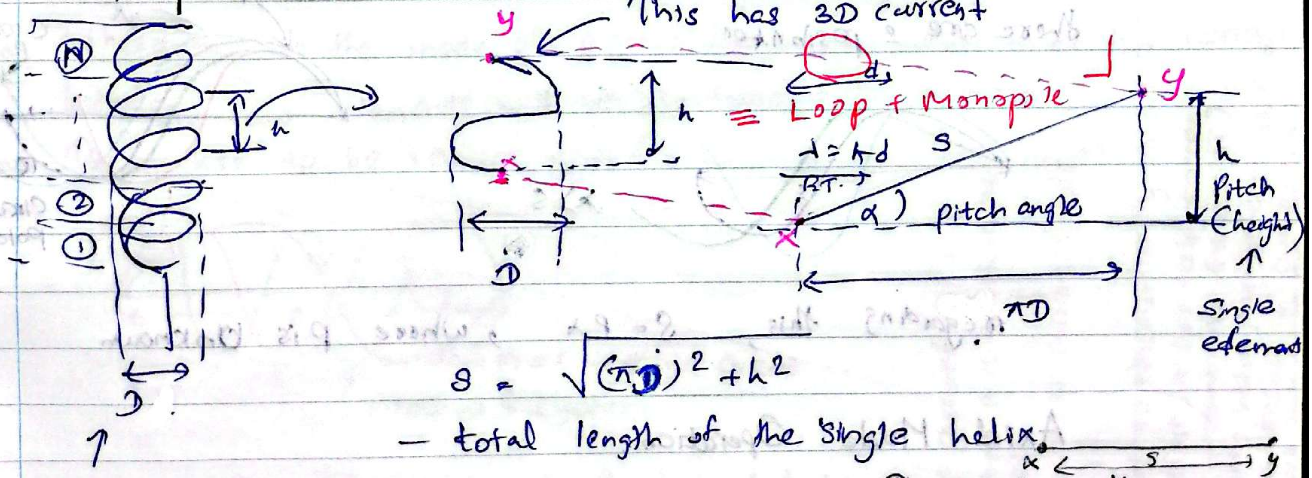
Analizitis





- The z axis is along the axis of the helix
- Anlyzed
  energ field is along z direction
- All electrical field and Magnetic field have to be in x-y direction

Helix parameters



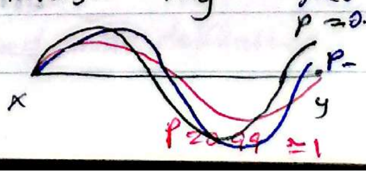
$$S = \sqrt{(\pi D)^2 + h^2}$$

– total length of the single helix

$$h = \lambda/4$$

x ← S → y

- The signal in the antenna must have a single cycle through x-y



For circular polarization,

$$|E_V| = |E_H|$$

$$|\angle E_V - \angle E_H| = \pi/2$$

If not, elliptical polarization

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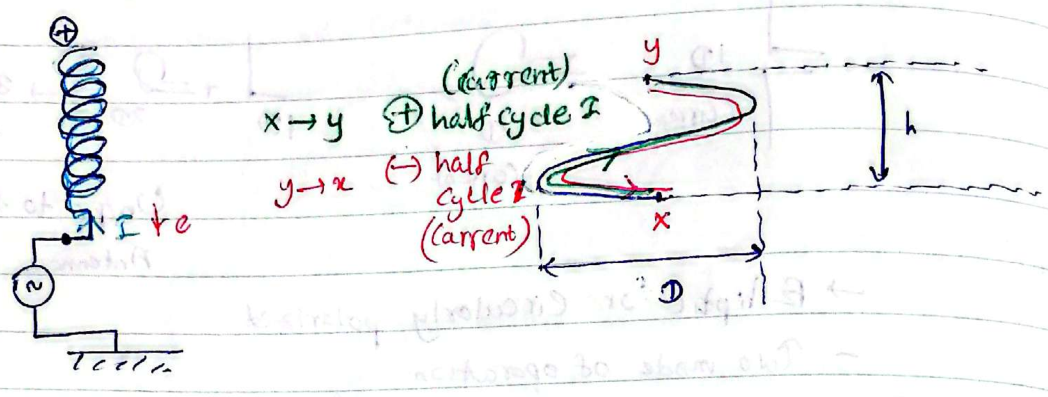
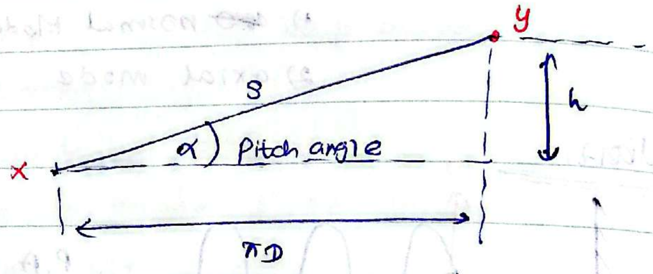
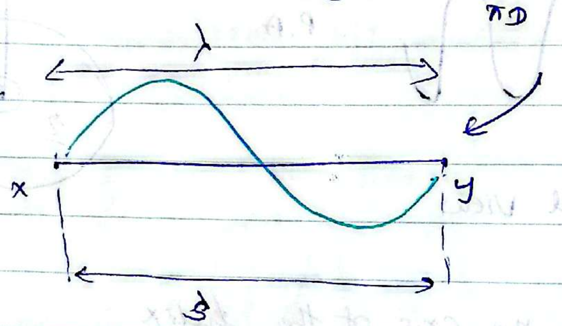
Helix Parameters

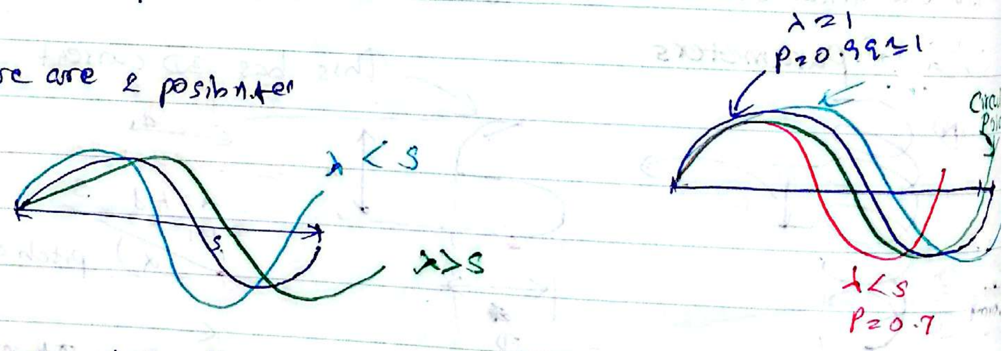






Ideally $S = \lambda$, but practically $S \neq \lambda$, but typically $S \simeq \lambda$ is preffered

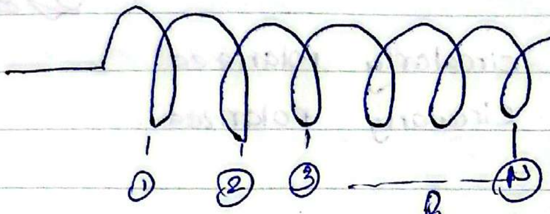
there are 2 posibilites

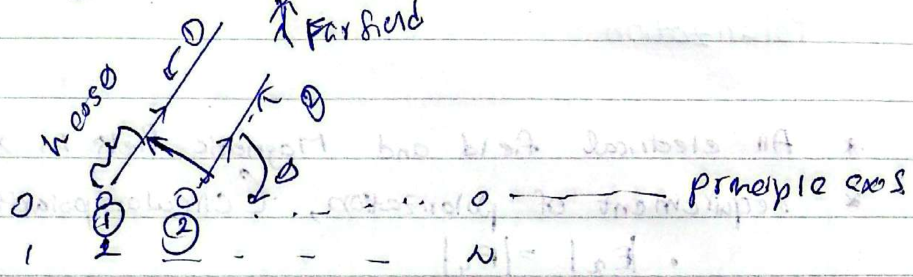


regarding this, $S = P\lambda$, where P is Unknown

Axial Mode Operation.

