EN4333 Microwave Engineering
Resonant Cavities

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Outline

1. Introduction
2. Cavities
3. Modes
4. Wall Currents
5. Conclusion

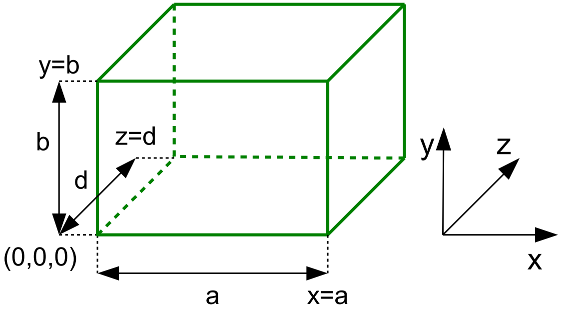
Introduction

Preliminaries

A cavity has boundaries for the EM wave in all three dimensions.

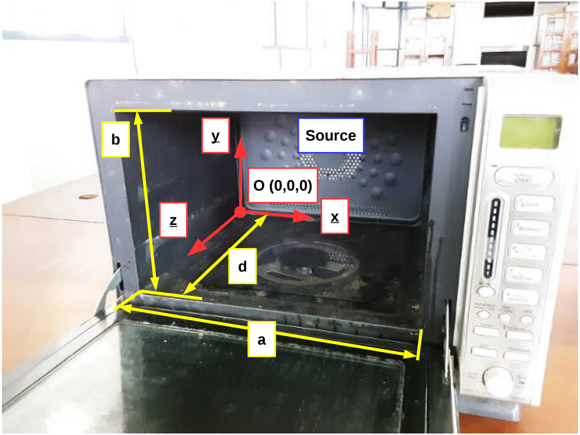
- The EM wave is contained
- Boundary conditions
- Free space or medium
  - Generally air or gas
  - No free charge
  - No conduction
- Results in electromagnetic standing waves for all three dimensions

Cavity Parameters



The axes are generally selected such that $a > b > d$.

Microwave Oven



Microwave Oven (Contd..)

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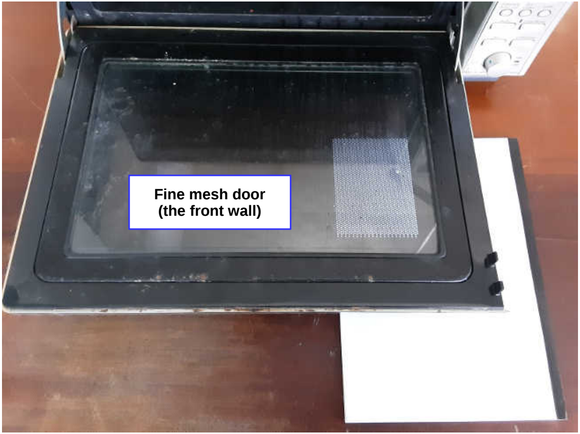
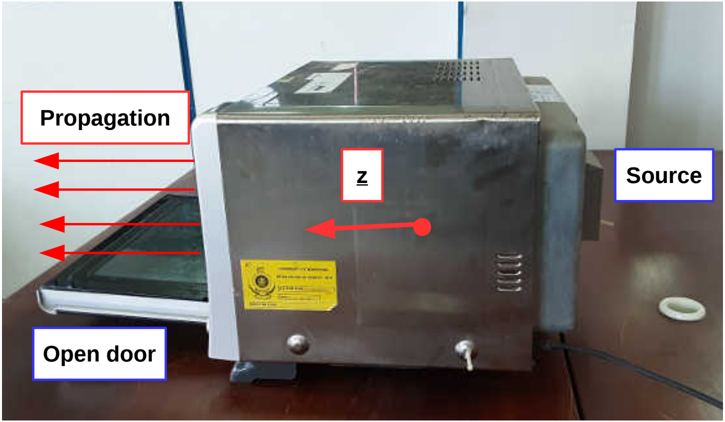


Microwave Oven (Contd..)



Propagation in Cavities

Analysis

The E and H fields within the cavity:

$$
\begin{aligned}
E &= E_x\underline{x} + E_y\underline{y} + E_z\underline{z} \\
H &= H_x\underline{x} + H_y\underline{y} + H_z\underline{z}
\end{aligned}
$$

From Helmholtz equations:

$$
\begin{aligned}
\nabla^2 E + k^2 E &= 0 \\
\nabla^2 H + k^2 H &= 0
\end{aligned}
$$

Where $k = \omega\sqrt{\mu\varepsilon} = 2\pi/\lambda$ for the given medium.

By considering the individual components of E and H in the $\underline{x}, \underline{y}, \underline{z}$ directions 3 separate Helmholtz equations can be obtained for E:

$$
\begin{aligned}
\nabla^2 E_x + k^2 E_x &= 0 \\
\nabla^2 E_y + k^2 E_y &= 0 \\
\nabla^2 E_z + k^2 E_z &= 0
\end{aligned}
$$

Same for H. Now E_x, E_y, E_z can be solved independantly by *separation of variables*

Analysis (Contd..)

From Faraday's Law,

$$\nabla \times E = \begin{vmatrix} \underline{x} & \underline{y} & \underline{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu \left(H_x \underline{x} + H_y \underline{y} + H_z \underline{z} \right)$$

By taking the individual components in the x,y and z directions,

$$\frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y = -j\mu\omega H_x \quad (1)$$

$$-\frac{\partial}{\partial z} E_x + \frac{\partial}{\partial x} E_z = j\mu\omega H_y \quad (2)$$

$$\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x = -j\mu\omega H_z \quad (3)$$

Analysis (Contd..)

