Lecture 8: Channel Coding

Channel coding Theorem

Problem:

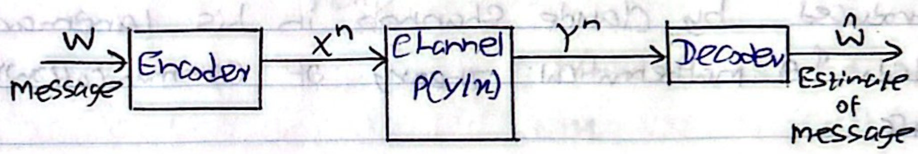
finding the maximum number of distinguishable signals for n uses of a communication channel.

The maximum number of distinguishable signals (M) for multi-level modulation (M-ary) is $M = 2^k$.

This number grows exponentially with n.

Mathematical model

The mathematical analog of a physical signaling system is shown



- Problem: two different input sequences may give rise to the same output sequence; the inputs are confusable
- We show that we can choose a "nonconfusable" subset of input sequences so that with high probability there is only one highly likely input that could have caused the particular output.

Definitions discrete channel: $(X, p(Y/X), Y)$

A system consisting of an input alphabet X and output alphabet Y (finite sets) and a probability transition matrix $P(Y/X)$ that expresses the probability of observing the output symbol Y given that we send the symbol X. The channel is said to be memoryless if the probability distribution of the output depends only on the input at that time and is conditionally independent of previous channel inputs or outputs

Definition "information" channel capacity of a discrete memoryless channel:

$$C = \max_{P(x)} I(X;Y),$$

C is the channel capacity in bits per channel use.

$I(X;Y)$ is the mutual information between the input random variable X and output random variable Y.

Information Capacity

The information capacity theorem, also known as the Shannon's channel capacity Theorem or Shannon's Theorem, is one of the fundamental results in information theory. It was introduced by Claude Shannon in his landmark paper titled, "A mathematical Theory of Communication" published in 1948.

$$C = \max I(X;Y)$$

~~Mutual Information~~

Q: What is mutual Information and how it is related to channel capacity. for a standard voice band communication channel the signal to noise ratio is 30dB and transmission bandwidth is 3kHz. what will be the Shannon limit for information in bits/sec. ~~[AKTU 2017-18, NEC-602]~~

Mutual Information $I(X:Y)$

$I(X:Y)$ represents the uncertainty about the channel input that is resolved by observing the channel o/p.

$$I(X;Y) = \text{Initial uncertainty} - \text{Final uncertainty}$$
$$= H(X) - H(X|Y) \quad \text{bits/symbol}$$

where $H(X) \Rightarrow$ represent the uncertainity about the channel

No: ______ Date: __/__/____

I/P before the channel o/p is observed.

$H(X/Y) \rightarrow$ represent the uncertainty about the channel I/P after the channel o/p is observed.

properties of $I(X;Y)$

i) $I(X:Y) = I(Y:X)$

ii) $I(X:Y) \geq 0$

iii) $I(X;Y) = H(Y) - H(Y/X)$

iv) $I(X;Y) = H(X) + H(Y) - H(X,Y)$

Channel capacity C of Discrete memoryless channel is defined by

$$C_s = \max_{P(x_i)} I(X;Y) \quad b/symbol$$

For lossless channel

$$I(X;Y) = H(X)$$

$$C_s = \max_{P(x_i)} H(X) = \log_2 m$$

where m is no. of symbol in x

$$\left(\frac{S}{N}\right)_{dB} = 30\,dB \qquad B = 3\,kHz \qquad R_{max} = ?$$

$$C_s = \max_{P(x_i)} I(X;Y)$$

Channel capacity for AWGN Channel

$$C_s = \max_{(f_X(x))} I(X;Y) = \frac{1}{2}\log_2\left(1+\frac{S}{N}\right) \; b/symbol$$

Now if channel BW is fixed then

$$C = 2BC_s$$

2B - Nyquist rate

$$= 2B \times \frac{1}{2}\log_2\left(1+\frac{S}{N}\right)$$

$$C = B\log_2\left(1+\frac{S}{N}\right) \quad bits/symbol$$

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\frac{S}{N} = 30$$

$$\log_{10}\frac{S}{N} = 3 \qquad S/N = 10^3$$

$$C = 3\times10^3 \log_2(1+1000)$$
$$= 3\times10^3 \log_2(1001)$$
$$= 3\times10^3 \frac{\log_{10}(1001)}{0.3010}$$

30 kbps

$= 29.90\times10^3$ b/p $= 29.90$ kbps $\approx$ ~~30 kpbs~~

Mutual Information

- Mutual information is a fundamental concept in information theory that measures the amount of information shared between two random variables. It quantifies how much knowing the value of one random variable reduces the uncertainty about the other random variable.

