ET3122 Antennas and Propagation
Antenna Basics

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November 25, 2020

Outline

1. Introduction
2. Antenna Parameters
3. Scalar and Vector Potentials
4. Spherical Polar Coordinates
5. Conclusion

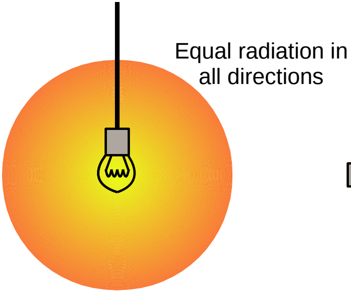
Introduction

Introduction

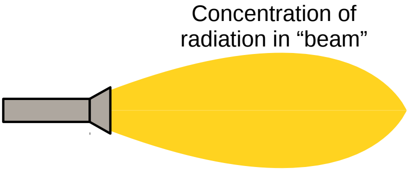
- An antenna is the *interface* between the electrical signal and radiated EM wave
  - Identical characteristics when used as a transmitter and receiver (*reciprocity*)
- Has to be an efficient radiator
  - Matched to the transmission line
  - Operational bandwidth

Antenna Parameters

Radiation Sources



a) An Isotropic Radiator

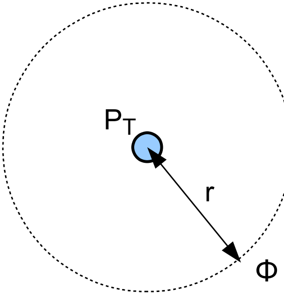


b) An Anisotropic Radiator

Isotropic Radiator

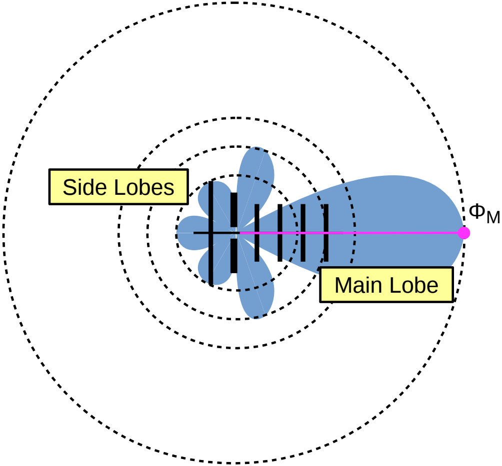
- An ideal point source
- Radiates energy equally in all directions
- For a power P_T at a distance r yeilds a power density Φ

$$\Phi = \frac{P_T}{4\pi r^2}$$

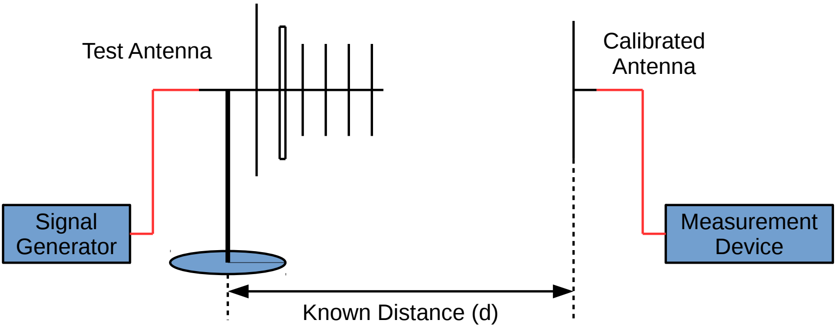


- Cannot be realized and mainly used as a *benchmark*
- All practical antennas are *anisotropic*

