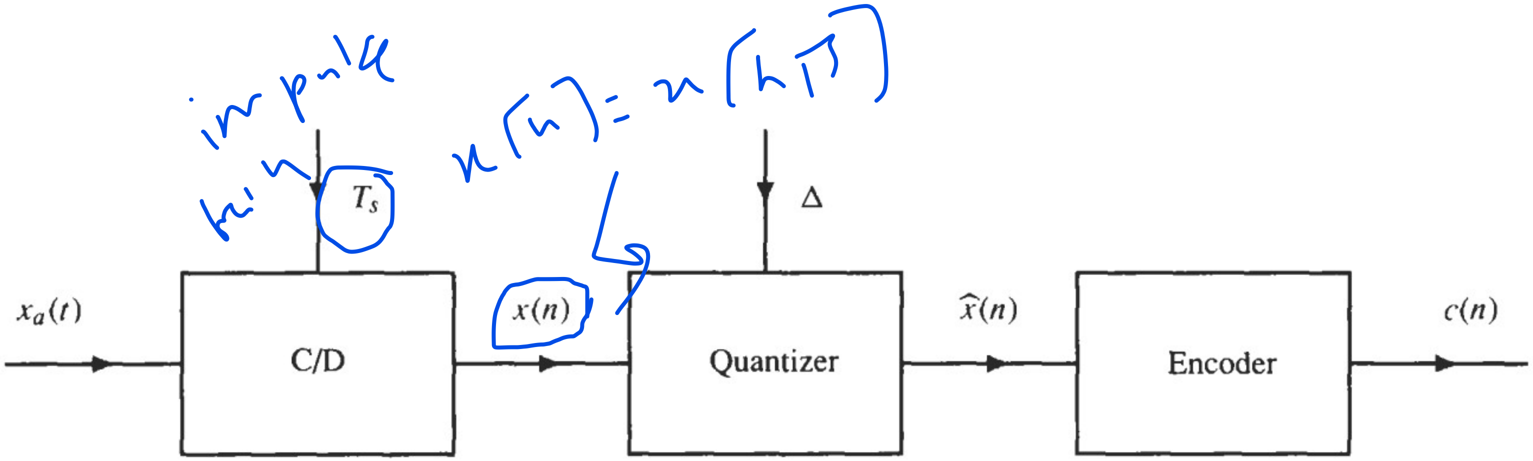
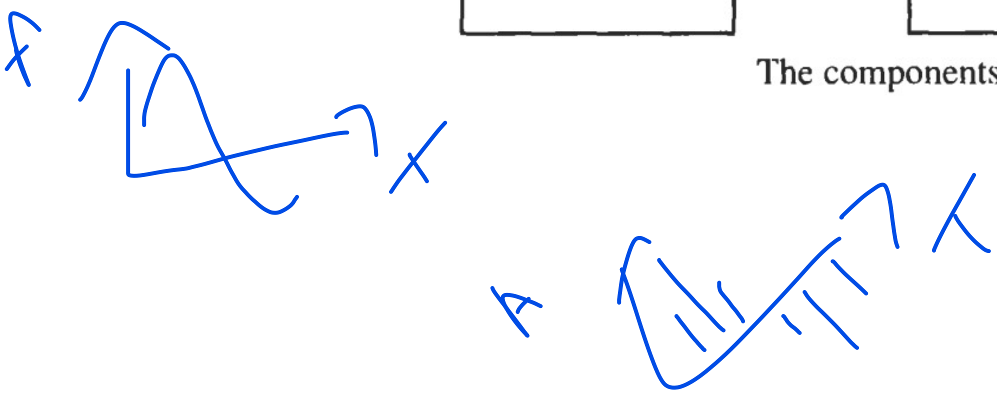
Sampling – A/D Conversion

