General Sir John Kotelawala Defence University

ET3212 Microwave Engineering
Transmission Lines

Upeka Premaratne

Department of Electronic and Telecommunication Engineering
University of Moratuwa

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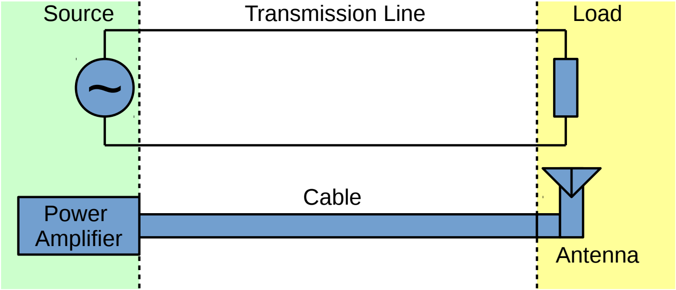
Outline

Introduction

Introduction

- Transmission lines are used to transmit power in high frequency systems



Transmission Line Theory

Frequency	Paradigm	Characteristic
0-100kHz	Circuit Theory	Lumped Components
$>$ 0.1MHz	Transmission Line Theory	Distributed Components
$>$1GHz	Electromagnetic Theory	Guided Propagation

- A transmission line is a *distributed component*
  - Analyzed using Heaviside's distributed component model
  - Can also be explained using EM theory

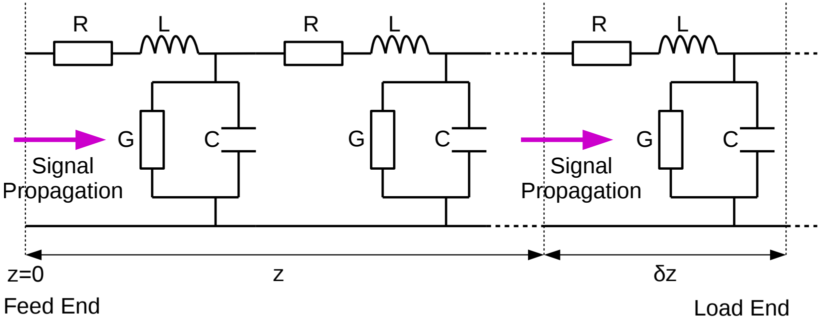
Practical Transmission Lines

- Always have *two* (or rarely more) conductors
- Symmetric (balanced) as in the twisted pair or asymmetric (unbalanced) as in the coaxial cable or microstripline
  - In an unbalanced transmission line the *current distribution* differs on each conductor
- The *skin depth* applies
  - Usually the signal propagates within a few microns of the surface

Transmission Line Parameters

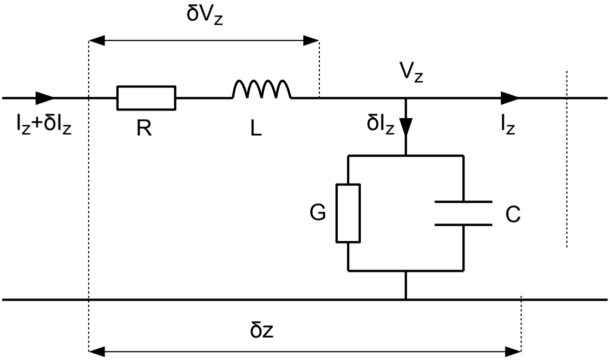
Distributed Component Model

- The distributed component (Heaviside) model of a transmission line



Distributed Component Model

- Small δz section of a transmission line



Line Voltage and Current

For the transmission line

$$\begin{aligned} Z &= R\delta z + j\omega L\delta z \\ Y &= G\delta z + j\omega C\delta z \\ \delta V_z &= (I_z + \delta I_z)(R\delta z + j\omega L\delta z) = (I_z + \delta I_z)(R + j\omega L)\delta z \\ \delta I_z &= V_z(G + j\omega C)\delta z \end{aligned}$$

Therefore the rate of change w.r.t. z:

$$\begin{aligned} \frac{dV_z}{dz} &= -(R + j\omega L)I_z \\ \frac{dI_z}{dz} &= -(G + j\omega C)V_z \end{aligned}$$

- Both voltage and current decrease with z.