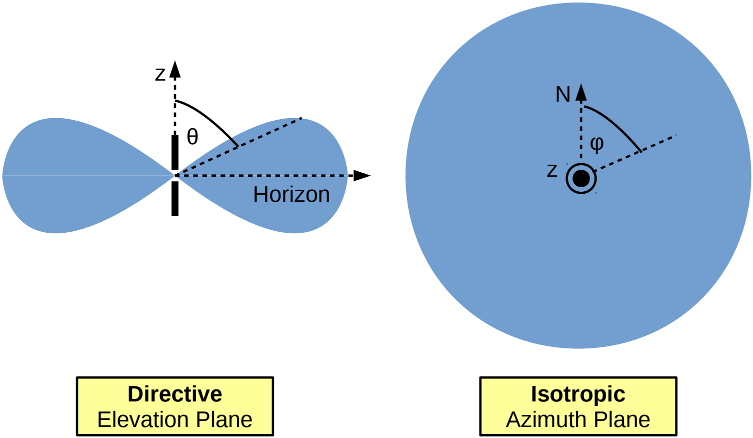
Anistotropic Radiators



Radiation Pattern Measurement



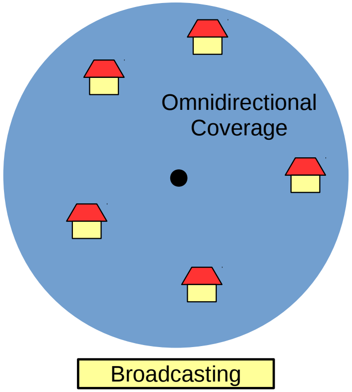
Omnidirectional Radiation Patterns



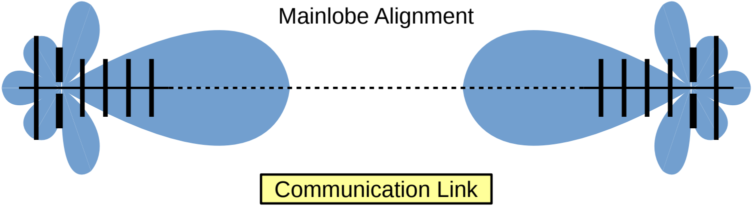
Radiation Pattern Utility

- What is the radiation pattern of the antenna useful for?
  - Depends on its characteristics
  - Can it be focused? etc.
- Some radiation patterns may have no apparent use
  - "Radiation" engineering to make them useful

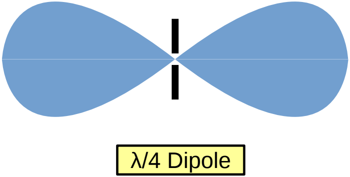
Radiation Pattern Utility (Contd..)

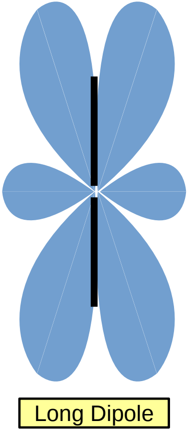


Radiation Pattern Utility (Contd..)



Radiation Pattern Utility (Contd..)

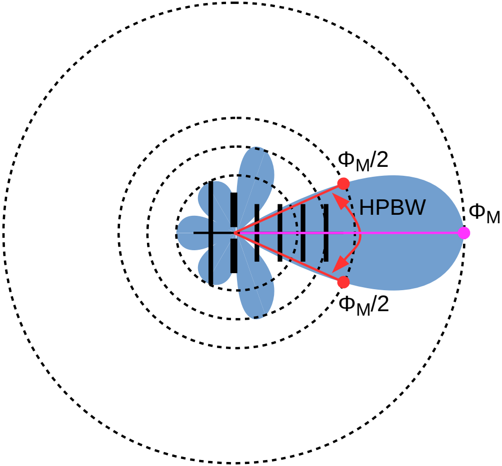




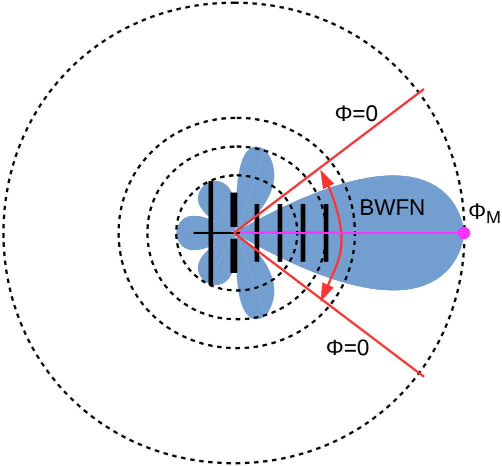
Radiation Pattern Metrics

- In order to select the best antenna for a particular application it is necessary to compare the radiation patterns
- Common metrics
  - Half Power Beamwidth (HPBW)
  - Beamwidth Between First Nulls (BWFN)
  - Sidelobe Suppression Ratio (SSR)

