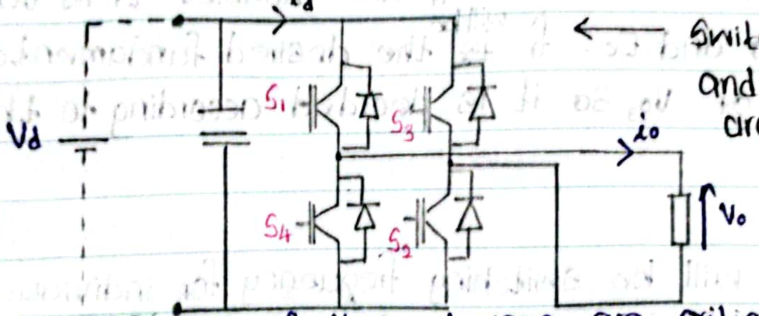
b) Square - wave PWM.

Square wave PWM offers voltage controlabirty but not harmonic reduction. So, we do not filter V_o, in view of seperating the fundamental with a square wave PWM inverter.

- Note that whenever the term "square wave" becomes part of the name, the inverter does not give harmonic reduction in V_o.

(i) Single - phase inverter



Switching devices, i.e IGBTs and diodes in this circuit are designed to operate at higher switching frequencies, for example 25kHz. Switching times and reverse recovery ~~are~~ characteristics of these devices are critically looked of in the design.

We have 2 alternative switching schemes to obtain square wave PWM.

(i) Bipolar switching schemes.
(ii) Unipolar switching schemes.

(i) Bipolar Switching PWM.

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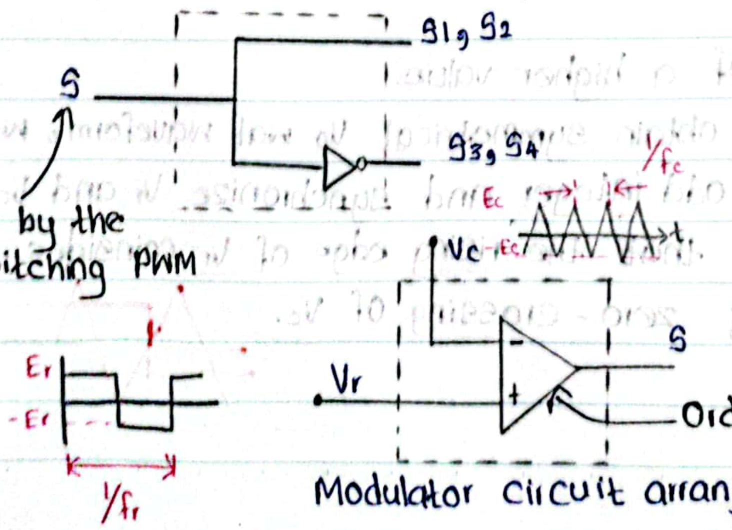
With this PWM, each half cycle of V_o has voltage pulses going between $+V_d$ and $-V_d$.

produced by the bipolar switching PWM modulator.



Modulator circuit arrangement

V_c = Carrier signal input.

This is a symmetrical triangular signal at peak value E_c and frequency f_c.
Both E_c and f_c are fixed for a given implementation.
Also, f_c is a high frequency.

Ex: $E_c = 5V$
$f_c = 10kHz.$

V_r = Reference signal input.

This is a square wave signal of amplitude E_r and frequency f_r. Both E_r and f_r are variables. E_r is variable between 0 and E_c. f_r ~~is~~ will be the desired fundamental frequency of V_o, so it is decided according to the need.

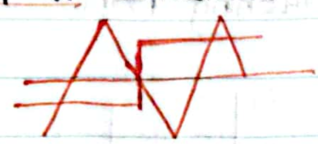
We will see f_c will be switching frequency for individual IGBTs. f_r will be fundamental frequency of V_o. and E_r will determine ~~(V_o)~~ $(V_{o,fund})_{rms}$.

