2: ELECTRIC AND MAGNETIC FIELDS

Steven W. Ellingson
Virginia Polytechnic Institute and State University

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CHAPTER OVERVIEW

2: Electric and Magnetic Fields

2.1: What is a Field?

The quantity that the field describes may be a scalar or a vector, and the scalar part may be either real- or complex-valued.

Definition: Field

A field is the continuum of values of a quantity as a function of position and time.

In electromagnetics, the electric field intensity $\mathbf{E}$ is a real-valued vector field that may vary as a function of position and time, and so might be indicated as "$\mathbf{E}(x, y, z, t)$," "$\mathbf{E}(\mathbf{r}, t)$," or simply "$\mathbf{E}$." When expressed as a phasor, this quantity is complex-valued but exhibits no time dependence, so we might say instead "$\widetilde{\mathbf{E}}(\mathbf{r})$" or simply "$\widetilde{\mathbf{E}}$."

An example of a scalar field in electromagnetics is the electric potential, V; i.e., $\mathrm{V}(\mathbf{r}, \mathrm{t})$.

A wave is a time-varying field that continues to exist in the absence of the source that created it and is therefore able to transport energy.

2.2: Electric Field Intensity

Electric field intensity is a vector field we assign the symbol $\mathbf{E}$ and has units of electrical potential per distance; in SI units, volts per meter (V/m). Before offering a formal definition, it is useful to consider the broader concept of the electric field.

Imagine that the universe is empty except for a single particle of positive charge. Next, imagine that a second positively-charged particle appears; the situation is now as shown in Figure $2.2.1$. Since like charges repel, the second particle will be repelled by the first particle and vice versa. Specifically, the first particle is *exerting force* on the second particle. If the second particle is free to move, it will do so; this is an expression of kinetic energy. If the second particle is somehow held in place, we say the second particle possesses an equal amount of potential energy. This potential energy is no less "real," since we can convert it to kinetic energy simply by releasing the particle, thereby allowing it to move.

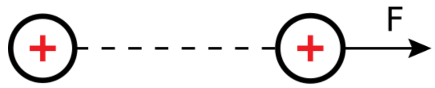

Figure $2.2.1$: A positively-charged particle experiences a repulsive force $\mathbf{F}$ in the presence of another particle which is also positively-charged. (CC BY SA 4.0; M. Goldammer).

Now let us revisit the original one particle scenario. In that scenario, we could make a map in which every position in space is assigned a vector that describes the force that a particle having a specified charge q would experience if it were to appear there. The result looks something like Figure $2.2.2$. This map of force vectors is essentially a description of the electric field associated with the first particle.

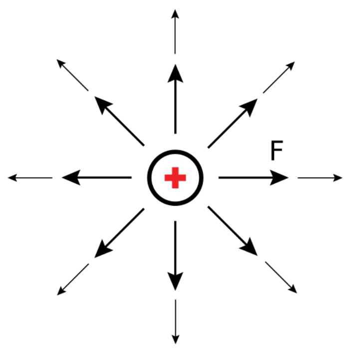

Figure $2.2.2$: A map of the force that would be experienced by a second particle having a positive charge. Here, the magnitude and direction of the force is indicated by the size and direction of the arrow. (CC BY SA 4.0; M. Goldammer).

There are many ways in which the electric field may be quantified. Electric field intensity $\mathbf{E}$ is simply one of these ways. We define $\mathbf{E}(\mathbf{r})$ to be the force $\mathbf{F}(\mathbf{r})$ experienced by a test particle having charge q, divided by q; i.e.,

$$\mathbf{E}(\mathbf{r}) \triangleq \lim_{q \to 0} \frac{\mathbf{F}(\mathbf{r})}{q} \tag{2.2.1}$$

Note that it is required for the charge to become vanishingly small (as indicated by taking the limit) in order for this definition to work. This is because the source of the electric field is charge, so the test particle contributes to the total electric field. To accurately measure the field of interest, the test charge must be small enough not to significantly perturb the field. This makes Equation 2.2.1 awkward from an engineering perspective, and we'll address that later in this section.

