

Electromagnetic Waves in a Medium

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Electromagnetic Waves in a Medium

Introduction

- The behavior of an EM waves in a medium
 - How different parameters of the wave vary with the medium
 - Attenuation of amplitude
 - Phase behavior

Classification of Media

Conductivity

- Perfect dielectrics ($\sigma \approx 0$)
 - Air, inert gases, hydrocarbon gases, glass etc.
- Low loss dielectrics (σ small but significant)
 - Fresh water, sea water, semiconductors etc.
- Good conductors (σ very large)
 - Metals

Other Parameters

- Permittivity
- Permeability

Maxwell's Equations for Sinusoidal Fields

Sinusoidal E field,

$$\nabla^2 E = \mu\epsilon\omega^2 \left(-1 + j\frac{\sigma}{\omega\epsilon} \right) E + \nabla \left(\frac{\rho}{\epsilon} \right)$$

Sinusoidal H field,

$$\nabla^2 H = \mu\epsilon\omega^2 \left(-1 + j\frac{\sigma}{\omega\epsilon} \right) H$$

Perfect Dielectric

For a perfect dielectric,

$$\sigma = 0$$

$$\rho = 0$$

Therefore,

$$\nabla^2 E = -\mu\epsilon\omega^2 E$$

$$\nabla^2 H = -\mu\epsilon\omega^2 H$$

$$k^2 = \mu\epsilon\omega^2$$

Perfect Dielectric (Contd..)

Velocity,

$$\frac{dz}{dt} = v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_r\varepsilon_r}\sqrt{\mu_0\varepsilon_0}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

Since it is most likely that both μ and ε are greater than the corresponding values for a vacuum,

$$\frac{1}{\sqrt{\mu\varepsilon}} < \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$
$$v < c$$

Perfect Dielectric (Contd..)

Properties of Propagation

- Velocity will be slower than in a vacuum
- The frequency will not change
- The wavelength will change accordingly
- There will be negligible attenuation

Impedance

The impedance of a perfect dielectric,

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \eta_0 \sqrt{\frac{\mu_r}{\epsilon_r}} = 377 \sqrt{\frac{\mu_r}{\epsilon_r}} \Omega$$

Exercise 1

The E field of a plane wave has an amplitude of 5.2 Vm^{-1} and has a wavelength of 4.5 cm in free space. It propagates through a hydrocarbon gas with a relative permittivity of 2.1 and relative permeability of 1 .

Find

1. The velocity and wavelength within the medium
2. The wave equation of the E field
3. The impedance of the medium
4. The wave equation of the H field

Neglect the polarization of the wave.

Exercise 1 (Contd..)

Velocity,

$$v = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.1}} = 2.0702 \times 10^8 \text{ ms}^{-1}$$

Frequency,

$$f = \frac{c}{\lambda_{\text{freespace}}} = \frac{3 \times 10^8}{0.045} = 6.67 \text{ GHz}$$

$$\omega = 2\pi f = 4.18 \times 10^{10} \text{ rads}^{-1}$$

Exercise 1 (Contd..)

Therefore the wavelength in the medium,

$$\lambda = \frac{v}{f} = \frac{2.0702 \times 10^8}{6.67 \times 10^9} = 3.11 \text{ cm}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.0311} = 202.3378 \text{ m}^{-1}$$

E field,

$$E = 5.2 \cos[(4.18 \times 10^{10})t - (202.3378)z] \text{ Vm}^{-1}$$

Exercise 1 (Contd..)

Medium impedance,

$$\eta = 377 \sqrt{\frac{\mu_r}{\epsilon_r}} = 377 \sqrt{\frac{(1)}{(2.1)}} = 260.1547 \Omega$$

H field,

$$\begin{aligned} H &= \frac{5.2}{260.1547} \cos[(4.18 \times 10^{10})t - (202.3378)z] \text{ Am}^{-1} \\ &= (0.02) \cos[(4.18 \times 10^{10})t - (202.3378)z] \text{ Am}^{-1} \end{aligned}$$

Propagation Coefficient

For a non-conducting medium, the propagation coefficient (γ) is given by,

$$k^2 = \mu\epsilon\omega^2 = -\gamma^2$$

Since k is real, γ is purely imaginary.

