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Department of Mathematics

Mathematics - MA 1103
Tutorial 06 - Probability & Statistics

Year: 2021   Intake: 38 - 03$^{\text{rd}}$ Batch   Semester: 01
Learning Outcomes Covered: LO4, LO5

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(1) Two balls are drawn in succession without replacement from an urn containing 4 red balls and 3 black balls. List the sample points for the possible outcomes and find the values x of the random variable X, where X is the number of red balls.

(2) Suppose that the error in the reaction temperature, in ^{0}C, for a controlled laboratory experiment is a continuous random variable X having the probability density function

$$f(x) = \begin{cases} \frac{x^2}{3} & \text{if} \quad -1 < x < 2 \\ 0, & \text{elsewhere.} \end{cases}$$

     (a) Show that $f(x)$ is a probability density function.

     (b) Find $P(0 < X \leq 1)$.

     (c) Find the cumulative distribution function $F(x)$ and use it to evaluate $P(0 < X \leq 1)$.

     (d) Find the mean and variance of $g(X) = 4X + 3$.

(3) A continuous random variable X has the density function

$$f(x) = \begin{cases} \frac{5}{8b} & \text{if} \quad \frac{2b}{5} \leq x \leq 2b \\ 0, & \text{elsewhere.} \end{cases}$$

     Find $F(x)$ and use it to evaluate $P(X \leq b)$.

(4) Let X be the random variable that denotes the life in hours of a certain electronic device. The probability density function is

$$f(x) = \begin{cases} \frac{20000}{x^3} & \text{if} \quad x > 100 \\ 0, & \text{elsewhere.} \end{cases}$$

     Find the expected life of this type of device.

(5) The weekly demand for Pepsi, in thousands of litres, from a local chain of efficiency stores, is a continuous random variable X having the probability density function

$$f(x) = \begin{cases} 2(x-1) & \text{if} \quad 1 < x < 2 \\ 0, & \text{elsewhere.} \end{cases}$$

    (a) Find the mean and variance of X.

    (b) Find the expected value of $g(X) = X^2 + X - 2$.

(6) Given a standard normal distribution, find the area under the curve that lies

    (a) to the right of $z = 1.84$, and

    (b) between $z = -1.97$ and $z = 0.86$.

(7) A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution and can be measured to any degree of accuracy,

    (a) what percentage of resistors will have a resistance exceeding 43 ohms?

    (b) If resistance is measured to the nearest ohm, find the percentage of resistances exceeding 43 ohms.

(8) The average grade for an exam is 74, and the standard deviation is 7. If 12% of the class is given A's, and the grades are curved to follow a normal distribution,

    (a) what is the lowest possible A and the highest possible B?

    (b) Find the 6th decile.

(9) Given the normally distributed variable X with mean 18 and standard deviation 2.5, find

    (a) $P(X < 15)$;

    (b) the value of k such that $P(X < k) = 0.2236$;

    (c) the value of k such that $P(X > k) = 0.1814$;

    (d) $P(17 < X < 21)$.