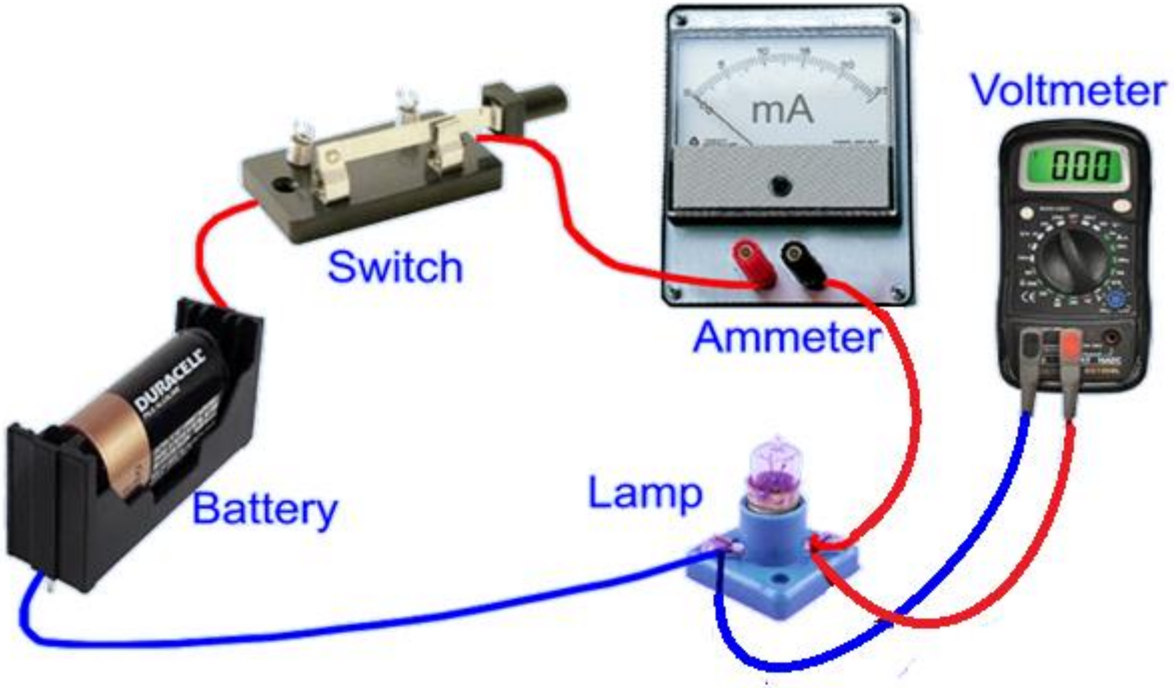
EE 1102 – Fundamentals of Electrical Engineering



DC Circuit Calculations

Prof Rohan Lucas

Learning outcomes

After successful completion of this module, you should be able to

1. Identify the basic electrical properties of resistors, inductors and capacitors
2. State Ohm’s law and Kirchoff’s voltage and current laws
3. Analysis dc circuits using the basic laws

Outline Syllabus

Overview of Electrical Engineering (2 hrs)

Role of Electrical Engineer, Introduction to Generation, Transmission, Distribution and Utilisation.

SI Units (2 hrs)

Basic and supplementary units, Derived units, Symbols

Basic DC circuit analysis (4 hrs)

Circuit elements, Circuit laws, Circuit solutions with DC.

Network Theorems (4 hrs)

Ohm's Law and Kirchhoff's Laws, Other network theorems.

Alternating Current theory (8 hrs)

Sinusoidal waveform, phasor and complex representation, Impedance and Admittance, Power and Energy, Power factor. Solution of simple R, L, and C circuit problems by phasor and complex variables.

Electrostatic and Electromagnetic theory (4hrs)

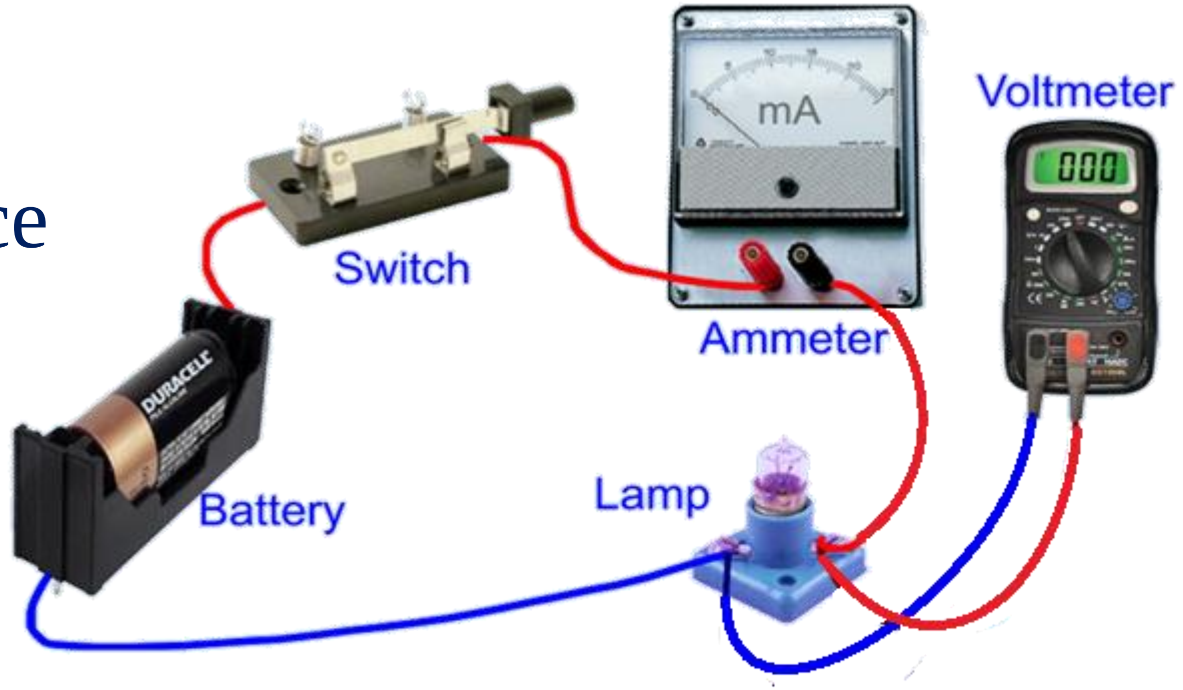
Basic Laws, Calculation of field and force

Electrical Installations (4 hrs)

Fuses, miniature circuit breakers, earth leakage circuit breakers, residual current circuit breakers, earthing, electric shock. Wiring regulations, basic domestic installations.

