Encoder

- The digitization of analog signal is done by the encoder. It designates each quantized level by a binary code.

Regenerative Repeater

- This section increases the signal strength. The output of the channel also has one ~~regerative~~ regenerative repeater circuit, to compensate the signal loss and reconstruct the signal and also to increase its strength.

Decoder

- The decoder circuit decodes the pulse coded waveform to reproduce the original signal. This circuit acts as the demodulator.

Reconstruction Filter

- After the digital-to-analog conversion is done by the regenerat circuit and the decoder, a low-pass filter is employed, called as the reconstruction filter. to get back the original signal.
- Hence, the pulse code Modulator circuit digitizes the give analog signal, codes it and samples it, and then transmits it in an analog form. The whole process is repeated in a reverse pattern to obtain the original signal.

No : ________ Date: __/__/__

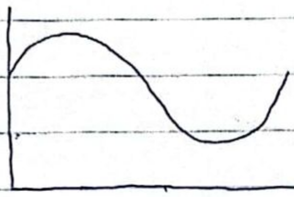
Quantization Process

- The method of sampling chooses a few points on the analog signal and then these points are joined to round off the value to a near stabilized value. Such a process is called as quantization.

Quantizing an Analog signal

analog-to-digital converters $\xrightarrow{\text{create}}$ A series of digital values out of given analog signal.

signal → digital → sampling and quantizing



quanting of an analog signal → descretizing the signal with a number of quantization levels

Quantization

- Sampled values of the amplitude by a finite set of levels,
- converts continuous amplitude sample → discrete time signal

- The figure in the slide shows how an analog signal gets quantized.

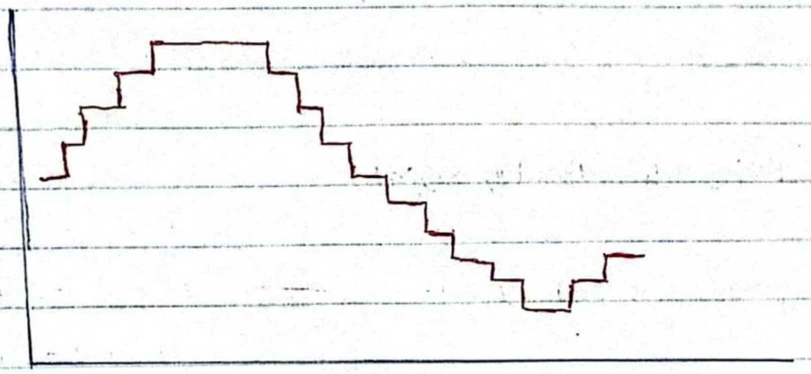
$\boxed{\text{Sampling and Quantization} \rightarrow \text{loss of information}}$

$\boxed{\text{The quality of a quantizer output} \rightarrow \text{the number of quantization levels used.}}$

$\boxed{\text{discrete amplitudes of the quantized output} \rightarrow \text{representation levels or reconstruction levels}}$

- The spacing between the two adjacent representation levels is called a quantum or step size.

Resultant Quantized signal → digital form for the given analog signal



Stair-case waveform

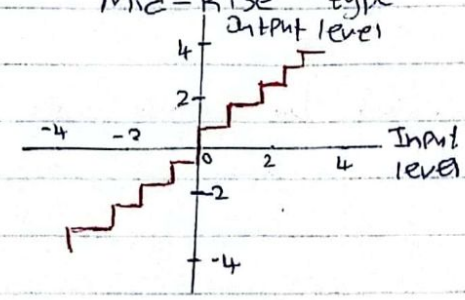
Types of quantization

Uniform quantization — Quantization levels are uniformly spaced

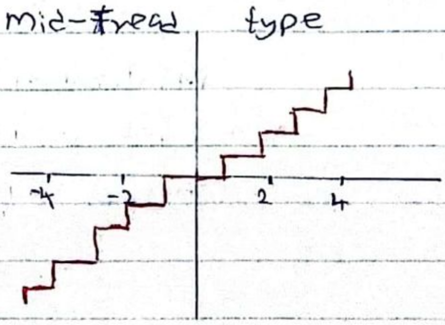
Non-Uniform quantization — Quantization levels are unequal

Types of uniform quantization

Mid-Rise type



Mid-tread type



Origin lies in the middle of a raising part of the stair-case like graph

Origin lies in the middle of a tread of the stair-case like graph

Quantization levels → even in number

Quantization levels → odd in number

mid-rise and mid-tread type of uniform quantizers are symmetric

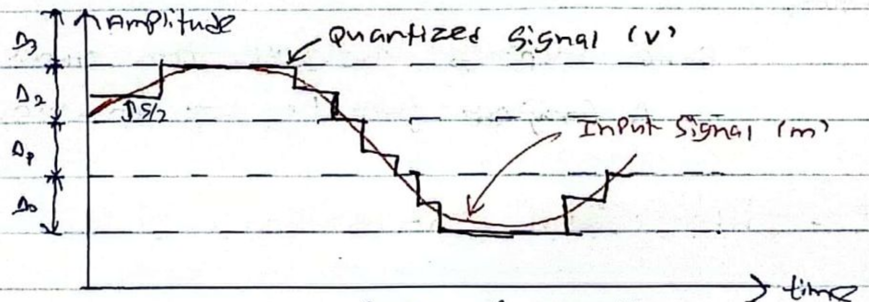
Quantization Error

The processing of the system results in an error, which is the difference of those values.

