Communication Theory II

Lecture 2: Baseband Pulse Transmission : Inter Symbol Interference

Most of the content are adapted from Communication Systems (chapter 3), Simon_Haykin.

Baseband Pulse Transmission

What is Modulation?

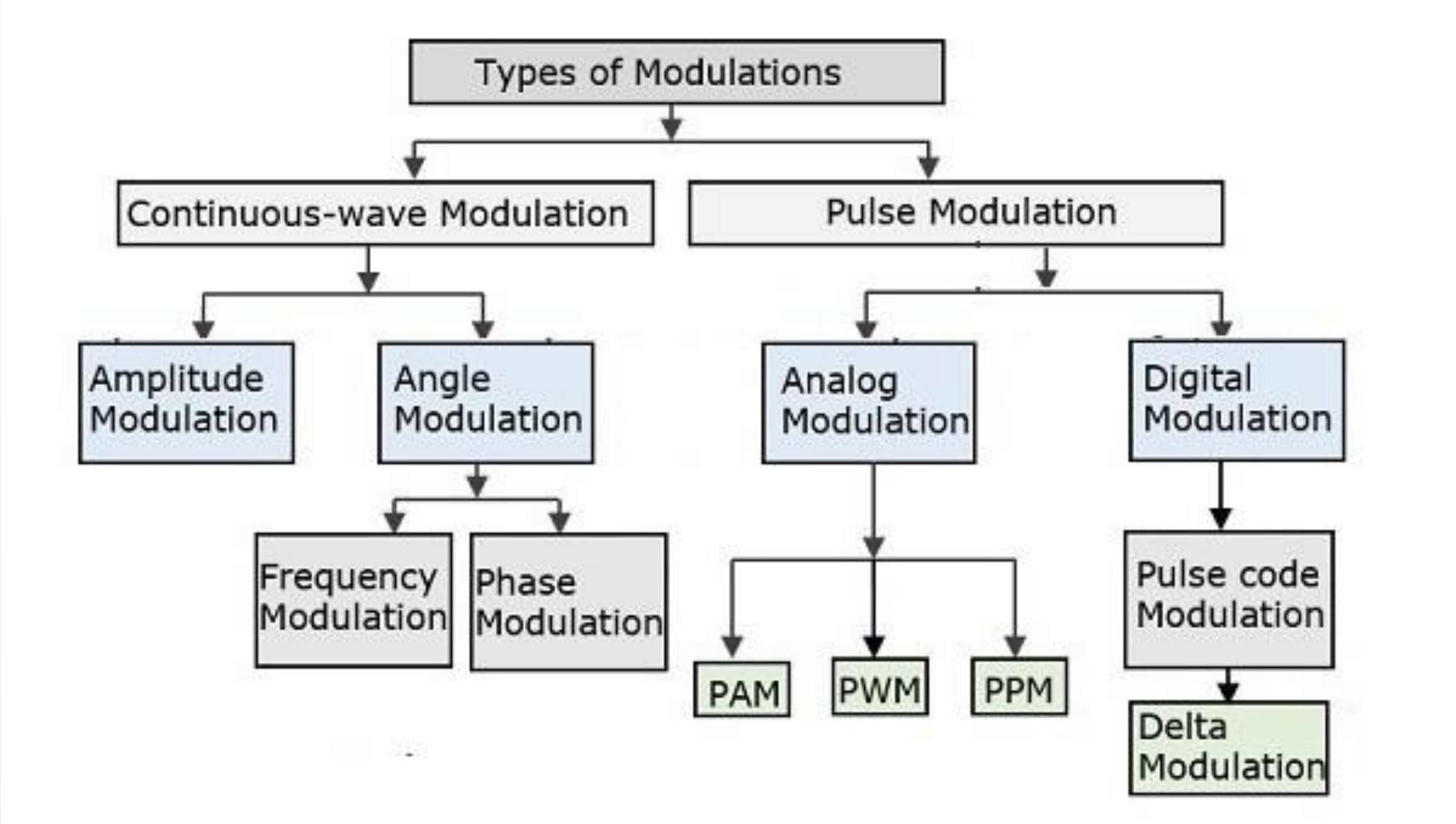
Modulation refers to the process of modifying a carrier signal, typically a high-frequency waveform, to carry information or data.

In modulation, the information signal (also called the baseband signal) is superimposed or impressed onto the carrier signal, altering its properties such as amplitude, frequency, or phase. The resulting modulated signal, known as the modulated carrier, carries the information to be transmitted.

Why we need modulation?

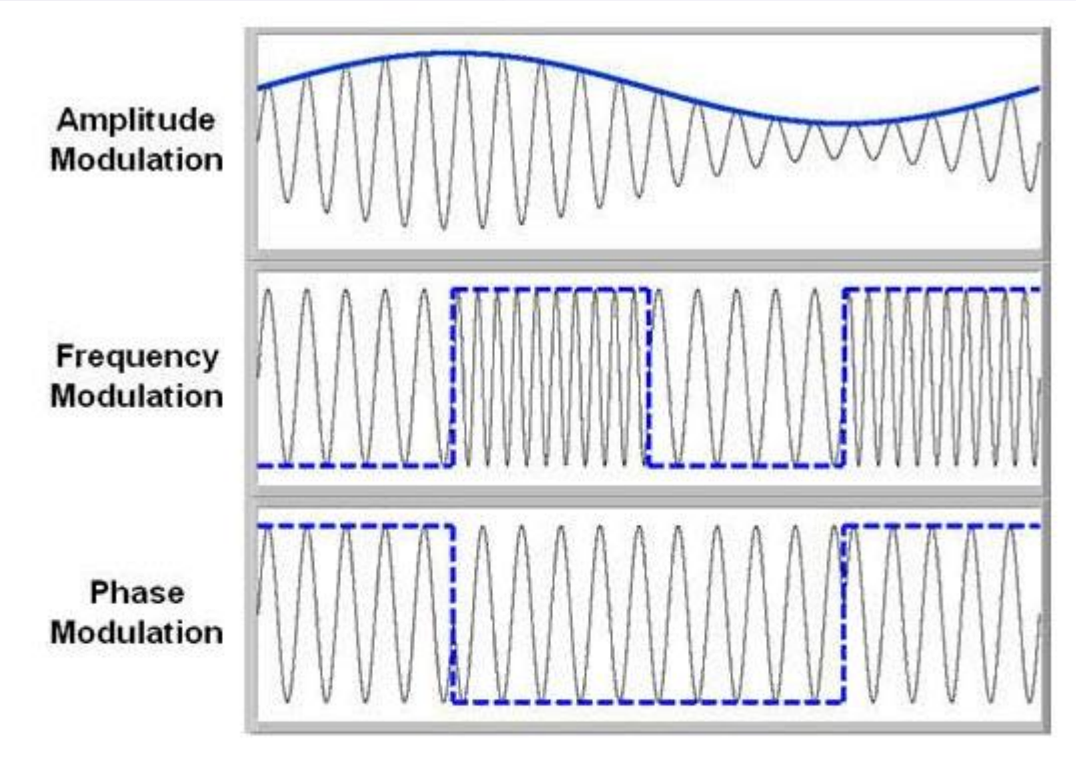
- **Transmission Efficiency**: Modulation enables the efficient use of the available bandwidth of communication channels. By shifting the frequency spectrum of the baseband signal to higher frequencies, multiple signals can be transmitted simultaneously without interference.
- **Signal Integrity**: Modulating the carrier signal helps in preserving the integrity of the information being transmitted. By utilizing specific modulation schemes, the modulated signal can be robust against noise, interference, and channel impairments, ensuring reliable transmission and reception of the information.
- **Compatibility**: Modulation allows for compatibility between different communication systems and devices. By using standardized modulation techniques, communication systems can interoperate, enabling seamless communication between different technologies and platforms.