What is an electric circuit?

- A Electric circuit
  - interconnection of electrical elements supplied from a source
- Basis of
  - electric power,
  - electric machines,
  - control,
  - electronics,
  - computers,
  - communications and
  - instrumentation



Terminology

Electric charge q - most basic quantity. [Unit - *coulomb (C)*]

Law of conservation of charge

"Charge can neither be created nor destroyed".

Algebraic sum of the charges in a system does not change.

The charge on an electron is -1.602×10^{-19} C.

Electric current *i* is rate of flow of electric charges (electrons) $i = \dfrac{dq}{dt}$

Electric current, by convention, flows in opposite direction to electrons. [Unit - ampere (A)]

Charge transfer from t_o and t

$$q = \int_{t_o}^{t} i\,dt$$

To move an electron in a conductor in a particular direction, or to create a current, requires work / energy.

Electro-motive force (*emf*) is the characteristic of any energy source capable of driving electric charge around a circuit. It is measured in volt.

Voltage is measured with reference to selected point (such as *earth*). [Unit - *volt (V)*]

Voltage $$v_{ab} = \frac{dw}{dq}$$ between two points *a* and *b* is the *energy* required (or work done) ***w*** to move a unit charge ***q*** from ***a*** to ***b*.**

Note:

Suffix ***ab*** need not be written when there is no ambiguity.

Power is the rate of doing work or transferring energy [Unit - *watt (W)*]

Thus $$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i$$ i.e. $p = v.i$

Energy is the capacity to do work. [Unit - *joule (J)*]

Energy transferred from time t_o and t is $$w = \int_{t_o}^{t} p\,dt = \int_{t_o}^{t} v \cdot i\,dt$$

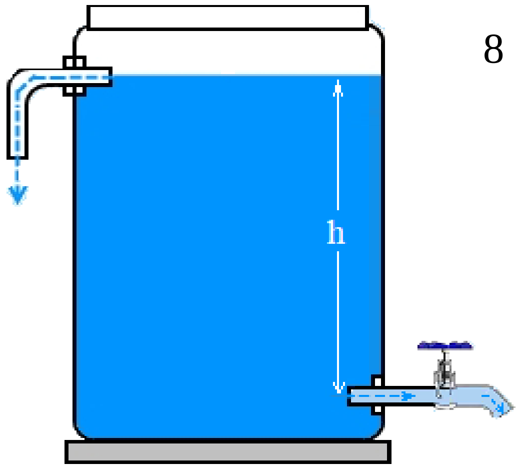
Joule's law When an electric current passes through a conductor, heat H is produced.

H $\propto$ resistance R of the conductor

$\propto$ duration of current flow, time t

$\propto$ square of the magnitude of current I.

An Analogy



An overhead tank supplies a tap in the garden.

Potential energy ***mg h*** joule.

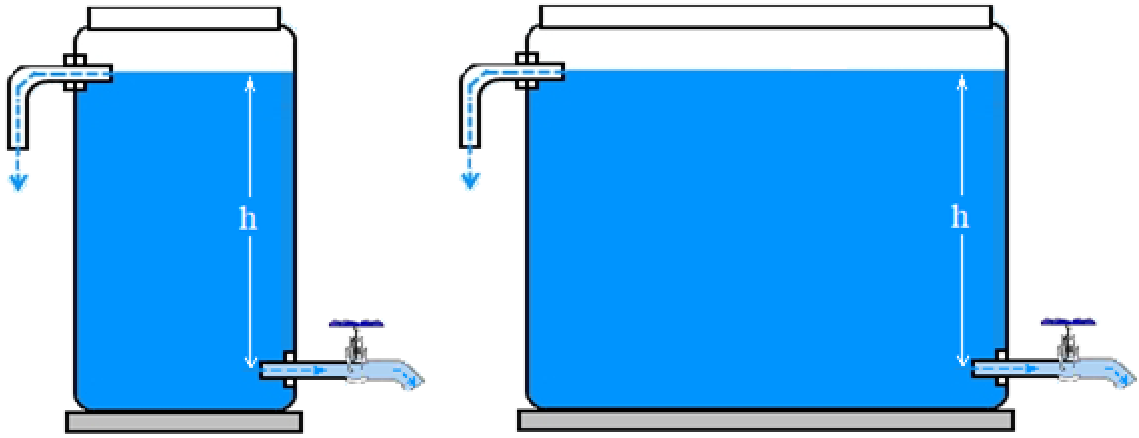
Has a potential of ***h*** metre.

[Similar to battery with energy capacity ***E.i.t*** and emf ***E***].

Ex. Car battery with emf of **12V** and capacity of **60Ah** or $12{\times}60{\times}3600$ J = **2.592 MJ** or $12{\times}60$ Wh = **0.72 kWh**

Consider two different tanks of the same height:

- same potential (***h***)
- different volume or mass (***m***)
- different potential energy (***mgh***)



Similarly different batteries could have the same emf (or potential) but have different capacities or energy.

Ex: a "12 V car battery" and "8 pen-torch batteries in series" would have the same emf, but obviously a completely different energy capacity].

Connecting a battery to a circuit,

- depending on the resistance of the circuit
- current coming out will differ.

[Similar to how much a tap is opened

- resistance to water flow in the path changes
- water will come out at different rate.

]

Maximum pressure available at the tap

- when the flow is a minimum
- there is no head loss due to friction in the pipe
- corresponds to the potential of the tank ***h***.

We can never get a pressure of more than ***h*** (except momentarily when we perhaps put our finger to partly block the flow of water).

Similarly, maximum potential

- available from a battery to a connected load is ***E***
- when no current is taken out of the battery (open circuit)
- there is no voltage drop in internal resistance of the battery and wire.

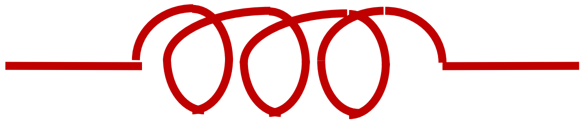
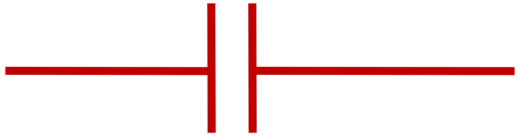
We can never get a potential of more than ***E*** (except during a transient operation, and inductance and/or capacitance is there in the circuit)].