Line Voltage and Current (Contd..)

From further differentiation:,

$$
\begin{aligned}
\frac{d^2V_z}{dz^2} &= -(R+j\omega L)\frac{dI_z}{dz} = (R+j\omega L)(G+j\omega C)V_z \\
\frac{d^2I_z}{dz^2} &= -(G+j\omega C)\frac{dV_z}{dz} = (G+j\omega C)(R+j\omega L)I_z
\end{aligned}
$$

By defining γ such that,

$$
\begin{aligned}
\gamma &= \sqrt{(G+j\omega C)(R+j\omega L)} \\
\frac{d^2V_z}{dz^2} &= \gamma^2 V_z \\
\frac{d^2I_z}{dz^2} &= \gamma^2 I_z
\end{aligned}
$$

Line Voltage and Current (Contd..)

The solution therefore becomes,

$$\begin{aligned} V_z &= Ae^{-\gamma z} + Be^{\gamma z} \\ I_z &= \frac{\gamma}{(R + j\omega L)} \left(Ae^{-\gamma z} - Be^{\gamma z} \right) \end{aligned}$$

Both V_z and I_z decrease when z increases, therefore $B = 0$,

$$\begin{aligned} V_z &= Ae^{-\gamma z} \\ I_z &= \frac{\gamma Ae^{-\gamma z}}{(R + j\omega L)} \end{aligned}$$

Characteristic Impedance

The **characteristic impedance** of a transmission line:

$$z_0 = \frac{V_z}{I_z} = \frac{(R + j\omega L)}{\gamma} = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = \sqrt{\frac{Z}{Y}}$$

At high frequencies, the reactive components become very large when compared to the resistive components (i.e., $j\omega C >> G$ and $j\omega L >> R$). Such transmission lines can then be taken as *lossless*.

$$z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \approx \sqrt{\frac{L}{C}}$$

Propagation Coefficient

The term γ is known as the propagation coefficient of the transmission line. It is a complex number given by,

$$\gamma = \alpha + j\beta$$

- The term α describes the amplitude behaviour of the transmission line (amplitude propagation coefficient)
- The term β describes the phase behavior (phase propagation coefficient)

Attenuation

The voltage and current through the transmission line are time harmonic. Therefore,

$$V_z = Ae^{-\gamma z}e^{j\omega t} = Ae^{-(\alpha+j\beta)z}e^{j\omega t} = Ae^{-\alpha z}e^{j(\omega t-\beta z)}$$

Therefore the amplitude change of V_z with respect to z is given by,

$$|V_z| = Ae^{-\alpha z}$$

where

$$\alpha = \frac{1}{(z_2 - z_1)} ln\left|\frac{V_1}{V_2}\right| \rightarrow \alpha = ln\left|\frac{V_1}{V_2}\right| \approx \frac{R}{2z_0} + \frac{Gz_0}{2}$$

Phase

The phase propagation:

$$\beta = \frac{2\pi}{\lambda} = \omega\sqrt{LC}$$

The phase velocity:

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

Group velocity:

$$v_g = \frac{d\omega}{d\beta}$$

The two values are related by $v_p v_g \leq c^2$.

- Generally v_p is around $0.6c - 0.7c$.

