GENERAL SIR JOHN KOTELAWALA DEFENCE UNIVERSITY

Faculty of Engineering

Department of Electrical, Electronic and Telecommunication Engineering

BSc Engineering Degree

Semester 8 Examination – November/ December 2022

(Intake 36 – ET)

ET4222 – COMMUNICATION THEORY III

Time allowed: 2 hours 12th December, 2022

ADDITIONAL MATERIAL PROVIDED

1. Queuing Formula
2. Earlang B Table

INSTRUCTIONS TO CANDIDATES

This paper contains 4 questions on 4 pages

Answer ALL FOUR questions

This is an **CLOSED book examination**

Write all the steps in the calculations. For missing steps, marks will be deducted.

This examination accounts for 70% of the module assessment. A total maximum mark obtainable is 100. The marks assigned for each question and parts thereof are indicated in square brackets

If you have any doubt as to the interpretation of the wordings of a question, make your own decision, but clearly state it on the script

Assume reasonable values for any data not given in or provided with the question paper, clearly make such assumptions made in the script

All examinations are conducted under the rules and regulations of the KDU

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Question 1

(a) Explain in what situations Earlang B table and Earlang C table are used.

(4 Marks)

(b) Explain quality of service from the aspect of telecommunication operator

(3 Marks)

(c) A telecommunication operator needed to compare the behavior of the voice traffic profile before pandemic situation and during the pandemic situation. Assume the month of January 2020 is a pre-covid period and the month of February 2020 is a pandemic situation. Consider a communication site located in Pettah and sketch the busy hour traffic profile for both months. Explain the key variations of the graph. (If you consider any assumptions clearly state them).

(9 Marks)

(d) A grocery shop is attended by one person. Apparently, the arrival pattern of customers during Saturdays follows a Poisson process with an arrival rate of 10 persons/hour. Customers are attended following a FIFO order and, due to the prestige of the shop, once they arrive, they are willing to wait for the service. The service time is distributed exponentially, with a mean time of 4 minutes. Determine,

(i) Probability of waiting in line.

(3 Marks)

(ii) Average length of the waiting line.

(3 Marks)

(iii) Average waiting time.

(3 Marks)

Question 2

(a) Explain the reason for accurate estimation for network performance is essential.

(4 Marks)

(b) Explain 4 different service disciplines used in a queuing system.

(4 Marks)

(c) Explain steady state of a queuing system and its' importance.

(3 Marks)

(d) A bank considers the possibility of installing a teller network in one of its branches. Given that they don't know the people inflow demanding the service, they only allocate single teller for a month. They collect data daily about the arrival time of the customers, as well as the service time. If the branch is in a district where there is no such service, the customer that arrives waits until he/she can use the teller when it is busy. After the corresponding data analysis, it is estimated that,

1. The arrivals follow a Poisson process.
2. The distribution of the service time is exponential.
3. The mean time between consecutive arrivals is 7.5 minutes.
4. The mean service time is 5 minutes/customer.

(i) Mean waiting time that each customer must spend in the queue.

(3 Marks)

(ii) Average number of waiting customers.

(4 Marks)

(iii) Average queue length when it is not empty and probability that when going to the teller there will be somebody in the queue.

(4 Marks)

Question 3

(a) A system is being designed. The inter-arrival times of the customers are expected to be exponentially distributed with mean $1/\lambda = 50$ ms. Three options were considered, during the design process.

(i) One single-server queue with infinite buffer space. The service times are exponentially distributed with mean $1/\mu = 20$ ms.

(ii) Two single-server queues, each with infinite buffer space. Customers are randomly dispatched to each queue with an equal probability. The service times are exponentially distributed with mean $1/\mu = 40$ ms at each server.

(iii) One two-server queue with infinite buffer space. The service times are exponentially distributed with mean $1/\mu = 40$ ms at each server.

Calculate the response time in each option using queuing analysis.

(15 Marks)

(b) Draw the state diagrams for case (i) and case (ii) in part (a).