- The mutual information is always non-negative, and it is zero if and only if X and Y are independent random variables.

- When $I(X;Y)$ is positive, it indicates that there is some dependency or relationship between X and Y. The larger the mutual information, the stronger the association or correlation between the two random variables.

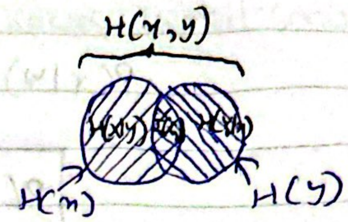
- In the context of communication systems and channel capacity, mutual information plays a crucial role. It measures how much information is conveyed from the input to the output of the communication channel.

- To achieve high channel capacity, it is desirable to maximize the mutual information between the transmitted symbols and the received symbols, which indicates efficient use of the channel and better resistance to noise and distortion.

$$I(x;y) = \underbrace{H(x)}_{\text{Uncertainty before observing Y}} - \underbrace{H(x|y)}_{\text{Uncertainty after observing Y}}$$

Conditional entropy

Mutual Information



Non-Negativity

$$I(x;y) \geq 0$$

Symmetry

$$I(x;y) = I(y;x)$$

Connection to Entropy

$$I(x;y) = H(X) - H(X|Y) = H(Y) - H(Y/x)$$

$$I(x;y) = H(x) + H(y) - H(x,y)$$

$$H(x,y) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} P(x_j, y_k) \log_2\left(\frac{1}{P(x_j, y_k)}\right)$$

Joint entropy is a measure of the total uncertainty or average information content in two or more random variables considered together.

$x = \{x_0, x_1, \ldots, x_{J-1}\}$, $y = \{y_0, y_1, \ldots, y_{K-1}\}$

Joint Entropy

Entropy of Joint probability distribution

$P(X,Y)$ = Joint prob. Distribution

$X \& Y$ = Random variable

$p(X,Y)$ = Probability that X and Y simultaneously assume particular value

$$H(x,Y) = \sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \log_2\left[\frac{1}{P(x_i, y_j)}\right]$$

Conditional probability

$P(X|Y)$ = conditional probability

$$\boxed{\begin{aligned} P(X,Y) &= P(X|Y)\,P(Y) \\ &= P(Y|X)\,P(X) \end{aligned}}$$

If X and Y are independent

Then

$$P(X,Y) = P(X)\,P(Y)$$

Average conditional self information is called as conditional entropy

$$H(X|Y) = \sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i\, y_j) \log_2 \left[\frac{1}{P(x_i/y_j)}\right]$$

$$H(Y|X) = \sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i\, y_j) \log_2 \left[\frac{1}{P(y_j/x_i)}\right]$$

Relationship

$$\begin{aligned} H(x,Y) &= H(X/Y) + H(Y) \\ &= H(Y/X) + H(X) \end{aligned}$$

Example

A source generates information with probabilities $P_1 = 0.1$, $P_2 = 0.2$, $P_3 = 0.3$ and $P_4 = 0.4$. Find the entropy of the system. What percentage of maximum possible information is being generated by this source?

Solution

$$H = \sum_{k=1}^{4} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$= (0.1)\log_2\left(\frac{1}{0.1}\right) + (0.2)\log_2\left(\frac{1}{0.2}\right) + (0.3)\log_2\left(\frac{1}{0.3}\right) + (0.4)\log_2\left(\frac{1}{0.4}\right)$$

$$H = 1.8464 \text{ bits/message}$$

Observing Y   Observing ?

