GENERAL SIR JOHN KOTELAWALA DEFENCE UNIVERSITY

Faculty of Engineering

Department of Mathematics

BSc Engineering Degree

Semester 1 Examination – July 2022

(Intake 39 –2^{nd} batch – All Streams)

MA 1103 – MATHEMATICS

Time allowed: **3 (Three)** hours 14th July, 2022

INSTRUCTIONS TO CANDIDATES

- This paper contains 4 questions from Page 3 to Page 5.
- Answer **all** questions.
- This is a closed book examination.
- You are permitted to use a calculator.
- This examination accounts for 70% of the module assessment. A total maximum mark obtainable is 100. The marks assigned for each question and parts thereof are indicated in square brackets
- If you have any doubt as to the interpretation of the wordings of a question, make your own decision, but clearly state it on the script.
- All examinations are conducted under the rules and regulations of the KDU.

This Page is Intentionally Left Blank.

1. (a) Find the intersection point of the line $\vec{r}(t) = \langle 1, 1, 0 \rangle + t\langle 0, 2, 4 \rangle$ with the plane $x + y + z = 14$.

[5 marks]

(b) Find the distance from the point $Q = (1, 2, 2)$ to the plane given by

$$\vec{n} \cdot \langle x, y, z \rangle = 3,$$

where $\vec{n} = \langle \frac{3}{5}, \frac{4}{5}, 0 \rangle$.

[5 marks]

(c) The polynomial $x^3 - mx + 15$ has three real roots. Two of these roots sum to 5. What is $|m|$?

[5 marks]

(d) The matrices $A = \begin{pmatrix} a & 1 \\ -2 & b \end{pmatrix}$ and $B = \frac{1}{25}\begin{pmatrix} 114 & 48 \\ 48 & 86 \end{pmatrix}$ have the same eigenvalues. Find all the possible values for a and b.

[5 marks]

(e) Calculate the area of the parallelogram spanned by $\vec{a} = -2\vec{i} + \vec{j} + 4\vec{k}$ and $\vec{b} = 3\vec{i} + 2\vec{j} + 5\vec{k}$.

[5 marks]

(f) Evaluate

$$\left(\frac{4+3i}{3-4i}\right)^{2022}.$$

[5 marks]

(g) The age distribution (in years) of a group of 100 individual is given below.

Ages (x)	2-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39
No of individuals (f)	8	11	21	20	17	10	10	3

Find the median value.

[5 marks]

(h) The probability that a thirty-year-old man will survive a fixed length of time is 0.995. The probability that he will die during this time is therefore 0.005. An insurance company will sell him a \$20,000 life insurance policy for this length of time for a premium of \$200. What is the expected gain for the insurance company?

[5 marks]

2. (a) What are the possible angles θ between two unit vectors $\vec{e}$ and $\vec{f}$ if

$$\left\|\vec{e} \times \vec{f}\right\| = \frac{1}{2}?$$

Show the calculations.

[6 marks]

(b) If $x + y + z = 0$, then find the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}.$$

[7 marks]

(c) An intelligent agent knows that 60 aircraft, consisting of fighter planes and bombers, are stationed at a certain secret airfield. The agent wishes to determine how many of the 60 are fighter planes and how many are bombers. There is a type of rocket carried by both sorts of plains; the fighter carries six of these rockets, the bomber only two. The agent learns that 250 rockets are required to arm every plane at this airfield. Furthermore, the agent overhears a remark that there are twice as many fighter planes as bombers at the base. Formulating a linear system, determine whether the agent information is correct or not.

[7 marks]

3. (a) Let α and β be the roots of the polynomial $3x^2 + x - 1$. Find the value of

$$3\left(\alpha^3 + \beta^3\right) + \left(\alpha^2 + \beta^2\right) - \alpha - \beta.$$

[5 marks]

(b) Use De Moivre's formula to derive the trigonometric identity

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta.$$

[5 marks]

(c) The loci C_1 and C_2 are given by $|z+1+3i| = |z-5-7i|$ and $\arg z = \frac{\pi}{4}$, respectively.

i. Find the loci C_1 and C_2. Then sketch on a single Argand diagram.

ii. Shade the region on a Argand diagram that satisfies

$$|z+1+3i| \leq |z-5-7i|$$

and

$$\frac{\pi}{4} \leq \arg z \leq \frac{\pi}{2}.$$

[2×5 marks]

4. (a) Give an example for ***"continuous random variable"*** and ***"discrete random variable"***.

[4 marks]

(b) A person has undertaken a mining job. The probabilities of completion of job on time with and without rain are 0.42 and 0.90 respectively. If the probability that it will rain is 0.45, then determine the probability that the mining job will be completed on time.

[6 marks]

(c) Suppose the random variable X has an exponential distribution with parameter $\lambda > 0$. If the probability density function of X is

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

then prove that

i. $E(X) = \frac{1}{\lambda}$,

ii. $V(X) = \frac{1}{\lambda^2}$.

[2×5 marks]

--------------------End of the question paper------------------------