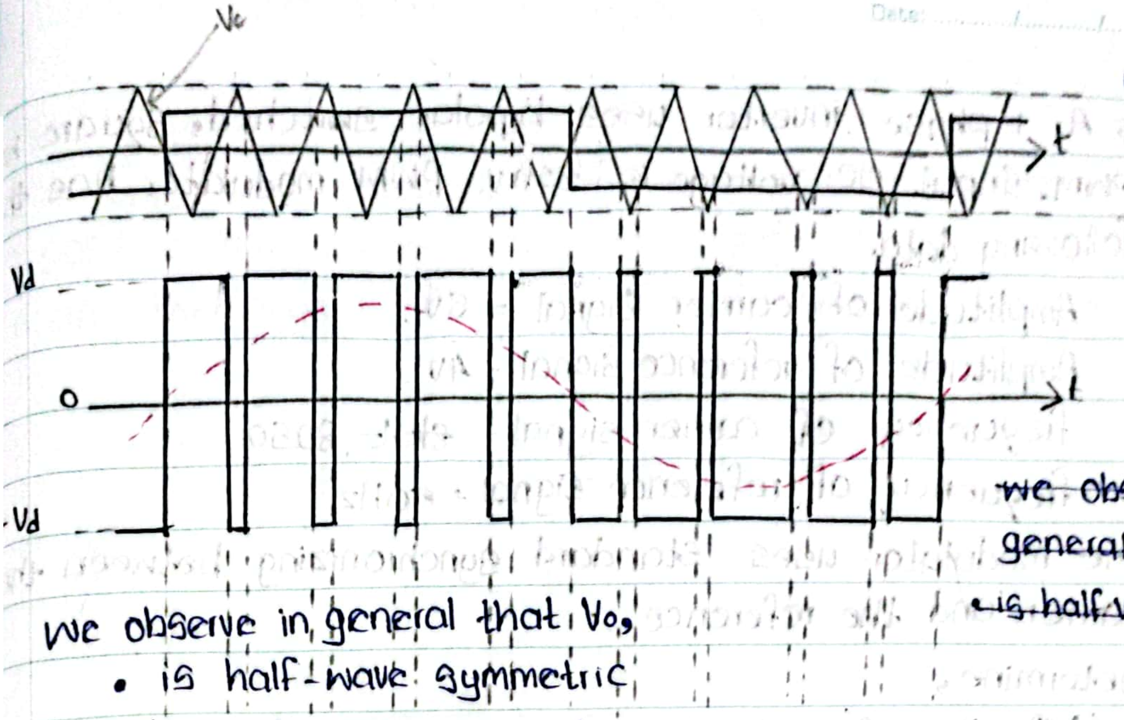
$f_c = f_s$
$f_r = f_{fund.}$

$$m = \frac{E_r}{E_c} = \text{Depth of modulation.}$$

$$p = \frac{f_c}{f_r} = \text{Carrier ratio.}$$

$m = 0 \sim 1$
p = integer of a higher value.
In order to obtain symmetrical V_o ~~wef~~ waveform, we select p to be an odd integer and synchronize V_r and V_c in such a way that the rising edge of V_r coinsides a negative-going zero-crossing of V_c.





~~we observe in general that V_o, is half wave symmetric~~

We observe in general that V_o,

- is half-wave symmetric
- is quater wave symmetric
- has p no. of pulses per cycle.

Using a fourier expansion of V_o, we can show, (when p is larger),

$$(V_o, fund)_{rms} \approx m\left(\frac{\sqrt{8}\,V_d}{\pi}\right)$$

← So, by varying m, we can vary $(V_o, fund)_{rms}$.

$$\begin{pmatrix}\text{Order of harmonics} \\ \text{in } V_o\end{pmatrix} = 3, 5, 7, 9, \ldots\ldots +$$ (High frequency harmonics due to carrier of fc and its multiplier).

Thus, harmonically, sq. wave PWM does not give us any advantage.

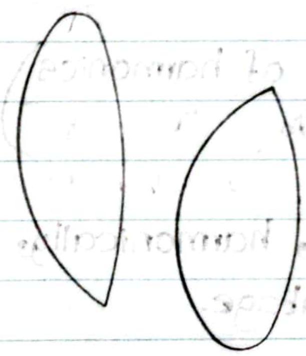
Ex: A 1 phase inverter uses bipolar switched, square wave PWM. Input DC voltage is 320 V. PWM modulator has the following data.

Amplitude of carrier signal = 6V.
Amplitude of reference signal = 4V.
frequency of carrier signal = ~~8kHz~~ 8250
frequency of reference signal = 50Hz.

