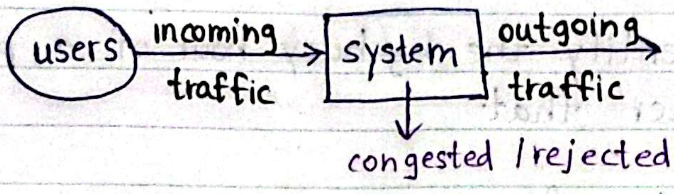
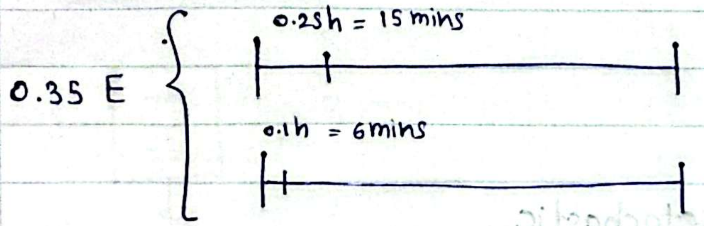
Communication Theory.



The incoming and outgoing traffics are different if the system doesn't have enough resources to cater them

Ear lang - measurement of tele traffic
(In a one circuit. if the connection stayed for an one hour it is an earlang)



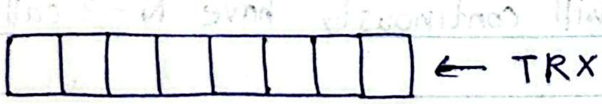
Quality of Service

* Voice.
  - Grade of service.

0.1% → from 1000 calls one call will be ~~dropped~~ blocked.

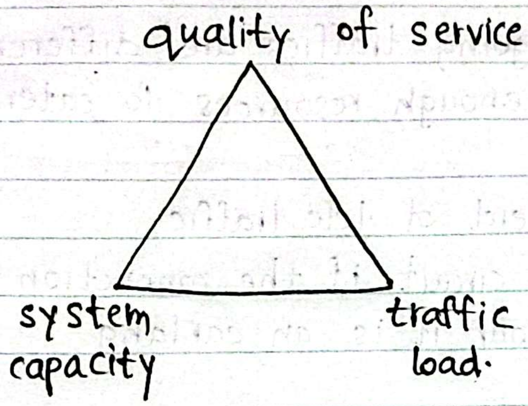
The company can decide how much grade of service should be in an area.

System Dimension:



Maximum Traffic Load.

- We have to identify the ~~by~~ busy hour of a particul. area and cater that



Teletraffic models.

Traffic models are stochastic

dimensioning
optimization
performance analysis

} Network planning.

Traffic concept.

If a network receives over a given period permanent demand of 1 call per second, with each call lasting for 3 seconds, the network will continously have $N = 3$ calls coexisting.

$$A = \frac{1}{T}\int_0^T n(t)\,dt$$

A = traffic intensity.
T = period of time.
$n(t)$ = number of busy resourc

OMNET + +

λ - mean arrival number of calls per unit time.

$A = \lambda t_m$ $\qquad$ t_m = holding time.

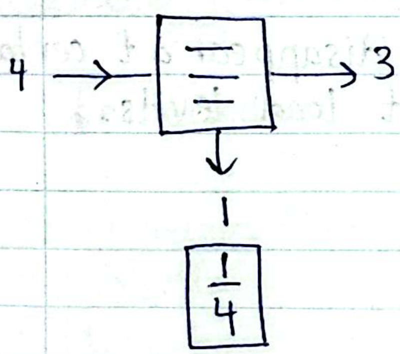
26/08/2025

Blocking.

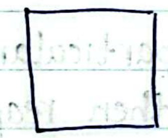
Call blocking - number of calls which were blocked.

Time blocking - fraction of time which were blocked.

Call blocking



Time blocking.



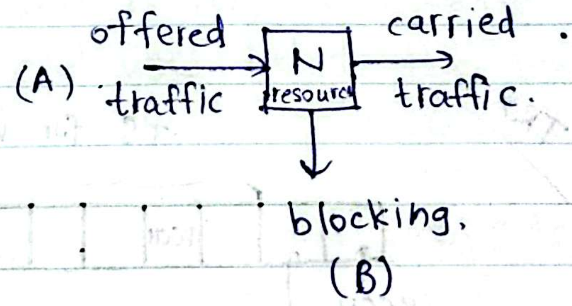
1hour → 15 mins. all channels were occupied.

$$\boxed{\frac{15}{60} = \frac{1}{4}}$$

To measure QOS → call blocking.