According the definition of Equation 2.2.1, the units of $\mathbf{E}$ are those of force divided by charge. The SI units for force and charge are the newton (N) and coulomb (C) respectively, so $\mathbf{E}$ has units of N/C. However, we typically express $\mathbf{E}$ in units of V/m, not N/C. What's going on? The short answer is that 1 V/m = 1 N/C:

$$\frac{\mathrm{N}}{\mathrm{C}} = \frac{\mathrm{N}\cdot\mathrm{m}}{\mathrm{C}\cdot\mathrm{m}} = \frac{\mathrm{J}}{\mathrm{C}\cdot\mathrm{m}} = \frac{\mathrm{V}}{\mathrm{m}}$$

where we have used the fact that 1 N·m = 1 joule (J) of energy and 1 J/C = 1 V.

Electric field intensity is a Vector Field

Electric field intensity ($\mathbf{E}$, N/C or V/m) is a vector field that quantifies the force experienced by a charged particle due to the influence of charge not associated with that particle.

The analysis of units doesn't do much to answer the question of why we should prefer to express $\mathbf{E}$ in V/m as opposed to N/C. Let us now tackle that question.

Figure $2.2.3$ shows a simple thought experiment that demonstrates the concept of electric field intensity in terms of an electric circuit. This circuit consists of a parallel-plate capacitor in series with a 9 V battery. The effect of the battery, connected as shown, is to force an accumulation of positive charge on the upper plate, and an accumulation of negative charge on the lower plate. If we consider the path from the position labeled "A," along the wire and through the battery to the position labeled "B," the change in electric potential is +9 V. It must also be true that the change in electric potential as we travel from B to A through the capacitor is −9 V, since the sum of voltages over any closed loop in a circuit is zero. Said differently, the change in electric potential between the plates of the capacitor, starting from node A and ending at node B, is +9 V. Now, note that the spacing between the plates in the capacitor is 1 mm. Thus, the rate of change of the potential between the plates is 9 V divided by 1 mm, which is 9000 V/m. This is literally the electric field intensity between the plates. That is, if one places a particle with an infinitesimally-small charge between the plates (point "C"), and then measures the ratio of force to charge, one finds it is 9000 N/C pointing toward A. We come to the following remarkable conclusion:

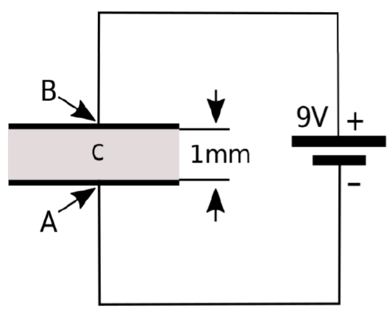

Figure $2.2.3$: A simple circuit used to describe the concept of electric field intensity. In this example, E at point C is 9000 V/m directed from B toward A. (CC BY 3.0; Y. Qin).

$\mathbf{E}$ points in the direction in which electric potential is most rapidly decreasing, and the magnitude of $\mathbf{E}$ is the rate of change in electric potential with distance in this direction.

The reader may have noticed that we have defined the electric field in terms of what it *does*. We have have not directly addressed the question of what the electric field *is*. This is the best we can do using classical physics, and fortunately, this is completely adequate for the most engineering applications. However, a deeper understanding is possible using quantum mechanics, where we find that the electric field and the magnetic field are in fact manifestations of the same fundamental force, aptly named the *electromagnetic force*. (In fact, the electromagnetic force is found to be one of just four fundamental forces, the others being *gravity*, the *strong nuclear force*, and the *weak nuclear force*.) Quantum mechanics also facilitates greater insight into the nature of electric charge and of the photon, which is the fundamental constituent of electromagnetic waves. For more information on this topic, an excellent starting point is the video "Quantum Invariance & The Origin of The Standard Model" referenced at the end of this section.

