GENERAL SIR JOHN KOTELAWALA DEFENCE UNIVERSITY

Faculty of Engineering
Department of Mathematics

BSc Engineering Degree
Semester 5 Examination - May 2024
(Intake 39 - All Engineering Streams)

MA 3102 - APPLIED STATISTICS

Time : 2 hours 10 May, 2024

ADDITIONAL MATERIAL PROVIDED

List of formulas

Statistical tables

INSTRUCTIONS TO CANDIDATES

This paper contains 4 questions from Page 3 to Page 4

Answer **ALL** questions

This is a closed-book examination

This examination accounts for 70% of the module assessment. The marks assigned for each question and parts thereof are indicated in square brackets

If you have any doubt as to the interpretation of the wordings of a question, make your own decision, but clearly state it on the script

Assume reasonable values for any data not given in or provided with the question paper, clearly make such assumptions made in the script

All examinations are conducted under the rules and regulations of the KDU

DETAILS OF ASSESSMENT

Learning Outcome (LO)	Questions that assess LO	Marks allocated (Total 70%)
LO1	Q1, Q2	12
LO2	Q1, Q2	12
LO3	Q2	12
LO4	Q3	17
LO5	Q4	17

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Question 1

(a) An officer in the army randomly selected 49 soldiers in his camp and found the average weight was 60 Kg. If the population standard deviation for the weight of the soldiers (σ) is 20 Kg, find the 95% confidence interval for the mean weight of soldiers. [5]

(b) Nine randomly selected aircrafts in a camp had the mean age of 6 years, and the standard deviation of 2.5 years. Find the 99% confidence interval of the mean age of the aircrafts. Assume the age of the aircrafts are normally distributed. [5]

(c) Suppose the flying time between two cities is approximately normally distributed, with a mean of 50 minutes and a population standard deviation of 5 minutes.

(i) What is the probability that on a random day the flying time will be, between 45 to 55 minutes? [5]

(ii) If a random sample of 16 flights is selected, what is the probability that the mean flying time will be more than 52 minutes? [5]

(iii) How large the sample must be selected if you want to be 95% confident that the true mean flying time will differ from the sample mean by 1.2 minutes? [5]

[25 Marks]

Question 2

(a) A company claims that the average waiting time for customer service calls is less than 5 minutes. To test this claim, a random sample of 16 customer service calls is selected, and the average waiting time is found to be 4.6 minutes with a standard deviation of 0.8 minutes.

(i) State the null and alternative hypotheses. [5]

(ii) Calculate the t-test value. [5]

(iii) Write your final conclusion by justifying the claim at $\alpha = 0.025$. [5]

(b) A study claims that the average monthly expenses of households in a city exceed Rs. 40,000. A researcher believes that the average monthly expenses are less than Rs. 40,000. To test this claim, a random sample of 30 households is selected, and their average monthly expenses are found to be Rs. 38,000, with a population standard deviation of Rs. 8,000. Test his Claim at a 1% significance level. [5]

(c) A researcher wants to estimate, with 95% confidence, the proportion of students at a university who own a car. A previous survey indicates that 40% of students at this university own a car. The researcher wants the estimate to be accurate within 5% of the true proportion. Find the minimum sample size needed for this study. [5]

[25 Marks]

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Question 3

(a) The electricity bill (Y) of a household in a city in Rupees is given by the formula:

$$Y = 2,000 + 50X$$

where X = represents the number of units of electricity consumed.