$H_{max} = \log M = \log_2 4 = 2$ bits/message

$\uparrow$ Total symbols

Percentage of maximum possible information

$$= \frac{H}{H_{max}} \times 100\%$$

$$= \frac{1.8464}{2} \times 100\%$$

$$= 92.32\%$$

Example 2

An event has six possible outcomes with the probabilities $P_1 = 1/2$, $P_2 = 1/4$, $P_3 = 1/8$, $P_4 = 1/16$, $P_5 = 1/32$, $P_6 = 1/32$. Find the entropy of the system. Also find the rate of information (R) if there are 16 outcomes (r) per seconds

Solution

$$H = \sum_{k=1}^{6} P_k \log\left(\frac{1}{P_k}\right)$$

$$= \frac{1}{2}\log(2) + \frac{1}{4}\log(4) + \frac{1}{8}\log(8) + \frac{1}{16}\log(16) + \frac{1}{32}\log(32) + \frac{1}{32}\log(32)$$

$$\boxed{H = \frac{31}{16} \text{ bits/message}}$$

~~R = rxH~~

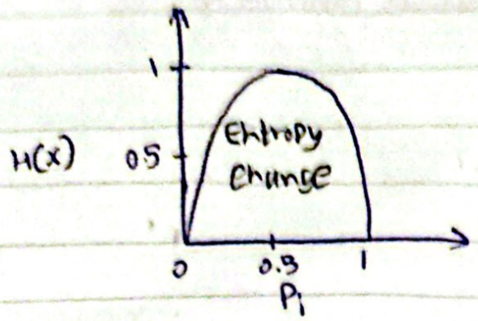
$$R = r \times H$$

$$= 16 \times \frac{31}{16}$$

$$\boxed{R = 31 \text{ bits/symbol}}$$

H(x) - Entropy

$$H(X) = E[I(x_i)] = -\sum_{i=1}^{N} P_i \log_2 P_i$$



Case 1 $A = \{0, 1\}$

$P(X=0) = 0$

$P(X=1) = 1$

$$H(X) = -\sum_{i=1}^{N} P_i \log_2 P_i$$

$$= -[(0 \times 1) + (1 \times 0)] = 0$$

The maximum entropy occurs when the probability is equal

$0 \leq H(x) \leq \log_2 N$ $A = \{\underbrace{0, 1, \ldots}_{N \text{ symbols}}\}$

for all $P_i = \frac{1}{N}$

- If has N number of Symbols in the alphabet. The maximum entropy come when every symbols has same probability
- Hence maximum entropy

$$H(x) = -\sum_{i=1}^{N} \frac{1}{N} \log_2 \frac{1}{N} = -\log_2 \frac{1}{N}$$

$$= \log_2 N \text{ (highest entropy that could occur)}$$

Entropy of a binary source - the binary source that generates independent symbols 0 and 1, with probabilities equal to P and (1-P)

Source entropy:

$$H(X) = -[P \log_2 P + (1-P) \log_2 (1-P)]$$

No: ________ Date: __/__/____

Channel model

$x_i \rightarrow \text{(comm channel)} \rightarrow y_j$
noise

Memoryless channel → output depends on present ~~it~~ input only

- Channel is desired in terms of input, output and set of transition probabilities

$p(y_j/x_i)$ → Conditional probabilityes

x_i, y_j if $i=j$ then no error

if $i \neq j$ error

Communication channel

- Transition probabilities of the channel can be represented by a matrix:

channel matrix:

$$\begin{array}{c|cccc} & y_1 & y_2 & \cdots & y_m \\ \hline x_1 & p(y_1/x_1) & p(y_2/x_1) & \cdots & p(y_m/x_1) \\ x_2 & p(y_1/x_2) & p(y_2/x_2) & \cdots & p(y_m/x_2) \\ \vdots & \vdots & \vdots & & \vdots \\ x_n & p(y_1/x_n) & p(y_2/x_n) & \cdots & p(y_m/x_n) \end{array} \rightarrow 1$$

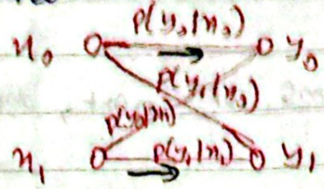
$$\boxed{\sum_{j=1}^{m} p(y_j/x_i) = 1}$$

$$\boxed{p(y_j) = \sum_{i=1}^{n} p(y_j/x_i)\,p(x_i)}$$

$p(y_1) = p(y_1/x_1)\,p(x_1) + p(y_1/x_2)\,p(x_2) \ldots \ldots p(y_1/x_n)\,p(x_n)$

Binary communication channel

Consider a discrete channel with only two symbols transmitted



$$\begin{array}{c|cc} & y_0 & y_1 \\ \hline x_0 & P(y_0/x_0) & P(y_1/x_0) \\ x_1 & P(y_0/x_1) & P(y_1/x_1) \end{array}$$