2.3: Permittivity

Permittivity describes the effect of material in determining the electric field in response to electric charge. For example, one can observe from laboratory experiments that a particle having charge q gives rise to the electric field

$$\mathbf{E} = \hat{\mathbf{R}}\, q\, \frac{1}{4\pi R^2}\, \frac{1}{\epsilon}$$

where R is distance from the charge, $\hat{\mathbf{R}}$ is a unit vector pointing away from the charge, and ϵ is a constant that depends on the material. Note that $\mathbf{E}$ increases with q, which makes sense since electric charge is the source of $\mathbf{E}$. Also note that $\mathbf{E}$ is inversely proportional to $4\pi R^2$, indicating that $\mathbf{E}$ decreases in proportion to the area of a sphere surrounding the charge – a principle commonly known as the *inverse square law*. The remaining factor $1/\epsilon$ is the constant of proportionality, which captures the effect of material. Given units of V/m for $\mathbf{E}$ and C for Q, we find that ϵ must have units of farads per meter (F/m). (To see this, note that 1 F $=$ 1 C/V.)

Permittivity

Permittivity (ϵ, F/m) describes the effect of material in determining the electric field intensity in response to charge.

In free space (that is, a perfect vacuum), we find that $\epsilon = \epsilon_0$ where:

$$\epsilon_0 \cong 8.854 \times 10^{-12} \text{ F/m}$$

The permittivity of air is only slightly greater, and usually can be assumed to be equal to that of free space. In most other materials, the permittivity is significantly greater; that is, the same charge results in a weaker electric field intensity.

It is common practice to describe the permittivity of materials relative to the permittivity of free space. This *relative permittivity* is given by:

$$\epsilon_r \triangleq \frac{\epsilon}{\epsilon_0}$$

Values of ϵ_r for a few representative materials is given in Appendix A1. Note that ϵ_r ranges from 1 (corresponding to a perfect vacuum) to about 60 or so in common engineering applications. Also note that relative permittivity is sometimes referred to as *dielectric constant*. This term is a bit misleading, however, since permittivity is a meaningful concept for many materials that are not dielectrics.

2.4: Electric Flux Density

Electric flux density, assigned the symbol $\mathbf{D}$, is an alternative to electric field intensity ($\mathbf{E}$) as a way to quantify an electric field. This alternative description offers some actionable insight, as we shall point out at the end of this section.

First, what is electric flux density? Recall that a particle having charge q gives rise to the electric field intensity

$$\mathbf{E} = \hat{\mathbf{R}}\, q \,\frac{1}{4\pi R^2}\,\frac{1}{\epsilon} \tag{2.4.1}$$

where R is distance from the charge and $\hat{\mathbf{R}}$ points away from the charge. Note that $\mathbf{E}$ is inversely proportional to $4\pi R^2$, indicating that $\mathbf{E}$ decreases in proportion to the area of a sphere surrounding the charge. Now integrate both sides of Equation 2.4.1 over a sphere $\mathcal{S}$ of radius R:

$$\oint_{\mathcal{S}} \mathbf{E}(\mathbf{r}) \cdot d\mathbf{s} = \oint_{\mathcal{S}} \left[\hat{\mathbf{R}}\, q \,\frac{1}{4\pi R^2}\,\frac{1}{\epsilon}\right] \cdot d\mathbf{s}$$

Factoring out constants that do not vary with the variables of integration, the right-hand side becomes:

$$q\,\frac{1}{4\pi R^2}\,\frac{1}{\epsilon}\oint_{\mathcal{S}} \hat{\mathbf{R}} \cdot d\mathbf{s}$$

Note that $d\mathbf{s} = \hat{\mathbf{R}} ds$ in this case, and also that $\hat{\mathbf{R}} \cdot \hat{\mathbf{R}} = 1$. Thus, the right-hand side simplifies to:

$$q\,\frac{1}{4\pi R^2}\,\frac{1}{\epsilon}\oint_{\mathcal{S}} ds$$

The remaining integral is simply the area of $\mathcal{S}$, which is $4\pi R^2$. Thus, we find:

$$\oint_{\mathcal{S}} \mathbf{E}(\mathbf{r}) \cdot d\mathbf{s} = \frac{q}{\epsilon} \tag{2.4.2}$$