Water coming out from the tap could be
(a) absorbed by the ground (lost) or
(b) collected in a bucket
 (stored and can be put back into the tank).

This is similar to the current going into a
(a) resistor 

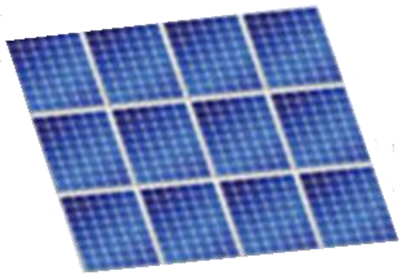
where energy gets dissipated (or lost) as heat to surroundings

(b) either an inductor or capacitor

where energy is stored in electromagnetic or electrostatic form and can be retrieved later and is not lost.

2.1.2 Basic Circuit Elements



Two basic types of elements

- *active elements* and *passive elements*.

Active element is capable of generating electrical energy.

Principle of Conservation of Mass and Energy

Generating/producing electrical energy

- conversion of non-electrical form energy to electrical energy.

Energy loss or dissipation

- conversion of electrical energy to to a non-useful form of energy

Examples of active elements are

- *voltage source* (such as a battery or generator)
- *current source*.

Dependant Sources

Most sources are independent of other circuit variables.

Some source elements are *dependant* (modelling elements such as transistors and operational amplifiers require dependant sources).

Active elements may be ***ideal*** sources.

- terminal voltage (or current) would be independent of connected circuit.

Passive element is one which does not generate electricity but either consumes it or stores it.

Resistors, *Inductors* and *Capacitors* are simple passive elements. Diodes, transistors are also passive elements.

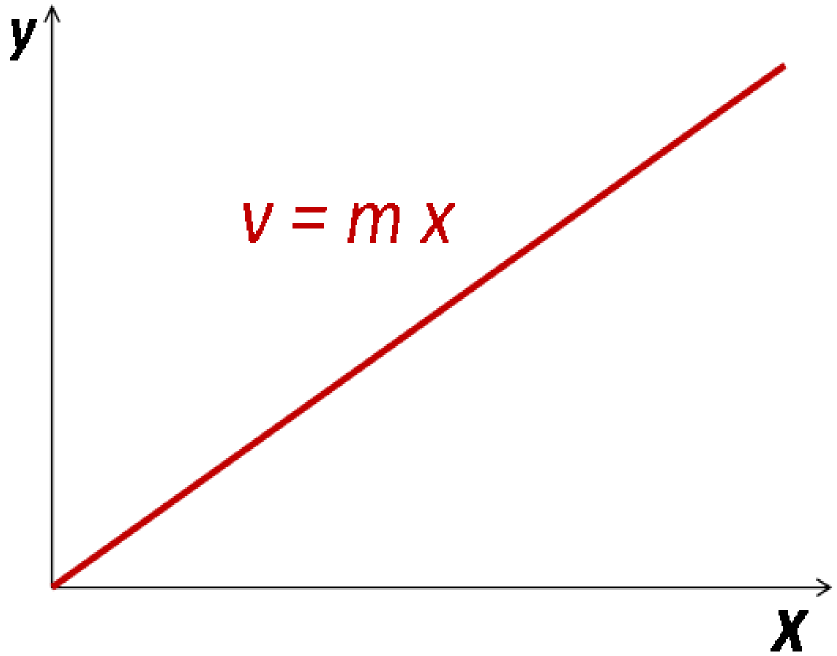
Passive elements may either be *linear* or *non-linear*.

Linear elements

Obey straight line law passing through origin.

- A linear resistor has a linear *voltage vs current* relationship (V = R.I).
- A linear inductor has a linear *flux vs current* relationship ($\phi = k\ I$).
- A linear capacitor has a linear *charge vs voltage* relationship (q = CV).

[R, k and C are constants of proportionality].



Resistors, inductors and capacitors may be linear or non-linear, while diodes and transistors are always non-linear.

Branch and Node

Branch

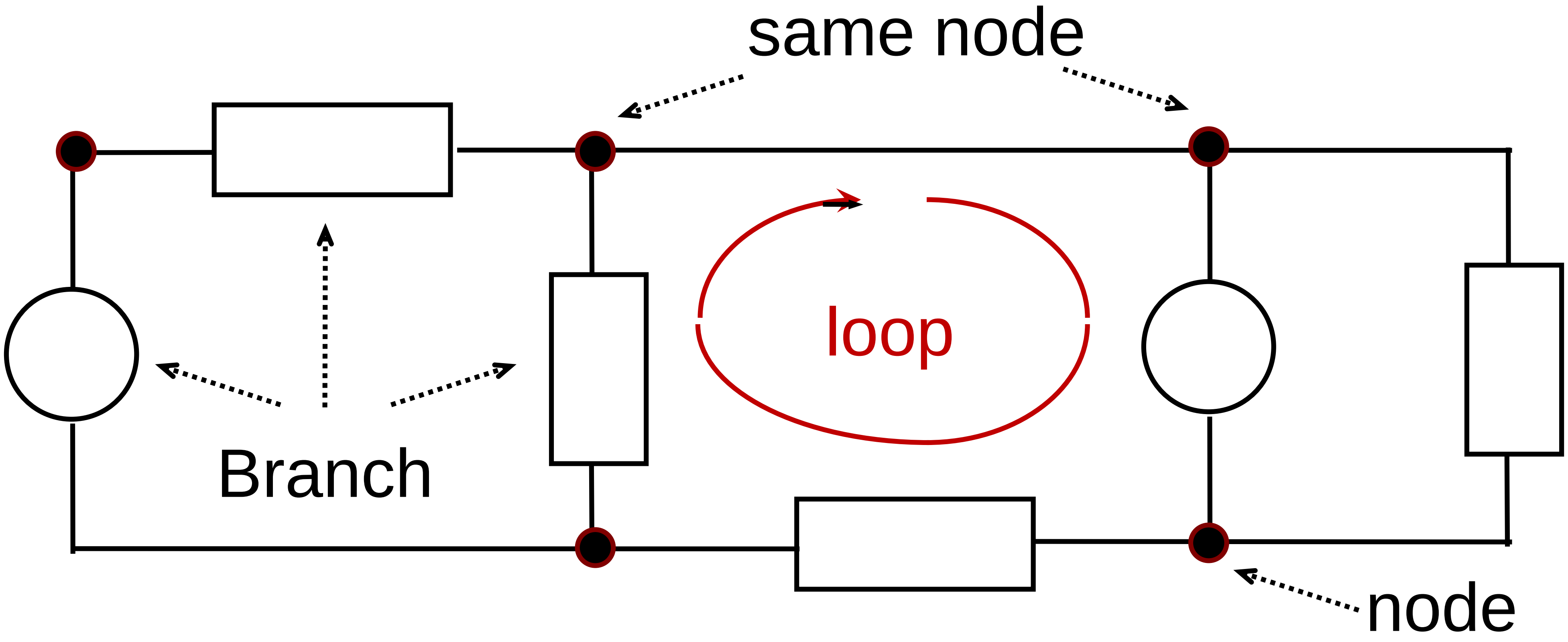
A branch is a 2 terminal element (with 2 ends).
A *branch* usually represents a single element, such as resistor or battery.

