EE1102 – Fundamentals of Electrical Engineering

5.0 Waveform Characteristics

Prof J. Rohan Lucas
lucas@kdu.ac.lk

Department of Electrical, Electronic & Telecommunication Engineering
General Sir John Kotelawala Defence University
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Waveform Characteristics (2 hrs)

Learning Outcome:

Perform circuit calculations of mean, peak and instantaneous value; frequency, rms value, average value for different waveforms.

Content:

- Direct and Alternating Waveforms
- Instantaneous, peak and mean values
- Rectified average value
- Effective value
- Form factor and Peak factor
- Calculations of values and factors

DC and AC Waveforms

Direct waveform, sinusoidal alternating and other waveforms

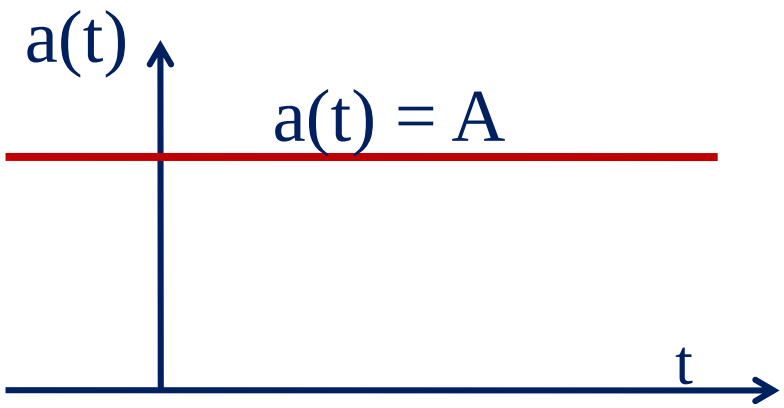


Figure 1(a) – direct waveform