The integral of a vector field over a specified surface is known as *flux* (see "Additional Reading" at the end of this section). Thus, we have found that the flux of $\mathbf{E}$ through the sphere $\mathcal{S}$ is equal to a constant, namely q/ϵ. Furthermore, this constant is the same regardless of the radius R of the sphere. Said differently, the flux of $\mathbf{E}$ is constant with distance, and does not vary as $\mathbf{E}$ itself does. Let us capitalize on this observation by making the following small modification to Equation 2.4.2:

$$\oint_{\mathcal{S}} [\epsilon\mathbf{E}] \cdot d\mathbf{s} = q$$

In other words, the flux of the quantity $\epsilon\mathbf{E}$ is equal to the enclosed charge, regardless of the radius of the sphere over which we are doing the calculation. This leads to the following definition:

electric flux density

The electric flux density $\mathbf{D} = \epsilon\mathbf{E}$, having units of C/m^2, is a description of the electric field in terms of flux, as opposed to force or change in electric potential.

It may appear that $\mathbf{D}$ is redundant information given $\mathbf{E}$ and ϵ, but this is true only in homogeneous media. The concept of electric flux density becomes important – and decidedly not redundant – when we encounter boundaries between media having different permittivities. As we shall see in Section 5.18, boundary conditions on $\mathbf{D}$ constrain the component of the electric field that is perpendicular to the boundary separating two regions. If one ignores the "flux" character of the electric field represented by $\mathbf{D}$ and instead considers only $\mathbf{E}$, then only the *tangential* component of the electric field is constrained. In fact, when one of the two materials comprising the boundary between two material regions is a perfect conductor, then the electric field is *completely determined* by the boundary condition on $\mathbf{D}$. This greatly simplifies the problem of finding the electric field in a region bounded or partially bounded by materials that can be modeled as perfect conductors, including many metals. In particular, this principle makes it easy to analyze capacitors.

We conclude this section with a warning. Even though the SI units for $\mathbf{D}$ are C/m^2, $\mathbf{D}$ describes an electric field and *not* a surface charge density. It is certainly true that one may describe the amount of charge distributed over a surface using units of C/m^2.

However, $\mathbf{D}$ is not necessarily a description of *actual* charge, and there is no implication that the source of the electric field is a distribution of surface charge. On the other hand, it *is* true that $\mathbf{D}$ can be interpreted as an *equivalent* surface charge density that would give rise to the observed electric field, and in some cases, this equivalent charge density turns out to be the actual charge density.

2.5: Magnetic Flux Density

Magnetic flux density is a vector field which we identify using the symbol $\mathbf{B}$ and which has SI units of tesla (T). Before offering a formal definition, it is useful to consider the broader concept of the *magnetic field*.

Magnetic fields are an intrinsic property of some materials, most notably permanent magnets. The basic phenomenon is probably familiar, and is shown in Figure $2.5.1$. A bar magnet has “poles” identified as “N” (“north”) and “S” (“south”). The N-end of one magnet attracts the S-end of another magnet but repels the N-end of the other magnet and so on. The existence of a vector field is apparent since the observed force acts at a distance and is asserted in a specific direction. In the case of a permanent magnet, the magnetic field arises from mechanisms occurring at the scale of the atoms and electrons comprising the material. These mechanisms require some additional explanation which we shall defer for now.

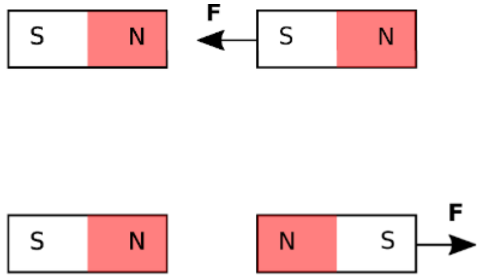

Figure $2.5.1$: Evidence of a vector field from observations of the force perceived by the bar magnets on the right in the presence of the bar magnets on the left. (CC BY 3.0; Y. Qin).