The modulator uses standard synchronizing between the carrier and the reference.

Determine,
(i) Depth of modulation.
(ii) Carrier ratio.
(iii) Switching frequency.
(iv) fund. frequency of output voltage.
(v) rms value of the fund. component of output voltage.
(vi) Two lowest order harmonics present in the output voltage.

Ans:



(i) $m = \dfrac{E_r}{E_c} = \dfrac{4}{6} = 0.67$ //

(ii) $p = \dfrac{f_c}{f_r} = \dfrac{\cancel{8000}\ 8250}{50} = 160$ //

(iii) $f_s = f_c = 8250\text{Hz}$ //

(iv) $f_o = f_r = 50\text{Hz}$ //

(v) $(V_{o,fund})_{rms} = 0.67\left(\dfrac{\sqrt{8} \times 320}{\pi}\right)$

$= 192.2\,V$ //

(vi) Two lowest order harmonics of V_o are 3^{rd} an 5^{th}.
It is 150 Hz and 250 Hz //

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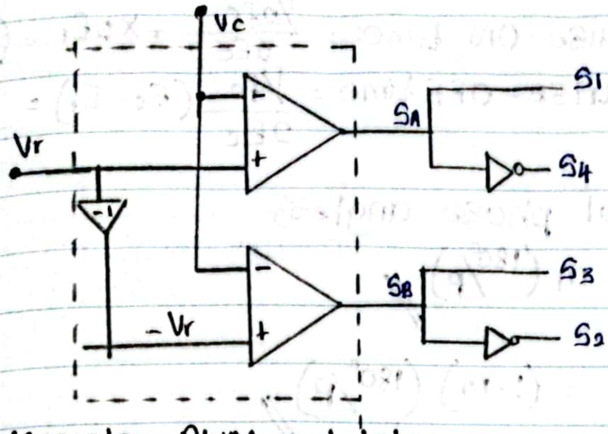
(ii) Unipolar Switching PWM

This PWM produces an output voltage of which the positive half cycle has pulses going between $+V_d$ and 0, and the negative half cycle going between 0 and $-V_d$.

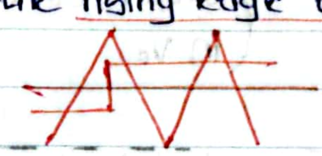


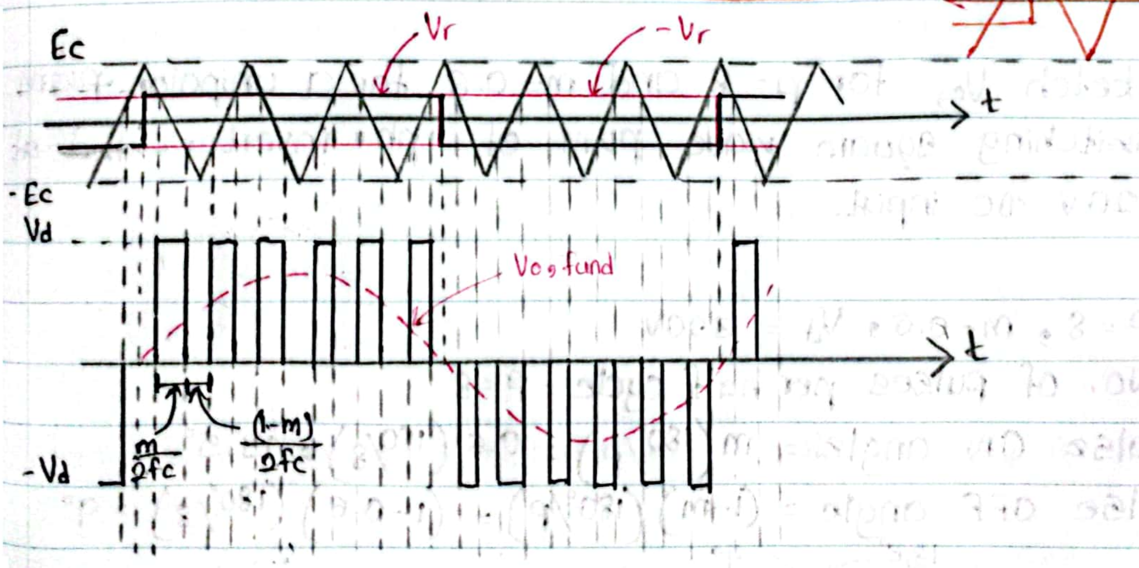
Vc and Vr signal-inputs are similar to those reffered to in the bipolar PWM.