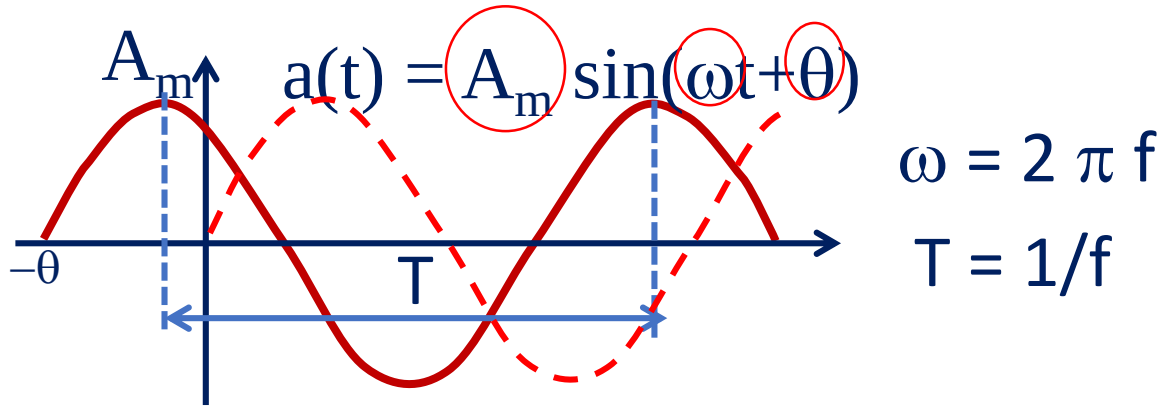


Figure 1(b) – sinusoidal waveform

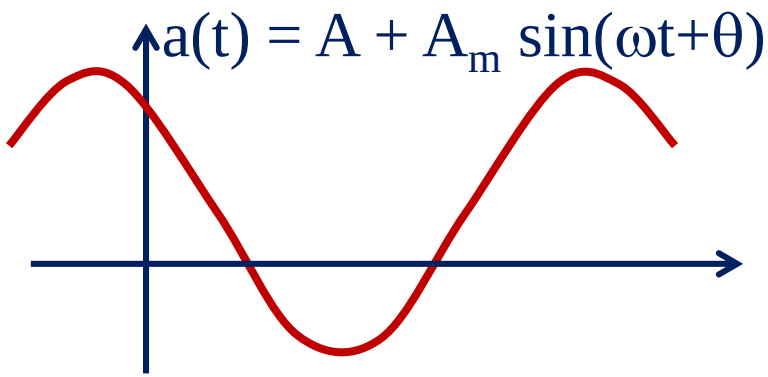


Figure 1(c) – ac + dc waveform

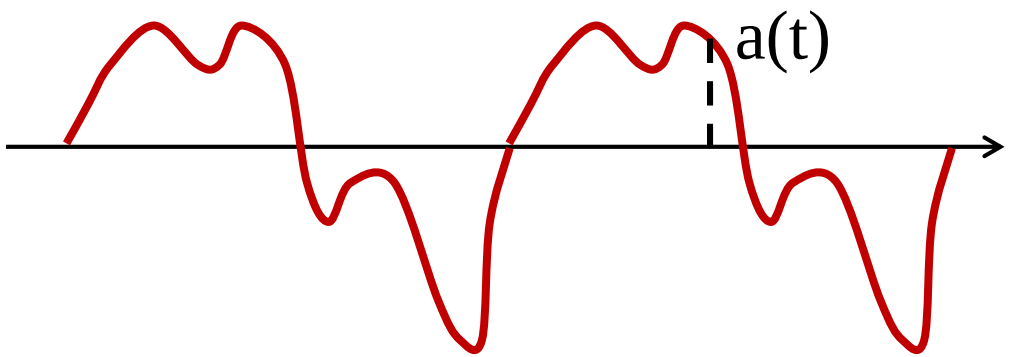


Figure 1(d) – other waveform

Significant magnitudes of waveforms

Instantaneous value - value a(t) at given instant of time t

For a sinusoid, $v(t) = V_m \sin(\omega t + \phi)$

Peak value - maximum instantaneous value,

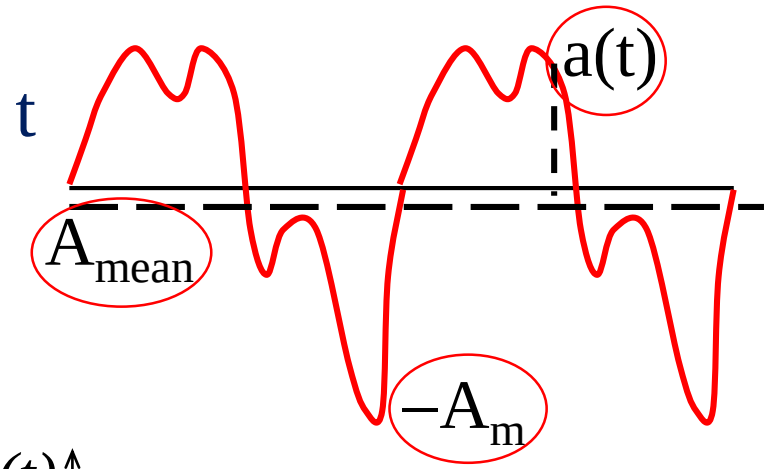
For sinusoidal a.c., **peak value** = V_m

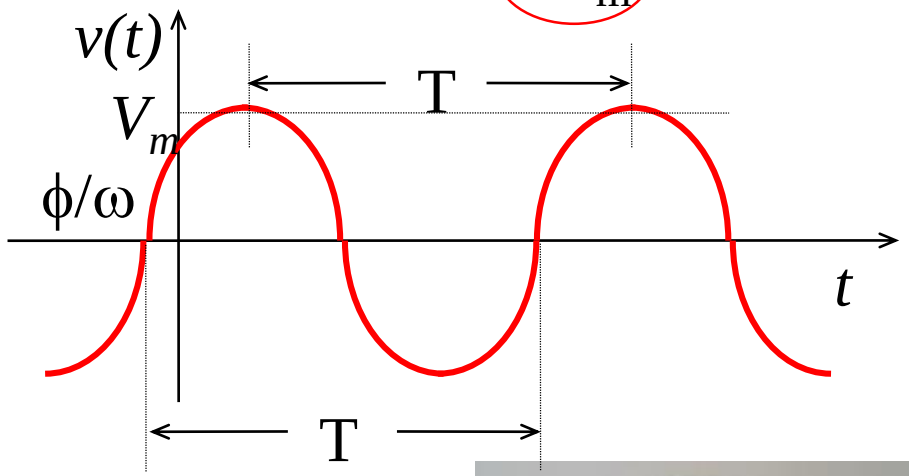
Mean value – mean value of waveform

$$A_{mean} = \frac{1}{T}\int_{T_0}^{T_0+T} a(t).dt = \text{direct component}$$

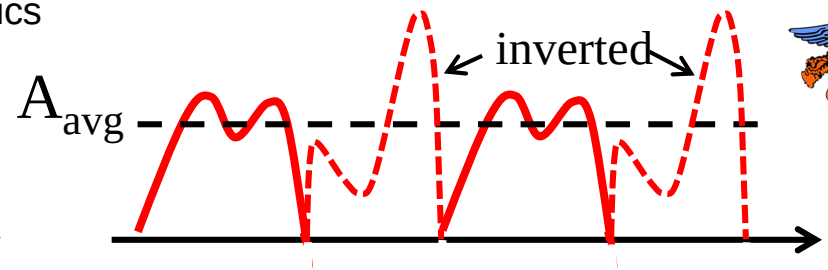
For a pure sinusoid *(Positive area = Negative area)*

$$\textbf{mean value} = \frac{1}{T}\int_{T_0}^{T_0+T} A_m \sin(\omega t + \emptyset).dt = 0$$