Types of Modulation

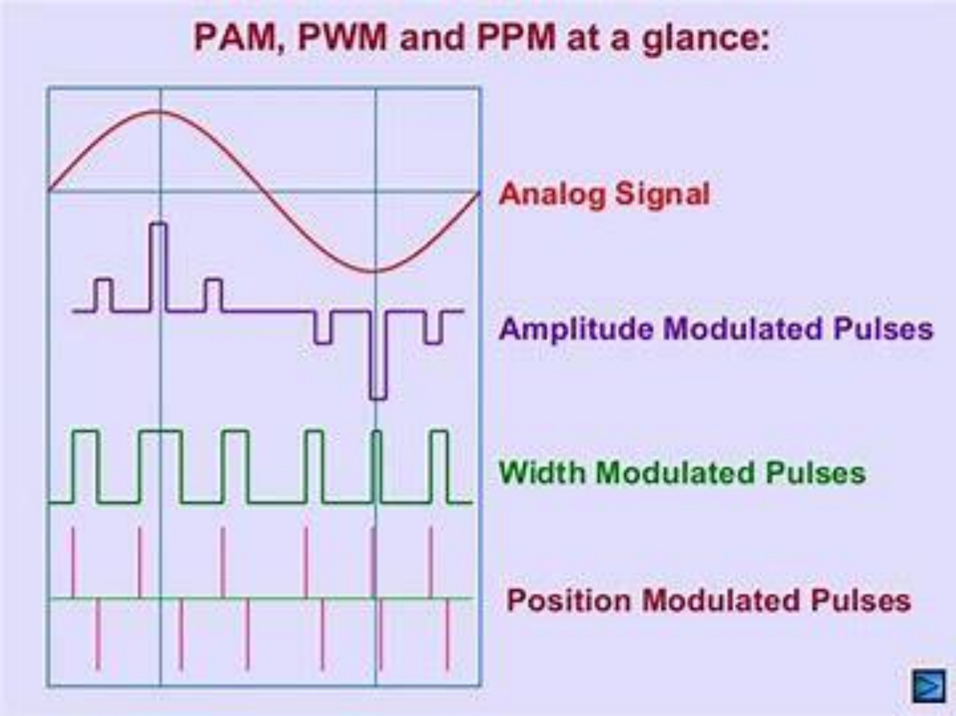


Types of Modulation

CW Modulation



Analog pulse Modulation



Types of Modulation

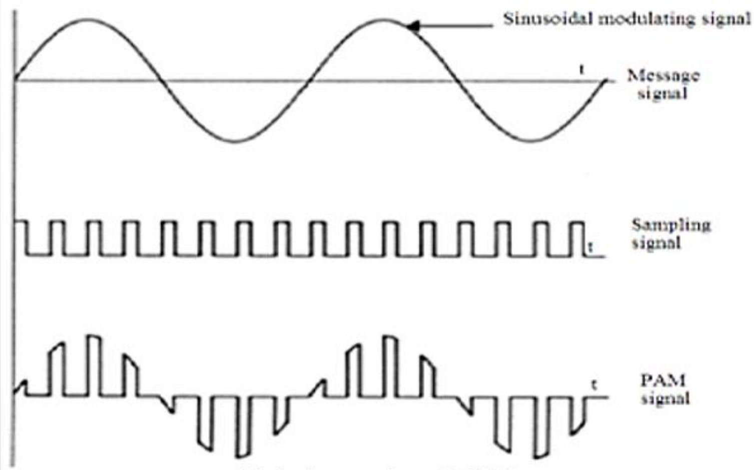


Fig1. Generation of PAM

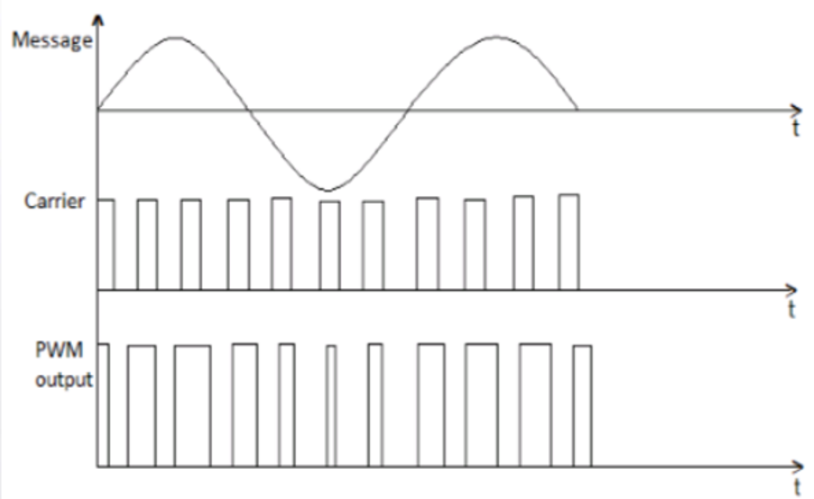


Fig2. Generation of PWM

Pulse Width Modulation (PWM)

Pulse Amplitude Modulation (PAM)

Pulse Position Modulation (PPM)

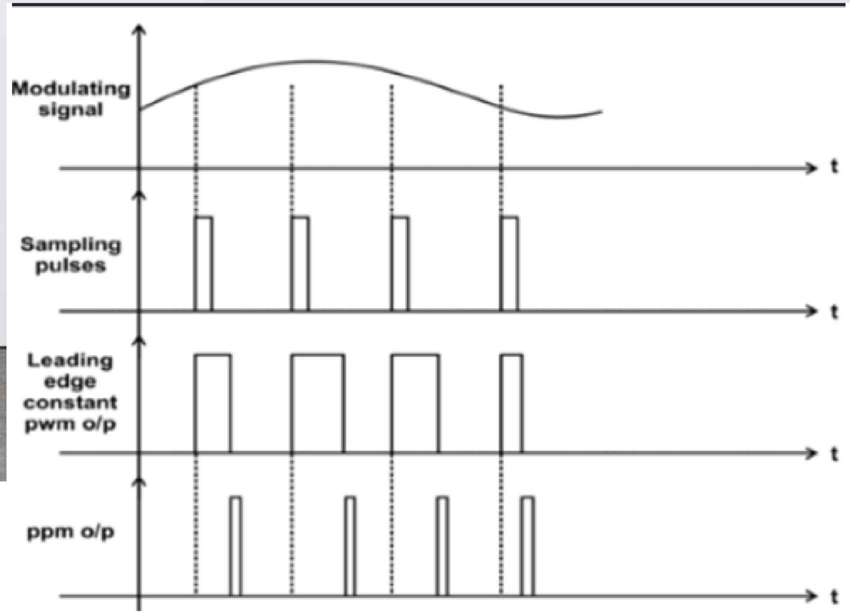


Fig3. Generation of PPM

Types of Modulation

Pulse code Modulation

Pulse Code Modulation (PCM) is a digital modulation technique used for encoding analog audio or voice signals into a digital format for transmission or storage. It is widely used in telecommunication systems, audio recording, and playback devices.

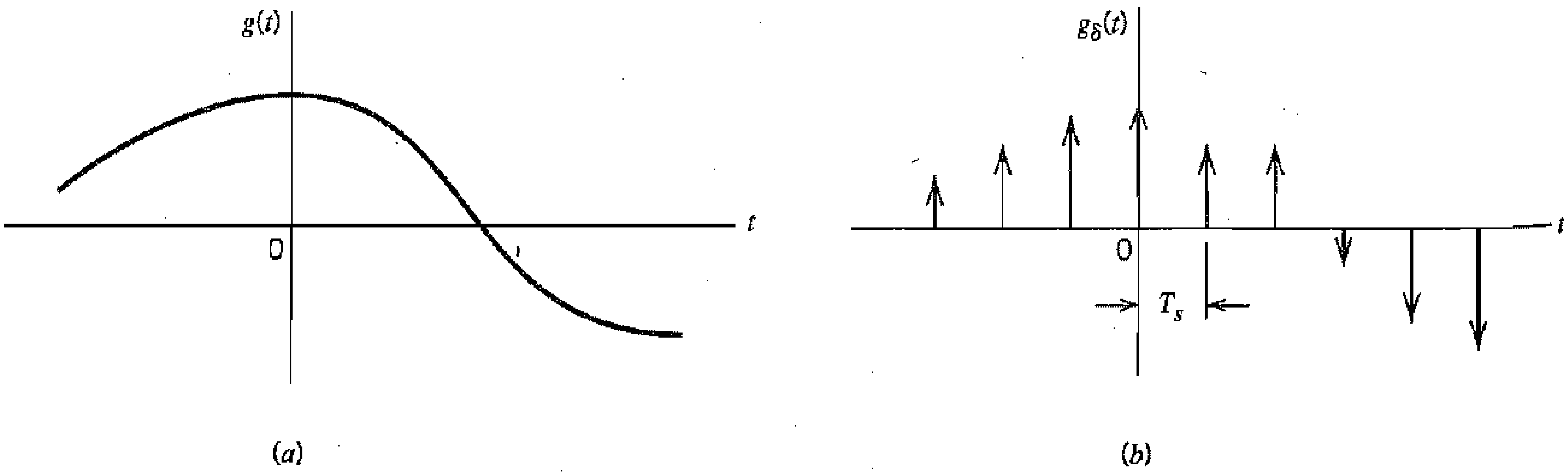
The PCM process involves the following steps:

- Sampling
- Quantization
- Encoding
- Transmission or Storage

During playback or reception, the PCM process is reversed:

- Decoding
- Digital-to-Analog Conversion
- Filtering and Reconstruction

Sampling Process



The sampling process. (*a*) Analog signal. (*b*) Instantaneously sampled version of the analog signal.