Earlang Loss Formula.

$$\boxed{E(N,A) = B = \frac{\frac{A^N}{N!}}{\sum_{j=0}^{N} \frac{A^j}{j!}}}$$



only for voice traffic which doesn't have waiting time

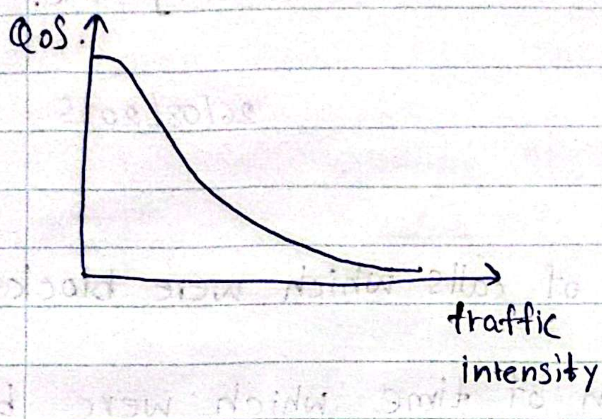
can't use in data as it has que.

Earlang B table

$1 - B_c(n) = 1 - Erl(n, a)$

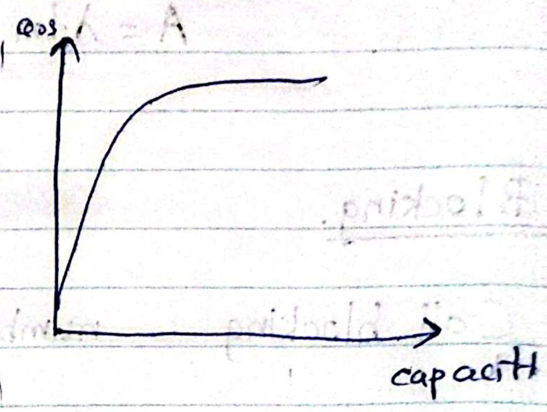
$n = 10$ channels (constant)

$QoS = 1 - B_c$



$a = 10$ traffic intensity

$QoS = 1 - B_c$



Load profiles.

During a particular day demand may disappear at certain times and then reappear, with different load levels.

peak hours and low intensity hours.

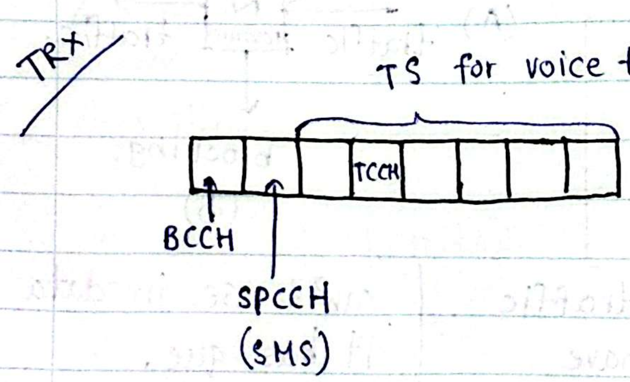
BHCA - Busy Hour Call Attempts.

Always a network is designed to cater the busy hour.

$$N_{BHCA} = \frac{A_{Erlang}}{T_{seconds}} \times 3600$$

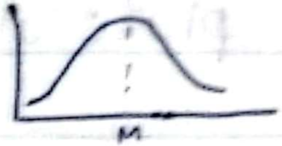
A - traffic intensity

T - mean duration of the request in time.



operators can change these allocations.

Characterisations of service

- Call arrival law (usually Poisson) 
- Call duration (usually exponential) 
- A constant bit rate.

- The voice in a packet network
- Packet flows of fixed length.

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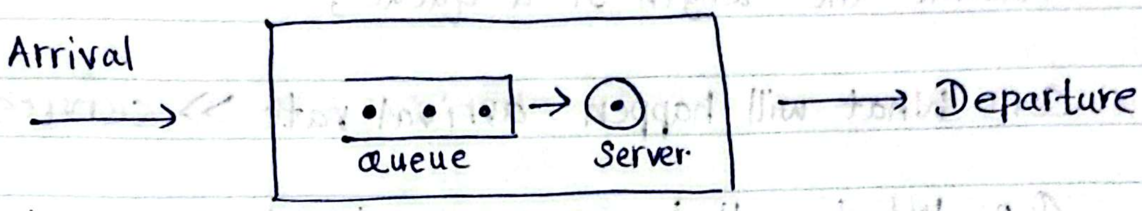
Queueing Theory

waiting, congestion } Based on the queue length these parameters change.