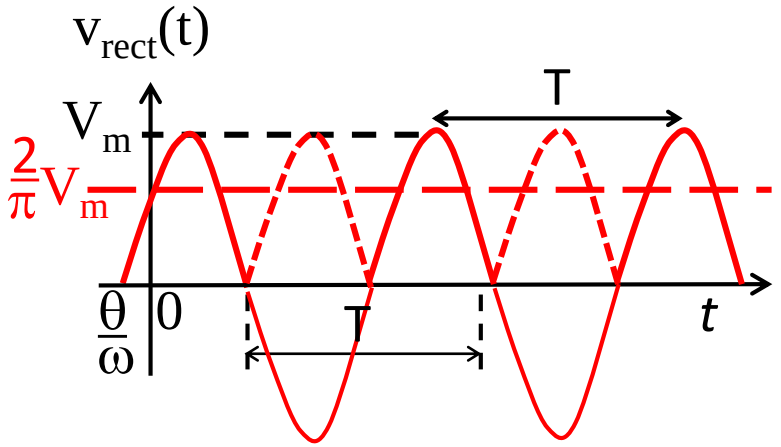
Average value



Average value of full-wave rectified waveform

$$\frac{1}{T}\left[\underset{\text{positive half cycle}}{\int a(t).dt} - \underset{\text{negative half cycle}}{\int a(t).dt}\right]$$

for sinusoidal a.c. if $T_o = \theta/\omega$,



$$\textbf{average value} = \frac{1}{T}\left[\int_0^{T/2} A_m \sin\omega t.dt - \int_{T/2}^{T} A_m \sin\omega t.dt\right]$$