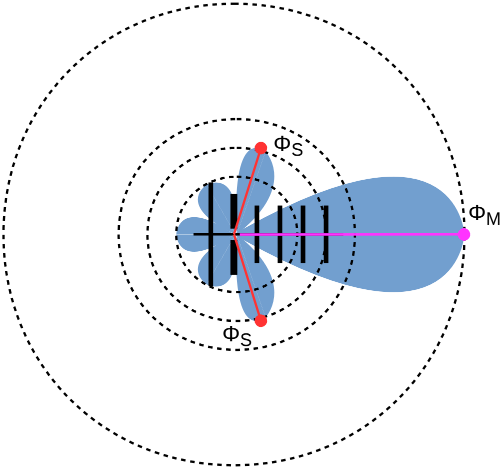
Half Power Beamwidth



Beamwidth Between First Nulls



Sidelobe Suppression Ratio



Metric Definitions

- The HPBW is the angle between the two points where the maximum radiation pattern power density (Φ_M) of the mainlobe is halved
  - If the radiation pattern is in terms of the E field its the point where the field becomes $E_M/\sqrt{2}$
- The BWFN is the angle between the two points nearest to the mainlobe with zero power density
  - Includes the entire mainlobe
  - In a measured radiation pattern can be taken as the minimum radiation point between the main lobe and the nearest pair of sidelobes
- The SSR is simply Φ_S/Φ_M
  - Can be given in terms of decibels

Directivity

- The ratio between the maximum power density of the antenna (Φ_m) and the power density of an isotropic radiator (Φ_i) for the same transmitted power P_T.

$$D = \frac{\Phi_m}{\Phi_i}$$

- Usually expressed in decibels

Calculation of Directivity

For an anisotropic radiator the *normalized* power density:

$$\Phi = \Phi_m \frac{F(\theta, \phi)}{F_{max}(\theta, \phi)}$$

Therefore, the radiated power

$$\begin{aligned} P_T &= \int_0^{2\pi} \int_0^{\pi} \Phi r^2 sin(\theta) d\theta d\phi \\ &= \frac{\Phi_m}{F_{max}(\theta, \phi)} \int_0^{2\pi} \int_0^{\pi} F(\theta, \phi) r^2 sin(\theta) d\theta d\phi \end{aligned}$$

Calculation of Directivity (Contd..)

For an isotropic radiator:

$$P_T = \Phi_i(4\pi r^2)$$

Therefore, the directivity

$$D = \frac{\Phi_m}{\Phi_i} = \frac{4\pi F_{max}(\theta,\phi)}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi) sin(\theta) d\theta d\phi}$$

Gain

The ratio between the maximum power density of the antenna and the maximum power density of a reference antenna.

$$G = \frac{(\Phi_m)_{ant}}{(\Phi_m)_{ref}}$$

It is also equal to the ratio of directivities of the two antennas.

$$G = \frac{D}{D_{ref}}$$

- Expressed in decibels
- The reference antenna is usually a standard half wave dipole (*preferred*) or an isotropic radiator

Antenna Aperture

The antenna aperture or *effective area* is defined as,

$$A_E = \frac{P_R}{\Phi}$$

Where, P_R is the power received by the antenna and Φ is the power density. This can also be expressed in terms of the antenna gain (G) and the wavelength λ.

$$A_E = \frac{\lambda^2 G}{4\pi}$$

- Can be readily used for *aperture* antennas

Radiation Impedance

The ratio between the input (feed) voltage and input current.

$$Z = \frac{V_F}{I_F} = R + jX$$

- R is the *radiation resistance* and X is the *radiation reactance*
- R can be calculated from

$$R = \frac{P_{eff}}{I_{rms}^2} = \frac{P_{rad}}{I_F^2} = \frac{1}{2I_F^2} \oint_S (E \times H^*) ds$$

