

General Sir John Kotelawala Defence University

# ET3122 Antennas and Propagation

## Antenna Arrays

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# Outline

## 1 Introduction

## 2 Special Array Formulations

- Beam Steering
- Image Arrays
- Earth Effects

## 3 The Yagi-Uda Array

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# Introduction

# Motivation

- Single element antennas generally have low directivity
  - ▶ Directive antennas like very long dipoles or longwire antennas can have unusable radiation patterns with multiple major lobes
- Combining unusable radiation patterns can result in better radiation patterns
  - ▶ Discone antenna
  - ▶ V and rhombic antennas
- Can combining radiation patterns be put to better use?

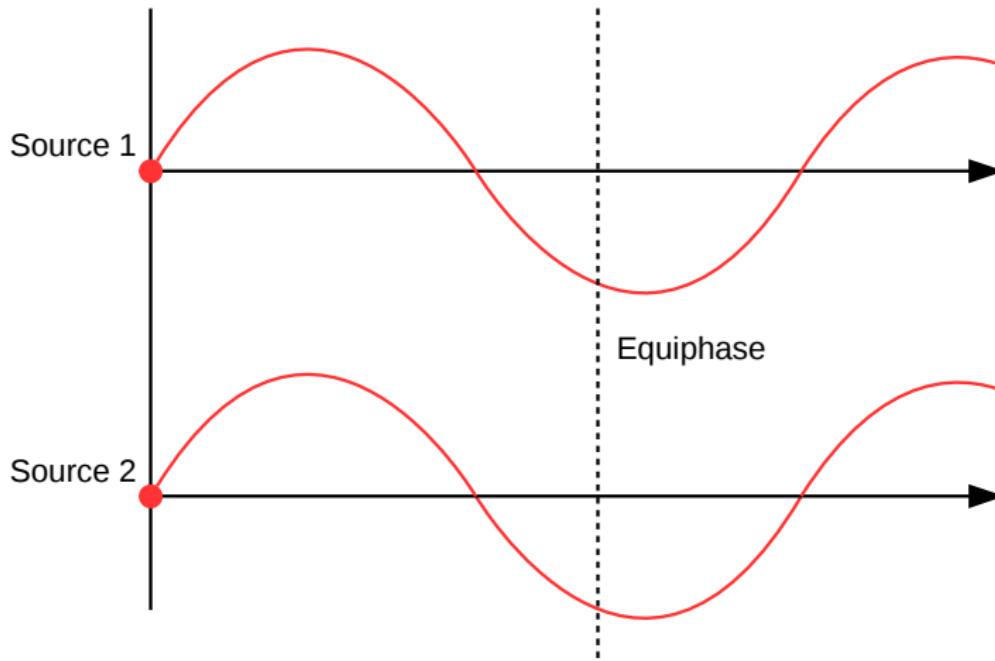
# Antenna Arrays

- An antenna array is a system that consists of multiple ( $\geq 2$ ) antennas operating at *the same frequency*
  - ▶ A log-periodic antenna is therefore not an array
- Each antenna is known as an element
  - ▶ An element can be active or passive (parasitic)
- Generally array elements are of the same type of radiator

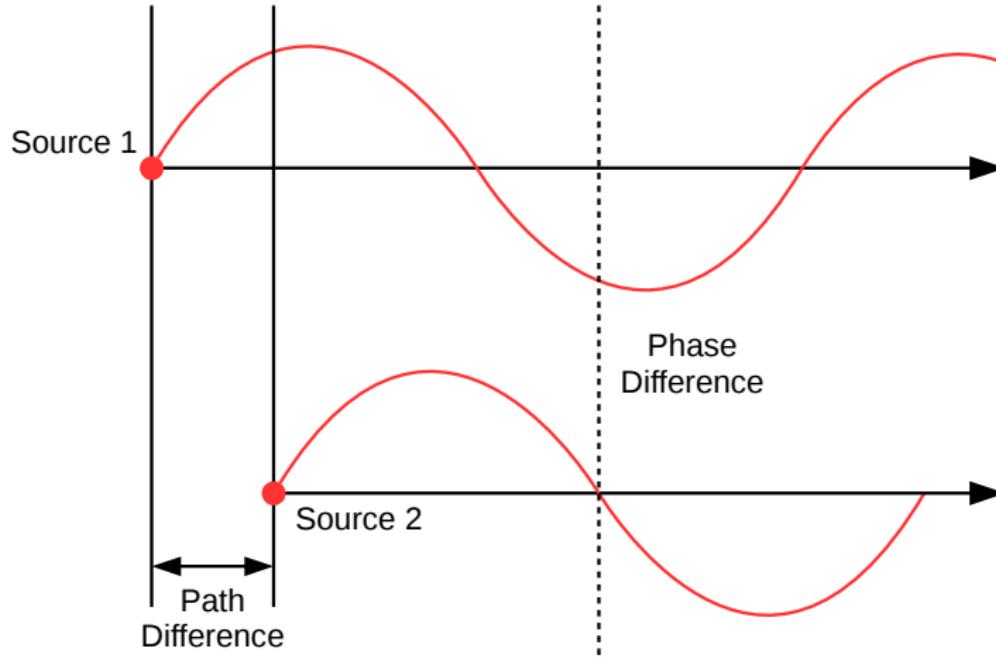
# Operating Principle

- Interference
- Caused by the spatial path difference between the transmitted waves of multiple sources
  - ▶ Results in a *phase difference* between the sources