(i) Explain the coefficients 2,000 and 50 in real terms. [5]

(ii) Estimate the electricity bill for a household if they consumed 100 units of electricity in a given month. [5]

(b) A study was conducted to compare the effectiveness of two types of fertilizer on plant growth. Half of the plants were treated with Fertilizer A (observations of X), and the other half were treated with Fertilizer B (observations of Y). The height of the plants was measured after a certain period. Assume that the distributions of X and Y are $N(\mu_X, \sigma^2)$ and $N(\mu_Y, \sigma^2)$, respectively. The following heights were recorded in centimeters:

$X: 20, 22, 19, 21, 18, 23, 20, 21, 22, 19$ n=10

$Y: 23, 24, 22, 20, 21, 25, 22, 23, 24, 21$ m=10

Test the null hypothesis $H_0: \mu_X = \mu_Y$ against $H_a: \mu_X < \mu_Y$ at $\alpha = 0.05$ and state your conclusion. [15]

[25 Marks]

Question 4

(a) Volume of the medicine pack (in ml) of a product is to be monitored by control charts using a sample size of $n = 5$. Data for 10 preliminary samples are shown below. Set up a control chart on this process and state whether the process is in statistical control concerning each chart.

[15]

	Observation				
Sample No	I	II	III	IV	V
1	10	11	10	11	13
2	12	12	10	15	10
3	9	9	8	10	10
4	12	11	10	10	12
5	12	12	11	14	11
6	9	10	12	8	11
7	14	14	12	12	13
8	10	12	10	11	12
9	10	9	11	10	10
10	12	10	10	12	11

(b) Write a short note on acceptance sampling and list the advantage of sampling compared to the 100% inspection. [10]

[25 Marks]

End of the Question Paper

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List of Formulas

- Confidence Interval $= \overline{x} \pm 1.96\,\dfrac{\sigma}{\sqrt{n}}$

- Confidence Interval $= \overline{x} \pm t_{\alpha/2,n-1}\,\dfrac{s}{\sqrt{n}}$

- $Z = \dfrac{X-\mu}{\sigma}$

- $Z = \dfrac{\overline{X}-\mu}{\sigma/\sqrt{n}}$

- $t = \dfrac{\overline{X}-\mu}{s/\sqrt{n}}$

- $S_p = \sqrt{\dfrac{(n_x-1)s_x^2+(n_y-1)s_y^2}{n_x+n_y-2}}$ $\quad t = \dfrac{\overline{X}-\overline{Y}}{S_p\sqrt{\frac{1}{n}+\frac{1}{m}}}$

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H_0	H_1	Critical Region
$\mu=\mu_0$	$\mu>\mu_0$	$z \geq z_\alpha$ or $\overline{x} \geq \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$
$\mu=\mu_0$	$\mu<\mu_0$	$z \leq -z_\alpha$ or $\overline{x} \leq \mu_0 - z_\alpha \frac{\sigma}{\sqrt{n}}$
$\mu=\mu_0$	$\mu\neq\mu_0$	$\lvert z\rvert \geq z_{\alpha/2}$ or $\lvert\overline{x}-\mu_0\rvert \geq z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

- $P\left(\dfrac{\overline{X}-\mu}{\sigma/\sqrt{n}} \leq z_\alpha\right) = 1-\alpha$

- $P\left[\overline{X} - z_\alpha\left(\dfrac{\sigma}{\sqrt{n}}\right) \leq \mu\right] = 1-\alpha$

- $Y \sim \text{Binomial}(n,p)$

  $E(Y) = np,\qquad Var(Y) = np(1-p)$

  $E\left(\dfrac{Y}{n}\right) = p,\qquad Var\left(\dfrac{Y}{n}\right) = \dfrac{p(1-p)}{n}$

  $\dfrac{Y-np}{\sqrt{np(1-p)}} = \dfrac{Y/n-p}{\sqrt{p(1-p)/n}}$

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- Confidence Intervals Difference Proportions

$$E\left(\frac{Y_1}{n_1}\right) = p_1, \qquad Var\left(\frac{Y_1}{n_1}\right) = \frac{p_1(1-p_1)}{n_1}$$

$$E\left(\frac{Y_2}{n_2}\right) = p_2, \qquad Var\left(\frac{Y_2}{n_2}\right) = \frac{p_2(1-p_2)}{n_2}$$

$$\frac{\left(\left(\frac{Y_1}{n_1}\right) - \left(\frac{Y_2}{n_2}\right)\right) - [p_1 - p_2]}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} \sim N(0,1)$$

$$\frac{y_1}{n_1} - \frac{y_2}{n_2} \pm Z_{\alpha/2}\sqrt{\frac{\frac{y_1}{n_1}\left(1-\frac{y_1}{n_1}\right)}{n_1} + \frac{\frac{y_2}{n_2}\left(1-\frac{y_2}{n_2}\right)}{n_2}}$$