Propagation Coefficient (Contd..)

For a conducting medium,

$$\sigma \neq 0$$

$$\rho = 0$$

Therefore,

$$\nabla^2 E = \mu \epsilon \omega^2 \left(-1 + j \frac{\sigma}{\omega \epsilon} \right) E$$

$$\nabla^2 H = \mu \epsilon \omega^2 \left(-1 + j \frac{\sigma}{\omega \epsilon} \right) H$$

$$k_c^2 = \mu \epsilon \omega^2 \left(1 - j \frac{\sigma}{\omega \epsilon} \right)$$

Propagation Coefficient (Contd..)

The propagation coefficient,

$$\begin{aligned}\gamma^2 &= -k_c^2 \\ \gamma &= jk_c = j\omega\sqrt{\mu\varepsilon} \left(1 - j\frac{\sigma}{\omega\varepsilon}\right)^{\frac{1}{2}} \\ &= \alpha + j\beta\end{aligned}$$

The coefficient is complex.

Propagation Coefficient (Contd..)

Since γ is complex,

$$\begin{aligned}E_x(z, t) &= E_0 e^{-\gamma z} e^{j\omega t} = E_0 e^{-(\alpha + j\beta)z} e^{j\omega t} = E_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \\|E_x(z, t)| &= E_0 e^{-\alpha z}\end{aligned}$$

- The amplitude is attenuated by a factor of $e^{-\alpha z}$ with z (attenuation coefficient)
- β gives the phase behavior (phase coefficient)

$$\beta = \frac{2\pi}{\lambda}$$

Low Loss Dielectric

In such a medium the conductivity is small but significant,

$$\omega\epsilon \gg \sigma$$

From the Taylor series, γ can be approximated by,

$$\begin{aligned}\gamma &\approx j\omega\sqrt{\mu\epsilon} \left(1 - j\frac{\sigma}{2\omega\epsilon} + \frac{1}{8} \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right) \\ &= \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} + j\omega\sqrt{\mu\epsilon} \left[1 + \frac{1}{8} \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right]\end{aligned}$$

Low Loss Dielectric (Contd..)

From this,

$$\alpha = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} = \frac{\eta\sigma}{2}$$
$$\beta = \omega\sqrt{\mu\epsilon} \left[1 + \frac{1}{8} \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right]$$

By taking $\eta_0 = 377\Omega$,

$$\alpha = \frac{\eta_0\sigma}{2} \sqrt{\frac{\mu_r}{\epsilon_r}} = 188.5\sigma \sqrt{\frac{\mu_r}{\epsilon_r}}$$

Exercise 2

Silicon and glass are both amorphous materials. However, glass is transparent and silicon is opaque. Show how this can be explained in terms of the conductivity of the two materials.

- For silicon, $\sigma_{Si} = 1.2 \times 10^{-3} Sm^{-1}$ and $\epsilon_r = 11.68$.
- For glass, $\sigma_{Glass} = 5 \times 10^{-15} Sm^{-1}$ and $\epsilon_r = 4.7$.
- For both materials, $\mu_r \approx 1$.

Exercise 2 (Contd..)

$$\alpha = 188.5\sigma\sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$\alpha_{Si} = 188.5(1.2 \times 10^{-3})\sqrt{\frac{(1)}{(11.68)}} = 0.0662$$

$$\alpha_{Glass} = 188.5(5 \times 10^{-15})\sqrt{\frac{(1)}{(4.7)}} = 4.3474 \times 10^{-13} \approx 0$$

$$\alpha_{Si} \gg \alpha_{Glass}$$

Electromagnetic Waves in Water

- For seawater, $\sigma = 4.8 S m^{-1}$ and $\epsilon_r = 80$.
- Therefore, $\alpha = 88.5148$ (a very high value)
- This is why,
 - A submarine cannot use radio waves for communication when underwater. Instead it should use sonar.
 - In the sea plants and creatures that need sunlight (coral, plankton) are limited to very shallow areas.

