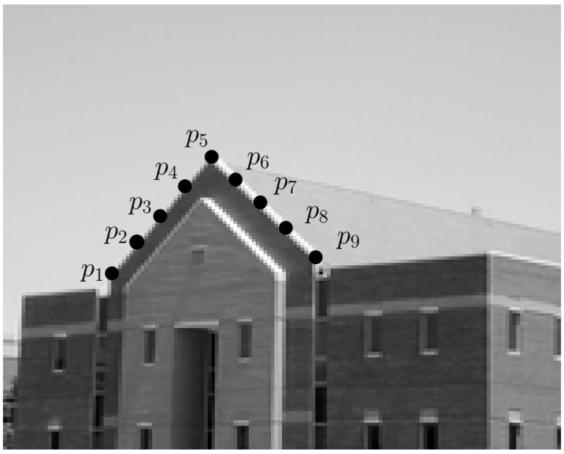
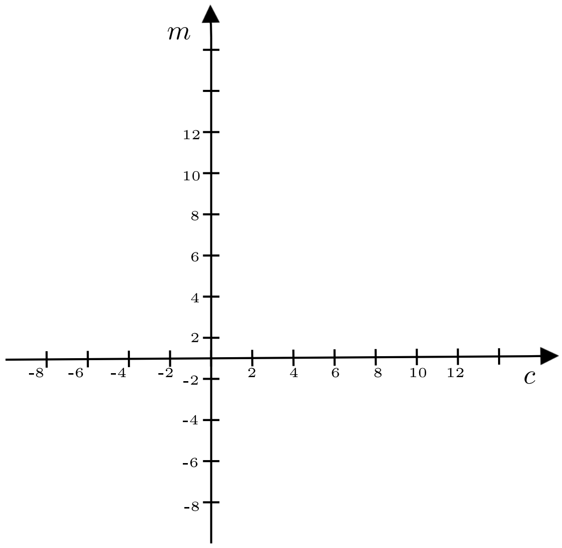
Q1. (a) Draw the Hough parameter space corresponds to the image pixels p_1 to p_9 of Figure Q1(a)i. For the drawing use Figure Q1(a)ii. Note that the pixels p_1 to p_5 fits a line of $y = 2x+1$ and pixels p_5 to p_9 fits a line of $y = -2x+10$. [**5**]



i Image space.



ii Hough parameter space.

Figure Q1(a): Image space and Hough parameter space for **Q1**a.

(b) To calculate the angle of inclination of the roof in the house depicted in Figure Q1(a)i, a dataset is formed by selecting pixels p_5 to p_9. It should be noted that the process of extracting the pixel positions is not perfect and some pixel locations may have been altered. These pixel points are shown in Figure Q1(b) which are known to form a line. The resulting dataset consists of a set of points with their corresponding x and y coordinates are given below.

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_5 & y_5 \end{bmatrix} = \begin{bmatrix} 0 & 12 \\ 1 & 10 \\ 2 & 8 \\ 3 & 10.5 \\ 4 & 4 \end{bmatrix}.$$

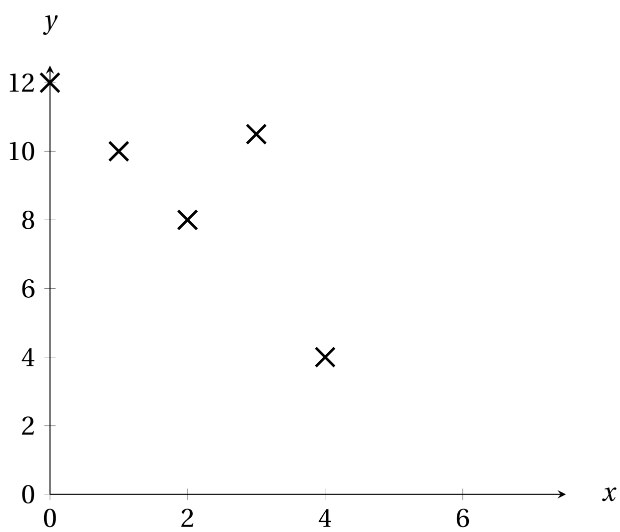


Figure Q1(b): Scatter plot of points for line fitting.