- Wavelength is small compared to helix dimensions
- Helix element taken as an element of an N element array





$\phi = \Delta\alpha$

2 different elements have path difference of $h\cos\theta$

$\phi = \frac{2\pi d}{\lambda}$

Path difference ⟶ phase difference (ϕ)

$\Delta S = d = h\cos\theta$

$d \qquad \frac{2\pi d}{\lambda}$

$\phi = \frac{2\pi h\cos\theta}{\lambda}$

Phase difference due to path difference and propagation

$\varphi = \phi_{\text{Path difference}} + \mathcal{E}_{\text{field difference}}$

$\longrightarrow \varphi = \Delta\alpha + C\mathcal{E} = \frac{2\pi h\cos\theta}{\lambda} - \frac{2\pi S}{p\lambda} = \frac{2\pi}{\lambda}\left(h\cos\theta - \frac{S}{p}\right)$

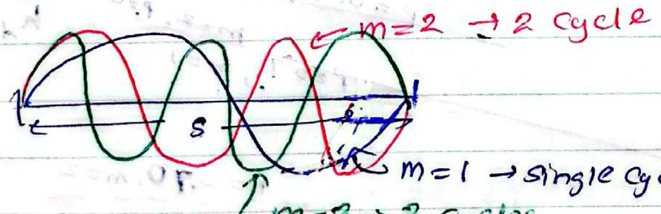
For end-fire operation,

$$\varphi = \frac{2\pi}{\lambda}\left(h\cos(\theta) - \frac{S}{p}\right) = -2\pi m$$

Therefore, when $\theta = 0$

$$p = \frac{S}{h + m\lambda} \simeq \boxed{0.7 \sim 0.99} \leftarrow \text{hard accounting for.}$$

* Where m is the mode of current distribution when m=1 the current between two elements will ~~be~~ undergo a single cycle
* Design has to be robust to P.



m=1 ⟶ operating in fundamental Mode

m>1 ⟶ higher order mode

⟶ higher frequency.

⟶ Parameter P is getting more and more difficult.

Effect of Parameter P,
- Parameter P is a tolerance parameter
  (how much current circulate within helix)
- Difficult to theoretically model (therefore, it has to be taken into account into the design)
- It affects helix parameters d and h and can result in overlapping modes which have to be avoided

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RHCP → Right hand circularly polarized
LHCP → Left hand circularly polarized.

Polarization

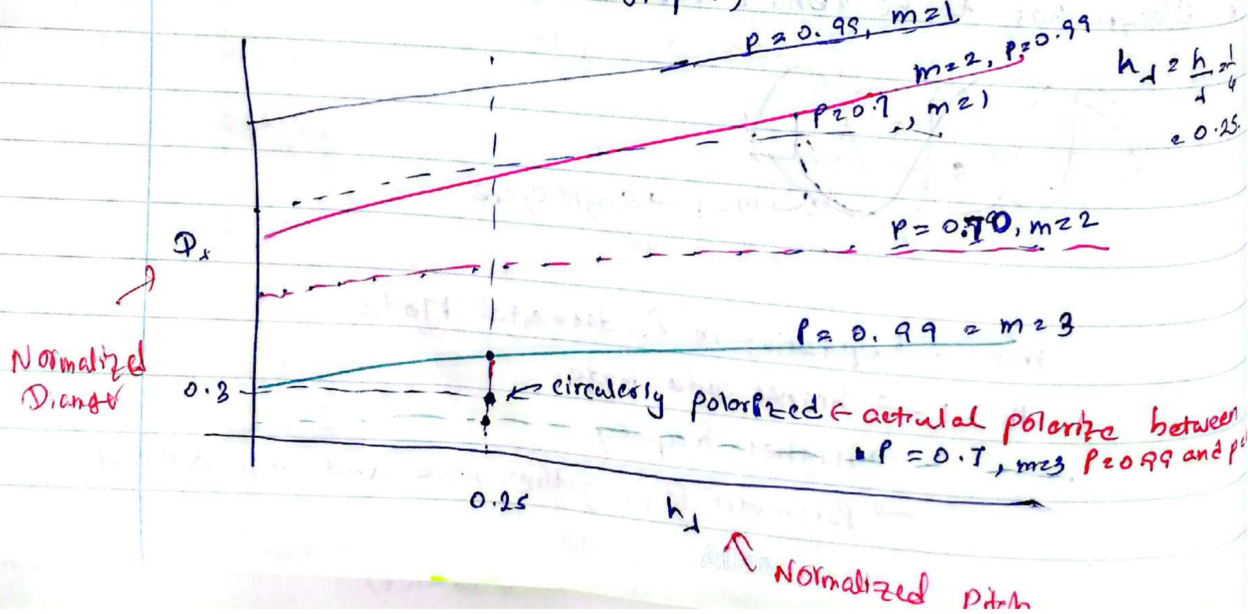
* All electrical field and Magnetic fiels in x-y directions
* requirement of polarization, (circular polarization)
  - $|E_x| = |E_y|$
  - phase difference of $\pm \pi/2$

* Therefore $\frac{E_x}{E_y} = \pm j = j/k \rightarrow k = \pm 1$ $\quad \boxed{\pm j = e^{\pm j\pi/2}}$

* if, $|E_x| \neq |E_y|$ then phace $= \pm\pi/2$, we here ~~ell~~ elliptical pol.
* When the polarization is elliptic

* When we realized helical antenna we avoid $m > 1$
* most convenient $m=1$ to aelive circular polarization when $m=1$
* $m=1 \leftarrow$ fundermental mode oparation
* Two conditions for circular polarization
  ① $h = \lambda/4$
  ② $\pi D = \lambda$

Effect of perameter p (graph)



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When the polarization is elliptic, $K \neq \mp 1$ where

$$K = h_\lambda - \frac{S_\lambda}{p}$$

$$K = h_\lambda - \frac{\sqrt{h^2 + (\pi D)^2}}{p}$$

From this is possible define the eccentricity of the polarization.

$$x = \begin{cases} \sqrt{1 - K^2} & -1 < K < 0 \\ \sqrt{1 - 1/K^2} & K < -1 \end{cases}$$

Interms of wave length λ

$$K = h/\lambda - \frac{\sqrt{h^2 + \pi D^2}}{p\lambda}$$

Ex:-

By taking $p = 0.85$ as a normal Value of p, determine the values of K and ratio $|E_x|/|E_y|$ of the resulting elliptical polarization of a realized helical antenna, when $h = \lambda/4$ at the extreme Values of $p = 0.7$ and $p = 0.99$. Assume that the antenna operates at fundermental mode

Find K for $p = 0.7$, $p = 0.99$

$$K = h_\lambda - \frac{S_\lambda}{p}$$

$$S_\lambda = \sqrt{h_\lambda^2 + (\pi D_\lambda)^2}$$

when $p = 0.7$

$D_\lambda = 0.27$ ← from graph

$S_\lambda = \sqrt{0.25^2 + (\pi \times 0.27)^2}$

$= 0.88$

$K = 0.25 - \frac{0.88}{0.7}$

$K = -1.007$

when $p = 0.99$

$D_\lambda = \cancel{0.345}\ \cancel{39}\ 0.39$

$S_\lambda = \sqrt{0.25^2 + (\pi \times 0.39)^2}$

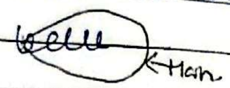
$= 1.825$

$K = 0.25 - \frac{1.825}{0.99}$

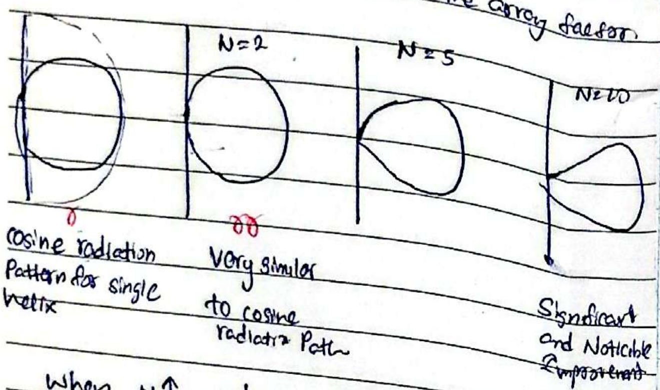
$K = \cancel{-1.08}\ -1.012$

Ideally $K = \pm 1$ but practically $K \neq \pm 1$ but be very close to it.

The helix has cosine radiation pattern and Main lobe of the ~~cosine~~ helix in cosine.



The helix cosine lobe means one lobe (main), and we can completely Neglect the one lobe of the array factor.



when N ↑, baseband bandwidth (bandwidth of the helics) ↓ bits.

