Infinite Impulse Response (IIR) Chebyshev Filters
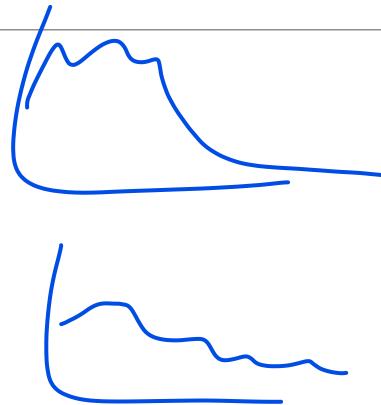
LECTURE 5

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Chebyshev Filters

Type-I Chebyshev Filters
Type-II Chebyshev Filters

It has ripples in the passband and no ripple in the stop band

It has the following transfer function

$$|H(\Omega)|^2 = \frac{1}{1+\varepsilon^2 C_N^2(\Omega)}$$

$C_N(x)$ is N^{th} order Chebyshev polynomial defined as

$$C_N(x) = \begin{cases} \cos(N \cos^{-1} x) & |x| \leq 1 \\ \cosh(N \cosh^{-1} x) & |x| > 1 \end{cases}$$

The Chebyshev polynomial can be generated by recursive technique

$$C_0(\Omega) = 1 \quad \text{and} \quad C_1(\Omega) = \Omega$$

$$C_{k+1}(\Omega) = 2\Omega C_k(\Omega) - C_{k-1}(\Omega)$$

Type-I Chebyshev Filters
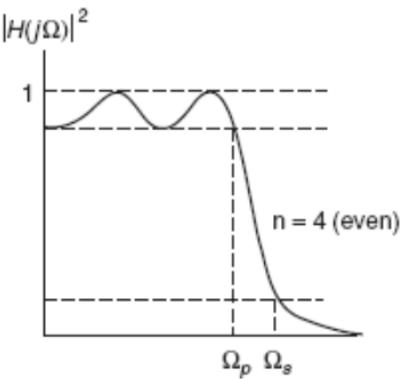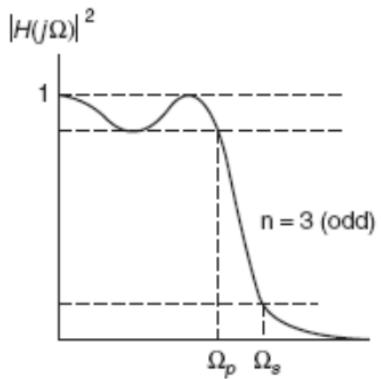
Chebyshev polynomial characteristics

$|C_N(\Omega)| \le 1$ for all $|\Omega| \le 1$

$C_N(1) = 1$ for all N

All the roots of $C_N(\Omega)$ occurs in the interval $-1 \le x \le 1$

The filter parameter ε is related to the ripple as shown in the figures

$$|H(0)|^2 = \begin{cases} 1 & N \text{ odd} \\ \dfrac{1}{1+\varepsilon^2} & N \text{ even} \end{cases}$$

$$|H(1)|^2 = \frac{1}{1+\varepsilon^2}$$

Chebyshev Polynomial

Step 1 Choose the type of transformation.
(Bilinear or impulse invariant transformation)

Step 2 Calculate the attenuation constant .

$$\varepsilon = \left[\frac{1}{A_1^2} - 1 \right]^{\frac{1}{2}}$$

Design procedure

Step 3 Calculate the ratio of analog edge frequencies Ω_2/Ω_1.

For bilinear transformation,

$$\frac{\Omega_2}{\Omega_1} = \frac{\dfrac{2}{T}\tan\dfrac{\omega_2}{2}}{\dfrac{2}{T}\tan\dfrac{\omega_1}{2}} = \frac{\tan\dfrac{\omega_2}{2}}{\tan\dfrac{\omega_1}{2}}$$

Step 4 Decide the order of the filter N such that
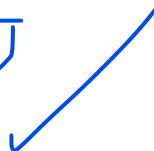
$$N \geq \frac{\cosh^{-1}\left\{\dfrac{1}{\varepsilon}\left[\dfrac{1}{A_2^2}-1\right]\right\}}{\cosh^{-1}\left\{\dfrac{\Omega_2}{\Omega_1}\right\}}$$

esign

rocedure

Step 5 Calculate the analog cutoff frequency Ω_c

For bilinear transformation,

$$\Omega_c = \frac{\Omega_1}{\left[\dfrac{1}{A_1^2} - 1\right]^{1/2N}} = \frac{\dfrac{2}{T}\tan\dfrac{\omega_1}{2}}{\left[\dfrac{1}{A_1^2} - 1\right]^{1/2N}}$$

Step 6 Determine the analog transfer function $H_a(s)$ of the filter.
When the order N is even, $H_a(s)$ is given by

$$H_a(s) = \prod_{k=1}^{N/2} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

When the order N is odd, $H_a(s)$ is given by

$$H_a(s) = \frac{B_0 \Omega_c}{s + c_0 \Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

where

$$b_k = 2y_N \sin\left(\frac{(2k-1)\pi}{2N}\right)$$

$$c_k = y_N^2 + \cos^2\frac{(2k-1)\pi}{2N}$$

$$c_0 = y_N$$

Design procedure

$$y_N = \frac{1}{2} \left\{ \left[\left(\frac{1}{\varepsilon^2} + 1 \right)^{\frac{1}{2}} + \frac{1}{\varepsilon} \right]^{\frac{1}{N}} - \left[\left(\frac{1}{\varepsilon^2} + 1 \right)^{\frac{1}{2}} + \frac{1}{\varepsilon} \right]^{\frac{-1}{N}} \right\}$$

For even values of N and unity dc gain filter, find $B_k\text{'s}$ such that

$$H_a(0) = \frac{1}{(1 + \varepsilon^2)^{1/2}}$$

For odd values of N and unity dc gain filter, find $B_k\text{'s}$ such that

$$\prod_{k=0}^{\frac{N-1}{2}} \frac{B_k}{C_k} = 1$$

(It is normal practice to take $B_0 = B_1 = B_2 = \ldots = B_k$)

Step 7   Using the chosen transformation, transform $H_a(s)$ to $H(z)$, where $H(z)$ is the transfer function of the digital filter.

[The high-pass, band pass and band stop filters are obtained from low-pass filter design by frequency transformation].

Design procedure