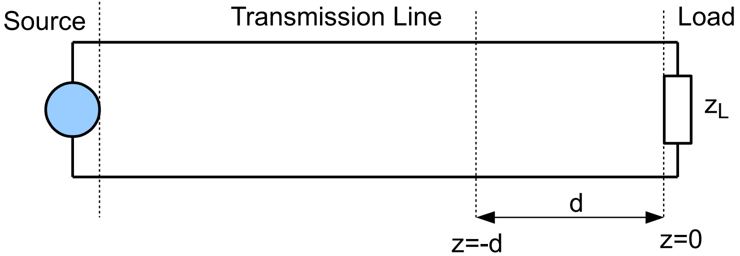
Distortionless Transmission Lines

- In a distortion free line
  - The α is independent of frequency
  - The β is linearly dependent on frequency
- The required condition is $RC = GL$ (Heaviside condition)

$$
\begin{aligned}
\gamma &= \sqrt{(G+j\omega C)(R+j\omega L)} = \sqrt{RG\left(1+\frac{j\omega C}{G}\right)\left(1+\frac{j\omega L}{R}\right)} \\
&= \sqrt{RG}\left(1+\frac{j\omega C}{G}\right) = \underbrace{\sqrt{RG}}_{\alpha} + \underbrace{j\omega\sqrt{LC}}_{\beta}
\end{aligned}
$$

Wave Reflection

Wave Reflection



In a transmission line with a finite length a reflected wave will exist (i.e., $B \neq 0$):

$$
\begin{aligned}
V_z &= Ae^{-\gamma z} + Be^{\gamma z} \\
I_z &= \frac{A}{z_0}e^{-\gamma z} - \frac{B}{z_0}e^{\gamma z}
\end{aligned}
$$

Wave Reflection (Contd..)

Consider a lossless transmission line connected to a load (z_L) at $z = 0$. Then the impedance at a distance $z = -d$ is given by,

$$z = \left(\frac{V}{I}\right)_{z=-d} = \frac{Ae^{j\beta d} + Be^{-j\beta d}}{\frac{A}{z_0}e^{j\beta d} - \frac{B}{z_0}e^{-j\beta d}} = z_0\left(\frac{Ae^{j\beta d} + Be^{-j\beta d}}{Ae^{j\beta d} - Be^{-j\beta d}}\right)$$

At the load $(d = 0)$, $z = z_L$. Therefore,

$$z_L = z_0\left(\frac{A+B}{A-B}\right)$$

The voltage reflection coefficient at a distance d from the load is given by,

$$\rho = \frac{V_r}{V_i} = \frac{Be^{-j\beta d}}{Ae^{j\beta d}} = \frac{B}{A}e^{-j2\beta d}$$

Wave Reflection (Contd..)

At the load (d = 0),

$$\rho_0 = \frac{B}{A}$$

Therefore,

$$z_L = z_0 \left(\frac{1 + \frac{B}{A}}{1 - \frac{B}{A}} \right) = z_0 \left(\frac{1 + \rho_0}{1 - \rho_0} \right)$$

Therefore,

$$\rho_0 = \frac{z_L - z_0}{z_L + z_0}$$

Wave Reflection (Contd..)

When the transmission line is short circuited:

$$z_L = 0 \rightarrow \rho_0 = -1$$

When the transmission line is open circuited:

$$z_L \longrightarrow \infty \rightarrow \rho_0 = 1$$

When the load of the transmission line is matched to its characteristic impedance:

$$z_L = z_0 \rightarrow \rho_0 = 0$$

Wave Reflection (Contd..)

The reflection coefficient at any distance can be expressed as,

$$\rho \;=\; \frac{B}{A}e^{-j2\beta d} = \rho_0 e^{-j2\beta d} = \left(\frac{z_L - z_0}{z_L + z_0}\right) e^{-j2\beta d}$$

Since ρ_0 can be a complex value due to a reactive load,

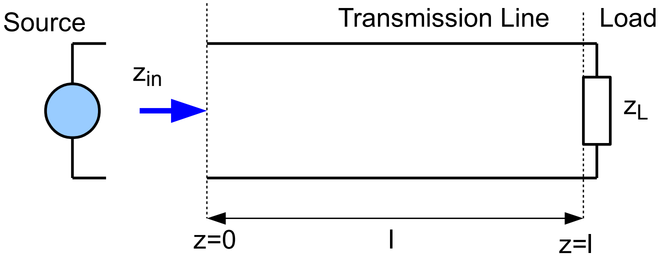
$$\rho_0 \;=\; |\rho_0|\, e^{j\theta}$$

Therefore,

$$\rho \;=\; |\rho_0|\, e^{j\theta} e^{-j2\beta d} = |\rho_0|\, e^{j(-2\beta d+\theta)}$$