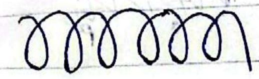
Helical → Low

	Bandwidth
N = 10 →	100 kb/s
N = 20 →	20 kb/s

* Decrese in basband bandwith is nonlinear and it degrees faster than incresing number of turns.
* If the time period of the ~~single~~ signal fed to the helix is τ have to wait for Nτ until the signal propagation to the end of the helix

→ A single element has a basic

Radiation pattern.

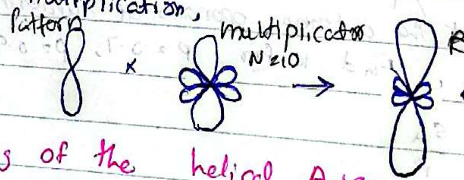


If the time period of the signal fed to the helix is have to wait for Nτ untill the signal proportional to the end of the helix

- A single element has a basic cosine radiation pattern
- This is pattern multiplied with the array factor.
- N turn helix radiation pattern.

$$F = \cos(\theta)\left[\frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right]$$

Pattern multiplication,



Matching of the helical Antenna

- Krauss formula for feed impedance

$$\boxed{Z_{in} = R = \frac{140\pi D}{\lambda}}$$

$$\frac{E_x}{E_y} = \frac{j}{k}$$

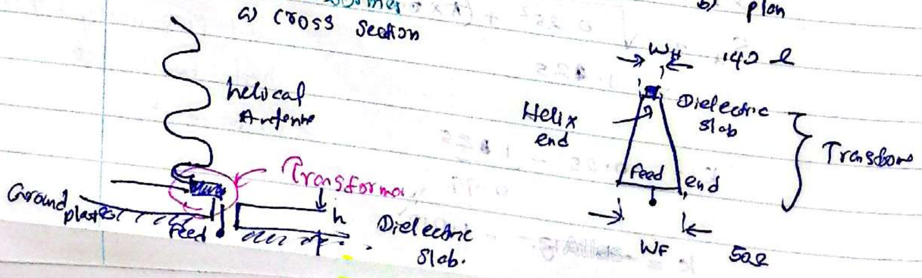
- for circular polarization

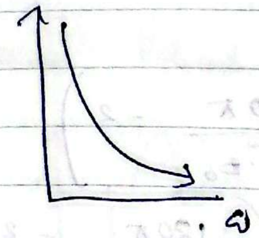
$$\pi D = \lambda \rightarrow D \simeq \lambda/\pi \rightarrow R \simeq 140\Omega$$

$$h \simeq \lambda/4$$

This will give a pitch angle of around 14° (0.245 rad) but a fixed feed impedance.

Matching transformer





- The ~~belt~~ tapering matching transformer matches to the 50 Ω transmission line at the whide end and the 140 Ω antenna at the narrow end
- width is given by,

$$\omega = t\left(\frac{120\pi}{\sqrt{\varepsilon_r Z_0}} - 2\right)$$

$\omega > 0$

$$\frac{120\pi}{\sqrt{\varepsilon_r Z_0}} - 2 > 0$$

$\sqrt{\varepsilon_r Z_0} < 120\pi$

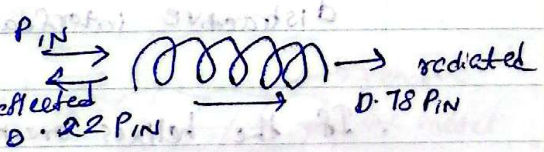
$\sqrt{\varepsilon_r} < \frac{180}{140}$

To be feasible,

$\sqrt{\varepsilon_r Z_0} < 60\pi$

exersize ②

An axial mode helical antenna has $D = \lambda/4 \Rightarrow 140\ \Omega$ Determine the widths of either end of the required matching transformer if the dielectric slab above the ground plane has a thickness of 4 mm and ε_r of 1.2



$Z_0 = 50\ \Omega$

$Z_{IN} = 140\ \Omega$

$$|\rho| = \left|\frac{Z_{IN} - Z_0}{Z_{IN} + Z_0}\right| = \left|\frac{140 - 50}{140 + 50}\right| = 0.47$$

$|\rho|^2$ - power (reflected)

$|\rho|^2 = 0.22 \longrightarrow$ 22% power wasted

$\varepsilon_r = 1.2$

thickness = 4 mm

~~W_H = 2 mm~~

$$W = \left(\frac{120\pi}{\sqrt{1.2 \times 140}} - 2\right) \times 4 \times 10^{-3}$$

$$= \left(\frac{120\pi}{153.4} - 2\right) 4 \times 10^{-3}$$

$$= 0.46 \times 4 \times 10^{-3}$$

$$= 1.84 \text{ mm}$$

$$\simeq 2 \text{ mm}$$

$$W_f = t\left(\frac{120\pi}{\sqrt{\varepsilon_r} \cdot Z_0} - 2\right)$$

$$= 4 \times 10^{-3}\left(\frac{120\pi}{\sqrt{1.2} \times 50} - 2\right)$$

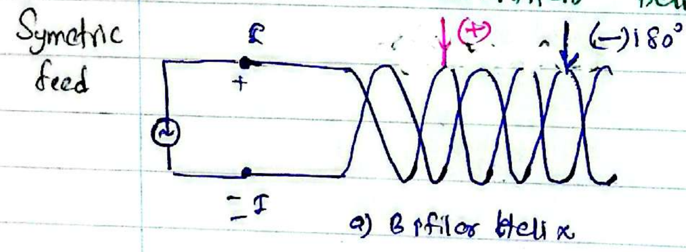
$$= \left(\frac{120\pi}{54.8} - 2\right) \times 4 \times 10^{-3}$$

$$= 4.88 \times 4 \times 10^{-3}$$

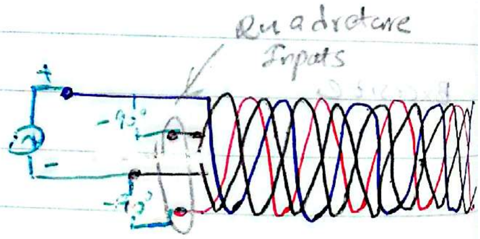
$$= 19.52 \times 10^{-3}$$

$$= 19.5 \text{ mm}$$

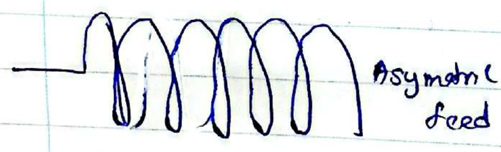
Bifilar and Quadrifilar Helical Antennas.



a) Bifilar Helix



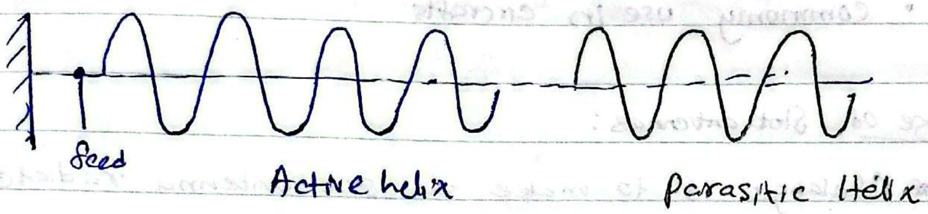
Quadrifilar Helix



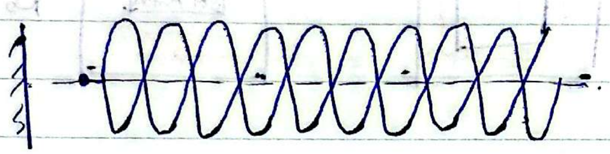
- Need perfect allignment. Otherwise it produce distractive interfearance.

- If the helixes are not perfectly allign if not it will cause distriptive interfearance.
- Quadrifilar antennas are rare due to the above reason.

Parasitic Helical Antenna

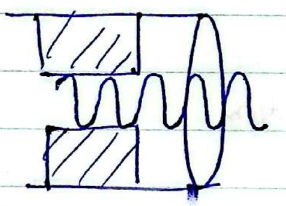


a) Ground mounted configuration

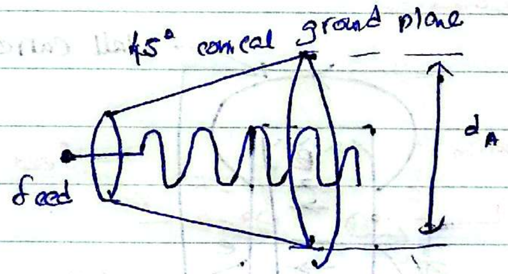


b) Prasitic Bifilar Configuration,

Cylindrical Waveguide Accessories.



