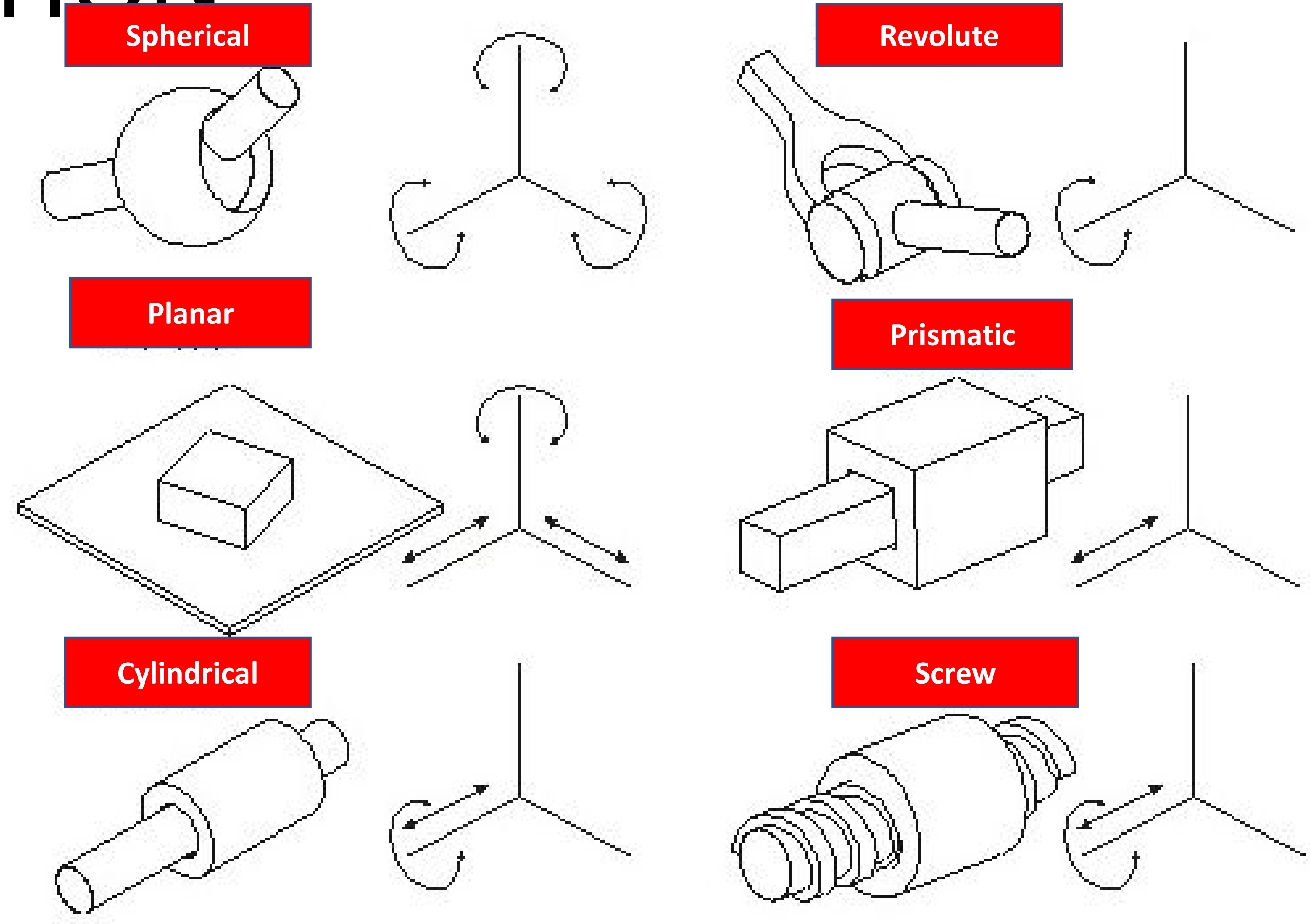
Manipulator kinematics

INTRODUCTION

- Kinematics is the **science of motion** that treats the subject **without regard to the forces** that cause it.
- The science of kinematics, studies the **position**, the **velocity**, the **acceleration**, and higher order derivatives of the position variables
- Hence, the study of the kinematics of manipulators refers to all the geometrical and time-based properties of the motion

LINK DESCRIPTION

- A manipulator may be thought of as a set of *bodies connected* in a chain by **joints**.
- These bodies are called **links**.
- Lower pair is used to describe the connection between a pair of bodies when the relative motion is characterized by two surfaces sliding over one another



The six possible lower-pair joints

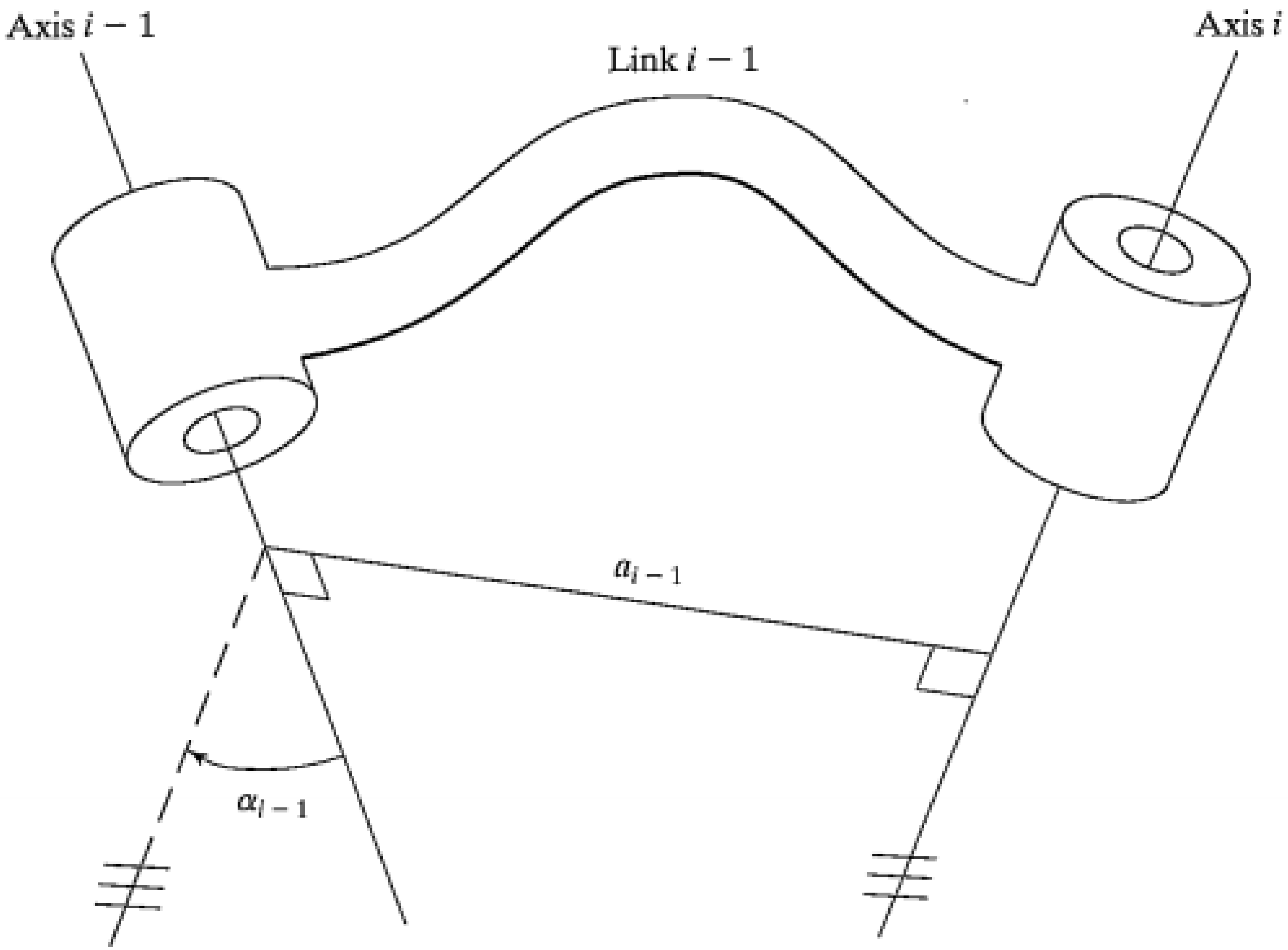
Typical link mechanical design attributes:

- The type of material used,
- The strength and stiffness of the link,
- The location and type of the joint bearings,
- The external shape,
- The weight and inertia, and more.

For obtaining kinematic equation

- *A link is considered only as a rigid body that defines the relationship between two neighboring joint axes of a manipulator*.

Link Description



The kinematic function of a link is to maintain a ***fixed relationship*** *between the* ***two joint axes*** *it supports.*

This relationship can be described with two parameters: the link length, a, *and the link twist,* α.

