ET3122 Antennas and Propagation
Dipole Antenna Basics

Upeka Premaratne

Department of Electronic and Telecommunication Engineering
University of Moratuwa

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Outline

Introduction

Introduction

- The dipole is the simplest form of antenna
  - Radiates by a current standing wave
- The $\lambda_e/4$ dipole is the most efficient radiator
  - The total length induces the maximum electric and magnetic field for efficient radiation
- A very useful antenna at microwave frequencies
  - As a standalone antenna
  - As a feed antenna for arrays

Types of Dipole

- Based upon length can be classified as
  - Short dipoles (approximately triangular current distribution)
  - Long dipoles (sinusoidal current distribution)
- Can be analyzed using an infinitessimal Hertzian radiator
- Numerous design considerations

Current Distribution

Current Distribution

- For an antenna of section length l (total length $2l$) and $k = 2\pi/\lambda_e$ the current distribution is given by

$$J(z) = \begin{cases} I_0 \sin\left[k(l-z)\right] & \text{when } z > 0 \\ I_0 \sin\left[k(l+z)\right] & \text{when } z < 0 \end{cases}$$

- Must always satisfy the boundary condition of
  - $J(z) = 0$ at $z = \pm l$
- The spatial current distribution will sinusoidally vary with time at angular frequency ω

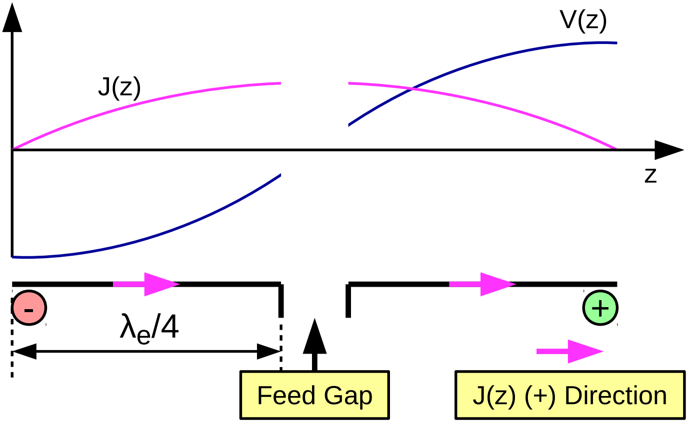
$$J(z) = \begin{cases} I_0 \sin\left[k(l-z)\right]\cos(\omega t) & \text{when } z > 0 \\ I_0 \sin\left[k(l+z)\right]\cos(\omega t) & \text{when } z < 0 \end{cases}$$

Potential Distribution

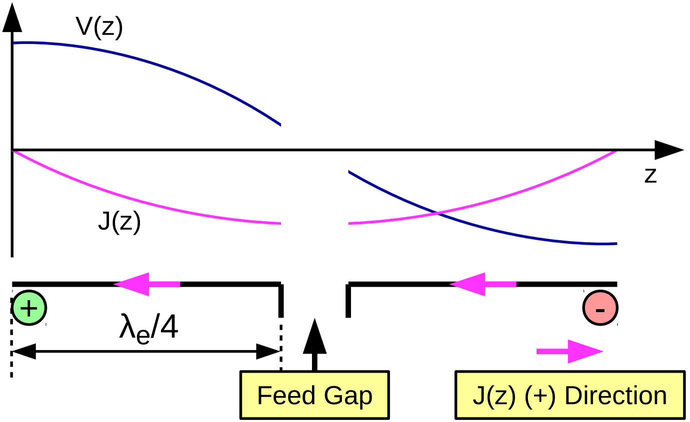
- Due to the resultant charge distribution of the antenna
- $\pi/2$ out of phase with the current

$$V(z) = \begin{cases} -V_0 \cos\left[k(l-z)\right]\cos(\omega t) & \text{when } z > 0 \\ V_0 \cos\left[k(l+z)\right]\cos(\omega t) & \text{when } z < 0 \end{cases}$$