Magnetic fields also appear in the presence of current. For example, a coil of wire bearing a current is found to influence permanent magnets (and vice versa) in the same way that permanent magnets affect each other. This is shown in Figure $2.5.2$. From this, we infer that the underlying mechanism is the same – i.e., the vector field generated by a current-bearing coil is the same phenomenon as the vector field associated with a permanent magnet. Whatever the source, we are now interested in quantifying its behavior.

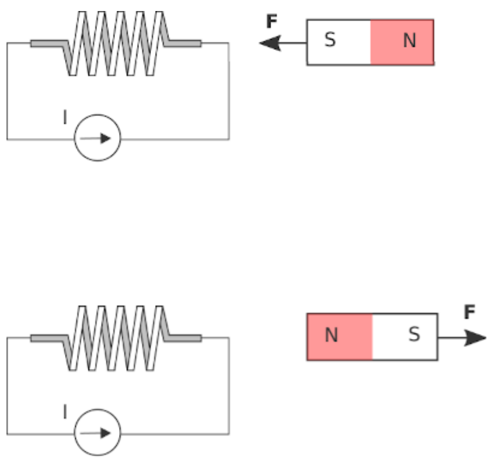

Figure $2.5.2$: Evidence that current can also create a magnetic field. (CC BY 4.0; Y. Qin).

To begin, let us consider the effect of a magnetic field on a electrically-charged particle. First, imagine a region of free space with no electric or magnetic fields. Next, imagine that a charged particle appears. This particle will experience no force. Next, a magnetic field appears; perhaps this is due to a permanent magnet or a current in the vicinity. This situation is shown in Figure $2.5.2$ (top). Still, no force is applied to the particle. In fact, nothing happens until the particle in set in motion. Figure $2.5.3$ (bottom) shows an example. Suddenly, the particle perceives a force. We’ll get to the details about direction and magnitude in a moment, but the main idea is now evident. A magnetic field is something that applies a force to a charged particle in motion, distinct from (in fact, in addition to) the force associated with an electric field.

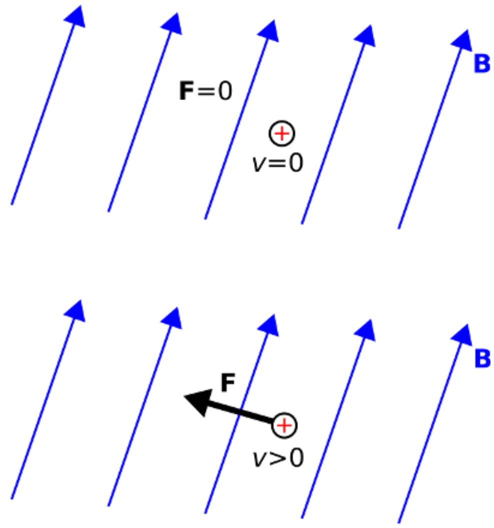

Figure 2.5.3: The force perceived by a charged particle that is (top) motionless and (bottom) moving with velocity $\mathbf{v} = \hat{\mathbf{z}}v$, which is perpendicular to the plane of this document and toward the reader (CC BY 4.0; Y. Qing).

Now, it is worth noting that a single charged particle in motion is the simplest form of current. Remember also that motion is required for the magnetic field to influence the particle. Therefore, not only is current the source of the magnetic field, the magnetic field also exerts a force on current. Summarizing:

> The magnetic field describes the force exerted on permanent magnets and currents in the presence of other permanent magnets and currents.

So, how can we quantify a magnetic field? The answer from classical physics involves another experimentally-derived equation that predicts force as a function of charge, velocity, and a vector field $\mathbf{B}$ representing the magnetic field. Here it is: The force applied to a particle bearing charge q is

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \tag{2.5.1}$$

where $\mathbf{v}$ is the velocity of the particle and "$\times$" denotes the cross product. The cross product of two vectors is in a direction perpendicular to each of the two vectors, so the force exerted by the magnetic field is perpendicular to both the direction of motion and the direction in which the magnetic field points.