Medium Impedance

The impedance of the medium is a complex value.

$$\begin{aligned}E_x(z, t) &= A_x e^{-\gamma z} e^{j\omega t} \\ \frac{\partial}{\partial z} E_x(z, t) &= -\gamma E_x(z, t) = -j\omega\mu H_y \\ \frac{E_x}{H_y} &= \frac{j\omega\mu}{\gamma} \\ &= \frac{j\omega\mu}{j\omega\sqrt{\mu\varepsilon} \left(1 - j\frac{\sigma}{\omega\varepsilon}\right)^{\frac{1}{2}}} = \sqrt{\frac{\mu}{\varepsilon}} \left(1 - j\frac{\sigma}{\omega\varepsilon}\right)^{-\frac{1}{2}}\end{aligned}$$

Phase Velocity

The phase velocity the wave is given by,

$$\begin{aligned} v_p &= \frac{\omega}{\beta} \\ &= \frac{\omega}{\omega \sqrt{\mu \epsilon} \left[1 + \frac{1}{8} \left(\frac{\sigma}{\omega \epsilon} \right)^2 \right]} \\ &= \frac{1}{\sqrt{\mu \epsilon} \left[1 + \frac{1}{8} \left(\frac{\sigma}{\omega \epsilon} \right)^2 \right]} \end{aligned}$$

Group Velocity

The group (envelope) velocity of the wave is given by,

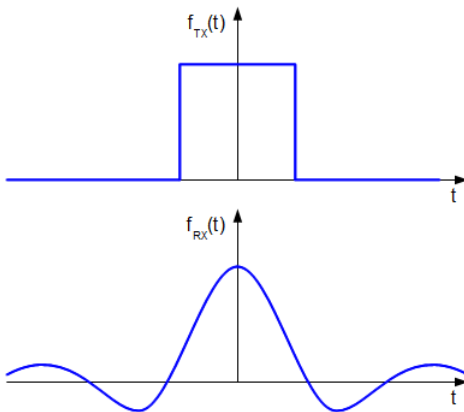
$$u_g = \frac{1}{\frac{d\beta}{d\omega}}$$

- If the phase coefficient of the medium is not a linear function of the ω , waves of different frequencies will propagate at different velocities
- Will result in dispersion (a delay in propagation)

Normal Dispersion

- In normal dispersion shorter the wavelength, the more the delay
 - Higher frequencies will be delayed more (i.e. travel slower) than lower frequencies
- Result in pulse spreading (*pulse broadening*) because the harmonics will travel slower than the fundamental