From modified Ampere's Law,

$$\nabla \times H = \begin{vmatrix} \underline{x} & \underline{y} & \underline{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon \left(E_x \underline{x} + E_y \underline{y} + E_z \underline{z} \right)$$

By taking the individual components in the x,y and z directions,

$$\frac{\partial}{\partial y} H_z - \frac{\partial}{\partial z} H_y = j\varepsilon\omega E_x \qquad (4)$$

$$-\frac{\partial}{\partial z} H_x + \frac{\partial}{\partial x} H_z = -j\varepsilon\omega E_y \qquad (5)$$

$$\frac{\partial}{\partial x} H_y - \frac{\partial}{\partial y} H_x = j\varepsilon\omega E_z \qquad (6)$$

Separation of Variables

Consider the component E_x of E:

Within the cavity $E_x := E_x(x, y, z)$ (a spatial function). Take the solution for E_x as a *product* of the form $E_x(x, y, z) = X_{\underline{x}}(x)Y_{\underline{x}}(y)Z_{\underline{x}}(z) = X_{\underline{x}}Y_{\underline{x}}Z_{\underline{x}}$.

From the Laplacian (for E_x):

$$Y_{\underline{x}}Z_{\underline{x}}\frac{\partial^2}{\partial x^2}X_{\underline{x}} + Z_{\underline{x}}X_{\underline{x}}\frac{\partial^2}{\partial y^2}Y_{\underline{x}} + X_{\underline{x}}Y_{\underline{x}}\frac{\partial^2}{\partial z^2}Z_{\underline{x}} = -k^2 X_{\underline{x}}Y_{\underline{x}}Z_{\underline{x}}$$

Separation of Variables (Contd..)

Since k^2 is a constant, it can be taken as $k^2 = k_x^2 + k_y^2 + k_z^2$.

$$Y_{\underline{x}} Z_{\underline{x}} \frac{\partial^2}{\partial x^2} X_{\underline{x}} + Z_{\underline{x}} X_{\underline{x}} \frac{\partial^2}{\partial y^2} Y_{\underline{x}} + X_{\underline{x}} Y_{\underline{x}} \frac{\partial^2}{\partial z^2} Z_{\underline{x}} = -(k_x^2 + k_y^2 + k_z^2) X_{\underline{x}} Y_{\underline{x}} Z_{\underline{x}}$$

This results in individual harmonic solutions

$$\begin{aligned}
\frac{\partial^2}{\partial x^2} X_{\underline{x}} = -k_x^2 X_{\underline{x}} \quad &\rightarrow \quad X_{\underline{x}} = A_{x\underline{x}} \cos(k_x x) + B_{x\underline{x}} \sin(k_x x) \\
\frac{\partial^2}{\partial y^2} Y_{\underline{x}} = -k_y^2 Y_{\underline{x}} \quad &\rightarrow \quad Y_{\underline{x}} = A_{y\underline{x}} \cos(k_y y) + B_{y\underline{x}} \sin(k_y y) \\
\frac{\partial^2}{\partial z^2} Z_{\underline{x}} = -k_z^2 Z_{\underline{x}} \quad &\rightarrow \quad Z_{\underline{x}} = A_{z\underline{x}} \cos(k_z z) + B_{z\underline{x}} \sin(k_z z)
\end{aligned}$$

Boundary Conditions

For the cavity, $E^{\parallel} = 0$, $H^{\perp} = 0$ at the boundaries and $\nabla \cdot E = 0$.

- There are *more* boundary conditions than needed
- The significance of this will be apparent later on

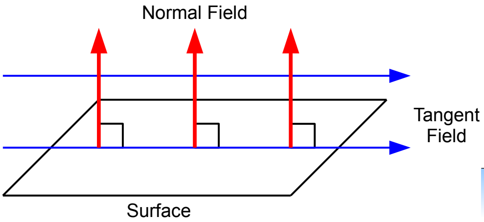
Therefore for E_x:

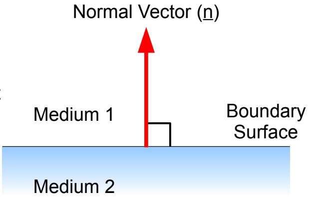
- $E_x = X_{\underline{x}} Y_{\underline{x}} Z_{\underline{x}} = 0$ at $y = 0, b$ and $z = 0, d$.
- $X_{\underline{x}} = 0$ *cannot* be used as a boundary condition (why?).

Boundary Conditions (Contd..)

Definitions

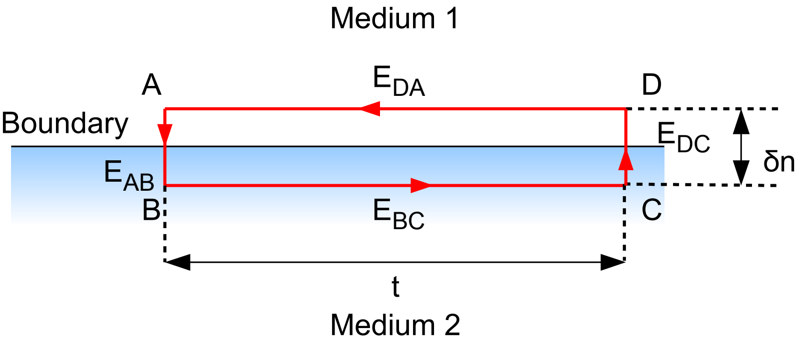
- The **Tangent Field** is the field which is parallel or in the direction of the tangent of a surface.
  - Denoted by the superscript $\|$ $(F^{\|})$.
- The **Normal Field** is a field which is normal to a surface.
  - Denoted by the superscript $\perp$ $(F^{\perp})$.





Boundary Conditions (Contd..)

For tangential E:



Boundary Conditions (Contd..)