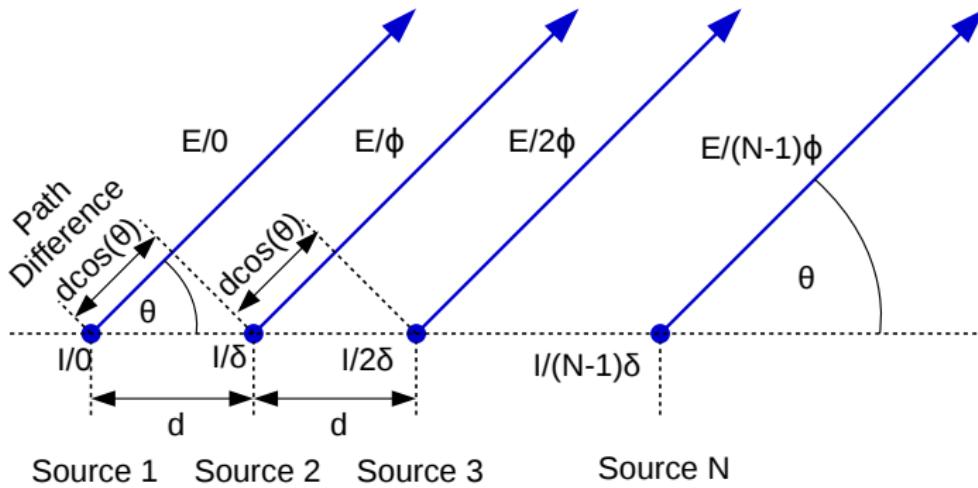
# Identical Path Propagation



# Propagation Path Difference



# Uniform Linear Isotropic Array



# Uniform Linear Isotropic Array (Contd..)

At the far field, the path difference between two adjacent sources,

$$\Delta d = d \cos(\theta)$$

Therefore, the resulting phase difference,

$$\Delta\phi = \frac{2\pi\Delta d}{\lambda} = \frac{2\pi d \cos(\theta)}{\lambda}$$

The total phase difference due to both the path difference and the phase difference between feed currents thus becomes,

$$\phi = \delta + \Delta\phi = \delta + \frac{2\pi d \cos(\theta)}{\lambda}$$

# Uniform Linear Isotropic Array (Contd..)

From superposition, the resultant E field is given by,

$$\begin{aligned}E_N &= E + Ee^{j\phi} + Ee^{j2\phi} + \cdots + Ee^{j(N-1)\phi} \\&= E \left[ 1 + e^{j\phi} + e^{j2\phi} + \cdots + e^{j(N-1)\phi} \right] \\&= E \left[ \frac{1 - e^{jN\phi}}{1 - e^{j\phi}} \right] = E \left( \frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)} \right) e^{j\frac{(N-1)\phi}{2}}\end{aligned}$$

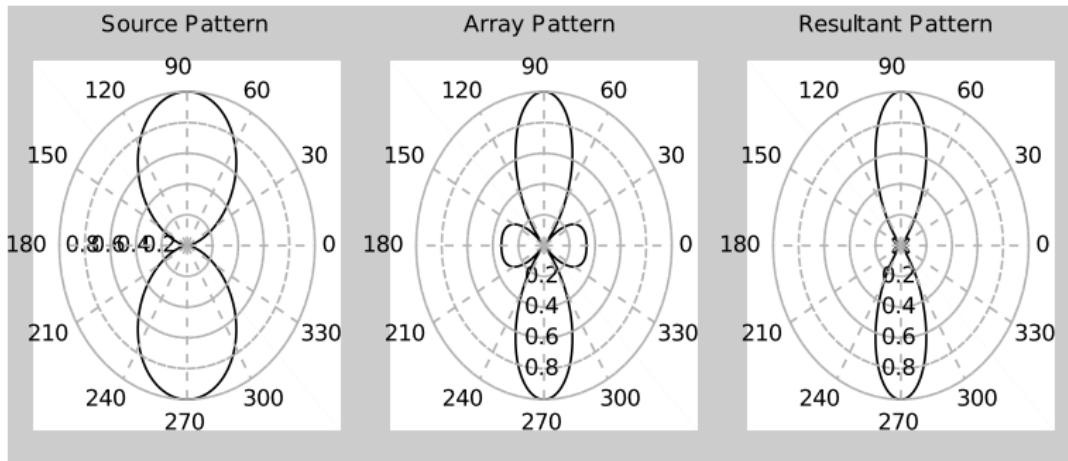
Therefore the amplitude variation of the array is given by,

$$|E_N| = E \left( \frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)} \right)$$

- $|E_N|$  is also known as the *array factor*

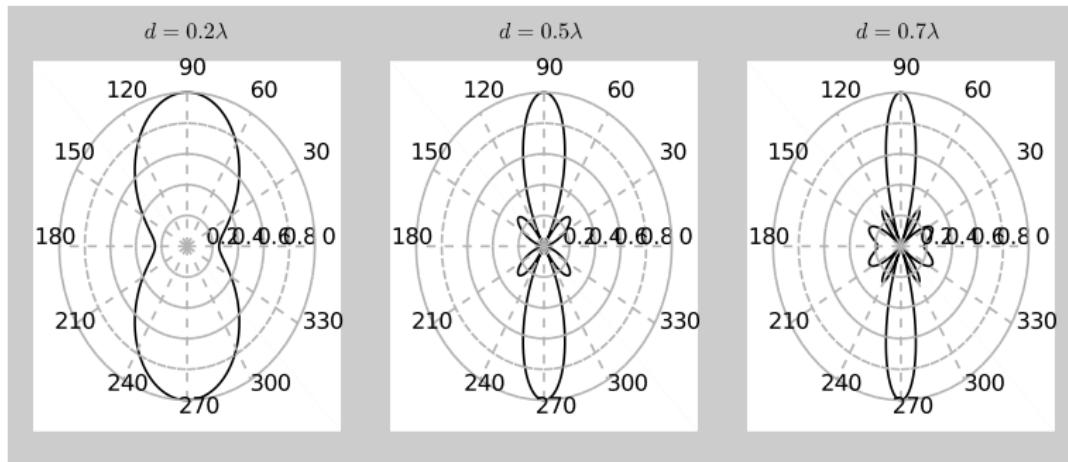
# Uniform Arrays of Anisotropic Sources

- For an anisotropic source ( $E_S(\theta)$ ), the *approximate* radiation pattern is given by  $E_R(\theta) = E_S(\theta) \times |E_N(\theta)|$  where  $E_N(\theta)$  is the pattern of the isotropic array with  $N$  elements.
- This is known as *pattern multiplication*



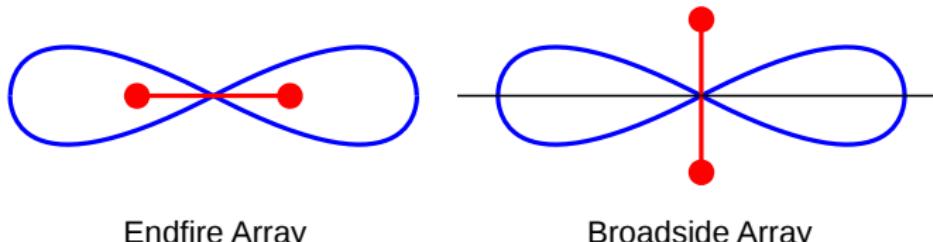
# Special Array Formulations

# Variation of Distance (N=4)



- Increased distance results in better directivity
  - ▶ Sidelobe energy increases with distance
  - ▶ Antenna also gets physically larger
- Usually  $d = \lambda/2$