(6 Marks)

(c) You are given the task to predict the receive signal strength at a given distance from a transmitter. Explain which simulation method will you use for the simulation.

(4 Marks)

Question 4

(a) Consider a queuing system with a single server. The arrival events can be modeled with Poisson distribution, but two customers arrive at the system at each arrival event. Each customer requires an exponentially distributed service time. Deduce the state diagram for the above scenario.

(5 Marks)

(b) A telephone switch has 10 output lines and a large number of incoming lines. Upon arrival a call on the input line is assigned an output line if such line is available – otherwise the call is blocked and lost. The output line remains assigned to the call for its entire duration which is of exponentially distributed length. Assume that 180 calls / hour arrive in Poisson fashion whereas the mean call duration is 110 seconds.

(i) Write the Kendall notation for the above scenario.

(ii) Calculate the offered traffic (utilization).

(iii) Obtain the blocking probability of the system.

(15 Marks)

(c) Explain the difference between open queuing system and close queuing system.

(5 Marks)

End of question paper

Symbols

P_n - Steady state probability that there are n customer in the system.

ρ - The traffic intensity or offered load. (Utilization) $\rho = \lambda\overline{S}$.

λ - Mean arrival rate of customers into the system

μ - Mean service rate per server, that is, the mean rate of service completions while the server is busy.

$\overline{N}$ - Expected steady state number of customers in the queuing system

Q - Random variable describing the steady state number of customers in the queue.

$F_T(t)$ - The distribution function of T , $F_T(t) = P[T < t]$

$\overline{T}$ - Expected steady state time a customer spends in the system, $\overline{T} = E[T] = \overline{W} + \overline{S}$

W - the time a customer spends in the queue before service begins.

$\overline{W}$ – Expected steady state time a customer spends in the queue before service begins.

S - customer service time

$\overline{S}$ - Expected customer service time.

a - Server utilization

a_i - Utilization of component i in a queuing network.

c - Number of servers in in a service facility.

K - Maximum number of customers allowed in queuing system. Also size of population in finite population models.

M/M/1/K Formulas

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{(1-\rho^{K+1})} & \text{if } \lambda \neq \mu \\ \dfrac{1}{K+1} & \text{if } \lambda = \mu \end{cases}$$

$n = 0, 1, \ldots, K$, where $\rho = \lambda \overline{S}$.

$\overline{\lambda} = (1 - P_K)\lambda$, Mean arrival rate into system.

$$\overline{N} = \begin{cases} \dfrac{\rho[1-(K+1)\rho^K + K\rho^{K+1}]}{(1-\rho)(1-\rho^{K+1})} & \text{ha } \lambda \neq \mu \\ \dfrac{K}{2} & \text{ha } \lambda = \mu. \end{cases}$$

$\overline{Q} = \overline{N} - (1 - P_0)$, $\Pi_n = \dfrac{P_n}{1 - P_K}, n = 0, 1, \ldots, K-1.$

$$F_T(t) = 1 - \sum_{n=0}^{K-1} \Pi_n Q[n; \mu t],$$

where

$$Q[n; \mu t] = e^{-\mu t} \sum_{k=0}^{n} \frac{(\mu t)^k}{k!}.$$

$$\overline{T} = \frac{\overline{N}}{\overline{\lambda}}, \quad \overline{W} = \frac{\overline{Q}}{\overline{\lambda}}.$$

$$F_T(t) = 1 - \sum_{n=0}^{K-2} \Pi_{n+1} Q[n; \mu t].$$

$$\mathbb{E}[W | W > 0] = \frac{\overline{W}}{1 - \Pi_0}, \quad a = (1 - P_K)\rho.$$

