General Sir John Kotelawala Defence University

ET3212 Microwave Engineering
Microwave Antennas

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Outline

Introduction

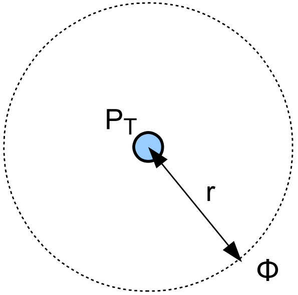
Introduction

- An antenna is the *interface* between the electrical signal and radiated EM wave
  - Identical characteristics when used as a transmitter and receiver (*reciprocity*)
- Has to be an efficient radiator
  - Matched to the transmission line
  - Operational bandwidth
- Microwave antennas
  - Short wavelengths result in small antenna dimensions
  - Large effective apertures therefore high gain

Isotropic Radiator

- An ideal point source
- Radiates energy equally in all directions
- For a power P_T at a distance r yeilds a power density Φ

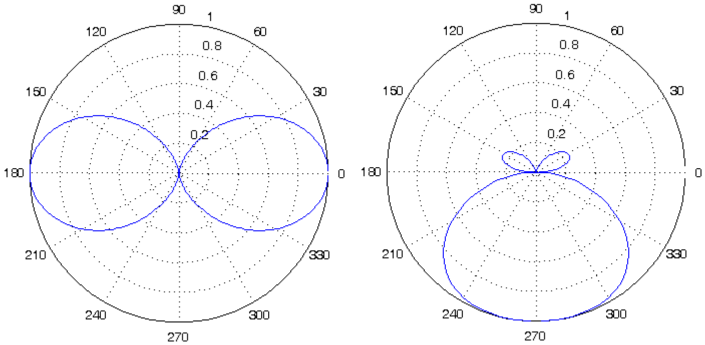
$$\Phi = \frac{P_T}{4\pi r^2}$$



- Cannot be realized and mainly used as a *benchmark*
- All practical antennas are *anisotropic*

Anistotropic Radiators

- The radiation pattern *varies* with observation angle



- Generally a *mainlobe* and *sidelobes*

Directivity

- The ratio between the maximum power density of the antenna (Φ_m) and the power density of an isotropic radiator (Φ_i) for the same transmitted power P_T.

$$D = \frac{\Phi_m}{\Phi_i}$$

- Usually expressed in decibels

Calculation of Directivity

For an anisotropic radiator the *normalized* power density:

$$\Phi = \Phi_m \frac{F(\theta,\phi)}{F_{max}(\theta,\phi)}$$

Therefore, the radiated power

$$\begin{aligned} P_T &= \int_0^{2\pi}\int_0^{\pi} \Phi r^2 sin(\theta) d\theta d\phi \\ &= \frac{\Phi_m}{F_{max}(\theta,\phi)} \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi) r^2 sin(\theta) d\theta d\phi \end{aligned}$$

Calculation of Directivity (Contd..)

For an isotropic radiator:

$$P_T = \Phi_i(4\pi r^2)$$

Therefore, the directivity

$$D = \frac{\Phi_m}{\Phi_i} = \frac{4\pi F_{max}(\theta,\phi)}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi) sin(\theta) d\theta d\phi}$$

Gain

The ratio between the maximum power density of the antenna and the maximum power density of a reference antenna.

$$G = \frac{(\Phi_m)_{ant}}{(\Phi_m)_{ref}}$$

It is also equal to the ratio of directivities of the two antennas.

$$G = \frac{D}{D_{ref}}$$

- Expressed in decibels
- The reference antenna is usually a standard half wave dipole (*preferred*) or an isotropic radiator

Antenna Aperture

The antenna aperture or *effective area* is defined as,

$$A_E = \frac{P_R}{\Phi}$$

Where, P_R is the power received by the antenna and Φ is the power density. This can also be expressed in terms of the antenna gain (G) and the wavelength λ.

$$A_E = \frac{\lambda^2 G}{4\pi}$$

- Can be readily used for *aperture* antennas

Radiation Impedance

The ratio between the input (feed) voltage and input current.

$$Z = \frac{V_F}{I_F} = R + jX$$

- R is the *radiation resistance* and X is the *radiation reactance*
- R can be calculated from

$$R = \frac{P_{eff}}{I_{rms}^2} = \frac{P_{rad}}{I_F^2} = \frac{1}{2I_F^2}\oint_S (E \times H^*) ds$$

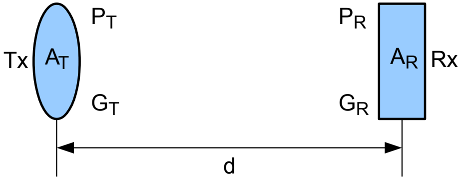
- X is difficult to obtain analytically

Bandwidth and EIRP

- The bandwidth of an antenna is the range of frequencies for which its characteristic parameters are maintained
  - Beyond the antenna bandwidth the characteristics deteriorate
- The *Effective Isotropically Radiated Power* or EIRP of an antenna is the apparent power radiated by the antenna had it been an isotropic radiator

$$EIRP = GP_T$$

Friis Formula



The power at the received antenna,

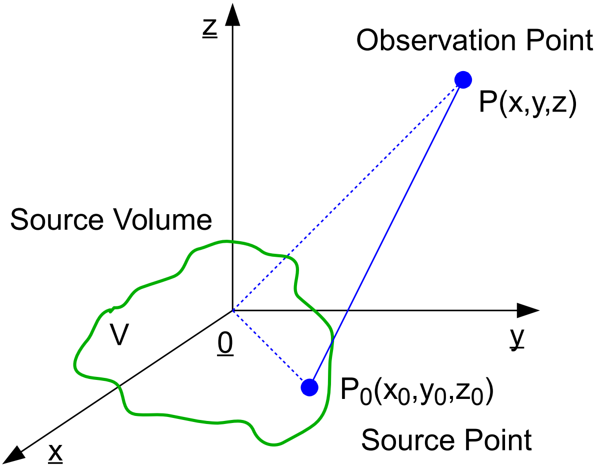
$$(P_R)_{dBm} = (P_T)_{dBm} + (G_T)_{dB} + (G_R)_{dB} - \underbrace{20log_{10}\left(\frac{4\pi d}{\lambda}\right)}_{\text{Free Space Loss}}$$