Unipolar PWM modulator.

Here in order to obtain a symmetrical Vo waveform, we use,

- an Even integer valued p.
- ~~the rising~~ a synchronising where the rising edge of Vr coincides positive peak of Vc.





SA	SB	Vo
0	0	0
0	1	$-V_d$
1	0	V_d
1	1	0

We observe generally that,

- V_o is half-wave symmetric.
- V_o is quater-wave symmetric.
- V_o has p no. of voltages pulses in each half cycle.

In each half cycle, pulse-ON time $= \frac{1/2f_c}{2E_c} \times 2E_r = \left(m/2f_c\right)$ //

pulse-OFF time $= \frac{1/f_c}{2E_c}(E_c - E_r) = \frac{(1-m)}{2f_c}$ //

In terms of fundamental phase angles,

pulse-ON angle $= m\left(180°/p\right)$ //

$\theta = \omega_r t = 2\pi f_r t$
$= 360° f_r \left(\frac{m}{2f_c}\right)$

pulse-OFF angle $= (1-m)\left(180°/p\right)$ //

Here too, we can show,

~~(V_o, rms)~~ $(V_o, \text{fund})_{rms} = m\left(\frac{\sqrt{8}\, V_d}{\pi}\right)$; assuming p to be high.

$\begin{pmatrix}\text{Order of harmonics} \\ \text{in } V_o\end{pmatrix} = 3, 5, 7, 9 \ldots + \begin{pmatrix}\text{Higher order harmonics} \\ \text{of } f_r \text{ and its multiples}\end{pmatrix}$

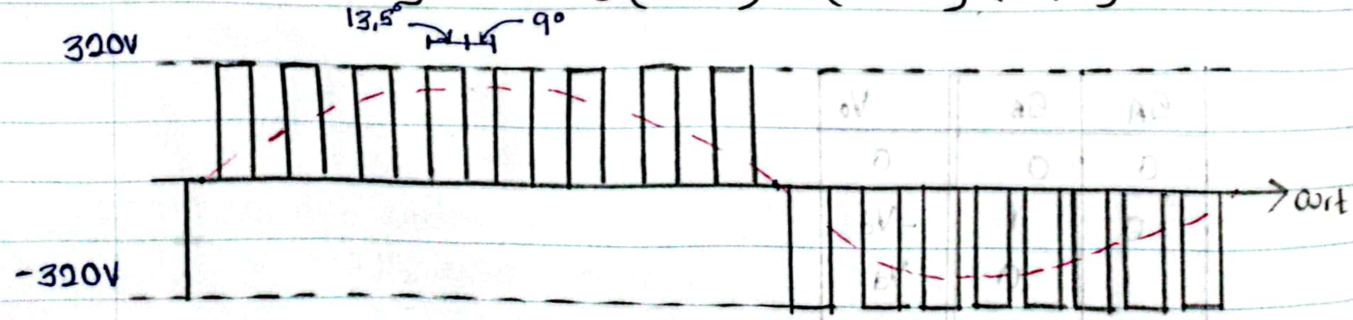
Ex: Sketch V_o, for $p = 8$ and $m = 0.6$ for a unipolar ~~PWM~~ switching square wave PWM of 1ph inverter, fed at 320V DC input.