M/M/c Formulas

$$\rho = \lambda \overline{S}, \quad a = \frac{\rho}{c}$$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!(1-a)} \right]^{-1} = \frac{c!(1-a)P[N \geq c]}{\rho^c}.$$

$$P_n = \begin{cases} \dfrac{\rho^n}{n!} P_0, & \text{if } n \leq c \\ \dfrac{\rho^n}{c! c^{n-c}} P_0, & \text{if } n \geq c. \end{cases}$$

$$P[N \geq n] = \begin{cases} P_0 \left[\sum_{k=n}^{c-1} \dfrac{\rho^k}{k!} + \dfrac{\rho^c}{c!(1-a)} \right] & \text{if } n < c, \\ P_0 \left[\dfrac{a^c a^{n-c}}{c!(1-a)} \right] = P[N \geq c] a^{n-c} & \text{if } n \geq c \end{cases}$$

$$\overline{Q} = \overline{\lambda} \cdot \overline{W} = \frac{\rho P[N \geq c]}{c(1-a)},$$

where

$$P[N \geq c] = C[c, \rho] = \frac{\dfrac{\rho^c}{c!}}{(1 - \dfrac{\rho}{c}) \sum_{n=0}^{c-1} \dfrac{\rho^n}{n!} + \dfrac{\rho^c}{c!}}.$$

$$Var(Q) = \frac{aC[c,\rho][1 + a - aC[c,\rho]]}{(1-a)^2}.$$

$$\overline{N} = \overline{\lambda} \cdot \overline{T} = \overline{Q} + \rho.$$

$$Var(N) = Var(Q) + \rho(1 + P[N \geq c]).$$

$$\overline{W}[0] = 1 - P[N \geq c], \quad F_T(t) = 1 - P[N \geq c] \exp[-c\mu t(1-a)],$$

$$\overline{W} = \frac{P[N \geq c]\overline{S}}{c(1-a)}.$$

M/M/c/K Formulas

$$\rho = \lambda \overline{S}.$$

$$P_0 = \left[\sum_{n=0}^{c} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \sum_{n=1}^{K-c} \left(\frac{\rho}{c} \right)^n \right]^{-1}.$$

$$P_n = \begin{cases} \dfrac{\rho^n}{n!} P_0 & \text{if } n = 1, 2, \ldots, c, \\ \dfrac{\rho^n}{c!} \left(\dfrac{\rho}{c} \right)^{n-c} P_0 & \text{if } n = c+1, \ldots, K. \end{cases}$$

The average arrival rate of customers who actually enter the system is $\overline{\lambda} = \lambda(1 - P_K)$.

The actual mean server utilization, a, is given by:

$$a = \frac{\overline{\lambda}\,\overline{S}}{c}.$$

$$\overline{Q} = \frac{\rho^c r P_0}{c!(1-r)^2} \left[1 + (K-c) r^{K-c+1} - (K-c+1) r^{K-c} \right],$$

where

$$r = \frac{\rho}{c}.$$

$$\overline{N} = \overline{Q} + \mathbb{E}[N_s] = \overline{Q} + \sum_{n=0}^{c-1} n P_n + c \left(1 - \sum_{n=0}^{c-1} P_n \right).$$

By Little's Law

$$\overline{W} = \frac{\overline{Q}}{\overline{\lambda}}, \qquad \overline{T} = \frac{\overline{N}}{\overline{\lambda}}.$$