Difference between an input value and its quantized value is called a **quantization Error**

Quantization Noise

- A type of quantization error, which usually occurs in analog audio signal, while quantizing it to digital.
- For example, in music, the signals keep changing continuously, where a regularity is not found in errors.
- Such errors create a wide band noise called as quantization Noise.



Process of quantization

$\Delta_0 = \Delta_1 = \Delta_2 = \Delta_3 = S$

S = step size

Quantized signal (v) is an approximation of (m)

$$\boxed{S = \frac{V_{RP}}{L}}$$

The difference between (m) & (v) is called Quantization error or quantization noise $\boxed{\varepsilon = V - m}$ (Shaded portion)

So, $\varepsilon \downarrow \approx S \downarrow \approx L \uparrow$

maximum Error $\boxed{\varepsilon_{max} = \pm \frac{S}{2}}$ where S = step size

Note: Each quantization Level (L) $\xrightarrow[\text{into}]{\text{convert}}$ Unique N bit digital word

$$\boxed{\text{No of } L = 2^N}$$

If $N=4$ i.e 4 bits/word

Then no. of quantization level $L = 2^4 = 16$

Encoding

The output of the quantizer is one of M possible signal levels.
- If we want to use a binary transmission system, then we need to map each quantized sample into an n bit binary word.

$$M=2, \quad N = \log_2(M)$$

- Encoding is the process of representing each quantized sample by an v bit code word.
- The mapping is one-to-one so there is no distortion introduced by encoding.
- ~~Some mappings are better than others.~~

~~A Gray code gives the best end-to-end performance.~~

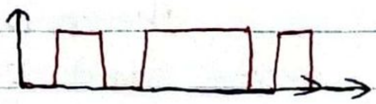
What is Line Coding?

- A line code is the code used for data transmission of a digital signal over a transmission line.
- This process of coding is chosen so as to avoid overlap and distortion of signal such as inter-symbol interference.
- Mapping of binary information sequence that enters the channel.
  Ex. "1" maps to +A pulse, "0" to -A pulse

Information: Fundamentally discrete in nature
Transmitted over-band limited channel: Signal gets dispersed
Causes: overlap and distortion
Distortion: Intersymbol Interference (ISI)

Introduction to line coding

Binary data: 1 and 0
Line coding: A group of binary data to represent symbols

0 1 0 1 1 1 0 1 → Line coding → 

- There are 2 major categories: return-to-zero (RZ) and nonreturn-to-zero (NRZ).
- With RZ coding, the waveform returns to a zero-volt level for a portion (usually one-half) of the bit interval.

Why Line Coding?

Line code selected to meet system requirements:
Transmitted power: Power consumption
Bit timing: Transitions in signal help timing recovery
Bandwidth efficiency: Reduce excessive transitions wastes
Low frequency content: Some channels block low frequencies
long periods of +A or of -A causes signal to "droop"
waveform should not have low-frequency content

Error detection: Ability to detect errors.

Complexity / Cost : Code implementable in chip at high speed

Properties of Line Coding

Transmission Bandwidth: more bits transmit on a single signal

power efficiency: given BW and probability of error

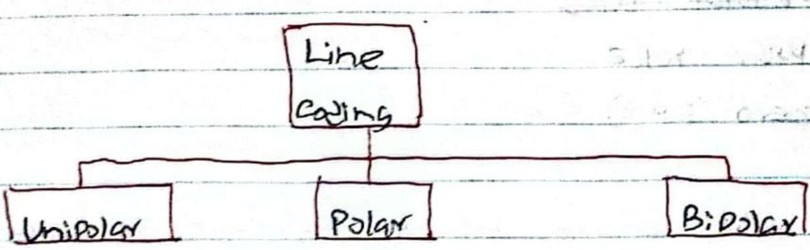
Error Detection and Correction capability:

Favorable power spectral density: dc = 0

Adequate timing content: Extract timing from pulses

Transparency: prevent of 0s or 1s.

