Module 5

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Department of Mathematics

MA 3102 APPLIED STATISTICS

Hypothesis Tests

Introduction

Hypothesis test for population mean one sample

Hypothesis test for difference of population means

Hypothesis test for proportions

Hypothesis test for paired data

The F test for equality of variance

Chi-square test

Tests of Hypotheses

- Hypotheses - Any statement about one or more population or population parameter(s)
- Null Hypotheses (H_0)- the hypothesis under test (null: no effect or not different)
- Alternative Hypotheses (H_1) - any hypothesis other than the null hypothesis
- A test is one-sided if the H_1 is one-sided (often marked by, $>$ upper tail, or $<$ lower tail), and a test is two-sided if the H_1 is two-sided marked by $\neq$)
- Based on sample data attempt is made to reject the null hypothesis.

Hypothesis test for population mean one sample

Case_I: σ^2 Known

Example 1:

Let X equal the breaking strength of a steel bar. If the bar is manufactured by process, X is $N(50, 36)$, i.e., X is normally distributed with $\mu = 50$ and $\sigma^2 = 36$. It is hoped that if a new process is used, X will be different. To test the new process, a sample of 16 steel bars produced through the new process were tested for strength and yield the sample mean of 53.

Hypotheses:

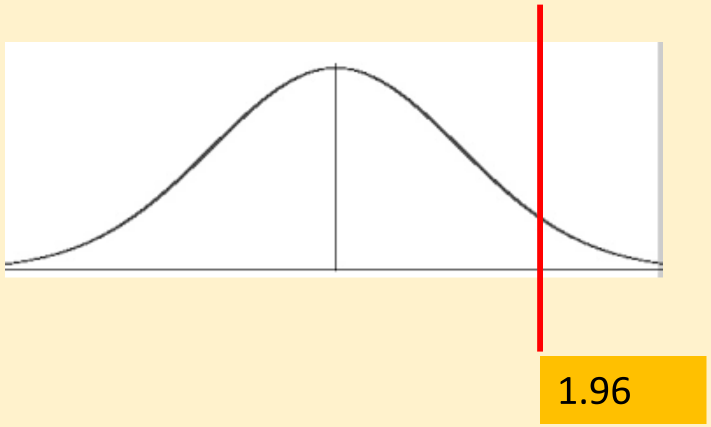
$$H_0: Mean\ strength\ of\ the\ new\ process\ is\ 50 \qquad \mu = 50$$

$$H_1: Mean\ strength\ of\ the\ new\ process\ is\ different\ from\ 50 \qquad \mu \neq 50$$

Test Statistic:

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{53 - 50}{6 / \sqrt{16}} = 2$$

Decision:



If the significance level α=0.05, then

Z calculated (2) > Z table value (1.96)

reject H_0

and conclude

$\underline{Mean\ strength\ of\ the\ new\ process\ is\ different\ from\ 50}$

<u>Example 2:</u>

Let X equal the breaking strength of a steel bar. If the bar is manufactured by process, X is $N(50, 36)$, i.e., X is normally distributed with $\mu = 50$ and $\sigma^2 = 36$. It is hoped that if a new process is used, X will be greater. To test the new process, a sample of 16 steel bars produced through the new process were tested for strength and yield the sample mean of 53.

<u>Hypotheses:</u>

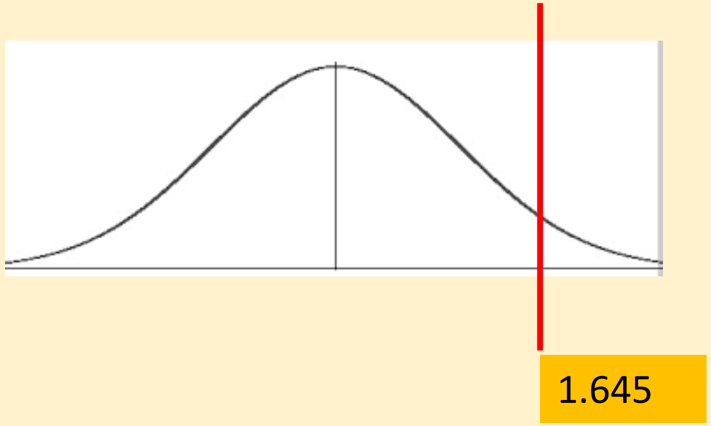
$$H_0: Mean\ strength\ of\ the\ new\ process\ is\ 50 \qquad \mu = 50$$

$$H_1: Mean\ strength\ of\ the\ new\ process\ is\ greater\ than\ 50 \qquad \mu > 50$$

Test Statistic:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{53 - 50}{6/\sqrt{16}} = 2$$

Decision:



If the significance level α=0.05, then

Z calculated (2) > Z table value (1.645)

reject H_0

$$\mathit{and\ conclude}$$

$$\underline{\mathit{Mean\ strength\ of\ the\ new\ process\ greater\ than}\ 50}$$