Ans: $p = 8$, $m = 0.6$, $V_d = 320V$

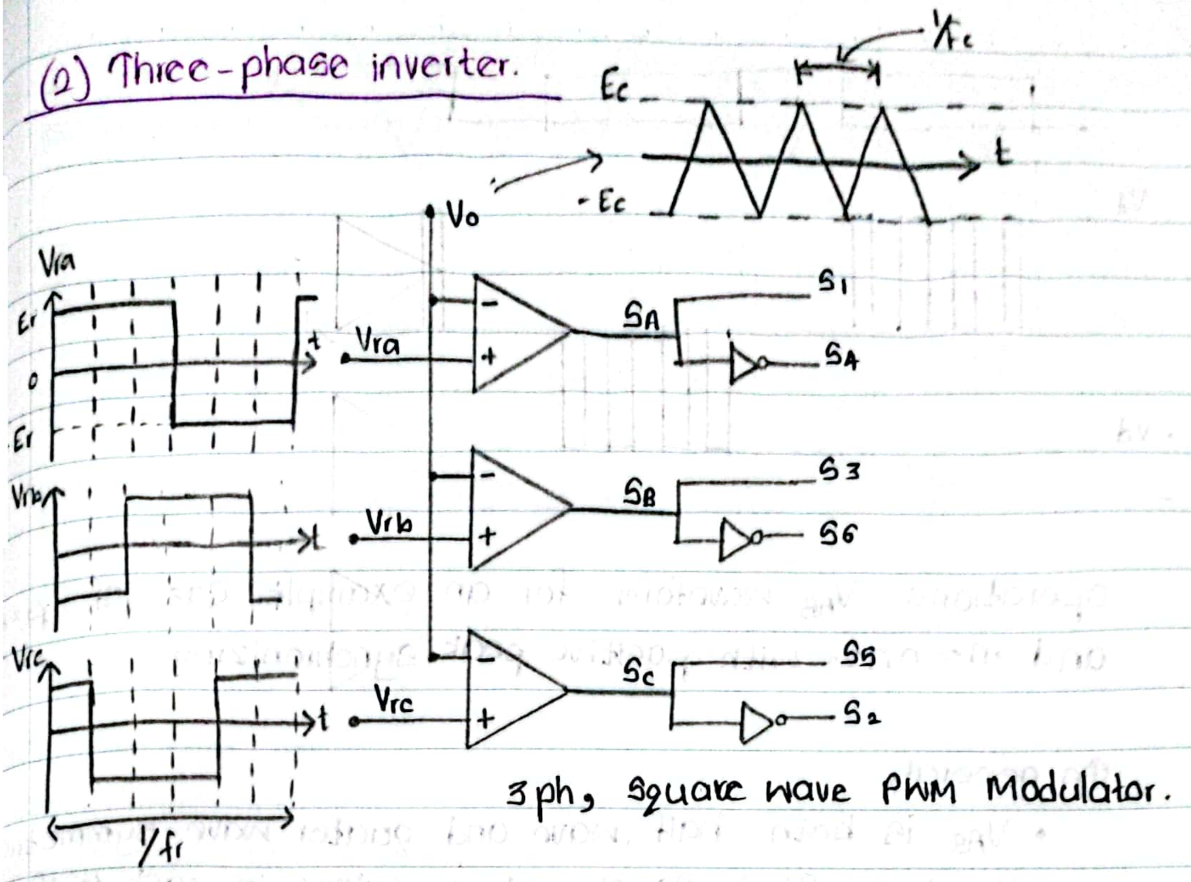
No. of pulses per half cycle $= p = 8$

pulse ON angle $= m\left(180°/p\right) = 0.6\left(180/8\right) = 13.5°$

pulse OFF angle $= (1-m)\left(180°/p\right) = (1-0.6)\left(180°/8\right) = 9°$



(2) Three-phase inverter.



3ph, square wave PWM Modulator.

V_{ra}, V_{rb} and V_{rc} are balanced, 3 phase square-wave signals.

V_o is a normal triangular carrier.

E_c and f_c are fixed.

E_r and f_r variable ($E_r = 0 \sim E_c$, f_r as required)