Probabilities of getting wrong outputs are $P(y_1/x_0)$ and $P(y_0/x_1)$

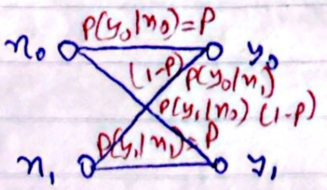
$$P(y_0) = P(y_0|x_0)\,P(x_0) + P(y_0|x_1)\,P(x_1)$$

$$P(y_1) = P(y_1|x_0)\,P(x_0) + P(y_1|x_1)\,P(x_1)$$

$$\begin{bmatrix} P(y_0) \\ P(y_1) \end{bmatrix} = \begin{bmatrix} P(x_0) & P(x_1) \end{bmatrix} \begin{bmatrix} P(y_0|x_0) & P(y_1|x_0) \\ P(y_0|x_1) & P(y_1|x_1) \end{bmatrix}$$

Binary symmetric channel

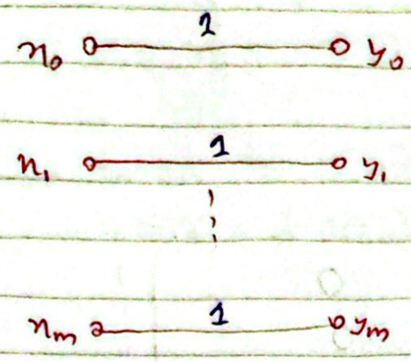
Binary communication channel is said to be symmetric if $P(y_0|x_0) = P(y_1|x_1) = P$



$$\begin{array}{c|cc} & y_0 & y_1 \\ \hline x_0 & P & 1-P \\ x_1 & 1-P & P \end{array}$$

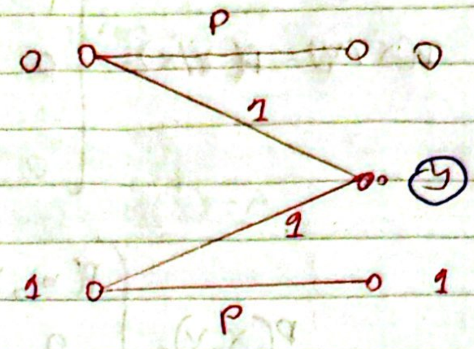
$$\begin{bmatrix} P(y_0) \\ P(y_1) \end{bmatrix} = \begin{bmatrix} P(x_0) & P(x_1) \end{bmatrix} \begin{bmatrix} P & 1-P \\ 1-P & P \end{bmatrix}$$

Noise free channel



$$P(Y/X) = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Binary Eraser channel



The symbol y represents that, due to noise no deterministic decision can be made as to whether the received symbol is 0 or 1.
ie output is erased. (y)

Noise Free channel

- Fig 1 shows a Noise-Free channel
- There is one-to-one correspondence between input and output
  - Each input symbol is received as one and the only one output symbol

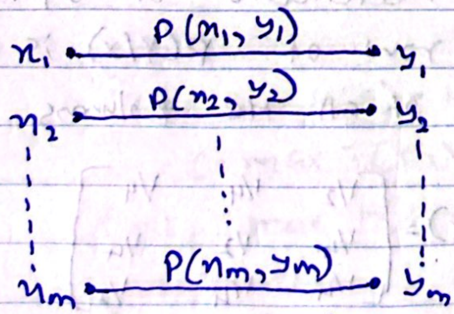


Fig 1. Noise-free channel

- The joint Probability matrix $P(X, Y)$ is of the diagonal form

$$P(X/Y) = P(Y/X) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$P(X,Y) = \begin{bmatrix} P(x_1, y_1) & 0 & \cdots & 0 \\ 0 & P(x_2, y_2) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & P(x_m, y_m) \end{bmatrix}$$

$$H(XY) = -\sum_{j=1}^{m} \sum_{k=1}^{n} p(x_j, y_k) \log p(x_j, y_k) = -\sum_{j=1}^{m} p(x_j, y_j) \log p(x_j, y_j)$$