Types of Line Coding

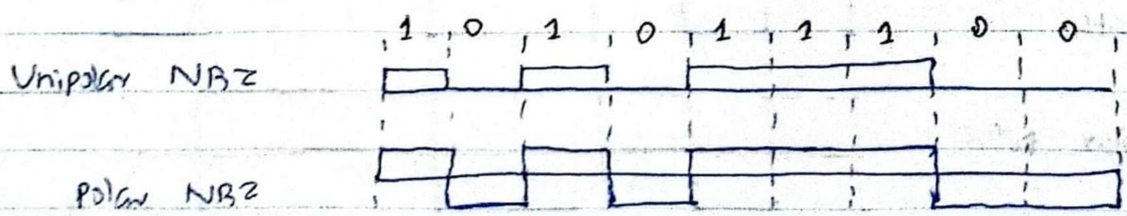


Signal Types

1. Non-Return to zero (NRZ)
2. Return to zero (RZ)

Unipolar & Polar

Non-Return-to-zero (NRZ)



Unipolar NRZ (Single voltage)
(on-off keying i.e. OOK)

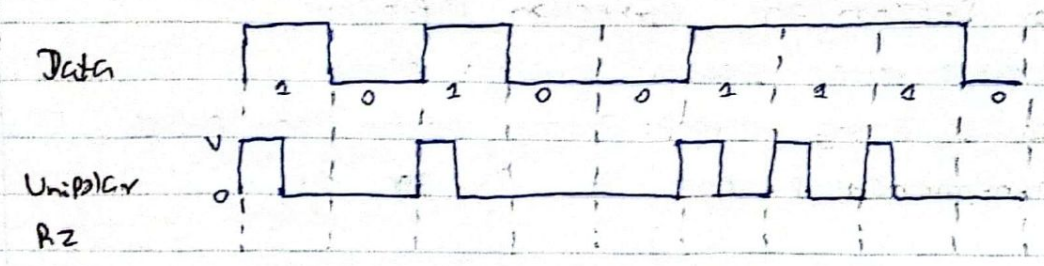
- "1" maps to +A pulse
- "0" maps to no pulse
- Long strings of A or 0
  - poor timing
  - Low-frequency content
- Simple

Polar NRZ (Two voltage)

- "1" maps to +A/2 pulse
- "0" maps to −A/2 pulse
- Long strings of +A/2 or −A/2
  - Poor timing
  - Low-frequency content
- Simple

Unipolar Return to zero (RZ):

- A High in data, though represented by a Mark Pulse, its duration T_0 is less than the symbol bit duration.
- Half of the bit duration remains high but it immediately returns to zero and shows the absence of Pulse during the remaining half of the bit duration.



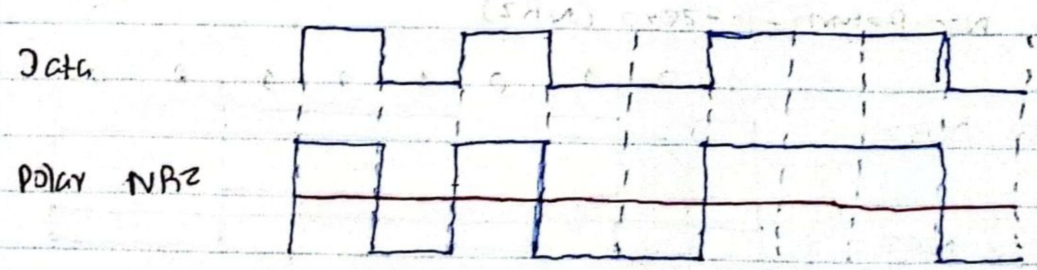
~~Unipolar NRZ~~

~~Unipol~~ Polar NRZ

Non-Return-to-zero (NRZ):

In polar NRZ encoding, we use two levels of voltage amplitude:

- In this type of polar signaling, a High in data is represented by a positive pulse, while a Low in data is represented by a negative pulse.
- If there is no change, the bit is 0; if there is a change, the bit is 1.



Polar RZ

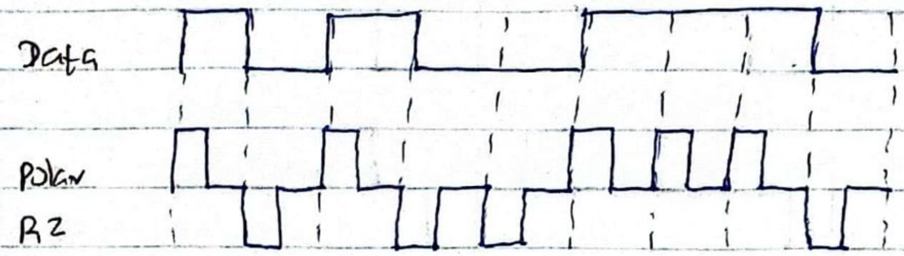
Return-to-zero (RZ)

• The main problem ~~of~~ with NRZ encoding occurs when the sender and receiver clocks are not synchronized. The receiver does not know when one bit has ended and the next bit is starting.

• One solution is the return-to-zero (RZ) scheme, which uses three values: positive, negative and zero.

• In RZ, the signal changes not between bits but during the bit. In the following figure, we see that the signal goes to 0 in the middle of each bit.

• It remains there until the beginnings of the next bit, proper synchronization.