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\ \delta(t - nT_s)$$

We refer to $g_\delta(t)$ as the *ideal sampled signal*. The term $\delta(t - nT_s)$ represents a delta function positioned at time $t = nT_s$.

Sampling Signals-Basics- Prof. Lain

[Sampling Signals - YouTube]

Sampling Process

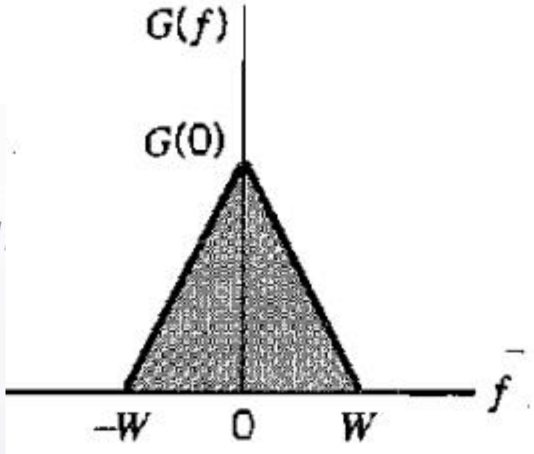
$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, \delta(t - nT_s)$$

Taking Fourier transform in both sides

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s) \exp(-j2\pi nfT_s)$$

This relation is called the *discrete-time Fourier transform*. It may be viewed as a complex Fourier series representation of the periodic frequency function $G_\delta(f)$, with the sequence of samples $\{g(nT_s)\}$ defining the coefficients of the expansion.

Sampling Process



- Suppose that g(t) is strictly band limited
  - With no frequency components higher than frequency W
  - G(f)=0 for $|f| \geq W$

  - Suppose we chose T_s=1/2W.

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s) \exp(-j2\pi n f T_s)$$

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right)$$

Sampling Process

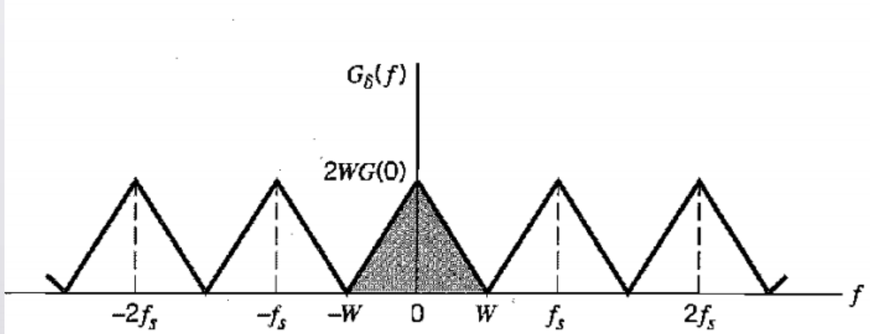
Using the table of Fourier-transform pairs, we may write

$$g_\delta(t) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f - mf_s)$$

where $G(f)$ is the Fourier transform of the original signal $g(t)$, and f_s is the sampling rate. Equation (3.2) states that *the process of uniformly sampling a continuous-time signal of finite energy results in a periodic spectrum with a period equal to the sampling rate.*

Fourier transform of $g_\delta(t)$ may also be expressed as

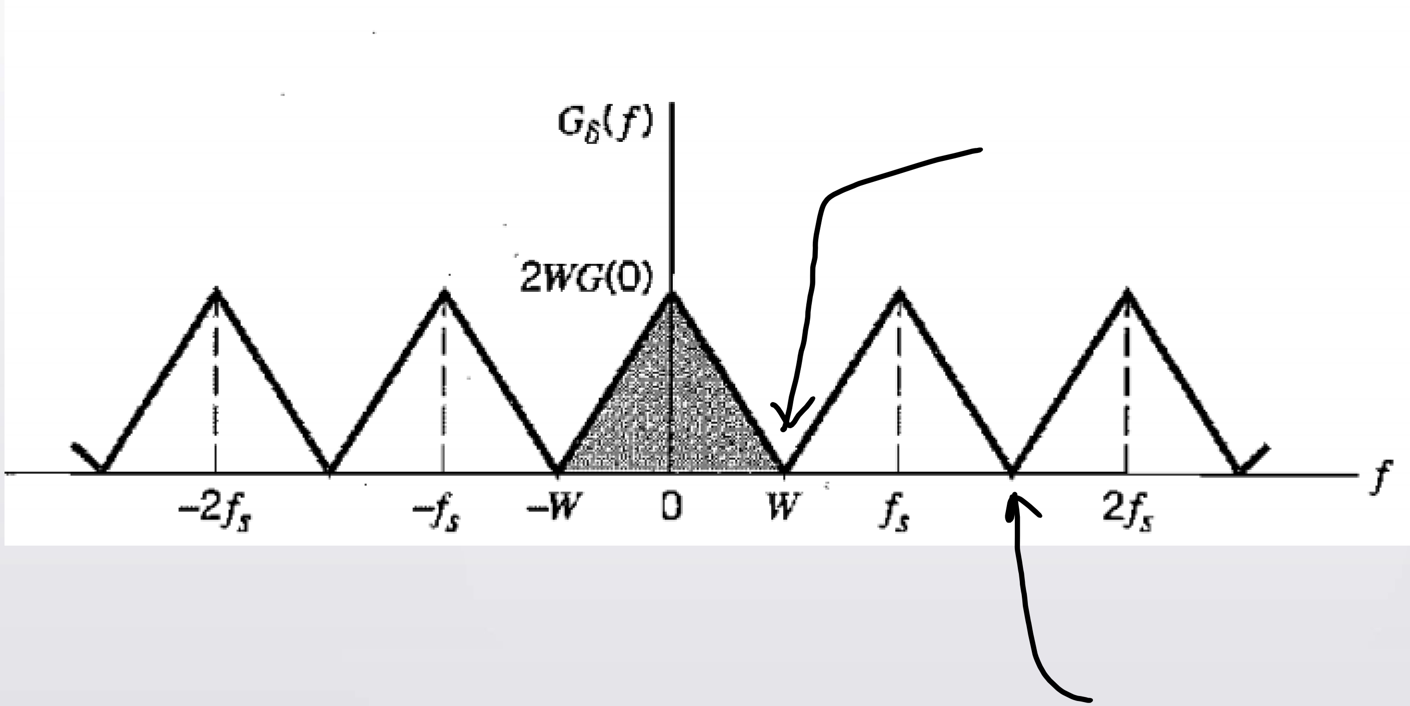
$$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} G(f - mf_s)$$



Fourier transformation fair

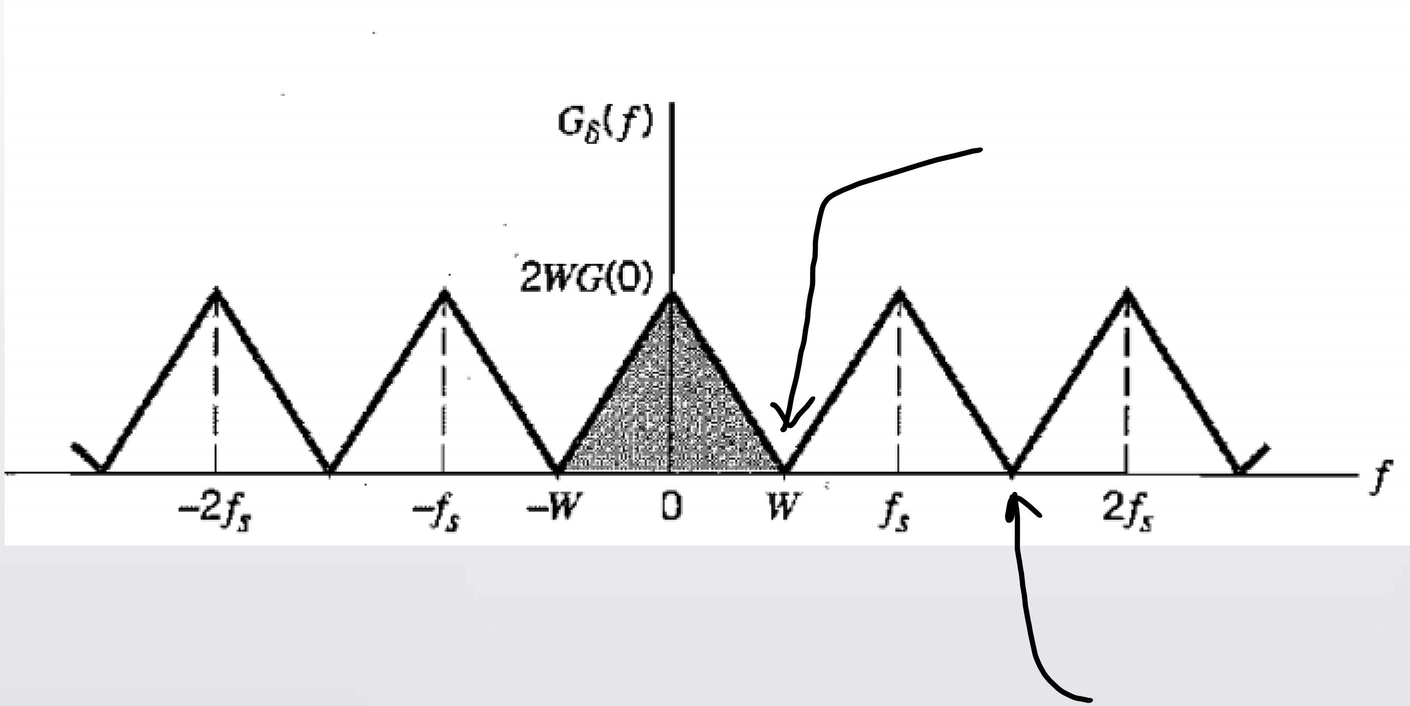
Time Function	*Fourier Transform*
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