LECTURE 1

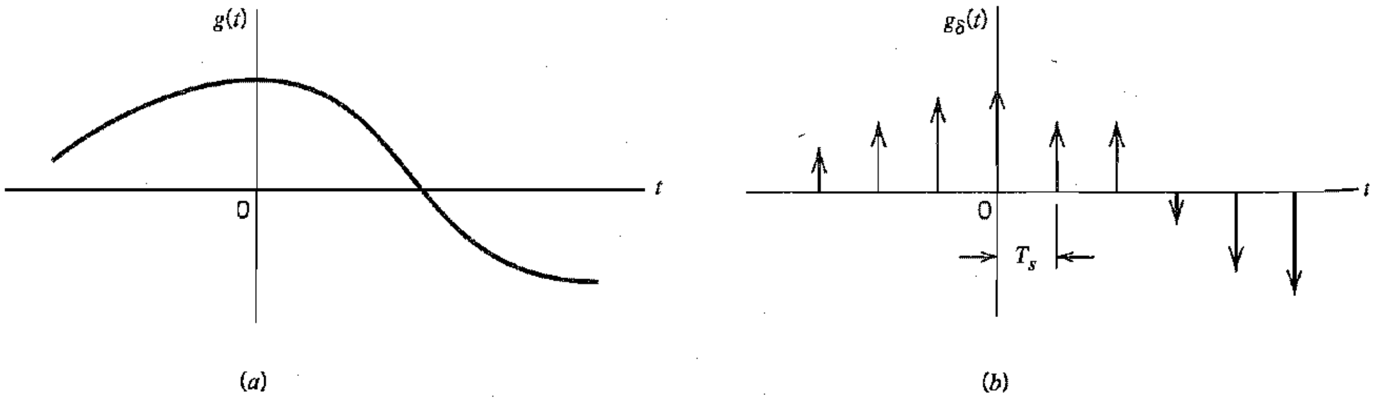
Analog to Digital Conversion (A/D)



The components of an analog-to-digital converter.