f_c will be f_s

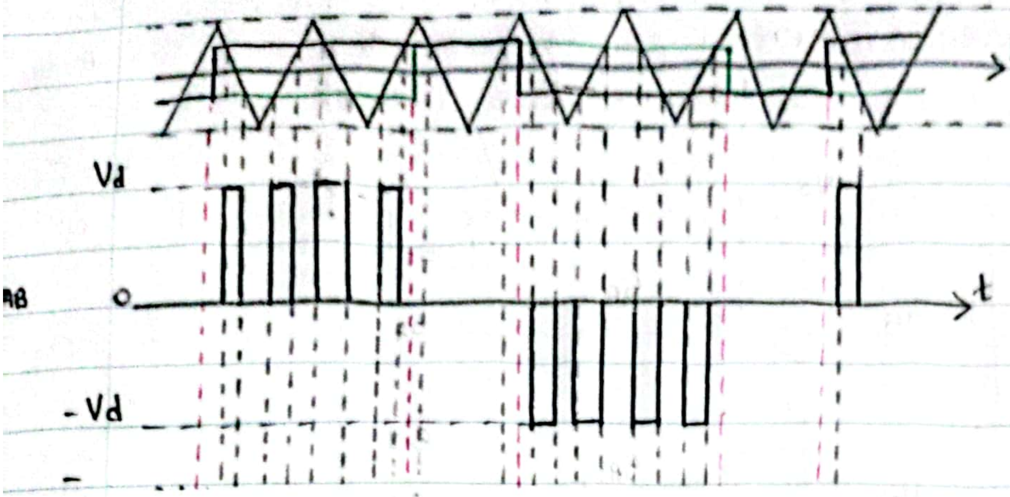
f_r will be f_o

$$m = \frac{E_r}{E_c} \quad \text{and} \quad m = 0 \sim 1$$

$$p = \frac{f_c}{f_r}$$

To obtain balanced, symmetrical, 3phase output voltage we use,

- p of odd ~~integer~~. or even ~~we~~ without a difference.
- Synchronizing similar to ~~bipolar~~, Unipolar, 1ph . PWM. without a difference.
- p should be a multiple of 3.



operations V_{AB} waveform for an example case of $p=6$ and $m \approx 0.75$, with positive peak synchronizing.

In general,

- V_{AB} is both half wave and quater wave symmetric.
- V_{AB} has $2/3\,p$ no. of voltage pulses in each half cycle, ~~peaked~~ packed inside 120 span, flancked by two half-off at two ends.
- pulse duty factor is same as the depth of modulation m.

  pulse ~~on~~ ON angle is $\frac{180^\circ \cdot m}{p}$ and OFF-angle $\frac{180^\circ(1-m)}{p}$.

We can show that,

$$(V_{AB,fund})_{rms} \approx m\left(\frac{\sqrt{6}\,V_d}{\pi}\right)$$; assuming a higher p.

$\begin{pmatrix}\text{order of harmonics} \\ \text{in } V_{AB}\end{pmatrix} = 1, 5, 7, 11, 13 \ldots + $ (higher order harmonics of fc and its multiples).

Ex: Construct phase-voltage V_{AN} for a 3ph. square-wave PWM inverter with $p = 6$, $m = 0.75$ and $V_d = 380V$.

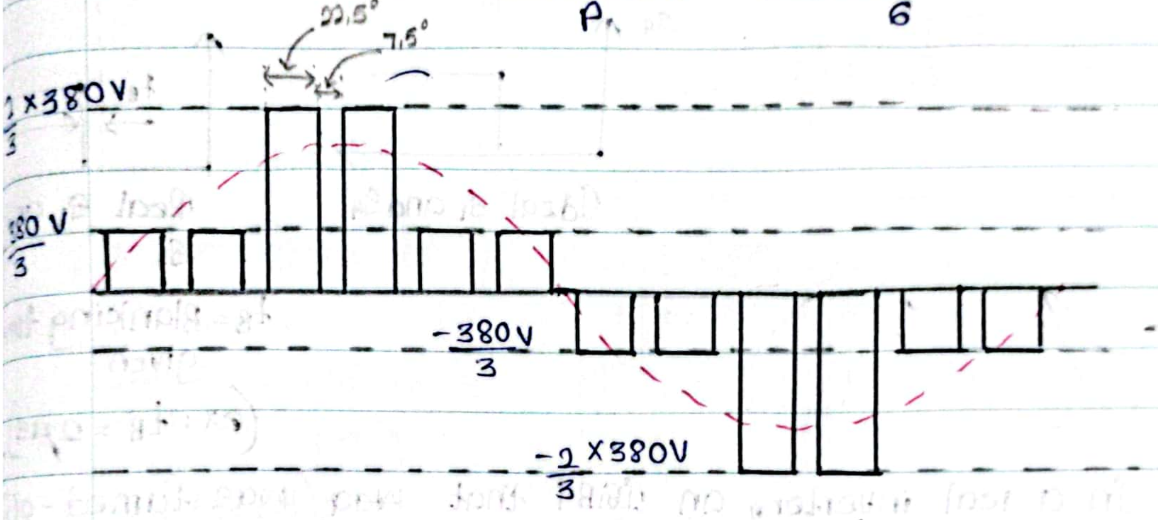
Ans: $p = 6$

$m = 0.75$

$V_d = 380V$.

