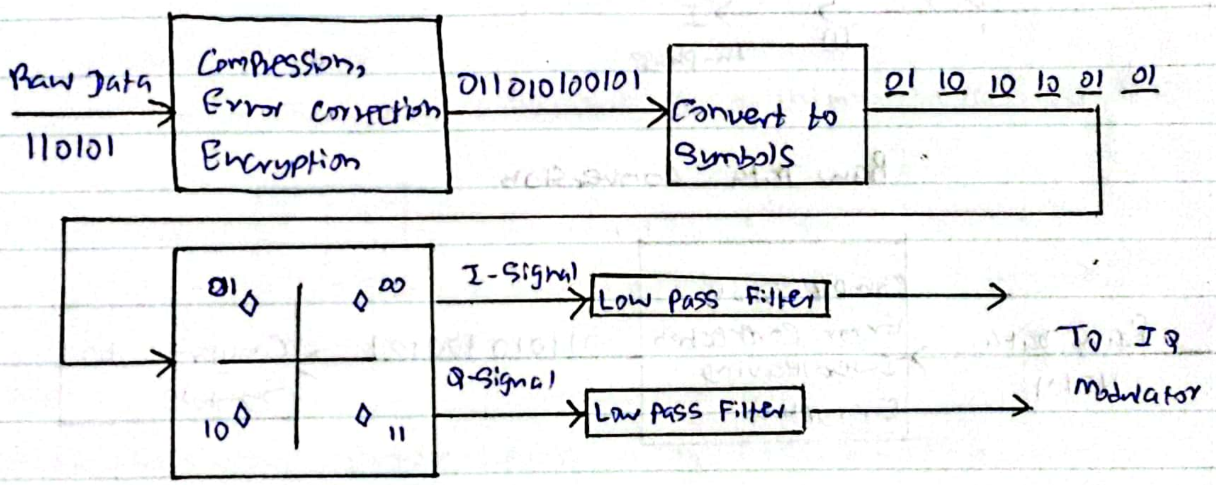
# Lecture 5:- Modulation on M-ary

## M-ary Modulation

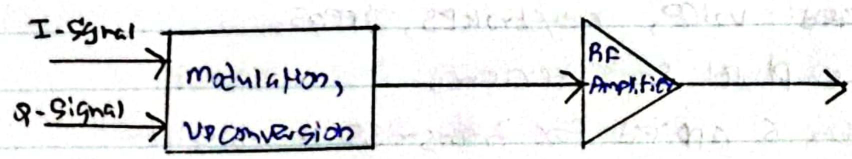
- M-ary (or multi-Symbol) modulation schemes include
    - i. multi-phase
    - ii. multi-amplitude
    - iii. combined multi-phase/multi-amplitude

- They are commonly used in telephone, microwave, WiFi, cellular mobile phones and satellite communications to achieve higher spectrum efficiency.

### M-ary Modulation Block Diagram




Modulation mapping




I - In phase Q - Quadrature

I/Q ~~data~~ modulation involves multiplying I/Q waveforms by ~~regarding~~ modulating signals that can have negative voltage values

I/Q data is used to modulate a carrier frequency

# In-Phase and quadrature components

- It is possible to create an arbitrarily phase-shifted sine wave by mixing together two sine waves that are 90° out of phase in different proportions

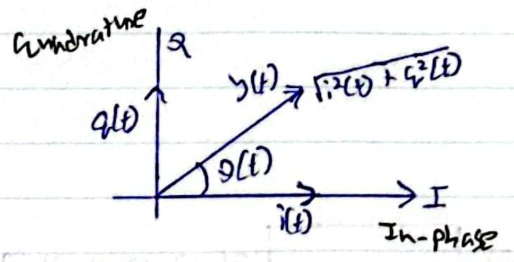
Complex baseband signal as $s(t) = i(t) + j\, q(t)$

In-Phase quadrature

Transmit signal $y(t) = i(t)\cos(2\pi f_c t) + q(t)\cos(2\pi f_c t + \pi/2)$

$$y(t) = i(t)\cos(2\pi f_c t) - q(t)\sin(2\pi f_c t) = \mathrm{Re}\left(y(t)\, e^{j2\pi f_c t}\right)$$

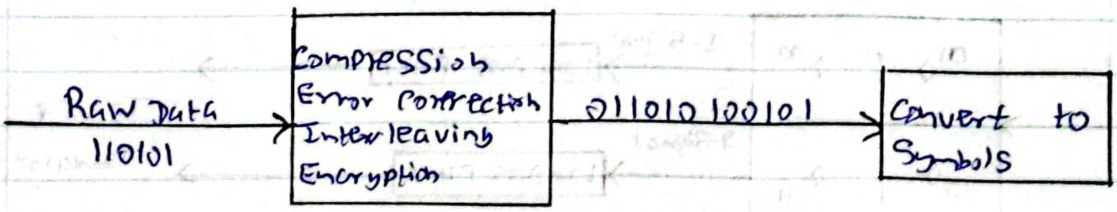
$$y(t) = \sqrt{i^2(t) + q^2(t)}\ \cos(2\pi f_c t + \theta(t))$$




~~theta~~

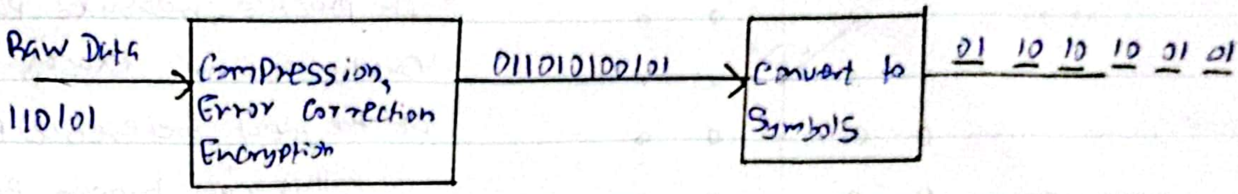
$\theta(t) = \tan^{-1}\left(q(t)/i(t)\right)$

## Raw Data Conversion




- Raw data come from the user
  Digitized voice, keystrokes, jpegs, ...
- Compression is employed for efficiency.
- Error Correction is applied for transmission quality.
- Interleaving creates signal-dropout resistance
- Encryption is applied for security.