Antenna Categories

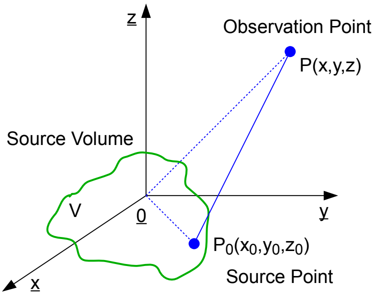
- Wired antennas
  - Radiate energy from a feed current
  - Generally used for low frequencies
  - Relatively low gain
- Aperture antennas
  - Modify an existing EM field
  - Aperture dimensions are practical for high frequencies
  - Medium to very high gain
- Antenna arrays
  - Multi element antennas

Wire Antennas

Retarded Potentials



Retarded Potentials (Contd..)



The *potential* due to a charge or current distribution within volume v

$$V(r) = \frac{1}{4\pi\varepsilon}\int_v \frac{\rho(r_0)}{|r-r_0|}dv$$

$$A(r) = \frac{\mu}{4\pi}\int_v \frac{J(r_0)}{|r-r_0|}dv$$

- The resulting EM waves will take time to travel from P_0 to P
  - The generating charge or current would have changed by then
  - Thus, the potential at P due to P_0 is *retarded* (i.e., lagged)
  - V is a *scalar* potential while A is a *vector* potential

Retarded Potentials (Contd..)

Due to the time lag

$$\begin{aligned} V(r,t) &= \frac{1}{4\pi\varepsilon}\int_V \frac{\rho\left(r_0, t-\frac{|r-r_0|}{c}\right)}{|r-r_0|}dv \\ A(r,t) &= \frac{\mu}{4\pi}\int_V \frac{J\left(r_0, t-\frac{|r-r_0|}{c}\right)}{|r-r_0|}dv \end{aligned}$$

- The resulting potential is a function of t

Retarded Potentials (Contd..)

By considering the *phase difference* due to the travel time from P to P_0, t can be eliminated

$$
\begin{aligned}
V(r) &= \frac{1}{4\pi\varepsilon}\int_V \frac{\rho(r_0)e^{-jk|r-r_0|}}{|r-r_0|}dv \\
A(r) &= \frac{\mu}{4\pi}\int_V \frac{J(r_0)e^{-jk|r-r_0|}}{|r-r_0|}dv
\end{aligned}
$$

where $k = 2\pi/\lambda$.

- The *far field* is when $r - r_0 \approx r$ and the *near field* is when otherwise
- Near field formulae are used for calculating the *radiation impedance* while far field formulae give the radiation pattern

Radiated Fields

- Antennas have no free charge so $V(r)$ can be ignored
- In *wire* antennas the potential is due to a current distribution
  - The radiated H and E fields are given by

$$\begin{aligned} H &= \frac{\nabla \times A}{\mu} \\ E &= \frac{\nabla \times H}{j\omega\varepsilon} \end{aligned}$$

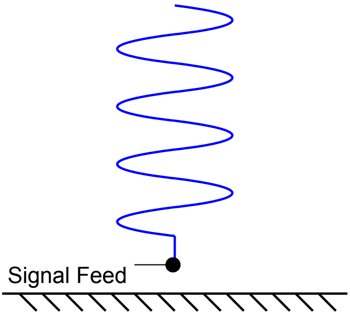
- In *aperture* antennas, the radiated fields can be found directly

Dipole Antennas

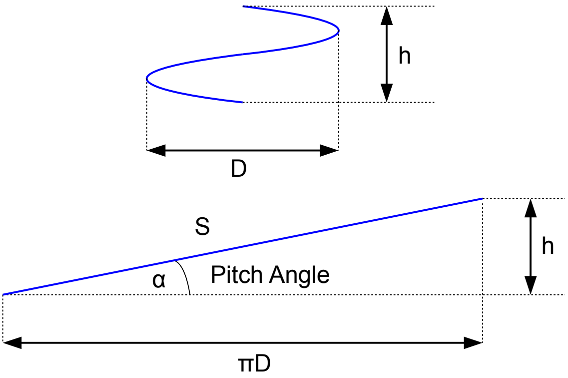
- Omnidirectional
- Dipole antennas can be used up to around 1GHz
  - Beyond this, the radiation pattern deteriorates
  - Usually have to be connected to coaxial lines
  - Linearly polarized
- Monopoles can be used for frequencies up to 3GHz

Helical Antennas

- 3D current distribution
- Elliptic or circularly polarized
- Two modes of operation
  - Normal mode
  - Axial mode