$$= \frac{1}{\omega T}\cdot 2A_m = \frac{2}{\pi}A_m$$

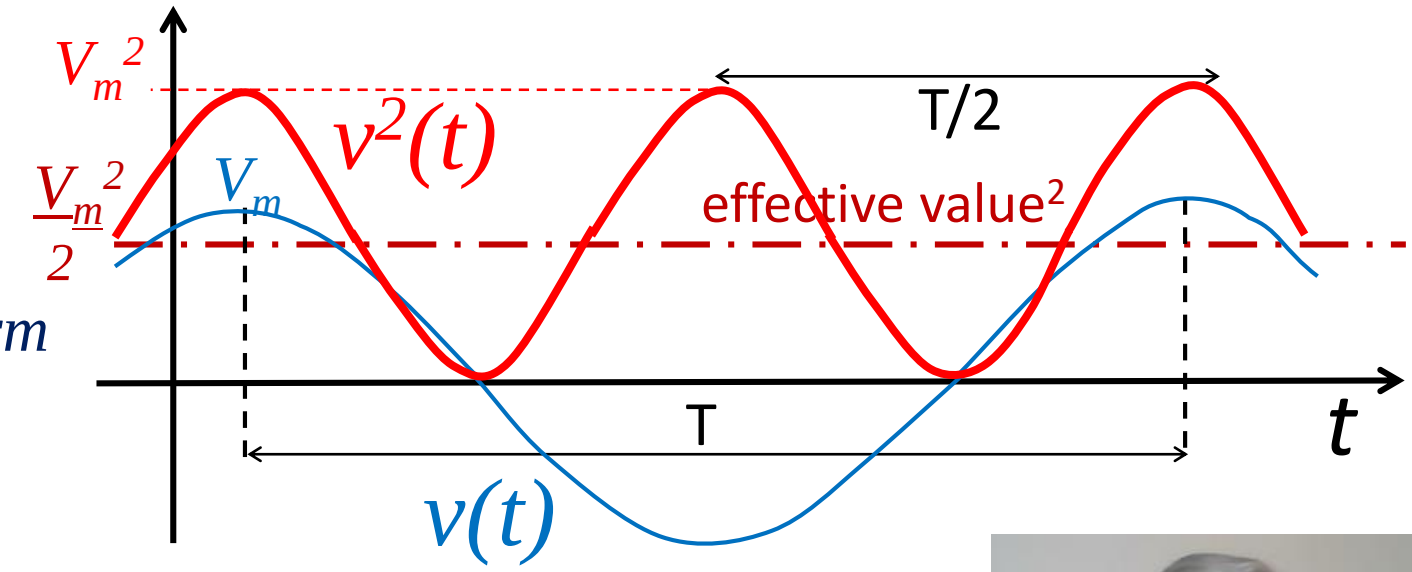
Effective value of waveform

Defined based on power equivalence

$$P = I^2_{effective}.R \text{ or } \frac{V^2_{effective}}{R}$$

$$\frac{V^2_{effective}}{R} \cdot T = \int_{T_o}^{T_o+T} \frac{v^2(t)}{R} \cdot dt, \text{ or } V^2_{effective} = \frac{1}{T} \int_{T_o}^{T_o+T} v^2(t) \cdot dt$$

$$V_{effective} = \sqrt{\frac{1}{T} \int_{T_o}^{T_o+T} v^2(t) \cdot dt}$$



*square **root** of **mean** of **squared** waveform*

- **root-mean-square** or **rms**

for sinusoidal a.c. waveform

$$\textbf{rms value} = \frac{1}{\sqrt{2}} V_m$$

rms value is specified for ac voltage or current waveforms.

Dependence of defined values on each other

For given waveform, *peak*, *average* and *rms values* are dependant.

$$\textbf{Form Factor} = \frac{rms \quad value}{average \quad value}$$

for a sinusoid, **Form Factor** =

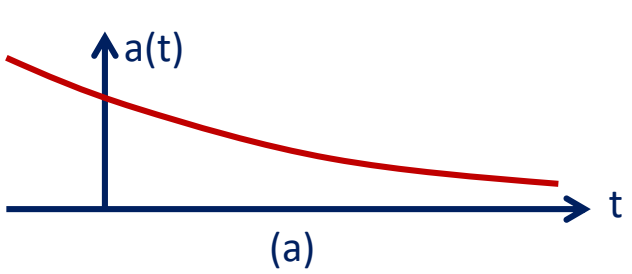
$$\frac{\frac{V_m}{\sqrt{2}}}{\frac{2V_m}{\pi}} = \frac{\pi}{2\sqrt{2}} = 1.1107 \cong 1.111$$

$$\textbf{Peak Factor} = \frac{peak \quad value}{rms \quad value}$$

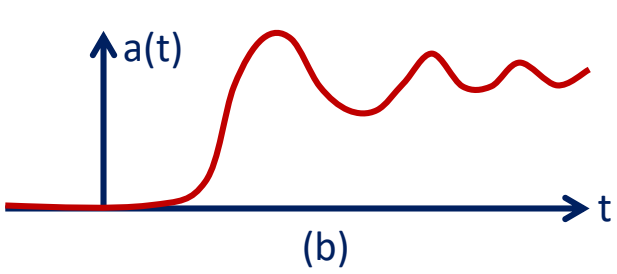
for a sinusoid, **Peak Factor** =

$$\frac{V_m}{\frac{V_m}{\sqrt{2}}} = \sqrt{2} = 1.4142$$

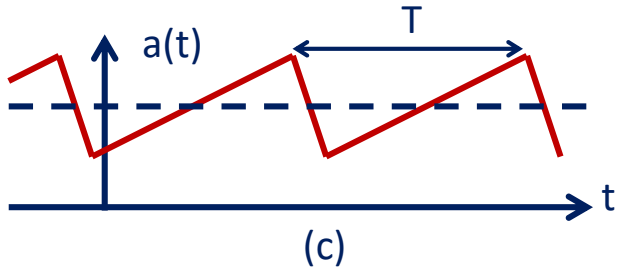
Types of Waveforms



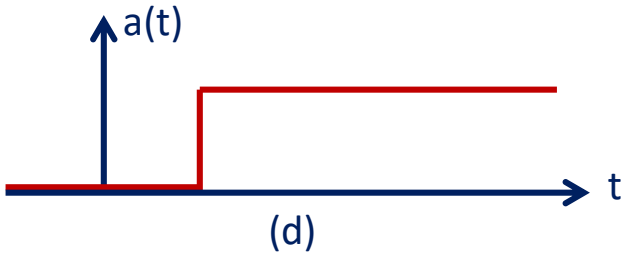
(a)



