# Resonant Devices and Antennas

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# Outline

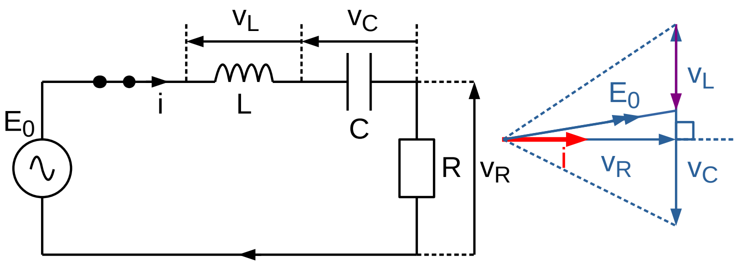
# Introduction

# Overview

- Resonance occurs when an electrical circuit introduces no phase shift between voltage and current of the exciting signal
  - i.e., there is no net reactance
- An important property at microwave frequencies
  - Cavity resonators
  - Oscillators
  - Filters
  - Resonant antennas

# Resonance

# Series RLC Circuit



# Resonance from Phasor Diagram

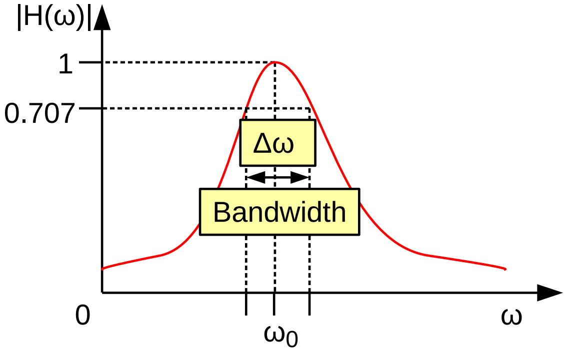
- When $V_L + V_C = 0 \Rightarrow X_L + X_C = 0$ the circuit will resonate
  - $E_0 = V_R$
  - The voltage across $R$ will be at its maximum
- The angular frequency of resonance

$$X_L + X_C = 0 \Rightarrow j\omega L + \frac{(-j)}{\omega C} = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

- The frequency is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

# Frequency Response



# Frequency Response (Contd..)

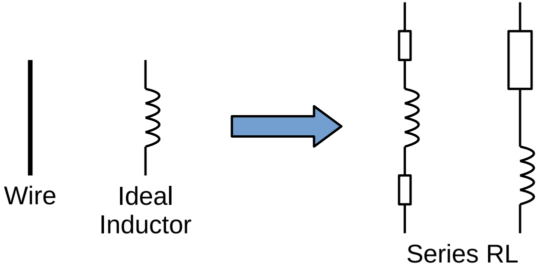
- The bandwidth of the filter is $\Delta\omega$
- Which is given by

$$\Delta\omega = \frac{\omega_0}{Q}$$

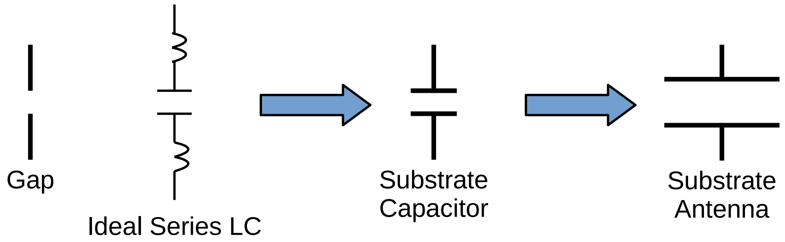
where $Q$ is the quality factor of the resonance circuit given by

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow \Delta\omega = \frac{R}{L}$$

# High Frequency Inductors

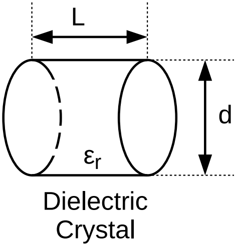


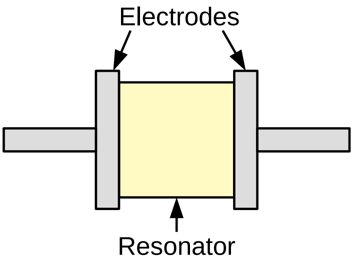
# High Frequency Capacitors



# Resonant Devices

# Dielectric Resonators




- Commonly used as circuit oscillators at microwave frequencies
- The material has a very high $\epsilon_r$
  - Typicall derived from complex titanium oxides of barium, magnesium, calcium and rare earth metals

