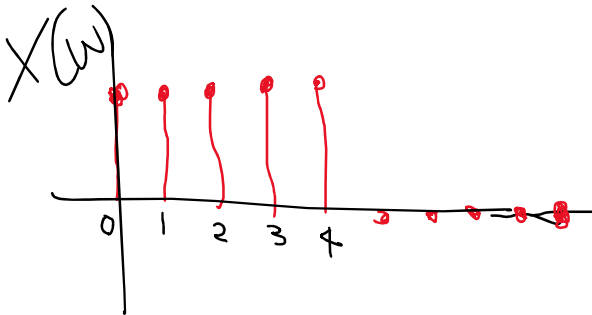


Example : DTFT



$$\begin{aligned}
 X(\omega) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \\
 &= \sum_{n=0}^5 e^{-j\omega n} \Rightarrow \frac{1-e^{-5j\omega}}{1-e^{-j\omega}} \Rightarrow \frac{e^{-5j\omega/2} \left[e^{5j\omega/2} - e^{-5j\omega/2} \right]}{e^{-j\omega/2} \left[e^{j\omega/2} - e^{-j\omega/2} \right]} \\
 &= e^{-2j\omega} \frac{2j \sin(5\omega/2)}{2j \sin(\omega/2)} \Rightarrow e^{-2j\omega} \frac{\sin(5\omega/2)}{\sin(\omega/2)}
 \end{aligned}$$

Let's take DFT of length of 10

$$\begin{aligned}
 X[k] &= \sum_{n=0}^9 x[n] \underbrace{e^{-j\frac{2\pi}{10}kn}}_{W_{10}} \\
 &= \sum_{n=0}^4 e^{-j\frac{2\pi}{10}kn} = \sum_{n=0}^4 W_{10}^{kn} \\
 &= \frac{1-W_{10}^{5k}}{1-W_{10}} \Rightarrow \frac{1-e^{-j\frac{10\pi}{10}k}}{1-e^{-j\frac{2\pi}{10}k}} \\
 &= \frac{\begin{bmatrix} e^{-j\frac{\pi}{10}k} & e^{-j\frac{9\pi}{10}k} \end{bmatrix} \begin{bmatrix} e^{-j\frac{\pi}{10}k} \end{bmatrix}}{\begin{bmatrix} e^{-j\frac{\pi}{10}k} \end{bmatrix} \begin{bmatrix} e^{j\frac{\pi}{10}k} & e^{-j\frac{\pi}{10}k} \end{bmatrix}} = e^{-j\frac{4\pi}{10}k} \frac{\sin(k\pi/2)}{\sin(k\pi/2)}
 \end{aligned}$$

From DTFT to DFT :

$$\text{Let's take } \omega = \frac{2\pi k}{10} = e^{j\frac{2\pi k}{10}} \frac{\sin\left(\frac{\pi k}{2}\right)}{\sin\left(\frac{\pi k}{10}\right)}$$

So what I wanted to show is when I sample DTFT I get DFT values.