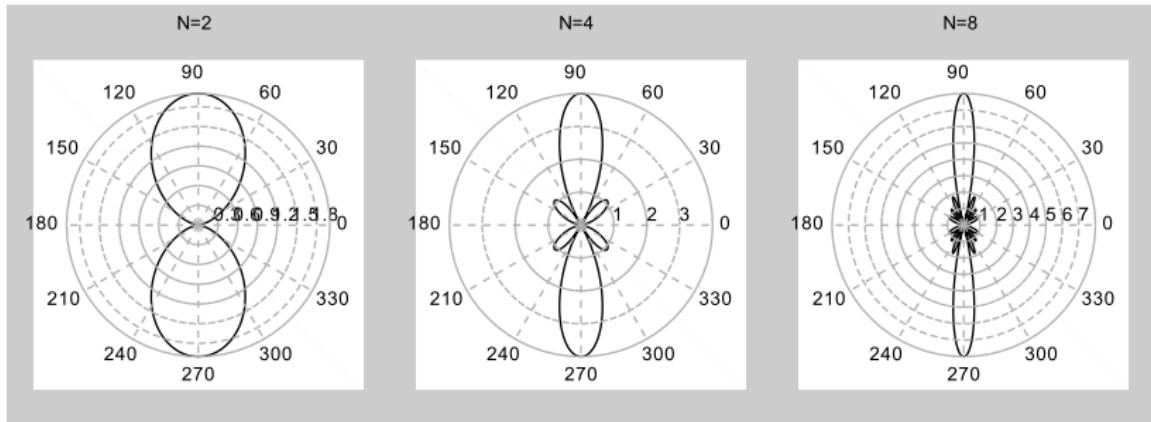
# Broadside and Endfire Arrays



When  $d = \lambda/2$ ,

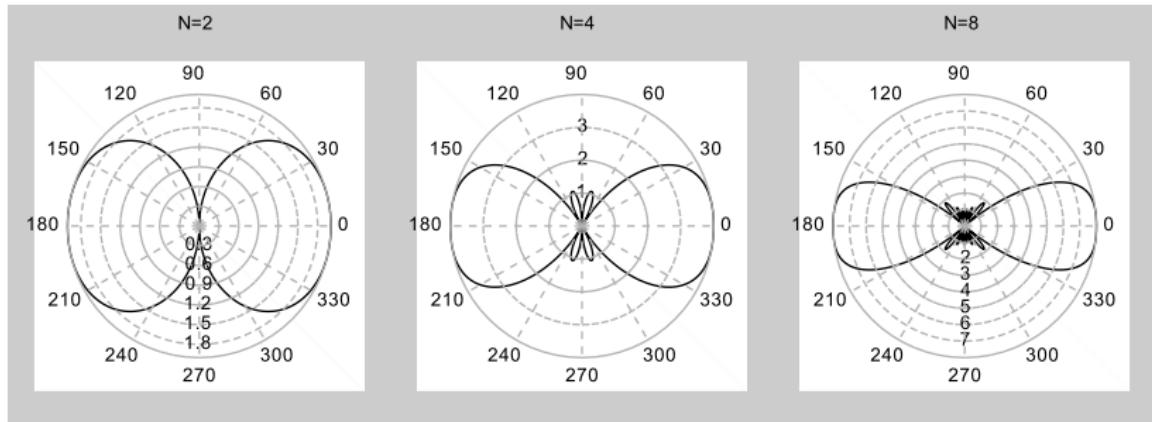
- Taking  $\delta = \pi$  results in an *endfire array* where the major lobes are along the array elements
- When  $\delta = 0$ , the array is *broadside* and the major lobes are perpendicular to the array elements

# Broadside Arrays



- Increased sidelobes and directivity with  $N$

# Endfire Arrays



- Increased sidelobes and directivity with  $N$
- Lesser directivity than broadside arrays