Example :Sampling Process

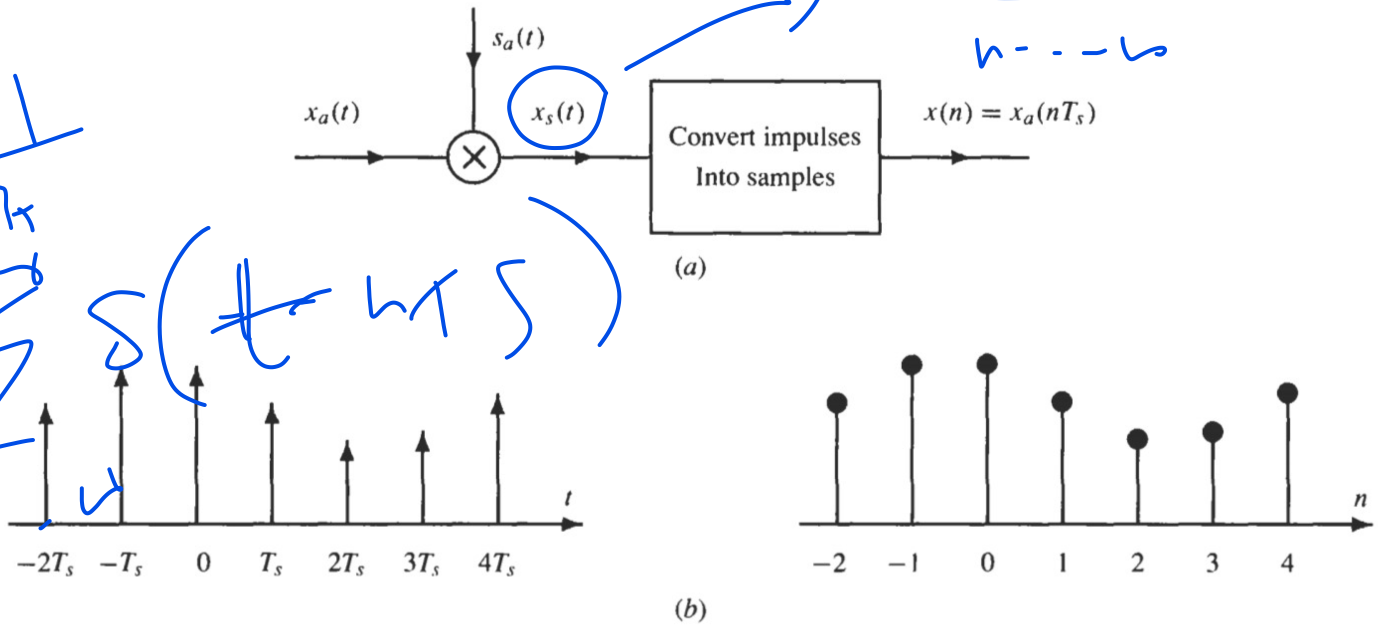


The sampling process. (*a*) Analog signal. (*b*) Instantaneously sampled version of the analog signal.

Periodic Sampling

$$\sum_{n=-\infty}^{\infty} x_a(nT_s)\,\delta(t-nT_s)$$

$$s_a(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$



Continuous-to-discrete conversion. (a) A model that consists of multiplying $x_a(t)$ by a sequence of impulses, followed by a system that converts impulses into samples. (b) An example that illustrates the conversion process.

Periodic Sampling contd.

The sample spacing T_s, is the sampling period, and f, = l/ T_s, is the sampling frequency in samples per second.

First, the continuous-time signal is multiplied by a periodic sequence of impulses,

$$s_a(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Periodic Sampling contd.

Then form the sampled signal

$$x_s(t) = x_a(t)s_a(t) = \sum_{n=-\infty}^{\infty} x_a(nT_s)\delta(t - nT_s)$$

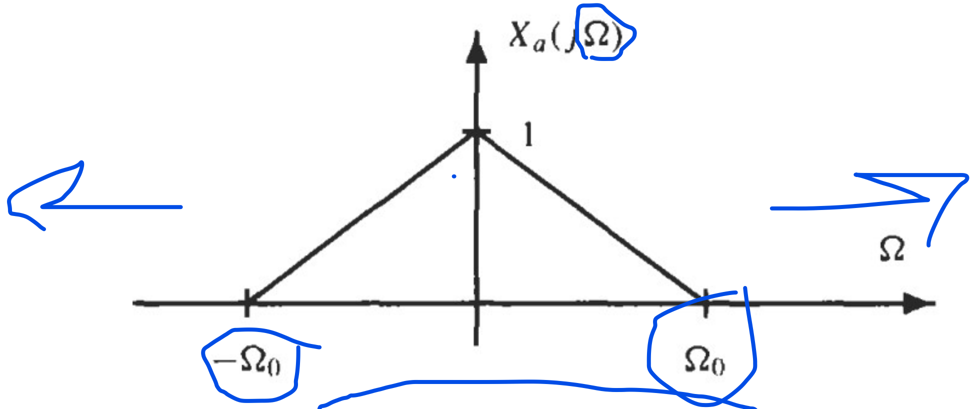
Then, the sampled signal is converted into a discrete-time signal by mapping the impulses that are spaced in time by Ts into a sequence x(n) where the sample values are indexed by the integer variable n:

$$x(n) = x_a(nT_s)$$

The sample spacing T_s, is the sampling period, and f, = I/ T_s, is the sampling frequency in samples per second.