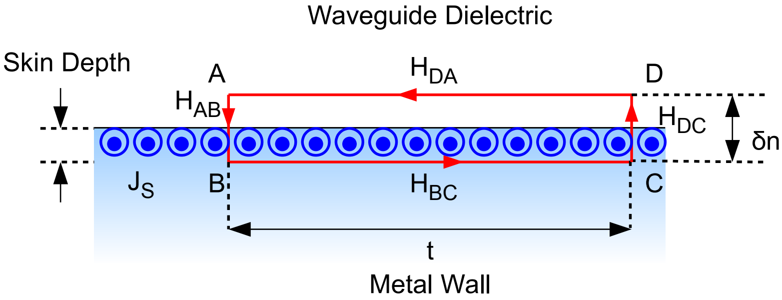
Consider a rectangular loop ABCD. Assume δn is small. Since the electric field is conservative,

$$\begin{aligned} \oint_c Edl = \int_A^B Edl + \int_B^C Edl + \int_C^D Edl + \int_D^A Edl &= 0 \\ \underbrace{E^{\perp}_{AB}\delta n}_{\approx 0} + E^{\parallel}_{BC}t - \underbrace{E^{\perp}_{CD}\delta n}_{\approx 0} - E^{\parallel}_{DA}t &= 0 \\ E^{\parallel}_{BC}t - E^{\parallel}_{DA}t &= 0 \end{aligned}$$

- Therefore $E_1^{\parallel} = E_2^{\parallel}$
  - Since E cannot exist within a metal at the boundary $E^{\parallel} = 0$.

Boundary Conditions (Contd..)

For tangential H:



Boundary Conditions (Contd..)

Consider the same rectangular loop ABCD. From Ampere's law,

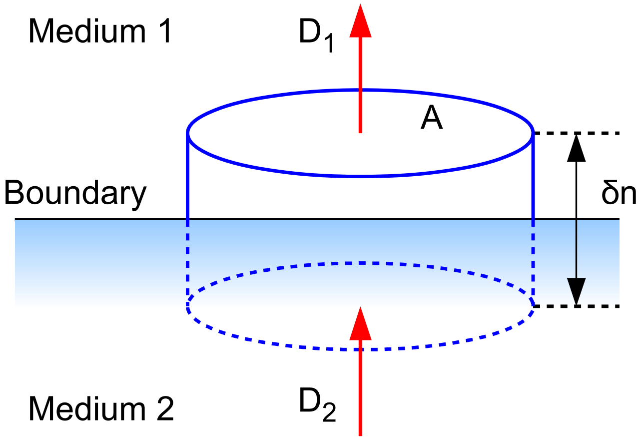
$$\begin{aligned}
\oint_c H dl = \int_A^B H dl + \int_B^C H dl + \int_C^D H dl + \int_D^A H dl &= I \\
\underbrace{H_{AB}^{\perp}\delta n}_{\approx 0} + H_{BC}^{\parallel} t - \underbrace{H_{CD}^{\perp}\delta n}_{\approx 0} - H_{DA}^{\parallel} t &= I \\
H_{BC}^{\parallel} t - H_{DA}^{\parallel} t &= I
\end{aligned}$$

Therefore $H_1^{\parallel} - H_2^{\parallel} = J_S$ where J_S is the surface current density. When this is rewritten in vector form $\underline{n} \times (H_1^{\parallel} - H_2^{\parallel}) = J_S$.

- Due to the **skin depth (δ)** within the metal beyond δ $H \approx 0$:
  - This results in $\underline{n} \times H^{\parallel} = J_S$ at the boundary
  - From $\nabla \cdot H = 0 \rightarrow H^{\perp} = 0$

Boundary Conditions (Contd..)

Normal electric field D



Boundary Conditions (Contd..)

Take δn as negligibly small. From Gauss' law,

$$\begin{aligned} \oint_S D dS &= Q \\ D_A A - D_B A &= Q \\ D_A - D_B &= \frac{Q}{A} = \rho_A \end{aligned}$$

Where ρ_A is the surface charge density. Therefore,

$$\begin{aligned} D_1^{\perp} - D_2^{\perp} &= \rho_A \\ \underline{n} \cdot (D_1 - D_2) &= \rho_A \\ \underline{n} \cdot (\varepsilon_1 E_1 - \varepsilon_2 E_2) &= \rho_A \end{aligned}$$

The Solution

$E_x = 0$ at $y = 0, b \Rightarrow Y_{\underline{x}} = 0$ at $y = 0, b$

- Therefore $A_{y\underline{x}} = 0$ and $k_y b = n\pi$

$E_x = 0$ at $z = 0, d \Rightarrow Z_{\underline{x}} = 0$ at $z = 0, d$

- Therefore $A_{z\underline{x}} = 0$ and $k_z d = l\pi$

This results in

$$E_x = A_0[\underbrace{A_{x\underline{x}}\cos(k_x x) + B_{x\underline{x}}\sin(k_x x)}_{\text{Boundary condition for this?}}]\sin(k_y y)\sin(k_z z)$$

where A_0 is a constant. Same procedure for E_y and E_z.

The Solution (Contd..)

Components of E

$$
\begin{aligned}
E_x &= A_0[A_{x\underline{x}}\cos(k_x x) + B_{x\underline{x}}\sin(k_x x)]\sin(k_y y)\sin(k_z z) \\
E_y &= B_0\sin(k_x x)[A_{y\underline{y}}\cos(k_y y) + B_{y\underline{y}}\sin(k_y y)]\sin(k_z z) \\
E_z &= C_0\sin(k_x x)\sin(k_y y)[A_{z\underline{z}}\cos(k_z z) + B_{z\underline{z}}\sin(k_z z)]
\end{aligned}
$$

From $\nabla \cdot E = 0$ i.e., $\frac{\partial}{\partial x}E_x + \frac{\partial}{\partial y}E_y + \frac{\partial}{\partial z}E_z = 0$:

$$
\begin{aligned}
\frac{\partial}{\partial x}E_x &= k_x A_0[-A_{x\underline{x}}\sin(k_x x) + B_{x\underline{x}}\cos(k_x x)]\sin(k_y y)\sin(k_z z) \\
\frac{\partial}{\partial y}E_y &= k_y B_0\sin(k_x x)[-A_{y\underline{y}}\sin(k_y y) + B_{y\underline{y}}\cos(k_y y)]\sin(k_z z) \\
\frac{\partial}{\partial z}E_z &= k_z C_0\sin(k_x x)\sin(k_y y)[-A_{z\underline{z}}\sin(k_z z) + B_{z\underline{z}}\cos(k_z z)]
\end{aligned}
$$

The Solution (Contd..)