- A rotating vector

Feed Impedance



General voltage and current

$$V_z = Ae^{-\gamma z} + Be^{\gamma z}$$
$$I_z = \frac{A}{z_0}e^{-\gamma z} - \frac{B}{z_0}e^{\gamma z}$$

Feed Impedance (Contd..)

Therefore, at $z = l$ (a transmission line with a load at l):

$$\begin{aligned} A &= \frac{1}{2}(V_z + I_z z_0)e^{\gamma z} = \frac{1}{2}(V_l + I_l z_0)e^{\gamma l} \\ B &= \frac{1}{2}(V_z - I_z z_0)e^{-\gamma z} = \frac{1}{2}(V_l - I_l z_0)e^{-\gamma l} \end{aligned}$$

General feed impedance (lossy line)

$$z_{in} = \left(\frac{V}{I}\right)_{z=0} = z_0\left(\frac{A+B}{A-B}\right) = z_0\left(\frac{z_l + z_0\tanh(\gamma l)}{z_0 + z_l\tanh(\gamma l)}\right)$$

Feed Impedance (Contd..)

When the line is lossless ($\gamma = j\beta$),

$$z_{in} = z_0 \left(\frac{z_l + j z_0 \tan(\beta l)}{z_0 + j z_l \tan(\beta l)} \right)$$

Therefore,

- A loaded, short circuited or open circuited transmission line of length l can be used to *modify* an impedance.
- l has to be precisely selected and depends on ω if $z_0 \neq z_l$
- The principle of *stub matching*
  - Now rare in low frequency use
  - Common in microwave frequencies

Voltage Standing Wave Ratio

The voltage along the transmission line is given by,

$$V = Ae^{j\beta d} + Be^{-j\beta d} = A\left[1 + \frac{B}{A}e^{-j2\beta d}\right]e^{j\beta d} = A\left[1 + |\rho_0|\, e^{j(-2\beta d+\theta)}\right]$$

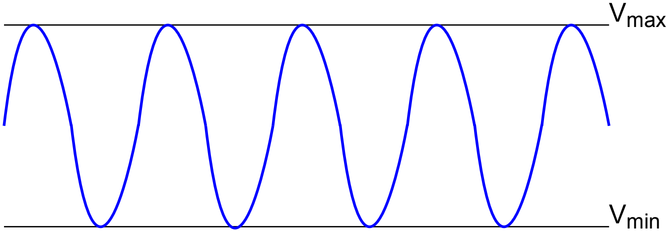
Therefore, the amplitude variation is given by,

$$|V| = A\left[1 + |\rho_0|\, e^{j(-2\beta d+\theta)}\right]$$

From this, the maximum and minimum amplitude of the voltage is obtained from,

$$\begin{aligned} V_{max} &= A\left[1 + |\rho_0|\right] \\ V_{min} &= A\left[1 - |\rho_0|\right] \end{aligned}$$

Voltage Standing Wave Ratio (Contd..)



Voltage Standing Wave Ratio (VSWR) is the ratio between the maximum and minimum voltage amplitude within the transmission line:

$$VSWR \;=\; \frac{V_{max}}{V_{min}} = \left[\frac{1+|\rho_0|}{1-|\rho_0|}\right]$$

Voltage Standing Wave Ratio (Contd..)