## Data Bits to Symbols



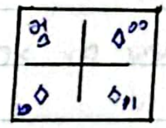

- Symbols are represented by the possible states of digital modulation.
- Higher order modulation allows more bits per symbol.
- What in the the world does that mean?

  Mapping symbols to I and Q

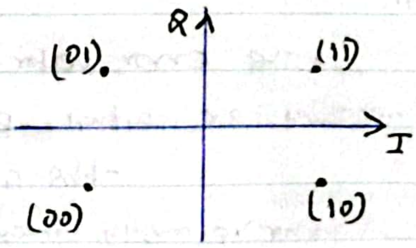
$$\text{Symbol rate} = \frac{\text{bit rate}}{\text{number of bits transmitted with each symbol}}$$ rs

## IQ Mapping

- What is Mapping:
  - Translate a symbol to a point in the IQ space



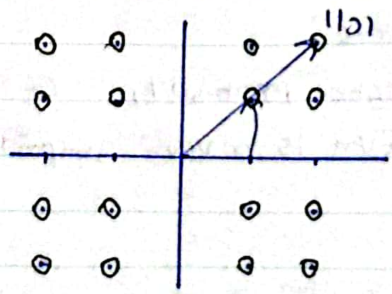

modulation mapping



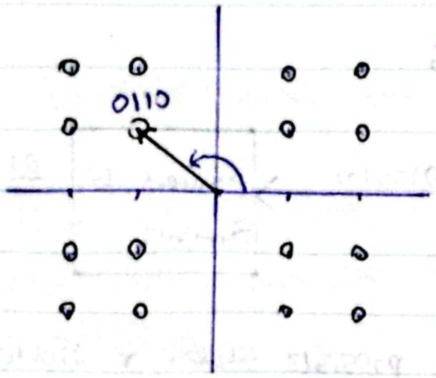

## Understanding Error vector Magnitude (EVM)

About digital modulation

Symbols are unique phase and/or amplitude combinations

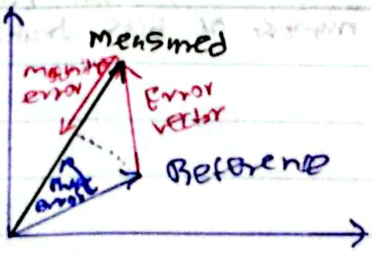



- Each symbols represents certain bits.




- In practice, measured points usually are not exactly at the ideal (reference) points
  - This can happen for many reasons

- If points are too far off, bit errors will occur
- One way to quantify this difference between actual and ideal point is error vector magnitude (EVM)




- Part of the difference due to magnitude error or phase error
- The error vector connects the ideal or reference point with the measured or actual point
  - Has a magnitude and a direction
- We're mostly interested in the magnitude of the error vector, not its direction
  - This is the error vector magnitude
- Measured at each symbol time
- Larger values of EVM indicate greater distance between measured and ideal points
  - Higher probability of bit errors
- Minimizing EVM is a very important goal

## Contributors to EVM

**Amplitude effects**
- Compression, non-linearity
- Noise (low SNR)
- Frequency response
- Intersymbol interference
- External interference, spurs
- Multipath, Fading

**Phase effects**
- Phase noise
- Phase response

**IQ imperfections**
- Gain imbalance
- Quadrature offset
- Carrier feed through

**Configuration issues**
- Mismatched filter types/ Parameters
- Mismatched symbol rates

## EVM and modulation order

- Higher modulation orders (more symbols)
  - Better throughput (more bits/symbol)

16 QAM
4 bits/symbol

64 QAM
6 bits/symbol

256 QAM
8 bits/symbol

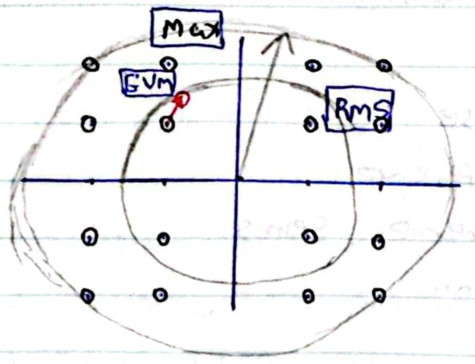
1024 QAM
10 bits/symbol

4096 QAM
12 bits/symbol

- However, more symbols → symbols closer together → greater chance of errors
- Therefore higher order modulation generally require better (i.e. lower) EVM values
  - Maximum allowable EVM often included in wireless specifications (e.g. 802.11 or cellular)
  - Function of modulation order and encoding

## Calculating EVM

- EVM is the magnitude (length) of a vector connecting the ideal vector endpoint with the received or measured vector endpoint.

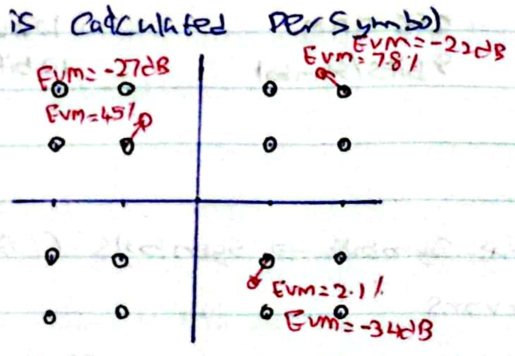



- The magnitude is reported as a normalized quantity
  - Relative to maximum constellation power ($EVM_{max}$)
  - Relative to RMS constellation power ($EVM_{RMS}$)

- Important to use same the normalization (max or RMS) when comparing EVM values
- EVM can be expressed:
  - As a percentage (%)
  - In decibels (dB)