# Dielectric Resonators (Contd..)

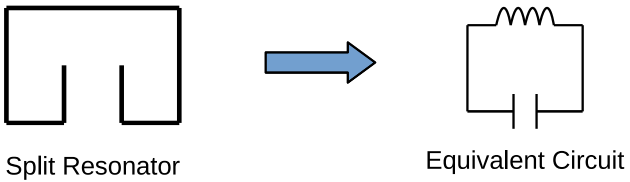
- Kajfez and Guillon empirical formula for the resonance frequency of mode $TE_{011}$ is given by

$$f_{GHz} = \frac{34}{d\sqrt{\varepsilon_r}}\left(\frac{d}{L} + 3.45\right)$$

where $0.5 < \frac{d}{L} < 2$ and $30 < \varepsilon_r < 50$

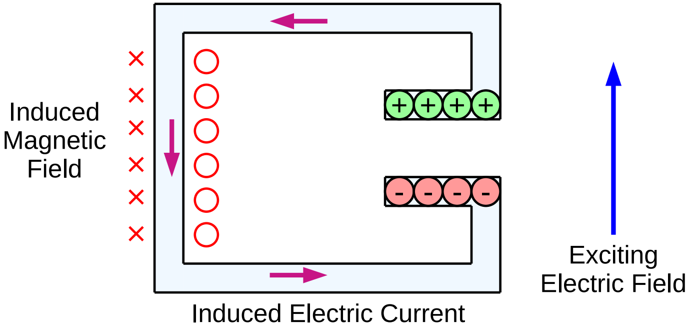
- Valid for cylindrical crystals

# Split-Ring Resonators



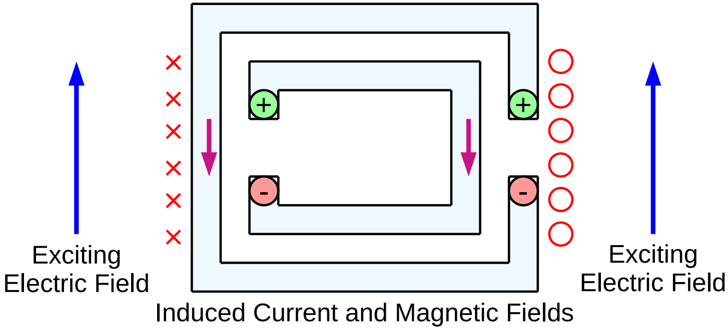

- Inductive and capacitive elements that form a resonator
- Excited by an external electromagnetic wave
- Can act as point sources for filters and metamaterials

# Split-Ring Resonators (Contd..)



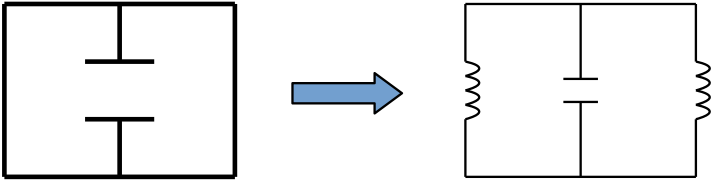

- A current and magnetic field is induced in the ring when excited by an external field

# Split-Ring Resonators (Contd..)



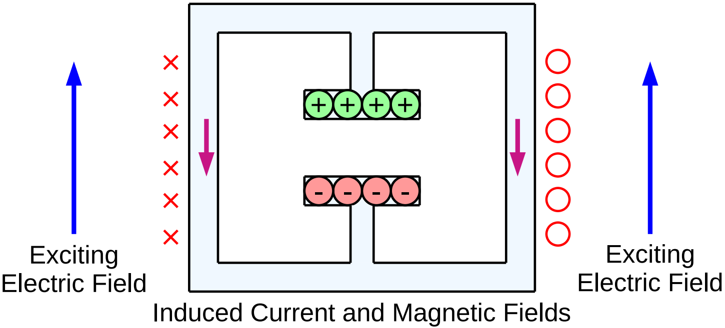

- The dual split ring arrangement cancels out the magnetic field within the rings