Sampling Process



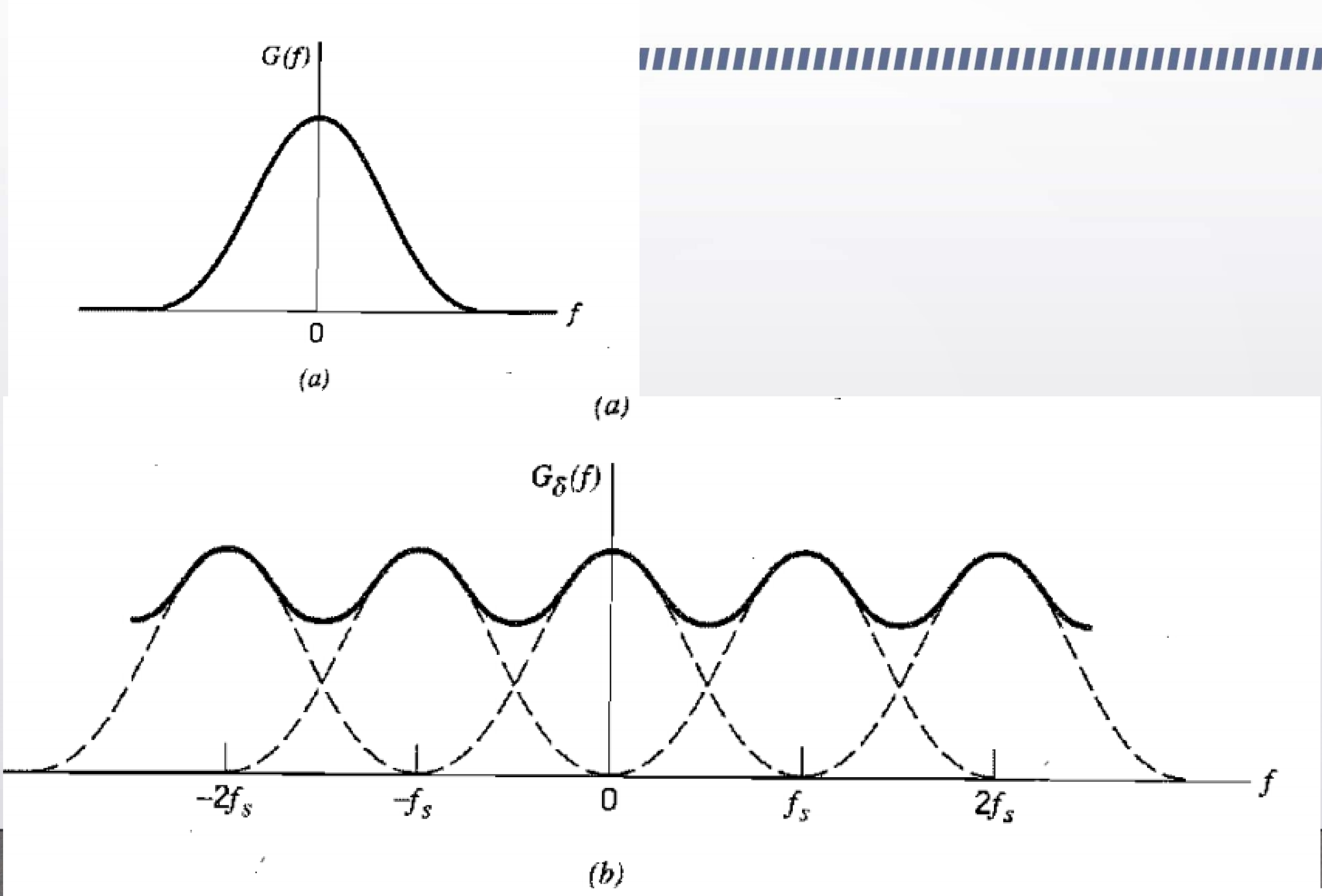
Fs >=?

Sampling Process



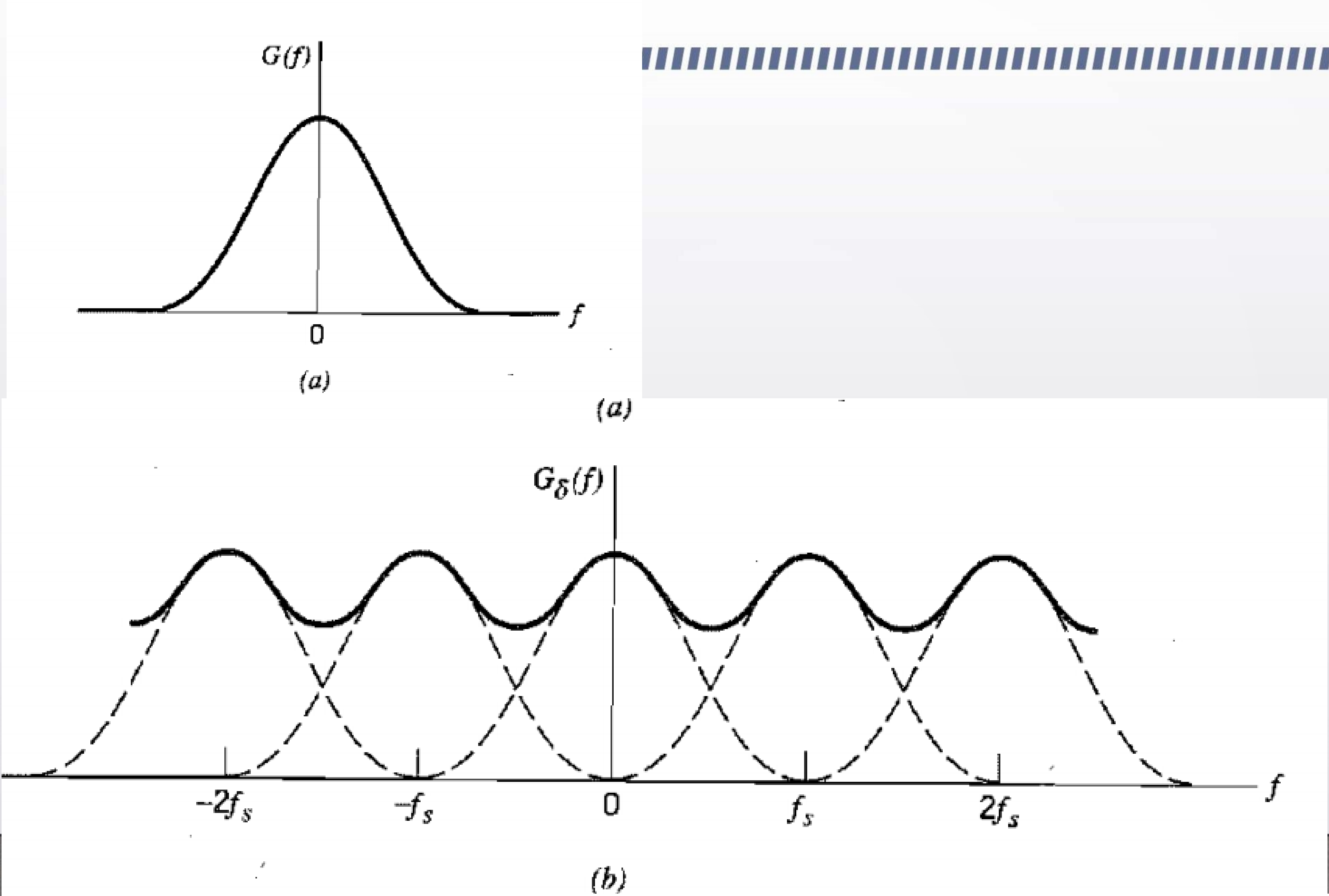
Fs < 2W?