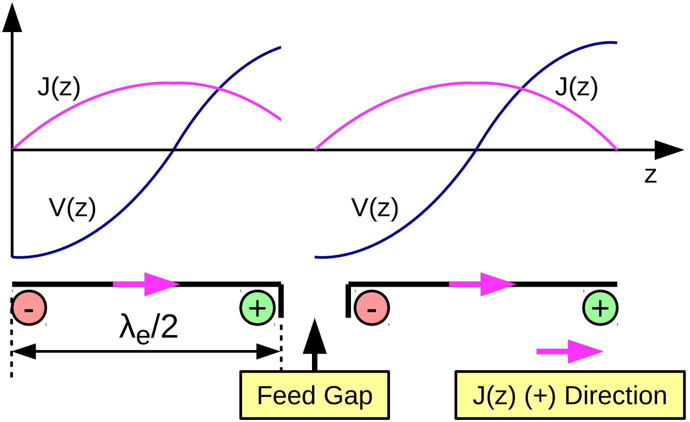
$\lambda/4$ Dipole (Positive Half Cycle)



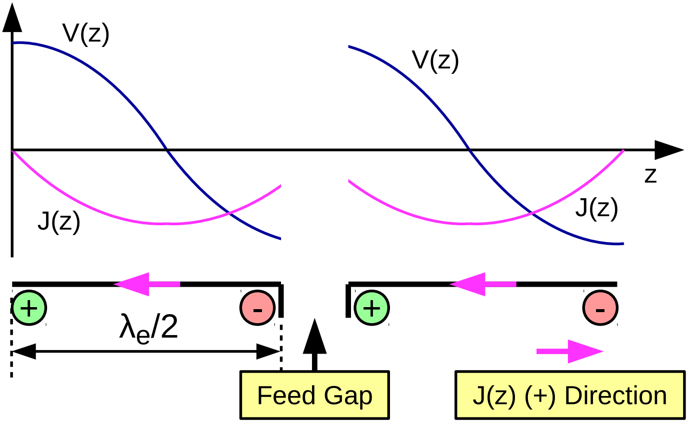
$\lambda/4$ Dipole (Negative Half Cycle)



$\lambda/2$ Dipole (Positive Half Cycle)



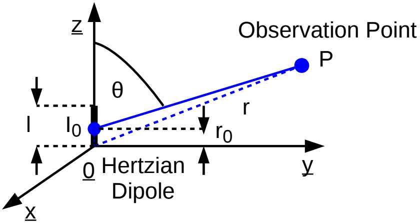
$\lambda/2$ Dipole (Negative Half Cycle)



Radiation Pattern

The Hertzian Dipole

- The Hertzian or infinitessimal dipole is the “building block” of the radiation pattern of a dipole antenna
- Not a true dipole because the current is constant
  - Violates the boundary condition of $J(z) = 0$ when $z = \pm l$
- Since the antenna is negligibly short for an observer in the far field $r \gg r_0$



The Hertzian Dipole (Contd..)

- Taking $J(z) = I_0\underline{z}$ results in the vector potential of

$$A(r) = \frac{\mu}{4\pi}\int_V \frac{J(r_0)e^{-jk|r-r_0|}}{|r-r_0|}dv = \frac{\mu}{4\pi}\int_0^l \frac{(I_0\underline{z})e^{-jk|r-r_0|}}{|r-r_0|}dz$$

- Since $r \gg r_0$,

$$A(r) = \frac{\mu I_0}{4\pi}\int_0^l \left(\frac{e^{-jkr}}{r}dz\right)\underline{z} = \frac{\mu I_0 l}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\underline{z}$$

- By converting to spherical polar coordinates,

$$A(r) = \left[\frac{\mu I_0 l}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\right](\cos(\theta)\underline{r} - \sin(\theta)\underline{\theta})$$

The Hertzian Dipole (Contd..)

- From this the H field is obtained from,

$$\begin{aligned} H &= \frac{1}{\mu}(\nabla \times A) = \frac{1}{\mu}\left[\frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r)\right)\right]\underline{\phi} \\ &= \frac{I_0 l}{4\pi}\left[\left(\frac{e^{-jkr}\sin(\theta)}{r}\right)\left(\frac{1}{r} + jk\right)\right]\underline{\phi} \end{aligned} \tag{1}$$

- The E field is obtained from,

$$\begin{aligned} E &= \frac{1}{j\omega\varepsilon}(\nabla \times H) \\ &= \frac{1}{j\omega\varepsilon}\left[\frac{1}{r\sin(\theta)}\left(\frac{\partial}{\partial \theta}(\sin(\theta)H_\phi)\right)\underline{r} + \frac{1}{r}\left(-\frac{\partial}{\partial r}(rH_\phi)\right)\underline{\theta}\right] \end{aligned}$$

The Hertzian Dipole (Contd..)