## EVM results

- EVM is calculated per symbol



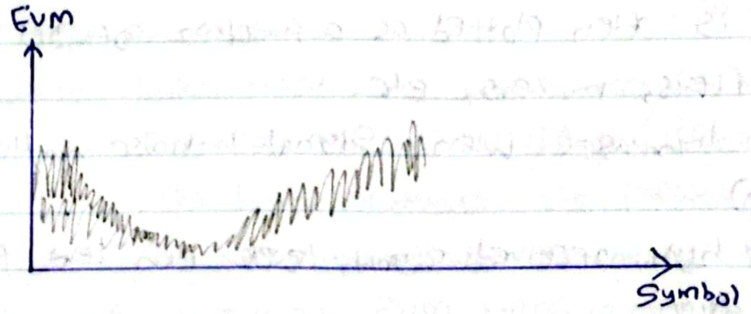

- EVM is reported over a number of symbols
  - max, min, average, etc. values

| $EVM_{max}$ | $EVM_{min}$ | $EVM_{avg}$ |
|---|---|---|
| 7.8% | 2.1% | 4.8% |
| -22dB | -34dB | -28dB |

- Lower EVM indicates better modulation accuracy
  - Smaller percentage values
  - Lower (more negative) dB values

EVM vs time (symbol)

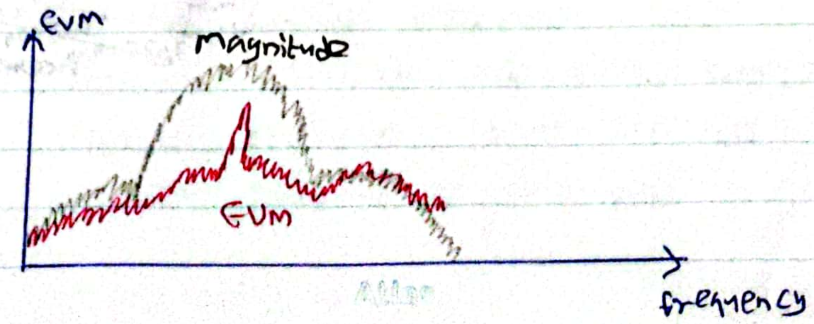
- EVM values can be plotted as a function of time (succesive symbols)
- Can provide useful information about sources of error or inaccuracy




- Slight differences in $T_x$ and $R_x$ symbol rate appear as a V-shaped curve
- EVM may be higher at the beginning or end of a bursted or pulsed signal
- EVM may increases when amplitude peaks or goes (close) to zero

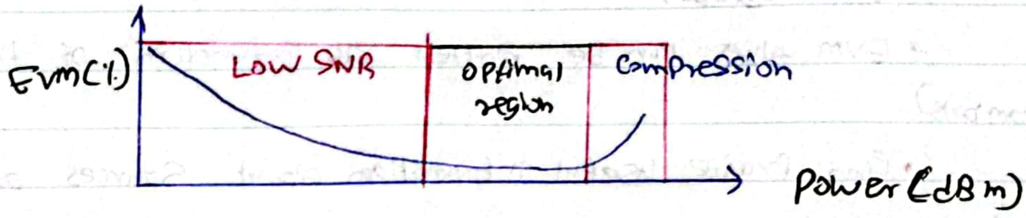
EVM vs. frequency

- EVM can also be plotted vs frequency
  - Also called error vector (magnitude) spectrum
  - Created by taking FFT of EVM vs time
- Useful in finding spurs/interferers that cannot be (easily) detected by standard power vs. frequency traces



-Combination of desired and undesired signal cau increased EVM at the frequency of the spurious signal

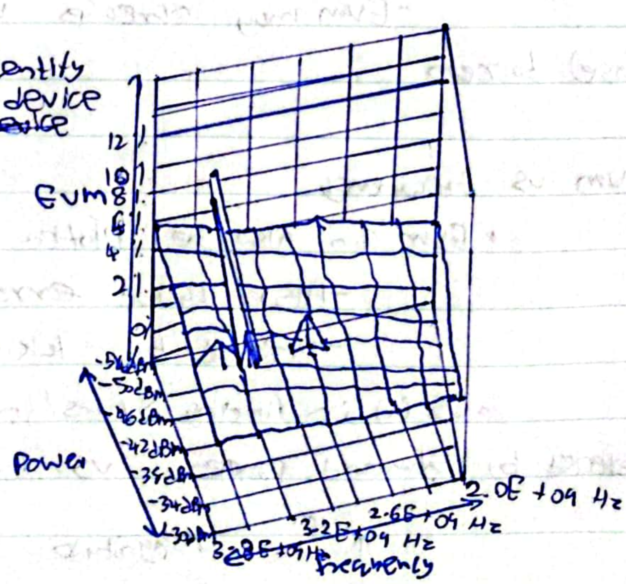
Evm. vs Power




- Evm is often plotted as a function of DUT input power for amplifiers, mixers, etc.
- Evm increases when signal-to-noise ratio decreases (low SNR> poor Evm)
- Very high received signal levels can lead to compression and degrade Evm.

Evm. vs power vs. frequency

- Evm can also be plotted as a function of both power and frequency
- These graphs can help identify trends or problem regions for a device under test



Instruments for measuring EVM

- Usually measured on a spectrum/signal analyzer
  - ~~Sometimes~~ Sometimes in combination with a vector signal generator
- Measurement setup should have a lower EVM than the DUT
  DUT – Device Under Test
  At least 5–10dB
  The more margin, the better

Measurement best practices

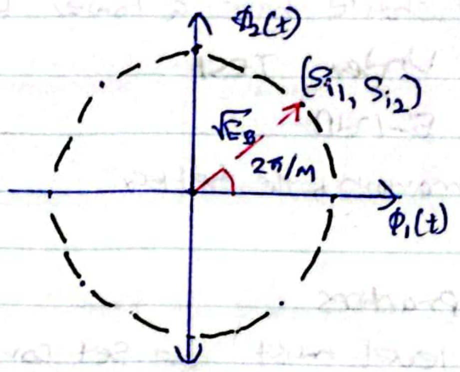
- Reference level must be set correctly
  - Too low causes clipping / distortion
  - Too high increases the influence of noise
  - Can be automatically determined
- Number of averaged EVM measurements:
  - High enough to obtain ~~repeatedly~~ repeatability
  - low enough to not excessive increase test time
- Enable equalization and/or frequency response correction
- Use a common frequency reference