$$\Pi_n = \frac{P_n}{1 - P_K}, \quad n = 0, 1, 2, \ldots, K-1,$$

Erlang B Traffic Table

Maximum Offered Load Versus B and N

B is in %

N/B	0.01	0.05	0.1	0.5	1.0	2	5	10	15	20	30	40
1	.0001	.0005	.0010	.0050	.0101	.0204	.0526	.1111	.1765	.2500	.4286	.6667
2	.0142	.0324	.0458	.1054	.1526	.2235	.3813	.5954	.7962	1.000	1.449	2.000
3	.0868	.1517	.1938	.3490	.4555	.6022	.8994	1.271	1.603	1.930	2.633	3.480
4	.2347	.3624	.4393	.7012	.8694	1.092	1.525	2.045	2.501	2.945	3.891	5.021
5	.4520	.6486	.7621	1.132	1.361	1.657	2.219	2.881	3.454	4.010	5.189	6.596
6	.7282	.9957	1.146	1.622	1.909	2.276	2.960	3.758	4.445	5.109	6.514	8.191
7	1.054	1.392	1.579	2.158	2.501	2.935	3.738	4.666	5.461	6.230	7.856	9.800
8	1.422	1.830	2.051	2.730	3.128	3.627	4.543	5.597	6.498	7.369	9.213	11.42
9	1.826	2.302	2.558	3.333	3.783	4.345	5.370	6.546	7.551	8.522	10.58	13.05
10	2.260	2.803	3.092	3.961	4.461	5.084	6.216	7.511	8.616	9.685	11.95	14.68
11	2.722	3.329	3.651	4.610	5.160	5.842	7.076	8.487	9.691	10.86	13.33	16.31
12	3.207	3.878	4.231	5.279	5.876	6.615	7.950	9.474	10.78	12.04	14.72	17.95
13	3.713	4.447	4.831	5.964	6.607	7.402	8.835	10.47	11.87	13.22	16.11	19.60
14	4.239	5.032	5.446	6.663	7.352	8.200	9.730	11.47	12.97	14.41	17.50	21.24
15	4.781	5.634	6.077	7.376	8.108	9.010	10.63	12.48	14.07	15.61	18.90	22.89
16	5.339	6.250	6.722	8.100	8.875	9.828	11.54	13.50	15.18	16.81	20.30	24.54
17	5.911	6.878	7.378	8.834	9.652	10.66	12.46	14.52	16.29	18.01	21.70	26.19
18	6.496	7.519	8.046	9.578	10.44	11.49	13.39	15.55	17.41	19.22	23.10	27.84
19	7.093	8.170	8.724	10.33	11.23	12.33	14.32	16.58	18.53	20.42	24.51	29.50
20	7.701	8.831	9.412	11.09	12.03	13.18	15.25	17.61	19.65	21.64	25.92	31.15
21	8.319	9.501	10.11	11.86	12.84	14.04	16.19	18.65	20.77	22.85	27.33	32.81
22	8.946	10.18	10.81	12.64	13.65	14.90	17.13	19.69	21.90	24.06	28.74	34.46
23	9.583	10.87	11.52	13.42	14.47	15.76	18.08	20.74	23.03	25.28	30.15	36.12
24	10.23	11.56	12.24	14.20	15.30	16.63	19.03	21.78	24.16	26.50	31.56	37.78
25	10.88	12.26	12.97	15.00	16.13	17.51	19.99	22.83	25.30	27.72	32.97	39.44
26	11.54	12.97	13.70	15.80	16.96	18.38	20.94	23.89	26.43	28.94	34.39	41.10
27	12.21	13.69	14.44	16.60	17.80	19.27	21.90	24.94	27.57	30.16	35.80	42.76
28	12.88	14.41	15.18	17.41	18.64	20.15	22.87	26.00	28.71	31.39	37.21	44.41
29	13.56	15.13	15.93	18.22	19.49	21.04	23.83	27.05	29.85	32.61	38.63	46.07
30	14.25	15.86	16.68	19.03	20.34	21.93	24.80	28.11	31.00	33.84	40.05	47.74
31	14.94	16.60	17.44	19.85	21.19	22.83	25.77	29.17	32.14	35.07	41.46	49.40
32	15.63	17.34	18.21	20.68	22.05	23.73	26.75	30.24	33.28	36.30	42.88	51.06
33	16.34	18.09	18.97	21.51	22.91	24.63	27.72	31.30	34.43	37.52	44.30	52.72
34	17.04	18.84	19.74	22.34	23.77	25.53	28.70	32.37	35.58	38.75	45.72	54.38
35	17.75	19.59	20.52	23.17	24.64	26.44	29.68	33.43	36.72	39.99	47.14	56.04
36	18.47	20.35	21.30	24.01	25.51	27.34	30.66	34.50	37.87	41.22	48.56	57.70
37	19.19	21.11	22.08	24.85	26.38	28.25	31.64	35.57	39.02	42.45	49.98	59.37
38	19.91	21.87	22.86	25.69	27.25	29.17	32.62	36.64	40.17	43.68	51.40	61.03
39	20.64	22.64	23.65	26.53	28.13	30.08	33.61	37.72	41.32	44.91	52.82	62.69
40	21.37	23.41	24.44	27.38	29.01	31.00	34.60	38.79	42.48	46.15	54.24	64.35
41	22.11	24.19	25.24	28.23	29.89	31.92	35.58	39.86	43.63	47.38	55.66	66.02
42	22.85	24.97	26.04	29.09	30.77	32.84	36.57	40.94	44.78	48.62	57.08	67.68
43	23.59	25.75	26.84	29.94	31.66	33.76	37.57	42.01	45.94	49.85	58.50	69.34