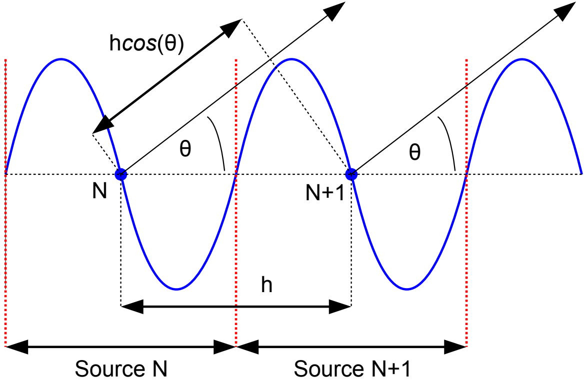
Helix Parameters



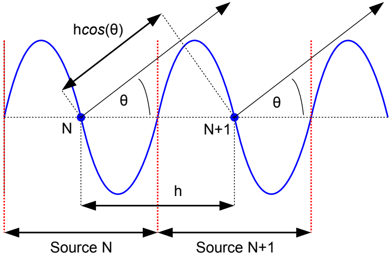
Take the number of turns as N

Axial Mode Operation

- Wavelength is small compared to helix dimensions
- Helix element taken as an element of an N element array



Axial Mode Operation (Contd..)



- Path difference $\Delta S = h\cos(\theta)$
- Phase difference due to *path difference* and *propagation*

$$\psi \quad = \quad \Delta\alpha + \Delta\zeta = \frac{2\pi hcos(\theta)}{\lambda} - \frac{2\pi S}{p\lambda} = \frac{2\pi}{\lambda}\left(hcos(\theta) - \frac{S}{p}\right)$$

Axial Mode Operation (Contd..)

Radiation Pattern

$$E = \cos(\theta) \left[\frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \right] \text{ (by pattern multiplication)}$$

For end-fire operation,

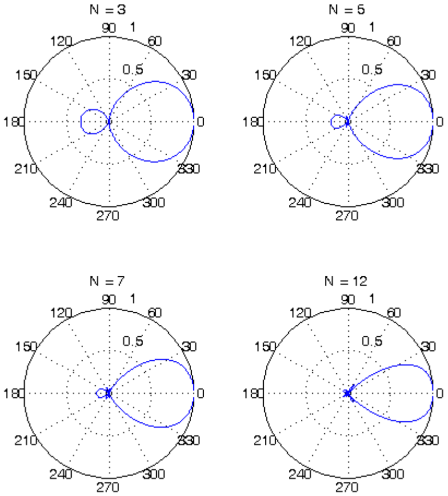
$$\psi = \frac{2\pi}{\lambda}\left(h cos(\theta) - \frac{S}{p}\right) = -2\pi m$$

Therefore, when $\theta = 0$,

$$p = \frac{S}{h + m\lambda} \approx 0.7 \sim 0.99$$

where m is the mode of current distribution. When $m = 1$, the current between two elements will undergo a single cycle.

Axial Mode Operation (Contd..)



Axial Mode Operation

Empirical Values

- Feed impedance

$$R = \frac{140\pi D}{\lambda}$$

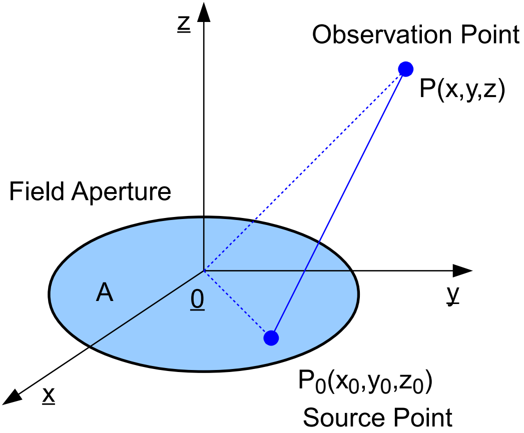
- For circular polarization

$$D \approx \frac{\lambda}{\pi}$$
$$h \approx \frac{\lambda}{4}$$

This will give a pitch angle of around 14^o (0.245rad).

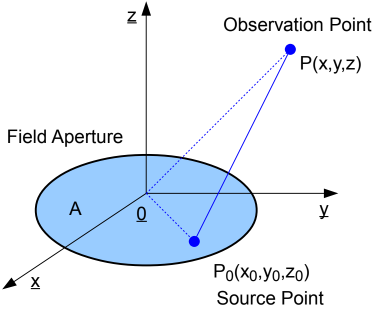
Aperture Antennas

Stutzmann's Method



Stutzmann's Method (Contd..)

The use of Huygen's principle (individual point sources) to find a radiation pattern.



$$E(r) = \frac{jk}{2\pi}\int_S E_A(r_0)\frac{e^{-jk|r-r_0|}}{|r-r_0|}ds$$

Complementary Wire Antennas

Babinet's Principle

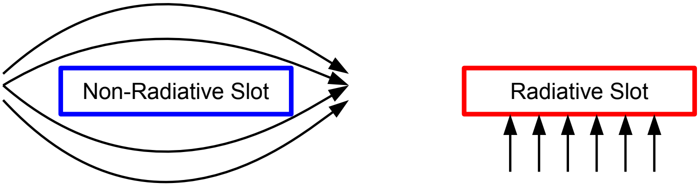
- The radiation pattern of an aperture is similar to the *complementary* wired structure

Booker's Law

- The feed impedance of an aperture (z_A) is related to that of the *complementary* wired structure (z_C) by

