General Sir John Kotelawala Defence University
Faculty of Engineering
Department of Mathematics

Mathematics - MA 1103

Tutorial 05 - Complex Numbers

Year: 2021   Intake: 38 - 03$^{\text{rd}}$ Batch   Semester: 01

Learning Outcomes Covered: LO3

Name of the Instructor Prepared: Ashani AG

---

(01) Express the following in the form $x + iy,$ with $x, y \in \mathbb{R}$.

(a) $\dfrac{3}{1+i}$

(b) $\dfrac{4i}{(1+2i)^2}$

(c) $(2-3i)(-2+i)$

(d) $\left(\sqrt{2}-i\right) - i\left(1 - \sqrt{2}i\right)$

(e) $\dfrac{i}{1-i} + \dfrac{1-i}{i}$

(f) $(2+i)^4 + (2-i)^4$

(g) $\dfrac{1+i}{1-i} - (1+2i)(2+2i) + \dfrac{3-i}{1+i}$

(h) $2i(i-1) + \left(\sqrt{3}+i\right)^3 + (1+i)\overline{(1+i)}$

(02) Express the following in Polar form ($[r, \theta]$).

(a) $\left(\dfrac{3-i}{2+i}\right)^2$

(b) $\dfrac{(1-i)}{(1+i)\left(1+\sqrt{3}i\right)}$

(c) $\dfrac{1-i}{1+i} - \dfrac{1+i}{1-i}$

(d) $\dfrac{1 - (1-i)^2}{1+2i}$

(03) Use the Exponential form to express the following in the form $x + iy,$ with $x, y \in \mathbb{R}$.

(a) $(1+i)^6$

(b) $(3+3i)^8$

(c) $\left(\sqrt{3}+i\right)^{50}$

(d) $\left(1+\sqrt{3}i\right)^{2011}$

(e) $\left(\dfrac{i+1}{\sqrt{2}}\right)^{1337}$

(04)   (a) Prove the following identities.

   i. $\cos 3\theta = \cos^3 \theta - 3\cos\theta \sin^2 \theta$.

   ii. $\dfrac{\cos 2\theta + i\sin 2\theta}{\cos 3\theta + i\sin 3\theta} = \cos\theta - i\sin\theta$

(b) Find real θ such that $\quad \dfrac{3 + 2i\sin\theta}{1 - 2i\sin\theta}\quad$ is

   i. real

   ii. imaginary

(05) (a) Solve the equations below:

   i. $z^3 = 27$

   ii. $z^3 = 8i$

   iii. $z^2 = 5 - 12i$

   iv. $z^5 = 4 - 4i$

   v. $(z - 1)^4 = -1$

   vi. $z^2 - i\,(z - 2) = (z - 2)$

(b) The following cubic equation is given

$$z^3 + pz^2 + 6z + q = 0$$

where, $p, q \in \mathbb{R}$.

One of the three solutions of the above cubic equation is $\quad 5 - i$.

   i. Find the other two solutions of the equation.

   ii. Determine the values of p, and q.

(06) The point P represents the complex number z on the Argand diagram. Sketch the locus of z which satisfy the equations below:

(a)   i. $\mathrm{Im}(z) = -1$

    ii. $|z| = |z - 4|$

    iii. $|z - 1| = |z + i|$

    iv. $|z| + |z - 4| = 6$

(b)   i. $\arg z = \dfrac{5\pi}{6}$

    ii. $\arg\,(z - 2 + 3i) = \dfrac{-\pi}{4}$