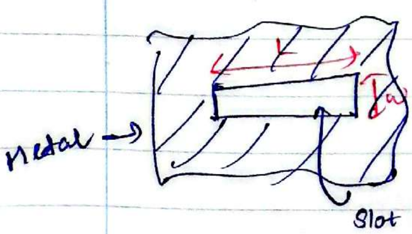
a) Cylindrical Waveguide parasitic Helix



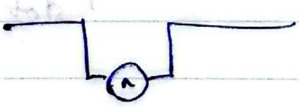
b). Helicone antenna

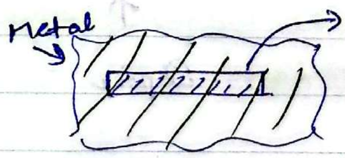
Slot Antenna

- Aperture Antenna
  - Basic form is the rectangular Slot
  - Can consider it as a "complementary" or "Inverse" dipole antenna
  - Apeture has a length aswell as Width



$W << L$





remove the metal then It give the

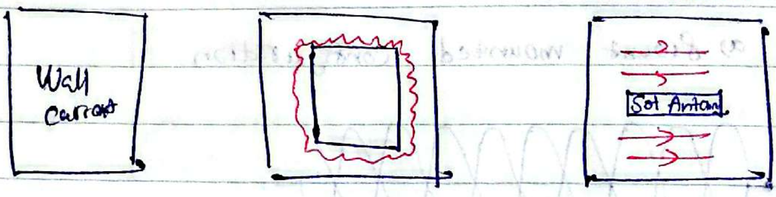
to get slot antenna we ne...

No:

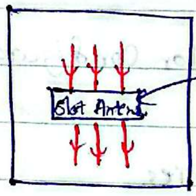
* Benifits of Slot antennas.
  - Can be Integrated into a metel casing
  - commonly use in aircrafts.

* Chalange of Slot antennas:
  * Chalange is to make the slot antenna radiate

Wall current is parallel to the slot



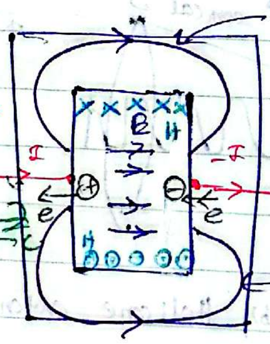
No radiation



Wall current is Prependicular (⊥) to the Slot.

→ radiation

radiative slot



Wall current

Feed excitation

Feed excitation

Wall current → generated by the feed ~~Amplifier~~. excitation.

Babinet's Principle

Z_C - feed Impedance of the Slot

$$Z_A \times Z_C = \frac{\eta_0^2}{4}$$

Impedance of the antenna ↑ Dipole

Impedance of the Compement ↑ slot

Date: 2025/06/11

ex: A dipole antenna has an impedance of $73 + j40\ \Omega$, using Babinet Principle find the impedance of the complementary slot. Take $\eta_0 = 120\pi\ \Omega$ or $377\ \Omega$

$$Z_A \times Z_c = \frac{\eta_0^2}{4}$$

$$(73 + 40j) \times Z_c = \frac{(120\pi)^2}{4} = (60\pi)^2$$

$$Z_c = \frac{377\ \Omega}{4} \times \frac{1}{73 + 40j}$$

$$= 374 - j205\ \Omega$$

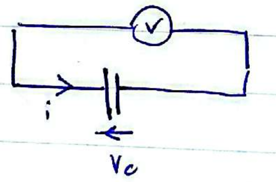
Resonant Devices and Antennas.

* ~~Respons~~ Resonance occurs when an electrical circuit introduce no phase shift between Voltage and Current of the exciting Signal.

ex. there is no net reactance

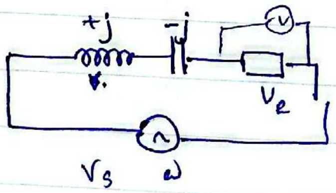
* Important property at microwave frequencies
  - → cavity resonators
  - → oscillators
  - → filters
  - → Resonant antennas.

* in, Capacitor circuit, if we need to calculate V_c, we need to integrate current first



$$V_c = \frac{1}{c} \int i \, dt$$

* phase

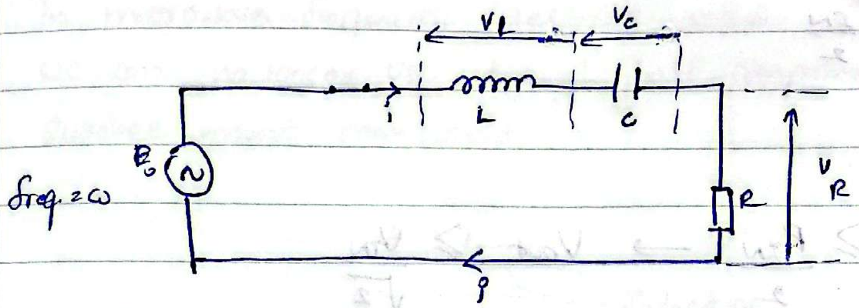


- no phase difference between V_s and V_R
- But phase shift between C and L

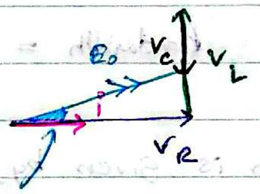
* Since there is no phase difference between V_s and V_R, there is no phase shift between Voltage and the Current in the circuit.

* Patch antenna is an example of ~~an~~ the Resonant antenna

Series RLC circuit



- i, is common to R, L, C
- i is the reference →
- i is in phase with V_R →
- since L is postive j ↑
- next C, and it is negative j ↓



phase difference = ϕ

if in any chance, $V_L + V_c = 0$, $\phi = 0$, so E_0, V_R are in phase so it is resonance

$$j\omega L I - \frac{jI}{\omega C} = 0$$

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega = \frac{1}{\sqrt{LC}}$$

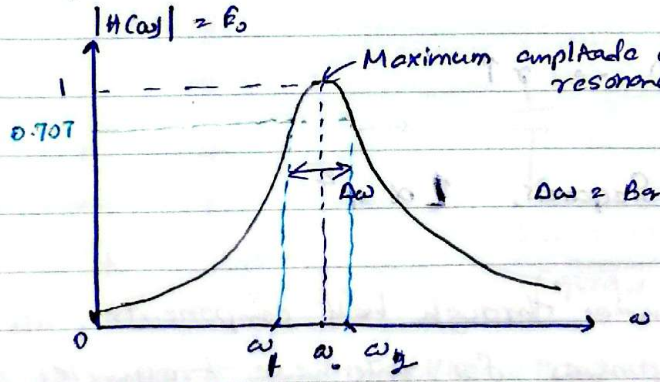
So, the resonance frequency is depend on L and C

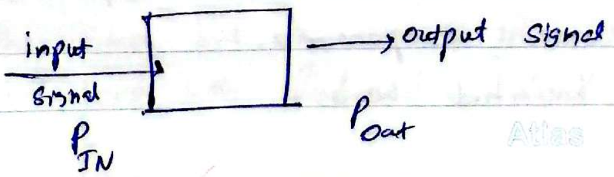
$$E = V_R = R\,i$$

Frequency Response

$E_0 = |E_0| \cos(\omega t)$

Amplitude can be change as we like





in the circuit, Band width is when

$P_{out} \geq \frac{P_{IN}}{2}$

$$P_{OUT} \geq \frac{P_{IN}}{2}$$

if $P \propto v^2$,

$$P_{OUT} \geq \frac{P_{IN}}{2} \longrightarrow V_{out} \geq \frac{V_{IN}}{\sqrt{2}}$$

$$\simeq 0.707\, V_{IN}$$

$$\text{Bandwidth} = \Delta\omega = \omega_2 - \omega_1.$$

Bandwidth is given by,

$$\Delta\omega = \frac{\omega_0}{Q} \longleftarrow Q \text{ factor / Quality factor}$$

$$Q = \frac{\omega_0}{\Delta\omega}$$

where the, Q is the Quality factor of the resonance circuit given by,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \longrightarrow \Delta\omega = \frac{R}{L}$$

if we need to have narrow BW,

↳ twining $Q\uparrow$, so R should be $R\uparrow$ and $L > C$

$L \propto N^2 \leftarrow N =$ number of the turns in coil

r = Internal resistance $\rightarrow r \propto N$

So, when $L\uparrow \rightarrow r\uparrow$

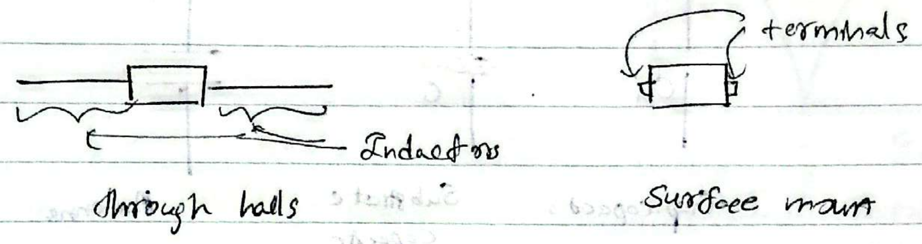
So in microwave frequencies, $L \propto r^2$

★ At microwave frequencies, through hall components will be never used, components for microwave frequencies has to be surface mount components.