At the locus $(x = 0, y, z)$:

$$\frac{\partial}{\partial y}E_y = k_y B_0 \underbrace{\sin(k_x x)}_{=0}[-A_{y\underline{y}}\sin(k_y y) + B_{y\underline{y}}\cos(k_y y)]\sin(k_z z) = 0$$

$$\frac{\partial}{\partial z}E_z = k_z C_0 \underbrace{\sin(k_x x)}_{=0}\sin(k_y y)[-A_{z\underline{z}}\sin(k_z z) + B_{z\underline{z}}\cos(k_z z)] = 0$$

$$\frac{\partial}{\partial x}E_x = k_x A_0 \underbrace{[-A_{x\underline{x}}\sin(k_x x) + B_{x\underline{x}}\cos(k_x x)]}_{\text{Has to be zero to satisfy } \nabla\cdot E = 0}\underbrace{\sin(k_y y)\sin(k_z z)}_{\text{Can take any value}}$$

Therefore when $x = 0$:

$$[-A_{x\underline{x}}\sin(k_x x) + B_{x\underline{x}}\cos(k_x x)] = 0 \rightarrow B_{x\underline{x}} = 0$$

Similarly $B_{y\underline{x}} = B_{z\underline{x}} = 0$

The Solution (Contd..)

At the locus $(x = a, y, z)$:

$$\frac{\partial}{\partial y} E_y = k_y B_0 \underbrace{\sin(k_x a)}_{=0} [-A_{y\underline{y}} \sin(k_y y)] \sin(k_z z) = 0$$

$$\frac{\partial}{\partial z} E_z = k_z C_0 \underbrace{\sin(k_x a)}_{=0} \sin(k_y y) [-A_{z\underline{z}} \sin(k_z z)] = 0$$

Also

$$\frac{\partial}{\partial x} E_x = k_x A_0 \underbrace{[-A_{x\underline{x}} \sin(k_x a)]}_{=0} \sin(k_y y) \sin(k_z z)$$

Therefore $k_x = \frac{m\pi}{a}$

The Solution (Contd..)

Therefore the solution for $E = E_x\underline{x} + E_y\underline{y} + E_z\underline{z}$ is

$$\begin{aligned} E_x &= X_E \cos(k_x x)\sin(k_y y)\sin(k_z z) \\ E_y &= Y_E \sin(k_x x)\cos(k_y y)\sin(k_z z) \\ E_z &= Z_E \sin(k_x x)\sin(k_y y)\cos(k_z z) \end{aligned} \tag{7}$$

From Faraday's Law for $H = H_x\underline{x} + H_y\underline{y} + H_z\underline{z}$

$$\begin{aligned} H_x &= X_H \sin(k_x x)\cos(k_y y)\cos(k_z z) \\ H_y &= Y_H \cos(k_x x)\sin(k_y y)\cos(k_z z) \\ H_z &= Z_H \cos(k_x x)\cos(k_y y)\sin(k_z z) \end{aligned} \tag{8}$$

For both fields $k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{a}$ and $k_z = \frac{l\pi}{d}$

Modes

Modes

From

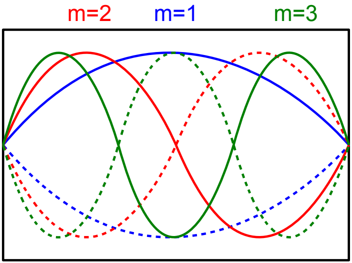
$$
\begin{aligned}
E_x &= X_E \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \sin\left(\frac{l\pi z}{d}\right) \\
E_y &= Y_E \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{l\pi z}{d}\right) \\
E_z &= Z_E \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{l\pi z}{d}\right)
\end{aligned}
$$

- m, n, l give the number of *peaks* in the $\underline{x}, \underline{y}, \underline{z}$ directions respectively

  Note: If a peak occurs at the boundary it is considered as a *half peak*. Half peaks at both boundaries will become a single peak.

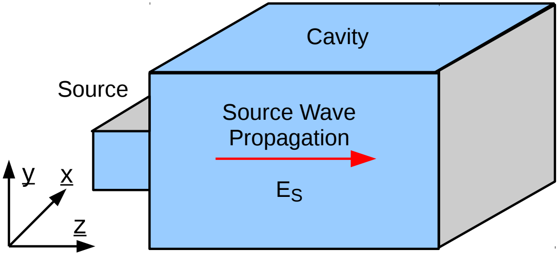
- It is also known as a *mode* for the cavity.

Modes (Contd..)



- The peak occurs in the amplitude (i.e., envelope) of the E or H field only
  - Both fields are time harmonic ($E = \left[E_x \underline{x} + E_y \underline{y} + E_z \underline{z}\right] e^{j\omega t}$)
  - All modes have the same frequency
  - A mode is denoted by the subscript notation $[\text{Mode Name}]_{nml}$.
- How can all field components of (7) and (8) have a common degree of freedom for amplitude?

Amplitude Relationship



- A practical cavity has to be excited by an external source
- This can be
  - A tube like a magnetron
  - An iris or resonant filter
  - An electrical signal

Amplitude Relationship (Contd..)