LINK-CONNECTION DESCRIPTION

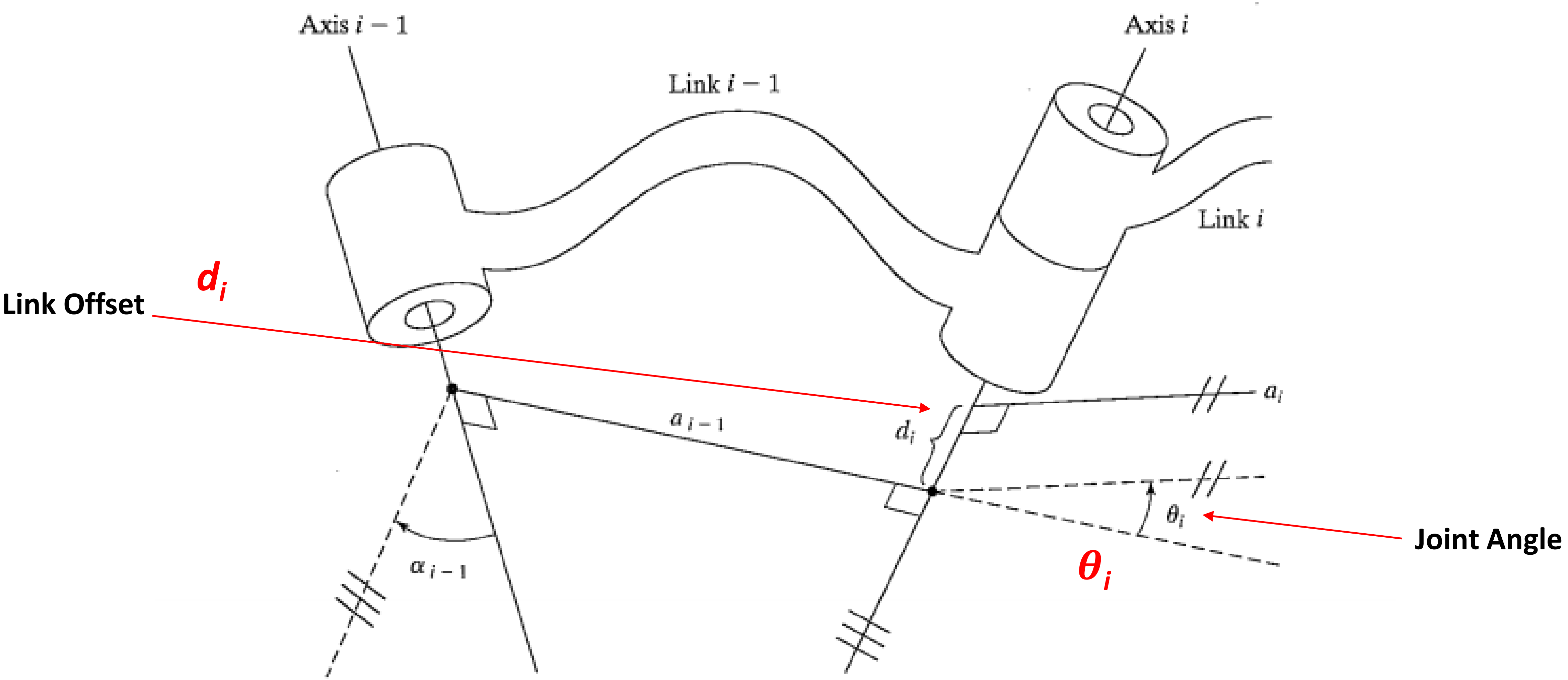


FIGURE 3.4: The link offset, d, and the joint angle, θ, are two parameters that may be used to describe the nature of the connection between neighboring links.

First and last links in the chain

- Link length, a_i and link twist $\boldsymbol{\alpha_i}$**,** depend on joint axes *i* and *i + 1*.
- a_1 through a_{n-1}, and α_1 *to* α_{n-1} are defined
- At the ends of the chain, assign zero to these quantities
- $a_0 = a_n = 0.0$ *and* $\alpha_0 = \alpha_n = 0$

Link parameters

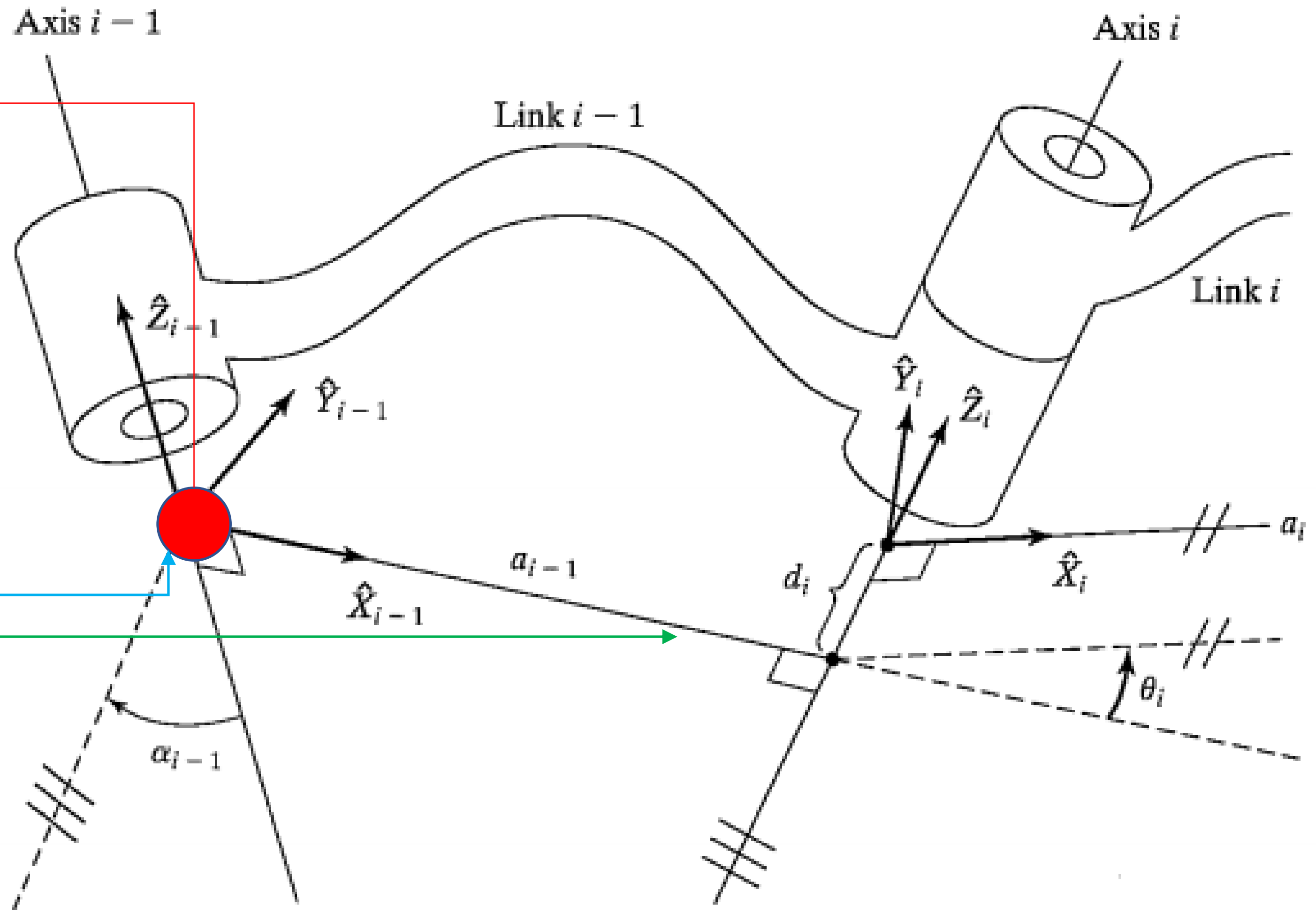
- Any robot can be described kinematically by giving the values of *four quantities* for each link
- Two describe the link itself, and two describe the link's connection to a neighboring link
- For a *revolute joint*, $\boldsymbol{\theta}_i$ is called the *joint variable*, and the other three quantities would be fixed link parameters.
- For *prismatic joints*, $\mathbf{d}_i$ is the *joint variable*, and the other three quantities are fixed link parameters.
- The ***definition*** of mechanisms by means of these quantities is a convention usually called the ***Denavit-Hartenberg*** notation

CONVENTION FOR AFFIXING FRAMES TO LINKS

- *In order to describe the location of each link relative to its neighbors,*
- *we define a frame attached to each link*
- *The link frames are named by number according to the link to which they are attached.*
- ***That is, frame {i } is attached rigidly to link i.***

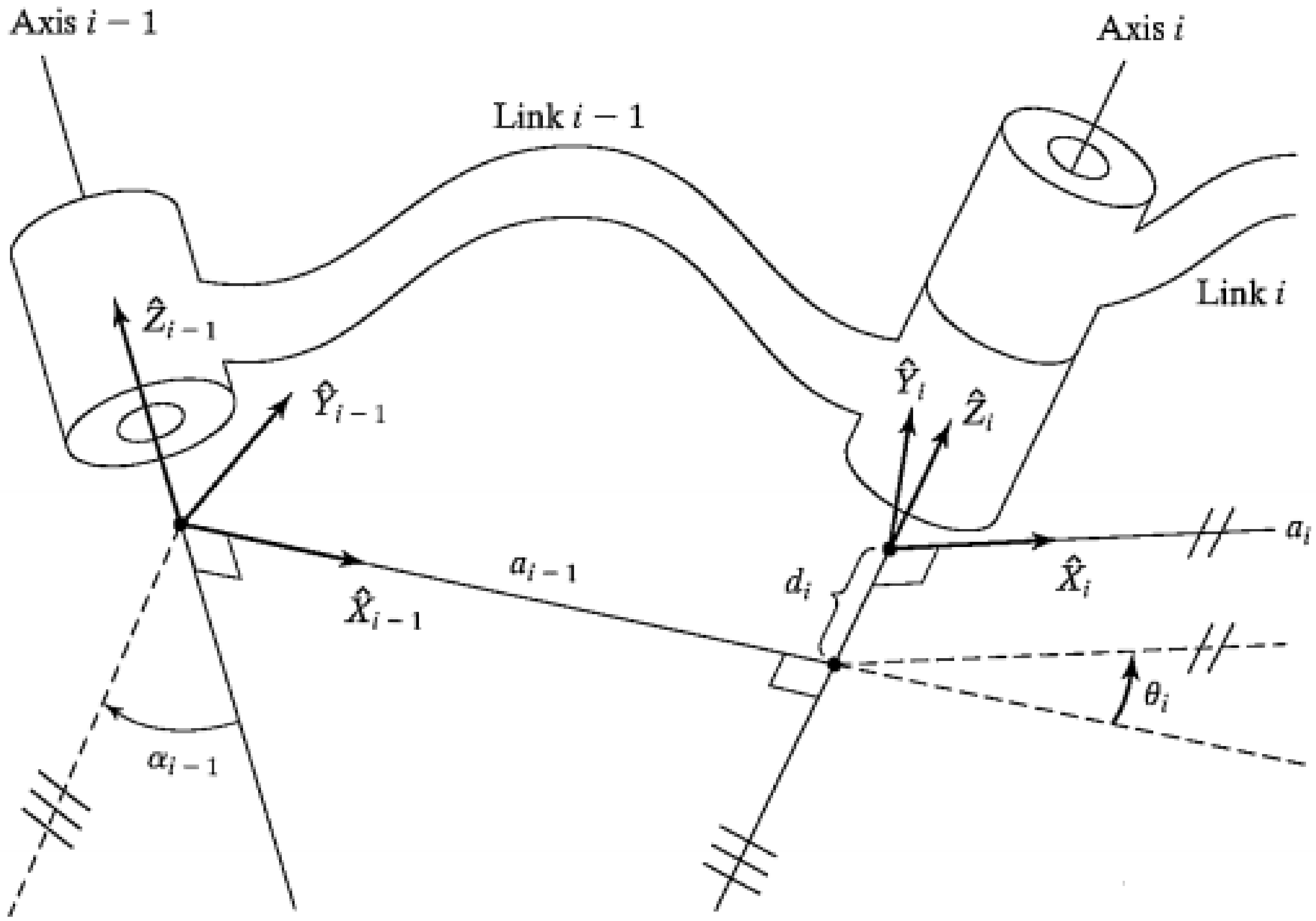
Intermediate links in the chain

- The Z-axis of frame **{i}**, called Z_i, is coincident with the joint axis X_i.
- The origin of frame **{i}** is located where the perpendicular intersects the joint *i* axis.
- X_i points along $\mathbf{a}_i$ in the direction from joint *i* to joint *i + 1*.