ISI- Inter symbol Interference



(*a*) Spectrum of a signal. (*b*) Spectrum of an undersampled version of the signal exhibiting the aliasing phenomenon.

ISI- Inter symbol Interference



(*a*) Spectrum of a signal. (*b*) Spectrum of an undersampled version of the signal exhibiting the aliasing phenomenon.

Sampling Process- Reconstruction

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right), \qquad -W < f < W$$

$$\begin{aligned} g(t) &= \int_{-\infty}^{\infty} G(f) \exp(j2\pi f t)\, df \\ &= \int_{-W}^{W} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right) \exp(j2\pi f t)\, df \end{aligned}$$

Interchanging the order of summation and integration:

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{1}{2W} \int_{-W}^{W} \exp\left[j2\pi f\left(t - \frac{n}{2W}\right)\right] df$$

The integral term in Equation is readily evaluated, yielding the final result

$$\begin{aligned} g(t) &= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{\sin(2\pi W t - n\pi)}{(2\pi W t - n\pi)} \\ &= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \operatorname{sinc}(2Wt - n), \qquad -\infty < t < \infty \end{aligned}$$

Sampling Process- Reconstruction

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{\sin(2\pi Wt - n\pi)}{(2\pi Wt - n\pi)}$$

$$= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \operatorname{sinc}(2Wt - n), \qquad -\infty < t < \infty$$

$$T_s = 1/2W.$$

What is this?

provides an *interpolation formula* for reconstructing the original signal $g(t)$ from the sequence of sample values $\{g(n/2W)\}$, with the sinc function $\operatorname{sinc}(2Wt)$ playing the role of an *interpolation function*. Each sample is multiplied by a delayed version of the interpolation function, and all the resulting waveforms are added to obtain $g(t)$.

Sampling Signals: Introduction Lecture- Prof. Iain
[Sampling Signals: Introduction Lecture - YouTube]

How are Signals Reconstructed from Digital Samples?- Prof. Iain
[How are Signals Reconstructed from Digital Samples? - YouTube]

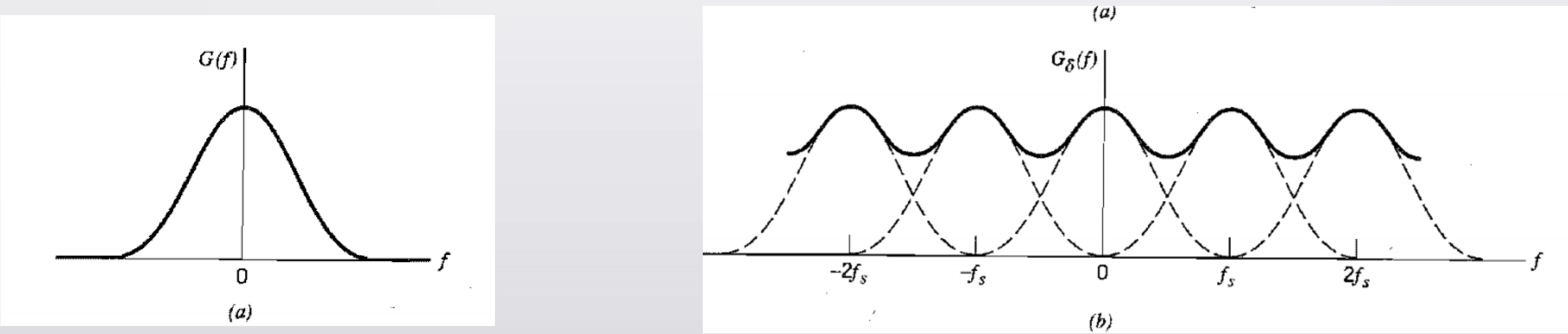
Sampling Theorem

1. *A band-limited signal of finite energy, which has no frequency components higher than W Hertz, is completely described by specifying the values of the signal at instants of time separated by 1/2W seconds.*
2. *A band-limited signal of finite energy, which has no frequency components higher than W Hertz, may be completely recovered from a knowledge of its samples taken at the rate of 2W samples per second.*

The sampling rate of $2W$ samples per second, for a signal bandwidth of W Hertz, is called the *Nyquist rate*; its reciprocal $1/2W$ (measured in seconds) is called the *Nyquist interval.*

ISI

The derivation of the sampling theorem, as described herein, is based on the assumption that the signal $g(t)$ is strictly band limited. In practice, however, an information-bearing signal is *not* strictly band limited, with the result that some degree of undersampling is encountered. Consequently, some *aliasing* is produced by the sampling process. Aliasing refers to the phenomenon of a high-frequency component in the spectrum of the signal seemingly taking on the identity of a lower frequency in the spectrum of its sampled version

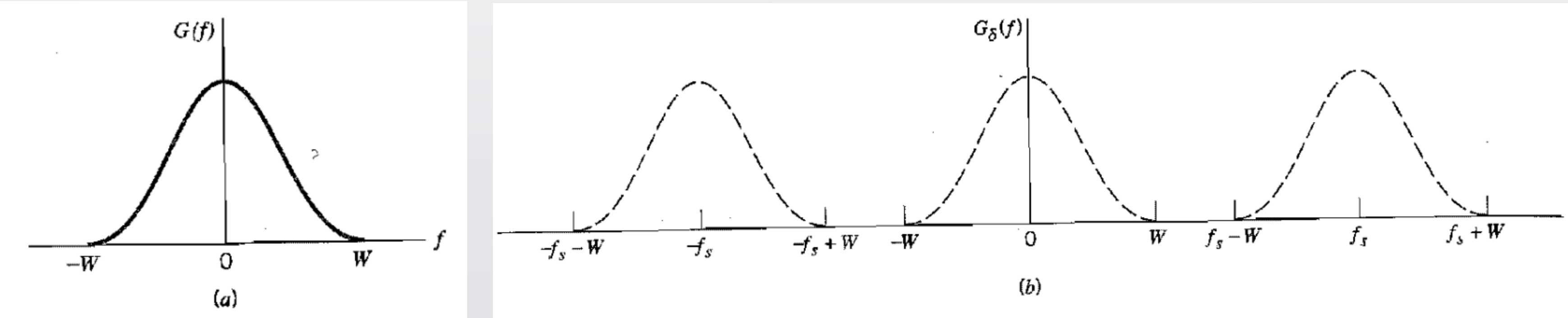


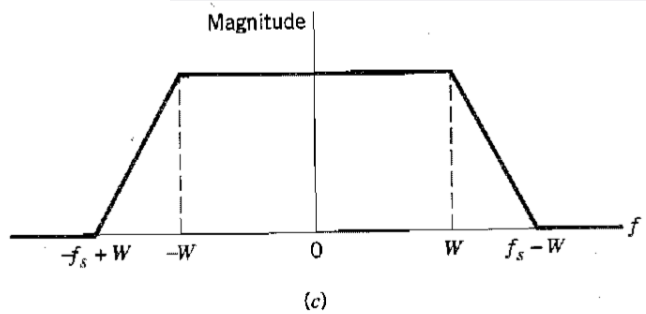
(*a*) Spectrum of a signal. (*b*) Spectrum of an undersampled version of the signal exhibiting the aliasing phenomenon.

ISI

To combat the effects of aliasing in practice, we may use two corrective measures, as described here:

1. Prior to sampling, a low-pass *anti-aliasing filter* is used to attenuate those high-frequency components of the signal that are not essential to the information being conveyed by the signal.
2. The filtered signal is sampled at a rate slightly higher than the Nyquist rate.





(*a*) Anti-alias filtered spectrum of an information-bearing signal. (*b*) Spectrum of instantaneously sampled version of the signal, assuming the use of a sampling rate greater than the Nyquist rate. (*c*) Magnitude response of reconstruction filter.