There is a reason behind here.

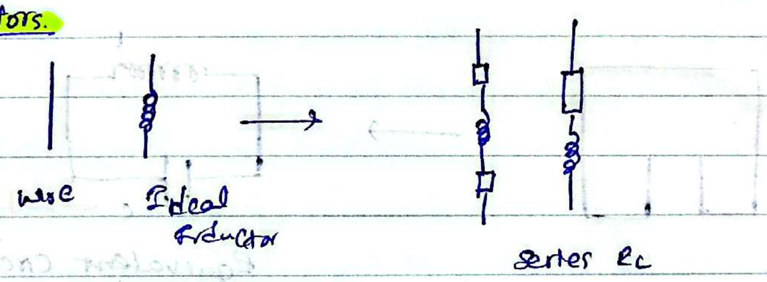
No : Date: / /

in microwave frequencies, wires and [illegible] act as Inductors, so we can no longer use through hall components. and we need surface mount components



* Surface mount components are must at microwave frequencies

HF Inductors.



So, wire has a inductor, and we actually have inductor ~~and~~ and its internal resistance

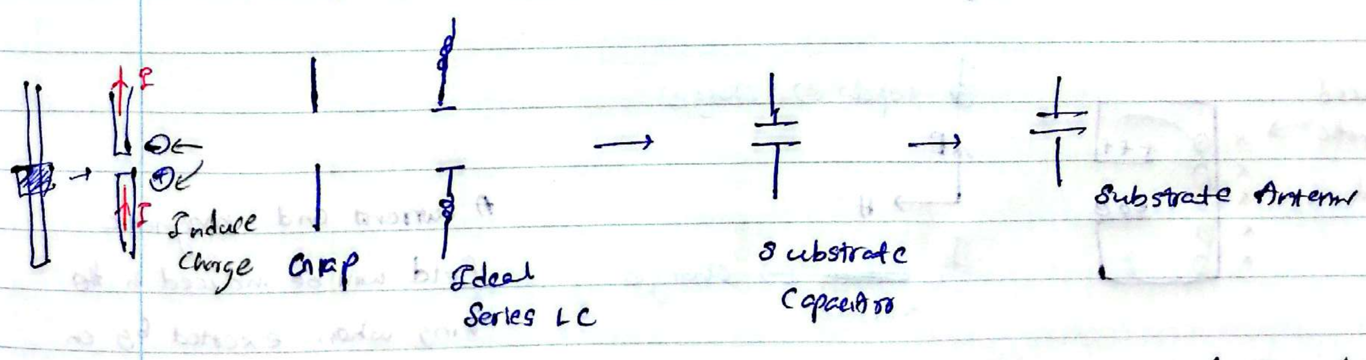
$r \propto l$ ← length of the wire $= l$

$L \propto l$

$L \propto r$

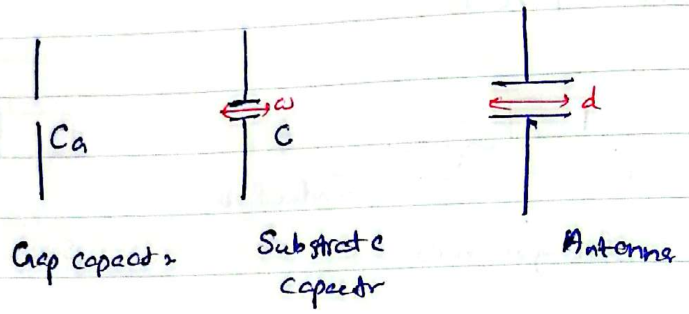
HF Capacitors

When it comes to a capacitor, we started off with wire and we make a gab in that wire



even in here, because of HF flow, we have created Gap ~~betwe~~ In the wire, but the current flow will not stop and create the capacitor here with Induced charged dipoles

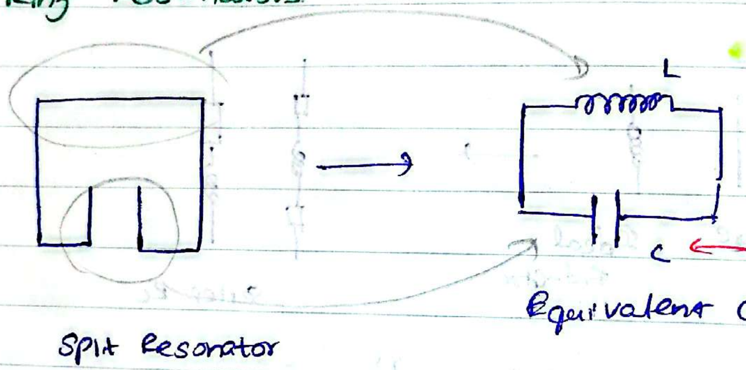
Gap Capacitor, has ~~and~~ low capacity than Substrate capacitor, and w[hen] we increse width of capacitor it can be act as antenna



$d > w$

$C > C_a$

Split-Ring Resonators



This act as Capa[citor] and also Anten[na]

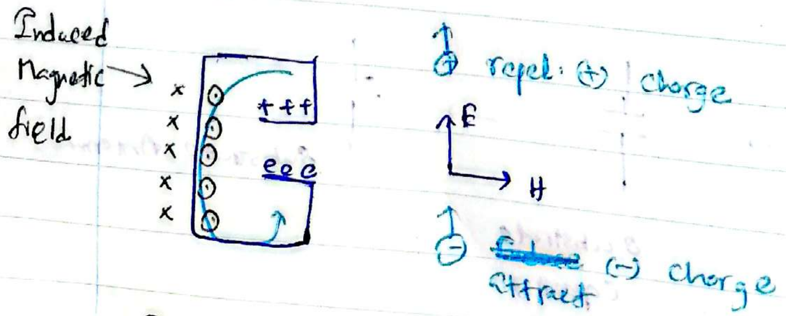
- Inductive and Capacitive elements that from a resonator
- Excited by an ~~elem~~ external electromagnetive wave
- Can act as point sourse for filters and metamaterials

metamaterials are mostly imaginary materials, in these meta[materials]

$\varepsilon_r < 0$

$\mu_r < 0$ These metamaterials can be implemented

$\varepsilon_r \mu_r < 0$ Using this split-ring resonators



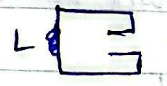
A current and magnetic field will be induced in the ring when excited by an external field

In this induced M. field, $\mu < 0$

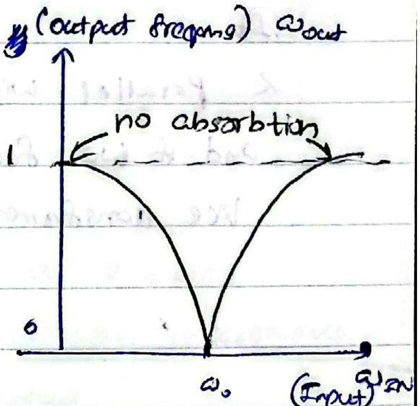
No :

Date: / /

In this, split-Ring resonator,

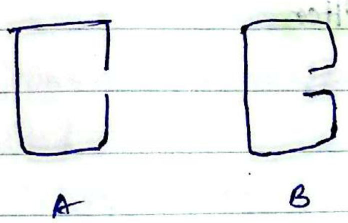
$\xrightarrow{}$ ω Input  $\longrightarrow$ ω output

resonante frequency $= \omega_0 = \frac{1}{\sqrt{LC}}$



- Maximum absorbtion of energy of the Input wave (resonance freq)
- So, the output will be zero

