Digital Systems and Designs

Tutorial 1

1. Consider the discrete time sequence

$$x(n) = \cos\left(\frac{n\pi}{8}\right)$$

Find two different continuous-time signals that would produce this sequence when sampled at a frequency of $f_s = 10Hz.$

2. If the Nyquist rate for $x_a(t)$ is Ω_s, what is the Nyquist rate for each of the following signals that are derived from $x_a(t)$?
   a. $\frac{dx_a(t)}{dt}$
   b. $x_a(2t)$
   c. $x_a^2(t)$
   d. $x_a(t)\cos(\Omega_0 t)$

3. A continuous-time signal $x_a(t)$ is known to be uniquely recoverable from its samples $x_a(nT_s)$ when T_s= 1 ms. What is the highest frequency in $X_a(f)$?

4.

A continuous-time signal $x_a(t)$ is bandlimited with $X_a(j\Omega) = 0$ for $|\Omega| > \Omega_0$. If $x_a(t)$ is sampled with a sampling frequency $\Omega_s \geq 2\Omega_0$, how is the energy in $x(n)$,

$$E_d = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

related to the energy in $x_a(t)$,

$$E_a = \int_{-\infty}^{\infty} |x_a(t)|^2\, dt$$

and the sampling period T_s?

5.

Given a real-valued bandpass signal $x_a(t)$ with $X_a(f) = 0$ for $|f| < f_1$ and $|f| > f_2$, the Nyquist sampling theorem says that the minimum sampling frequency is $f_s = 2f_2$. However, in some cases, the signal may be sampled at a lower rate.

(*a*) Suppose that $f_1 = 8$ kHz and $f_2 = 10$ kHz. Make a sketch of the discrete-time Fourier transform of $x(n) = x_a(nT_s)$ if $f_s = 1/T_s = 4$ kHz.

(*b*) Define the bandwidth of the bandpass signal to be

$$B = f_2 - f_1$$

and the center frequency to be

$$f_c = \frac{f_2 + f_1}{2}$$

Show that if $f_c > B/2$ and f_2 is an integer multiple of the bandwidth B, no aliasing will occur if $x_a(t)$ is sampled at a sampling frequency $f_s = 2B$.

(*c*) Repeat part (*b*) for the case in which f_2 is not an integer multiple of the bandwidth B.

nyquist rate = $\Omega_s = \frac{2\pi}{T_s} > 2\Omega$

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6.

Determine the minimum sampling frequency for each of the following bandpass signals:

(a) $x_a(t)$ is real with $X_a(f)$ nonzero only for 9 kHz $< |f| <$ 12 kHz.

(b) $x_a(t)$ is real with $X_a(f)$ nonzero only for 18 kHz $< |f| <$ 22 kHz.

(c) $x_a(t)$ is complex with $X_a(f)$ nonzero only for 30 kHz $< f <$ 35 kHz.

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