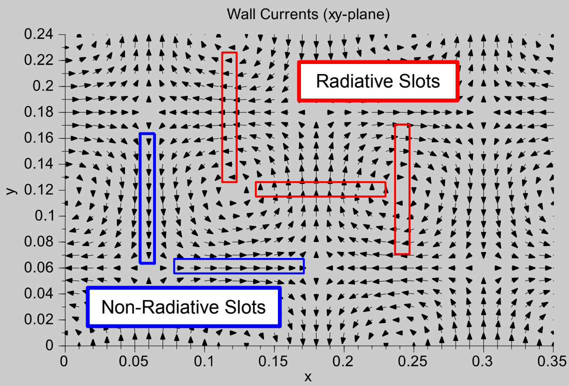
$$z_A z_C = \frac{\eta^2}{4}$$

Radiating Slots



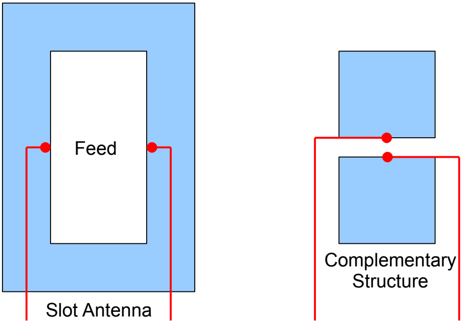
- A slot is a removal of metal from a waveguide or cavity wall
- If a slot is placed such that the current can easily bypass it, it is *non-radiative*
  - Has to be thin so that the parallel current can flow around it
  - If not, (i.e., current is normal) the slot is *radiative*
  - Even in a non-radiative slot a small amount of radiation can leak out
- Used for mode filters in waveguides

Slot Placement



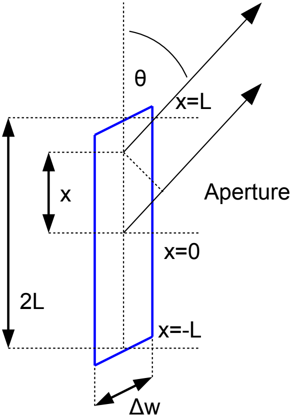
- Possible slot openings on a cavity wall for an antenna
  - A suitable location may not be practicable

Slot Feeds



- A feed is used when the metal structure is not a cavity
  - Commonly employed in aircraft, cellular phones etc.

Radiation Pattern



Path difference between two waves is given by,

$$\Delta \quad = \quad xcos(\theta)$$

Therefore,

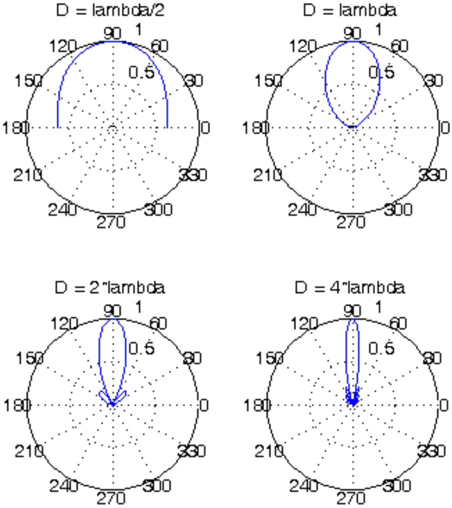
$$|r - r_0| \quad \approx \quad r - xcos(\theta)$$

Radiation Pattern (Contd..)

From Stutzmann's principle

$$\begin{aligned}
E(r) &= \frac{jk}{2\pi}\int_{-L}^{L} E_A \frac{e^{-jk(r-xcos(\theta))}}{r}(\Delta w)dx \\
&= \frac{jk\Delta w E_A e^{-jkr}}{2\pi r}\int_{-L}^{L} e^{jkxcos(\theta)}dx \\
&= \frac{jk\Delta w E_A e^{-jkr}}{2\pi r}\left[\frac{e^{jkxcos(\theta)}}{jkcos(\theta)}\right]_{-L}^{L} \\
&= j\frac{\Delta w E_A e^{-jkr}}{\pi r}\left[\frac{sin\left(kLcos(\theta)\right)}{cos(\theta)}\right]
\end{aligned}$$

Radiation Pattern (Contd..)



Lens and Reflector Antennas

- Aperture antennas where the EM waves obey geometric optics
- Lens antennas use a dielectric material like a lens
  - Size limitation due to the inability to support a lens
  - Useful in providing protection to antennas and waveguides
- Reflector antennas use a metal to concentrate radio waves
  - Highest achievable effective aperture

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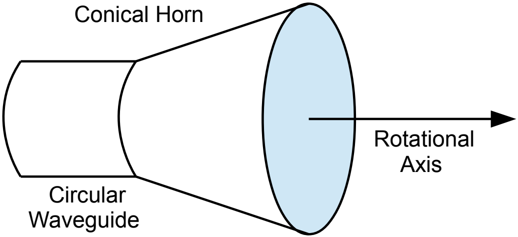
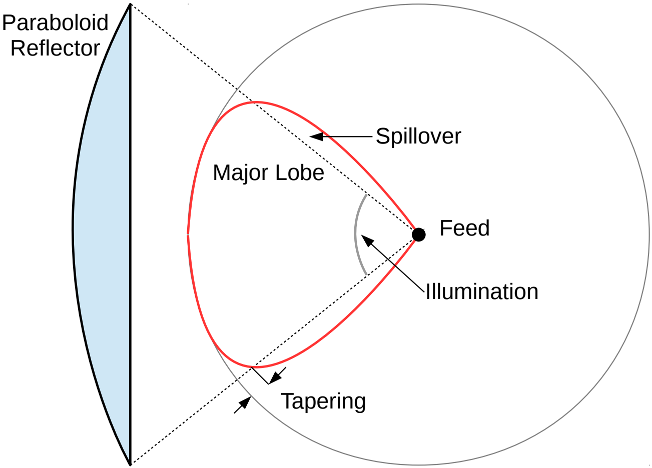
Body of Revolution (BOR) Antennas