Node

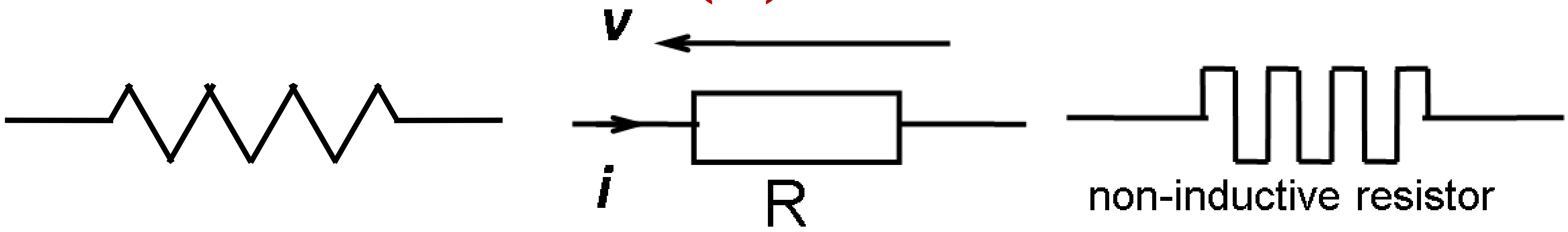
A *node* is the point connecting two or more branches.
The node is usually indicated by a *dot* (•) in a circuit.

Loop or mesh

A *loop* is any closed path in a circuit, formed by starting at a node, passing through a number of branches and ending up once more at the original node.



Resistance R [Unit: ohm (Ω)]



The relationship between voltage and current is given by

***v* = R *i*,** or ***i* = *G v*,** where G = conductance = 1/R

$$\boldsymbol{R = \frac{\rho\, l}{A}}$$

, where *r* = resistivity, *l* = length and *A*= material cross section

Power loss in a resistor = $\boldsymbol{R\, i^2}$.

Energy dissipated in a resistor $$\boldsymbol{w = \int R.i^2\, dt}$$

No storage of energy in a resistor.

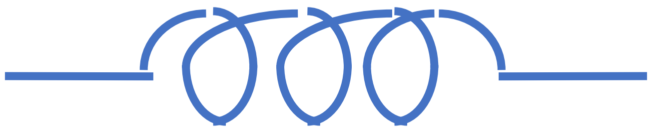
Circuit Component Materials

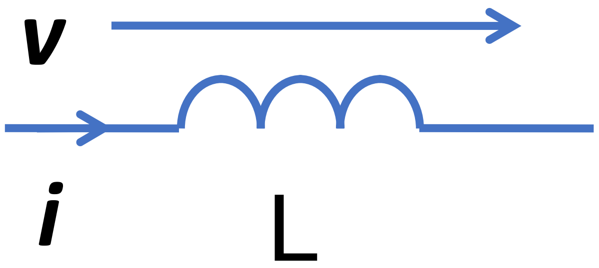
Usage	conductor			
Material	Silver	Copper	Gold	Aluminium
Resistivity (Ω m)	16.4×10^{-9}	17.2×10^{-9}	24.5×10^{-9}	28×10^{-9}

Usage	insulator			
Material	Paper	Mica	Glass	Teflon
Resistivity (Ω m)	10×10^{9}	0.5×10^{12}	10^{12}	3×10^{12}

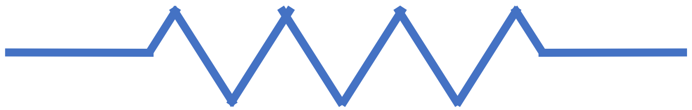
Usage	semi-conductor		
Material	Carbon	Germanium	Silicon
Resistivity (Ω m)	40×10^{-6}	0.47	640

Inductance L [Unit: henry (H)]





not commonly used



Relationship between voltage and current $v = N\dfrac{d\phi}{dt} = L\dfrac{di}{dt}$

$L = \dfrac{N^2 \mu A}{l}$ where m = permeability, N = number of turns,

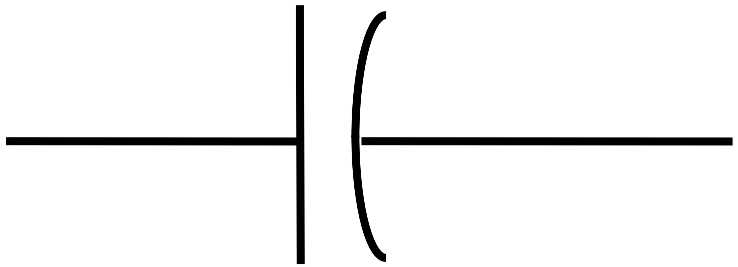
l = length and A = cross section of core

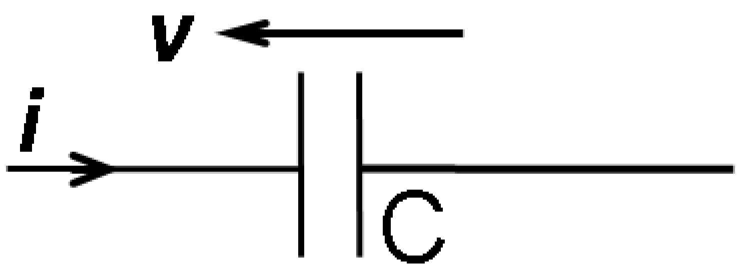
Energy stored in an inductor = $\frac{1}{2} L\, i^2$

No energy is dissipated in a pure inductor.