- Maximum error of the estimate $= \varepsilon = Z_{\alpha/2}\,\frac{\sigma}{\sqrt{n}}$

$$n = \frac{Z^2_{\alpha/2}\,\sigma^2}{\varepsilon^2}$$

- Control limits for the $\overline{x}$ chart can be determined as follows:

$$UCL = \mu + \frac{3\sigma}{\sqrt{n}} \qquad CL = \mu \qquad LCL = \mu - \frac{3\sigma}{\sqrt{n}}$$

$$\hat{\sigma} = \frac{\overline{R}}{d_2}$$

- Control limits for R chart

$$UCL = D_4\overline{R} \qquad CL = \overline{R} \qquad LCL = D_3\overline{R}$$

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

- The np chart

  It is possible t base a control chart on the number of nonconforming, rather than the fraction nonconforming.

  3σ approach:

$$UCL = np + 3\sqrt{np(1-p)} \qquad CL = np \qquad LCL = np - 3\sqrt{np(1-p)}$$

  Note: for unknown p, p can be replaced by $\overline{p}$.

Table E (continued)

Cumulative Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

For z values greater than 3.49, use 0.9999.

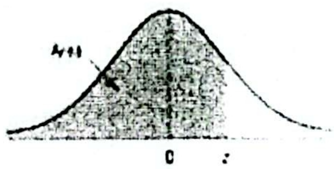

TABLE A-3 t Distribution: Critical t Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
	Area in Two Tails				
Degrees of Freedom	0.01	0.02	0.05	0.10	0.20
1	63.657	31.821	12.706	6.314	3.078
2	9.925	6.965	4.303	2.920	1.886
3	5.841	4.541	3.182	2.353	1.638
4	4.604	3.747	2.776	2.132	1.533
5	4.032	3.365	2.571	2.015	1.476
6	3.707	3.143	2.447	1.943	1.440
7	3.499	2.998	2.365	1.895	1.415
8	3.355	2.896	2.306	1.860	1.397
9	3.250	2.821	2.262	1.833	1.383
10	3.169	2.764	2.228	1.812	1.372
11	3.106	2.718	2.201	1.796	1.363
12	3.055	2.681	2.179	1.782	1.356
13	3.012	2.650	2.160	1.771	1.350
14	2.977	2.624	2.145	1.761	1.345
15	2.947	2.602	2.131	1.753	1.341
16	2.921	2.583	2.120	1.746	1.337
17	2.898	2.567	2.110	1.740	1.333
18	2.878	2.552	2.101	1.734	1.330
19	2.861	2.539	2.093	1.729	1.328
20	2.845	2.528	2.086	1.725	1.325
21	2.831	2.518	2.080	1.721	1.323
22	2.819	2.508	2.074	1.717	1.321
23	2.807	2.500	2.069	1.714	1.319
24	2.797	2.492	2.064	1.711	1.318
25	2.787	2.485	2.060	1.708	1.316

Observations in Sample, n	Chart for Averages: Factors for Control Limits			Chart for Standard Deviations: Factors for Center Line		Chart for Standard Deviations: Factors for Control Limits				Chart for Ranges: Factors for Center Line		Chart for Ranges: Factors for Control Limits				
	A	A_2	A_3	c_4	$1/c_4$	B_3	B_4	B_5	B_6	d_2	$1/d_2$	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.575
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.115
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.04230	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608

(continued overleaf)