The reader would be well-justified in wondering why the force exerted by the magnetic field should *perpendicular* to $\mathbf{B}$. For that matter, why should the force depend on $\mathbf{v}$? These are questions for which classical physics provides no obvious answers. Effective answers to these questions require concepts from quantum mechanics, where we find that the magnetic field is a manifestation of the fundamental and aptly-named *electromagnetic force*. The electromagnetic force also gives rises to the electric field, and it is only limited intuition, grounded in classical physics, that leads us to perceive the electric and magnetic fields as distinct phenomena. For our present purposes – and for most commonly-encountered engineering applications – we do not require these concepts. It is sufficient to accept this apparent strangeness as fact and proceed accordingly.

Dimensional analysis of 2.5.1 reveals that $\mathbf{B}$ has units of (N·s)/(C·m). In SI, this combination of units is known as the tesla (T).

We refer to $\mathbf{B}$ as *magnetic flux density*, and therefore tesla is a unit of magnetic flux density. A fair question to ask at this point is: What makes this a flux density? The short answer is that this terminology is somewhat arbitrary, and in fact is not even uniformly accepted. In engineering electromagnetics, the preference for referring to $\mathbf{B}$ as a "flux density" is because we frequently find ourselves integrating $\mathbf{B}$ over a mathematical surface. Any quantity that is obtained by integration over a surface is referred to as "flux," and so it becomes natural to think of $\mathbf{B}$ as a flux density; i.e., as flux per unit area. The SI unit for magnetic flux is the weber (Wb). Therefore, $\mathbf{B}$ may alternatively be described as having units of Wb/m^2, and 1 Wb/m^2 $= 1$ T.

> Magnetic flux density ($\mathbf{B}$, T or Wb/m^2) is a description of the magnetic field that can be defined as the solution to Equation 2.5.1.

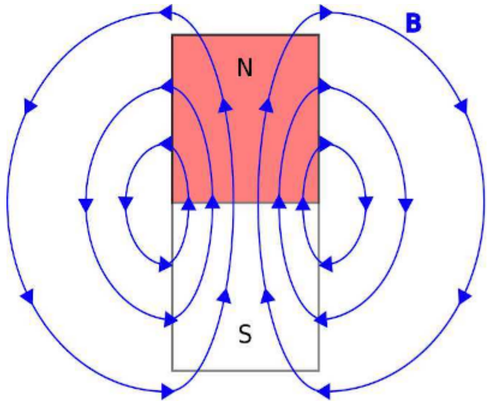

Figure $2.5.4$: The magnetic field of a bar magnet, illustrating field lines. (CC BY 4.0; Y. Qing).

When describing magnetic fields, we occasionally refer to the concept of a *field line*, defined as follows:

> A magnetic field line is the curve in space traced out by following the direction in which the magnetic field vector points.

This concept is illustrated in Figure $2.5.4$ for a permanent bar magnet and Figure $2.5.5$ for a current-bearing coil.

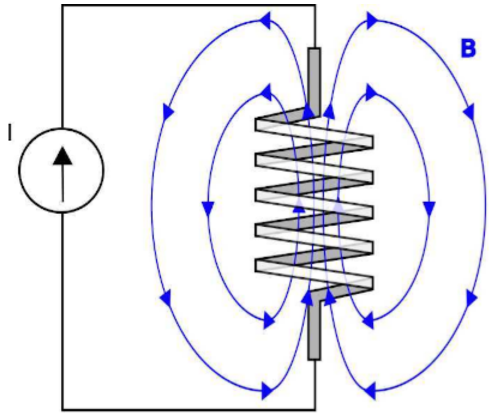

Figure $2.5.5$: The magnetic field of a current-bearing coil, illustrating field lines. (CC BY 4.0; Y. Qing).

Magnetic field lines are remarkable for the following reason:

> A magnetic field line always forms a closed loop.

This is true in a sense even for field lines which seem to form straight lines (for example, those along the axis of the bar magnet and the coil in Figures $2.5.4$ and $2.5.5$, since a field line that travels to infinity in one direction reemerges from infinity in the opposite direction.