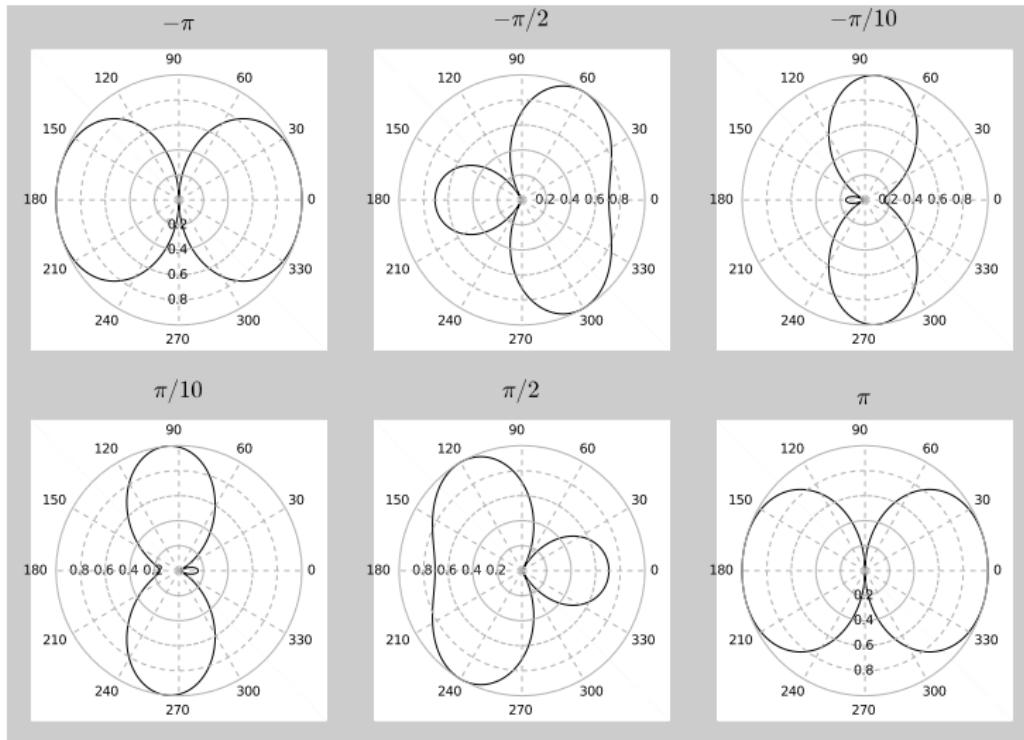
## Beam Steering

# Beam Steering

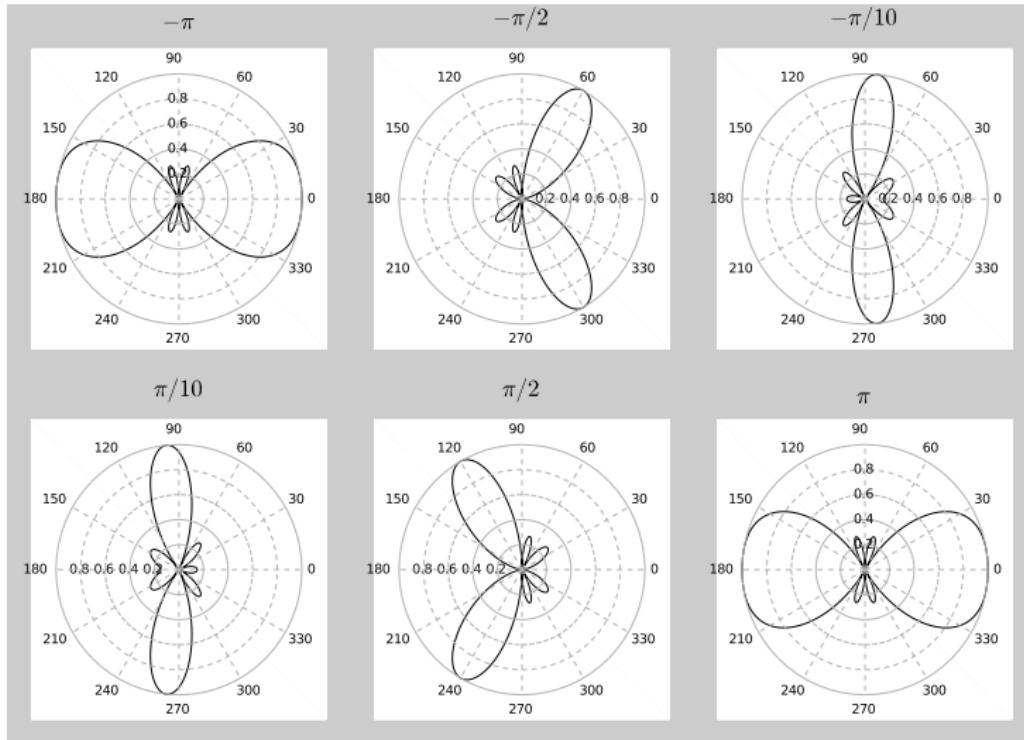
- Broadside ( $\delta = 0$ ) and endfire ( $\delta = \pi$ ) configurations are the two *extremes*
  - ▶ When  $\theta \in (0, \pi)$  the radiation pattern can be modified so that the major lobes are steered between the two extreme patterns
  - ▶ For large  $N$ , versatile steering can be achieved
- Enables the major lobes to be steered without physically rotating the antenna