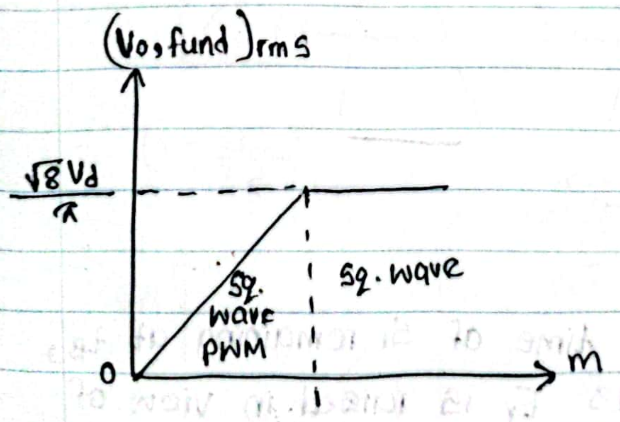
$$\text{pulse ON angle} = \frac{180^\circ m}{p} = \frac{180 \times 0.75}{6} = 22.5^\circ$$

$$\text{pulse OFF angle} = \frac{180^\circ(1-m)}{p} = \frac{180(1-0.75)}{6} = 7.5^\circ$$

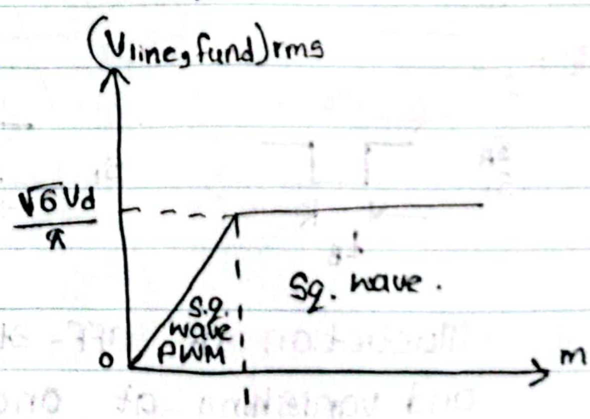


(iii) Over-Modulation

Over modulation means raising the depth of modulation above 1. This is used as a strategy to transfer the control from square-wave PWM to square wave.

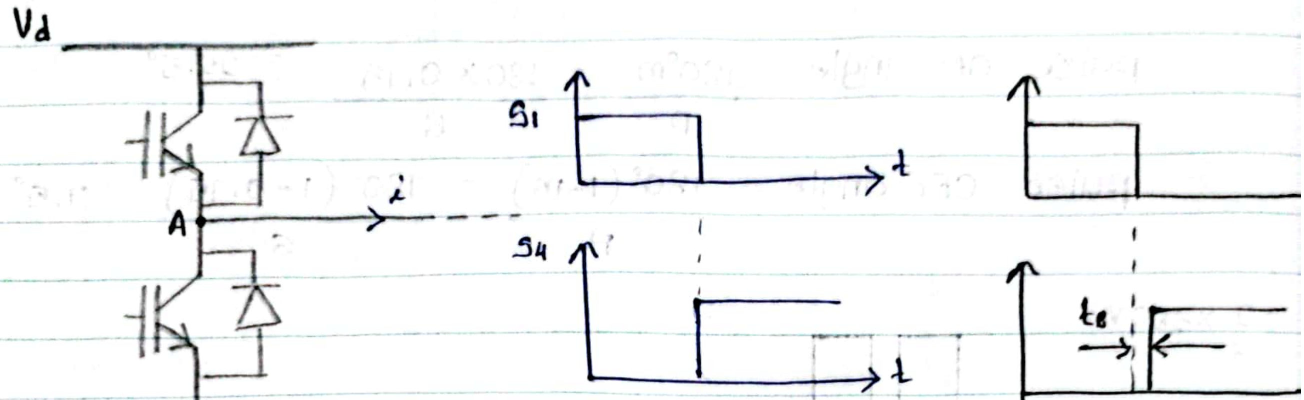


Ideal overmodulation case of 1-ph. inverter



Ideal overmodulation case of 3ph. inverter.