Quantization

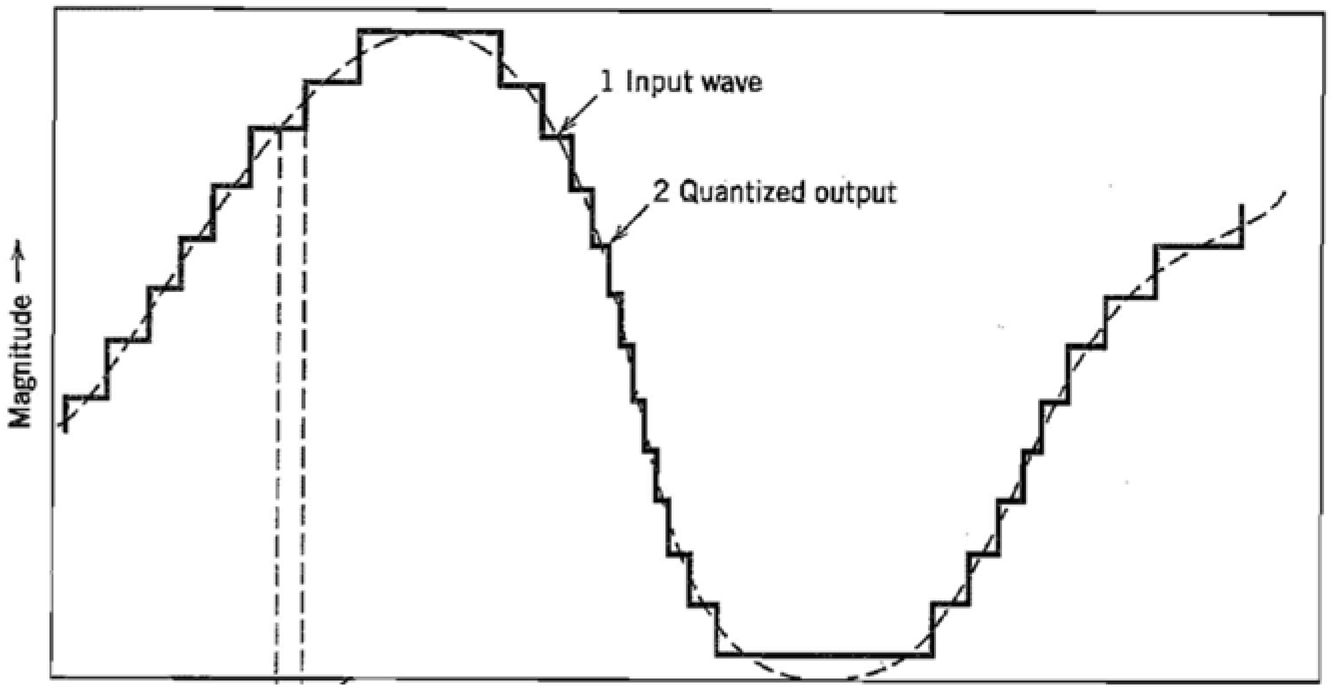
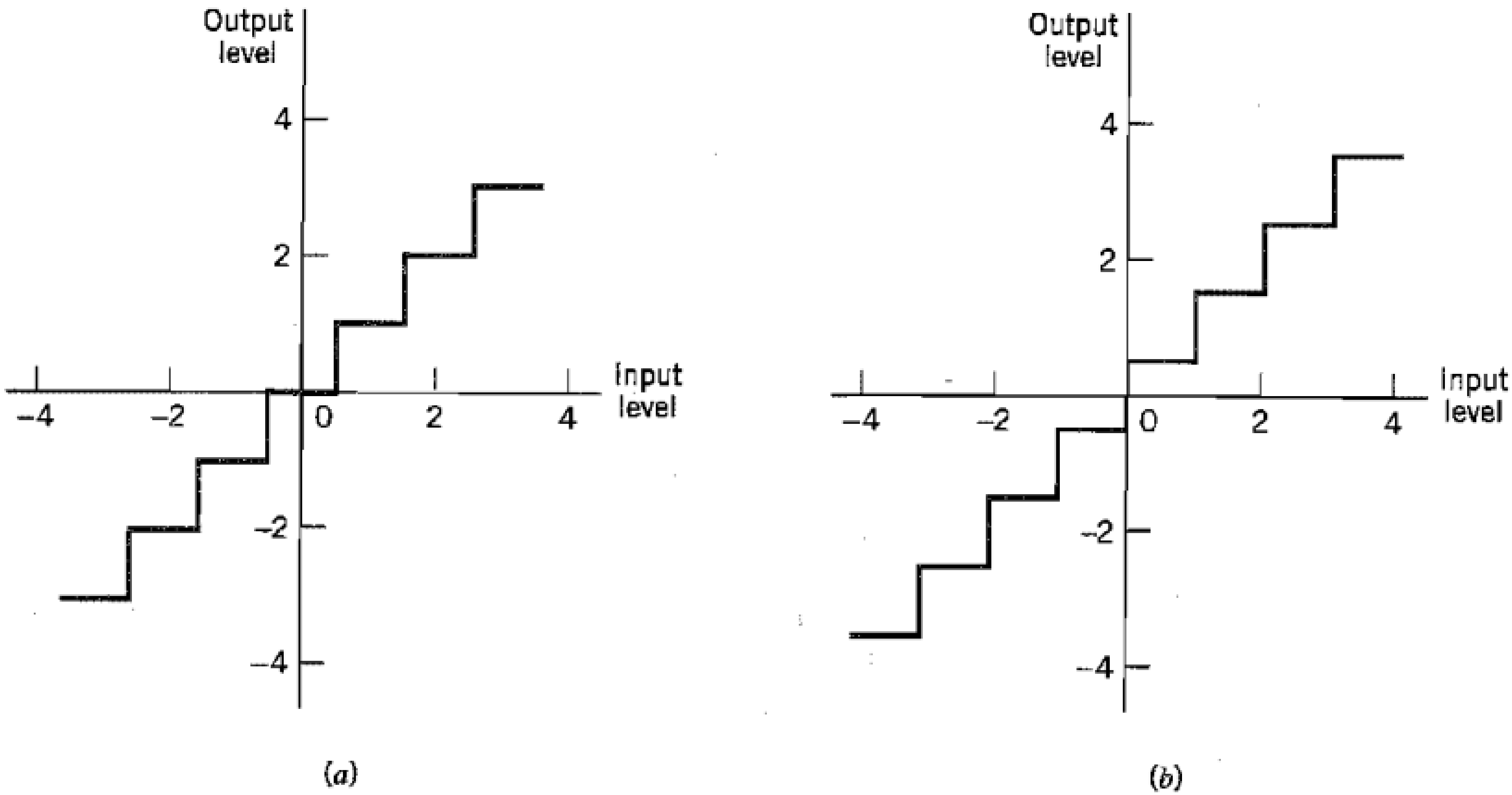


Illustration of the quantization process.

Quantization



Two types of quantization: (*a*) midtread and (*b*) midrise.

Quantization Noise

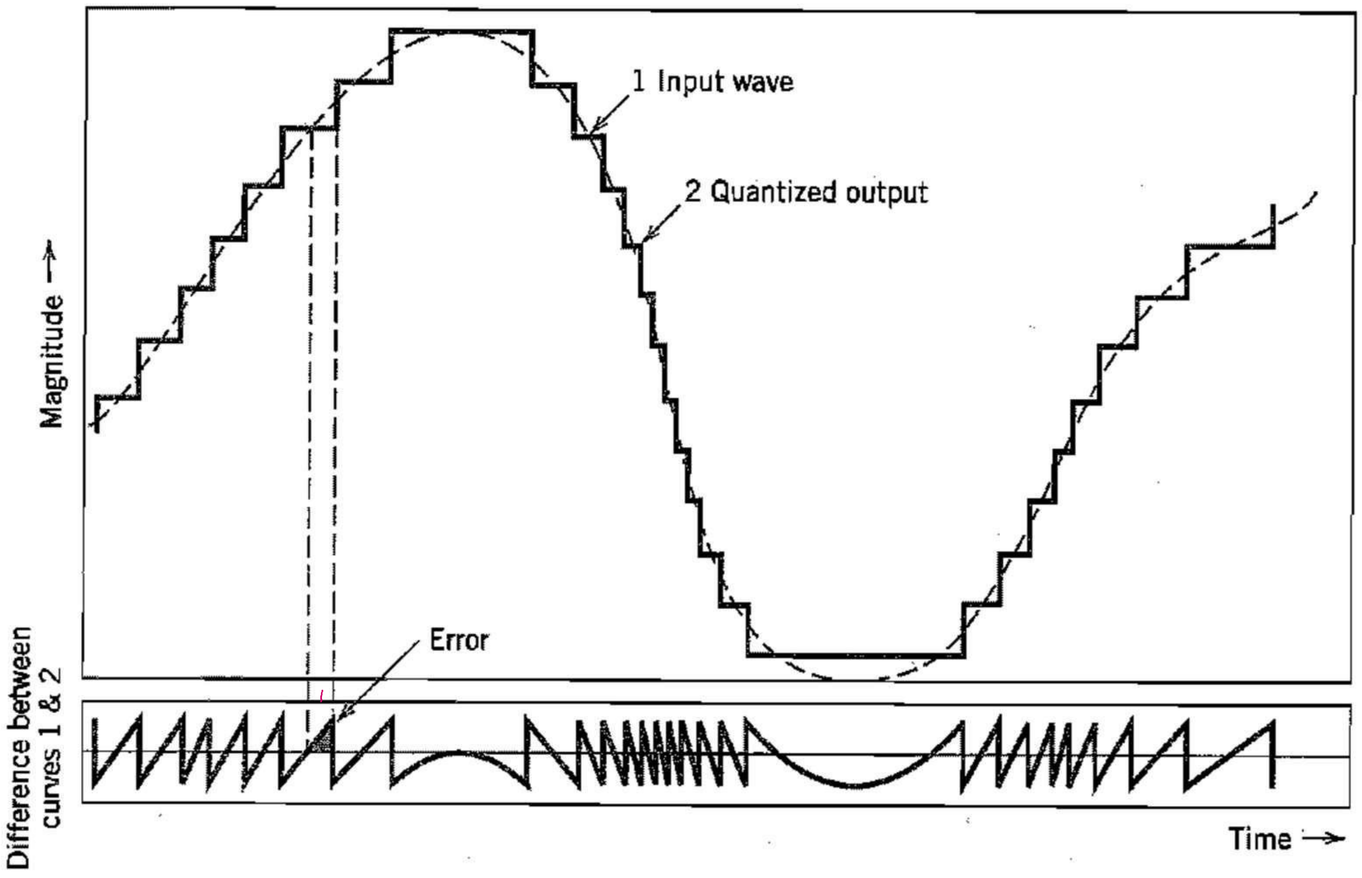


Illustration of the quantization process.