Summary

- EVM is the most important overall measurement of modulation quality
- Measures the relative magnitude of a vector connecting the ideal (reference) symbol and the measured symbol.
- Errors may be due to magnitude and/or phase errors
- Measured using a spectrum/signal analyzer
  - Sometimes together with a vector & signal generator
- Good EVM measurements require
  - Instruments with good EVM ~~perf~~ performance
  - Correct reference level settings
  - Correct number of averages

# M-ary Phase shift keying

- M-ary PSK is two-dimensional $(N=2)$, M signal points are equally spaced on a circle of radius $\sqrt{E}$ and centered at origin




- In M-ary PSK, the phase of carrier can take one of M possible values.

$$\theta_i = \frac{2(i-1)\pi}{M} \qquad i = 1, 2, \dots, M$$

- QPSK is a special case of M-ary PSK $(M=4)$
- The transmitted MPSK signal over duration T is

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right). \qquad i=1, \dots M$$

- E: Signal Energy per Symbol
  - $f_c = n_c/T$ is the carrier frequency, $n_c$ is an integer

The signal can be written as

$$S_i(t) = \sqrt{2E/T}\,\cos(2\pi f_c t)\cos(2\pi(i-1)/M) \mp \sqrt{2E/T}\,\sin(2\pi f_c t)\sin\left(\frac{2\pi(i-1)}{M}\right)$$

The orthogonal basis functions are

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t), \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t), \qquad 0 \le t \le T$$

Signal projections onto orthonormal basis functions

$$S_{i1} = \int_0^T S_i(t)\,\phi_1(t)\,dt = \sqrt{E}\cos[2\pi(i-1)/M]$$

$$S_{i2} = \int_0^T S_i(t)\,\phi_2(t)\,dt = \sqrt{E}\,\overset{\sin}{\cancel{\cos}}(2\pi(i-1)/M)$$

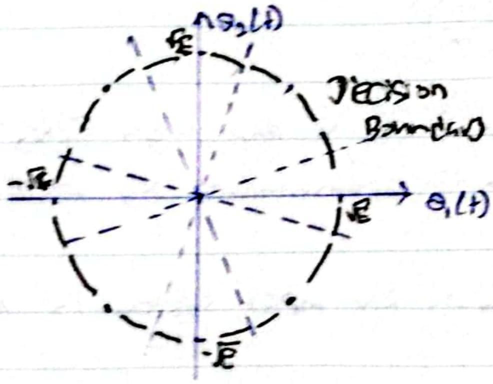
## M-ary Phase Shift Keying: Examples

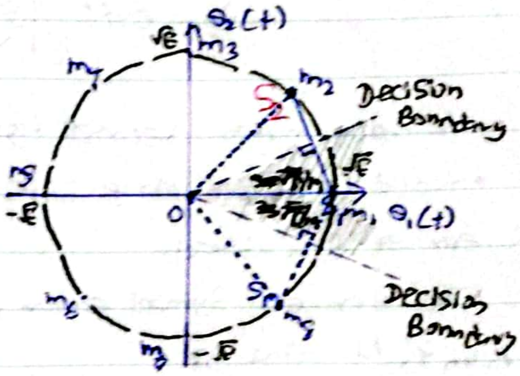
Signal-Space for MPSK

| Phase of MPSK (rad) | Coordinates of Signal Points | |
|---|---|---|
| | $s_{i1}$ | $s_{i2}$ |
| $\theta_i = \frac{2(i-1)\pi}{M}$ | $\sqrt{E}\cos\left(2\pi\frac{(i-1)}{M}\right)$ | $\sqrt{E}\sin\left(2\pi\frac{(i-1)}{M}\right)$ |

$i \in \{1, \ldots, M\}$

Exercise: Determine the signal constellation points for 8 & 16-PSK System




- Average error probability of symbol error for MPSK can be approximated using union bound

$$P_e \le \frac{1}{2}\sum_{\substack{k=1 \\ k\ne i}}^{M} \operatorname{erfc}\left(\frac{d_{ik}}{2\sqrt{N_0}}\right), \text{ for all } i.$$

- Assume $s_1$ is sent, $s_1 = (\sqrt{E} \quad 0)^T$, and that $E/N_0 \gg 1$
- The two nearest points $s_2$ and $s_m$ can be mistaken for $s_1$ due to channel noise
- The angle formed between $s_1$ and $s_2$ (or $s_1$ and $s_m$) is $2\pi/m$
- Decision boundary for $z_1$ and $z_2$ form an angle of $\pi/m$
- Euclidean distance of the points $s_1$ and $s_2$ (or $s_1$ and $s_m$)

$$d_{12} = d_{1m} = 2\sqrt{E}\sin(\pi/m)$$

Error Probability for mpsk

$$P_e \approx \text{erfc}\left(\sqrt{\frac{E}{N_0}}\,\sin\left(\frac{\pi}{m}\right)\right)$$

$$\frac{1}{2}\sum_{\substack{k=1\\k\neq 1}}^{M} \text{erfc}\left(\frac{d_{1k}}{2\sqrt{N_0}}\right) \approx \frac{1}{2}\,\text{erfc}\left(\frac{d_{12}}{2\sqrt{N_0}}\right) + \frac{1}{2}\,\text{erfc}\left(\frac{d_{1M}}{2\sqrt{N_0}}\right)$$

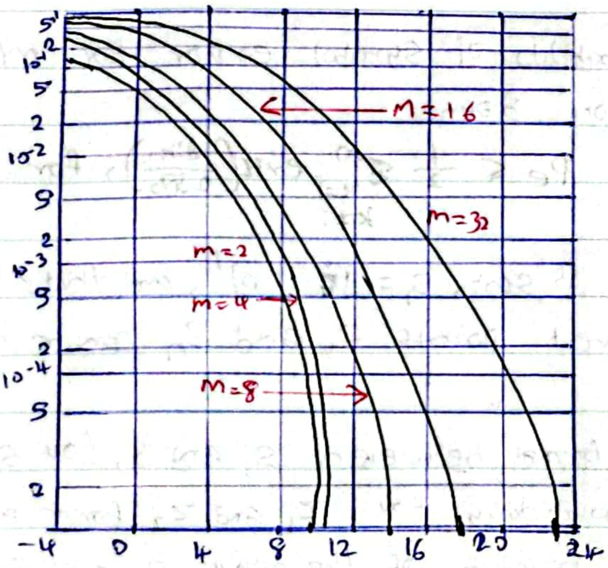
$$d_{12} = d_{1M} = 2\sqrt{E}\,\sin(\pi/M)$$

$$= \text{erfc}\left(\sqrt{\frac{E}{N_0}}\,\sin\left(\frac{\pi}{m}\right)\right)$$