Intermediate links in the chain

- We define α_i as being measured in the right-hand sense about $\boldsymbol{X_i}$
- $\boldsymbol{Y_i}$ is formed by the right-hand rule to complete the ***ith*** frame.



Summary of the link parameters in terms of the link frames

- *If the link frames have been attached to the links according to our convention, the following definitions of the link parameters are valid:*

$a_i =$ *the distance from* $\hat{Z}_i$ *to* $\hat{Z}_{i+1}$ *measured along* $\hat{X}_i$;

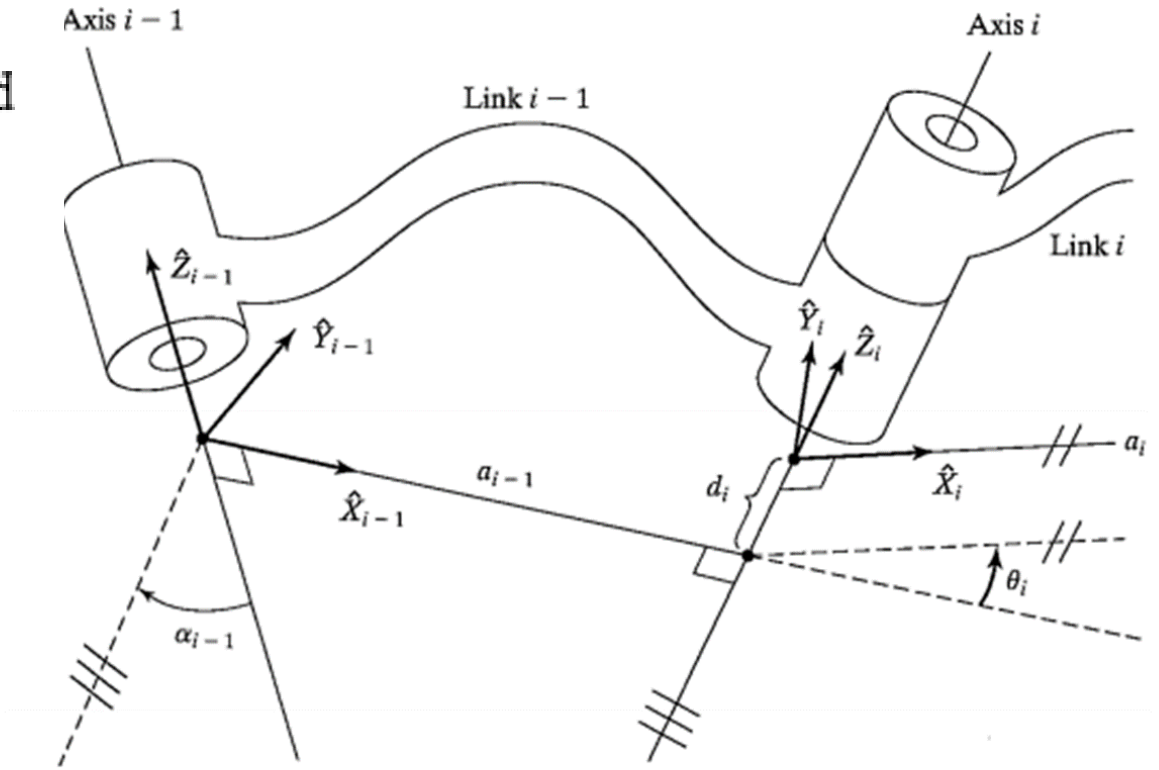
$\alpha_i =$ *the angle from* $\hat{Z}_i$ *to* $\hat{Z}_{i+1}$ *measured about* $\hat{X}_i$;

$d_i =$ *the distance from* $\hat{X}_{i-1}$ *to* $\hat{X}_i$ *measured along* $\hat{Z}_i$; and

$\theta_i =$ *the angle from* $\hat{X}_{i-1}$ *to* $\hat{X}_i$ *measured about* $\hat{Z}_i$.

$\boldsymbol{a, \alpha = link\ Parameters}$

$\boldsymbol{d, \theta = Joint\ parameters}$



$a_i =$ *the distance from* $\hat{Z}_i$ *to* $\hat{Z}_{i+1}$ *measured along* $\hat{X}_i$;

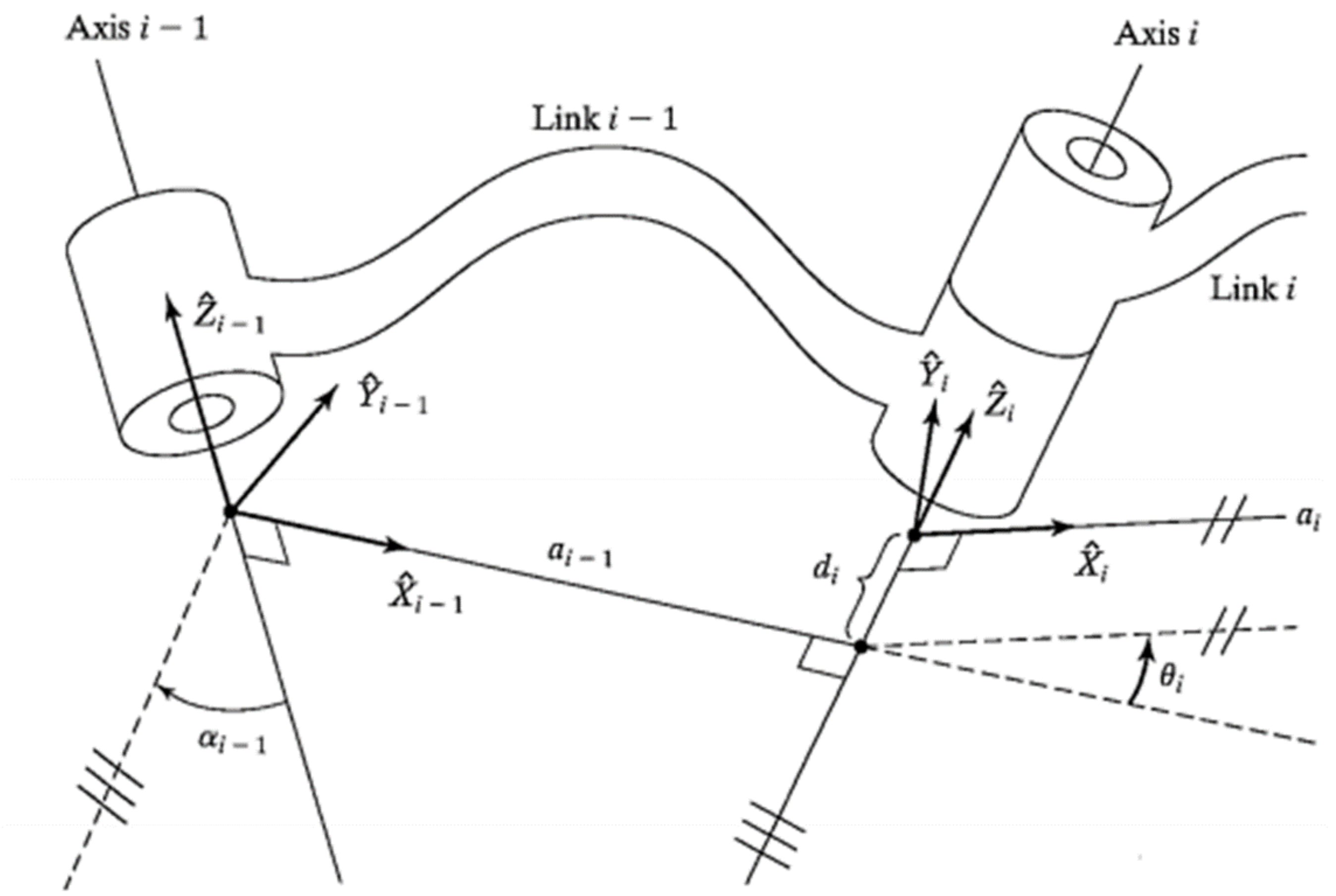
$\alpha_i =$ *the angle from* $\hat{Z}_i$ *to* $\hat{Z}_{i+1}$ *measured about* $\hat{X}_i$;

Link

$d_i =$ *the distance from* $\hat{X}_{i-1}$ *to* $\hat{X}_i$ *measured along* $\hat{Z}_i$; and

$\theta_i =$ *the angle from* $\hat{X}_{i-1}$ *to* $\hat{X}_i$ *measured about* $\hat{Z}_i$.