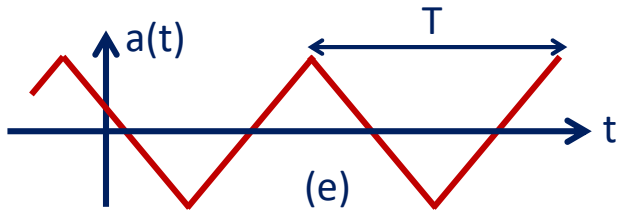
(b)



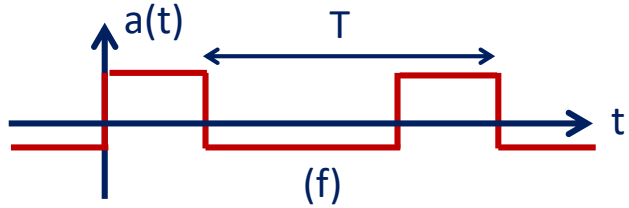
(c)



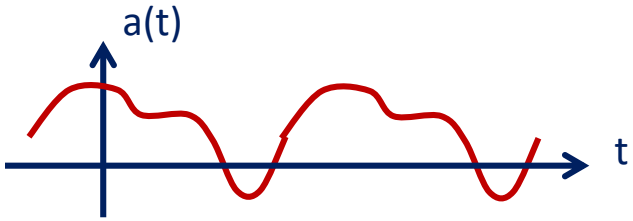
(d)



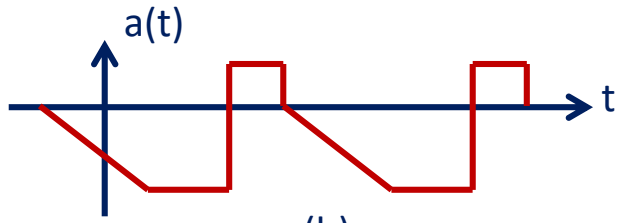
(e)



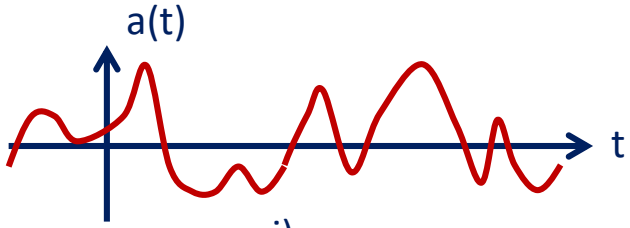
(f)



(g)



(h)

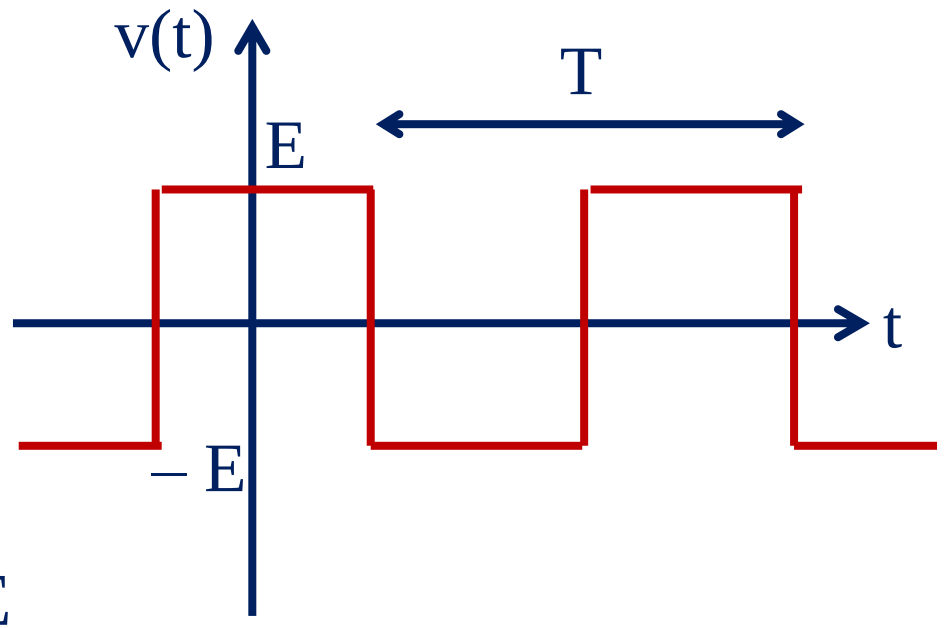


i)

(a), (b), (c) and (d) are uni-directional, although not purely direct.
(e) and (f) are repetitive with zero mean value,
(c), (g) and (h) are repetitive waveforms with finite mean values.
(i) is alternating but non-repetitive and mean value is non-zero.