Service facility.

scheduling policy → FIFO, LIFO, Random, Round robbing.

Client arrive - poisson, Gaussian, Bournali
Service Time - the probability distribution of its duration
Service Discipline - FIFO, LIFO, Random, Round robbing.

Arrival process

1. The time interval between successive arrivals.
   (the interarrival time $1/\lambda$)

2. The number of arrivals in a given time interval
   (the arrival rate λ)

$$\boxed{\lambda = \lim_{T \to \infty} \frac{n(T)}{T}}$$

time T
arrivals $n(T)$

$$A(t) = \frac{1}{\lambda} = \int_0^{\infty} t \cdot d\,A(t).$$

To decide the length of a queue,

Q1. What will happen arrival rate >> service

Q2 What will happen arrival rate >> service.

Q3 What will happen the space available to hold the queue is limited.

Poison Process:

The probability of observing k arrivals in an interval of length t:

$$P_K(t) = \frac{(\lambda t)^K}{K!} \cdot e^{-\lambda t}.$$

$$P(X=k) = \frac{(A)^K}{K!} \cdot e^{-A}$$ A -> is the mean traffic.

Mean $E[X] = \frac{1}{\lambda}$

Second moment $E[X^2] = \frac{2}{\lambda^2}$

Variance $Var[X] = \frac{1}{\lambda^2}$

Example

The arrival of jobs to a supercomputing center follows a Poisson distribution with a mean interarrival time of 15 minutes.

(a) Calculate the rate of arrivals.
(b) Probability of time between arrivals $\leq \tau$ hours.
(c) Probability of k arrivals in τ hours

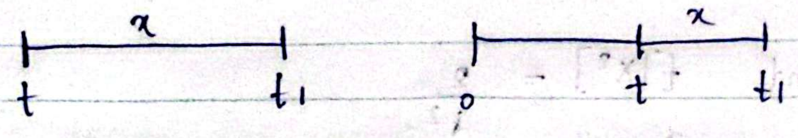
(a) $\frac{1}{\lambda} = 15$ $\lambda = \frac{1}{15} \times 60 = 4$

~~$\lambda = 0.067$ minutes~~

(b) $P(t \leq \tau) = 1 - e^{-4\tau}$

(c)

Exponential Distribution Function.



$P(x \le t) = 1 - P(X >$

$$\lambda(t) = \text{Prob}\{x \le t\} = 1 - P\{x > t\}$$

But $P\{x > t\} = P\{0 \text{ arrivaval in } (0, t]\}$

Kendall's Notation.

$A/B/m/K/n/D$.

A – distribution function of the interarrival times
B – service distribution
m – number of servers.
K – system capacity.
n – population size.
D – queue discipline.

Example.

$M/D/6$

* number of customers present in the system.

$$p_n = \text{Prob}\{N = n\}.$$

* average number in the system at stedy state is.

$$\therefore L = E[N] = \sum_{i=0}^{\infty} n p_n$$

* average number of customers in the queue.

$$L_q = E[N_q]$$

$L - L_q$ = number of customer in the servke.

* time that customer spends in the system.

$E[R]$ = The waiting time in the (W_q) + service time.

* U-Utilization is defined as the fraction of time that the server is busy. (for a single server).

$C = 1$ $\quad U\rho = \dfrac{\lambda}{\mu}.$ $\quad \lambda$ - arrival rate, μ - service rate.

$\dfrac{\lambda}{\mu} < 1 \leftarrow$ for a stable system.

$C > 1$ $\quad U\rho = \dfrac{\lambda}{C\mu}$

S..van

* traffic intensity.

$\bar{x}$ - average number of customers process per unit time.

$$\lambda \bar{x} = \frac{\lambda}{\mu}$$

single server $\longrightarrow U$

multiple server $\longrightarrow cU$

* mean number of customes in the system

$L = \lambda W.$ $W \rightarrow$ Response time.

$= T_s + T_Q$

Queue

$L_q = \lambda W_q$

Service.