So this can act as Band stop filter

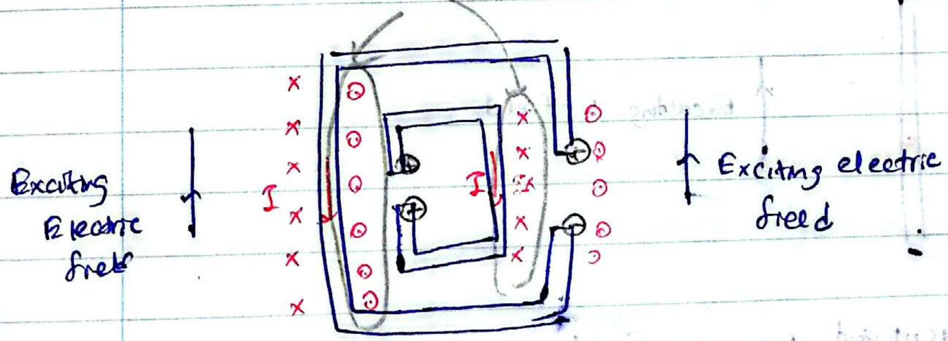


A → capacitive part is less and B is large,

So, Absorbtion of A is lower then B

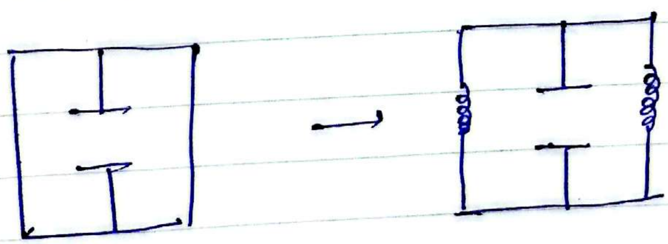
If we have 2 split-rings and one inside the other,

This two will cancel out

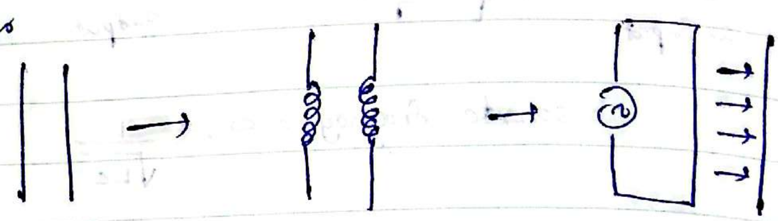


This is Asymetric Split-rings.

If we need Symmetric split ring resonator

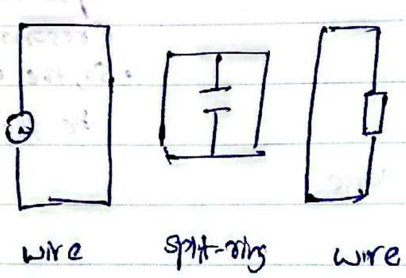


2 parallel wires act as transformer in High frequencies and if we fed to one wire, it will be leak to the other one like transformer



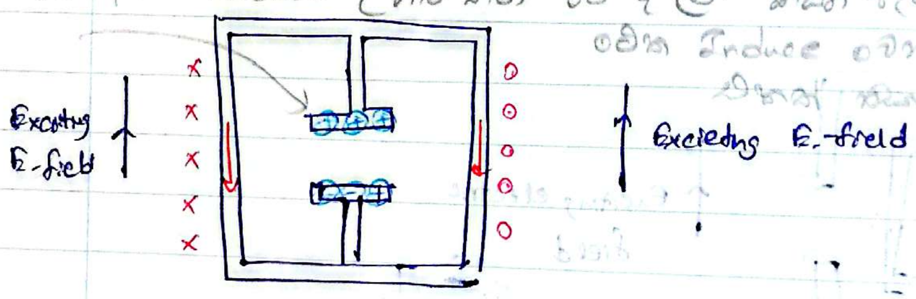
Band stop filter

Microwave ~~capp~~ coupler. or Band pass filter



$\omega = \omega_0 \rightarrow$ max. power transfer to the load.

එම ස්ථානයේ Induce [illegible] current [illegible] එවිට Induce වෙනවා external EM field [illegible]



Induced current and Magnetic field

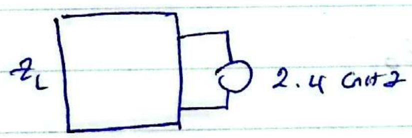
* This arrangement is perfectly semmetric and cancel out the magnetic fields within the ring
* Dimentions are in the millimeter / μm range

Resonant Cavities

- Main example → Microwave oven. ('hollow chamber)

• water molecules have the highest absorbtion at 2.4 GHz.
• most foods have water molecules and, it will absorb microwave energy and from that, it will make food heatup
• For these frequencies there is no licence and it is in ISM Band

- contain the microwaves and allow maximum energy transfer to the food.
- Sourse of the microwave in microwave oven called magnetron Sourse. and it is a oscillating microwave tube. it can have 1kW power.
- In this oven has transparent fine meash door Conductors Contained In and Sn.
- and Cavity has to be a metal.



$P_s \rightarrow Z_L$ cavity has to resonate.

* $\omega \neq 2.4$ GHz, energy from the magnitrom will end up reflected back to the magnitron and It will absorb by wall (Becouse of no resonance)
* If it resonate properly foods will absorb the power

Mode Summary,

From $k^2 = k_x^2 + k_y^2 + k_z^2$ the angular cutoff frequency ω_{mnl} can be obtained

$$f_{mnl} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{d}\right)^2} \longrightarrow (A)$$

2.4GHz,

a — length of cavity, b — height of cavity, d — width of Cavity

The ~~de~~ Resonance frequency depends on the physical dimensions of the Cavity

$$c \text{ (speed of the light)} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

c will come down, when relative permeability and permittivity is change

$$v = \frac{1}{\sqrt{\mu_r \mu_0 \varepsilon_r \varepsilon_0}}$$

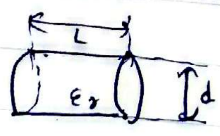
Fundermental Mode

The Fundermental mode of a cavity is the mode with the lowest frequency that satisfies above — (A) equation

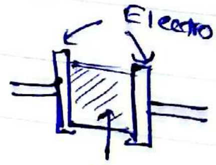
γ_2

Dielectric Resonators

* All resonators has very large ε_r (main common thing on Dielectric Resonators)



Dielectric Crystal



Resonator

- commonly used as circuit oscillators at microwave frequencies
- The material has a very large ε_r

[illegible]

- Kaifez and Guillon empirical formula for the resonance frequency of mode TE_{01} is given by.

$$f_{GHz} = \frac{34}{d\sqrt{\varepsilon_r}}\left(\frac{d}{L} + 3.45\right)$$ ← only valid for Cylindrical resonators

- where, $0.5 < \frac{d}{L} < 2$ and $30 < \varepsilon_r < 50$
- d, L are in (mm) Units.
- Valid for cylindrical Cristals

Ex: $L = 3\text{ mm}$, $d = 5\text{ mm}$, $\varepsilon_r = 40$.

$$f_{GHz} = \frac{34}{5\sqrt{40}}\left(\frac{5}{3} + 3.45\right)$$

$$= \frac{34}{5\sqrt{40}}\left(5/3 + 3.45\right)$$

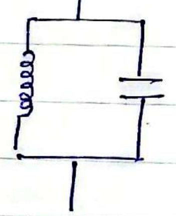
$$= 5.50 \text{ GHz}$$

Patch antennas only radiate for 180 (omni directional). But, slot antenna radiates for 360°.

Date 2025/06/18

Patch Antenna

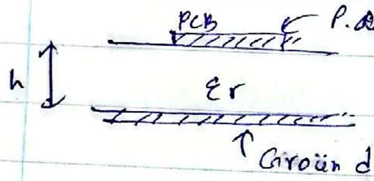
- A type of resonant antenna (resonant microstrip antenna)
- This is a substrate antenna (omni directional for 180°)



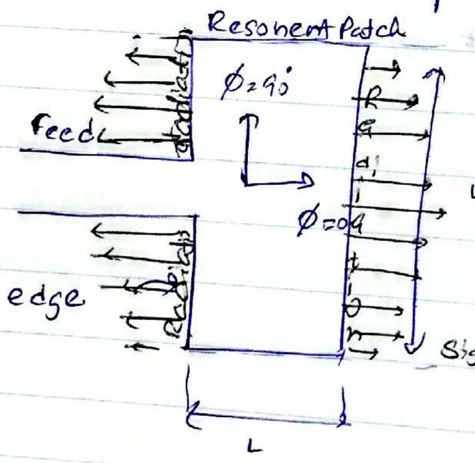
$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Q value → BW

මෙතනට තව කෑලි Add වෙන්න ඕන slider වලට



- edge fed rectangular patch antenna.