- Thus, by substituting (1)

$$
\begin{aligned}
E &= \left(\frac{1}{j\omega\varepsilon}\right)\left(\frac{I_0 l}{4\pi}\right)\left[\left(\frac{2}{r}\right)\left(\frac{e^{-jkr}}{r}\right)\left(\frac{1}{r}+jk\right)\cos(\theta)\underline{r}\right. \\
&\quad \left. +\sin(\theta)\left(k^2 e^{-jkr}-\frac{jke^{-jkr}}{r}-\frac{e^{-jkr}}{r^2}\right)\underline{\theta}\right] \\
&= \frac{I_0 l e^{-jkr}}{j4\pi\omega\varepsilon}\left[2\cos(\theta)\left(\frac{1}{r^3}+\frac{jk}{r^2}\right)\underline{r}\right. \\
&\quad \left. +\sin(\theta)\left(\frac{k^2}{r}-\frac{jk}{r^2}-\frac{1}{r^3}\right)\underline{\theta}\right] \qquad (2)
\end{aligned}
$$

The Hertzian Dipole (Contd..)

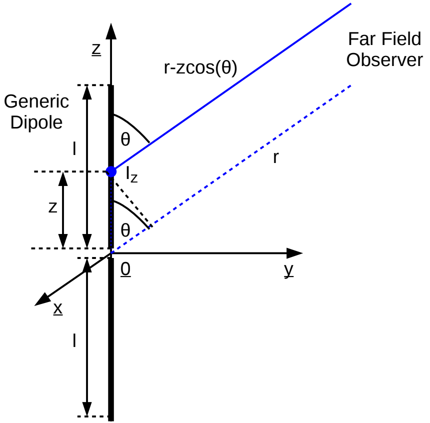
- By considering the far field approximation $kr \gg 1$
- From (2)

$$E \approx \left[\frac{j\eta k I_0 l e^{-jkr}\sin(\theta)}{4\pi r}\right]\underline{\theta} \quad (3)$$

- From (1)

$$H \approx \left[\frac{jk I_0 l e^{-jkr}\sin(\theta)}{4\pi r}\right]\underline{\phi} \quad (4)$$

Generic Dipole



Generic Dipole (Contd..)

- The far field radiation pattern can be considered as an integration of the individual Hertzian dipoles for the current distribution

$$E = \int_{-l}^{l} dE = \int_{-l}^{l} \frac{j\eta k(I_z dz)e^{-jk(r - z\cos(\theta))}\sin(\theta)}{4\pi(r - zcos(\theta))}$$

- For the amplitude $r - z\cos(\theta) \approx r$

$$E \approx \frac{j\eta k e^{-jkr}\sin(\theta)}{4\pi r} \int_{-l}^{l} \left[I_z e^{jkz\cos(\theta)} dz \right] \underline{\theta}$$

Generic Dipole (Contd..)

$$
\begin{aligned}
E &\approx \frac{j\eta k e^{-jkr}\sin(\theta)}{4\pi r}\left[\int_{-l}^{0} I_0 \sin\left[k(l+z)\right] e^{jkz\cos(\theta)}dz + \right. \\
&\quad \left. \int_{0}^{l} I_0 \sin\left[k(l-z)\right] e^{jkz\cos(\theta)}dz\right]\underline{\theta} \\
&\approx \frac{j\eta I_0 e^{-jkr}}{2\pi r}\left[\frac{cos\left(kL\cos(\theta)\right) - cos\left(kL\right)}{\sin(\theta)}\right]\underline{\theta} \qquad (5) \\
H &\approx \frac{j I_0 e^{-jkr}}{2\pi r}\left[\frac{cos\left(kL\cos(\theta)\right) - cos\left(kL\right)}{\sin(\theta)}\right]\underline{\phi} \qquad (6)
\end{aligned}
$$