- An antenna that has a mechanical structure that is rotationally invarient (i.e., symmetric around an axis of rotation)
  - For example, the conical horn antenna
  - Used as point source feeders for lens and reflector antennas
  - It is obviously directive in the azimuth plane

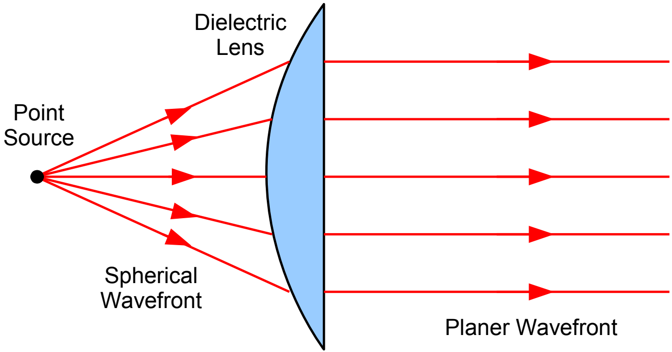
Feed Illumination



Feed Illumination (Contd..)

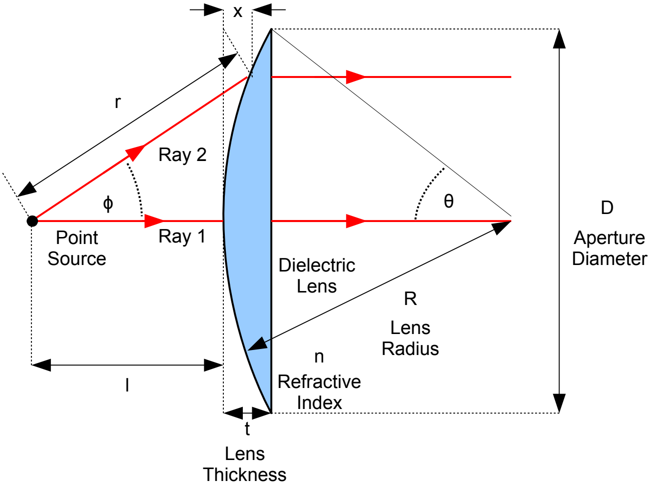
- Common to both lens and reflector antennas
- For the major lobe of the feed
  - The *illumination* is the radiation from the feed that gets reflected (or refracted) by the antenna
  - The *spillover* is the wasted radiation (excluding sidelobes)
  - The *tapering* is the reduction of illumination at the edges of the antenna
- The objective of feed design is to minimize tapering and spillover using a practical radiation pattern

Lens Antennas



- No interference between the source and transmitted waves

Lens Antenna Design



Lens Antenna Design (Contd..)

Design parameters

1. Distance of source from lens
2. Lens thickness
3. Lens radius
4. Refractive index of lens
5. Aperture diameter

Lens Antenna Design (Contd..)

For there to be no phase difference, the propagation times for both waves should be equal.

- Ray 1 travels a distance l in air and x in the lens
- Ray 2 travels a distance r in air

Therefore,

$$\frac{r}{c} = \frac{l}{c} + \frac{nx}{c} \rightarrow r = l + nx$$

Lens Antenna Design (Contd..)

From the Lensmaker's equation,

$$\frac{1}{l} = (n-1)\left[\frac{1}{R_N} - \frac{1}{R_F} + \frac{(n-1)t}{nR_NR_F}\right]$$

where R_N (near source side radius) is infinite and $R_F = R$ (far from source side radius). This results in,

$$R = (n-1)l \tag{1}$$

For a given aperture diameter D $(R > D/2)$, the lens thickness is given by,

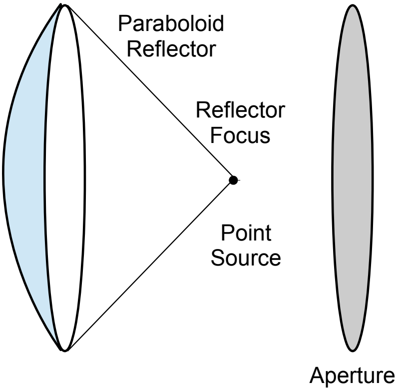
$$t = R - \sqrt{R^2 - \frac{D^2}{4}} \tag{2}$$

Lens Antenna Design (Contd..)

Design methodologies (according to requirement)

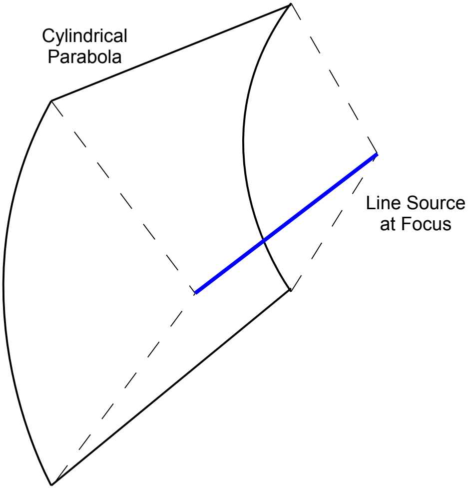
1. Start from focal length l
   - Obtain R for given n from (1)
   - Obtain t for required D from (2)
2. Start from D and t
   - Numerically obtain R for given t and D from (2)
   - Obtain l from (1) for given n
3. Other possibilities also exist
   - May be necessary to find required D or n first etc.