With next Link

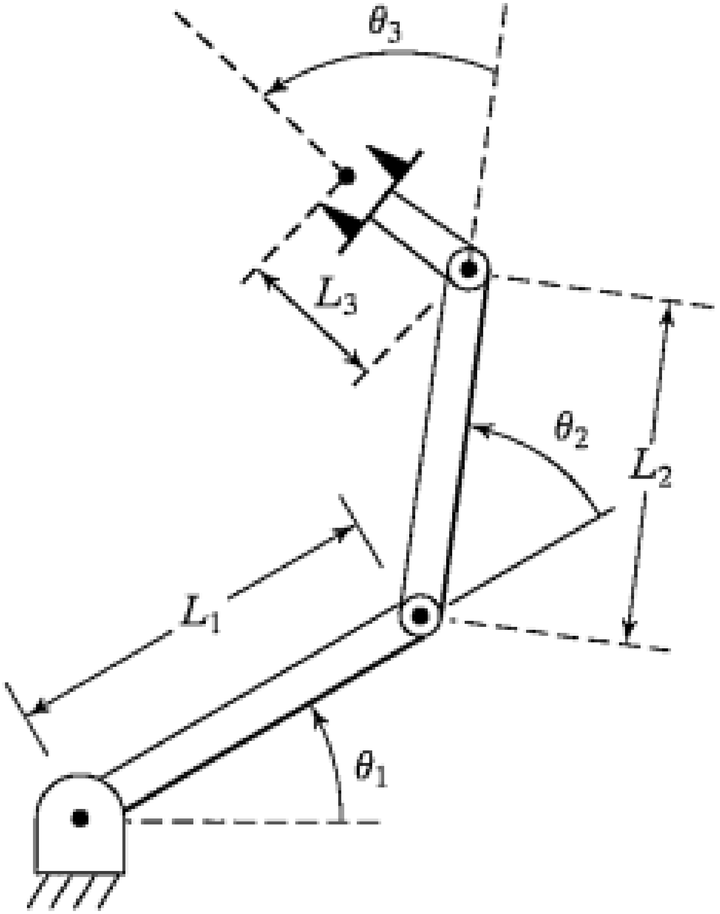


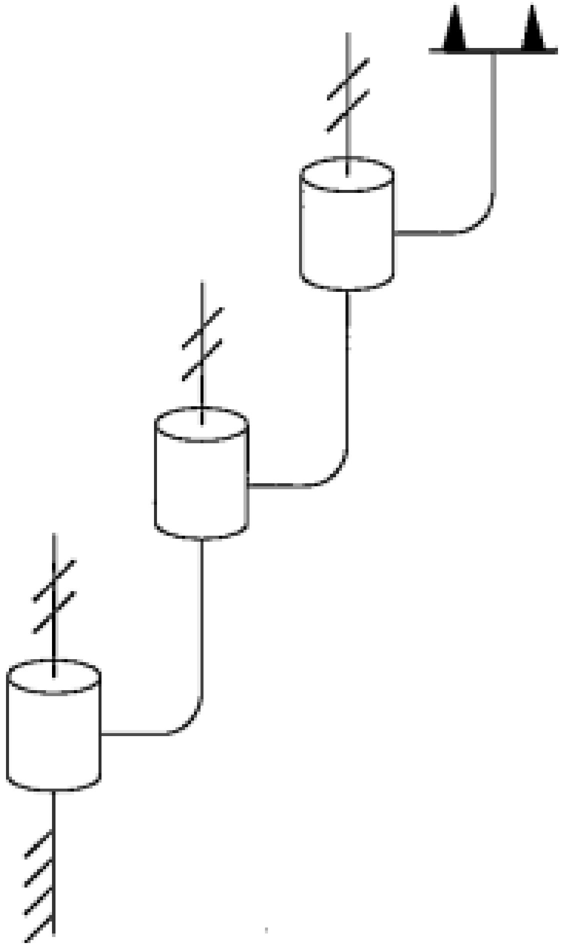
Summary of link-frame attachment procedure

1. Identify the joint axes and imagine (or draw) infinite lines along them. For steps 2 through 5 below, consider two of these neighboring lines (at axes i and $i+1$).
2. Identify the common perpendicular between them, or point of intersection. At the point of intersection, or at the point where the common perpendicular meets the ith axis, assign the link-frame origin.
3. Assign the $\hat{Z}_i$ axis pointing along the ith joint axis.
4. Assign the $\hat{X}_i$ axis pointing along the common perpendicular, or, if the axes intersect, assign $\hat{X}_i$ to be normal to the plane containing the two axes.
5. Assign the $\hat{Y}_i$ axis to complete a right-hand coordinate system.
6. Assign $\{0\}$ to match $\{1\}$ when the first joint variable is zero. For $\{N\}$, choose an origin location and $\hat{X}_N$ direction freely, but generally so as to cause as many linkage parameters as possible to become zero.

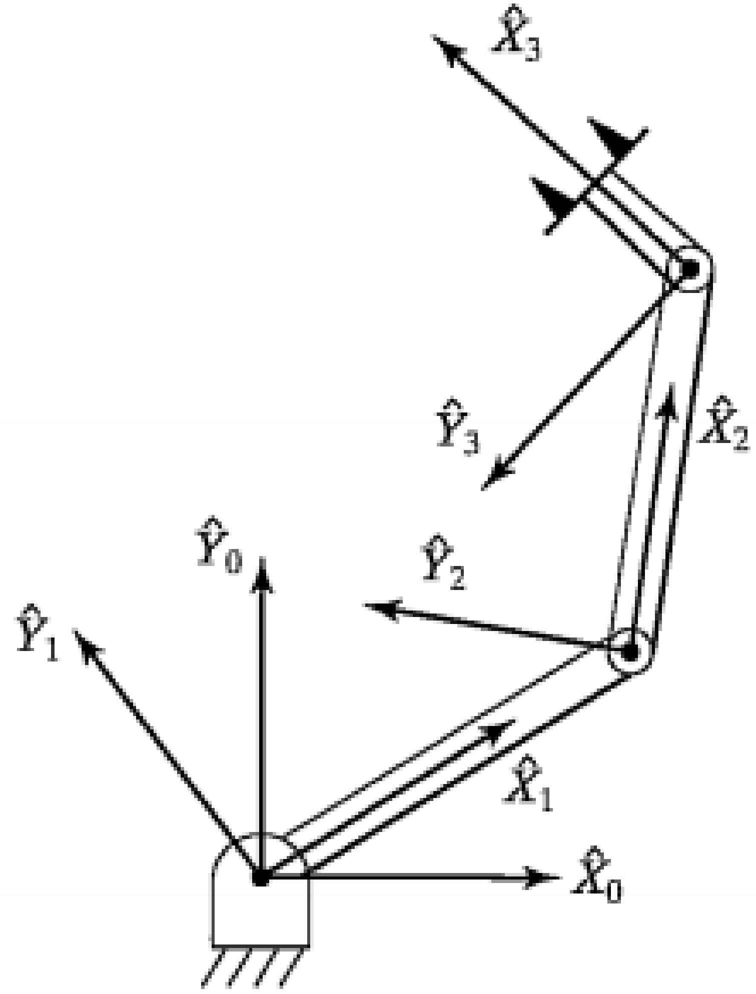
- *Figure shows a three-link planar arm.*
- *all three joints are revolute, this manipulator is sometimes called an RRR (or 3R) mechanism.*

- *A three-link planar arm.*
- *On the right, the same manipulator by means of a schematic notation.*
- *Hash marks on the axes indicate that they are mutually parallel.*





- *We start by defining the reference frame, frame {0}.*
- *It is fixed to the base and aligns with frame {1} when the first joint variable ($\theta 1$) is zero*
- *we position frame {0} as shown in Fig. with Z0 aligned with the joint-1 axis.*



$a_i =$ *the distance from* $\hat{Z}_i$ *to* $\hat{Z}_{i+1}$ *measured along* $\hat{X}_i$;

$\alpha_i =$ *the angle from* $\hat{Z}_i$ *to* $\hat{Z}_{i+1}$ *measured about* $\hat{X}_i$;

$i = i - 1$

$d_i =$ *the distance from* $\hat{X}_{i-1}$ *to* $\hat{X}_i$ *measured along* $\hat{Z}_i$; and

$\theta_i =$ *the angle from* $\hat{X}_{i-1}$ *to* $\hat{X}_i$ *measured about* $\hat{Z}_i$.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

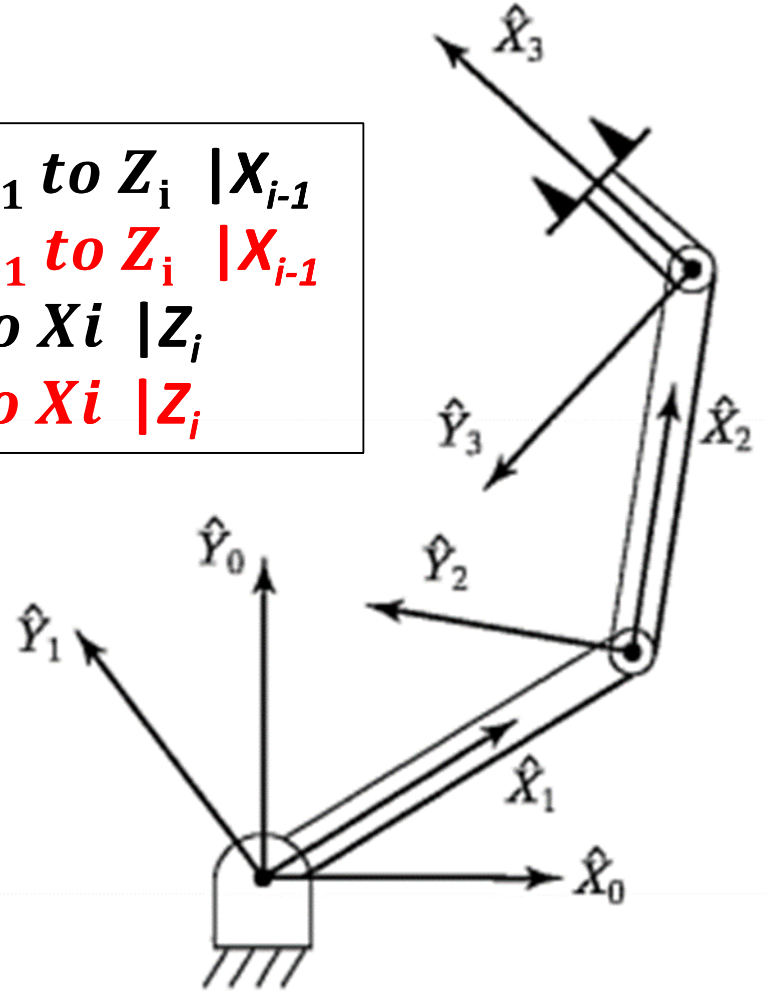
FIGURE 3.8: Link parameters of the three-link planar manipulator.