Integral Proof

$$\int e^{Az}\sin(Bx+C) = \frac{e^{Az}}{A^2+B^2}\left[A\sin(Bz+C) - B\cos(Bz+C)\right]$$

$$\begin{aligned} I &= \int_{-l}^{0} e^{jk\cos(\theta)z}\sin(kz+kl)dz \\ &= \frac{1}{k\sin^2(\theta)}\left[j\cos(\theta)\sin(kl) - \cos(kl)\right] + \frac{e^{-jkl\cos(\theta)}}{k\sin^2(\theta)} \\ &= \frac{e^{-jkl\cos(\theta)} - \cos(kl) + j\cos(\theta)\sin(kl)}{k\sin^2(\theta)} \end{aligned}$$

Integral Proof (Contd..)

$$\begin{aligned} J &= \int_0^l e^{jk\cos(\theta)z}\sin(kL-kz)dz = -\int_0^l e^{jk\cos(\theta)z}\sin(kz-kL)dz \\ &= \frac{e^{jkl\cos(\theta)}}{k\sin^2(\theta)} - \frac{1}{k\sin^2(\theta)}\left[j\cos(\theta)\sin(kl)+\cos(kl)\right] \\ &= \frac{e^{jkl\cos(\theta)} - \cos(kl) - j\cos(\theta)\sin(kl)}{k\sin^2(\theta)} \end{aligned}$$

For a dipole the integral becomes,

$$E = \frac{j\eta k e^{-jkr}\sin(\theta)}{4\pi r}(I+J)$$

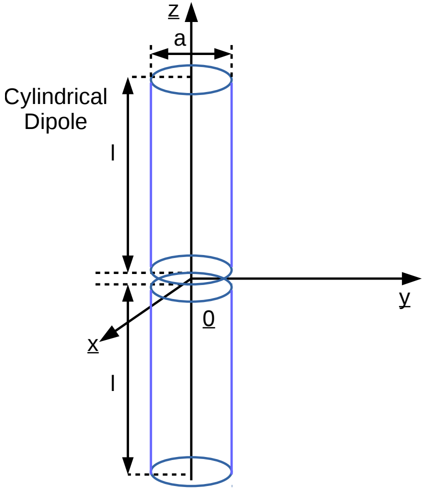
Feed Impedance

Feed Impedance

- The input impedance of the antenna as seen by the source or transmission line
- Has to be matched to the transmission line or else power will be reflected
- For a given load z_L and characteristic impedance z_0, the proportion of reflected power is given by $|\rho_0|^2$ where

$$\rho_0 = \frac{z_L - z_0}{z_L + z_0}$$

Feed Impedance (Contd..)



Feed Impedance (Contd..)

- For a dipole the formula for feed impedance is obtained using the induced EMF method (Carter, 1932)
  - Given in terms of the length of a section l as well as diameter a

$$Z_{IN} = R_{IN} + jX_{IN} = \frac{1}{\sin^2(kl)}\left[R_0 + jX_0\right]$$

Where

$$\begin{aligned} R_0 &= \frac{\eta_0}{2\pi}\Big\{\gamma_0 + \ln(2kl) - \mathsf{Ci}(2kl) + \frac{1}{2}\sin(2kl)\left(\mathsf{Si}(4kl) - 2\mathsf{Si}(2kl\right. \\ &\quad + \frac{1}{2}\cos(2kl)\left(\gamma_0 + \ln(kl) + \mathsf{Ci}(4kl) - 2\mathsf{Ci}(2kl)\right)\Big\} \end{aligned}$$

Feed Impedance (Contd..)