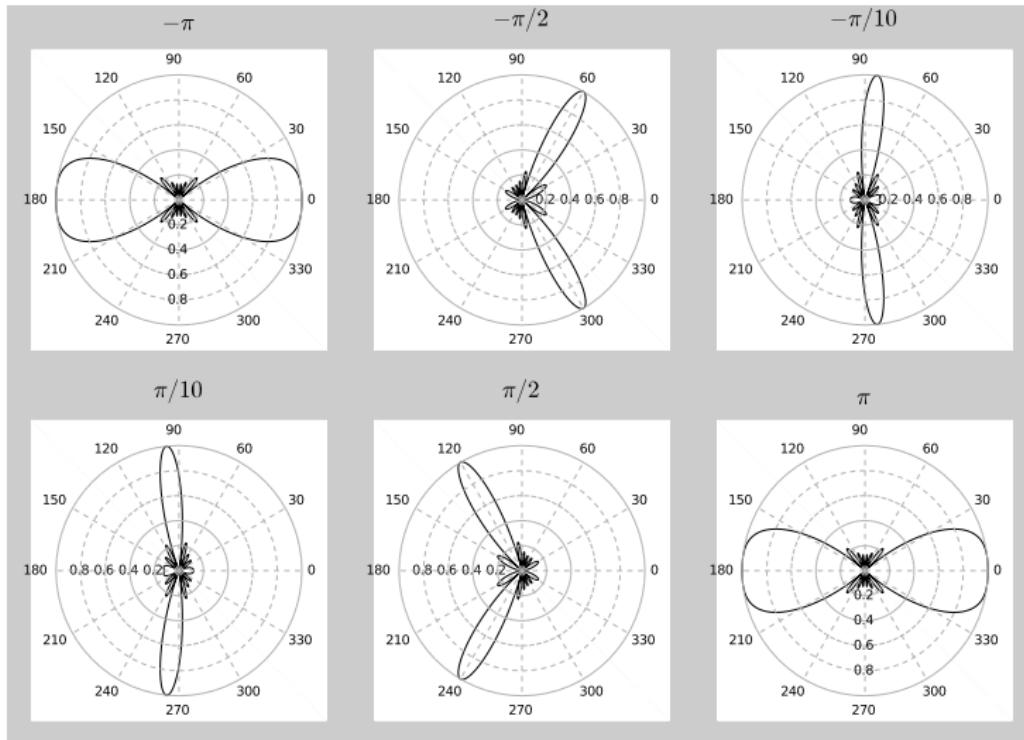
## Beam Steering

Beam Steering  $N = 2$ 

## Beam Steering

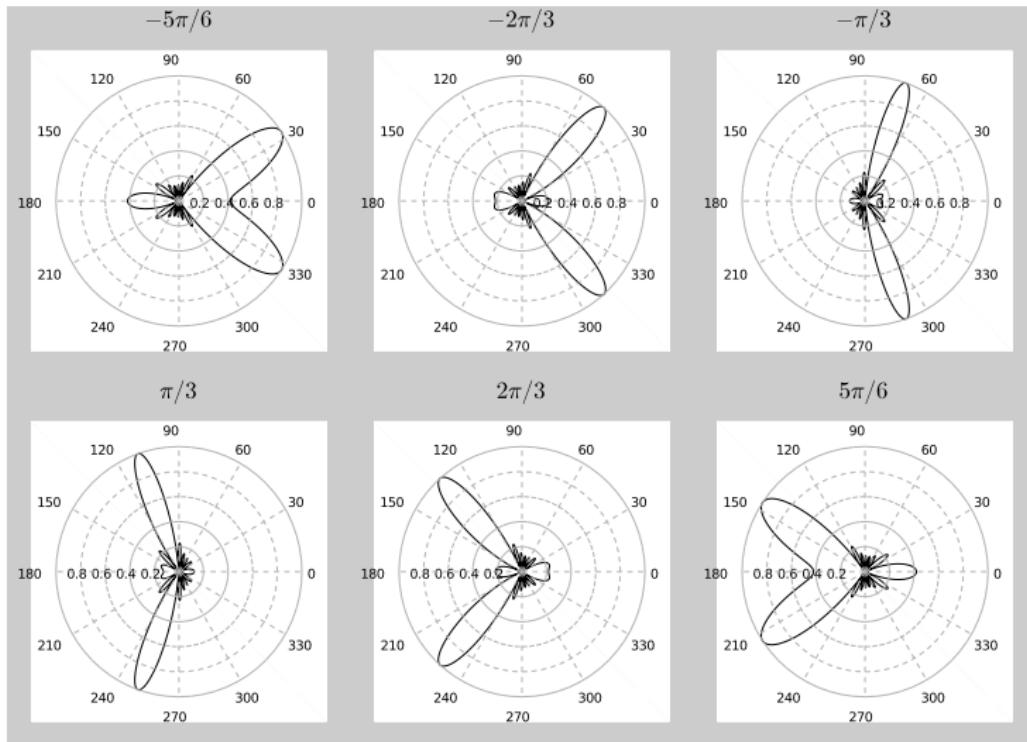
Beam Steering  $N = 4$ 

## Beam Steering

Beam Steering  $N = 8$ 

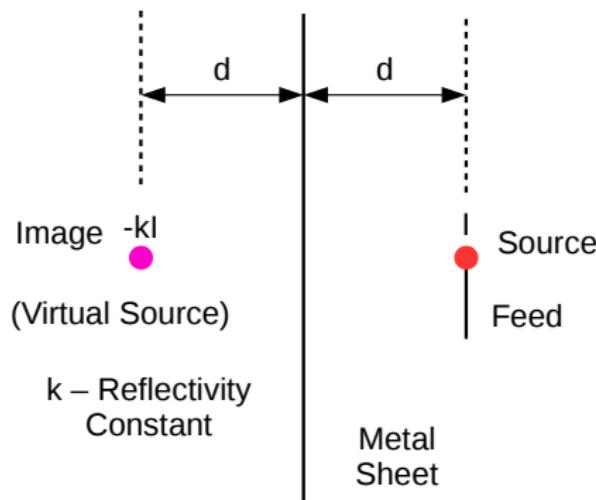
**Beam Steering**

# Beam Steering $N = 8$ (Contd..)



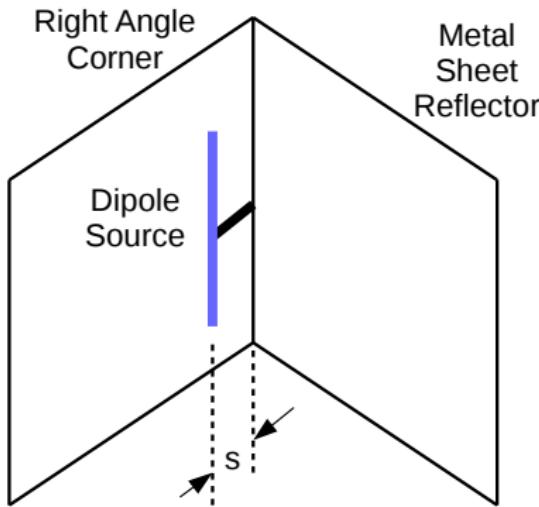
## Image Arrays

## Source Images



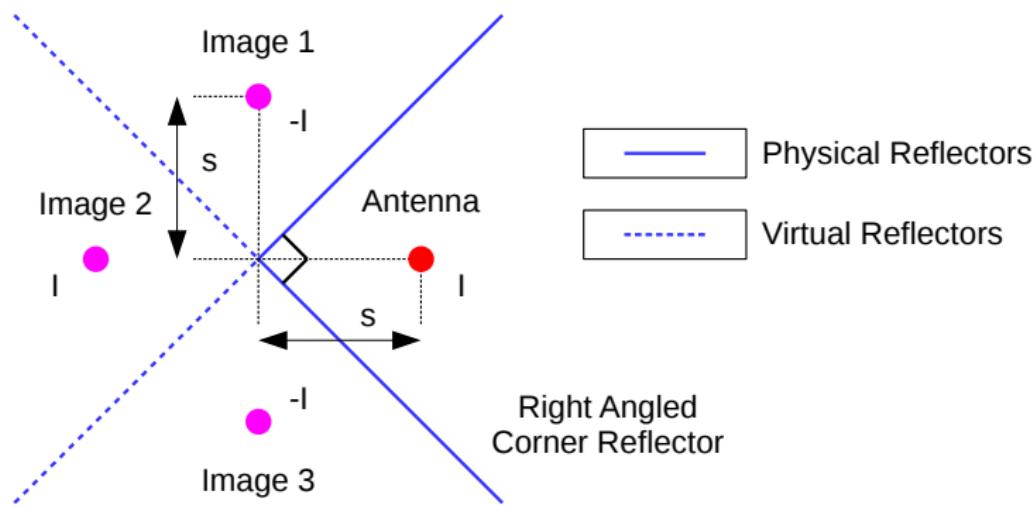
## Image Arrays

# The Corner Reflector



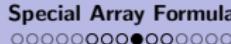
## Image Arrays

# Image Arrays (Corner Reflector)



- The antenna along with the images form two broadside arrays
  - ▶ Antenna and Image 2
  - ▶ Images 1 and 3

## Image Arrays



## Corner Reflector

Taking the Antenna and Image 2,

$$\begin{aligned}\phi_1 &= \frac{4\pi s \cos(\theta)}{\lambda} \\ E_1(\theta) &= E_0 \cos\left(\frac{\phi_1}{2}\right) = E_0 \cos\left(\frac{2\pi s \cos(\theta)}{\lambda}\right)\end{aligned}$$