$L_s = \lambda W_s$

Steady state solution for the M/M/1 queue is given by,

$$p_n = p^n(1-\rho) \quad \text{for } \rho = \frac{\lambda}{\mu} < 1.$$

The probability that the queue contains at least k customers.

$$\text{Prob}(n \geq k) = \sum_{i=k}^{\infty} P_i = (1-p)\sum_{i=k}^{\infty} \rho^i = (1-\rho)\left(\sum_{i=0}^{\infty}\rho^i - \sum_{i=0}^{k-1}\rho^i\right)$$

$$= (1-\rho)\left(\frac{1}{1-\rho} - \frac{1-\rho^k}{1-\rho}\right)$$

$$= \rho^k$$

Performance measures concerning the M/M/1 Queue

1. Mean number in System

$$L = (1-\rho)\frac{\rho}{(1-\rho)^2} = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$$

2. Variance of Number in System

$$\text{Var}(N) = = \rho\frac{1+\rho}{(1-\rho)^2} - \left(\frac{\rho}{1-\rho}\right)^2 = \frac{\rho}{(1-\rho)^2}$$

3. Mean queue length.

$$L_q = \rho L = L - \rho$$

Average Response Time.

$$E[N] = \lambda E[R] \quad (L = \lambda W)$$

$$E[R] = \frac{1}{\lambda} E[N] = \frac{1}{\lambda} \frac{\rho}{1-\rho} = \frac{1/\mu}{1-\rho} = \frac{1}{\mu-\lambda}$$

Average Waiting Time

$$L_q = \lambda W_q$$

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\rho}{\mu-\lambda}$$

Example 1

The arrival pattern of cars to the local oil change center follows a Poisson distribution at a rate of four per hour. If the time to perform an oil change is exponentially distributed and requires an average of 12 minutes to carry out, what is the probability of finding more than 3 cars waiting for the single available mechanic to service their car?

$\lambda = 4h^{-1}$ $\quad\quad$ $\mu = 5h^{-1}$

$\cancel{\mu = \frac{12}{60} = \frac{1}{}}$

$$P(n \geq 3) = \rho^3 = \left(\frac{4}{5}\right)^3 = 0.512$$

Ex 2

In a tool crib, workers come to take tools at 4/hours on the average. Waiting for them costs 10 $ per hour. The service time per worker is in the tool crib is 12 mins.
What will be total waiting cost of the workers per day if it is 8 hours a day?
Assume M/M/1 queueing system.

$\lambda = 4h^{-1}$ $\qquad L_q = \lambda W_q$

$\mu = 5h^{-1}$

$$= \frac{\lambda \rho}{(\lambda - \mu)} = \frac{4 \cdot 4/5}{5-4}$$

$$= 3.2$$

Total waiting cost $= 3.2 \times 10 \times 8 = 256$

Ex 3.

$\lambda = 2.5$

$\mu = 7.5$

$$L = \frac{\lambda}{\mu - \lambda} = \frac{2.5}{7.5 - 2.5} = 0.5 \Rightarrow 0.5 \times 8.5 \times 0.8$$

The M/M/c Queue.

Multi Server Systems.

$$\lambda_n = \lambda \quad \text{for all } n,$$

$$\mu_n = \begin{cases} n\mu & 1 \leq n \leq c, \\ c\mu & n \geq c. \end{cases}$$

$$P_n = P_0 \prod_{i=1}^{n} \frac{\lambda}{i\mu} = P_0 \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} \quad \text{if } 1 \leq n \leq c,$$

$$P_n = P_0 \prod \frac{\lambda}{c\mu}$$

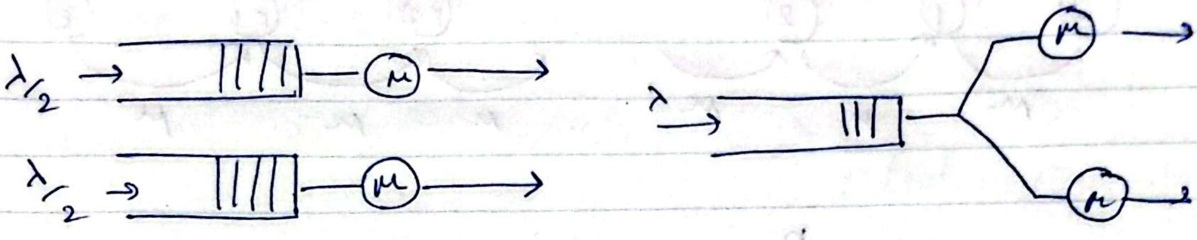
Define $\rho = \dfrac{\lambda}{(c\mu)}$ $\quad$ $\rho < 1$ stable system.

$$P_n = P_0 \frac{(c\rho)^n}{n!} \qquad n \leq c.$$

$$P_n = P_0 \frac{(c\rho)^n}{c^{n-c} c!} = \frac{P_0 \rho^n c^c}{c!} \quad \text{for } n \geq c.$$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!} \frac{1}{1-\rho} \right]^{-1}.$$

Comparing M/M/1 Queue with M/M/C Queue.