Bipolar code

• In bipolar encoding (something called multilevel binary), there are three voltage levels (1, 0, -1),

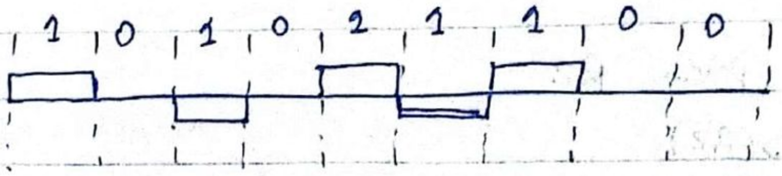
- positive
- negative and
- zero

• Even in this method, we have two types

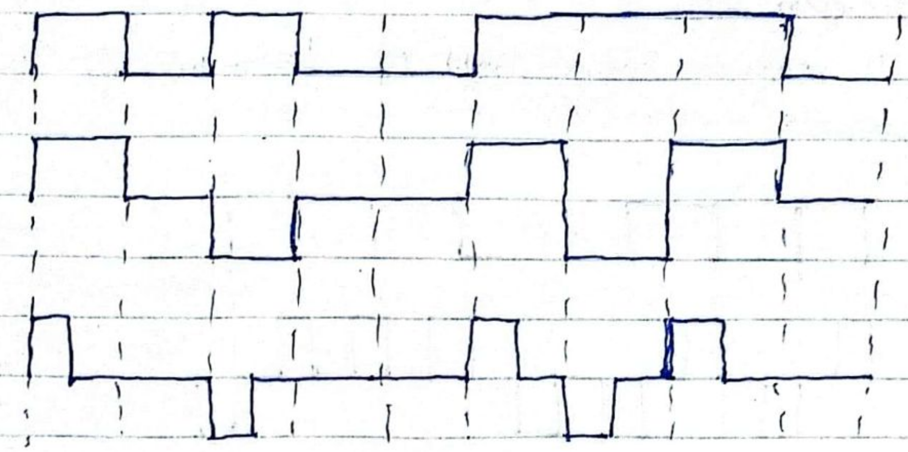
Bipolar NRZ

Bipolar RZ

• The voltage level for one data element is at zero, while the voltage level for the other element alternates between positive and negative.



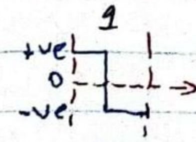
- Three signal levels: {-A, 0, +A}
- "1" maps to +A or -A in alternation
- "0" maps to no pulse
- - Every +pulse matched by -pulse so little content at low frequencies.
- ~~Strin~~ String of 1s produces a square wave



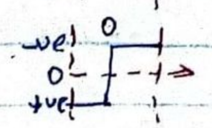
Split Phase Manchester

- In Manchester encoding, the duration of the bit is divided into two halves.
- The voltage remains at one level during the first half and moves to the other level during the second half.

A 'one' is +ve in 1st half and -ve in 2nd half



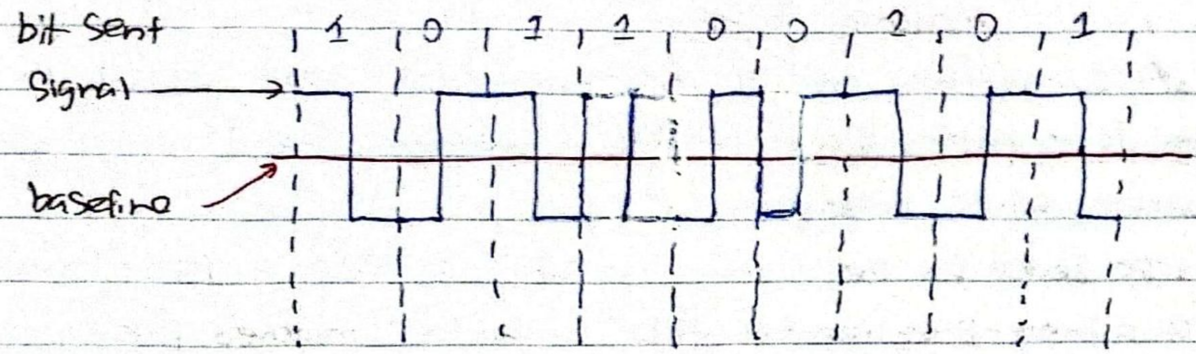
A 'zero' is -ve in 1st half and +ve in 2nd half