Since $p(x_j, y_k) = 0$ for $j \neq k$ $\qquad p(x_j, y_j) = p(x_j) = p(y_j)$

$H(X,Y) = H(X) = H(Y)$

$H(X/Y) = H(Y/X) = -m(1 \log 1) = 0$

Thus $I(X;Y) = H(X) - H(X/Y) = H(X) = H(Y) = H(X,Y)$

Symmetric Channel

- Is defined as the one for which
  - $H(Y/x_j)$ is independent of j; i.e. the entropy corresponding to each row of $P(Y/X)$ is the same
  - The sum of all the columns of $P(Y/X)$ is the same

$$P(Y/X) = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$$

$$P(Y/X) = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/2 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$$

$$I(X;Y) = H(Y) - H(Y|x)$$

$$I(x;Y) = H(Y) - \sum_{j=1}^{m} H(Y|x_j)p(x_j) = H(Y) - A\sum_{j=1}^{m} p(x_j)$$

where $A = H(Y/x_j)$ is independent of j

Also $\sum_{j=1}^{m} p(x_j) = 1$ Hence $I(x;Y) = H(Y) - A$

Channel capacity

- It is nothing but maximum of the mutual information
- Channel capacity C is given by,

  - $C = \max I(X;Y)$
  - Also $C = \max[H(X) - H(X/Y)]$

- we can say that, $I(X;Y)$ is the difference of two entropies and C is max $I(X;Y)$.
- Unit is bits/sec
- Transmission efficiency or Channel efficiency is given by,

$$\eta = \frac{\text{Actual Information}}{\text{Maximum Information}}$$

$$\eta = \frac{I(X;Y)}{\max I(X;Y)} = \frac{I(X;Y)}{C}$$

- Redundancy of the Channel is given by, $R = 1 - \eta$

- For Noise-free channel

$$I(X;Y) = H(X) - H(X/Y) = H(X) = H(Y) = H(X,Y)$$

$$C = \max I(X;Y) = \max H(X)$$

But $\max H(X) = \log M$ Bits/message

~~Thetto~~ Therefore $C = \log M$ Bits/message

- For Symmetric channel

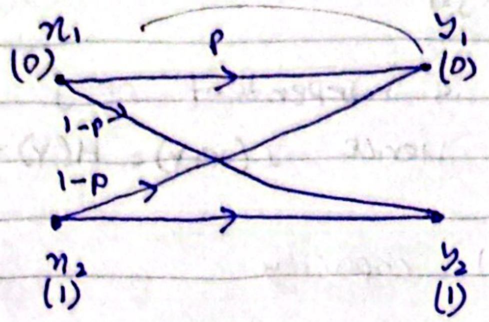
$$I(X;Y) = H(Y) - A$$

$$C = \max I(X;Y) = \max[H(Y) - A]$$

$$C = \max[H(Y)] - A = \log n - A$$

Example

Find the channel capacity for the (i) $P=0.9$, (ii) $P=0.8$, (iii) $P=0.7$, (iv) $P=0.6$ and (v) $P=0.5$



$$C = \max[H(Y)] - A = \log n - A$$

$$C = \log 2 - H(Y/x_j)$$

$$= \log 2 - \left[- \sum_{j=1}^{2} p(y_k/x_j) \log p(y_k/x_j) \right]$$

$$= \log 2 + p\log p + (1-p)\log(1-p)$$

For $P=0.9$

$C = 1 + 0.9\log 0.9 + 0.1\log 0.1 = 0.531$ bits/message

for $P=0.8$

$C = 1 + 0.8\log 0.8 + 0.2\log 0.2 = 0.278$ bits/message

For $P=0.7$

$C = 1 + 0.7\log 0.7 + 0.3\log 0.3 = 0.119$ bits/message

For $P=0.6$

$C = 1 + 0.6\log 0.6 + 0.4\log 0.4 = 0.029$ bits/message

for $P=0.5$

$C = 1 + 0.5\log 0.5 + 0.5\log 0.5 = 0$ bits/message

Joint and conditional Entropy - example

A discrete source transmits message x_1, x_2 and x_3 with the probabilities 0.3, 0.4 and 0.3. The source is connected to the channel given in fig.1. Calculate all the entropies