..and

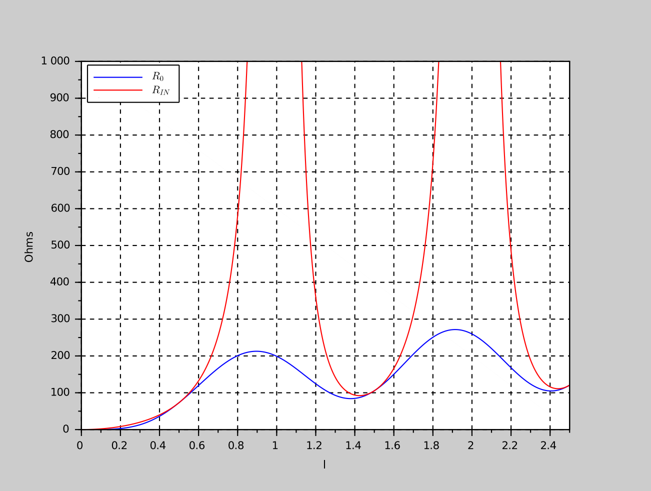
$$X_0 = \frac{\eta_0}{2\pi}\left\{\mathrm{Si}(2kl) + \frac{1}{2}\cos(2kl)\left(2\mathrm{Si}(2kl) - \mathrm{Si}(4kl)\right) + \frac{1}{2}\sin(2kl)\left(\mathrm{Ci}(4kl) - 2\mathrm{Ci}(2kl) - \mathrm{Ci}\left(\frac{ka^2}{4l}\right)\right)\right\}$$

$\gamma_0 = 0.5772156649$ (the Euler-Mascheroni constant)

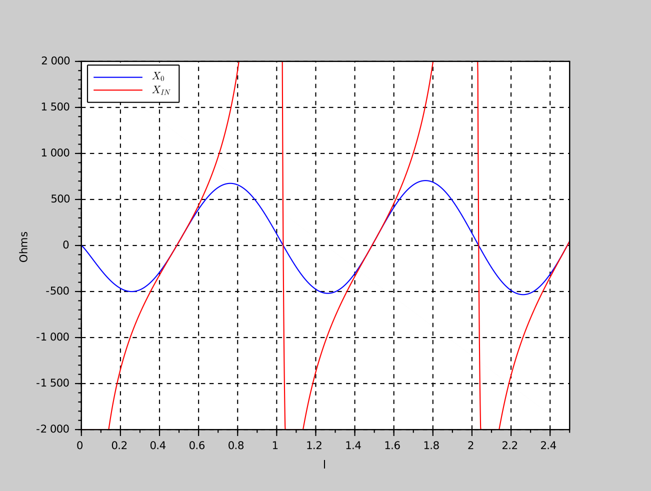
$\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos(t)}{t}dt$ (cosine integral)

$\mathrm{Si}(x) = -\int_0^x \frac{\sin(t)}{t}dt$ (sine integral)

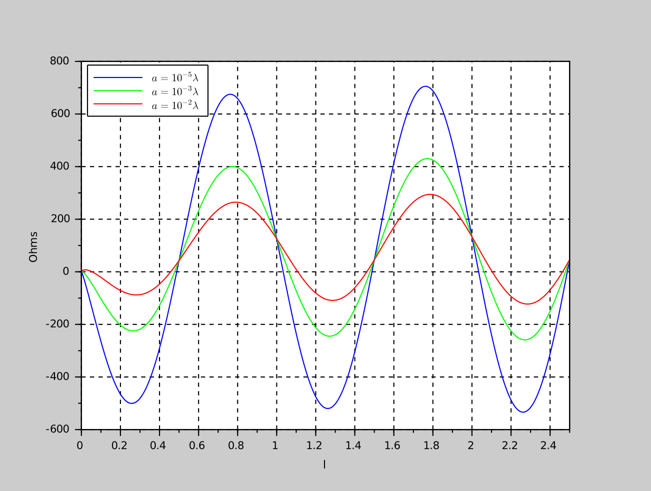
Dipole Feed Resistance



Dipole Feed Reactance



Dipole Feed Reactance (Contd..)



Questions

1. What is the desired reactance of a dipole for it to be matched to the transmission line?
2. Why is a dipole shorter than $\lambda_e/4$ said to be capacitive and a dipole longer than $\lambda_e/4$ said to be inductive?
3. Why is increasing a with respect to λ of a dipole preferred?
4. How can the bandwidth of a dipole antenna be determined?

Conclusion

Conclusion

- The dipole is the simplest radiator
- It radiates by the induction of two dipoles at each end
  - These dipoles in tern generate the E field
  - The H field is generated by the periodic current flow
- Its feed impedance is given by Carter's formula
- Widely used in its basic or derivative form