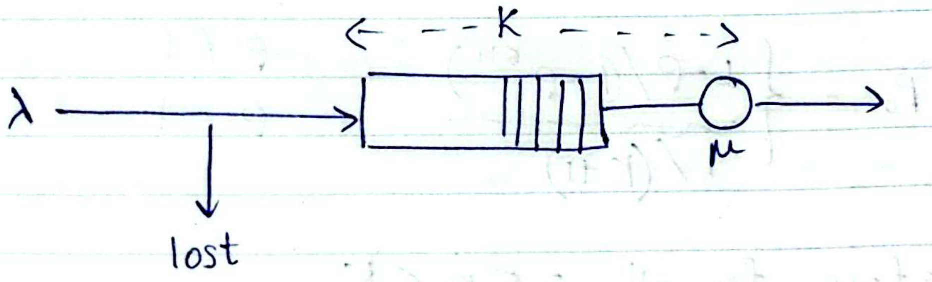


7/10/2023.

	M/M/1 × 2		M/M/2		M/M/1 (speed) $\mu = 2\mu$
average arrival time $E[N]$	$\frac{2\rho}{1-\rho}$	$\geq$	$\frac{2\rho}{1-\rho} \cdot \frac{1}{1+\rho}$	$\geq$	$\frac{\rho}{1-\rho}$
$E[R]$ average response time.	$\frac{2}{2\mu-\lambda}$	$\geq$	$\frac{1/\mu}{1-\rho^2}$	$\geq$	$\frac{1}{2\mu-1}$

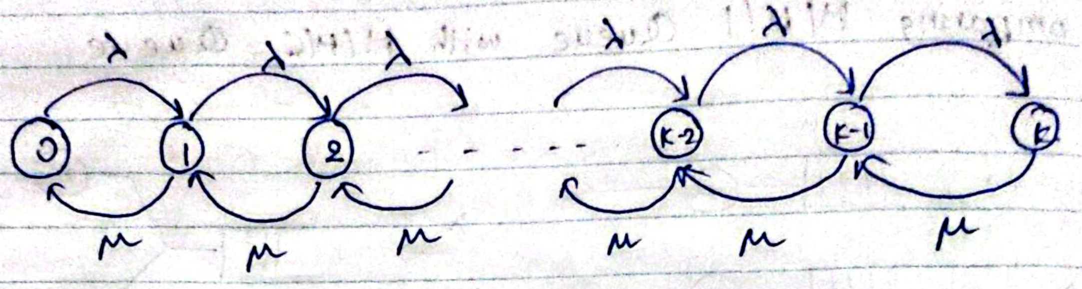
M/M/1/K Queue.

K - number of customers which can stay in the system with the service.



loss system

(network busy)



$$P_n = P_0 \prod_{i=1}^{n} \frac{\lambda_{i-1}}{\mu_i} \qquad n \geq 1$$

$$P_{n+1} = \frac{\lambda + \mu}{\mu} P_n - \frac{\lambda}{\mu} P_{n-1} \qquad 1 \leq n < K-1$$

$$P_K = \frac{\lambda}{\mu} P_{K-1}$$

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 = \rho^n P_0 \qquad 0 \leq n \leq K.$$

$$P_0 = \frac{1}{\sum_{n=0}^{K} \rho^n}$$

$$P_0 = \begin{cases} 1-\rho/(1-\rho^{K+1}) & \rho \neq 1 \\ 1/(K+1) & \rho = 1 \end{cases}$$

Therefore for all $0 \leq n \leq K$.

$$P_n = \frac{(1-\rho)\rho^n}{1-\rho^{K+1}} \qquad \rho \neq 1$$

$K \longrightarrow \infty$

$$\lim_{K \to \infty} \frac{1-\rho}{1-\rho^{K+1}} \rho^n = (1-\rho)\rho^n$$

M/M/1 Queue.

Q A Telephone switch has 10 output lines and large number of incoming lines. Upon arrival call ~~of the in~~ is assigned an output line if such line is available. Otherwise the call is blocked and lost. The output line remains assigned to the call for its entire duration which is of exponentially distributed.
Assume that 180 calls per hour arrival in Poisson passion whereas the mean call duration is 110 s.