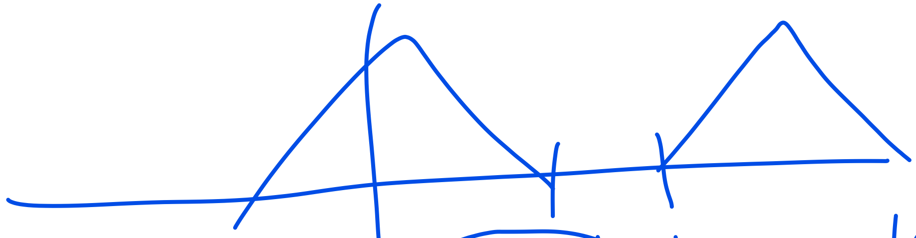
$w = \Omega T_s$

Aliasing



$x_a(t)$ is strictly bandlimited so that $X_a(j\Omega) = 0$ for $|\Omega| > \Omega_o$

Aliasing contd.

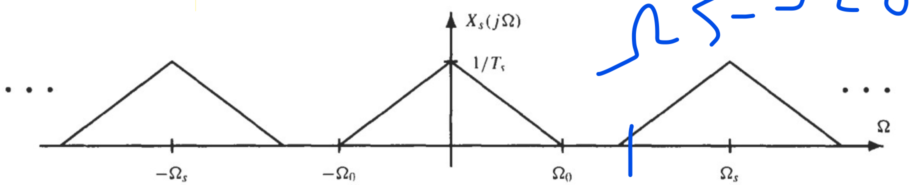


ω_m $\qquad \omega_s - \omega_m$

$\omega_s - \omega_m > \omega_m$

If $x_a(t)$ is sampled with a sampling frequency $\Omega_s \geq 2\Omega_0$, the Fourier transform of $x_s(t)$ is formed by periodically replicating $X_a(j\Omega)$.

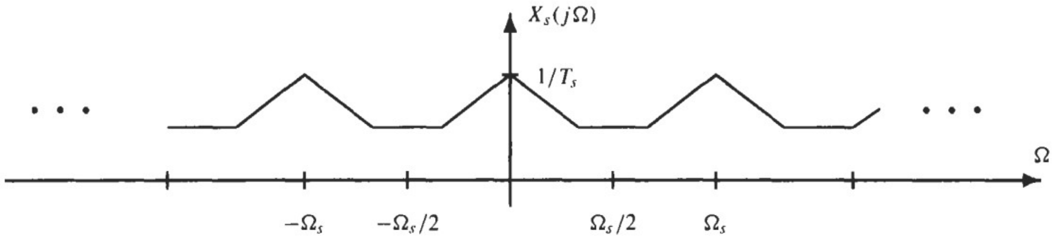
$\Omega_s - \Omega_0$



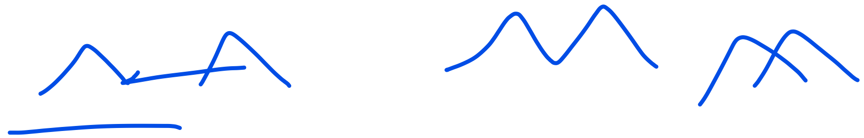
$\Omega_s - \Omega_0 > \Omega_0$

If $\Omega_s < 2\,\Omega_0$, the shifted spectra $X_a(j\Omega\text{-}jk\,\Omega_s)$ overlap, and when these spectra are summed to form $X_s(j\Omega)$.

$\omega_s > 2\omega_m$



Aliasing



Nyquist Sampling Theorem

$\omega_s \geq 2\omega_m$

$\omega_s > 2\omega_m$

If $x_a(t)$ is strictly bandlimited,

$$X_a(j\Omega) = 0 \qquad |\Omega| > \Omega_0$$

then $x_a(t)$ may be uniquely recovered from its samples $x_a(nT_s)$ if

$$\boxed{\Omega_s} = \frac{2\pi}{T_s} \geq 2\Omega_0$$

$\Omega_s - \Omega_0 > \Omega_0$

The frequency Ω_0 is called the *Nyquist frequency*, and the minimum sampling frequency $\Omega_s = 2\Omega_0$, is called the *Nyquist rate*.