At a point of maximum voltage where $d = d_1$,

$$\theta - 2\beta d_1 = 0$$

At a point of minimum voltage where $d = d_2$,

$$\theta - 2\beta d_2 = \pi$$

Therefore, the distance between an adjacent maximum and minimum becomes, At a point of maximum voltage where $d = d_1$,

$$\begin{aligned} 2\beta d_1 - 2\beta d_2 &= \pi \\ d_1 - d_2 &= \frac{\pi}{2\beta} = \frac{\pi}{2\left(\frac{2\pi}{\lambda}\right)} = \frac{\lambda}{4} \end{aligned}$$

Voltage Standing Wave Ratio (Contd..)

For a short or open circuited transmission line,

$$\begin{aligned} |\rho_0| &= 1 \\ VSWR &\longrightarrow \infty \end{aligned}$$

For a transmission line with a matched load,

$$\begin{aligned} |\rho_0| &= 0 \\ VSWR &= 1 \end{aligned}$$

Power Reflection

For a lossless transmission line,

$$P_I = P_L + P_R$$

Since $V_R = \rho V_I$ and $P \propto V^2$:

$$P_R \propto V_R^2 = \rho^2 V_I^2$$

Therefore,

$$P_R = \rho^2 P_I \rightarrow \frac{P_R}{P_I} = \rho^2$$

Microwave Transmission Lines

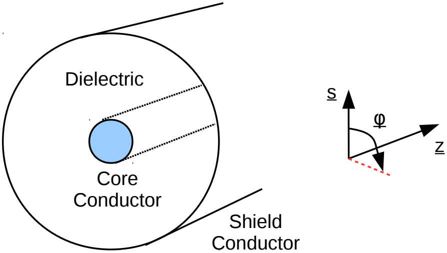
Microwave Transmission Lines

- Coaxial cables
- Substrate transmission lines
  - Slotlines
  - Microstrips
  - Striplines
- Many adaptations necessary for high frequencies

Coaxial Cable

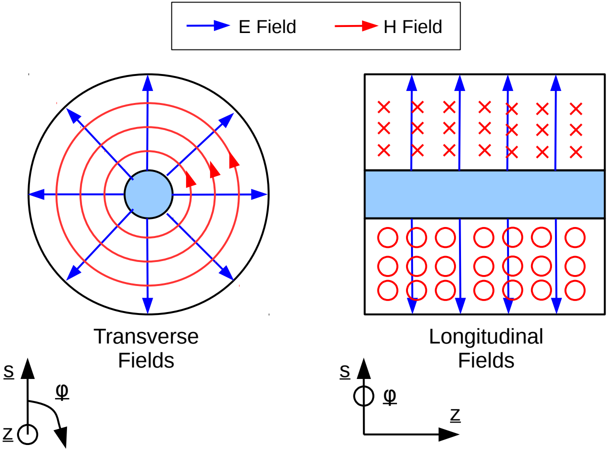
Per unit inductance and capacitance

$$L = \frac{\mu_0}{2\pi} ln\left(\frac{b}{a}\right)$$
$$C = \frac{2\pi\varepsilon_r\varepsilon_0}{\ln\left(\frac{b}{a}\right)}$$

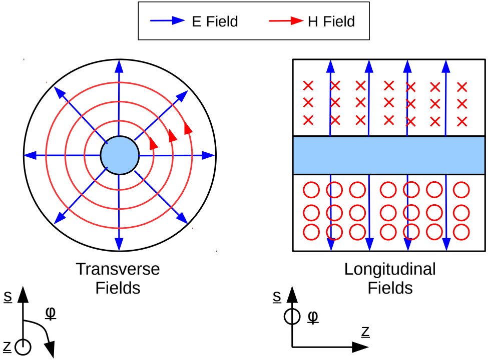


$$z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_r\varepsilon_0}} ln\left(\frac{b}{a}\right)$$

Coaxial Cable Fields



Coaxial Cable Fields (Contd..)