Calculation of Relevant Magnitudes of Repetitive Waveforms

Square waveform



Period = T, frequency f = 1/T, $\omega = 2\pi/T$

Peak value = E

Mean value = 0

Average value (rectified) = E

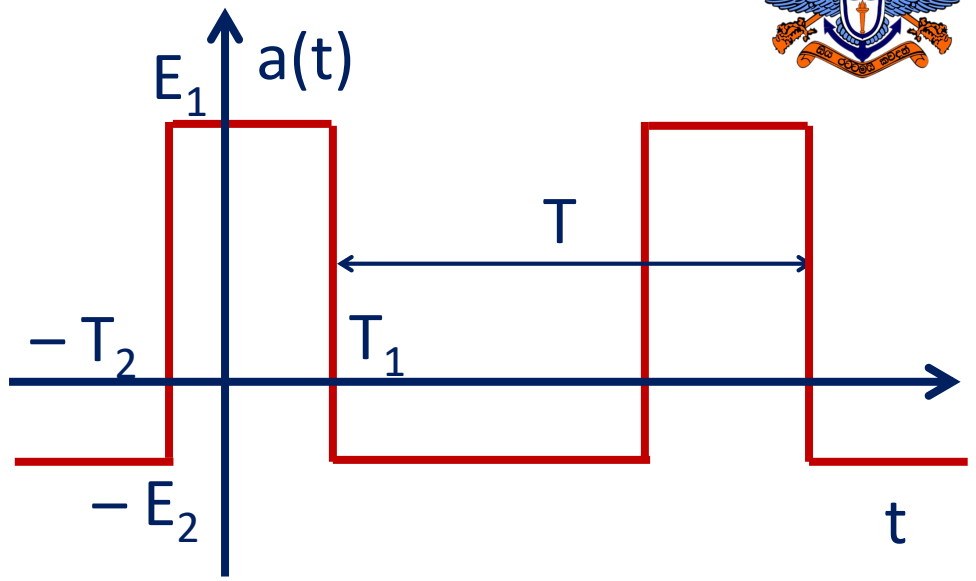
RMS value = $\sqrt{[E^2 \times ½T + (-E)^2 \times ½T]/T}$ = E

Form factor = rms value/average value = E/E = 1

Peak factor = peak value/rms value = E/E = 1

- The square waveform that has equal positive and negative half cycles have all its values equal.

Rectangular waveform



With magnitudes and durations as indicated

Positive Peak = E_1, Negative Peak = E_2

Mean value = $\frac{E_1.(T_1+T_2) - E_2.(T - T_1 - T_2)}{T}$

Average value (rectified) = $\frac{E_1.(T_1+T_2) + E_2.(T - T_1 - T_2)}{T}$

RMS value = $\sqrt{\frac{E_1^{\,2}.(T_1+T_2) + E_2^{\,2}.(T - T_1 - T_2)}{T}}$

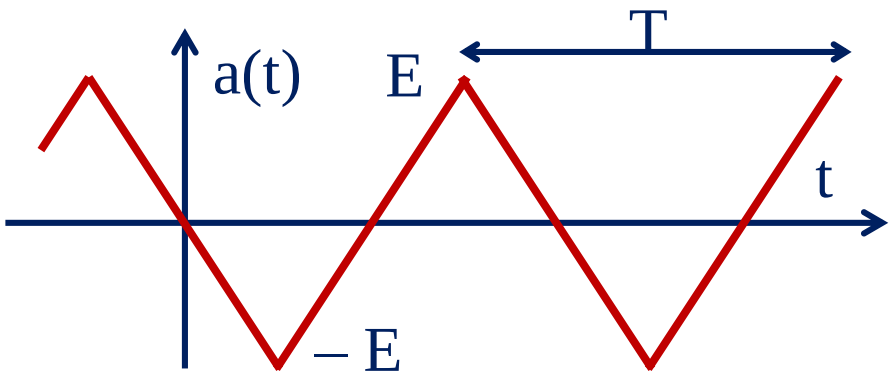
If E_1 = 100V, E_2 = 40V, T = 10s, T_1 = 4s, T_2 = 2s, T = 20s,

Mean = $\frac{100\times6 - 40\times14}{20}$ = 2V, Average = $\frac{100\times6 + 40\times14}{20}$ = 58V,