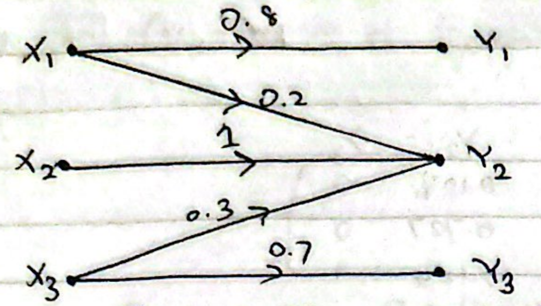


Fig.1. Conditional probability matrix

Now fig 1 gives the conditional probability matrix $P(Y/X)$ as

$$P(Y/X) = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & 0.8 & 0.2 & 0 \\ x_2 & 0 & 1 & 0 \\ x_3 & 0 & 0.3 & 0.7 \end{array}$$

$$P(x) = [0.3 \quad 0.4 \quad 0.3]$$

The joint probability matrix $P(X,Y)$ can be obtained by multiplying the rows of $P(Y/X)$ by $P(x_1)$, $P(x_2)$ and $P(x_3)$

$$P(X,Y) = P(X)\,P(Y/X)$$

$$= \begin{bmatrix} 0.8\times0.3 & 0.2\times0.3 & 0 \\ 0 & 1\times0.4 & 0 \\ 0 & 0.3\times0.3 & 0.7\times0.3 \end{bmatrix}$$

$$= \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & 0.24 & 0.06 & 0 \\ x_2 & 0 & 0.4 & 0 \\ x_3 & 0 & 0.09 & 0.21 \end{array} \rightarrow 1$$

The probabilities $P(Y_1)$, $P(Y_2)$ and $P(Y_3)$ can be obtained by adding columns of $P(X,Y)$.

$$P(Y_1) = 0.24$$
$$P(Y_2) = 0.06 + 0.4 + 0.09 = 0.55$$
$$P(Y_3) = 0.21$$

The conditional probability matrix $P(X/Y)$ can be obtained by dividing the columns of $P(X,Y)$ by $P(Y_1)$, $P(Y_2)$ and $P(Y_3)$

then

$$P(X/Y) = \begin{array}{c c} & \begin{array}{ccc} Y_1 & Y_2 & Y_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 1 & 0.109 & 0 \\ 0 & 0.727 & 0 \\ 0 & 0.164 & 1 \end{bmatrix} \end{array}$$

The entropies can now be calculated as follows

$$H(X) = -\sum_{j=1}^{3} P(x_j) \log P(x_j)$$

$$= -(0.3 \log 0.3 + 0.4 \log 0.4 + 0.3 \log 0.3)$$
$$= 1.571 \text{ bits/message}$$

$$H(Y) = -\sum_{k=1}^{3} P(Y_k) \log P(Y_k)$$

$$= -(0.24 \log 0.24 + 0.55 \log 0.55 + 0.21 \log 0.21)$$
$$= 1.441 \text{ bits/message}$$

$$H(X,Y) = -\sum_{j=1}^{3} \sum_{k=1}^{3} P(x_j, Y_k) \log P(x_j, Y_k)$$

$$= -(0.24 \log 0.24 + 0.06 \log 0.06 + 0.4 \log 0.4$$
$$\cancel{= 2.053 \text{ bits/message}} + 0.09 \log 0.09 + 0.21 \log 0.21)$$
$$= 2.053 \text{ bits/message}$$

$$H(X/Y) = \sum_{j=1}^{3} \sum_{k=1}^{3} P(x_j, Y_k) \log P(x_j / Y_k)$$

$$= -(0.24 \log 1 + 0.06 \log 0.109 + 0.4 \log 0.727$$
$$+ 0.09 \log 0.164 + 0.21 \log 1)$$
$$= 0.612 \text{ bits/message}$$

No: ______ Date: __/__/____

$$H(Y/X) = -\sum_{j=1}^{3}\sum_{k=1}^{3} P(x_j, y_k)\log P(y_k/x_j)$$

$$= -(0.24\log 0.8 + 0.06\log 0.2 + 0.4\log 1 + 0.09\log 0.3 + 0.21\log 0.7) = 0.482 \text{ bits/message}$$

~~A discrete source transmits message x_1, x_2, x_3 with the probabilities 0.3, 0.4~~

Channel capacity by Shannon-Hartley and It's proof

$$C = B\log_2\left[1 + \frac{S}{N}\right]$$

Where,

B = Bandwidth of Channel

S = Signal power

N = Noise power

Proof :- Received signal = Signal power (S) + Noise power (N) and it's mean square value is $\sqrt{S+N}$