- When a practical cavity is excited
  - The amplitude of the source E_S will be a *degree of freedom*
  - It can be changed arbitrarily (as practicable)
- The amplitude of the standing wave will depend on E_S
  - Therefore, the X_E, Y_E, Z_E, X_H, Y_H and Z_H have to be expressible in terms of E_S.
- For convenience can take the longitudinal direction as the direction the source transmits
  - Therefore $E_S = Z_E$ (or $H_S = Z_H$)

Amplitude Relationships (E Field)

From (1), (2) and (3)

$$
\begin{aligned}
-k_y Z_E + k_z Y_E &= j\mu\omega X_H && (9)\\
-k_z X_E + k_x Z_E &= j\mu\omega Y_H && (10)\\
-k_x Y_E + k_y X_E &= j\mu\omega Z_H && (11)
\end{aligned}
$$

From $\nabla \cdot E = 0$

$$X_E k_x + Y_E k_y + Z_E k_z = 0 \qquad (12)$$

- **Six unknowns** have to be reduced to a **single** DOF

Amplitude Relationships (H Field)

From (4), (5) and (6)

$$-k_y Z_H + k_z Y_H = j\epsilon\omega X_E \tag{13}$$
$$-k_z X_H + k_x Z_H = j\epsilon\omega Y_E \tag{14}$$
$$-k_x Y_H + k_y X_H = j\epsilon\omega Z_E \tag{15}$$

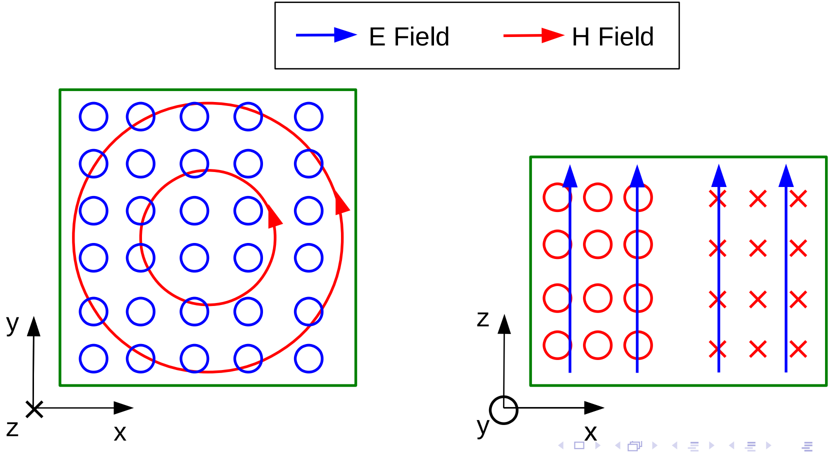
From $\nabla \cdot H = 0$

$$X_H k_x + Y_H k_y + Z_H k_z = 0 \tag{16}$$

- Again **six unknowns** have to be reduced to a **single** DOF

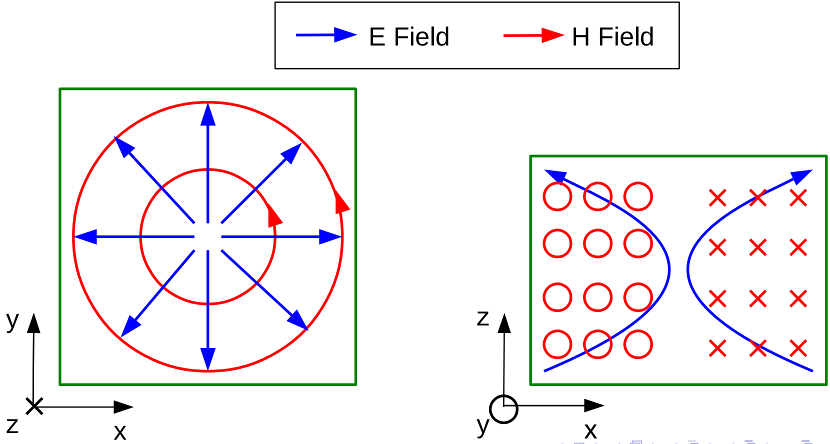
Sample Mode Fields 1

- Sample cavity mode 1 (note the E and H field components)



Sample Mode Fields 2

- Sample cavity mode 2 (note the E and H field components)



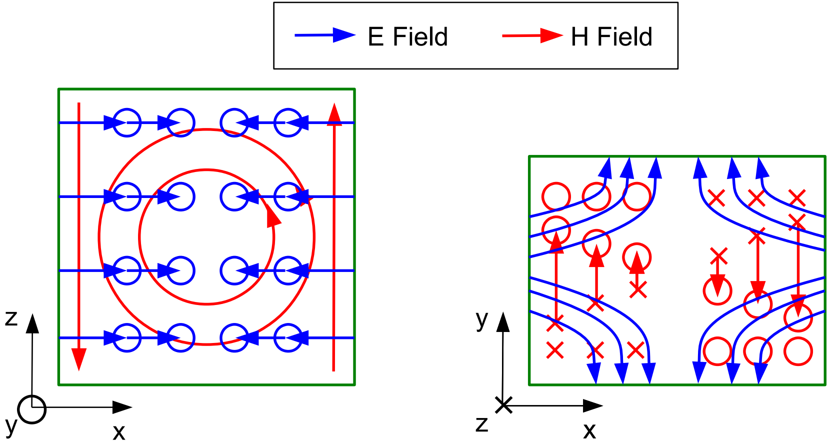
Transverse Magnetic Modes

From (11), taking $H_z = 0 \rightarrow Z_H = 0$ results in $k_x Y_E - k_y X_E = 0$. From this Z_E can be made the DOF.

$$
\begin{aligned}
E_x &= -\frac{Z_E k_z k_x}{k_x^2 + k_y^2} \cos(k_x x) \sin(k_y y) \sin(k_z z) \\
E_y &= -\frac{Z_E k_z k_y}{k_x^2 + k_y^2} \sin(k_x x) \cos(k_y y) \sin(k_z z) \\
E_z &= Z_E \sin(k_x x) \sin(k_y y) \cos(k_z z) \\
H_x &= \frac{j Z_E k_y}{\mu\omega} \left[\frac{k^2}{k_x^2 + k_y^2}\right] \sin(k_x x) \cos(k_y y) \cos(k_z z) \\
H_y &= \frac{j Z_E k_x}{\mu\omega} \left[\frac{k^2}{k_x^2 + k_y^2}\right] \cos(k_x x) \sin(k_y y) \cos(k_z z)
\end{aligned}
$$