RMS = $\sqrt{\frac{100^2\times6 + 40^2\times14}{20}}$ = 64.19V

Form factor = 64.19/58 = 1.1107, Peak factor = 100/64.19 = 1.558

Symmetrical Triangular waveform



Peak value = E,

Mean value = 0

$$\text{Average value} = \frac{\frac{1}{2}\times \mathrm{E}\times\left(\frac{1}{2}T\right)+\frac{1}{2}\times \mathrm{E}\times\left(\frac{1}{2}T\right)}{T} = 0.5\mathrm{E}$$

$$\text{r.m.s. value} = \sqrt{\frac{1}{\frac{1}{2}T}\int_{-\frac{1}{4}T}^{\frac{1}{4}T}\left(-\frac{4E}{T}t\right)^2 dt} = \sqrt{\frac{\left(\frac{4E}{T}\right)^2\times\left(\frac{\left(\frac{1}{2}T\right)^3}{3}\right)}{\frac{1}{2}T}} = \sqrt{\frac{E^2}{3}} = 0.5773\mathrm{E}$$

Form factor = 0.5773E/0.5E = 1.155,

Peak factor = E/0.5573E = 1.732

For asymmetrical triangular waveforms, the method is similar

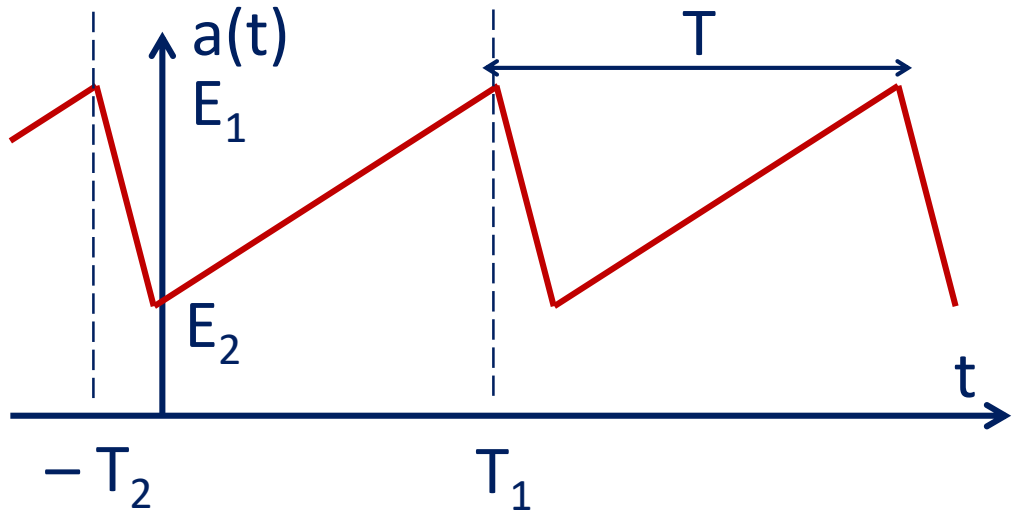
Saw-tooth waveform

Peak value = E_1

Mean value = $\frac{\frac{1}{2}\times(E_2+E_1)\times T}{T} = \frac{1}{2}(E_2 + E_1)$

Average value = mean value (*no negatives*)

RMS = $\sqrt{\frac{1}{T}\int_{-T_2}^{T_1} a^2(t)dt}$



If E_1 = 100V, E_2 = 40V, T_1 = 8s, T_2 = 2s, T= 10s, Mean = ½(100+40) = 70V

RMS = $\sqrt{\frac{1}{10}\int_{-2}^{0}(40-30t)^2dt + \int_{0}^{8}(40+7.5t)^2dt}$ =

$\sqrt{\frac{1}{10}\left[\frac{(40-30t)^3}{3\times(-30)}\Big|_{-2}^{0} + \frac{(40+7.5t)^2}{3\times(7.5)}\Big|_{0}^{8}\right]} = \sqrt{\frac{1}{225}\left[\frac{40^3}{-4} - \frac{100^3}{-4} + 100^3 - 40^3\right]}$=72.111 V

Form factor = 72.111/70 = 1.030, Peak factor = 100/72.111 = 1.387

Irregular waveform

Period = 20s, Peak value = 20V (*negative is higher*)

Mean value = $\frac{-(\frac{1}{2}\times 20\times 10+20\times 5)+10\times 5}{20}$ = – 7.5V

Rectified Average value = $\frac{\frac{1}{2}\times 20\times 10+20\times 5+10\times 5}{20}$ = 12.5V