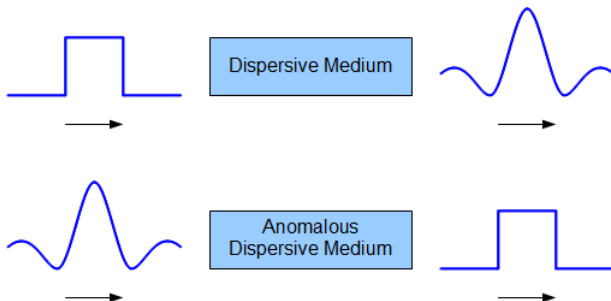
Pulse Spreading



Anomalous Dispersion

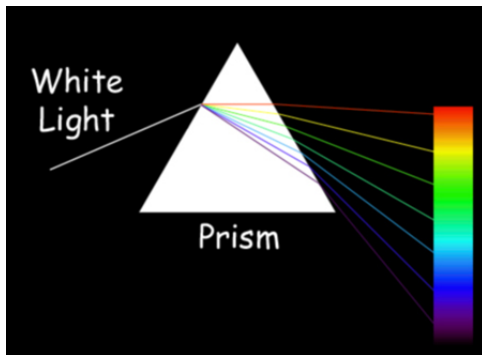
- In an anomalous dispersive medium, shorter wavelengths are delayed less than longer wavelengths
 - Higher frequencies will travel faster than lower frequencies
- Therefore it can be used to reverse the effect of spreading
 - Alternative sections of normal and anomalous dispersive fibers are used to preserve the pulse shape in long haul fiber optic links

Pulse Despreading



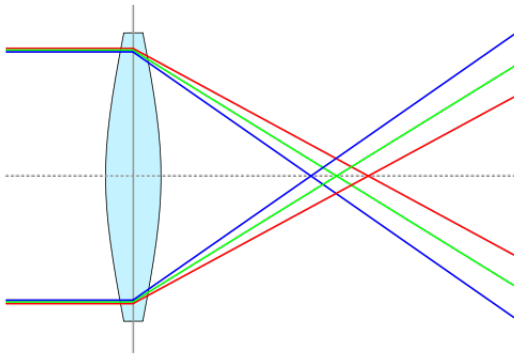
Splitting of White Light by a Prism

- Different colors have different velocities in glass
- Hence, white light splits into its component colours

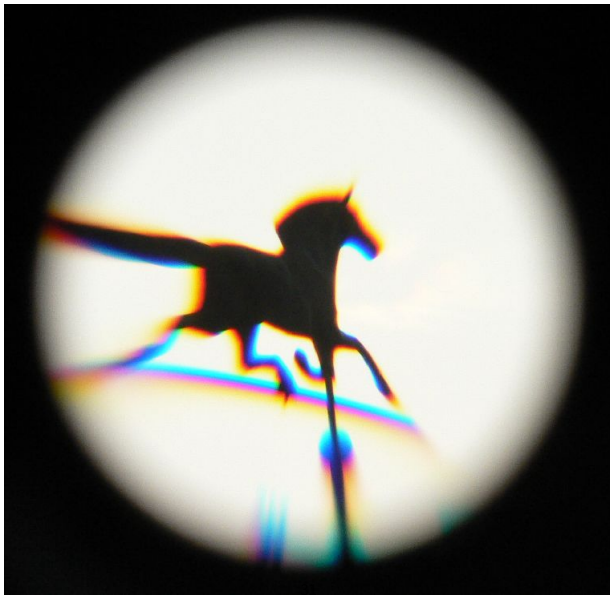


Chromatic Abberation

- The lens behaves like a prism
- Sharp edges of objects get colored
- Can be corrected by using special corrective lenses



Chromatic Abberation (Contd..)



Chromatic Abberation (Contd..)



Good Conductors

In a good conductor $\frac{\sigma}{\omega\epsilon} \gg 1$,

$$-1 + j\frac{\sigma}{\omega\epsilon} \approx j\frac{\sigma}{\omega\epsilon}$$

Therefore, the value of γ can be approximated to,

$$\begin{aligned}\gamma &\approx j\omega\sqrt{\mu\epsilon} \left(-j\frac{\sigma}{\omega\epsilon}\right)^{\frac{1}{2}} \\ &= \sqrt{j\omega\mu\sigma}\end{aligned}$$

Good Conductors (Contd..)