- Average error probability of Symbol error for MPSK

$$P_e = \text{erfc}\left(\sqrt{\frac{E}{N_0}}\,\sin\left(\frac{\pi}{m}\right)\right)$$

- The approximation becomes tight for fixed $M$ and high $E/N_0$
- Note that for $M=4$ (QPSK), $P_e$ is approximated by $\text{erfc}(\sqrt{E/2N_0})$ as earlier discussed.
- The average Symbol error probability for MPSK system for $M = 2, 4, 8, 16$ and $32$




Power spectra for M-ary PSK

- Symbol duration for MPSK is $T = T_b \log_2 M$
- $T_b$: bit duration
- As in the QPSK system, the PSD for MPSK is

$$S_B(f) = 2E\,\text{sinc}^2(fT) = 2E_b \log_2 M\,\text{sinc}^2(fT_b \log_2 M)$$

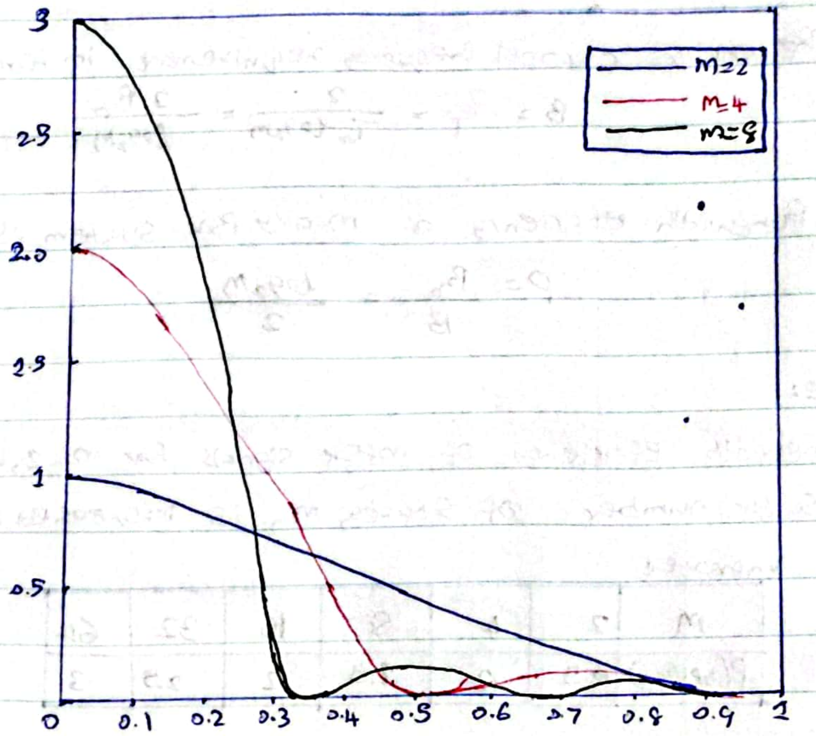
- Null-to-Null Bandwidth: Spectral width of the main lobe, which is bounded by spectral nulls

• Spectral nulls: frequencies at which the PSD is zero.

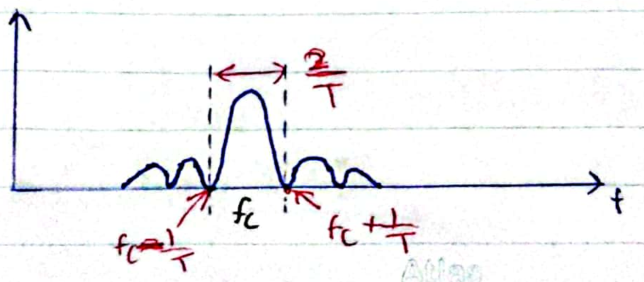
• Spectral width of main lobe in MPSK signals defines the null-to-null bandwidth; it contains most of the signal power.

• For bandpass basis functions, $\phi_1(t)$ and $\phi_2(t)$, channel requirement in terms of bandwidth is

$$B = \frac{2}{T}$$




```
m=[2 4 8];
f=0:0.01:1;
Tb=1
y=log2(m).*sinc((f*Tb)*log2(m)).^2;
plot(f,y)
legend('M=2','M=4','M=8')
xlabel('Normalized frequency')
ylabel('Normalized power spectral density,$_B(f)/2E_b$')
```



## Bandwidth Efficiency of M-ary PSK

$$\rho = \frac{\log_2 M}{2}$$

The bit rate in bits/sec (BPS) is

$$R_b = 1/T_b$$

where

$$T_b = \frac{T}{\log_2(M)} \Rightarrow T = T_b \log_2 M$$

Redefining channel frequency requirement in terms of $R_b$

$$B = \frac{2}{T} = \frac{2}{T_b \log_2 M} = \frac{2 R_b}{\log_2 M}$$

Bandwidth efficiency of M-ary PSK system is

$$\rho = \frac{R_b}{B} = \frac{\log_2 M}{2}$$

Example:

- Bandwidth efficiency of MPSK signals for $M = 2, 4, 8, 16, 32$ and $64$
- As the number of states, M, is increased, the bandwidth efficiency is improved.

| M | 2 | 4 | 8 | 16 | 32 | 64 |
|---|---|---|---|---|---|---|
| ρ(bps/Hz) | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |

What is the express of improvement in bandwidth efficiency in MPSK as M increases?