Paraboloid Reflector

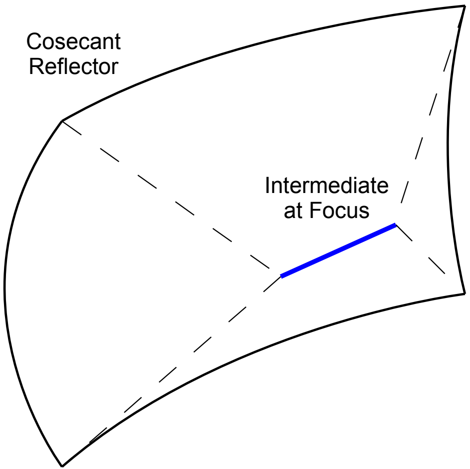


- Has to be used with a BOR feed antenna

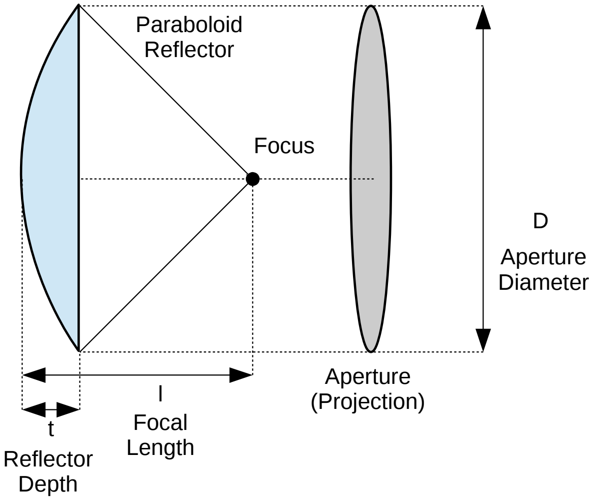
Cylindrical Parabolic Reflector



Cosecant Squared Reflector



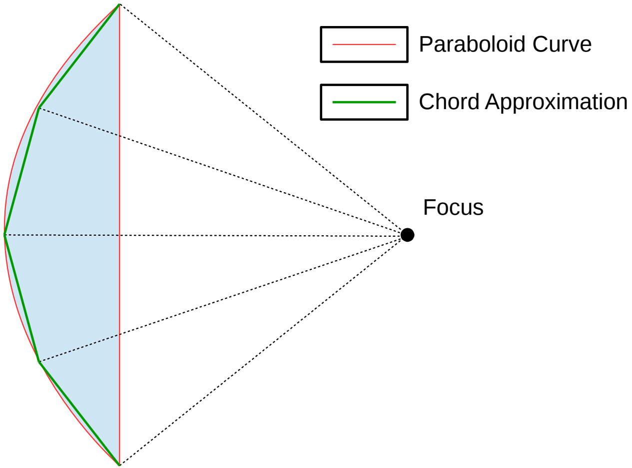
Reflector Design



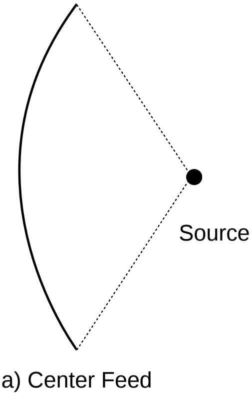
Reflector Design (Contd..)

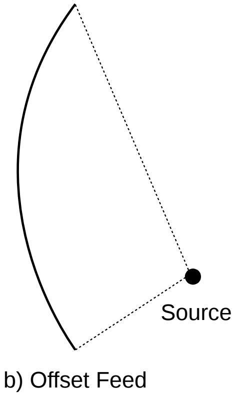
- The reflector depth t, focal length l and reflector diameter D are related by the parabolic equation $16lt = D^2$
  - Often a design starts with D (required aperture and gain)
  - Can also start with l
- By taking $D = 5.44l$ and $t = 1.85l$ it is possible to make the focus and center of mass to coincide
- It is also necessary to realize the antenna
  - It can be realized as one single piece
  - Chord approximation for large antennas
  - A structural engineer may be necessary to assess the strength of the physical structure

Reflector Design (Contd..)