Practical inductors have some wire resistance causing some power loss. Also be a small power loss in the magnetic core (if any).

Capacitance C [Unit: farad (F)]





Relationship between voltage and current $i = \dfrac{dq}{dt} = C\,\dfrac{dv}{dt}$

$C = \dfrac{\varepsilon A}{d}$ for a parallel plate capacitor;

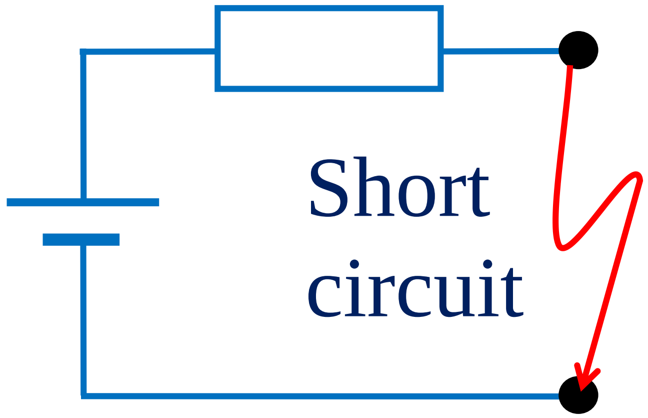
where ε = permittivity, d = spacing and = cross section of dielectric

Energy stored in an capacitor = $\frac{1}{2}\,C\,v^2$

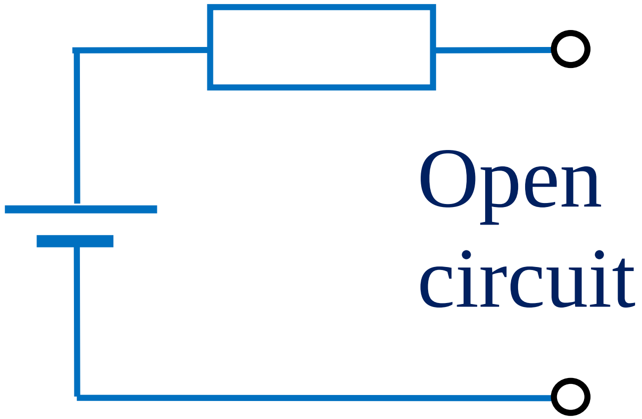
No energy is dissipated in a pure capacitor.

Practical capacitors also have a negligible power loss in dielectric.

Short-circuit and open circuit



A **short circuit** in a circuit element is when the resistance (and any other impedance) of the element approaches zero.

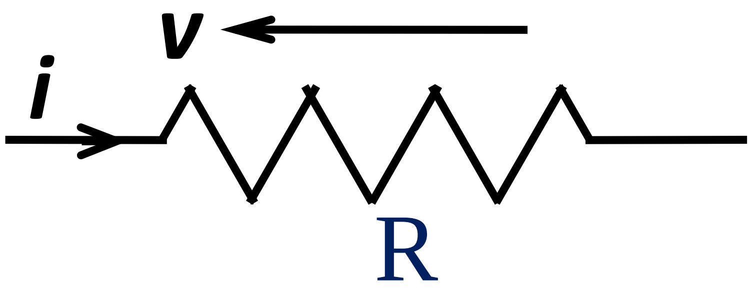


An **open circuit** in a circuit element is when the resistance (and any other impedance) of the element approaches infinity.

[The term impedance is similar to resistance but is used in alternating current theory for other components]

2.2 Fundamental Laws

Fundamental laws of electric circuits are Ohm's law and Kirchoff's laws.



2.2.1 Ohm's Law

Ohm's law states that the voltage ***v*** across a resistor is directly proportional to the current ***i*** flowing through it.

$$\boldsymbol{v \propto i, \quad v = R \,.\, i}$$

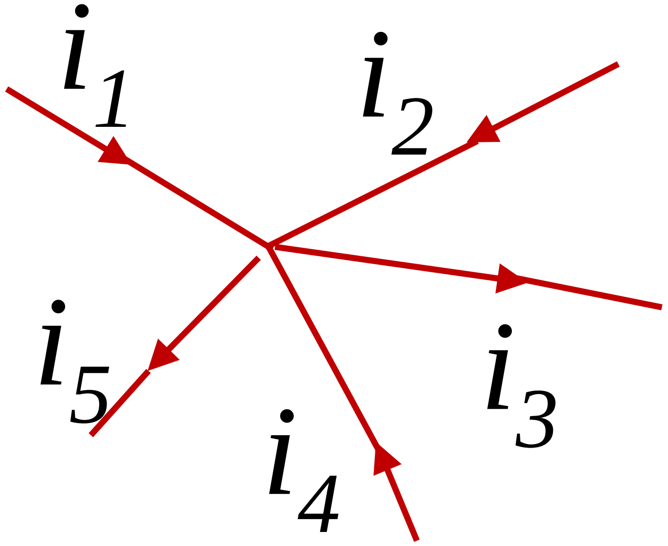
where R is the proportionality constant.