Quantization

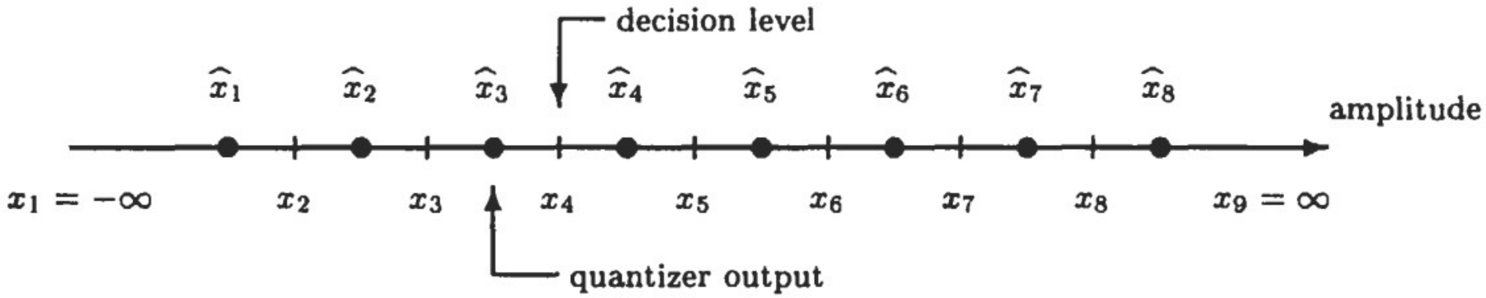
A quantizer is a nonlinear and noninvertible system that transforms an input sequence x(n) that has a continuous range of amplitudes into a sequence for which each value of x(n) assumes one of a finite number of possible values.

This operation is denoted by $\hat{x}(n)=Q[x(n)]$

The quantizer has $L + 1$ decision levels $x_1, x_2, \ldots x_{L+1}$ that divide the amplitude range for $x(n)$ into L intervals.

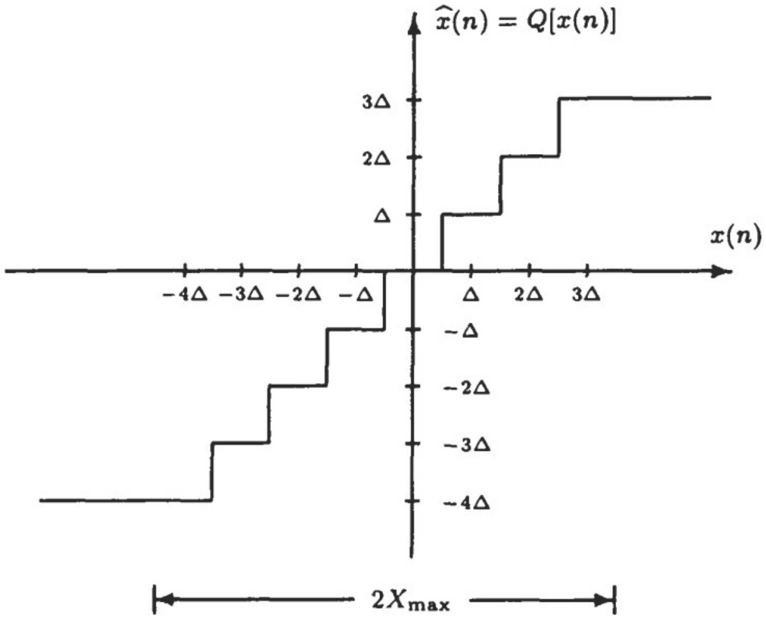
$L = 2^n$

Quantization contd.



A quantizer with nine decision levels that divide the input amplitudes into eight quantization intervals and eight possible quantizer outputs, $\hat{x}_k$.