Sample Mode Fields 3

- Sample cavity mode 2 (note the E and H field components)



Transverse Electric Modes

From (15), taking $E_z = 0 \rightarrow Z_E = 0$ results in $k_x Y_H - k_y X_H = 0$. Z_H can be made the DOF.

$$
\begin{aligned}
H_x &= -\frac{Z_H k_z k_x}{k_x^2 + k_y^2} \sin(k_x x)\cos(k_y y)\cos(k_z z) \\
H_y &= -\frac{Z_H k_z k_y}{k_x^2 + k_y^2} \cos(k_x x)\sin(k_y y)\cos(k_z z) \\
H_z &= Z_H \cos(k_x x)\cos(k_y y)\sin(k_z z) \\
E_x &= \frac{j Z_H k_y}{\mu\omega}\left[\frac{k^2}{k_x^2 + k_y^2}\right] \cos(k_x x)\sin(k_y y)\sin(k_z z) \\
E_y &= \frac{j Z_H k_x}{\mu\omega}\left[\frac{k^2}{k_x^2 + k_y^2}\right] \sin(k_x x)\cos(k_y y)\sin(k_z z)
\end{aligned}
$$

Mode Summary

- Depending on E_z and H_z, there can be different *modes* of standing waves within a cavity.
  - Same as different modes of propagation within a waveguide (metallic or dielectric) or transmission line

$E_z = 0$	$H_z = 0$	Transverse Electromagnetic (TEM)
$E_z = 0$	$H_z \neq 0$	Transverse Electric (TE)
$E_z \neq 0$	$H_z = 0$	Transverse Magnetic (TM)
$E_z \neq 0$	$H_z \neq 0$	Hybrid (HEM)

- The modes of a cavity (and waveguide) are TE and TM modes
  - In TE modes E occurs in the transverse plane only
  - In TM modes H occurs in the transverse plane only

Mode Summary (Contd..)

- From $k^2 = k_x^2 + k_y^2 + k_z^2$ the angular cutoff frequency ω_{mnl} can be obtained.

$$f_{mnl} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{d}\right)^2} \tag{17}$$

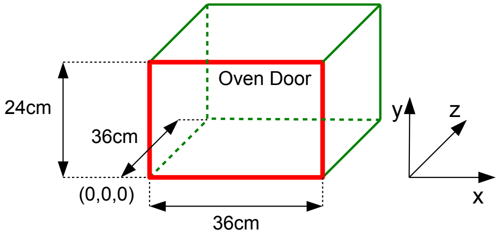
Fundamental Modes

The fundamental mode of a cavity is the mode with the lowest frequency that satisfies (17). From (7) and (8)

- For TE modes it is TE_{101} or TE_{011}
- For TM modes it is TM_{110}

Modes - Example 1

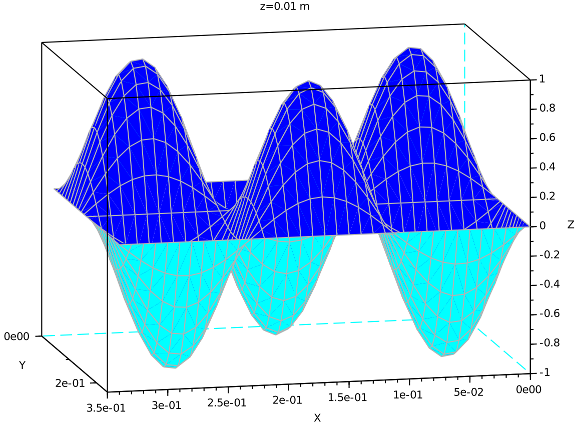
Take the cavity of a microwave oven.



Water molecules resonate at $2.45GHz$ which has a wavelength of $12cm$. The dimensions ($\underline{x}$, $\underline{y}$ and $\underline{z}$) of a cavity of a sample microwave oven are $36cm \times 24cm \times 36cm$. This results in the ratio $m : n : l = 3 : 2 : 3$.

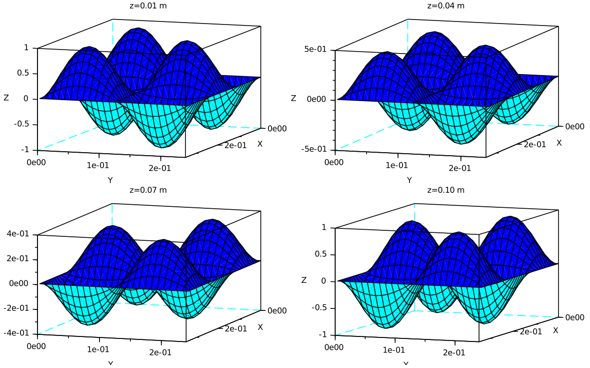
Modes - Example 1 (Contd..)

At a distance of $1cm$ from the door, the plot of E_z (i.e., along plane $z = 1cm$):



Modes - Example 1 (Contd..)

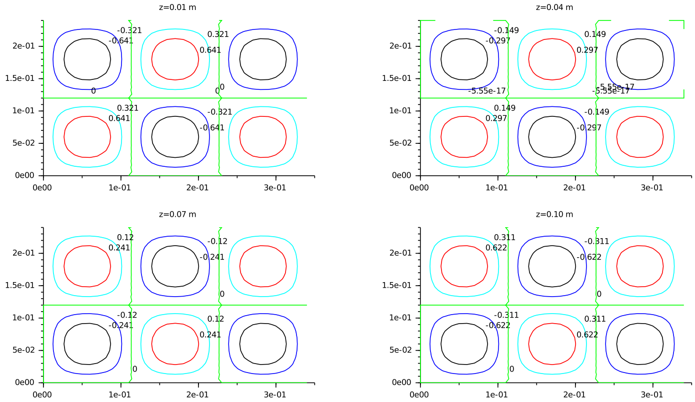
Plot of E_z for various values of z:



- Note the sinusoidal behavior of the peak amplitude
- The variation of the peak of z is also *sinusoidal*

Modes - Example 1 (Contd..)