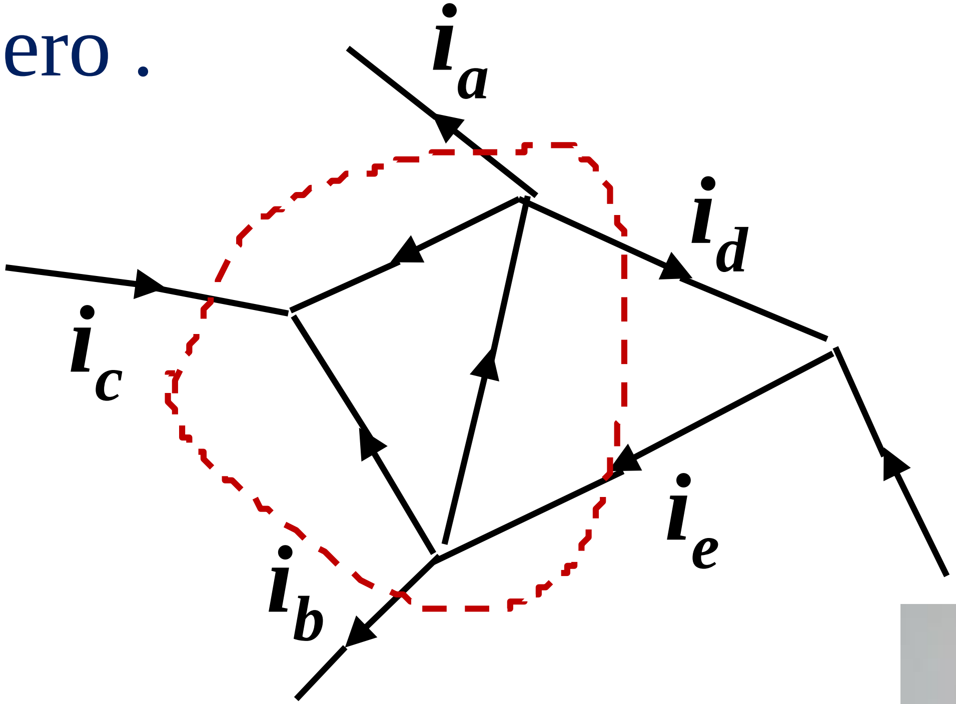
In addition to Ohm's law we need the

Kirchoff's voltage law and the **Kirchoff's current law** to analyse circuits.

2.2.2 Kirchoff's Current Law

Kirchoff's first law is based on the principle of conservation of charge.
Since charge is the integral of current,
Kirchoff's Current Law states that algebraic sum of currents entering a node (or a closed boundary) is zero .





$$\Sigma i = 0$$

$$i_1 + i_2 - i_3 + i_4 - i_5 = 0$$

$$i_a - i_b + i_c - i_d + i_e = 0$$

2.2.3 Kirchoff's Voltage Law

Kirchoff's second law is based on principle of conservation of energy. Requires that potential difference taken round a closed path is zero.

Kirchoff's Voltage Law states that the algebraic sum of all voltages around a closed path (or loop) is zero.

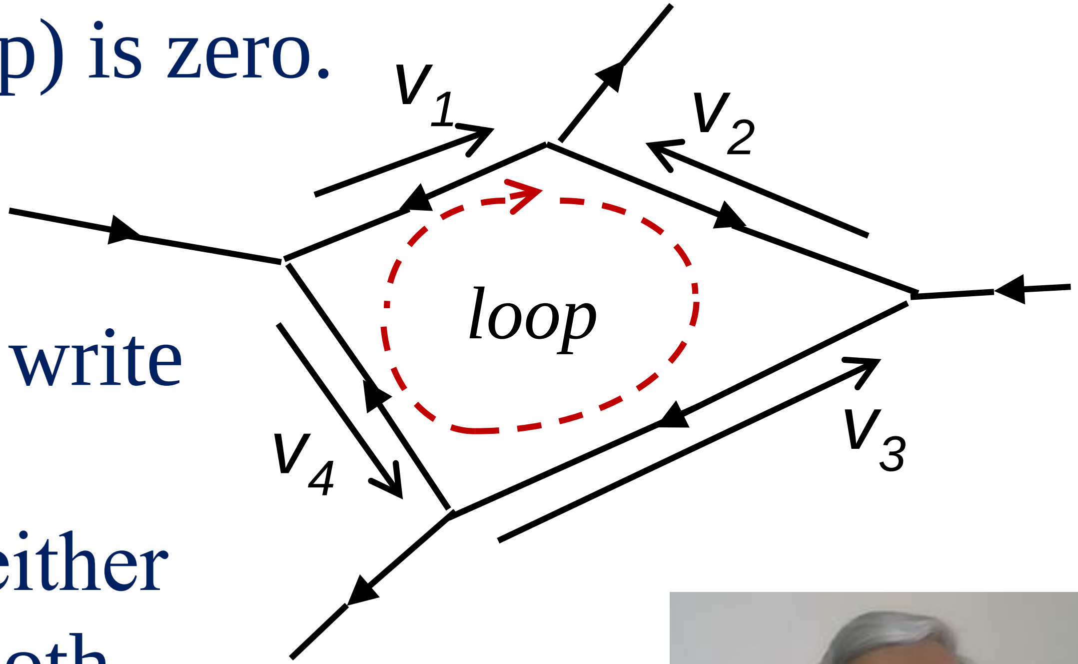
$$\Sigma v = 0$$

$$-v_1 + v_2 + v_3 + v_4 = 0$$

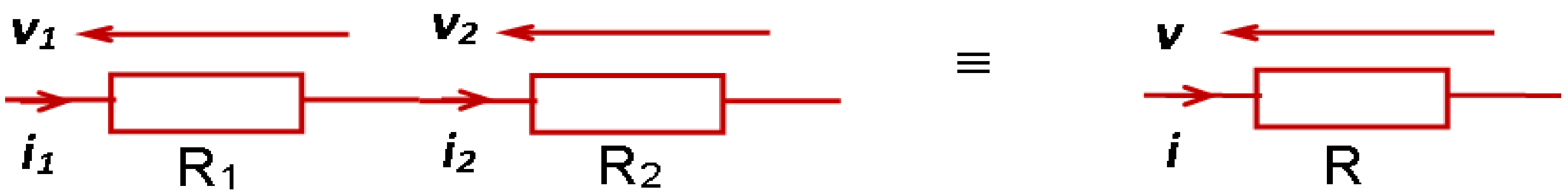
depending on convention, you may also write

$$v_1 - v_2 - v_3 - v_4 = 0$$

Note: v_1, v_2 … may be voltages across either active elements or passive elements or both and may be obtained using Ohm's law.



2.2.4 Series Circuits



From Kirchoff's current law, $i_1 = i_2 = i$

From Kirchoff's Voltage Law, $v_1 + v_2 = v$.