- The fields are contained by both conductors
- No E or H component in the longitudinal direction
  - This is a Transverse Electro Magnetic TEM Mode

Microwave Coaxial Cables

- The shield conductor has to be made flexible
  - Made out of mesh instead of solid metal
  - Higher radiation loss at microwave frequencies
  - Therefore microwave frequency coaxial cables require more rigid tightly spaced meshes
- Dielectric losses are also significant
  - Dielectric hysteresis due to polar dipoles can cause heating

Power Attenuation

Surface resistance

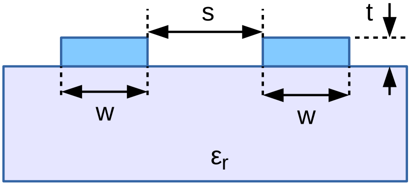
$$R_S = \frac{1}{\sigma_C \delta} = \sqrt{\frac{\omega \mu_C}{2\sigma_C}}$$

Attenuation coefficient

$$\alpha \approx \frac{R_S}{2Z_0} + \frac{Z_0 G(\omega)}{2} = \frac{1}{2Z_0}\sqrt{\frac{\omega \mu_C}{2\sigma_C}} + \frac{Z_0 G(\omega)}{2}$$

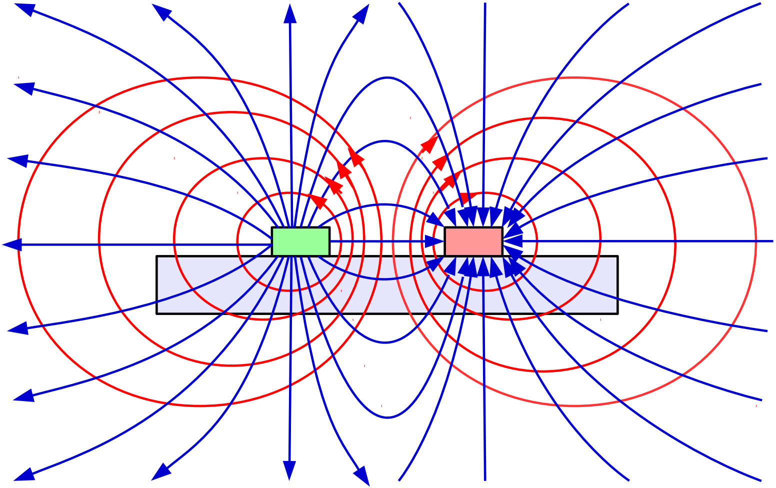
where $G(\omega)$ is the dielectric conductance at ω

Microwave Slotline



- A slotline consists of two parallel conducting strips on a dielectric substrate with relative permittivity ϵ_r
- The parameters are the separation (s) between the two conductors, the thickness (t) of the strip and the width (w) of each line

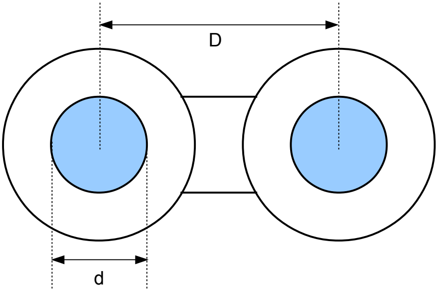
Slotline Fields



Twin Wire Model

Per unit inductance and capacitance

$$
\begin{aligned}
L &= \frac{\mu_0}{\pi} \cosh^{-1}\left[\frac{D}{d}\right] \\
C &= \frac{\pi \varepsilon_r \varepsilon_0}{\cosh^{-1}\left[\frac{D}{d}\right]}
\end{aligned}
$$



Therefore, from:

$$z_0 = \sqrt{\frac{L}{C}} = \frac{1}{\pi}\sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} cosh^{-1}\left(\frac{D}{d}\right)$$

Characteristic Impedance

Effective relative permittivity approximation

$$\epsilon_{eff} \approx \frac{\epsilon_r + 1}{2} \text{ (heuristic)}$$

Springfield approximation

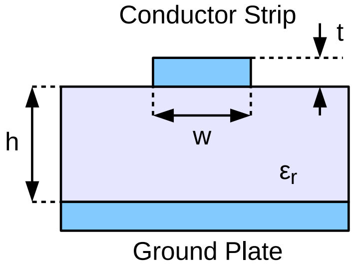
$$d \approx 0.67w\left(0.8 + \frac{t}{w}\right)$$