This has transition in middle of each bit period

This is used by IEEE 802.3 (Ethernet) LAN standard

Manchester Encoding



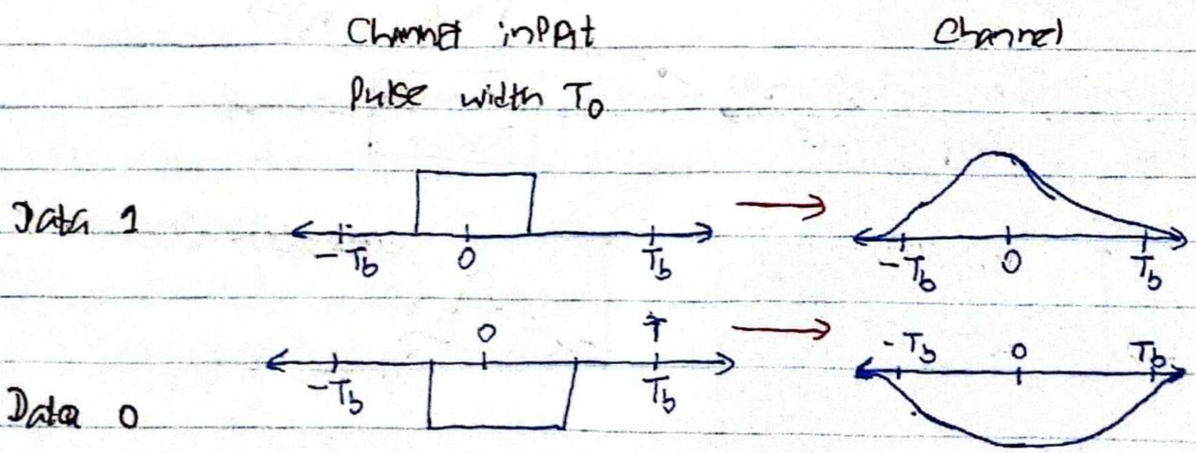
Inter Symbol Interference

- Transmission of digital data (bit stream) over a noisy baseband channel typically suffers two channel imperfections
  - Intersymbol interference (ISI)
  - Background noise (eg AWGN)

- These two interferences/noises often occur simultaneously
- However, for simplicity, they are often seperately considered in analysis

Intersymbol interference (ISI) occurs when a pulse spreads out in such a way that it interferes with adjacent at the sample instant

Example: assume polar NRZ line code. The channel outputs are shown as "smeared" (width T_b becomes $2T_b$) pulses (spreading due to bandlimited channel)



ISI - Inter Symbol Interference

• This is a form of distortion of a signal, in which one or more Symbols interfere with subsequent signals, causing ~~poor~~ noise or delivering a poor input.

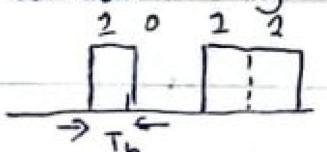
The main causes of ISI are

Multi path propagation

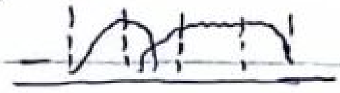
Non-linear frequency in channel

• The Problem of ISI exists Strongly in Telephone channels like Coaxial cables and optical fibers.

• The main objective is to study the effect of ISI, when digital data is transmitted through band limited channel to solution to overcome the degradation of waveform by properly shaping pulse.



Transmitted waveform



Pulse Dispersion

• In base band transmission best way is to map digits or symbols into pulse waveform. This waveform is generally termed as Line codes.

• To proceed with a mathematical study of intersymbol interference consider a baseband binary PAM system, a generic form of which is depicted in slide

• The term "baseband" → an information bearing signal whose spectrum extends from (or near)

• The pulse-amplitude modulator changes the input binary data stream $\{b_k\}$ into a new sequence of short pulses, short enough to approximate impulses.

• More specifically, the pulse amplitude a_k is represented in the polar form:

$$a_k = \begin{cases} +1 & \text{if } b_k \text{ is symbol } 1 \\ -1 & \text{if } b_k \text{ is symbol } 0 \end{cases}$$

• The sequence of short pulses, so produced is applied to a transmit filter whose impulse response is denoted by $g(t)$. The transmitted signal is thus defined by the sequence

$$s(t) = \sum_k a_k g(t - kT_b)$$

• ~~A binary~~ A binary data stream represented by the sequence $\{a_k\}$, where $a_k = +1$ for symbol 1 and $a_k = -1$ for symbol 0, modulates the basis pulse $g(t)$ and superposes linearly to form the transmitted signal $s(t)$.

• The signal $s(t)$ is naturally modified as a result of transmission through the channel whose impulse response is denoted by $h(t)$.

• The noisy received signal $x(t)$ is passed through a receive filter of impulse response $c(t)$.

• The resulting filter output $y(t)$ is sampled synchronously with the transmitter, with the sampling instants being ~~transmitted~~ determined by a clock or timing signal that is usually extracted from the receive-filter output.

- Finally, the sequence of samples thus obtained is used to reconstruct the original data sequence by means for a decision device.

- Specifically, the amplitude of each sample is compared with a zero threshold, assuming that the symbols 1 and 0 are equiprobable.
  - If the zero threshold is exceeded, a decision is made in favor of symbol 1; otherwise a decision is made in favor of symbol 0.
  - If the sample amplitude equals the zero threshold exactly, the receiver simply makes a random guess.

We may express the receive filter output as

$$y(t) = \sum_k a_k p(t - kT_b)$$

- Where the pulse $p(t)$ is to be defined.
- An arbitrary time delay t_0 should be included in the argument of the pulse $p(t-kT_b)$ to represent the effect of transmission delay through the system.