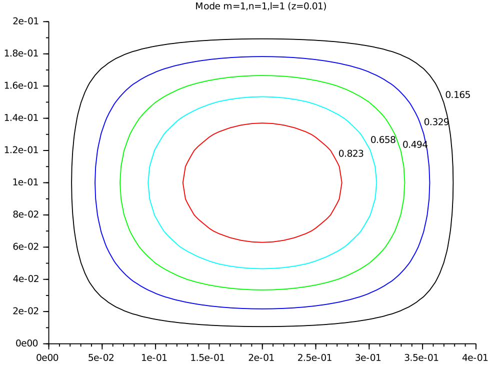
Contour plot of E_z for various values of z:



Modes - Example 2

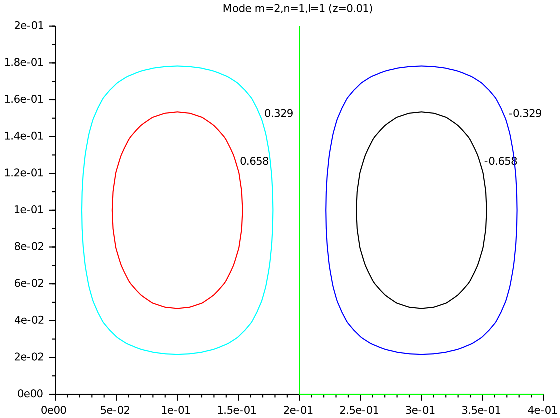
Modes of a cavity $40cm \times 20cm \times 20cm$ at $z = 1cm$.

n=1, m=1 and l=1



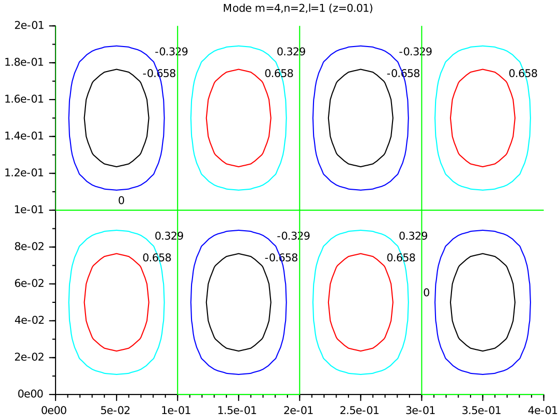
Modes - Example 2 (Contd..)

n=2, m=1 and l=1



Modes - Example 2 (Contd..)

n=4, m=2 and l=1

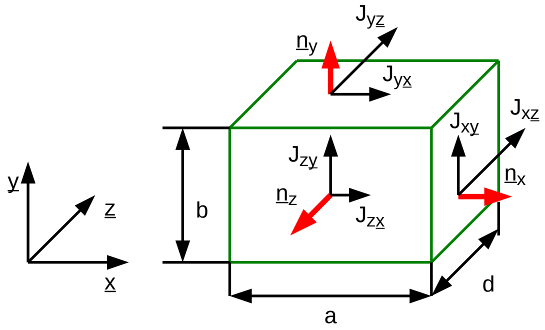


Wall Currents

Design Exercise 1

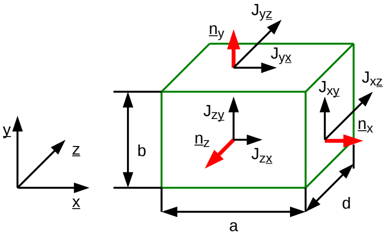
1. From the boundary condition $\underline{n} \times H^{\|} = J_S$ find the surface current equation for a cavity wall.
2. Based on this find the best location for positioning the conveyor belt in an industrial dryer.

Wall Currents



- For a practical conductor $\sigma < \infty$.
  - This results in a **skin depth** and **wall currents**

Wall Currents (Contd..)



Wall	Equation	$\underline{n}$
Bottom	$y=0$	$-\underline{y}$
Top	$y=b$	$\underline{y}$
Left	$x=0$	$-\underline{x}$
Right	$x=b$	$\underline{x}$
Front	$z=0$	$-\underline{z}$
Back	$z=d$	$\underline{z}$

Note: Left and right w.r.t. $\underline{z}$.

Taking $H = H_x\underline{x} + H_y\underline{y} + H_z\underline{z}$, for the top and bottom walls:

$$\begin{aligned} J_y &= \underline{n} \times H = \begin{vmatrix} \underline{x} & \underline{y} & \underline{z} \\ 0 & \pm 1 & 0 \\ H_x & H_y & H_z \end{vmatrix} \\ &= \pm(H_z\underline{x} - H_x\underline{z})\big|_{y=0,b} \end{aligned}$$

For the left and right walls:

$$\begin{aligned} J_x &= \pm(-H_z\underline{y} + H_y\underline{z})\big|_{x=0,a} \\ J_z &= \pm(-H_y\underline{x} + H_x\underline{y})\big|_{z=0,d} \end{aligned}$$

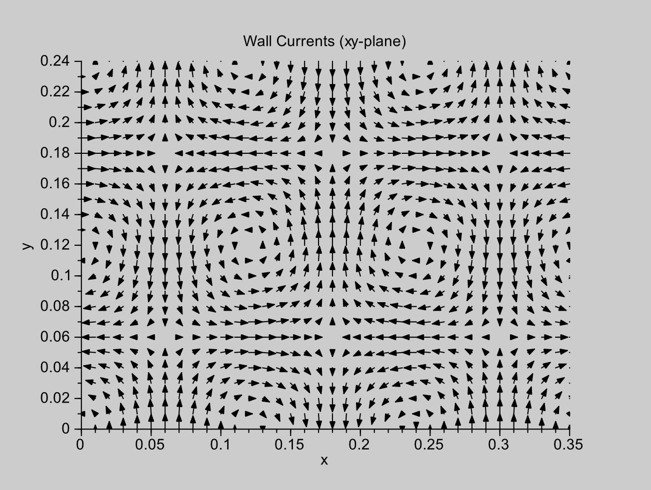
Wall Currents (Contd..)