Taking Image 1 and Image 3 as a broadside array,

$$\phi_2 = \frac{4\pi s \sin(\theta)}{\lambda}$$

Since both contain negative currents,

$$E_2(\theta) = -E_0 \cos\left(\frac{\phi_2}{2}\right) = -E_0 \cos\left(\frac{2\pi s \sin(\theta)}{\lambda}\right)$$

## Image Arrays



# Corner Reflector (Contd..)

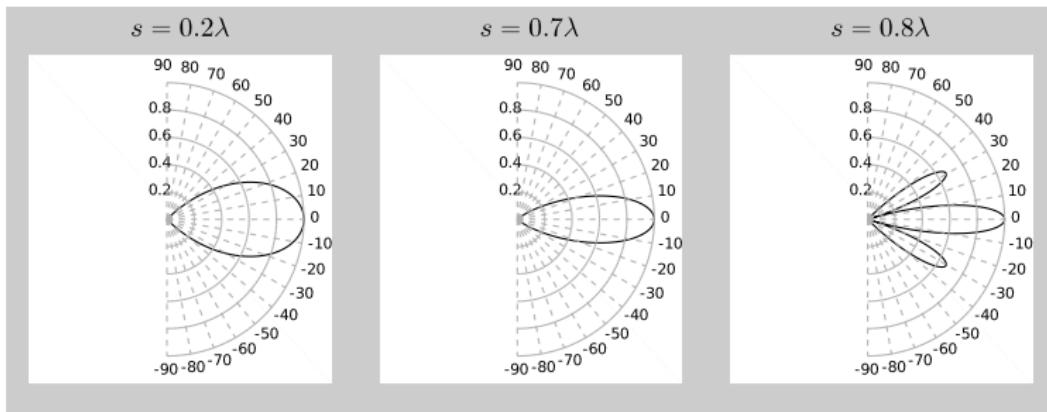
Therefore the resultant pattern is given by,

$$\begin{aligned} E(\theta) &= E_1(\theta) + E_2(\theta) \\ &= E_0 \left[ \cos\left(\frac{2\pi s \cos(\theta)}{\lambda}\right) - \cos\left(\frac{2\pi s \sin(\theta)}{\lambda}\right) \right] \end{aligned}$$

- Can make an omnidirectional antenna directive
- Dimensions will only be practical at high frequencies

## Image Arrays

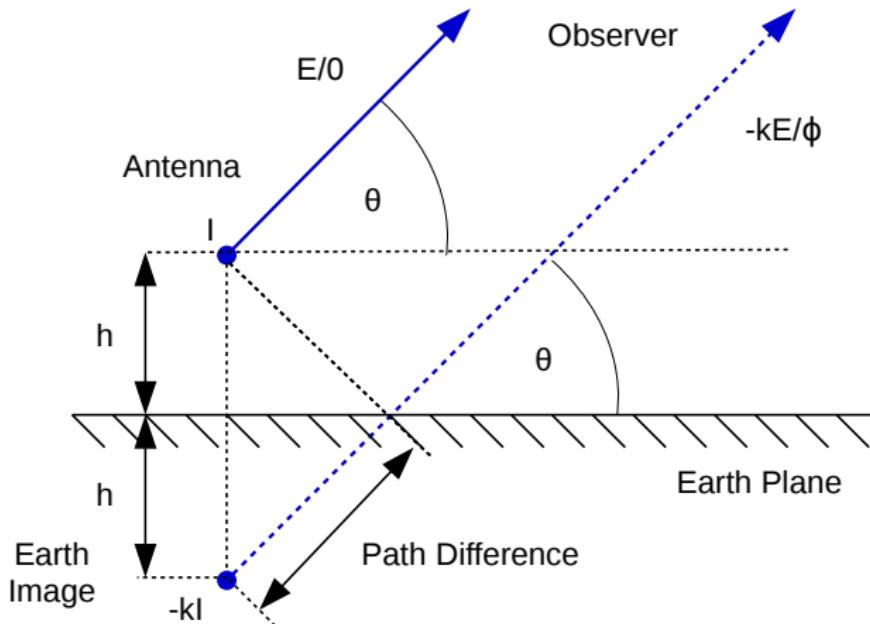
## Corner Reflector Radiation Patterns



- Directive radiation pattern
- Significant sidelobes emerge when  $s$  increases compared to  $\lambda$

## Earth Effects

## Earth Reflection



## Earth Effects

# Earth Reflection (Contd..)

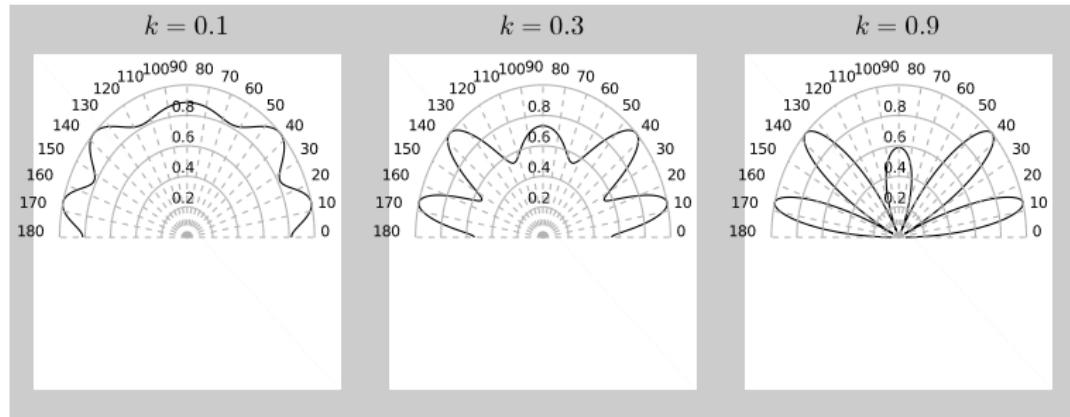
- The current in the antenna  $I$  and the image due to earth reflection  $-kI$  form an array
- Using first principles to obtain the radiation pattern

$$\begin{aligned}E_{ER} &= E \left( 1 - ke^{j\phi} \right) = E (1 - k\cos(\phi) - jk\sin(\phi)) \\&= E \sqrt{[1 + k^2 - 2k\cos(\phi)]} \\&= E \sqrt{\left[ 1 + k^2 - 2k\cos\left(\frac{4\pi h \sin(\theta)}{\lambda}\right) \right]}\end{aligned}$$