- We have put this delay equal to zero without loss of generality, then the channel noise is ignored.
- The scaled pulse $p(t)$ is obtained by a double convolution involving the impulse response $g(t)$ of the transmit filter, the impulse response $h(t)$ of the channel, and the impulse response $c(t)$ of the receive filter, as shown by

$$p(t) = g(t) * h(t) * c(t)$$

$* \rightarrow$ convolution

- The pulse $p(t)$ is normalized by setting which justifies the use of a scaling factor ~~to~~ with ~~all~~ amplitude changes ~~incu~~ with the course of signal transmission through the system.
- Convolution in the time domain $\rightarrow$ multiplication in the frequency domain

- Use Fourier transform to change into the equivalent form

$$P(f) = G(f)\,H(f)\,C(f)$$

- $P(f)$, $G(f)$, $H(f)$ and $C(f)$ are the Fourier transforms of $p(t)$, $g(t)$, $h(t)$, and $c(t)$, respectively

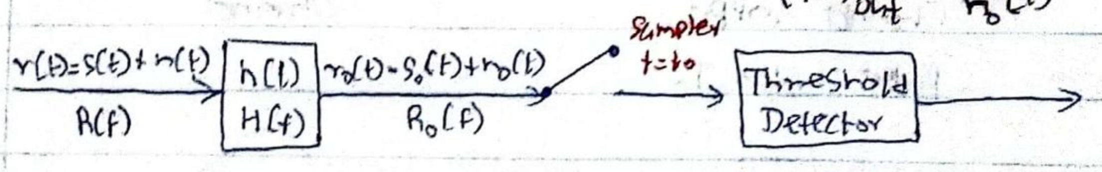
Matched Filter

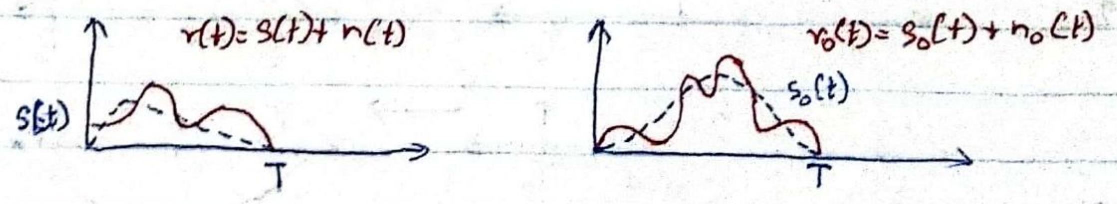
- A fundamental result in communication theory deals with the detection of a pulse signal of known waveform that is immersed in additive white noise.
- The device for the optimum detection of such a pulse involves the use of a linear-time-invarient filter known as matched filter.
- It is called a matched filter because its impulse response is matched to the pulse signal.

- A matched filter is a filter used in communications to "match" a particular transit waveform
- It passes all the signal frequency components while suppressing any frequency components where there is only noise and allows to pass the maximum amount of signal power.
- The purpose of the matched filter is to maximize the signal to noise ratio at the sampling point of a bit stream and to minimize the probability of undetected errors received from a signal.
- To achieve the maximum SNR, we want to allow through all the signal frequency components, but to emphasize more on signal frequency components that are large and so contribute more to improving the overall SNR.

The goal is maximize $(S/N)_{out}$

$$\left(\frac{S}{N}\right)_{out} = \frac{s_o^2(t)}{n_o^2(t)}$$





Design criterion

To find $h(t)$ such that the output signal-to-noise ratio SNR_0 is maximized

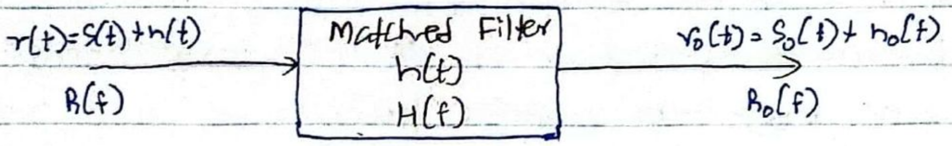
$$x(t) = g(t) + w(t) \quad \text{for} \quad 0 \leq t < T$$

$$\begin{aligned} y(t) &= x(t) * h(t) \\ &= [g(t) + w(t)] * h(t) \\ &= g(t) * h(t) + w(t) * h(t) \\ &= g_0(t) + n(t) \end{aligned}$$

$$SNR_0 = \frac{|g_0(T)|^2}{E[n^2(T)]}$$

The Matched Filter is the linear filter that maximizes:

$$\left(\frac{S}{N}\right)_{out} = \frac{S_0^2(t)}{n_0^2(t)}$$



Recall

$$y(t) = h(t) * x(t) \Leftrightarrow Y(f) = H(f)X(f)$$

$$S_y(f) = |H(f)|^2 S_x(f)$$

Where $|g_0(T)|^2$ is the instantaneous power in the output signal, E is the statistical expectation operator, and $E[n^2(t)]$ is a measure of the average output noise power.