- Noise power is N and it's mean square value is $\sqrt{N}$

- So number of levels can be seperated without error is

$$m = \frac{\sqrt{N+S}}{\sqrt{N}} = \sqrt{1 + \frac{S}{N}}$$

- So digital Information is

$$I = \log_2 m$$

$$= \log_2\sqrt{1 + S/N}$$

$$= \frac{1}{2}\log_2(1 + S/N)$$

- If channel transmittes k pulses per second then channel capacity is

$$C = Ik = \frac{k}{2}\log_2\left(1 + \frac{S}{N}\right)$$

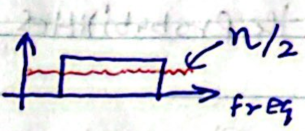
No: ____ Date: __/__/____

- Nyquist Bandwidth is ★ $K = 2B$

$$\boxed{C = B\log_2\left(1+\frac{S}{N}\right)}$$

• Practically if we increase B then noise N will also increase. So capacity of channel can not be infinite.

If $\eta/2$ is power density then,



$$N = \eta B$$

- So channel capacity is

$$C = B\log_2\left(1+\frac{S}{\eta B}\right)$$

$$= \frac{\eta B}{S}\left(\frac{S}{\eta}\right)\log_2\left(1+\frac{S}{\eta B}\right)$$

$$= \frac{S}{\eta}\left[\frac{\log_2\left(1+\frac{S}{\eta B}\right)}{\left(\frac{S}{\eta B}\right)}\right] \quad \underline{B\rightarrow\infty}$$

- For $\lim_{n\to 0} \frac{\log_2(1+n)}{(1/n)} = \log_2 e = 1.44$

$$C_{B\to\infty} = \frac{S}{\eta}\log_2 e = \boxed{1.44\frac{S}{\eta}}$$

Date: __/__/__

Examples on Channel capacity by Shannon-Hartley

For a typical telephone line with a signal to noise ratio of 30dB and an audio bandwidth 3KHz, max. data rate of

$SNR = 30dB = 10^3$

$B = 3kHz$

$C = ?$

dB

$10 \rightarrow 10$

$20 \rightarrow 10^2$

$30 \rightarrow 10^3$

$40 \rightarrow 10^4$

$$C = B\log_2(1+SNR)$$
$$= 3\times10^3 \log_2(1+10^3)$$
$$= 3\times10^3 \frac{\log 1001}{\log 2} = 3\times10^6 \text{ bPS} = 3 \text{ MbPS}$$

For a satellite TV channel with a signal to noise ratio of 20dB and a video bandwidth of 10MHz, find max. data rate?

$\rightarrow SNR = 20dB = 10^2$

$B = 10MHz = 10\times10^6 Hz$

$C = ?$

$$C = B\log_2(1+SNR)$$
$$= 10\times10^6 \log_2(1+10^2)$$
$$= 10^7 \frac{\log(101)}{\log 2}$$
$$= 6.6\times10^7 \text{ bPS} = 66 \text{ MbPS}$$

No: Date: / /

③ A Gaussian channel has 1MHz bandwidth. Calculate the channel capacity. If the signal power to noise spectral density ratio S/N is 10^5 Hz. Also find the max. Information rate.

$B = 1MHz = 10^6 Hz$

$SNR = 10^5$

$C = ?$

$C = B \log_2 (1 + SNR)$

$= 10^6 \frac{\log_2 (100001)}{\log 2}$

$= \cancel{16.6 \times 10^5}\ 16.6 \times 10^6$ bps

$C = 16.6$ Mbps

Calculate channel capacity with $B \to \infty$

- Channel capacity

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

- If we have noise power

$\boxed{N = \eta B}$ where $\frac{\eta}{2}$ is power density

$$C = B \log_2 \left(1 + \frac{S}{\eta B}\right)$$

$$= \frac{\eta B}{S} \times \frac{S}{\eta} \log_2 \left(1 + \frac{S}{\eta B}\right)$$

$$= \frac{S}{\eta} \left[\frac{\eta B}{S} \log_2 \left(1 + \frac{S}{\eta B}\right)\right]$$

Here $B \to \infty$

$$C = \frac{S}{\eta} \lim_{B \to \infty} \frac{\log_2 \left(1 + \frac{S}{\eta B}\right)}{\left(\frac{S}{\eta B}\right)}$$

$$C = \frac{S}{\eta} \log_2 e$$

$$\boxed{C_{B \to \infty} = (1.44) \frac{S}{\eta}}$$

Atlas