- For perfectly reflective earth  $k = 1$

## Earth Effects

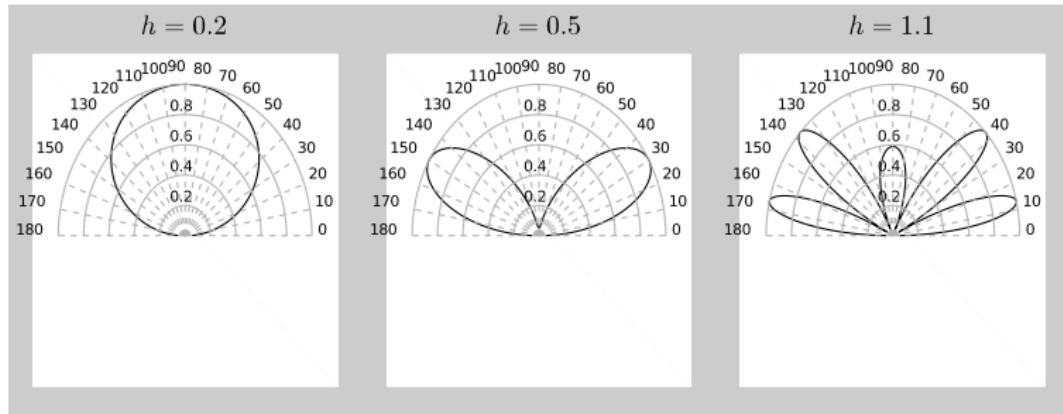
# Radiation Patterns for Different $k$ ( $h = 1.1\lambda$ )



- The omnidirectional radiation pattern deteriorates into a heavily lobed pattern with increased  $k$

## Earth Effects

# Radiation Patterns for Different $h$ ( $k = 0.9$ )



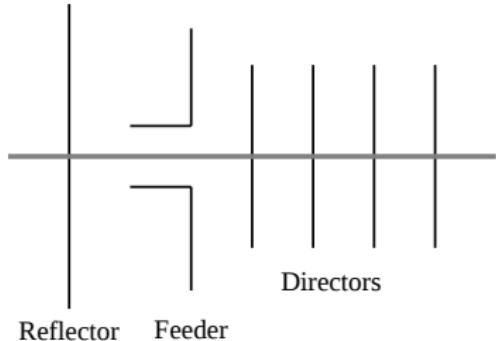
- The number of lobes increases with  $h$

# The Yagi-Uda Array

# Yagi-Uda Array

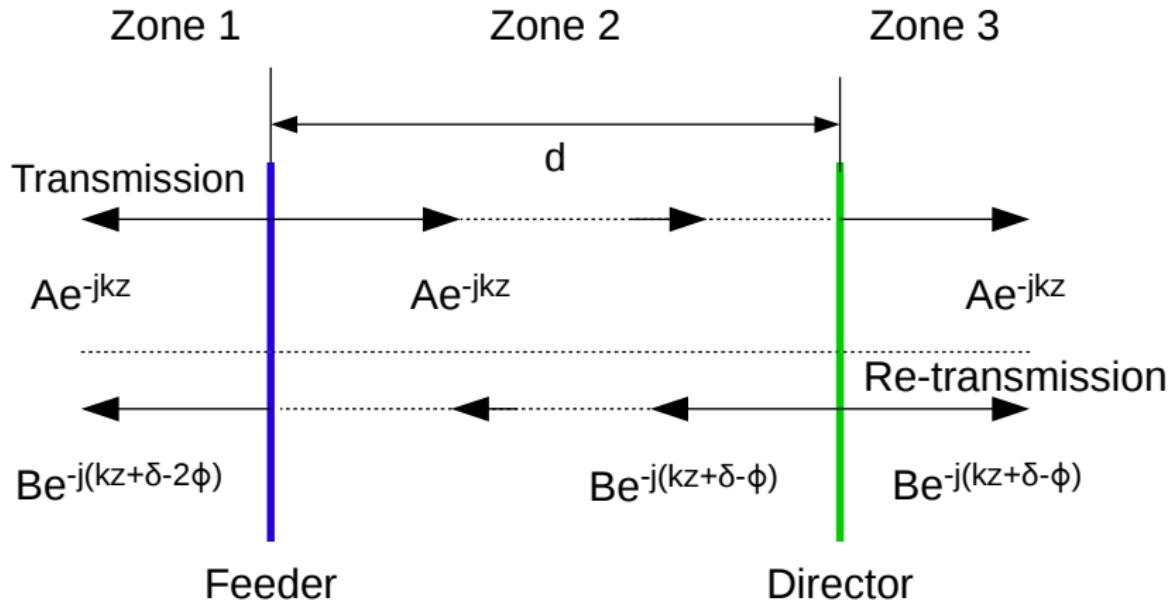
## Overview

- Single feeding (active) element
- Multiple parasitic (passive) elements
  - ▶ Directors and reflector (increase directivity)
  - ▶ Parasitic elements help modify the feed impedance
  - ▶ The end result is a highly directive antenna



- Feeding element ( $\approx 0.99\lambda$ )
  - ▶ Capacitive
- Directors ( $\approx 0.9\lambda - 0.95\lambda$ )
  - ▶ Capacitive
- Reflector ( $\approx 1.1\lambda$ )
  - ▶ Inductive

# Yagi-Uda Array Operation



# Yagi-Uda Array Operation (Contd..)

- Phase difference between feeder and director  $\phi = 2\pi d/\lambda$
- Phase difference due to director  $\pi + \delta$
- Resultant wave in **Zone 1**