The requirement is to specify the impulse response $h(t)$ of the filter such that the output signal-to-noise ratio is maximized

Signal Power

~~Let g(f) and H(f) denoted the Fourier~~ Transform of $g(t)$ and ~~h(t)~~

$$g_0(t) = \int_{-\infty}^{\infty} H(f) G(f) \exp(j2\pi f t)\, df$$

Signal Power

Let $G(f)$ denote the Fourier transform of the known signal $g(t)$, and $H(f)$ denote the frequency response of the filter. Then the Fourier transform of the output signal $g_0(t)$ is equal to $H(f)G(f)$, and $g_0(t)$ is itself given by the inverse Fourier transform:

$$g_0(t) = \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi ft)\,df$$

Hence, when the filter output is sampled at time $t = T$, the signal power will be:

$$\text{Signal power} = |g_0(T)|^2 = \left|\int_{-\infty}^{\infty} H(f)\,G(f)\exp(j2\pi fT)\,df\right|^2$$

Noise Power

- Consider next the effect on the filter output due to the noise $w(t)$ acting alone.
- The power spectral density $S_N(f)$ of the output noise $n(t)$ is equal to the power spectral density of the input noise $w(t)$ times the squared magnitude response $|H(f)|^2$
- Since $w(t)$ is white with constant power spectral density $N_0/2$, it follows that:

$$S_N(f) = \frac{N_0}{2}|H(f)|^2$$

- The average power of the output noise $n(t)$ is therefore

$$\text{The noise power} = E[n^2(t)] = \int_{-\infty}^{\infty} S_N(f)\,df$$

$$= \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\,df$$

S/N Ratio

- Thus the signal to noise ratio become

$$\eta = \frac{\left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)\,df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2\,df} \quad \ldots\ldots (1)$$

For a given $G(f)$, ~~what~~ the particular form of the frequency response $H(f)$ of the filter makes η a maximum.

To find the solution to this optimization problem, we apply a mathematical result known as Schwarz's inequality to the ~~numerat~~ numerator of Equation

Schwarz's inequality

If we have two complex ~~funct~~ functions $\phi_1(x)$ and $\phi_2(x)$ in the real variable x, satisfying the conditions:

$$\int_{-\infty}^{\infty} |\phi_1(x)|^2 dx < \infty \quad \text{AND} \quad \int_{-\infty}^{\infty} |\phi_2(x)|^2 dx < \infty$$

Then

$$\left|\int_{-\infty}^{\infty} \phi_1(x)\phi_2(x)\, dx\right|^2 \leq \int_{-\infty}^{\infty} |\phi_1(x)|^2 dx \int_{-\infty}^{\infty} |\phi_2(x)|^2 dx$$

This equality holds, if and only if, we have $\phi_1(x) = k\phi_2^*(x)$ where k is an arbitrary constant, and * denotes complex conjugation

Returning to the problem at hand, we readily see that by invoking Schwarz's inequality ~~Eq~~ and setting $\phi_1(x) = H(f)$ and $\phi_2(x) = G(f)\exp(j2\pi fT)$

Applying the Schwarz's inequality to the numerator of equation(1), we have

$$\left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)\, df\right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |G(f)|^2 df \quad \ldots\ldots (2)$$

(Note: $|e^{j2\pi fT}| = 1$)

Thus, the peak pulse signal-to-noise ratio is

Subtituting (2) into (1)

$\eta =$

The S/N ratio $\eta \leq \frac{2}{N}\int_{-\infty}^{\infty} |G(f)|^2 df$

Or

$$\eta \leq \frac{2E}{N_0}$$

where the energy $E = \int_{-\infty}^{\infty} |G(f)|^2 df$ is the input signal energy

The right-hand side of this relation does not depend on the frequency response $H(f)$ of the filter but only on the signal energy and the noise power spectral density.

Consequently, the peak pulse signal-to-noise ratio η will be a maximum when $H(f)$ is chosen so that the equality holds: that is,

$$H(f) = kG^*(f)\exp(-j2\pi fT)$$

where $G^*(f)$ is the complex conjugate of the Fourier transform of the input signal $g(t)$ and k is a scaling factor of appropriate dimensions. This relation states that, except for the factor $k\exp(-j2\pi fT)$, the frequency response of the optimum filter is the same as the complex conjugate of the Fourier transform of the input signal.