- Deterioration in error performance considering of fixed $E/N_0$

$$\boxed{P_e = \operatorname{erfc}\left(\sqrt{\frac{E}{N_0}} \sin\left(\frac{\pi}{M}\right)\right)}$$

# M-ary PSK:

BPSK $\Rightarrow$ 2 Symbols 0, 1

$\therefore$ phase shift keying in BPSK $= \frac{2\pi}{\text{no. of symbols}} = \frac{2\pi}{2} = 180° = \pi$

QPSK $\Rightarrow$ 4 Symbols

$\therefore$ phase shift in QPSK $= \frac{2\pi}{4} = \frac{\pi}{2} = 90°$

M-ary PSK $\Rightarrow$

If there are "N symbols"

$2^N = M$ possible Symbols

$\therefore$ phase shift $= \frac{2\pi}{M}$

$\therefore$ the duration of each bit will be '$NT_b$'

$$\boxed{T_S = NT_b}$$

Transmitted waveform,

$$\boxed{S(t) = \sqrt{2P_S}\cos(2\pi f_0 t + \phi_m)} \rightarrow (1)$$

$m = 0, 1, 2, \ldots, M-1$

$\phi_m \rightarrow$ Phase Angle $\boxed{\phi_m = (2m+1)\frac{\pi}{M}}$

## Signal phase Representation

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

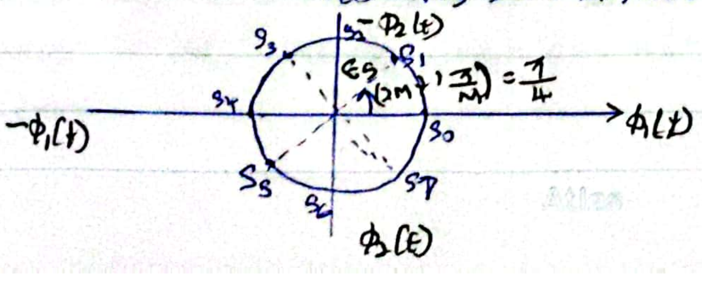
(1) $\Rightarrow$

$$S(t) = \sqrt{2P_S}\cos\phi_m\cos(2\pi f_0 t) - \sqrt{2P_S}\sin\phi_m\sin(2\pi f_0 t)$$

$\times \,\&\, \div \sqrt{2/T_S}$

$$S(t) = \sqrt{P_S T_S}\,\underbrace{\sqrt{\tfrac{2}{T_S}}\cos\phi_m\cos(2\pi f_0 t)}_{\phi_1(t)} - \sqrt{P_S T_S}\,\underbrace{\sqrt{\tfrac{2}{T_S}}\sin\phi_m\sin(2\pi f_0 t)}_{\phi_2(t)}$$

$$S(t) = \sqrt{P_S T_S}\cos\phi_m\,\phi_1(t) - \sqrt{P_S T_S}\sin\phi_m\,\phi_2(t)$$



M-Symbol

Signal points $S_0, S_1, S_2, \ldots S_{m-1}$

8 Symbol

Signal Points $S_0, S_1, S_2, \ldots S_7$

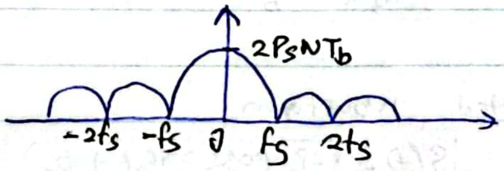
Distance of each Signal Point from origin $\sqrt{P_s T_s} = \sqrt{E_s}$

$E_s \rightarrow$ symbol energy

Power spectral density

PSD of QPSK $\Rightarrow S_{P(QPSK)}(f) = 2 P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$

Put $T_s = N T_b \Rightarrow \boxed{S_B(f) = 2 P_s N T_b \left[\frac{\sin(\pi f N T_b)}{\pi f N T_b}\right]^2}$




$N T_b = T_s \Rightarrow f_s = \frac{1}{T_s}$

Bandwidth of M-ary PSK

$BW = (HF - LF)$ in mainlobe

$BW = f_s - (-f_s) = 2 f_s$

$BW = 2 \frac{1}{T_s} = 2 \cdot \frac{1}{N T_b}$ $\quad T_b = f_b$

$\boxed{BW = \frac{2 f_b}{N}}$ $\quad$ if $N \uparrow$ $BW \downarrow$

## M-ARY Quadrature Amplitude Modulation (MQAM)

The direct modulation of carriers in quadrature (i.e. $\cos\omega_0 t$ and $\sin\omega_0 t$) is involved, therefore, this system is called as the quadrature amplitude phase shift keying i.e. QAPSK or simply QASK.

- It is also known as quadrature amplitude modulation (QAM)

- QAM is a combination of both amplitude modulation and phase modulation

- M-ary QAM is a two dimensional generalization of M-ary PAM, it involves two orthogonal basis functions, they are

$$\phi_1(t) = u_1(t) = \sqrt{\frac{2}{T_s}} \cos(\omega_c t)$$

and

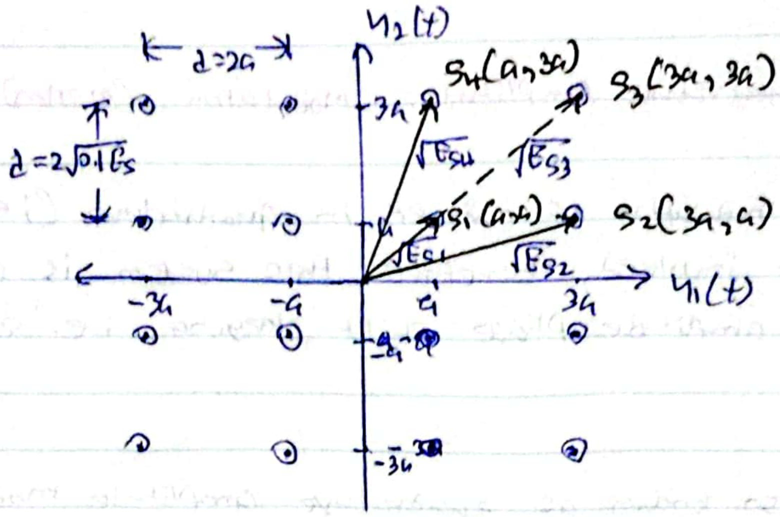
$$\phi_2(t) = u_2(t) = \sqrt{\frac{2}{T_s}} \sin(\omega_c t)$$

### Geometrical Representation of QASK

- Let us assume that using QASK, we want to transmit a symbol consisting of 4-bits. This means that $N=4$ and there are $2^4 = 16$ different possible symbols

- Now, let us assume that all the 16 signals are equally spaced. As these signals are placed symmetrically, we can determine the energy associated with a signal, by considering the four signals in the first quadrant.