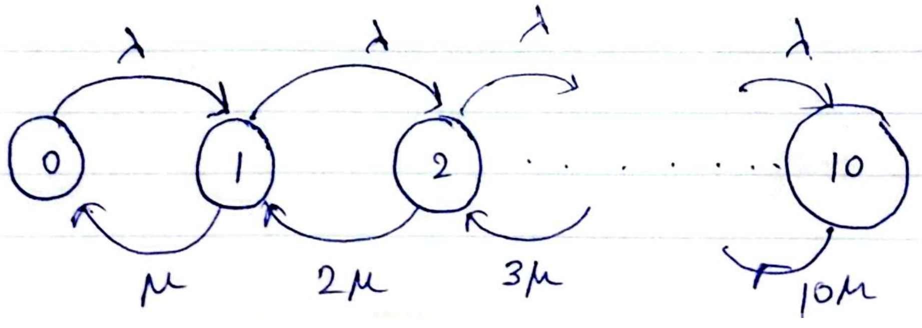
(i) Blocking probability.
(ii) How many calls are rejected per hour.
(iii) Draw the state diagram.

λ, μ, number of servers, queue size

$\lambda = 180\ h^{-1}$ $\qquad$ $\mu = \frac{60 \times 60}{110} = 32.73\ h^{-1}$

number of servers = 10

M/M/10/10



No :

offered load.

$$\rho = \frac{\lambda}{\mu} = 180 \times \frac{110}{3600} = 5.5$$

Lossy systems → no queue,
(Earlang B table)

→ has a queue
(Earlang C table)

it has
* # channels
* offered load
* blocking probability:

∴ Blocking probability 3 %

Rejected calls $= \frac{3}{100} \times 180$

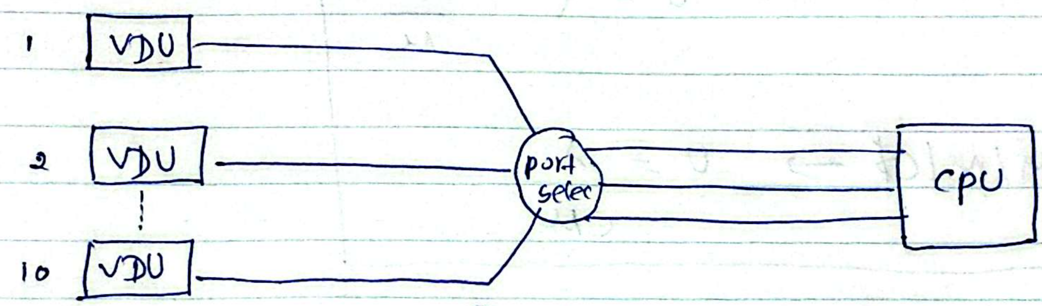
$= 5.4$

load handle by ~~the~~ one server.

$$\left(\frac{5.5\,(1 - 0.03)}{10} \right)$$

$= 0.5335$

Q A Group of 10 Video Display Units (VDUs) for transactions processing gain access to 3 computer ports through a port selector.
If the transaction generated by each VDU can be as a Poissons stream with rates of six transactions per hour, the length of each transaction is approximated exponentially distributed with the mean of 5 minutes.
Calculate
(i) The P that all three ports are engaged
(ii) when a VDU initiates a connection.
(iii) The average number of computer ports engaged



M/M/3/3.

Consider the case when $\rho = 1$
(M/M/1/K)

$$L_q = L - (1 - P_0) = L - \left(1 - \frac{1-\rho}{1-\rho^{k+1}}\right)$$

Effective arrival rates

$$\bar{\lambda} = \lambda(1 - P_K)$$

$$W = \frac{1}{\bar{\lambda}} L \qquad W_q = \frac{1}{\bar{\lambda}} L_q$$

$$W_q = W - \frac{1}{\mu}$$

Throughput $\quad X = \lambda(1 - P_K)$

$$\boxed{\begin{array}{l} M/M/1 \rightarrow U = \rho = \dfrac{\lambda}{\mu} \\[2em] M/M/C \rightarrow U = \dfrac{\lambda}{C\mu} \end{array}}$$

$$M/M/1/K \rightarrow U = 1 - P_0$$
$$= \frac{1}{\mu} X$$
$$= \rho(1 - P_K)$$