Mobile rada also have patch antenna (radar dosent want wide bandwidth) → HB100 → Dopler radar motion detector. (10.2 GHz) This also have cavity resonator. this can be adjust between 10.7 GHz – 10.2 GHz

Parameters of the antenna

Radiative ...
ΔL has to be ...
Clear of any
components

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* The patch has an effective length

$$L_{eff} = L + 2\Delta L$$

Find L W for a given ω_0.

Design according to the Carver and Mink equations for a required resonance frequency f_0.

The width of the patch is given by W,

$$\boxed{W = \frac{c}{2f_0}\sqrt{\frac{2}{\varepsilon_r + 1}}}$$

resonance frequency.

The effective primittivity along L is different to that of W due to fringe effects and is given by,

$$\boxed{\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2\sqrt{1 + \frac{12h}{W}}}}$$

for future example,

Step ① $h = 1.8\,mm$ $\varepsilon_r = 4.2$ $f_0 = 10.7\,GHz$.

find W,

$$W = \frac{3\times10^8}{2\times10.7\times10^9}\sqrt{\frac{2}{4.2+1}}$$

$$= 8.69\times10^{-3}\ m$$

$$= 8.69\ mm \simeq 8.7\ mm$$

Step ②, find L_{eff} for this firstly we need ε_{eff} and ΔL

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \underbrace{\frac{\varepsilon_r - 1}{2\sqrt{1 + \frac{12h}{W}}}}_{\text{for } \Delta L \text{ part}}$$

$$= \frac{4.2+1}{2} + \frac{4.2-1}{2\sqrt{1 + \frac{12\times1.8}{8.7\times10^{-3}}}}$$

$$\cancel{= 2.6 +}$$

$$= 3.457$$

Atlas

The resut in a fring corect

$$\Delta L = \frac{0.412h\left(\varepsilon_{eff} + 0.3\right)\left(w + 0.264h\right)}{\left(\varepsilon_{eff} - 0.258\right)\left(w + 0.8h\right)}$$

$$= \frac{0.412 \times 1.8 \times 10^{-3}\left(3.457 + 0.3\right)\left(8.7 + 0.264 \times 1.8\right)}{\left(3.457 - 0.258\right)\left(8.7 \times 10^{-3} + 0.8 \times 1.8\right)}$$

$$= \cancel{0.296\ mm}\ 0.78$$

The effictive length of the patch is, L_{eff}.

$$L_{eff} = L + 2\Delta L \longrightarrow L = \frac{c}{2f_0\sqrt{\varepsilon_{eff}}} - 2\Delta L$$

$$L = \frac{3 \times 10^8}{2 \times 10.7 \times 10^9 \sqrt{3.457}} - 2\Delta L.$$

$$= 7.53 \times 10^{-3}$$

$$= 7.54\ mm - 2 \times 0.78 = 5.98\ mm.$$

$$L_{eff} = 7.54\ mm$$

Step③ The feed impedance is approximately

$$Z_{IN} = \frac{90}{\varepsilon_r - 1}\left(\frac{\varepsilon_r L}{W}\right)^2$$

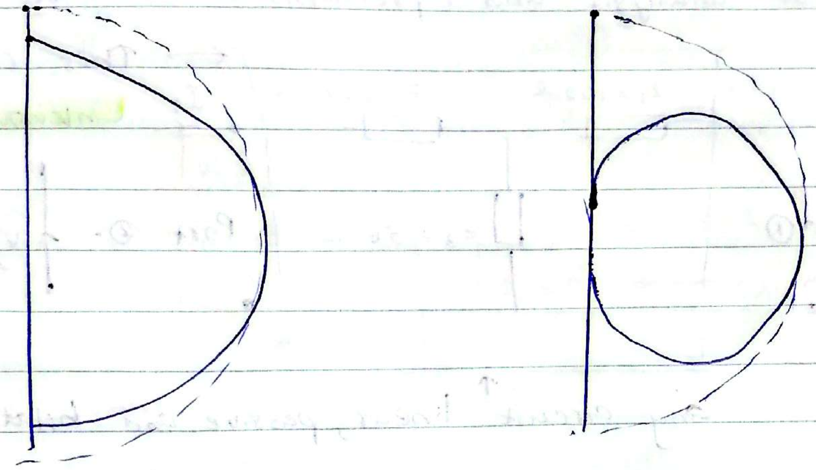
$$= \frac{90}{4.2 - 1}\left(\frac{4.2 \times 5.98 \times 10^{-3}}{8.7 \times 10^{-3}}\right)^2$$

$$\approx 234.39\ \Omega$$

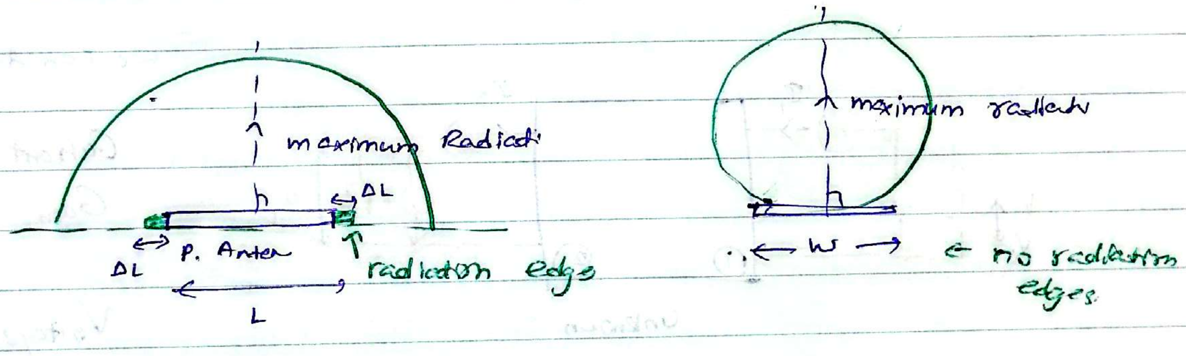
$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \qquad \boxed{\text{sinc}\,\theta \equiv \frac{\sin\theta}{\theta}}$$

$$\lim_{\theta \to 0} \frac{\cos\theta}{1} = 1$$

Radiation Pattern → ominidirectional Radiation pattern (180°)



$\phi = 0$ is along L $\phi = \pi/2$ is along W



ominidirectional cosine

* maximum radiation is prependicular to patch

Obtained from Hammor et al. for free space wavelength λ

where $k = \frac{2\pi}{\lambda}$

$$\psi_W = \frac{kW}{2} \sin\theta \sin\phi$$

$$\psi_L = \frac{kL}{2} \sin\theta \cos\phi$$

which results in

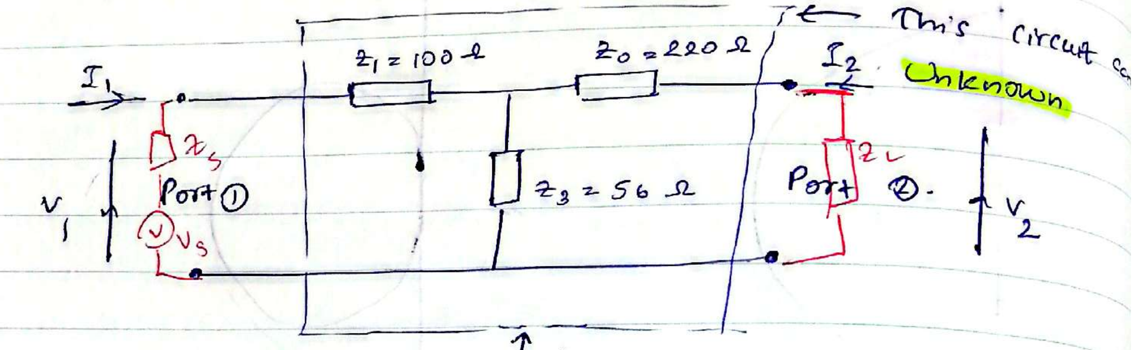
$$E_\theta(\theta,\phi) = E_h \,\text{sinc}(\psi_W) \cos(\psi_L) \cos\theta \sin\phi$$

$$E_\phi(\theta,\phi) = E_h \,\text{sinc}(\psi_W) \cos(\psi_L) \cos\phi$$

where $E_h = \cos(kh\cos\theta)$ is the slot image factor

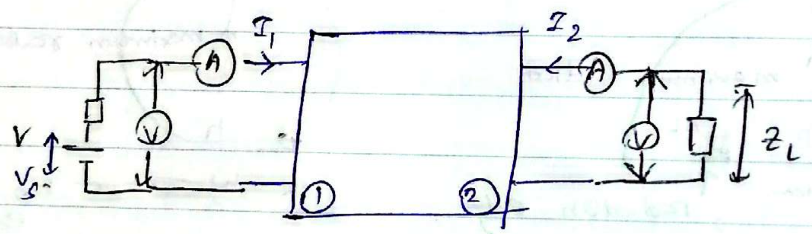
Two-Port Networks → Scattor parameters.