- The average normalized energy of each signal is given by the average of the energy associated with signals in the first quadrant.




Geometric representation of 16 signals in a QASK system (16-QAM)

Therefore, $$E_s = \frac{E_{s1} + E_{s2} + E_{s3} + E_{s4}}{4}$$

Looking at figure, we can write

$E_{s1} = (a^2+a^2)$, $E_{s2} = (9a^2+a^2)$

$E_{s3} = (9a^2+9a^2)$ and $E_{s4} = (a^2+9a^2)$

Substituting all these values into expression for $E_s$, we obtain

$$E_s = \frac{1}{4}\left[(a^2+a^2)+(9a^2+a^2)+(a^2+9a^2)+(9a^2+9a^2)\right]$$

$$E_s = 10a^2$$

Therefore, $a = \sqrt{0.1E_s}$

and $d = 2a = 2\sqrt{0.1E_s}$

where $E_s$ = Normalized Symbol Energy. In this System, because each Symbol consists of 4-bits, the normalized Symbol energy is given by,

$$E_s = 4E_b$$

where $E_b$ is the normalized energy per bit.

Therefore, $a = \sqrt{0.4E_b}$ and $d = 2\sqrt{0.4E_b}$

Depending on the number of possible symbols M, we have two different constellation diagrams.

1. Square constellation: In which number of bits per symbol is even
2. Cross constellation: In which number of bits per symbol is odd

## Types of QAM

- Depending on the number of bits per message, the QAM signals may be classified as under:

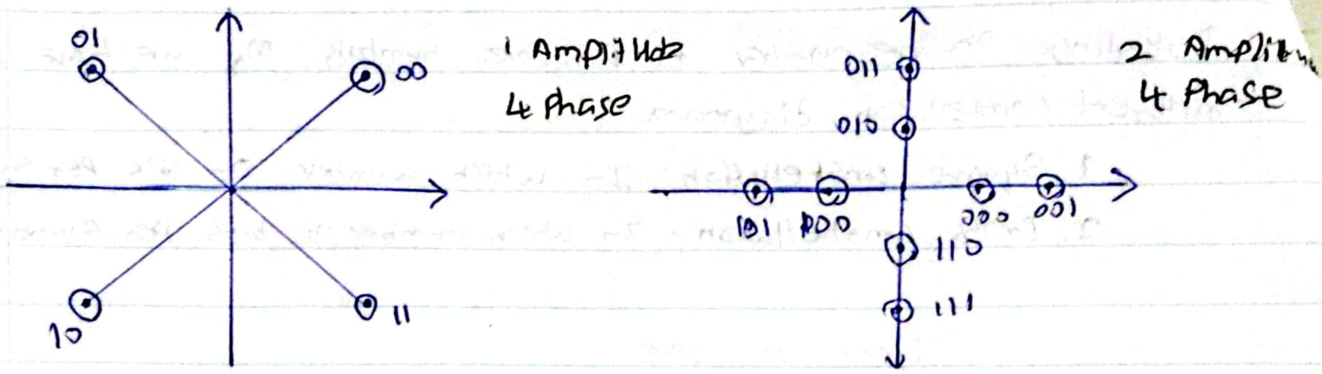
| Name of scheme | Bits per symbol | Number of symbols |
|---|---|---|
| 4 QAM | 2 | $2^2 = 4$ |
| 8 QAM | 3 | $2^3 = 8$ |
| 16 QAM | 4 | $2^4 = 16$ |
| 32 QAM | 5 | $2^5 = 32$ |
| 64 QAM | 6 | $2^6 = 64$ |

## 4 QAM and 8 QAM Systems

- The constellation of 4 QAM system is shown in figure. All the symbols have same amplitude but different phases.

- Figure (b) shows the constellation diagram of 8-QAM. Here, it may be noted that there are two amplitude levels and four phases involved.

- The time domain display of 8 QAM is shown in figure.




(a) 4 QAM (b) 8QAM

Fig. Constellation diagrams

## Bandwidth of QASK system

- The expression for the QASK output is very similar to that of a M-ary PSK.
- The spectrum of QASK is shown in figure, which is quite similar to that of a M-ary PSK.

Therefore, the bandwidth of QASK is given by,

$$BW = f_s - (-f_s) = 2f_s$$

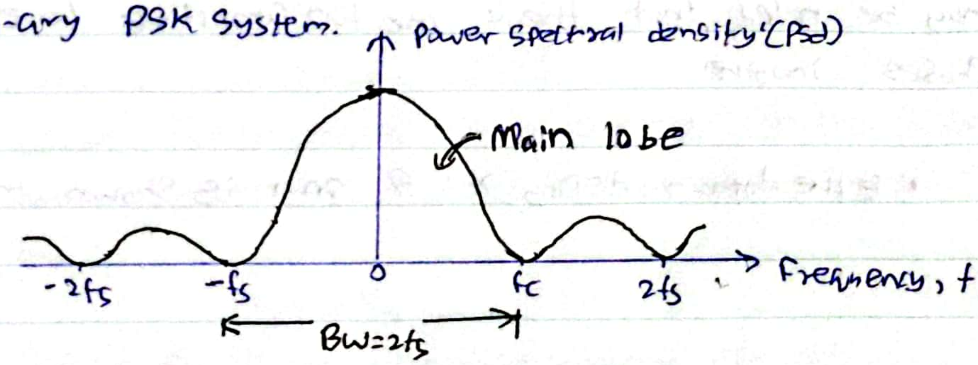
$$BW = \frac{2}{T_s} = \frac{2}{NT_b}$$

(Since $T_s = NT_b$)

Therefore,

$$BW = \frac{2f_b}{N} \quad \text{since } f_b = \frac{1}{T_b}$$

- Thus, the bandwidth of QASK system is same as that of an M-ary PSK system.