(c) Use all the points to find the least-squares-fit line and show graphically in the same scatter plot. [**3**]

(d) Write down the values of the slope (m) and intercept (c) of the line based on your least-squares solution. [**2**]

(e) To reduce the impact of outliers, a robust estimator is introduced. The robust estimator finds model parameters which minimize the following loss function

$$\sum_i \rho_\sigma(r(x_i;\theta)).$$

Here, $r(x_i;\theta)$ residual of x_i w.r.t. model parameters θ and $\rho_\sigma(r) = \frac{r^2}{r^2+\sigma^2}$ with $\sigma = 1$. Fill table Q1(e) for two models given below. [**10**]

i. $y = -2x + 10$,

ii. $y = -1.55x + 12$.

In table Q1(e), $\bar{\boldsymbol{y}} = m\boldsymbol{x} + c$ is the output of the model.

Table Q1(e): Table for question **Q1**e.

For $y = -2x + 10$					
$\boldsymbol{x}$	$\boldsymbol{y}$	$\bar{\boldsymbol{y}} = m\boldsymbol{x} + c$	$\boldsymbol{r} = \boldsymbol{y} - \bar{\boldsymbol{y}}$	$\boldsymbol{L}$	$\boldsymbol{L}_r$
$x_1 = 0$	$y_1 = 12$	$\bar{y}_1 =$	$r_1 = y_1 - \bar{y}_1 =$	$l_1 = (r_1)^2 =$	$l_{r,1} = \frac{(r_1)^2}{(r_1)^2+\sigma^2} =$
$x_2 = 1$	$y_2 = 10$	$\bar{y}_2 =$	$r_2 = y_2 - \bar{y}_2 =$	$l_2 = (r_2)^2 =$	$l_{r,2} = \frac{(r_2)^2}{(r_2)^2+\sigma^2} =$
$x_3 = 2$	$y_3 = 8$	$\bar{y}_3 =$	$r_3 = y_3 - \bar{y}_3 =$	$l_3 = (r_3)^2 =$	$l_{r,3} = \frac{(r_3)^2}{(r_3)^2+\sigma^2} =$
$x_4 = 3$	$y_4 = 10.5$	$\bar{y}_4 =$	$r_4 = y_4 - \bar{y}_4 =$	$l_4 = (r_4)^2 =$	$l_{r,4} = \frac{(r_4)^2}{(r_4)^2+\sigma^2} =$
$x_5 = 4$	$y_5 = 4$	$\bar{y}_5 =$	$r_5 = y_5 - \bar{y}_5 =$	$l_5 = (r_5)^2 =$	$l_{r,5} = \frac{(r_5)^2}{(r_5)^2+\sigma^2} =$
				$\left(\frac{1}{5}\right)\sum_{i=1}^{5} l_i =$	$\left(\frac{1}{5}\right)\sum_{i=1}^{5} l_{r,i} =$
For $y = -1.55x + 12$					
$\boldsymbol{x}$	$\boldsymbol{y}$	$\bar{\boldsymbol{y}} = m\boldsymbol{x} + c$	$\boldsymbol{r} = \boldsymbol{y} - \bar{\boldsymbol{y}}$	$\boldsymbol{L}$	$\boldsymbol{L}_r$
$x_1 = 0$	$y_1 = 12$	$\bar{y}_1 =$	$r_1 = y_1 - \bar{y}_1 =$	$l_1 = (r_1)^2 =$	$l_{r,1} = \frac{(r_1)^2}{(r_1)^2+\sigma^2} =$
$x_2 = 1$	$y_2 = 10$	$\bar{y}_2 =$	$r_2 = y_2 - \bar{y}_2 =$	$l_2 = (r_2)^2 =$	$l_{r,2} = \frac{(r_2)^2}{(r_2)^2+\sigma^2} =$
$x_3 = 2$	$y_3 = 8$	$\bar{y}_3 =$	$r_3 = y_3 - \bar{y}_3 =$	$l_3 = (r_3)^2 =$	$l_{r,3} = \frac{(r_3)^2}{(r_3)^2+\sigma^2} =$
$x_4 = 3$	$y_4 = 10.5$	$\bar{y}_4 =$	$r_4 = y_4 - \bar{y}_4 =$	$l_4 = (r_4)^2 =$	$l_{r,4} = \frac{(r_4)^2}{(r_4)^2+\sigma^2} =$
$x_5 = 4$	$y_5 = 4$	$\bar{y}_5 =$	$r_5 = y_5 - \bar{y}_5 =$	$l_5 = (r_5)^2 =$	$l_{r,5} = \frac{(r_5)^2}{(r_5)^2+\sigma^2} =$
				$\left(\frac{1}{5}\right)\sum_{i=1}^{5} l_i =$	$\left(\frac{1}{5}\right)\sum_{i=1}^{5} l_{r,i} =$

