GENERAL SIR JOHN KOTELAWALA DEFENCE UNIVERSITY

Faculty of Engineering

Department of Electrical, Electronic and Telecommunication Engineering

BSc Engineering Degree

End Semester Examination – June/July 2020

Semester 5 - Intake 35 (ET/MC)

ET 3142– DIGITAL SIGNAL PROCESSING

Time allowed: 2 hours

13th July 2020

ADDITIONAL MATERIAL PROVIDED

Fourier Transform Theorems (page 07)

INSTRUCTIONS TO CANDIDATES

- This paper contains **4** questions on **6** pages
- **Answer all questions**
- This is a closed book examination
- This examination accounts for 100% of the module assessment. A total maximum mark obtainable is 100. The marks assigned for each question and parts thereof are indicated in square brackets
- If you have any doubt as to the interpretation of the wordings of a question, make your own decision, but clearly state it on the script
- Assume reasonable values for any data not given in or provided with the question paper, clearly make such assumptions made in the script
- All examinations are conducted under the rules and regulations of the KDU

[Total 25 marks]

1

a) Consider that the input signal to a discrete-time signal is $x[n]$. The system output a signal $y[n]$ as highlighted in Figure-1 below.

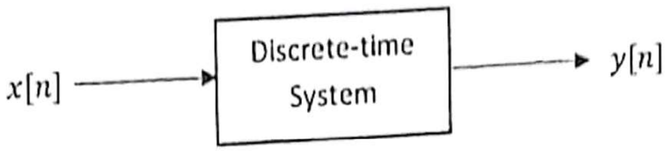


Figure 1

Determine whether the each of the systems below are linear and time-invariant

(5 marks)

i) $$y[n] = \sum_{k=n_0}^{n} x[k]$$

(5 marks)

ii) $$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$$

(5 marks)

iii) $y[n] = e^{x[n]}$

(5 marks)

iv) $y[n] = x[n] + u[n+1]$

(5 marks)

b) Determine the impulse response of the following system

$$y[n] = x[n - n_0] \; h(1).$$

2

[Total 25 marks]

a) The definition of the discrete-time Fourier transform of a sequence $x[n]$ is given by

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jwn}$$

Using the definition, determine the Fourier transform of the following sequences ($u[n]$ refers to the unit step function).

i) $x[n] = 0.5^{n}\, u[n]$ (5 marks)

ii) $x[n] = 2^{n}\, u[-n]$ (5 marks)

b) If the Fourier transform of a sequence x[n] is $X(e^{jw})$, determine the Fourier transform of following sequence (You may use the Fourier transform theorems given in the appendix). (8 marks)

$$x_0[n] = x[n] - x[n-3]$$

c) Write the Frequency response of the liner time-invariant system input and output satisfy the following difference equation. (7 marks)

$$y[n] - \frac{1}{2}\, y[n-1] = x[n] + 2\, x[n-1] + x[n-2]$$

3 [Total 25 marks]

a) The definition of the discrete-time Z-transform of a sequence $x[n]$ is given by

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

Using the definition, determine the Z-transform and the Region of convergence (ROC) of the following sequences ($u[n]$ refers to the unit step function and $\delta[n]$ *refers to impulse function*).

i) $x[n] = 0.5^n\, u[n]$ (5 marks)

ii) $x[n] = 0.5^n\, u[-n]$ (5 marks)

iii) $x[n] = 0.5^n\, (u[n] - u[n-10])$ (5 marks)

iv) $x[n] = \delta[n]$ (5 marks)

b) Consider the Z-transform $X(z)$ of a sequence whose Pole-zero plot is given by Figure-2 below. If the Fourier transform of the sequence exists, determine the Region-of-convergence (ROC) of the Z-transform. (5 marks)

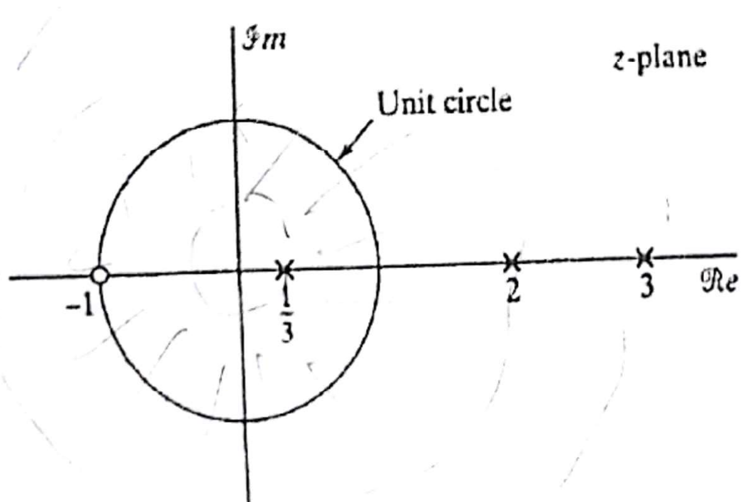


Figure 2

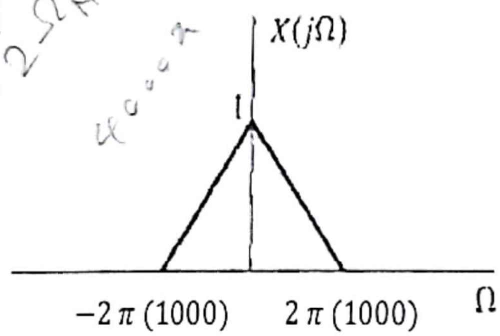
2 [Total 25 marks]

a) The continues-time signal (10 marks)

$$x_c(t) = sin\ (\ 2\pi\ (100)\ t)$$

Is sampled with sampling period $T = {}^{1}/_{400}$ second to obtain a discrete time signal $x[n]$. What is the resulting signal $x[n]$.

b) A signal has the Fourier transform given in Figure 3. What is the minimum sampling rate such that no aliasing occurs in the sampled signal? (10 marks)



c) The sequence (5 marks)

$$x[n] = cos\left(\frac{\pi\, n}{4}\right)$$

was obtained by sampling the continues-time signal

$$x_c(t) = cos(\Omega\, t)$$

with the sampling duration T. Write two possible values for Ω in terms of T, that results in the same sampled sequence

Appendix

FOURIER TRANSFORM THEOREMS

Sequence $x[n]$ $y[n]$	Fourier Transform $X(e^{j\omega})$ $Y(e^{j\omega})$
1. $ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
2. $x[n - n_d]$ (n_d an integer)	$e^{-j\omega n_d} X(e^{j\omega})$
3. $e^{j\omega_0 n} x[n]$	$X(e^{j(\omega-\omega_0)})$
4. $x[-n]$	$X(e^{-j\omega})$ $X^*(e^{j\omega})$ if $x[n]$ real.
5. $nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
6. $x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
7. $x[n]y[n]$	$\dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$