Therefore,

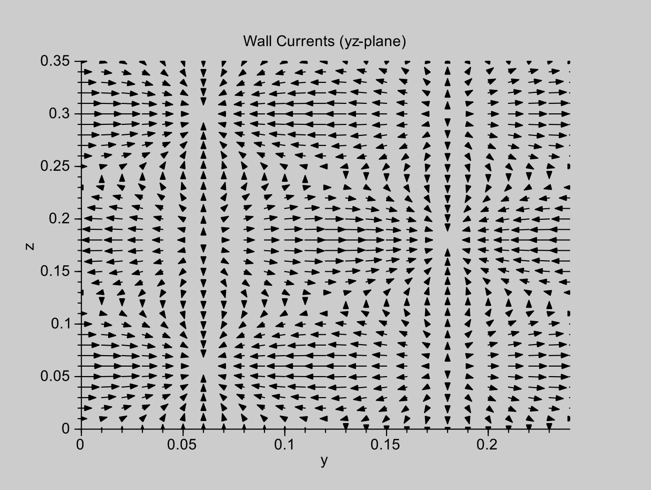
$$\begin{aligned}
J_x &= Z_H \left[\cos(k_y y)\sin(k_z z)\underline{y} + \frac{k_z k_y}{k_x^2+k_y^2}\sin(k_y y)\cos(k_z z)\underline{z}\right] \\
J_y &= Z_H \left[\cos(k_x x)\sin(k_z z)\underline{x} + \frac{k_z k_x}{k_x^2+k_y^2}\sin(k_x x)\cos(k_z z)\underline{z}\right] \\
J_z &= \frac{Z_H k_z}{k_x^2+k_y^2}\left[k_y\cos(k_x x)\sin(k_y y)\underline{x} - k_x\sin(k_x x)\cos(k_y y)\underline{y}\right]
\end{aligned}$$

Note: The $\pm$ depends on the sign of $\cos(q\pi)$ where $q = m, n, l$

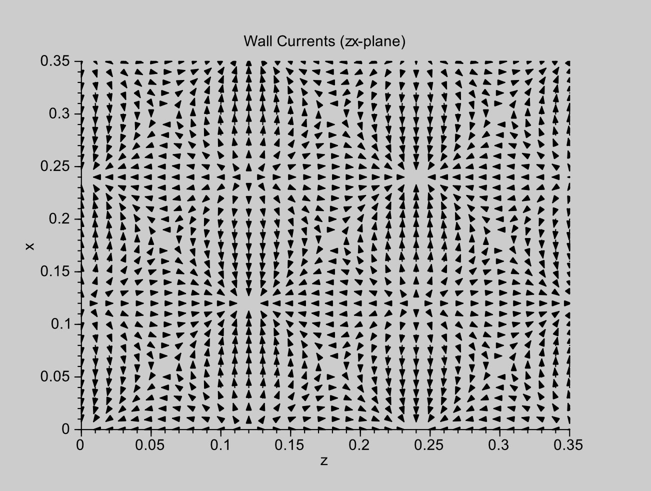
Microwave Oven TE Wall Currents - XY Plane



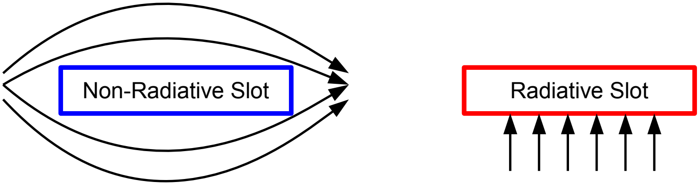
Microwave Oven TE Wall Currents - YZ Plane



Microwave Oven TE Wall Currents - ZX Plane

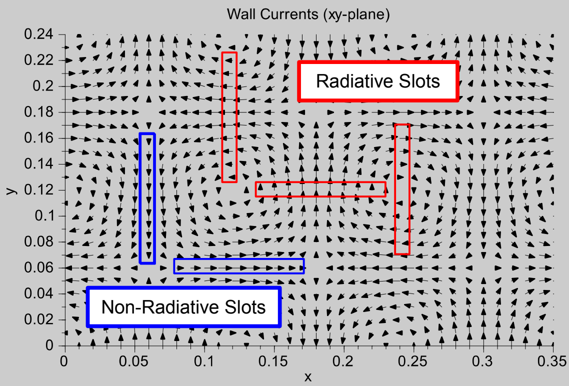


Slots



- A slot is a removal of metal from the cavity wall
- If a slot is placed such that the current can easily bypass it, it is *non-radiative*
  - Has to be thin so that the parallel current can flow around it
  - If not, (i.e., current is normal) the slot is *radiative*
  - Even in a non-radiative slot a small amount of radiation can leak out

Slot Placement



- Slot opening for the conveyor belt has be non-radiative
    - A suitable location may not be practicable

Conclusion

Summary

- In cavities the EM waves are contained by six boundaries
  - Cavities contain energy (e.g. as in the microwave oven)
- A mode is a possible solution for the E and H fields of a cavity
  - All modes have the same frequency
  - In a cavity (and waveguide) only transverse E or H modes can occur

Motivating Question

- What will happen to the EM wave within the cavity if two facing sides of the cavity are removed?

Next Topic..

Rectangular Waveguides