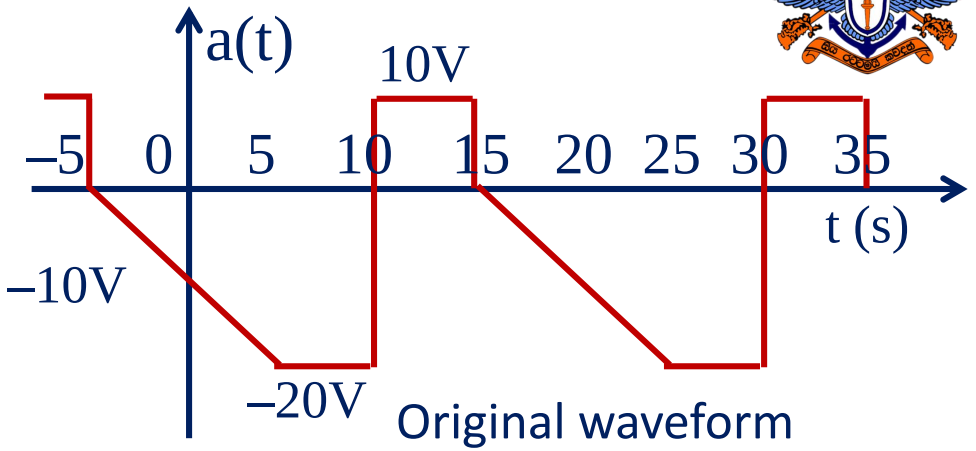
r.m.s. value = $\sqrt{\frac{1}{T}\int_{-T_2}^{T_1} a^2(t)dt}$ =

$$\sqrt{\frac{1}{20}\left[\int_{-5}^{5}(10+2t)^2dt+\int_{5}^{10}20^2dt+\int_{10}^{15}10^2dt\right]}$$

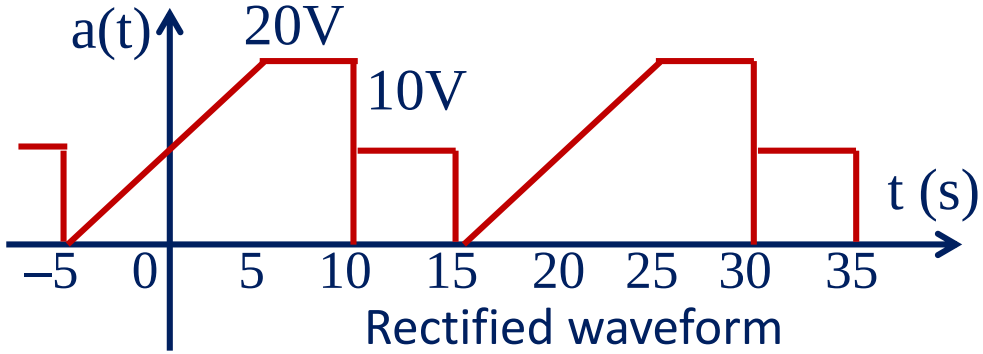
$$=\sqrt{\frac{1}{20}\left[\frac{(10+2t)^3}{3\times 2}\Big|_{-5}^{5}+20^2t\Big|_{5}^{10}+10^2t\Big|_{10}^{15}\right]}=13.844\text{V}$$

Form factor = 13.844/12.5 = 1.108,

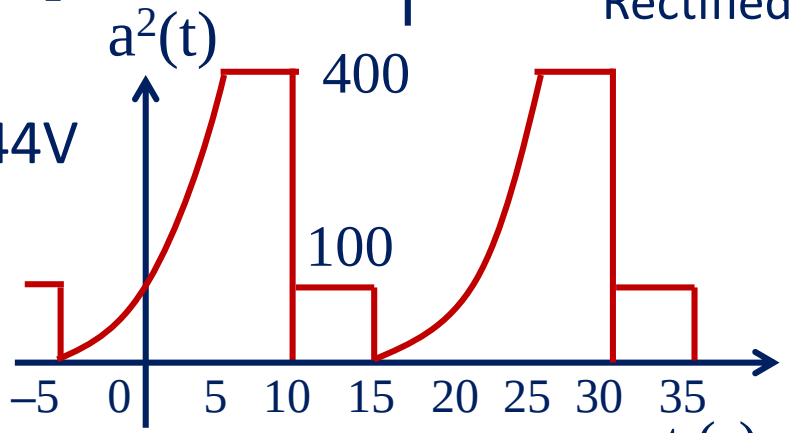
Peak factor = 20/13.844 = 1.445



Original waveform



Rectified waveform



Squared waveform

Recap of Waveform Characteristics

- Direct and Alternating Waveforms
- Instantaneous, peak and mean values
- Rectified average value
- Effective value
- Form factor and Peak factor
- Calculations of values and factors