FUNDAMENTALS OF CIVIL ENGINEERING

CE 1102

Dr. Shayani Mendis

Stress Resultants, Stresses and Strains

Stress Resultants (Internal Forces)

- Due to the loads acting on structures, stresses are developed within the structural components
- Resultant of these stresses are called stress resultants or internal forces

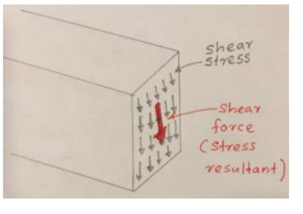
Stress Resultants (Internal Forces)

Followings are the most common stress resultants

Axial force (tension / compression)

Shear force

Bending moment



Stress Resultants (Internal Forces)

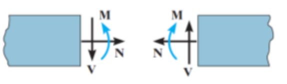
- In a beam, at different cross sections of the beam, there are different values for shear force and bending moment.
- In a truss member, as truss members are considered as weightless members, throughout the member at any cross section, same axial force exists.

Stress Resultants (Internal Forces)

- To calculate internal forces of any member;
  - First, external unknown forces should be found using equilibrium equation and the free body diagram.
  - Next, an imaginary cut needs to be made at the point where internal forces are to be found.

Stress Resultants (Internal Forces)

- To calculate internal forces of any member (contd.);
  - Now, the structure is in two parts. Draw the internal forces at the cut.



  - Consider the equilibrium of one part and find the unknown internal forces.

Sign Convention

Positive normal force

Tension is positive

Positive shear force

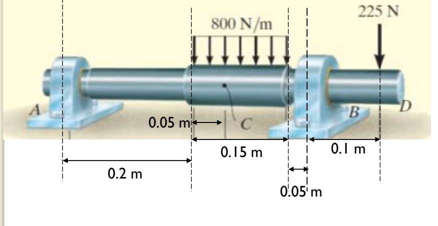
Clockwise shear is positive

Positive bending moment

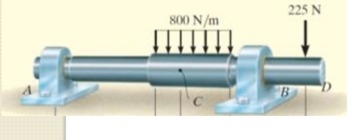
Sagging moment is positive

Example 1

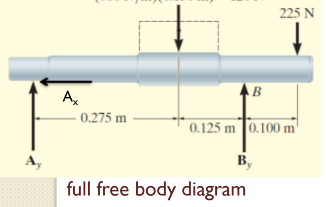
Figure below shows a machine shaft in which bearing at A can be considered as a pin and the bearing at B can be considered as a roller. Determine the internal forces acting on the cross section at point C.



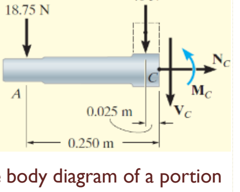
Answer



Actual Structure



full free body diagram



free body diagram of a portion of the structure

Answer (contd.)

for full free body diagram,

$\circlearrowleft + \Sigma M_B = 0;$

$$-A_y(0.400\ \text{m}) + 120\ \text{N}(0.125\ \text{m}) - 225\ \text{N}(0.100\ \text{m}) = 0$$

$$A_y = -18.75\ \text{N}$$

for the free body diagram of the portion considered,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad -18.75\ \text{N} - 40\ \text{N} - V_C = 0$$

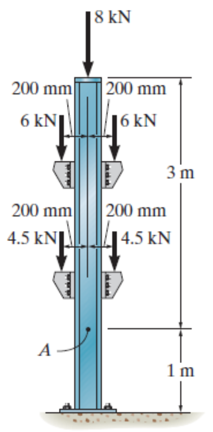
$$V_C = -58.8\ \text{N}$$

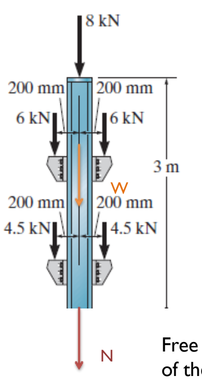
$$\circlearrowleft + \Sigma M_C = 0; \qquad M_C + 40\ \text{N}(0.025\ \text{m}) + 18.75\ \text{N}(0.250\ \text{m}) = 0$$

$$M_C = -5.69\ \text{N}\cdot\text{m}$$

Example 2

Determine the stress resultant acting on the cross section through point A if the column has a mass of 200 kg/m



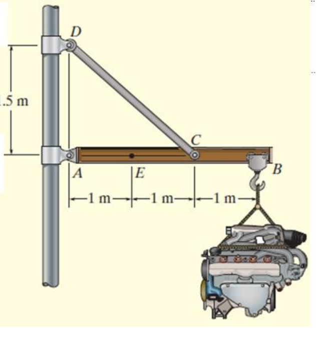


Answer:
N = -35 kN

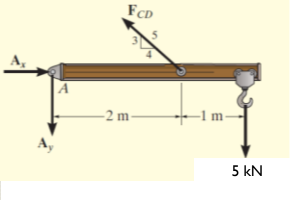
Free body diagram of the portion of column considered

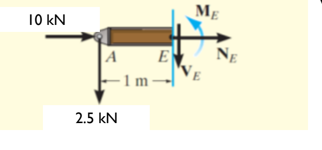
Example 3

The figure shows a crane boom carrying a load of 500 kg. Find the stress resultants of the cross section of boom at point E. Consider A, C and D as pins.



Answer





F_{CD} = 12500 N
A_x = 10000 N
A_y = 2500 N
M_E = -2500 Nm
N_E = - 10000 N
V_E = -2500 N

Example 3 (contd.)

Think …..

- If all the joints are pins, why the structure is not considered as a truss?
- Why DC is considered as a truss member?