Taking the square-root of j as,

$$\sqrt{j} = \frac{1+j}{\sqrt{2}}$$

$$\gamma = \frac{1+j}{\sqrt{2}} \sqrt{\omega\mu\sigma} = (1+j)\sqrt{\pi f\mu\sigma} = \alpha + j\beta$$

$$\alpha = \beta = \sqrt{\pi f\mu\sigma}$$

Medium Impedance

As with the case of the Low Loss Dielectric,

$$\frac{E_x}{H_y} = \frac{E_y}{H_x} = \frac{j\omega\mu}{\gamma}$$

Thus, the impedance of the medium is given by,

$$\begin{aligned}\eta &= \frac{j\omega\mu}{(1+j)\sqrt{\pi f\mu\sigma}} = (1+j)\sqrt{\frac{\pi f\mu}{\sigma}} \\ &= (1+j)\frac{\alpha}{\sigma}\end{aligned}$$

Phase Velocity

The phase velocity of a good conductor is,

$$\begin{aligned}u_p &= \frac{\omega}{\beta} \\&= \frac{\omega}{\sqrt{\frac{1}{2}\omega\mu\sigma}} \\&= \sqrt{\frac{2\omega}{\mu\sigma}}\end{aligned}$$

Wavelength

Within a conductor, the wavelength is given by,

$$\begin{aligned}\lambda &= \frac{2\pi}{\beta} \\ &= \frac{2\pi}{\sqrt{\frac{1}{2}\omega\mu\sigma}} = \frac{2\pi}{\sqrt{\pi f\mu\sigma}} \\ &= 2\sqrt{\frac{\pi}{f\mu\sigma}}\end{aligned}$$

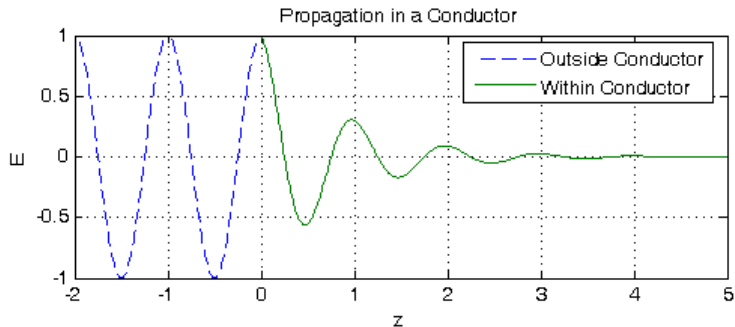
Skin Depth

Attenuation in a good conductor,

$$\begin{aligned} A(z) &= e^{-\alpha z} \\ &= e^{-\sqrt{\pi f \mu \sigma} z} \end{aligned}$$

- At high frequencies α can become very large
- Therefore an EM wave is rapidly attenuated

Skin Depth (Contd..)



Skin Depth (Contd..)

- The *skin depth* or *penetration depth* (δ) is defined as the distance a wave travels before it is attenuated by a factor of 0.368 (or e^{-1})

For this,

$$z = \delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

Exercise 3

Find the skin depth of aluminium and iron at 50Hz and 1GHz.
Take,

$$\mu_r(Fe) = 800$$

$$\mu_r(Al) \approx 1$$

$$\sigma_{Fe} = 1.03 \times 10^7 \text{ Sm}^{-1}$$

$$\sigma_{Al} = 3.78 \times 10^7 \text{ Sm}^{-1}$$

Exercise 3 (Contd..)

Take,

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi f \mu_r \mu_0 \sigma}} = \frac{503.2921}{\sqrt{f \mu_r \sigma}}$$

Answers,

	50Hz	1GHz
Al	1.16cm	2.589μm
Fe	0.8mm	0.175μm

Exercise 3 (Contd..)

Based on the results explain,

1. The reason for commonly using steel (highly magnetic) for constructing casings for biomedical equipment.
2. The reason for using aluminium for constructing the casings of cellular phones.

Exercise 3 (Contd..)

- Electrical signals produced by the human body are usually less than 100Hz
 - These signals have to be shielded from 50Hz ac line noise
 - At this frequency a casing of aluminum 1cm thick would not be sufficient
 - A 1mm casing of steel would provide reasonable shielding
- Cellular phones operate above 800MHz
 - At such frequencies the skin depth is negligible
 - Aluminum is preferred over steel because it is corrosion resistant