H_0	H_1	Critical Region
$\mu = \mu_0$	$\mu > \mu_0$	$z \geq z_\alpha$ or $\bar{x} \geq \mu_0 + z_\alpha \sigma/\sqrt{n}$
$\mu = \mu_0$	$\mu < \mu_0$	$z \leq -z_\alpha$ or $\bar{x} \leq \mu_0 - z_\alpha \sigma/\sqrt{n}$
$\mu = \mu_0$	$\mu \neq \mu_0$	$\lvert z \rvert \geq z_{\alpha/2}$ or $\lvert \bar{x} - \mu_0 \rvert \geq z_{\alpha/2} \sigma/\sqrt{n}$

Case_II: σ^2 Unknown

Example 3

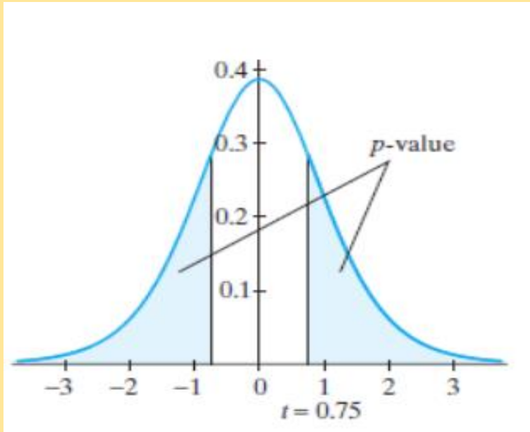
Let X (in millimeters) equal the growth in 15 days of a tumor induced in a mouse. Assume that the distribution of X is $N(\mu, \sigma^2)$. We shall test the null hypothesis H_0: $\mu = \mu_0 = 4.0$ mm against the two-sided alternative hypothesis H_1: $\mu \neq 4.0$. If we are given that $n = 9$, $\bar{X} = 4.3$, and $s = 1.2$, test the hypothesis at significance level of $\alpha = 0.10$.

Hypotheses:

H_0: $\mu = 4.0$
H_1: $\mu \neq 4.0$

Test Statistic:

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{4.3 - 4.0}{1.2/\sqrt{9}} = 0.75$$



Critical Value

$$t_{0.10/2,(9-1)} = 1.860$$

Decision:

$$|t| = |0.75| < 1.860$$

Therefore do not reject H_0

Growth in 15 days of a tumor induced in a mouse is not different from 4 mm

Example 4

In attempting to control the strength of the wastes discharged into a nearby river, a paper firm has taken a number of measures. Members of the firm believe that they have reduced the oxygen-consuming power of their wastes from a previous mean μ of 500 (measured in parts per million of permanganate). They plan to test H_0: $\mu = 500$ against H_1: $\mu < 500$, using readings taken on $n = 25$ consecutive days. The observed values of the sample mean and sample standard deviation were $\bar{X} = 308.8$ and $s = 115.15$. Testing at significance level of $\alpha = 0.01$

Hypotheses:

H_0: $\mu = 500$

H_1: $\mu < 500$

Test Statistic:

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{308.8 - 500}{115.5/\sqrt{25}} = -8.30$$

Critical Value:

$$t_{0.10,(25-1)} = -2.492$$

Decision:

$$-8.30 < -2.492$$

Therefore reject H_0

we clearly reject the null hypothesis and accept H_1: $\mu < 500$.

Hypothesis test for paired data

Example:

Trace metals in drinking water affect the flavor and an unusually high concentration can pose a health hazard. Ten pairs of data were taken measuring zinc concentration in bottom water and surface water. Does the data suggest that the true average concentration in the bottom water exceeds that of surface water?

Location	Zn at Bottom	Zn at Surface	Difference
1	5	4	1
2	8	7	1
3	9	9	0
4	11	11	0
5	10	10	0
6	14	13	1
7	15	12	3
8	10	8	2
9	20	15	5
10	10	8	2
			Sum 15

Std. Deviation 1.581

Hypotheses:

H_0: μ_D =0

$$H_1: \mu_D < 0$$

Test Statistics:

$$t = \frac{\overline{X_D} - \mu_D}{S_d / \sqrt{n}} = \frac{1.5 - 0}{1.581 / \sqrt{10}} = 3$$

Critical Value

$$t_{0.05,(10-1)} = 1.833$$

Decision:

$$|t| = |3| > 1.833$$

Therefore reject H_0…………

This test is called ***paired t test***

Home work

8.1-1. Assume that IQ scores for a certain population are approximately $N(\mu, 100)$. To test H_0: $\mu = 110$ against the one-sided alternative hypothesis H_1: $\mu > 110$, we take a random sample of size $n = 16$ from this population and observe $\overline{x} = 113.5$.

(a) Do we accept or reject H_0 at the 5% significance level?

(b) Do we accept or reject H_0 at the 10% significance level?

(c) What is the p-value of this test?

8.1-2. Assume that the weight of cereal in a "12.6-ounce box" is $N(\mu, 0.2^2)$. The Food and Drug Association (FDA) allows only a small percentage of boxes to contain less than 12.6 ounces. We shall test the null hypothesis H_0: $\mu = 13$ against the alternative hypothesis H_1: $\mu < 13$.

(a) Use a random sample of $n = 25$ to define the test statistic and the critical region that has a significance level of $\alpha = 0.025$.

(b) If $\overline{x} = 12.9$, what is your conclusion?

(c) What is the p-value of this test?