Stress

This is different for bending stresses

- Stress = Normal force / Area
- Units
  - Pascal (Pa) – N/m^2
  - Kilo Pascal (kPa) - kN/m^2 (10^3 N/m^2)
  - Mega Pascal (MPa) - MN/m^2 (10^6 N/m^2)

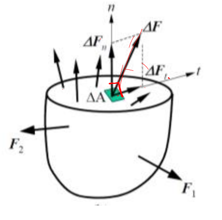
Stress
- Normal Stress
  - Tensile Stress
  - Compressive Stress
- Shear Stress (tangential)

Normal and Shear Stress

- Normal stress (σ) – Due to axial tension or compression

  $\sigma = \Delta F_n / \Delta A$
- Shear stress (τ) – Due to sliding effect of one layer of material over another layer

  $\tau = \Delta F_t / \Delta A$



Example 1

$\sigma = \frac{F}{A}$ $F = 80kg \times 9.8 = N$

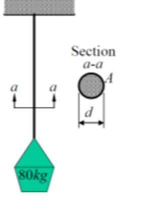
- 80 kg weight is hanged by a cable of diameter 3.15 mm. What is the stress in the cable.

$A = \pi d^2/4 = \pi \times (3.15 \times 10^{-3})^2/4 =$

Area = 7.793 x 10^{-6} m^2

Force = 80 x 9.8 = 784 N

Stress = 100.6 MPa



$A = 7.793 \times 10^{-6}\ m^2$ $\sigma = 100.6\ MPa$

Example 2

$\sigma_{allowable} = 50\ MPa$

Consider the same cable in example 1. Assume that the cable is made of copper. The allowable stress for copper is 50 MPa.

a) Determine the maximum weight that the cable can carry without exceeding the allowable stress limit.

b) What should be the minimum diameter of the cable, if it is to carry 80 kg weight?

$\sigma \propto W$ $\sigma < 50\ MPa$

$\frac{F}{A} < 50\ MPa$

$W \times 9.81 < 50 \times A$

$F < 39.76 kg$

(b) $\sigma = 50$

$F = 80 \times 9.81$

A ?

$A = \frac{F}{\sigma}$

$d = 4.47mm$

Example 2 (Ans.)

a)

$\sigma_{max} = F_{max} / A$

$50 \times 10^6\ Pa = F_{max} / 7.793 \times 10^{-6}\ m^2$

$F_{max} = 389.65\ N$

$W_{max} = 39.76\ kg$

b)

$\sigma_{max} = F / A_{min}$

$A_{min} = 80 \times 9.8 / 50 \times 10^6\ Pa$

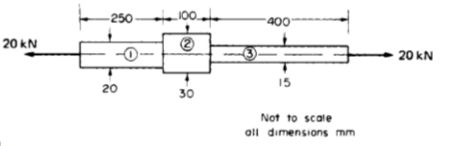
$d_{min} = 4.469 \times 10^{-3}\ m$

Assignment

- Do example 3 to 6 to be submitted as an assignment.
- Deadline 10/06/2022 at 4.00pm
- Please submit to (Instructor) Civil department 3rd Floor
- 50% deduction of marks for late submission
- No need to put a cover page, just write your Reg No and Name in the first page

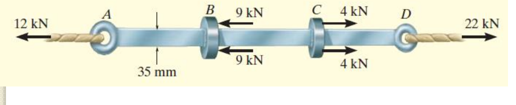
Example 3

- Determine the stress in each section of the bar shown in figure below. The bar is subjected to 20 kN tensile force. Section 1 and 3 are circular cross sections of 20 mm and 15 mm diameter respectively. Section 2 is a 30 mm square cross section. (Ans: 63.66 MPa/ 22.2 MPa/ 113.2 MPa)



Example 4

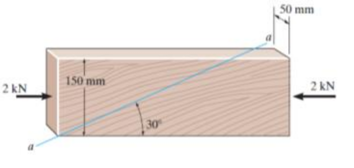
- The bar given in figure below has a constant cross section of width 35 mm and thickness 10 mm. Determine the maximum normal stress in the bar.



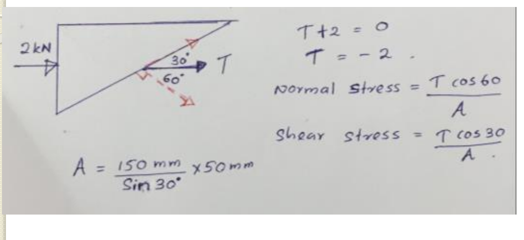
Ans: 85.7 MPa

Example 5

- Determine the normal stress and shear stress in the plane along section a-a of the block given below.

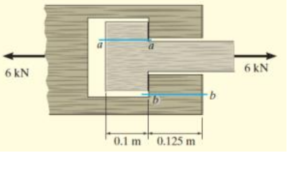


Example 5 (Hint)

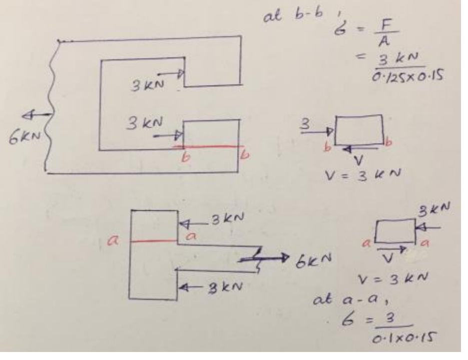


Example 6

- The wooden joint shown below has a width of 150 mm. Determine the shear stresses developed along plane a-a and b-b.



Example 1.6 (Hint)



Example 1.7

- The cable in example 1 was elongated 1.35 mm due to 80 kg weight. What would be the strain of the cable, if the original length of the cable is 1.5 m?

strain = 900 x 10^{-6} = 900 μm/m = 900 micro strain (με)

Strain = change in length/ original length