Also from Ohm's Law, $v_1 = R_1 i_1$, $v_2 = R_2 i_2$, $v = R\ i$

$\therefore\ R_1 i + R_2 i = R\ i$, or $R = R_1 + R_2$

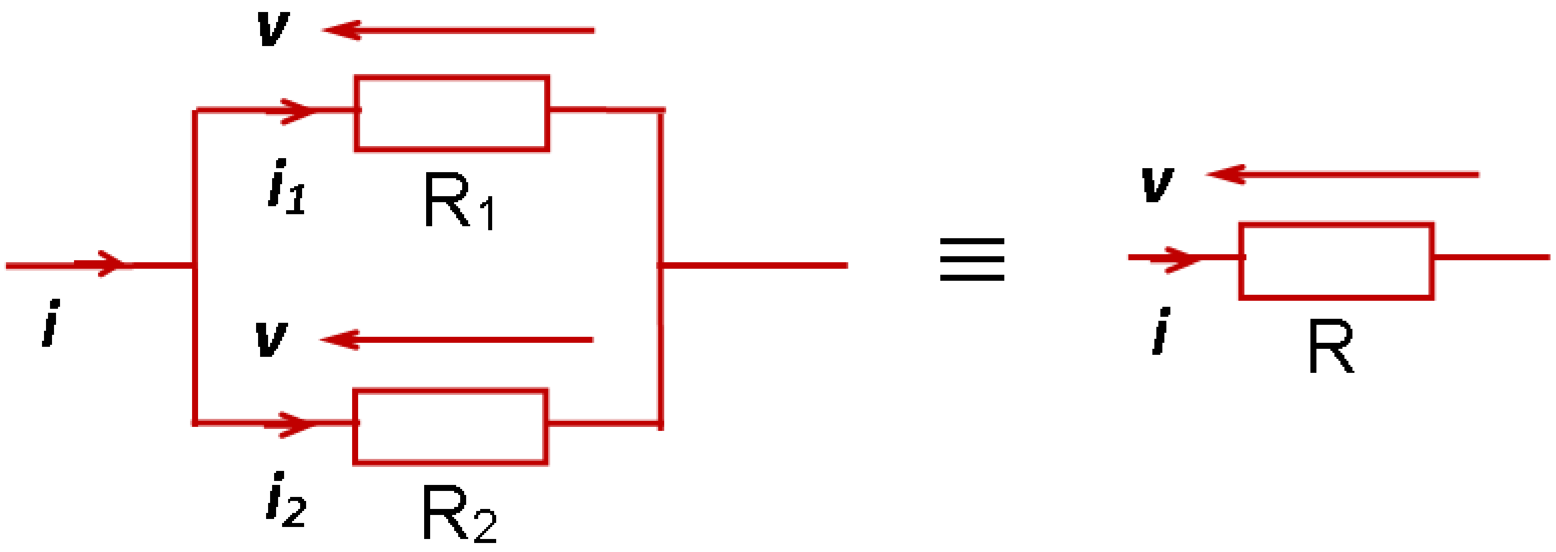
Also, voltage division rule

$$\frac{v_1}{v_2} = \frac{R_1\ i_1}{R_2\ i_2} = \frac{R_1\ i}{R_2\ i} = \frac{R_1}{R_2}\ , \text{ and } \frac{v_1}{v} = \frac{R_1}{R_1 + R_2},\quad \frac{v_2}{v} = \frac{R_2}{R_1 + R_2}$$

In series circuit, total resistance is sum of individual resistances

- voltage across individual element is directly proportional to element resistance.

2.2.5 Parallel Circuits



From Kirchoff's current law, $i_1 + i_2 = i$

From Kirchoff's Voltage Law, $v_1 = v_2 = v$.

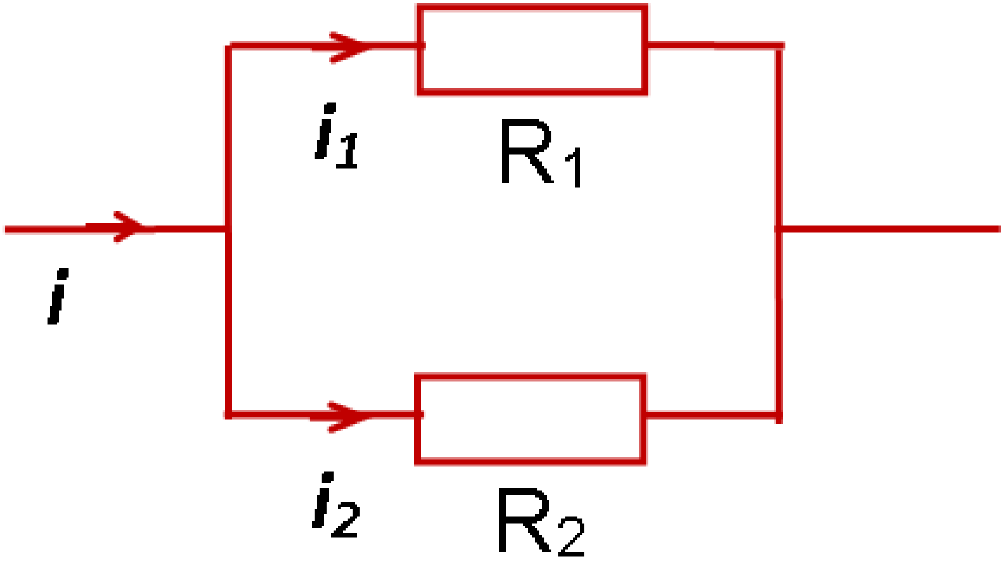
Also from Ohm's Law, $v_1 = R_1\, i_1, \quad v_2 = R_2\, i_2, \quad v = R\ i$

$$\therefore \quad \frac{v}{R_1} + \frac{v}{R_2} = \frac{v}{R} \quad \text{or} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R = \frac{R_1\, R_2}{R_1 + R_2}$$

Current division rule

$$\frac{i_1}{i_2} = \frac{v_1 / R_1}{v_2 / R_2} = \frac{R_2\ v}{R_1\ v} = \frac{R_2}{R_1}\ ,$$ and

$$\frac{i_1}{i} = \frac{R_2}{R_2 + R_1}, \quad \frac{i_2}{i} = \frac{R_1}{R_2 + R_1}$$



