

DFT:

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} \quad k = 0, 1, \dots, N-1$$

$$X[0] = x[0]W_N^0 + x[1]W_N^0 + x[2]W_N^0 + \dots + x[N-1]W_N^0$$

$$X[1] = x[0]W_N^0 + x[1]W_N^1 + x[2]W_N^2 + \dots + x[N-1]W_N^{N-1}$$

$$X[2] = x[0]W_N^0 + x[1]W_N^2 + x[2]W_N^4 + \dots + x[N-1]W_N^{2(N-1)}$$

\vdots

$$X[N-1] = x[0]W_N^0 + x[1]W_N^{N-1} + x[2]W_N^{2(N-1)} + \dots + x[N-1]W_N^{(N-1)^2}$$

Linear Algebra

$$\begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix} = \begin{bmatrix} W_N^0 & W_N^0 & \cdots & W_N^0 \\ W_N^0 & W_N^1 & \cdots & W_N^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ W_N^0 & W_N^{(N-1)} & \cdots & W_N^{(N-1)^2} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} \Rightarrow X = Fx$$

$N \times 1 \qquad N \times N \qquad N \times 1$

F is authogonal

columns all have the length of N

perpendicular

$$\begin{array}{c}
 \text{Diagram of } N \times 1 \text{ vector } x \\
 \text{Diagram of } N \times N \text{ matrix } F \\
 \text{Diagram of } N \times 1 \text{ vector } X
 \end{array}
 = \alpha[G] \quad \begin{array}{c}
 \text{Diagram of } N \times 1 \text{ vector } x \\
 \text{Diagram of } N \times N \text{ matrix } F \\
 \text{Diagram of } N \times 1 \text{ vector } X
 \end{array}
 + \chi[J] \quad \begin{array}{c}
 \text{Diagram of } N \times 1 \text{ vector } x \\
 \text{Diagram of } N \times N \text{ matrix } F \\
 \text{Diagram of } N \times 1 \text{ vector } X
 \end{array}
 + \alpha[n] \quad \begin{array}{c}
 \text{Diagram of } N \times 1 \text{ vector } x \\
 \text{Diagram of } N \times N \text{ matrix } F \\
 \text{Diagram of } N \times 1 \text{ vector } X
 \end{array}$$