The above equation specifies the optimum filter in the frequency domain. To characterize it in the time domain, we take the inverse Fourier transform of $H(f)$ to obtain the impulse response of the optimum filter as

$$h_o(t) = k\int_{-\infty}^{\infty} G^*(f)\exp[-2\pi f(T-t)]\,df$$

Since for a real signal $g(t)$ we have $G^*(f) = G(-f)$, we may rewrite equation as

$$h(t) = k\int_{-\infty}^{\infty} G(-f)\exp[-j2\pi f(T-t)]\,df$$

$$h(t) = kg(T-t)$$

The impulse response of the filter is the time-reversed and delayed version of the input signal $g(t)$. "Matched with the input signal"

Thus, the peak pulse signal-to-noise ratio will be:

$$\eta_{max} = \frac{2}{N_0}\int_{-\infty}^{\infty} |G(f)|^2\,df$$

According to Rayleigh's energy theorem, states that the integral of the squared magnitude spectrum of a pulse signal with respect to frequency is equal to the signal energy E

$$E = \int_{-\infty}^{\infty} g^2(t)\,dt = \int_{-\infty}^{\infty} |G(f)|^2\,df \quad \cancel{= k^2 N_0 E/2}$$

$$g_o(T) = kE$$

~~Hence~~

$$\cancel{g_0(T) = kE}$$

~~Substituting Equation~~

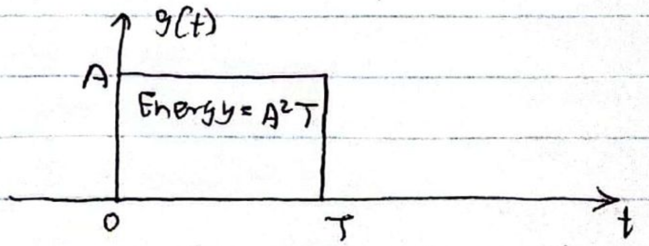
Therefore,

$$\eta_{max} = \frac{2E}{N_0} = \frac{(kE)^2}{(k^2 N_0 E/2)}$$

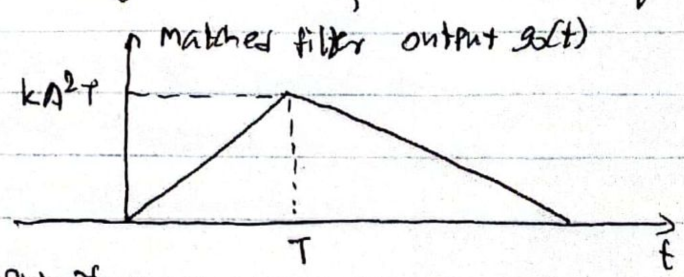
Thus, the peak pulse signal-to-noise ratio of a matched filter depends only on the ratio of the signal energy to the power spectral density of the white noise at the filter input.

Matched Filter for Rectangular Pulse

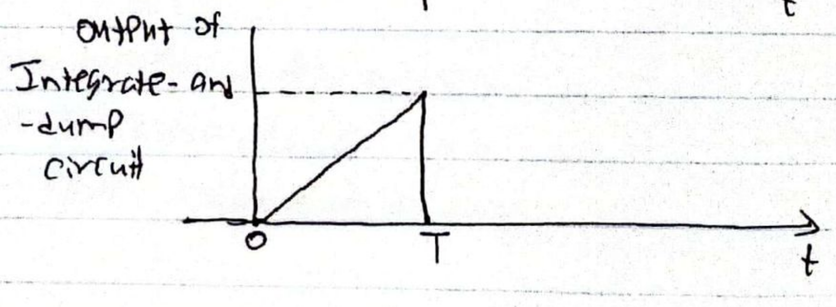
- Consider a signal $g(t)$ in the form of a rectangular pulse of amplitude A and duration T, as shown in Figure. In this example, the impulse response $h(t)$ of the matched filter has exactly the same waveform as the signal itself. The output signal $g_0(t)$ of the matched filter produced in response to the input signal $g(t)$ has a triangular waveform, as shown in Figure.
- The maximum value of the input signal $g_0(t)$ is equal to kA^2T, which is the energy of the input signal $g(t)$ scaled by the factor k; this maximum value occurs at $t=T$, as indicated.



Rectangular Pulse

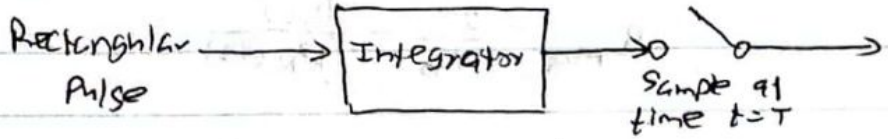


Matched Filter output



Integrator output

• For the special case of a rectangular pulse, the matched filter may be implemented using a circuit shown as the integrate-and-dump circuit, a block diagram of which is shown in Figure.



Integrate-and-dump circuit

• The integrator computes the area under the rectangular pulse, and the resulting output is then sampled at time $t=T$, where T is the duration of the pulse. Immediately after $t=T$, the integrator is restored to its initial condition; hence the name of the circuit.

• Integrator output shows the output waveform of the integrate-and-dump circuit for the rectangular pulse of Figure rectangular pulse.

• We see that for $0 \le t \le T$, the output of this circuit has the same waveform as that appearing at the output of the matched filter. The difference in the notations used to describe their peak values is of no practical significance.