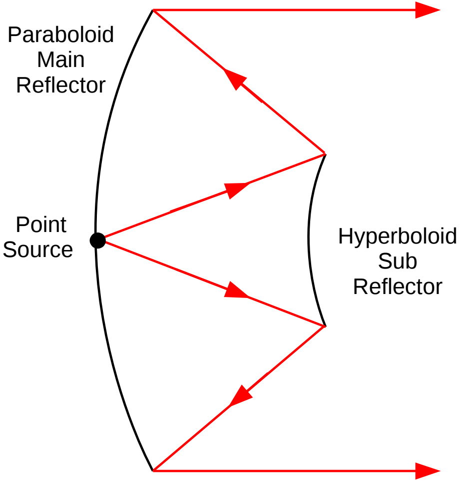


Reflector Feed





Cassegrain Feed



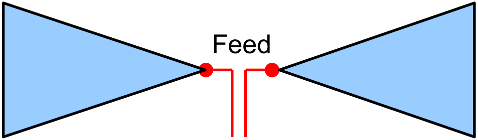
Wideband Antennas

Motivation

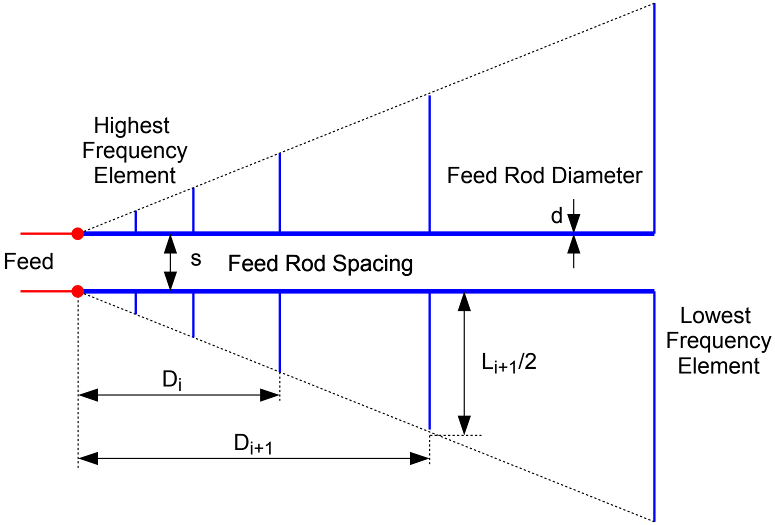
- Modern communication technologies require wide bandwidth
  - Mobile communication devices with backward compatibility
  - CDMA
  - Wideband satellite communication
- Most antennas have a limited bandwidth
  - Degradation of radiation pattern
  - Sensitive feed impedance
- How can wideband operation be achieved?

Rationale

- Use multiple elements of different length
  - Design so that the current distribution is frequency independent
  - When this happens, elements do not interfere with each other
- Can also use a continuous structure



Log Periodic Dipole Antenna



Design Procedure

Design Parameters

- Range f_{min} to f_{max}
- Number of elements N

Take the *geometric ratio* τ,

$$\tau = \sqrt[N-1]{\frac{f_{min}}{f_{max}}}$$

The spacing between two elements is given by $D_{i+1} - D_i$ where

$$\tau = \frac{D_i}{D_{i+1}}$$

This results in a total antenna length of D_N with D_1 chosen.

Design Procedure (Contd..)

The spacing factor (σ),

$$\sigma = \frac{D_{i+1} - D_i}{2L_{i+1}}$$

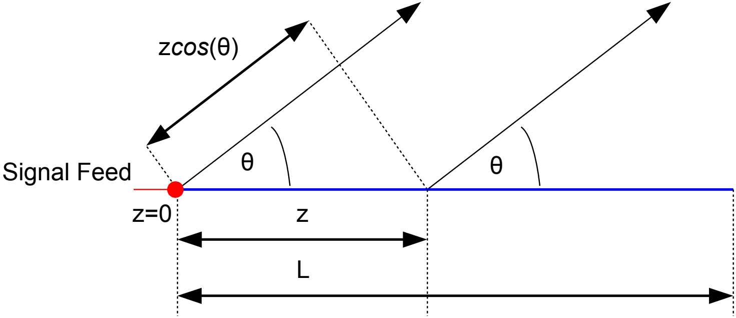
The iteration is started with $L_N = \lambda_{max}/2$ (for f_{min}).

The feed spacing s for a transmission line with characteristic impedance z_0 is given by,

$$s = d\cosh\left(\frac{z_0}{120}\right)$$

Traveling Wave Antennas

- Traveling wave current distribution like in a transmission line
- A standing wave may be generated depending on the reflection
- Derivations of the longwire antenna ($L \gg \lambda$)



Traveling Wave Antennas

Assuming the end of the line is perfectly matched and lossless,

$$J = I_0 e^{-\gamma z} = I_0 e^{-(\alpha + j\beta)z} = I_0 e^{-j\beta z}$$

Approximate the longwire as a summation of infinitesimal dipoles. Therefore, the path difference becomes,

$$R = r - zcos(\theta)$$

Therefore,

$$dE = \left[\frac{j\eta k sin(\theta) I_z e^{-jk(r - zcos(\theta))}}{4\pi r} dz \right] \underline{\theta}$$

Traveling Wave Antennas (Contd..)

Thus, the integral of the current distribution becomes,

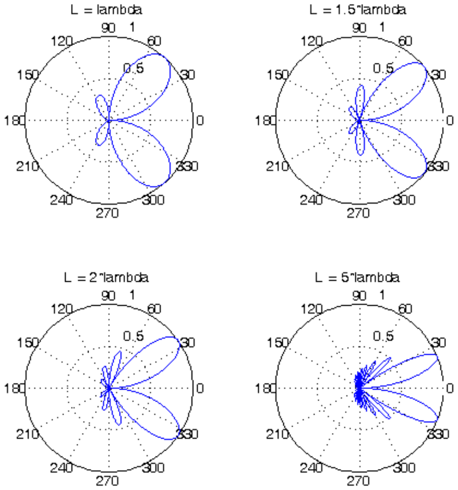
$$E = \left[\frac{j\eta k e^{-jkr} sin(\theta)}{4\pi r}\int_0^L I_0 e^{-jkz} e^{jkz cos(\theta)} dz\right] \underline{\theta}$$

This simplifies to,

$$E = \left[\frac{j\eta I_0 e^{-jk\left[r+\frac{L}{2}(1-cos(\theta))\right]} sin(\theta) sin\left[\frac{kL(cos(\theta)-1)}{2}\right]}{2\pi r(cos(\theta)-1)}\right] \underline{\theta}$$

$$H = \left[\frac{j I_0 e^{-jk\left[r+\frac{L}{2}(1-cos(\theta))\right]} sin(\theta) sin\left[\frac{kL(cos(\theta)-1)}{2}\right]}{2\pi r(cos(\theta)-1)}\right] \underline{\phi}$$