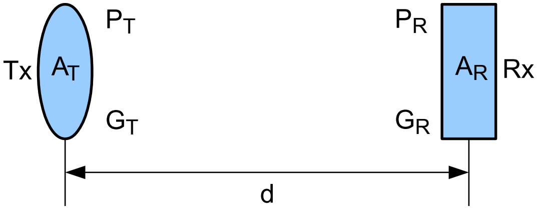
- X is difficult to obtain analytically

Bandwidth and EIRP

- The bandwidth of an antenna is the range of frequencies for which its characteristic parameters are maintained
  - Beyond the antenna bandwidth the characteristics deteriorate
- The *Effective Isotropically Radiated Power* or EIRP of an antenna is the apparent power radiated by the antenna had it been an isotropic radiator

$$EIRP = GP_T$$

Friis Formula



At the receiving antenna

$$\phi_R = \frac{G_T P_T}{4\pi d^2}$$

$$A_T = \frac{\lambda^2 G_T}{4\pi}$$

$$A_R = \frac{\lambda^2 G_R}{4\pi}$$

Friis Formula (Contd..)

The power received by the antenna becomes,

$$P_R = \phi_R A_R = \left(\frac{G_T P_T}{4\pi d^2}\right)\left(\frac{\lambda^2 G_R}{4\pi}\right) = G_T G_R \left(\frac{\lambda^2}{4\pi d^2} P_T\right)$$

Therefore, the transmission loss (L) is given by,

$$L = \frac{P_T}{P_R} = \frac{1}{G_T G_R}\left(\frac{4\pi d^2}{\lambda^2}\right)$$

When converted into dBm

$$(P_R)_{dBm} \quad = \quad (P_T)_{dBm} + (G_T)_{dB} + (G_R)_{dB} - \underbrace{20\log_{10}\left(\frac{4\pi d}{\lambda}\right)}_{\text{Free Space Loss}}$$

Free Space Loss Formula

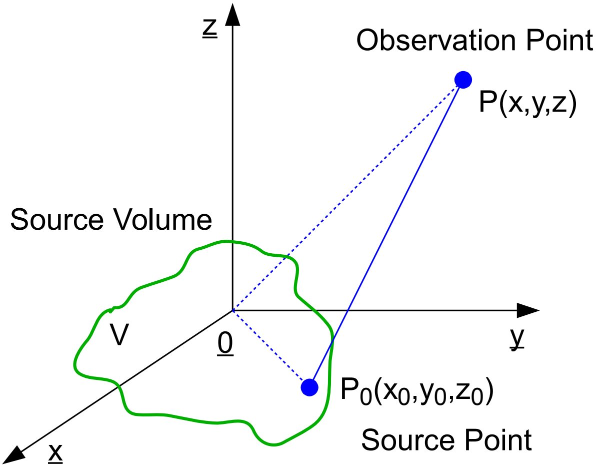
Convenient to apply formula for calculating the free space loss

$$L_{FS} = 20\log_{10}(d) + 20\log_{10}(f) + 92.45$$

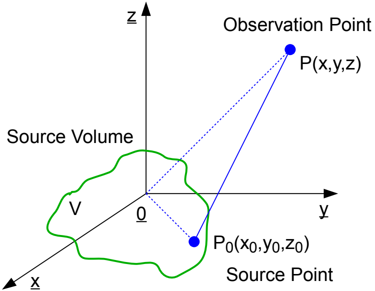
- d is the distance in km
- f is the frequency in GHz

Scalar and Vector Potentials

Retarded Potentials



Retarded Potentials (Contd..)



The *potential* due to a charge or current distribution within volume v

$$V(r) = \frac{1}{4\pi\varepsilon}\int_v \frac{\rho(r_0)}{|r-r_0|}dv$$

$$A(r) = \frac{\mu}{4\pi}\int_v \frac{J(r_0)}{|r-r_0|}dv$$

- The resulting EM waves will take time to travel from P_0 to P
  - The generating charge or current would have changed by then
  - Thus, the potential at P due to P_0 is *retarded* (i.e., lagged)
  - V is a *scalar* potential while A is a *vector* potential

Retarded Potentials (Contd..)