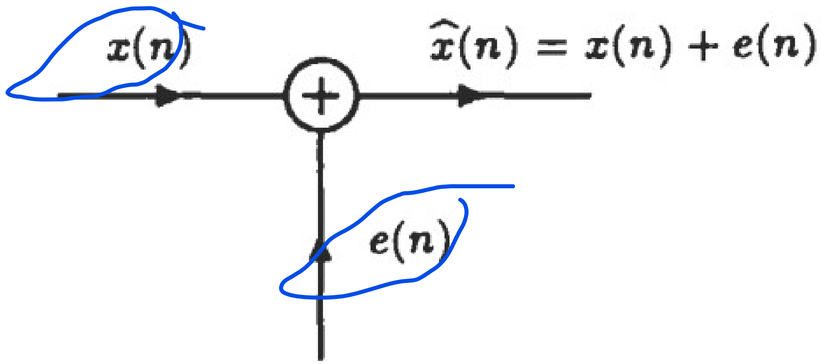
Example: Quantization Process



A 3-bit uniform quantizer.

2^3

8



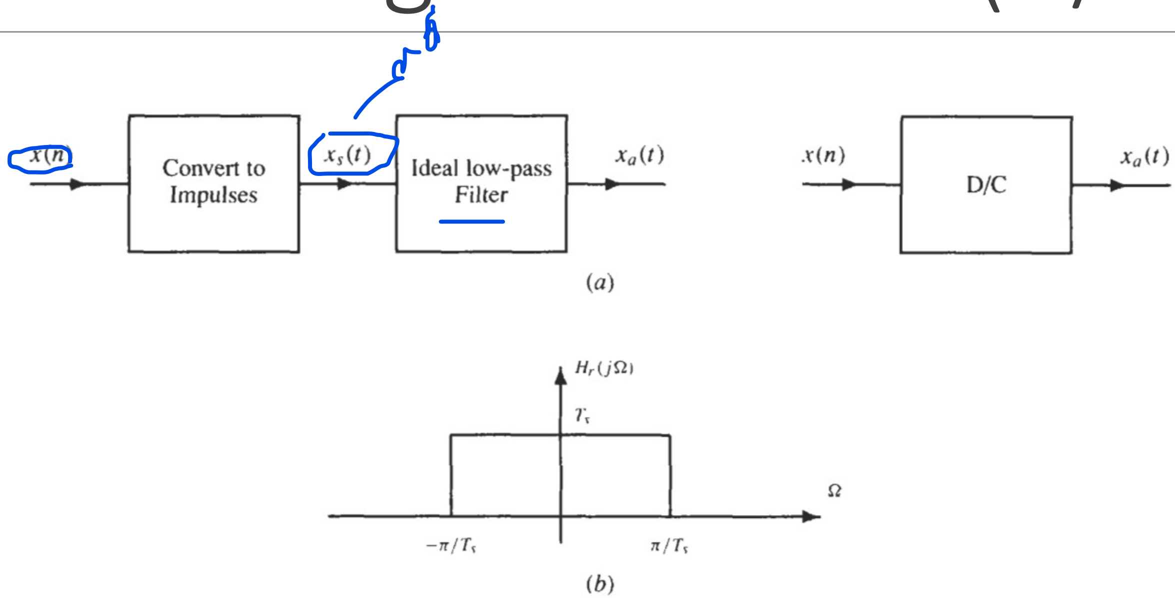
A quantization noise model.

Encoder

The output of the quantizer is sent to an encoder, which assigns a unique binary number (***codeword***) to each quantization level.

Binary Symbol	Numeric Value
0 1 1	$\frac{3}{4}$
0 1 0	$\frac{1}{2}$
0 0 1	$\frac{1}{4}$
0 0 0	0
1 1 1	$-\frac{1}{4}$
1 1 0	$-\frac{1}{2}$
1 0 1	$-\frac{3}{4}$
1 0 0	-1

Digital to Analog Conversion (D/A)



(*a*) A discrete-to-continuous converter with an ideal low-pass reconstruction filter. (*b*) The frequency response of the ideal reconstruction filter.