(f) Using this robust estimator, choose the best-fitted model among the models mentioned in **Q1**e for the given dataset. Justify your answer. [**2**]

(g) Suppose you have the freedom to remove one data point from the dataset. In that case, which data point would you choose to remove? Next, calculate the least-squares-fit line for the given data (with one data point omitted) and write down the values of the slope (m) and intercept (c) of the line based on the least-squares solution. [**2**]

(h) What must be done if there are outliers in the dataset? [**1**]

Q2. (a) List four visual recognition tasks for which machine learning or deep learning is widely used. [**4**]

(b) Backpropagation and stochastic gradient decent are the two main operations in training a neural network

i. Consider the single-variable function

$$f(w) = 3w^2 - 24w - 4.$$

Stating form $w_0 = 1$ run two iterations of gradient descent with a learning rate of 0.1 and write down the updated values of w and $f(w)$ after two iterations. [**4**]

(c) Figure Q2(c) shows an image, a CNN kernel, and a grid for displaying the output.

i. Apply the convolution operation on the image using the kernel shown in the same figure. Here, both padding and stride is set to 1. Show the output in the given grid. [**4**]

ii. Apply a ReLU activation function on the result. The ReLU activation function is defined as $f(x) = \max(0, x)$. [**2**]

0	10	20	30
10	0	20	200
40	20	20	200
190	180	190	200

i Image

$\frac{0}{5}$	$\frac{0}{5}$	$\frac{1}{5}$
$\frac{-1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
$\frac{-1}{5}$	$\frac{-1}{5}$	$\frac{0}{5}$

ii CNN Kernel

iii Grid for CNN Kernel output

iv Grid for ReLU output

Figure Q2(c): Image, CNN kernel, and output grid for **Q2**c.

(d) Consider a convolutional neural network (CNN) implementation for detecting hand written digits. There are 10 digits from 0 to 9. Here, assume that each digit image is 32×32 RGB (3 color channels). The CNN consists of a convolutional layer with five 3×3 kernels (padding=0 and stride=1) follows by a max-pooling layer of 2×2 pooling window (stride=2). Then, there is a flatten layer of l nodes. Note that the number of nodes in the flatten layer is determined by the size of the output from the preceding layer. Next, the flatten layer follows by a dense layer of k nodes (neurons) which has same dimension as the flatten layer. Finally there is a dense softmax output layer with m nodes (neurons).

i. How many learnable parameters in the convolutional layer ? [**2**]

ii. How many nodes must be there in the dense layers (k and m)? [**4**]

iii. Sketch the network. [**2**]

iv. Compute the total number of learnable parameters of this CNN. [**3**]