$a_{i-1} = Z_{i-1}$ ***to*** $Z_i \;|X_{i-1}$

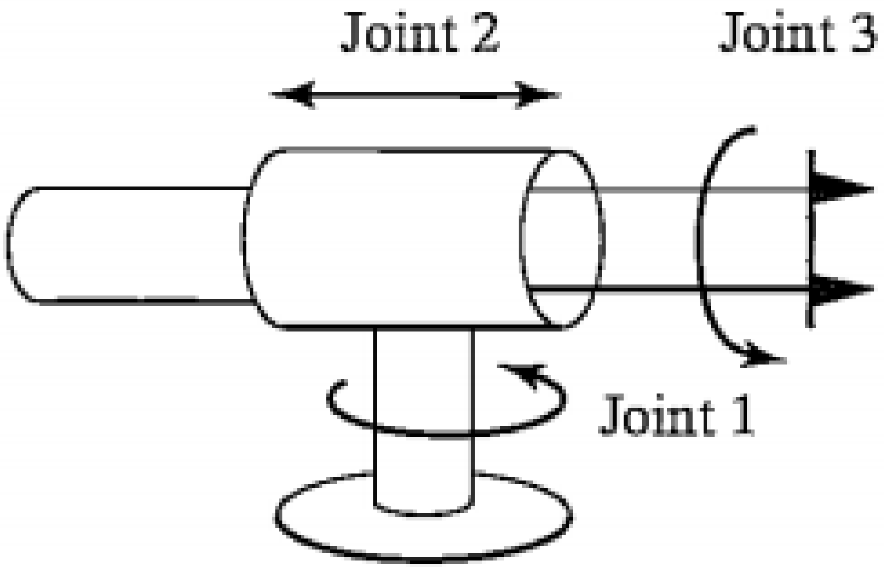
$\alpha_{i-1} = Z_{i-1}$ ***to*** $Z_i \;|X_{i-1}$

$d_i = X_{i-1}$ ***to*** $X_i \;|Z_i$

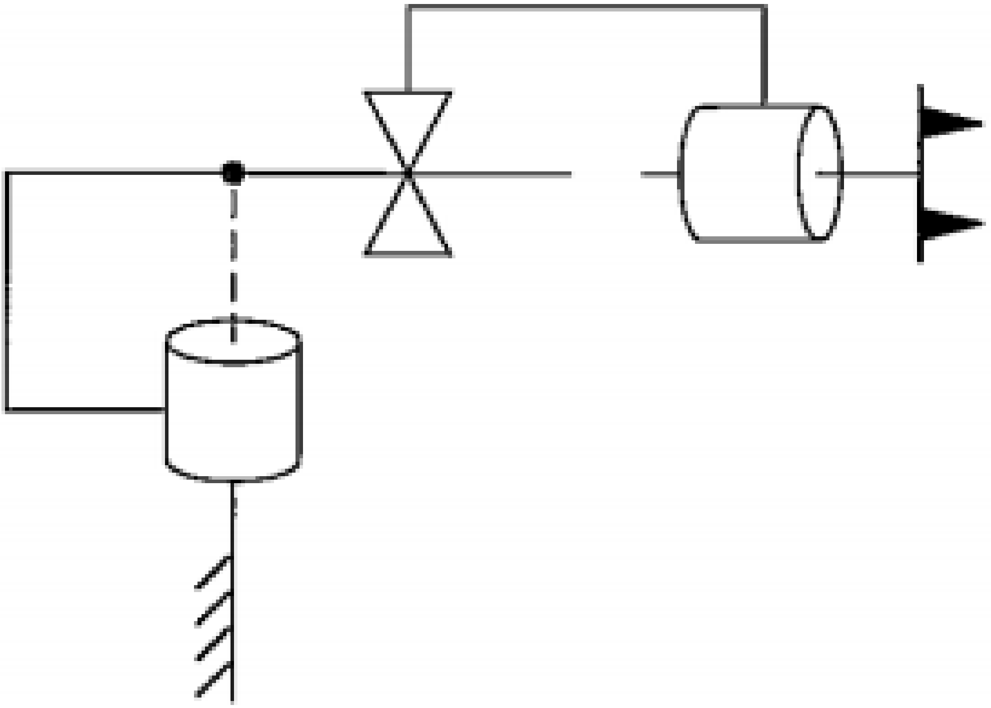
$\theta_i = X_{i-1}$ ***to*** $X_i \;|Z_i$



RPR mechanism



(a)



(b)

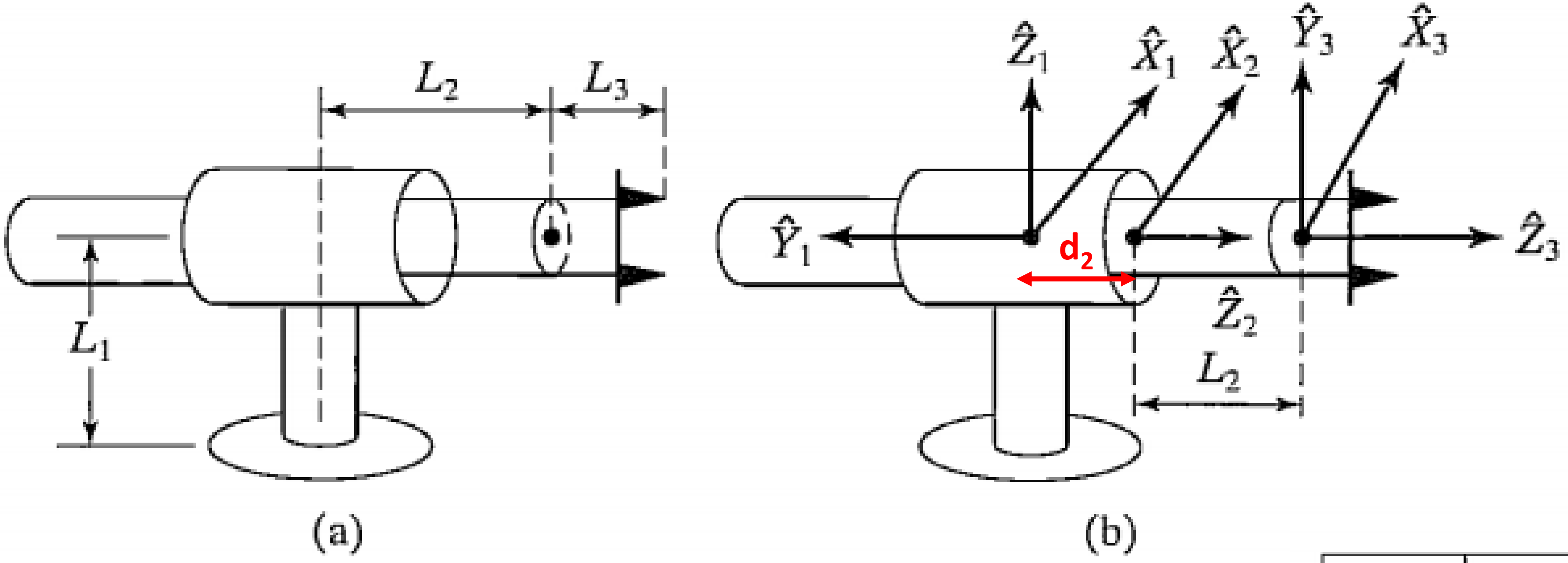
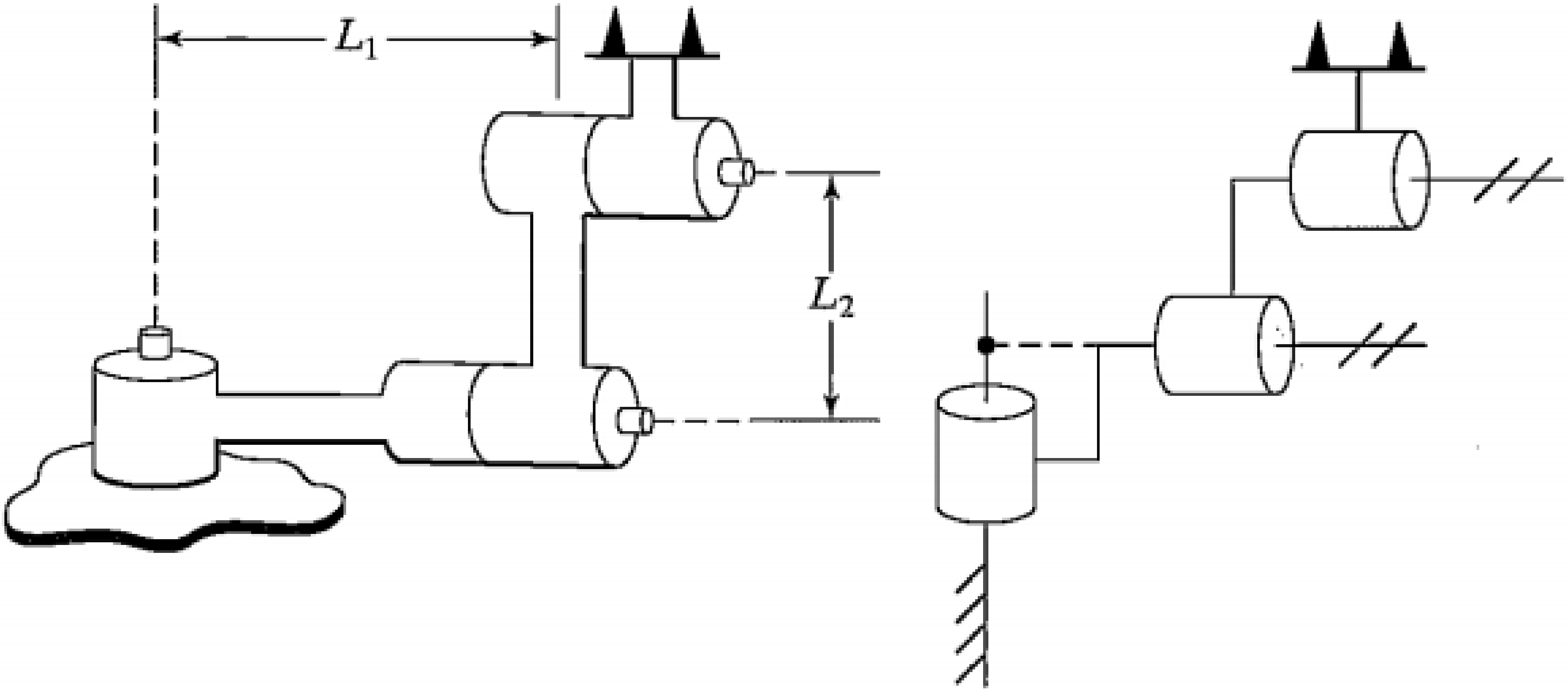