44	24.33	26.53	27.64	30.80	32.54	34.68	38.56	43.09	47.09	51.09	59.92	71.01
45	25.08	27.32	28.45	31.66	33.43	35.61	39.55	44.17	48.25	52.32	61.35	72.67
46	25.83	28.11	29.26	32.52	34.32	36.53	40.55	45.24	49.40	53.56	62.77	74.33
47	26.59	28.90	30.07	33.38	35.22	37.46	41.54	46.32	50.56	54.80	64.19	76.00
48	27.34	29.70	30.88	34.25	36.11	38.39	42.54	47.40	51.71	56.03	65.61	77.66
49	28.10	30.49	31.69	35.11	37.00	39.32	43.53	48.48	52.87	57.27	67.04	79.32
50	28.87	31.29	32.51	35.98	37.90	40.26	44.53	49.56	54.03	58.51	68.46	80.99
51	29.63	32.09	33.33	36.85	38.80	41.19	45.53	50.64	55.19	59.75	69.88	82.65
52	30.40	32.90	34.15	37.72	39.70	42.12	46.53	51.73	56.35	60.99	71.31	84.32
53	31.17	33.70	34.98	38.60	40.60	43.06	47.53	52.81	57.50	62.22	72.73	85.98
54	31.94	34.51	35.80	39.47	41.51	44.00	48.54	53.89	58.66	63.46	74.15	87.65
55	32.72	35.32	36.63	40.35	42.41	44.94	49.54	54.98	59.82	64.70	75.58	89.31
56	33.49	36.13	37.46	41.23	43.32	45.88	50.54	56.06	60.98	65.94	77.00	90.97
57	34.27	36.95	38.29	42.11	44.22	46.82	51.55	57.14	62.14	67.18	78.43	92.64
58	35.05	37.76	39.12	42.99	45.13	47.76	52.55	58.23	63.31	68.42	79.85	94.30
59	35.84	38.58	39.96	43.87	46.04	48.70	53.56	59.32	64.47	69.66	81.27	95.97
60	36.62	39.40	40.80	44.76	46.95	49.64	54.57	60.40	65.63	70.90	82.70	97.63
61	37.41	40.22	41.63	45.64	47.86	50.59	55.57	61.49	66.79	72.14	84.12	99.30
62	38.20	41.05	42.47	46.53	48.77	51.53	56.58	62.58	67.95	73.38	85.55	101.0
63	38.99	41.87	43.31	47.42	49.69	52.48	57.59	63.66	69.11	74.63	86.97	102.6
64	39.78	42.70	44.16	48.31	50.60	53.43	58.60	64.75	70.28	75.87	88.40	104.3
65	40.58	43.52	45.00	49.20	51.52	54.38	59.61	65.84	71.44	77.11	89.82	106.0
66	41.38	44.35	45.85	50.09	52.44	55.33	60.62	66.93	72.60	78.35	91.25	107.6
67	42.17	45.18	46.69	50.98	53.35	56.28	61.63	68.02	73.77	79.59	92.67	109.3
68	42.97	46.02	47.54	51.87	54.27	57.23	62.64	69.11	74.93	80.83	94.10	111.0
69	43.77	46.85	48.39	52.77	55.19	58.18	63.65	70.20	76.09	82.08	95.52	112.6
70	44.58	47.68	49.24	53.66	56.11	59.13	64.67	71.29	77.26	83.32	96.95	114.3
71	45.38	48.52	50.09	54.56	57.03	60.08	65.68	72.38	78.42	84.56	98.37	116.0
72	46.19	49.36	50.94	55.46	57.96	61.04	66.69	73.47	79.59	85.80	99.80	117.6
73	47.00	50.20	51.80	56.35	58.88	61.99	67.71	74.56	80.75	87.05	101.2	119.3
74	47.81	51.04	52.65	57.25	59.80	62.95	68.72	75.65	81.92	88.29	102.7	120.9
75	48.62	51.88	53.51	58.15	60.73	63.90	69.74	76.74	83.08	89.53	104.1	122.6
76	49.43	52.72	54.37	59.05	61.65	64.86	70.75	77.83	84.25	90.78	105.5	124.3
77	50.24	53.56	55.23	59.96	62.58	65.81	71.77	78.93	85.41	92.02	106.9	125.9
78	51.05	54.41	56.09	60.86	63.51	66.77	72.79	80.02	86.58	93.26	108.4	127.6
79	51.87	55.25	56.95	61.76	64.43	67.73	73.80	81.11	87.74	94.51	109.8	129.3
80	52.69	56.10	57.81	62.67	65.36	68.69	74.82	82.20	88.91	95.75	111.2	130.9
81	53.51	56.95	58.67	63.57	66.29	69.65	75.84	83.30	90.08	96.99	112.6	132.6
82	54.33	57.80	59.54	64.48	67.22	70.61	76.86	84.39	91.24	98.24	114.1	134.3
83	55.15	58.65	60.40	65.39	68.15	71.57	77.87	85.48	92.41	99.48	115.5	135.9
84	55.97	59.50	61.27	66.29	69.08	72.53	78.89	86.58	93.58	100.7	116.9	137.6
85	56.79	60.35	62.14	67.20	70.02	73.49	79.91	87.67	94.74	102.0	118.3	139.3
86	57.62	61.21	63.00	68.11	70.95	74.45	80.93	88.77	95.91	103.2	119.8	140.9
87	58.44	62.06	63.87	69.02	71.88	75.42	81.95	89.86	97.08	104.5	121.2	142.6
88	59.27	62.92	64.74	69.93	72.82	76.38	82.97	90.96	98.25	105.7	122.6	144.3
89	60.10	63.77	65.61	70.84	73.75	77.34	83.99	92.05	99.41	107.0	124.0	145.9
90	60.92	64.63	66.48	71.76	74.68	78.31	85.01	93.15	100.6	108.2	125.5	147.6