Quantization Noise

Consider then an input m of continuous amplitude in the range $(-m_{max}, m_{max})$. Assuming a uniform quantizer of the midrise type the step-size of the quantizer is given by

$$\Delta = \frac{2m_{max}}{L}$$

where L is the total number of representation levels. For a uniform quantizer, the quantization error Q will have its sample values bounded by $-\Delta/2 \leq q \leq \Delta/2$.

Quantization error

Suppose we have an analog signal with a range of voltage levels from -4V to +4V. We want to quantize this signal using 3-bit PCM, which means we will have 2^3 = 8 possible quantization levels. Let's assume the analog signal takes on a specific voltage level of +1.7V. What is the quantization error?

Quantization range: (+4V) - (-4V) = 8V
Quantization step size: 8V / 8 levels = 1V
The closest quantization level to this voltage is +2V.
Quantization level: +2V
Analog signal level: +1.7V
Quantization error: +2V - +1.7V = +0.3V

Calculate quantization error for 6 bits?

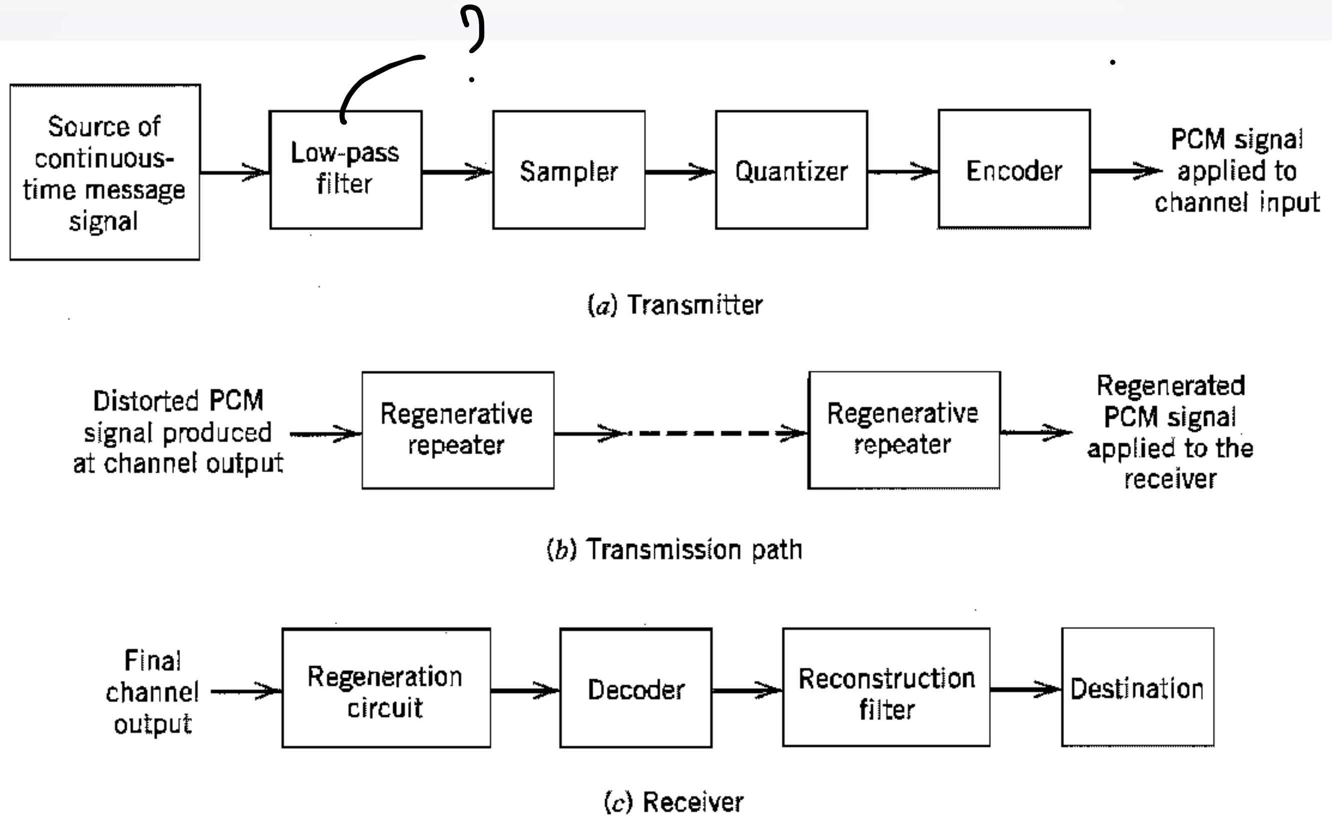
Quantization range: (+4V) - (-4V) = 8V
Quantization step size: 8V / 64 levels = 0.125V
Closest quantization level: +1.75V (nearest quantization level based on the step size)
Quantization error: +1.75V - +1.7V = +0.05V

Quantization error

Signal-to-(quantization) noise ratio for varying number of representation levels for sinusoidal modulation

Number of Representation Levels, L	Number of Bits per Sample, R	Signal-to-Noise Ratio (dB)
32	5	31.8
64	6	37.8
128	7	43.8
256	8	49.8

Pulse Code Modulation



The basic elements of a PCM system.

Encoding

- In combining the process of sampling and quantizing, the specification of a continuous message (baseband) signal becomes limited discrete set of values but not in the form best suited to transmission over telephone line or transmission line.

In summary, while sampling and quantization convert the continuous baseband signal into a discrete set of values, additional processing, such as modulation, is often required to prepare the signal for efficient transmission over telephone lines or transmission lines. Modulation techniques help optimize bandwidth utilization and enable reliable communication over the channel.