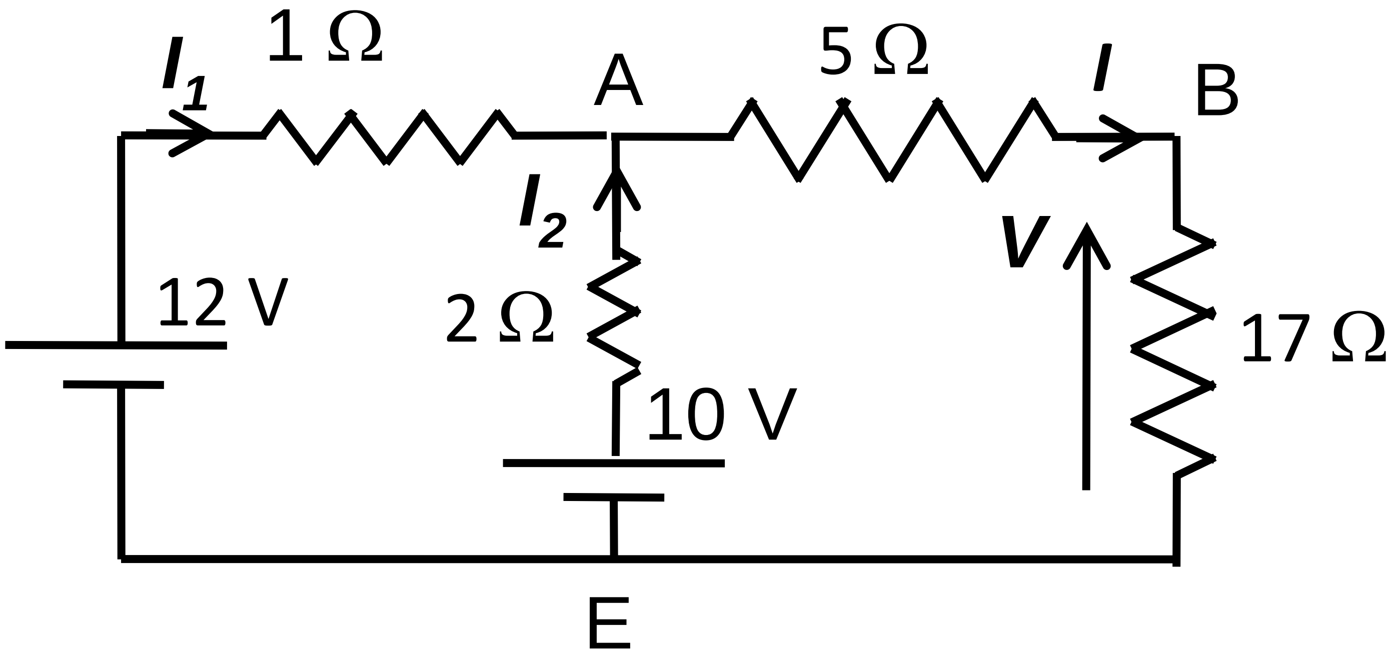
i.e. in a parallel circuit,

- total inverse resistance is sum of individual inverse resistances
- current through individual elements is inversely proportional to the resistance of that element.

Example

Using Ohm's Law and Kirchoff's Laws determine currents and voltages.

Solution



Using Kirchoff's current Law

$$I = I_1 + I_2\ ,$$

Using Kirchoff's voltage Law

$$12 = 1\,I_1 - 2\,I_2 + 10,$$

$$10 = 2\,I_2 + 5\,I + 17\,I$$

Simplifying gives

$$I_1 - 2\,I_2 = 2$$

$$22\,I_1 + 24\,I_2 = 10$$

Solving gives $34\,I_1 + 0.I_2 = 34 \;\rightarrow I_1 = \mathbf{1A}$

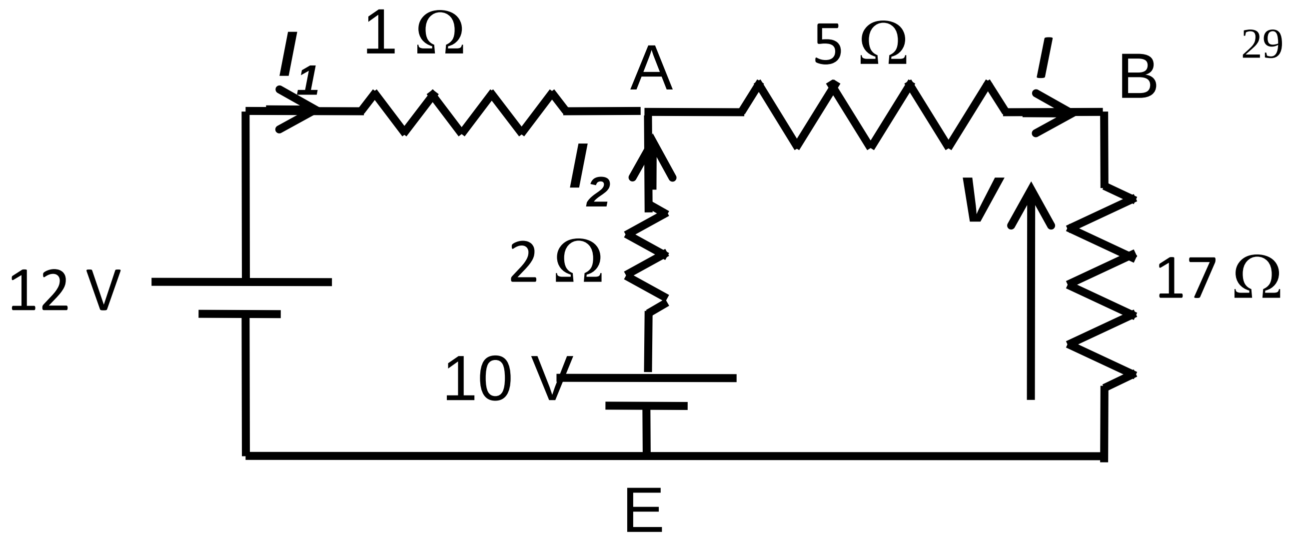
$2\,I_2 = I_1 - 2 = 1 - 2 = -1$

$I_2 = \mathbf{-0.5A}$

$I = I_1 + I_2 = 1 - 0.5 = \mathbf{0.5A}$

$V_{AE} = 12 - 1.\,I1 = 12 - 1 = \mathbf{11V}$

$V = 17 \times I = 17 \times 0.5 = \mathbf{8.5V}$



Power supplied from 12V source = 12 × 1 = **12W**

10V source = 10 × (-0.5) = – **5W**

$\left.\begin{array}{l}12\text{W}\\-5\text{W}\end{array}\right\}\ \mathbf{7W}$

Power loss in 1 Ω $= 1^2 \times 1 = \mathbf{1W}$, in 2 Ω $= (-0.5)^2 \times 2 = \mathbf{0.5W}$,

in 5 Ω $= (0.5)^2 \times 5 = \mathbf{1.25W}$

Power delivered to 17 Ω Load $= (0.5)^2 \times 17 = 4.25\mathbf{W}$

Total Consumption = 1 + 0.5 + 1.25 + 4.25 = **7 W**

END OF PRESENTATION