2.6: Permeability

Permeability describes the effect of material in determining the magnetic flux density. All else being equal, magnetic flux density increases in proportion to permeability.

To illustrate the concept, consider that a particle bearing charge q moving at velocity $\mathbf{v}$ gives rise to a magnetic flux density:

$$\mathbf{B}(\mathbf{r}) = \mu \frac{q\mathbf{v}}{4\pi R^2} \times \hat{\mathbf{R}} \tag{2.6.1}$$

where $\hat{\mathbf{R}}$ is the unit vector pointing from the charged particle to the field point $\mathbf{r}$, R is this distance, and "$\times$" is the cross product. Note that $\mathbf{B}$ increases with charge and speed, which makes sense since moving charge is the source of the magnetic field. Also note that $\mathbf{B}$ is inversely proportional to $4\pi R^2$, indicating that $|\mathbf{B}|$ decreases in proportion to the area of a sphere surrounding the charge, also known as the *inverse square law*. The remaining factor, μ, is the constant of proportionality that captures the effect of material. We refer to μ as the *permeability* of the material. Since $\mathbf{B}$ can be expressed in units of Wb/m^2 and the units of $\mathbf{v}$ are m/s, we see that μ must have units of henries per meter (H/m). (To see this, note that 1 H $\triangleq$ 1 Wb/A.)

> Permeability (μ, H/m) describes the effect of material in determining the magnetic flux density.

In free space, we find that the permeability $\mu = \mu_0$ where:

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

It is common practice to describe the permeability of materials in terms of their *relative permeability*:

$$\mu_r \triangleq \frac{\mu}{\mu_0}$$

which gives the permeability relative to the minimum possible value; i.e., that of free space. Relative permeability for a few representative materials is given in Appendix A2.

Note that μ_r is approximately 1 for all but a small class of materials. These are known as *magnetic materials*, and may exhibit values of μ_r as large as $\sim 10^6$. A commonly-encountered category of magnetic materials is *ferromagnetic* material, of which the best-known example is iron.

2.7: Magnetic Field Intensity

Magnetic field intensity $\mathbf{H}$ is an alternative description of the magnetic field in which the effect of material is factored out. For example, the magnetic flux density $\mathbf{B}$ (reminder: Section 2.5) due to a point charge q moving at velocity $\mathbf{v}$ can be written in terms of the *Biot-Savart Law*:

$$\mathbf{B} = \mu \frac{q\mathbf{v}}{4\pi R^2} \times \hat{\mathbf{R}} \tag{2.7.1}$$

where $\hat{\mathbf{R}}$ is the unit vector pointing from the charged particle to the field point $\mathbf{r}$, R is this distance, "$\times$" is the cross product, and μ is the permeability of the material. We can rewrite Equation 2.7.1 as:

$$\mathbf{B} \triangleq \mu\mathbf{H} \tag{2.7.2}$$

with:

$$\mathbf{H} = \frac{q\mathbf{v}}{4\pi R^2} \times \hat{\mathbf{R}} \tag{2.7.3}$$

so $\mathbf{H}$ in homogeneous media does not depend on μ.

Dimensional analysis of Equation 2.7.3 reveals that the units for $\mathbf{H}$ are amperes per meter (A/m). However, $\mathbf{H}$ does not represent surface current density, as the units might suggest. While it is certainly true that a distribution of current (A) over some linear cross-section (m) can be described as a current density having units of A/m, $\mathbf{H}$ is associated with the magnetic field and not a particular current distribution (the concept of *current density* is not essential to understand this section; however, a primer can be found in Section 6.2). Said differently, $\mathbf{H}$ can be viewed as a description of the magnetic field in terms of an equivalent (but not actual) current.

> The magnetic field intensity $\mathbf{H}$ (A/m), defined using Equation 2.7.2, is a description of the magnetic field independent from material properties.