Therefore,

$$Z_0 = \sqrt{\frac{L}{C}} \approx \frac{1}{\pi}\sqrt{\frac{2\mu_0}{(1+\varepsilon_r)\varepsilon_0}}\cosh^{-1}\left(\frac{s+w}{0.67(0.8w+t)}\right)$$

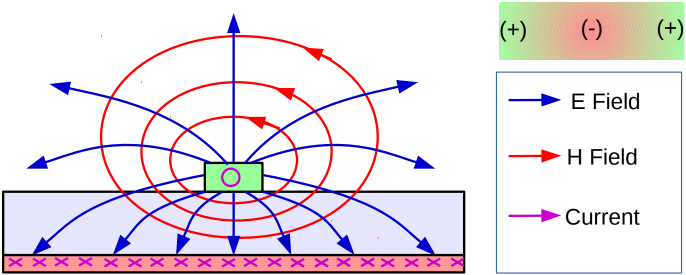
$$= \frac{120\sqrt{2}}{\sqrt{1+\varepsilon_r}}\cosh^{-1}\left(\frac{s+w}{0.67(0.8w+t)}\right)$$

Microstrip Transmission Line



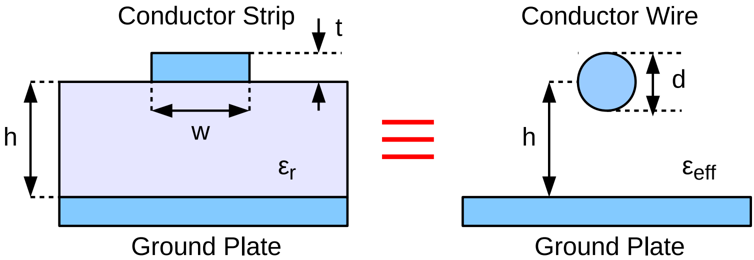
- A microstrip line is a conducting strip and ground plane separated by a dielectric substrate with relative permittivity ϵ_r
- The parameters are the separation (h) between the two conductors, the thickness (t) of the strip and the width (w) of the strip

Microstrip Fields



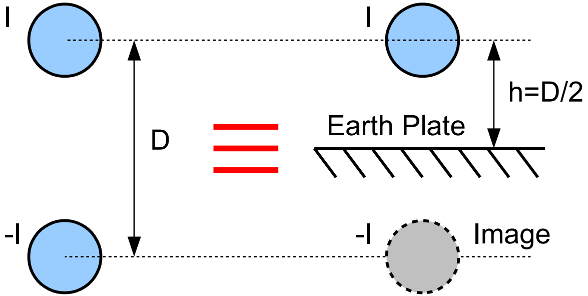
- This mode is known as a *quasi* TEM mode
  - A small longitudinal E field exists but it is very small compared to the transverse fields

Narrow Microstrip Approximation



- When $h \gg w$, it is considered as a *narrow* microstrip
  - Can be approximated to a conducting wire and ground plate
  - Using the *method* of images wire and ground plate can be transformed into a *twin wire* transmission line
  - An *effective* relative permittivity is needed

Method of Images



- A current (or charge) and its negative is equivalent to a single current and earth plate along the bisecting plane

Characteristic Impedance

Effective relative permittivity approximation

$$\begin{aligned}\epsilon_{eff} &\approx \frac{\epsilon_r + 1}{2} \text{ (heuristic)} \\ &\approx 0.475\epsilon_r + 0.67 \text{ (Digiacomo et al.)}\end{aligned} \tag{1}$$

Springfield approximation

$$d \approx 0.67w\left(0.8 + \frac{t}{w}\right) \tag{2}$$

Therefore from (1) and (2),

$$Z_0 = \sqrt{\frac{L}{C}} \approx \frac{120}{\sqrt{\epsilon_{eff}}}\cosh^{-1}\left[\frac{2h}{d}\right] \approx \frac{120}{\sqrt{\epsilon_{eff}}}\ln\left[\frac{4h}{d}\right] \text{ since } 2h \gg d$$

Characteristic Impedance (Contd..)