Due to the time lag

$$
\begin{aligned}
V(r,t) &= \frac{1}{4\pi\varepsilon}\int_V \frac{\rho\left(r_0, t-\frac{|r-r_0|}{c}\right)}{|r-r_0|}dv \\
A(r,t) &= \frac{\mu}{4\pi}\int_V \frac{J\left(r_0, t-\frac{|r-r_0|}{c}\right)}{|r-r_0|}dv
\end{aligned}
$$

- The resulting potential is a function of t

Retarded Potentials (Contd..)

By considering the *phase difference* due to the travel time from P to P_0, t can be eliminated

$$V(r) = \frac{1}{4\pi\varepsilon}\int_V \frac{\rho(r_0)e^{-jk|r-r_0|}}{|r-r_0|}dv$$

$$A(r) = \frac{\mu}{4\pi}\int_V \frac{J(r_0)e^{-jk|r-r_0|}}{|r-r_0|}dv$$

where $k = 2\pi/\lambda$.

- For the amplitude $r - r_0 \approx r$ can be used for the *far field*
  - For the phase this approximation may not be applicable if the antenna has a significant length

Radiated Fields

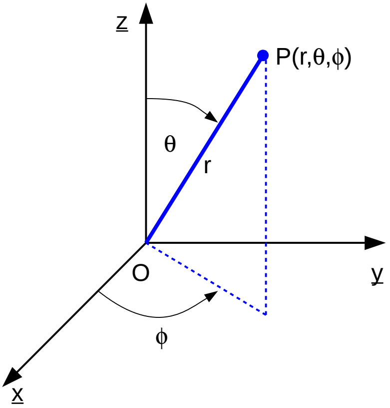
- Antennas have no free charge so $V(r)$ can be ignored
- In *wire* antennas the potential is due to a current distribution
  - The radiated H and E fields are given by

$$
\begin{aligned}
H &= \frac{\nabla \times A}{\mu} \\
E &= \frac{\nabla \times H}{j\omega\varepsilon}
\end{aligned}
$$

- In *aperture* antennas, the radiated fields can be found directly

Spherical Polar Coordinates

Spherical Polar Coordinates



Transformation Matrix

$(x, y, z) \Rightarrow (r, \theta, \phi)$

$$\begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} = T \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

Where,

$$T = \begin{pmatrix} \cos\phi\sin\theta & \sin\phi\sin\theta & \cos\theta \\ \cos\phi\cos\theta & \sin\phi\cos\theta & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix}$$

For $(r, \theta, \phi) \Rightarrow (x, y, z)$

$$T^{-1} = \begin{pmatrix} \cos\phi\sin\theta & \cos\phi\cos\theta & -\sin\phi \\ \sin\phi\sin\theta & \sin\phi\cos\theta & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix}$$

Operators

Gradient

$$\nabla f = \left(\frac{\partial f}{\partial r}\right)\underline{r} + \frac{1}{r}\left(\frac{\partial f}{\partial \theta}\right)\underline{\theta} + \frac{1}{r\sin\theta}\left(\frac{\partial f}{\partial \phi}\right)\underline{\phi}$$

Divergence

$$\nabla \cdot A = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(A_\phi)$$

Laplacian

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

Operators (Contd..)

Curl

$$\nabla \times A = \begin{vmatrix} \frac{r}{r^2 \sin\theta} & \frac{\theta}{r \sin\theta} & \frac{\phi}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin\theta A_\phi \end{vmatrix}$$

Conclusion

Summary

- An antenna is an interface between the electric signal and radiated EM wave
  - All practical antennas are anisotropic radiators
  - Most efficient within the bandwidth
  - Has to be matched to the transmission line
- A *good* antenna must have a useful radiation pattern and matching feed impedance
- In wired antennas, the radiation is due to a current distribution
- In an aperture antenna, the radiation is due to E and H fields