- Method of analysis and representaion of a circuit.



any circuit linear, passive and bilatterald.

for conventional we always get current flow in to the port.



Unknown

I_1, I_2, V_1, V_2 are measurable

We can define,

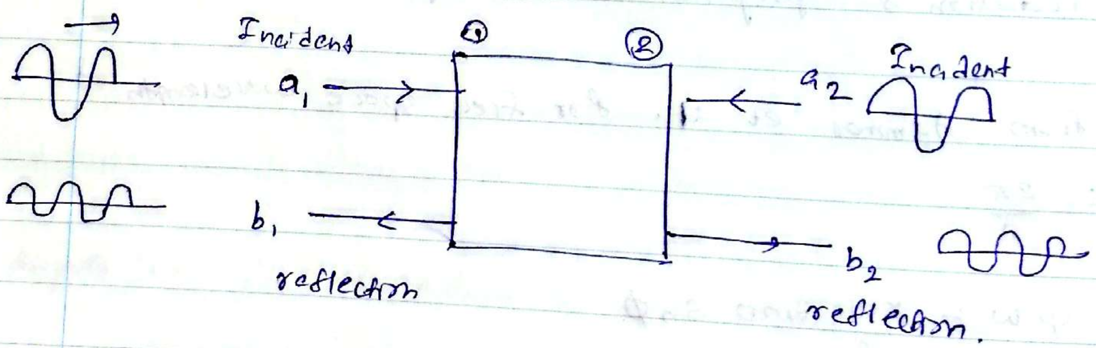
Current Gain $A_I =$

Voltage gain =

$A_V = \frac{V_2}{V_1}$

Overall Voltage gain

$A_S = \frac{V_2}{V_S}$



a_1, a_2 are incident voltage signals

b_1, b_2 are reflected voltage signals

Two-Port Network discription

$$V_{11} = Z_{11} I_1 + Z_{12} I_2 \quad \text{—} ①$$

$$V_{22} = Z_{21} I_1 + Z_{22} I_2 \quad \text{—} ②$$

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Finding $Z_{11}, Z_{12}, Z_{21}, Z_{22}$

$$V_{11} = Z_{11} I_1 + \underbrace{Z_{12} I_2}_{0} \leftarrow I_2 = 0 \text{ by making open circuit at port 2}$$

$V_1 = Z_{11} I_1$

from ① & ②

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = 156\,\Omega \leftarrow \text{Open circuit test for port ②}$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = 276\,\Omega$$

for above circuit,

$$Z_{11} = 100 + 56 = \cancel{106}\ 156\,\Omega = V_1/I_1$$

$$Z_{22} = 220 + 56 = 276\,\Omega$$

$$Z_{21} = \frac{V_2}{I_1}$$

$$Z_{21} = 56\,\Omega \qquad V_2 = \frac{56}{56+100} V_1$$

$$Z_{12} = 56\,\Omega \qquad V_2 = \frac{56}{156} V_1$$

③ — $V_2 = Z_L(-I_2)$ $\qquad \cancel{\frac{156 \times 2}{56}}$ $\qquad V_2 = 56 \times \frac{V_1}{156}$

from, ② = ③,

$$Z_L(-I_2) = Z_{21} I_1 + Z_{22} I_2 \qquad V_2 = 56 \times I_1$$

$$Z_{21} I_1 = (Z_{22} - Z_L) I_2 \qquad Z_{21} = Z_{12} = \frac{V_2}{I_1} = 56.$$

Current gain

$$A_I = \frac{I_2}{I_1} = \frac{-Z_{21}}{(Z_{22} - Z_L)}$$

if $Z_L = 1\,k\Omega$, $\quad A_I = \frac{-56}{276 + 1000}$

$$A_I = -0.0438 = \frac{I_2}{I_1} = \frac{I_2}{I_1}$$
$$= 0.044$$

$$\frac{V_2}{V_1} = \frac{Z_{21} I_1 + Z_{22} I_2}{Z_{11} I_1 + Z_{12} I_2}$$

$$\frac{Z_{21} I_1 + Z_{22} I_1 \left(\frac{-Z_{21}}{Z_{22} - Z_L}\right)}{Z_{11} I_1 + Z_{12} I_1 \left(\frac{-Z_{21}}{Z_{22} - Z_L}\right)} = \frac{56 \cancel{I_1} + 276 \times \cancel{I_1} (\bar{0}.044)}{156 \cancel{I_1} + 56 \times \cancel{I_1} (-0.044)}$$

$$A_v = \frac{V_2}{V_1} = \cancel{5.56}\ \underline{\underline{0.285}} \leftarrow \text{for } 1k\Omega \text{ Load resistor Voltage Gain}$$

$$\boxed{A_I = \frac{I_2}{I_1} = \left(\frac{-Z_{21}}{Z_{22}+Z_L}\right)}$$

$$A_V = 0.285 = \frac{V_2}{V_1}$$

$$A_V = \frac{Z_{21} + Z_{22}\left(\frac{-Z_{21}}{Z_{22}+Z_L}\right)}{Z_{11} + Z_{12}\left(\frac{-Z_{21}}{Z_{22}+Z_L}\right)}$$

$$\boxed{A_V = \frac{Z_{22} Z_L}{Z_{11}Z_{22} - Z_{12}Z_{21} + Z_{11}Z_L}}$$

$V_S \rightarrow ?$

$$A_S = \frac{V_2}{V_S}$$

$$V_S = Z_S I_1 + V_1$$

$$V_S - Z_S I_1 = V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_S - Z_S I_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_S = (Z_{11} + Z_S) I_1 + Z_{12} I_2$$

$$A_S = \frac{V_2}{V_S} = \frac{Z_{21} I_1 + Z_{22} I_2}{(Z_{11} + Z_S) I_1 + Z_{12} I_2}$$

$$= \frac{Z_{21} + Z_{22}\, I_2/I_1}{(Z_{11} + Z_S) + Z_{12}\, I_2/I_1}$$

$$= \frac{Z_{21} + Z_{22} A_I}{Z_{11} + Z_S + Z_{12} A_I}$$

$$= \frac{Z_{21} + Z_{22}\left(\frac{-Z_{21}}{Z_{22}+Z_L}\right)}{Z_{11} + Z_S + Z_{22}\left(\frac{-Z_{21}}{Z_{22}+Z_L}\right)}$$

$$= \frac{56 + 276 \times 0.044}{156 + 100 + 276 \times 0.046}$$

$$= \frac{Z_{21} Z_L}{(Z_{11} + Z_S)(Z_{22} + Z_L) - Z_{12} Z_{21}}$$

$$= \frac{56 \times 1000}{(156 + 100)(276 + 100) - 56 \times 56}$$

[illegible]

$$= 0.173$$

[illegible]

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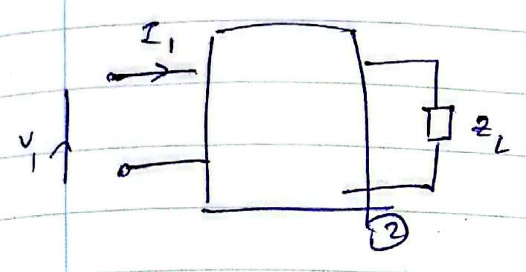

$$Z_{IN} = \frac{V_1}{I_1} \qquad (0 < Z_L < \infty)$$