From (1) and (2)

$$
\begin{aligned}
Z_0 &\approx \frac{120}{\sqrt{0.475\epsilon_r + 0.67}} \ln\left[\frac{4h}{0.67w\left(0.8 + \frac{t}{w}\right)}\right] \\
&\approx \frac{174}{\sqrt{\epsilon_r + 1.41}} \ln\left[\frac{6h}{0.8w + t}\right] \quad \text{when } h < 0.8w
\end{aligned}
$$

For wide microstrips ($w \gg h$), the Assadourian and Rimai approximation can be used

$$
Z_0 \approx \frac{120\pi}{\sqrt{\epsilon_r}} \frac{h}{w}
$$

Power Attenuation

Surface resistance

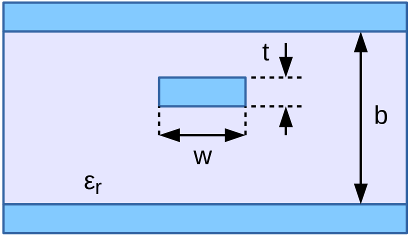
$$R_S = \frac{1}{\sigma_C \delta} = \sqrt{\frac{\omega \mu_C}{2\sigma_C}}$$

Assadourian and Rimai approximation for α of a microstrip

$$\alpha = \sqrt{\frac{\varepsilon_{eff}\varepsilon_0 \omega}{2\sigma_C}} \left[\frac{1 + \frac{h}{d}}{2\sqrt{h^2 - d^2}\cosh^{-1}\left(\frac{h}{d}\right)} \right]$$

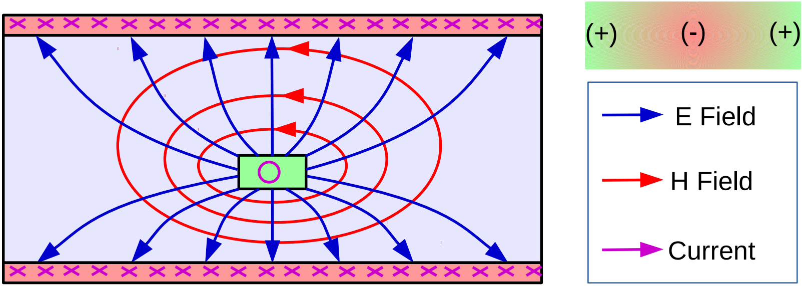
where σ_C is the conductivity and μ_C is the permeability of the microstrip conductor, d is the Springfield diameter

Microwave Striplines



- A stripline has a conducting strip within substrate with relative permittivity ϵ_r enclosed by two ground planes
- The parameters are the separation (b) between the two planes, the thickness (t) of the strip and the width (w) of the strip

Striplines Fields



- Fields are contained similar to that of a coaxial cable
  - Very low radiation (from the edges) and attenuation

Characteristic Impedance

Cohn's formula: uses a coaxial cable approximation

$$Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{4b}{\pi d}\right).$$

where d is the Springfield diameter. Subjected to the constraints

$$w \leq 0.35(b-t) \text{ and } 4t \leq b$$

Saliant Points

- Striplines have the least radiation loss of all substrate lines
  - Same power attenuation formula of coaxial cables can be used
- However
  - Involve complex designs
  - Difficult to connect to external components
  - Highly constrained design space

Conclusion

Summary

- Transmission lines transmit power at high frequencies by containing EM fields
- For maximum power transfer, the load has to be matched to the characteristic impedance of the transmission line
- Practical transmission lines have at least two conductors
  - Can have a balanced or unbalanced current distribution
- The operating frequency is limited by the efficiency of the shielding and dielectric loss