91	61.75	65.49	67.36	72.67	75.62	79.27	86.04	94.24	101.8	109.4	126.9	149.3
92	62.58	66.35	68.23	73.58	76.56	80.24	87.06	95.34	102.9	110.7	128.3	150.9
93	63.42	67.21	69.10	74.50	77.49	81.20	88.08	96.43	104.1	111.9	129.8	152.6
94	64.25	68.07	69.98	75.41	78.43	82.17	89.10	97.53	105.3	113.2	131.2	154.3
95	65.08	68.93	70.85	76.33	79.37	83.13	90.12	98.63	106.4	114.4	132.6	155.9
96	65.92	69.79	71.73	77.24	80.31	84.10	91.15	99.72	107.6	115.7	134.0	157.6
97	66.75	70.65	72.61	78.16	81.25	85.07	92.17	100.8	108.8	116.9	135.5	159.3
98	67.59	71.52	73.48	79.07	82.18	86.04	93.19	101.9	109.9	118.2	136.9	160.9
99	68.43	72.38	74.36	79.99	83.12	87.00	94.22	103.0	111.1	119.4	138.3	162.6
100	69.27	7~.25	75.24	80.91	84.06	87.97	95.24	104.1	112.3	120.6	139.7	164.3

N is the number of servers. The numerical column headings indicate blocking probability B in %. Table generated by Dan Dexter