# Split-Ring Resonators (Contd..)



- Symmetric split ring resonators

# Split-Ring Resonators (Contd..)



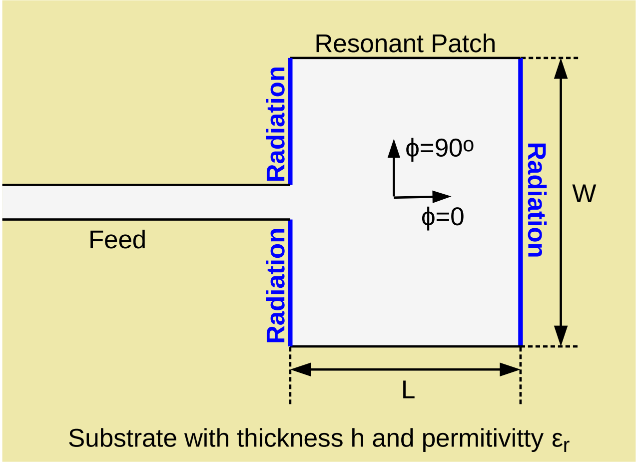

- The symmetric arrangement cancels out the magnetic field within the ring

# Patch Antennas

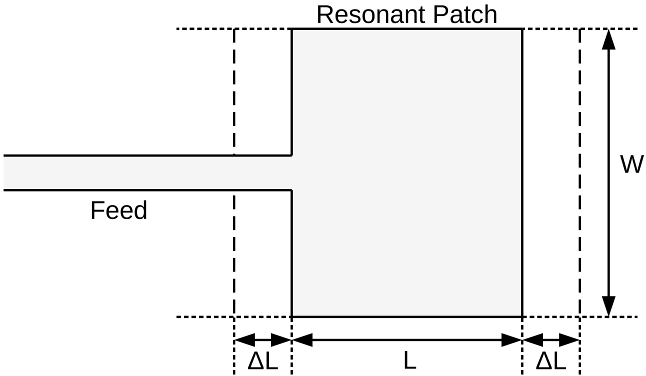
# Introduction

- The patch antenna is a resonant microstrip antenna
  - Modelled as a resonant cavity
  - Bandwidth depends on the $Q$ value
  - Not suitable for wideband applications
- Consists of a ground plate with substrate of thickness $h$ and relative permitivitty $\varepsilon_r$
- Most common patch is the rectangular patch

# Operation


Resonant Patch
Radiation
Radiation
Radiation
ϕ=90o
ϕ=0
W
Feed
L
Substrate with thickness h and permitivitty εr

# Parameters



# Patch Antennas Design

- Designed according to the Carver and Mink equations for a required resonance frequency $f_0$
- The width of the patch is given by

$$W = \frac{c}{2f_0}\sqrt{\frac{2}{\varepsilon_r + 1}}$$

- The effective permitivity along $L$ is different to that of $W$ due to fringe effects and is given by

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2\sqrt{1 + \frac{12h}{W}}}$$

# Patch Antennas Design (Contd..)

- This results in a fringe correction

$$\Delta L = \frac{0.412h(\varepsilon_{\text{eff}} + 0.3)(W + 0.264h)}{(\varepsilon_{\text{eff}} - 0.258)(W + 0.8h)}$$

- The effective length of the patch is given by

$$L_{\text{eff}} = L + 2\Delta L \Rightarrow L = \frac{c}{2f_0\sqrt{\varepsilon_{\text{eff}}}} - 2\Delta L$$

- The feed impedance is approximated by

$$Z_{IN} = \frac{90}{\varepsilon_r - 1}\left(\frac{\varepsilon_r L}{W}\right)^2$$

# Radiation Pattern

- Obtained from Hammer et al. for free space wavelenth $\lambda$ where $k = \frac{2\pi}{\lambda}$

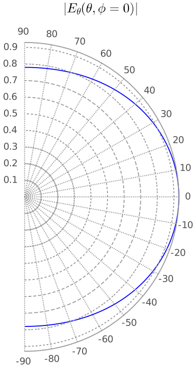
$$\psi_W = \frac{kW}{2} \sin(\theta) \sin(\phi)$$
$$\psi_L = \frac{kL}{2} \sin(\theta) \cos(\phi)$$

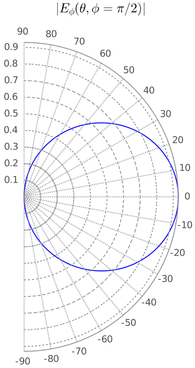
- Which results in

$$E_\theta(\theta, \phi) = E_h \mathsf{sinc}(\psi_W) \cos(\psi_L) \cos(\theta) \sin(\phi)$$
$$E_\phi(\theta, \phi) = E_h \mathsf{sinc}(\psi_W) \cos(\psi_L) \cos(\phi)$$

where $E_h = \cos(kh\cos(\theta))$ is the slot image factor

# Radiation Pattern (Contd..)




- $\phi = 0$ is along $L$ and $\phi = \pi/2$ is along $W$

# Exercise

Using the Carver and Mink equations, obtain the parameters and feed impedance for a patch antenna that will resonate at 10.7 GHz when fabricated on a PCB with $\varepsilon_r = 4.2$ and thickness of 1.8 mm.

# Conclusion

# Conclusion

- Resonance has multiple applications in microwave engineering
  1. As resonant cavities in microwave ovans
  2. As dielectric resonators in microwave circuits
  3. Split ring resonators as filters and metamaterials
  4. In resonant antennas such as the patch antenna
- Bandwidth is determined by the Q value