Frequency spectrum of QASK

# M-ary ASK Modulation Technique

- In M-ary Amplitude shift keying (M-ary ASK), there are M different amplitude levels of the carrier.
- The signal is represented by,

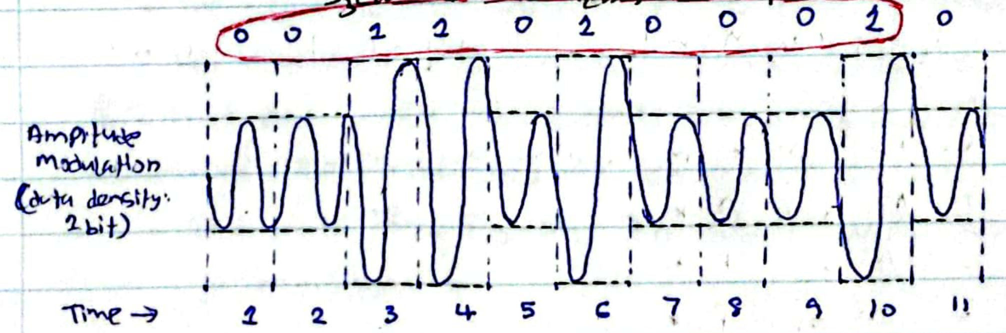
$$S_i(t) = A_i \cos(2\pi f_c t), \quad 0 \leq t \leq T_s \quad \text{for } i = 1, 2, \ldots, M$$

- $A_i = (2i - 1 - M)d$, where $2d$ is the difference between two consecutive signal amplitudes.
- Let $M=4$ and $d=1$, the four signal amplitudes will be $-3, -1, 1$ and $3$ V.

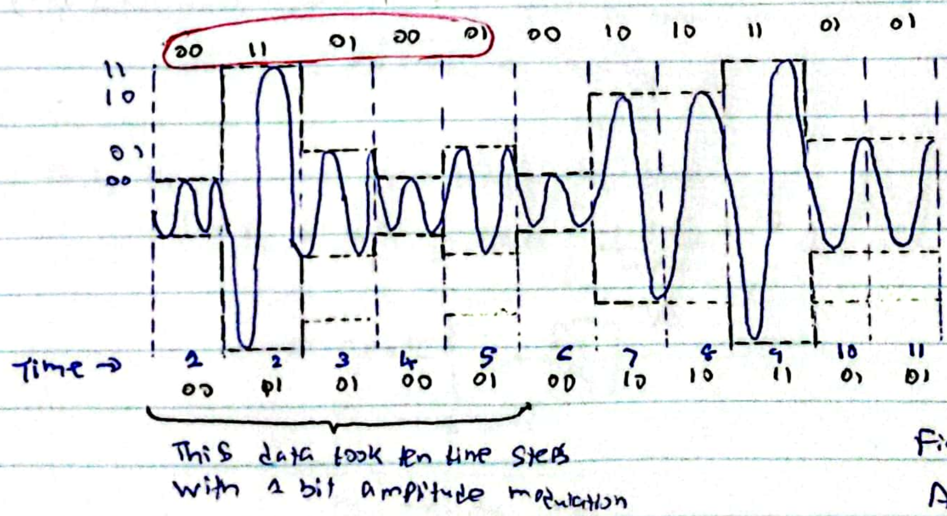
The M-ASK signals will be: for $0 \leq t \leq T_s$

$$S_1(t) = \cos(2\pi f_c t), \qquad S_2(t) = -\cos(2\pi f_c t),$$

$$S_3(t) = 3\cos(2\pi f_c t), \qquad S_4(t) = -3\cos(2\pi f_c t)$$




BASK : 2 Level
1 bit in 1 symbol duration




M-ASK : 4 Level
2 bit in 1 symbol duration

Fig 2: BASK and Mary ASK Signal Representation

## Bandwidth of QASK System

- The expression for the QASK system output is very similar to that of a M-ary PSK.
- The spectrum of QASK is shown in figure, which is quite similar ~~to th~~ to that of a M-ary PSK.

Therefore, the bandwidth of QASK is given by

$$BW = f_s - (-f_s) = 2f_s$$

$$BW = \frac{2}{T_s} = \frac{2}{NT_b}$$

## Error probability of 16 QAm (16 QASK)

- The signal space diagram of 16 QAm is shown in figure.
- Let us calculate the error probability for the symbol such as $r_6$ in figure which is located at $(a, a)$. This signal has the largest ~~for~~ probability of error.
- The signals $r_7$, $r_{10}$ and $r_{11}$ also will have the largest error probability

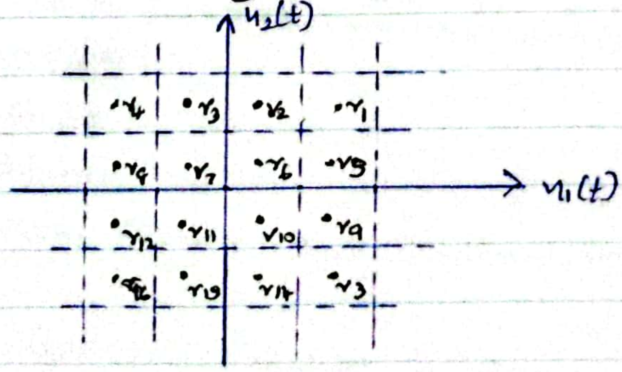
The minimum distance is given by

$$d = \sqrt{0.4 E_s} = \sqrt{1.6 E_b}$$

And, the error probability is given by

$$P_e \leq 4 \times \frac{1}{2} \operatorname{erfc}\left[\frac{1.6 E_b}{4 N_0}\right]^{-1/2}$$

$$= 2\operatorname{erfc}\left[0.4\frac{E_b}{N_0}\right]^{-1/2}$$




Signal space of 16 QAm