It may appear that $\mathbf{H}$ is redundant information given $\mathbf{B}$ and μ, but this is true only in homogeneous media. The concept of magnetic field intensity becomes important – and decidedly not redundant – when we encounter boundaries between media having different permeabilities. As we shall see in Section 7.11, boundary conditions on $\mathbf{H}$ constrain the component of the magnetic field which is tangent to the boundary separating two otherwise-homogeneous regions. If one ignores the characteristics of the magnetic field represented by $\mathbf{H}$ and instead considers only $\mathbf{B}$, then only the perpendicular component of the magnetic field is constrained.

The concept of magnetic field intensity also turns out to be useful in a certain problems in which μ is not a constant, but rather is a function of magnetic field strength. In this case, the magnetic behavior of the material is said to be *nonlinear*. For more on this, see Section 7.16.

2.8: Electromagnetic Properties of Materials

In electromagnetic analysis, one is principally concerned with three properties of matter. These properties are quantified in terms of *constitutive parameters*, which describe the effect of material in determining an electromagnetic quantity in response to a source. Here are the three principal constitutive parameters:

- *Permittivity* (ϵ, F/m) quantifies the effect of matter in determining the electric field in response to electric charge. Permittivity is addressed in Section 2.3.
- *Permeability* (μ, H/m) quantifies the effect of matter in determining the magnetic field in response to current. Permeability is addressed in Section 2.6.
- *Conductivity* (σ, S/m) quantifies the effect of matter in determining the flow of current in response to an electric field. Conductivity is addressed in Section 6.3.

> The electromagnetic properties of most common materials in most common applications can be quantified in terms of the constitutive parameters ϵ, μ, and σ.

To keep electromagnetic theory from becoming too complex, we usually require the constitutive parameters to exhibit a few basic properties. These properties are as follows:

- *Homogeneity*. A material that is *homogeneous* is uniform over the space it occupies; that is, the values of its constitutive parameters are constant at all locations within the material. A counter-example would be a material that is composed of multiple chemically-distinct compounds that are not thoroughly mixed, such as soil.
- *Isotropy*. A material that is *isotropic* behaves in precisely the same way regardless of how it is oriented with respect to sources and fields occupying the same space. A counter-example is quartz, whose atoms are arranged in a uniformly-spaced crystalline lattice. As a result, the electromagnetic properties of quartz can be changed simply by rotating the material with respect to the applied sources and fields.
- *Linearity*. A material is said to be *linear* if its properties are constant and independent of the magnitude of the sources and fields applied to the material. For example, capacitors have capacitance, which is determined in part by the permittivity of the material separating the terminals (Section 5.23). This material is approximately linear when the applied voltage V is below the rated working voltage; i.e., ϵ is constant and so capacitance does not vary significantly with respect to V. When V is greater than the working voltage, the dependence of ϵ on V becomes more pronounced, and then capacitance becomes a function of V. In another practical example, it turns out that μ for ferromagnetic materials is nonlinear such that the precise value of μ depends on the magnitude of the magnetic field.
- *Time-invariance*. An example of a class of materials that is not necessarily time-invariant is piezoelectric materials, for which electromagnetic properties vary significantly depending on the mechanical forces applied to them – a property which can be exploited to make sensors and transducers.

Linearity and time-invariance (LTI) are particularly important properties to consider because they are requirements for *superposition*. For example, in a LTI material, we may calculate the field $\mathbf{E}_1$ due to a point charge q_1 at $\mathbf{r}_1$ and calculate the field $\mathbf{E}_2$ due to a point charge q_2 at $\mathbf{r}_2$. Then, when both charges are simultaneously present, the field is $\mathbf{E}_1+\mathbf{E}_2$. The same is *not* necessarily true for materials that are not LTI. Devices that are nonlinear, and therefore not LTI, do not necessarily follow the rules of elementary circuit theory, which presume that superposition applies. This condition makes analysis and design much more difficult.

No practical material is truly homogeneous, isotropic, linear, and time-invariant. However, for most materials in most applications, the deviation from this ideal condition is not large enough to significantly affect engineering analysis and design. In other cases, materials may be significantly non-ideal in one of these respects, but may still be analyzed with appropriate modifications to the theory.