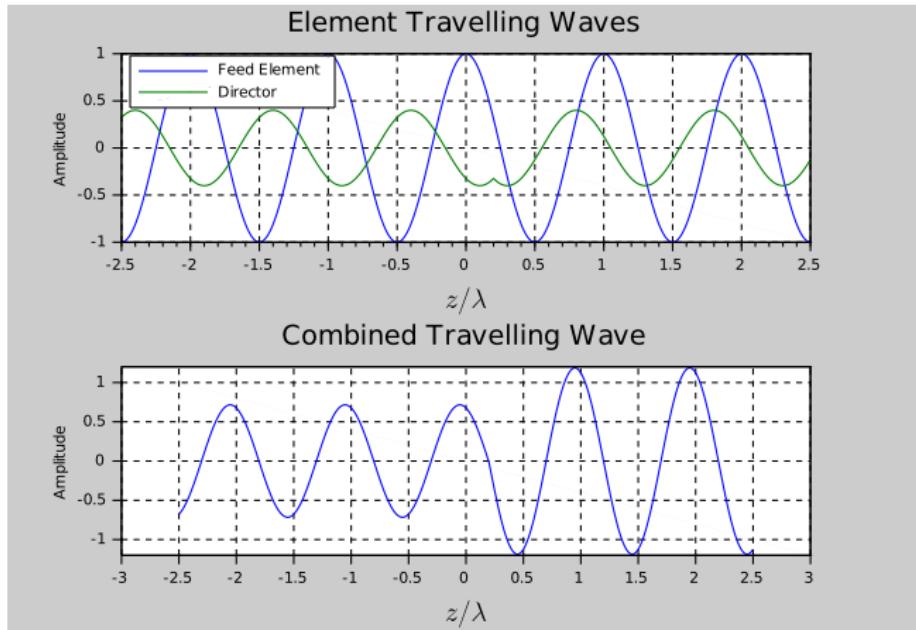
$$F_1(z) = Ae^{-jkz} + Be^{-j(kz+\psi_1)} = Ae^{-jkz} + Be^{-j(kz+\delta-2\phi)}$$

- Resultant wave in **Zone 3**

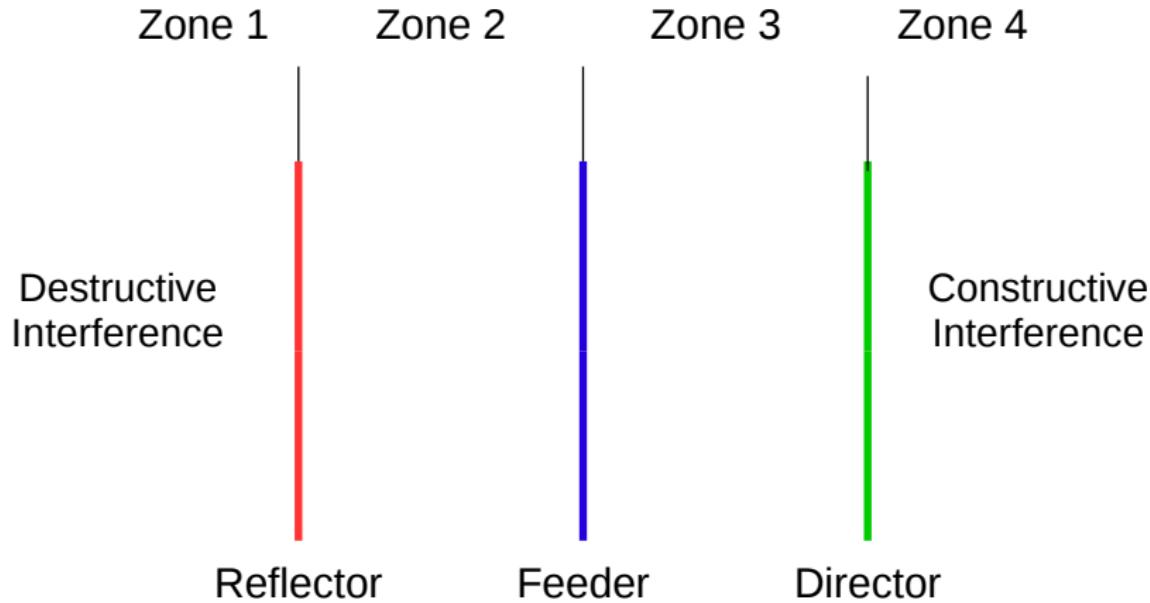
$$F_3(z) = Ae^{-jkz} + Be^{-j(kz+\psi_2)} = Ae^{-jkz} + Be^{-j(kz+\delta-\phi)}$$

- Select  $d$  such that  $\psi_1$  results in *destructive* interference in **Zone 1** and  $\psi_3$  results in *constructive* interference in **Zone 3**.

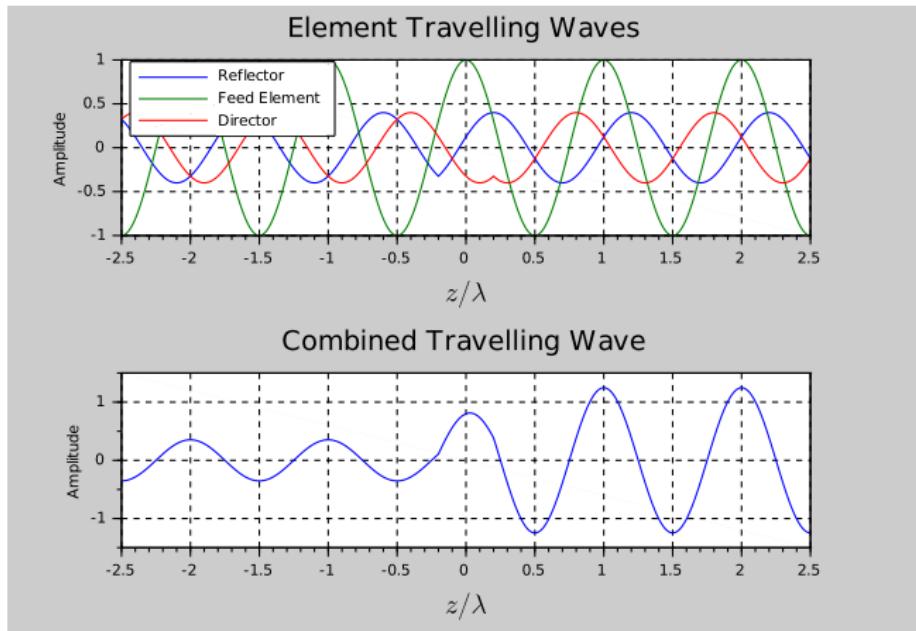
# Yagi-Uda Array Operation (Contd..)



# Yagi-Uda Array Operation (Contd..)



# Yagi-Uda Array Operation (Contd..)



# Conclusion

# Summary

- An array consists of multiple antenna elements
- Directivity improves with number of elements
  - ▶ Most effective at high frequencies due to smaller dimensions