FIGURE 3.10: Link-frame assignments.

$$a_{i-1} = Z_{i-1} \text{ to } Z_i \ |X_{i-1}$$
$$\alpha_{i-1} = Z_{i-1} \text{ to } Z_i \ |X_{i-1}$$
$$d_i = X_{i-1} \text{ to } X_i \ |Z_i$$
$$\theta_i = X_{i-1} \text{ to } X_i \ |Z_i$$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3



(a) (b)

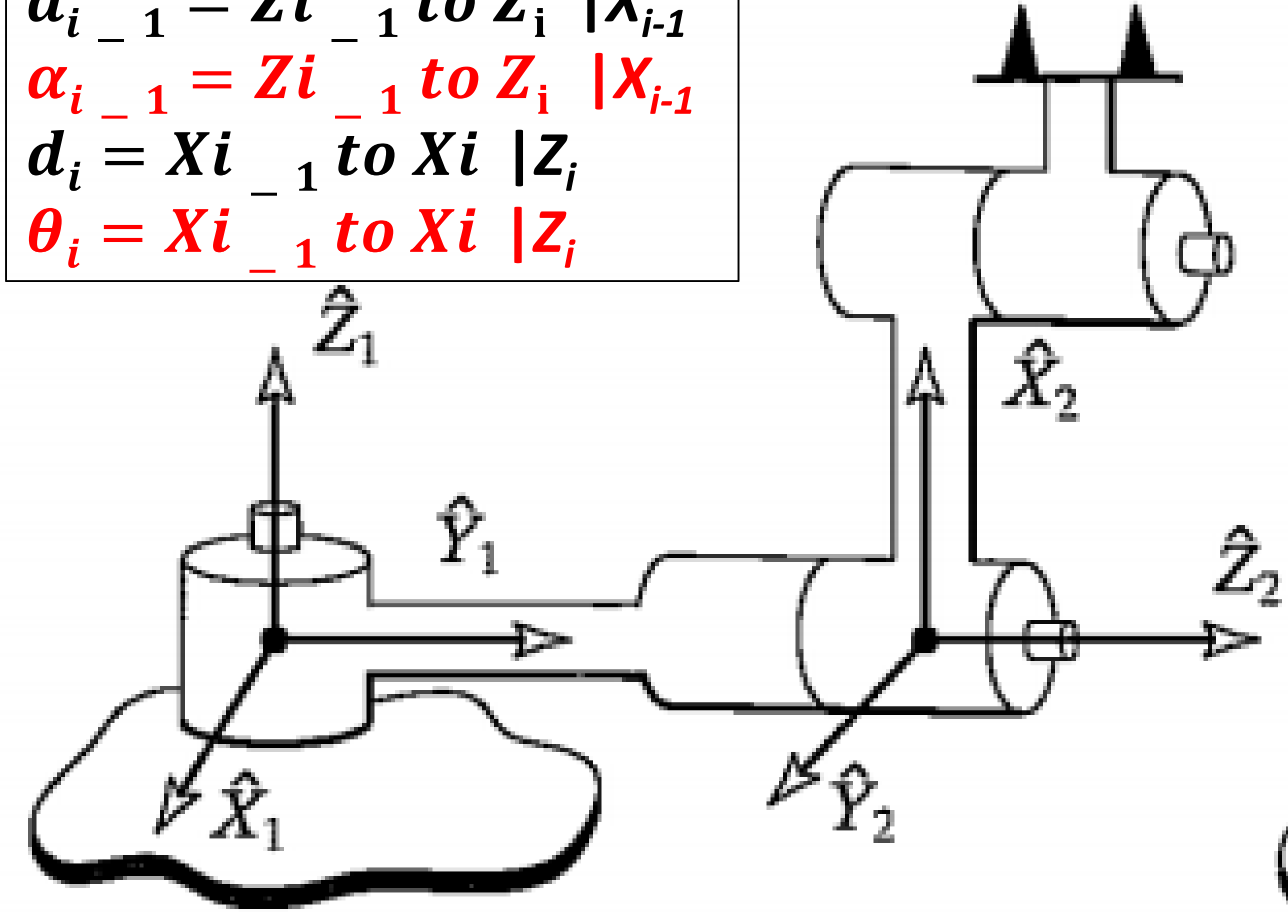
FIGURE 3.12: Three-link, nonplanar manipulator.

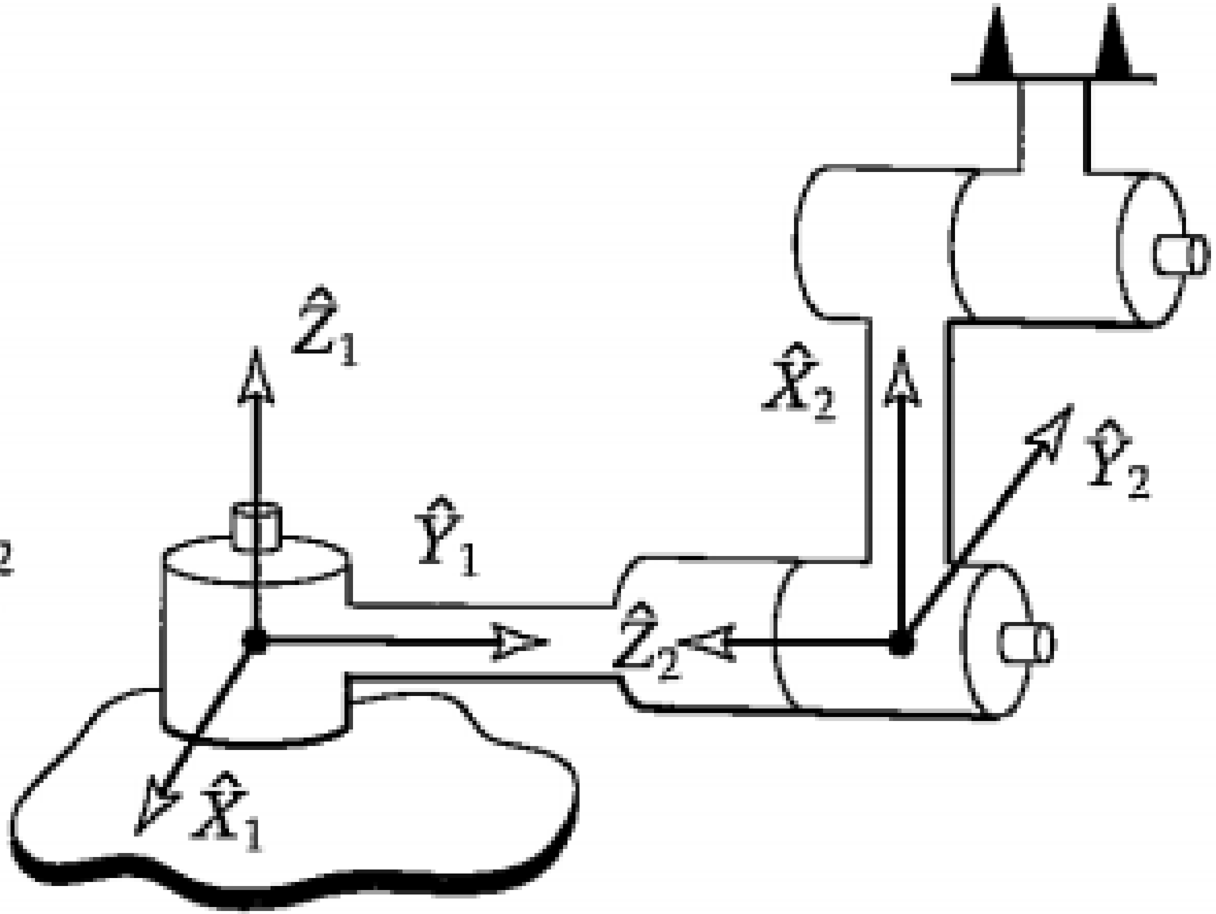
$a_{i-1} = Z_{i-1}$ to Z_i $|X_{i-1}$

$\alpha_{i-1} = Z_{i-1}$ to Z_i $|X_{i-1}$

$d_i = X_{i-1}$ to X_i $|Z_i$

$\theta_i = X_{i-1}$ to X_i $|Z_i$





$a_1 = 0$	$a_2 = L_2$	
$\alpha_1 = -90°$	$\alpha_2 = 0$	$\theta_2 = -90°$
$d_1 = 0$	$d_2 = L_1$	

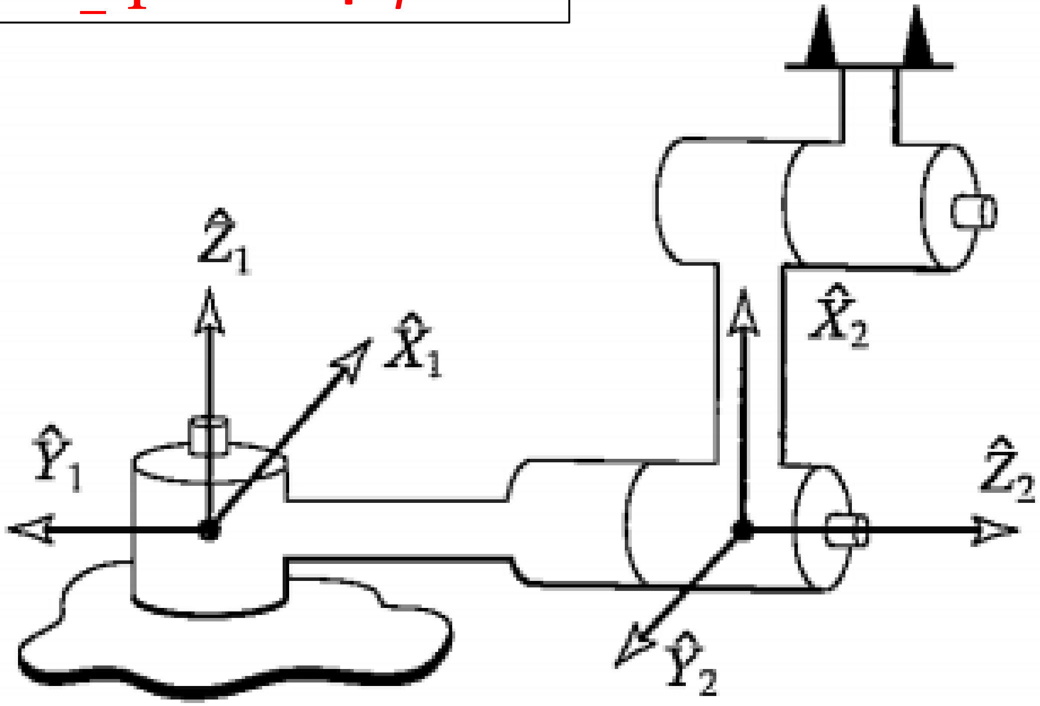
$a_1 = 0$	$a_2 = L_2$	
$\alpha_1 = 90°$	$\alpha_2 = 0$	$\theta_2 = 90°$
$d_1 = 0$	$d_2 = -L_1$	