Traveling Wave Antennas - Radiation Patterns



Standing Wave Antennas

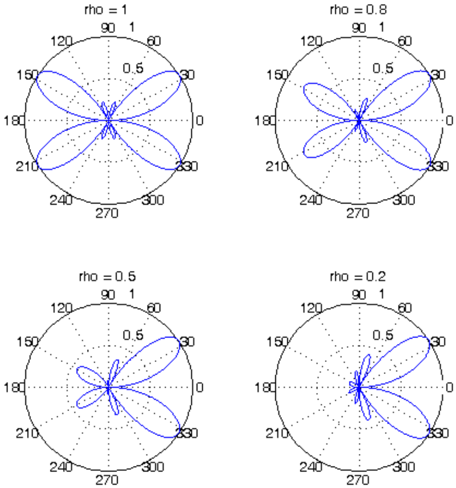
When the longwire has a reflection coefficient of ρ,

$$J = I_0 e^{-jkz} + \rho I_0 e^{jkz}$$

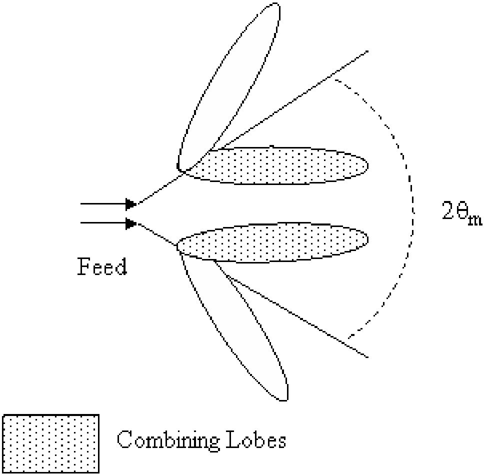
Therefore,

$$E = \left[\frac{j\eta k e^{-jkr} sin(\theta)}{4\pi r}\int_0^L \left(I_0 e^{-jkz} + \rho I_0 e^{jkz}\right) e^{jkzcos(\theta)} dz\right] \underline{\theta}$$

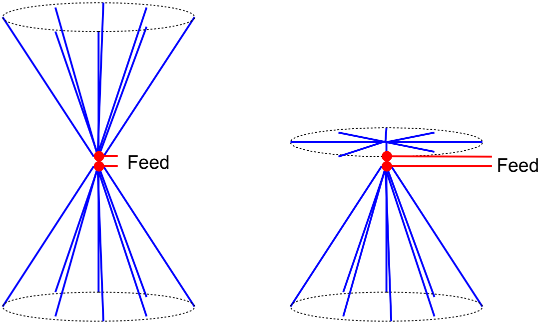
Standing Wave Antennas - Radiation Patterns



V Antenna



Biconical and Discone Antennas

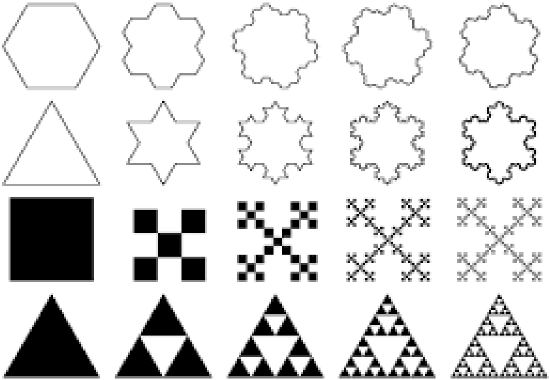


Wire Biconical Antenna

Wire Discone Antenna

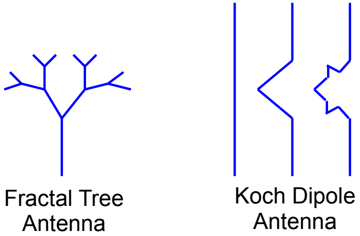
- A composite of multiple V antennas
- Can be made solid at microwave frequencies

Self Similarity and Fractals



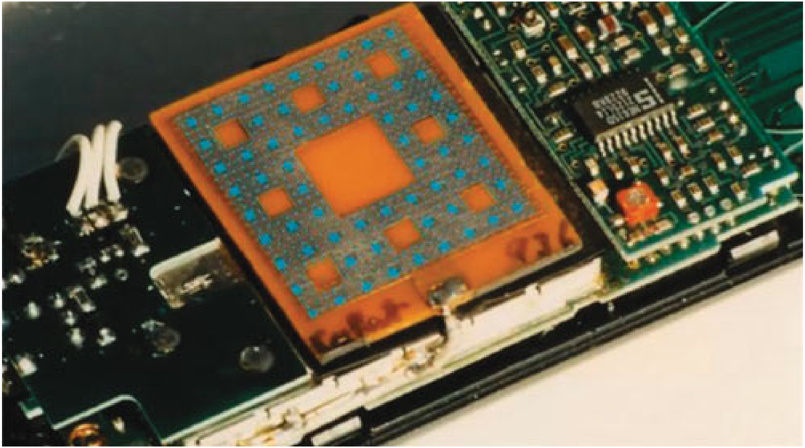
Wolfram Mathworld

Fractal Antennas



- Scale invariant
- Can be an iterative feed like the Fractal Tree or compacting like the Koch dipole

Fractal Antennas (Contd..)



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Conclusion

Summary

- An antenna is an interface between the electric signal and radiated EM wave
  - All practical antennas are anisotropic radiators
  - Most efficient within the bandwidth
  - Has to be matched to the transmission line
- A *good* antenna must have a useful radiation pattern and matching feed impedance
- At microwave frequencies
  - Short wavelengths result in small antenna dimensions
  - Large effective apertures therefore high gain