Binary Number System for R=4 bits/sample

Binary number system for R = 4 bits/sample

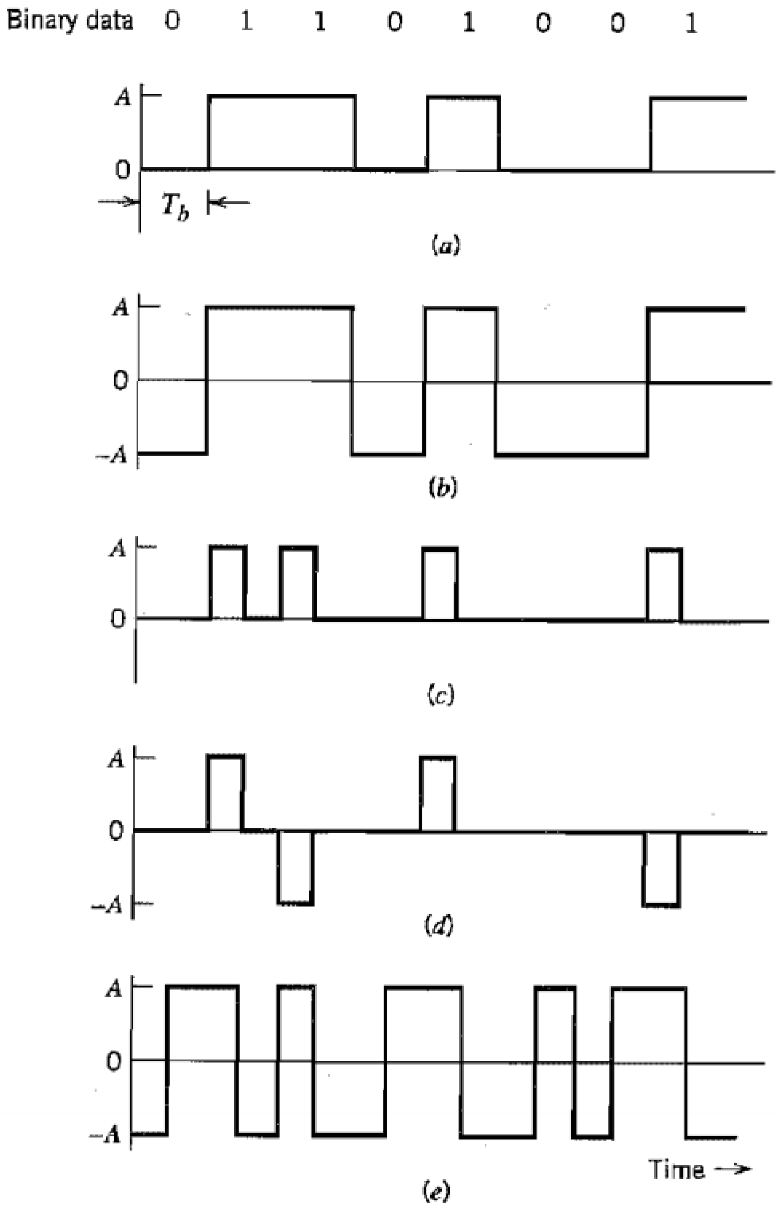
Ordinal Number of Representation Level	*Level Number Expressed as Sum of Powers of 2*	*Binary Number*
0		0000
1	2^0	0001
2	2^1	0010
3	$2^1 + 2^0$	0011
4	2^2	0100
5	$2^2 + 2^0$	0101
6	$2^2 + 2^1$	0110
7	$2^2 + 2^1 + 2^0$	0111
8	2^3	1000
9	$2^3 + 2^0$	1001
10	$2^3 + 2^1$	1010
11	$2^3 + 2^1 + 2^0$	1011
12	$2^3 + 2^2$	1100
13	$2^3 + 2^2 + 2^0$	1101
14	$2^3 + 2^2 + 2^1$	1110
15	$2^3 + 2^2 + 2^1 + 2^0$	1111

Any plan for representing each of this discrete set of values as a particular arrangement of discrete events is called a *code*. One of the discrete events in a code is called a *code element* or *symbol*. For example, the presence or absence of a pulse is a symbol. A particular arrangement of symbols used in a code to represent a single value of the discrete set is called a *code word* or *character*.

In a *binary code*, each symbol may be either of two distinct values or kinds, such as the presence or absence of a pulse. The two symbols of a binary code are customarily denoted as 0 and 1.

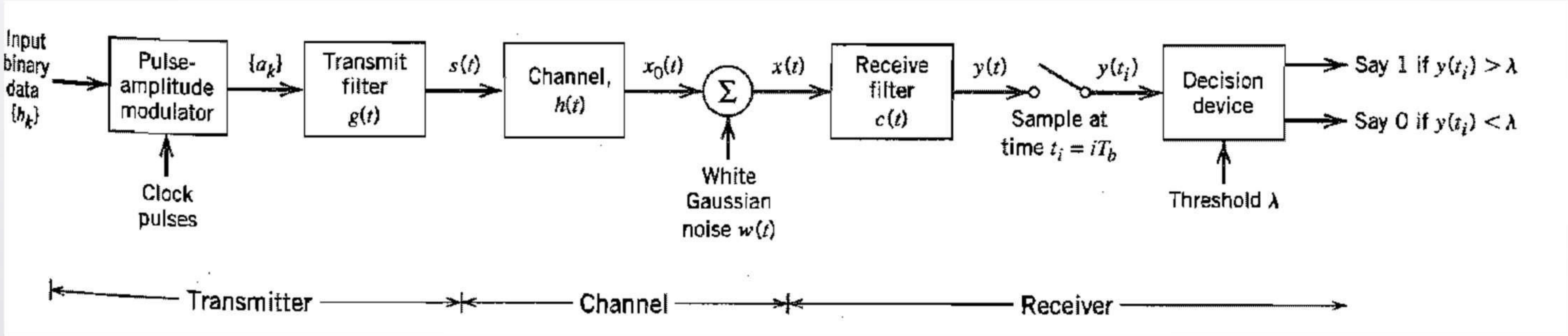
Line code

Any of several line codes can be used for the electrical representation of a binary data stream.

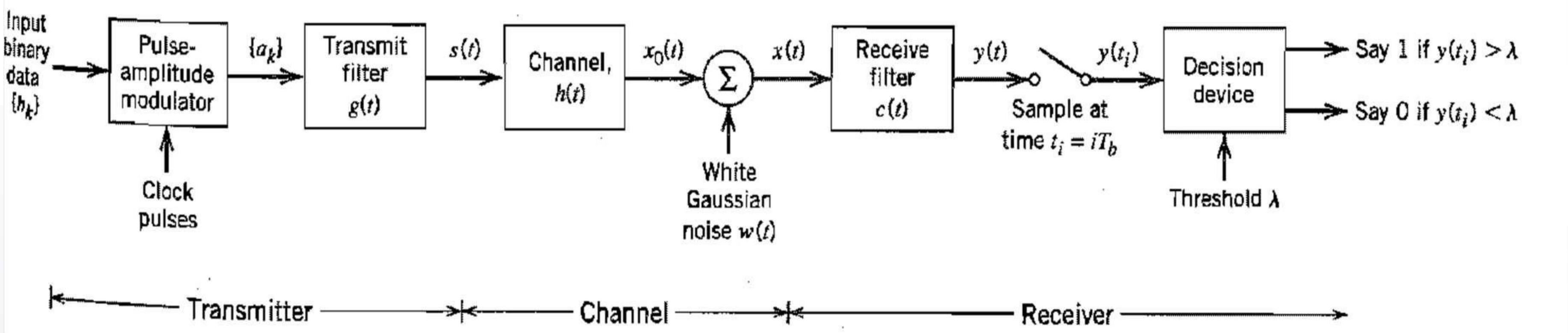


Line codes for the electrical representations of binary data. (*a*) Unipolar NRZ signaling. (*b*) Polar NRZ signaling. (*c*) Unipolar RZ signaling. (*d*) Bipolar RZ signaling. (*e*) Split-phase or Manchester code.

Baseband Binary Data Transmission System ISI with more detail



Baseband Binary PAM



$$a_k = \begin{cases} +1 & \text{if symbol } b_k \text{ is } 1 \\ -1 & \text{if symbol } b_k \text{ is } 0 \end{cases}$$

The sequence of short pulses so produced is applied to a *transmit filter* of impulse response $g(t)$, producing the transmitted signal

$$s(t) = \sum_k a_k g(t - kT_b)$$

$$y(t) = \mu \sum_k a_k p(t - kT_b) + n(t)$$

The scaled pulse $\mu p(t)$ is obtained by a double convolution involving the impulse response $g(t)$ of the transmit filter, the impulse response $h(t)$ of the channel, and the impulse response $c(t)$ of the receive filter, as shown by

$$\mu p(t) = g(t) \star h(t) \star c(t)$$

$$y(t) = \mu \sum_{k} a_k p(t - kT_b) + n(t)$$

The receive filter output $y(t)$ is sampled at time $t_i = iT_b$ (with i taking on integer values), yielding

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k p[(i - k)T_b] + n(t_i)$$

$$= \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p[(i - k)T_b] + n(t_i)$$

In Equation . the first term μa_i represents the contribution of the ith transmitted bit. The second term represents the residual effect of all other transmitted bits on the decoding of the ith bit; this residual effect due to the occurrence of pulses before and after the sampling instant t_i is called intersymbol interference (ISI). The last term $n(t_i)$ represents the noise sample at time t_i.

Absence of ISI and Noise

$$y(t_i) = \mu a_i$$

which shows that, under these ideal conditions, the ith transmitted bit is decoded correctly. The unavoidable presence of ISI and noise in the system, however, introduces errors in the decision device at the receiver output. Therefore, in the design of the transmit and receive filters, the objective is to minimize the effects of noise and ISI and thereby deliver the digital data to their destination with the smallest error rate possible.

When the signal-to-noise ratio is high, as is the case in a telephone system, for example, the operation of the system is largely limited by ISI rather than noise; in other words, we may ignore $n(t_i)$.

Condition for free ISI

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k p[(i-k)T_b] + n(t_i)$$
$$= \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p[(i-k)T_b] + n(t_i) \tag{4.48}$$

$$p(iT_b - kT_b) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases} \tag{4.49}$$

where $p(0) = 1$, by normalization. If $p(t)$ satisfies the conditions of Equation (4.49), the receiver output $y(t_i)$ given in Equation (4.48) simplifies to (ignoring the noise term)

$$y(t_i) = \mu a_i \qquad \text{for all } i$$

which implies zero intersymbol interference. Hence, the two conditions of Equation (4.49) ensure *perfect reception in the absence of noise.*

THANK YOU