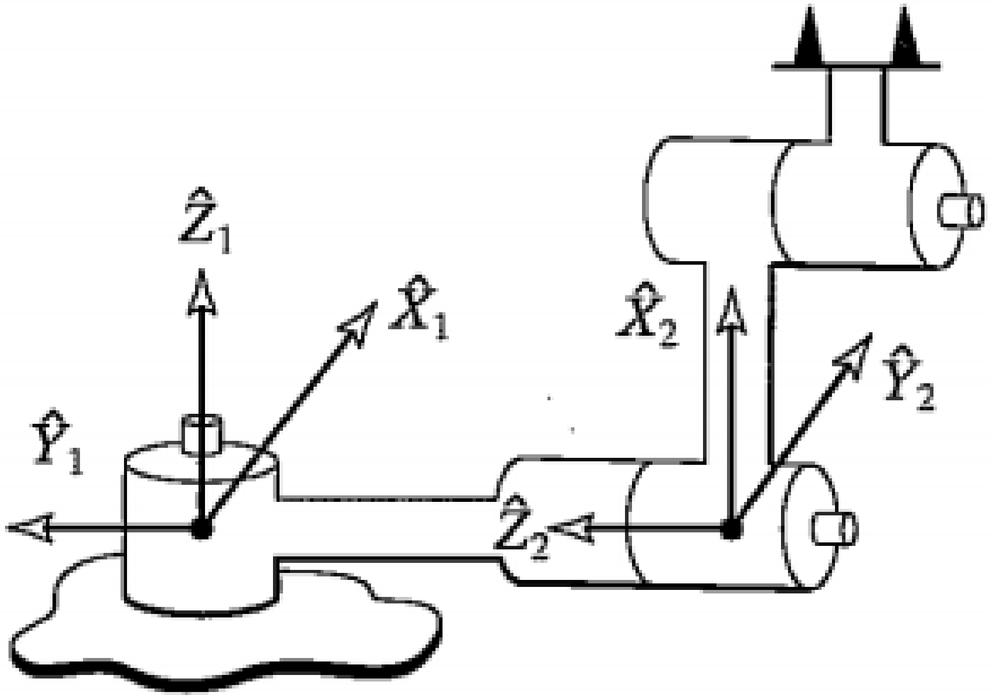
$a_{i-1} = Z_{i-1}$ to Z_i $|X_{i-1}$

$\alpha_{i-1} = Z_{i-1}$ to Z_i $|X_{i-1}$

$d_i = X_{i-1}$ to X_i $|Z_i$

$\theta_i = X_{i-1}$ to X_i $|Z_i$





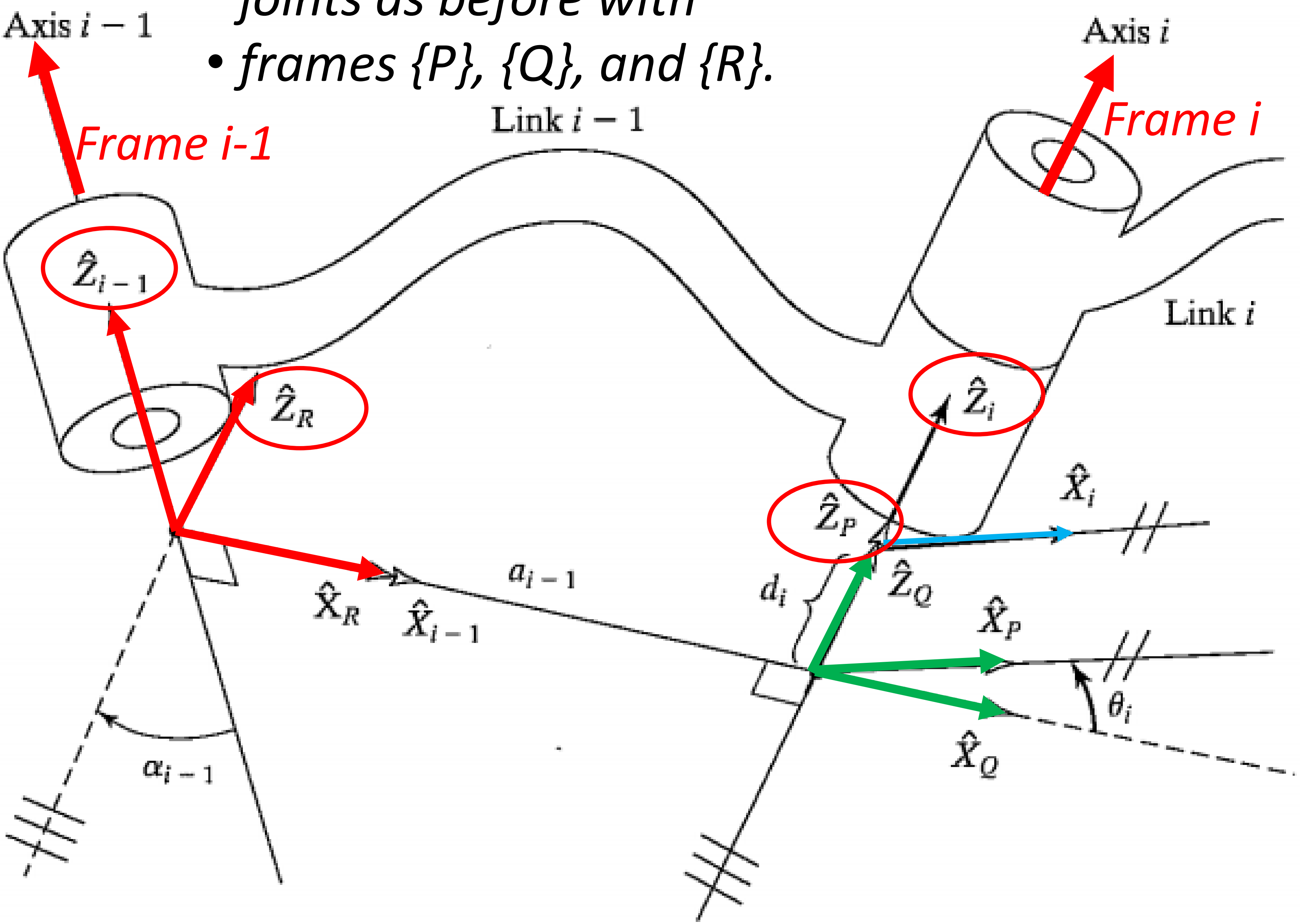
$a_1 = 0$	$a_2 = L_2$		$a_1 = 0$	$a_2 = L_2$	
$\alpha_1 = 90°$	$\alpha_2 = 0$	$\theta_2 = 90°$	$\alpha_1 = -90°$	$\alpha_2 = 0$	$\theta_2 = -90°$
$d_1 = 0$	$d_2 = L_1$		$d_1 = 0$	$d_2 = -L_1$	

Derivation of link transformations

- We wish to construct the ***Transform*** that defines frame **{*i* }** relative to the frame **{*i* — *1*}.**
- Function of the ***four*** link parameters.
- For any given robot, this transformation will be a function of ***only one variable***, the other three parameters being fixed by mechanical design.
- In order to solve each of these subproblems, ${}^{i-1}_{i}T$ we will further break each subproblem ***into four sub subproblems***.

- Figure shows the same pair of joints as before with
- frames {P}, {Q}, and {R}.

- *Frame {R} differs from frame {i-1} only by a rotation of α_{i-1}*
- *Frame {Q} differs from {R} by a translation a_{i-1}*
- *Frame {P} differs from {Q} by a rotation θ_i*
- *And frame {i} differs from {P} by a translation d_i.*



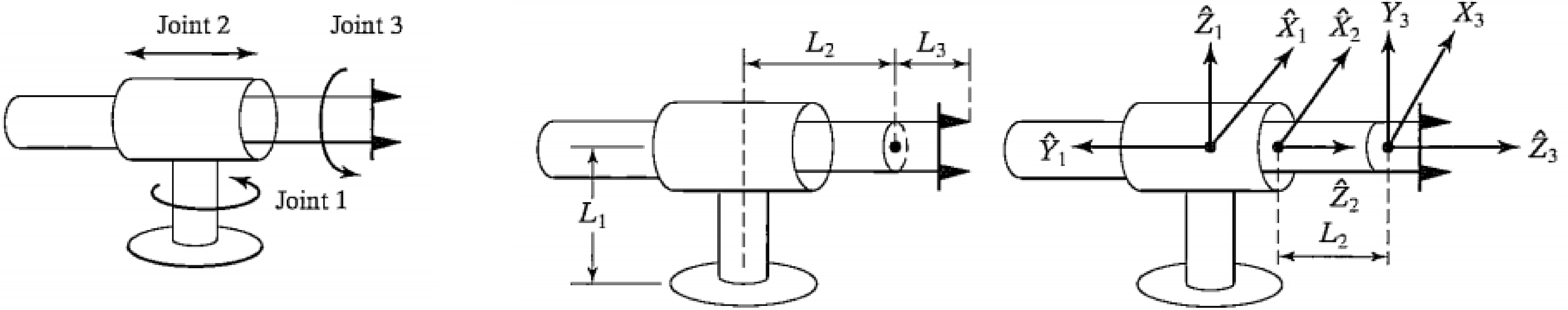
$${}^{i-1}P = {}^{i-1}_{R}T \; {}^{R}_{Q}T \; {}^{Q}_{P}T \; {}^{P}_{i}T \; {}^{i}P$$

$${}^{i-1}P = {}^{i-1}_{i}T \; {}^{i}P$$

$${}^{i-1}_{i}T = {}^{i-1}_{R}T \; {}^{R}_{Q}T \; {}^{Q}_{P}T \; {}^{P}_{i}T$$

$${}^{i-1}_{i}T = R_X(\alpha_{i\_1}) \, D_X(a_{i\_1}) \, R_Z(\theta_i) \, D_Z(d_i)$$

$${}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$



(a)

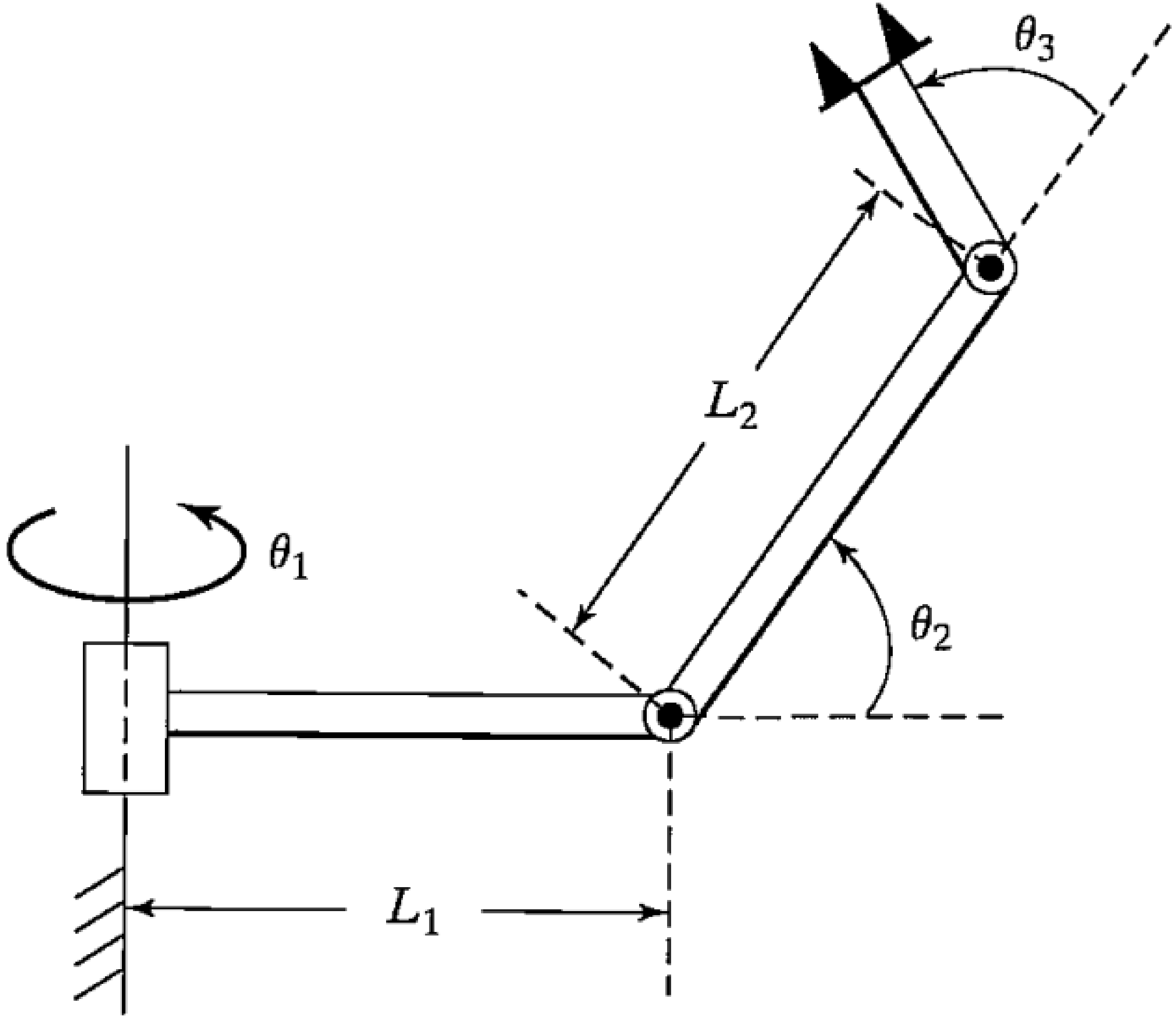
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

$$
{}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

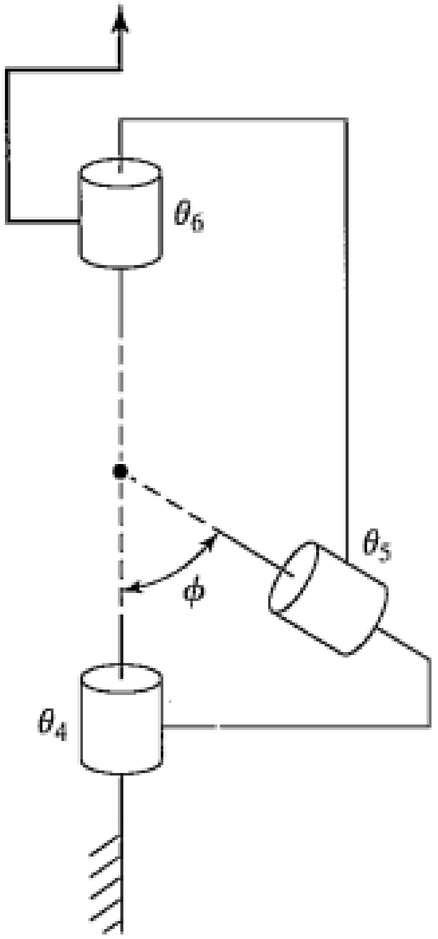
$$
{}^1_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

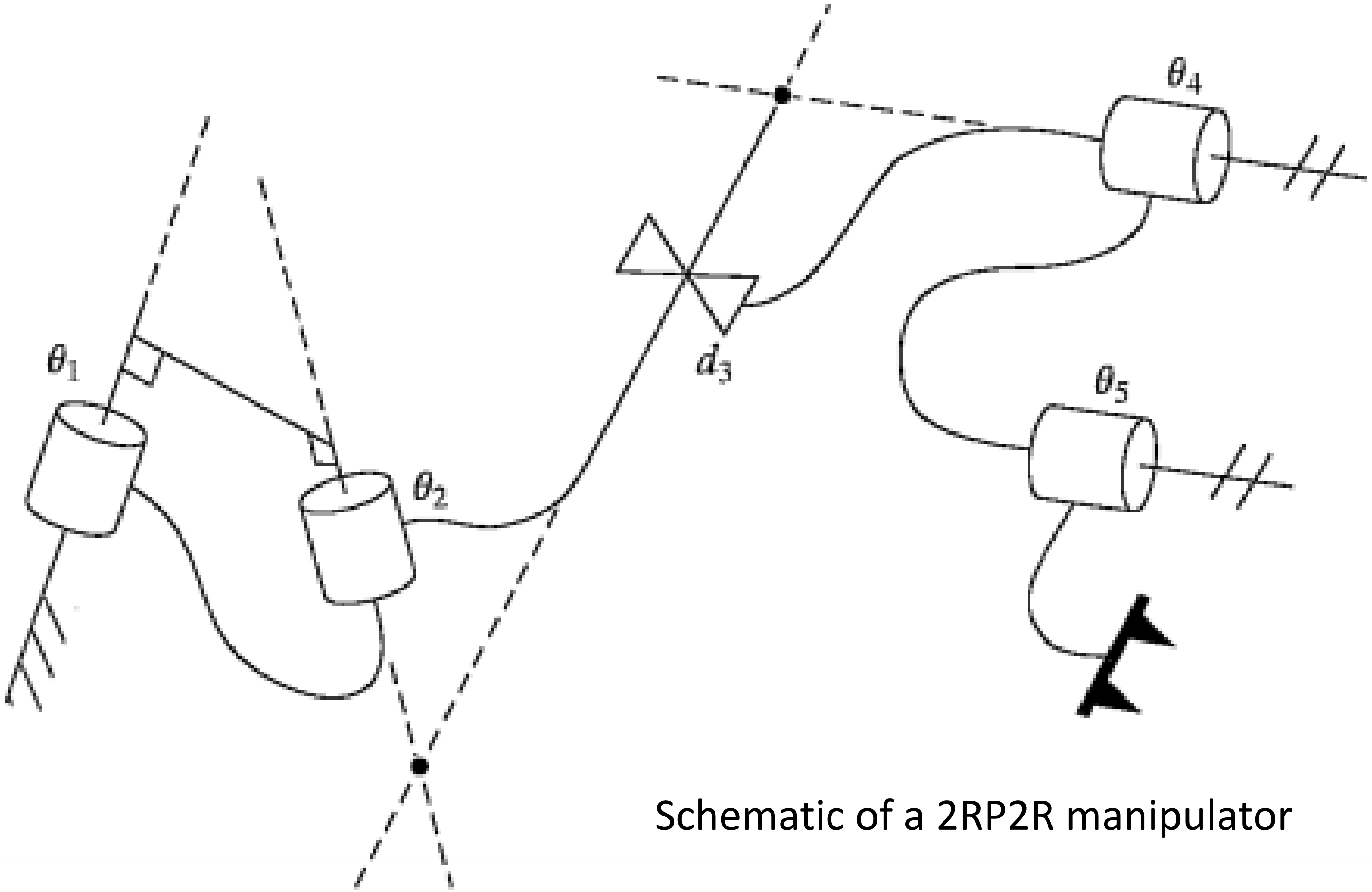


θ_3
L_2
θ_1
θ_2
L_1

Figure shows the schematic of a wrist which has three intersecting axes that are not orthogonal. Assign link frames to this wrist , and give the link parameters.



Show the attachment of link frames for the 3-DOF manipulator shown schematically in Fig.



Schematic of a 2RP2R manipulator

Show the attachment of link frames for the 3-DOF manipulator shown schematically in Fig.