In practice, however, the transition from sq. wave - PWM control to sq. wave control is not smooth, due to the real operation of the inverter. The ~~tr~~ transition occurs with a step jump of the voltage, due to the influence of inherent "blanking time" set between the upper and the lower IGBTs in each leg of the inverter.



Ideal S_1 and S_4

Real S_1 and S_4

t_B = Blanking time given
(ex: $t_B = 2\mu s$).

In a real inverter, an IGBT that ~~was~~ was turned-off, needs a minimum time t_B before it can be turned-on, again.

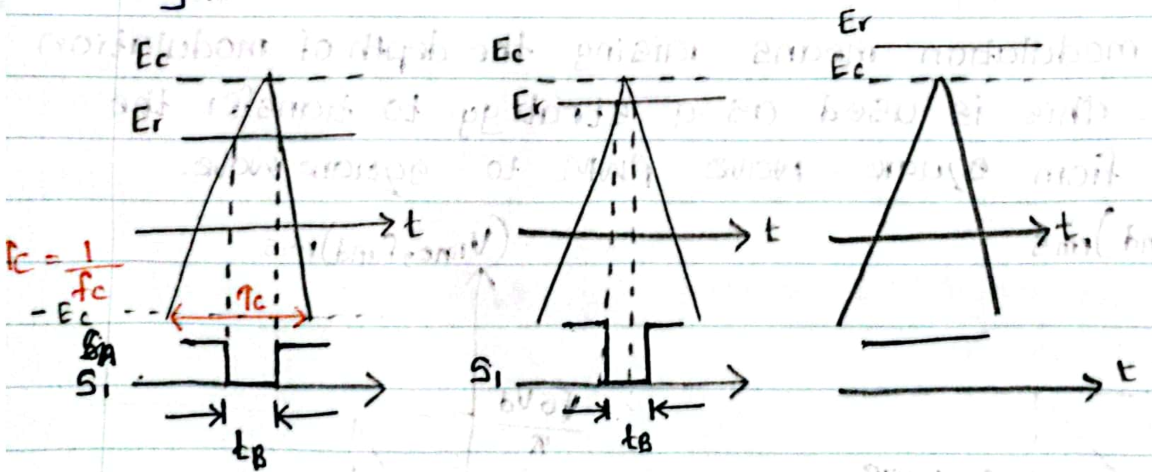


Illustration of OFF-state time of S_1 remaining at t_B, and vanishing at once as E_r is raised in view of over modulation.

Depth of modulation m, when off time of S_3 is t_B,

$$m = \frac{E_r}{E_c} \quad \text{——— ①}$$

$$t_B = \frac{T_c}{2E_c}(E_c - E_r)$$

$$= \frac{1}{2f_c}(1-m)$$

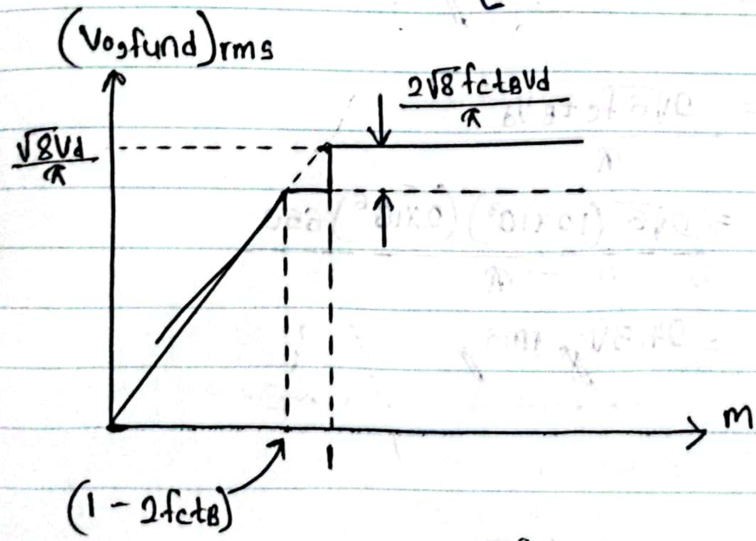
$$\therefore \; m = 1 - 2f_c t_B$$

$$(V_{o,fund})_{rms} = \frac{\sqrt{8}\, m V_d}{\pi} = \frac{\sqrt{8}(1-2f_c t_B) V_d}{\pi} \;;\; 1\text{ph. Inverter.}$$

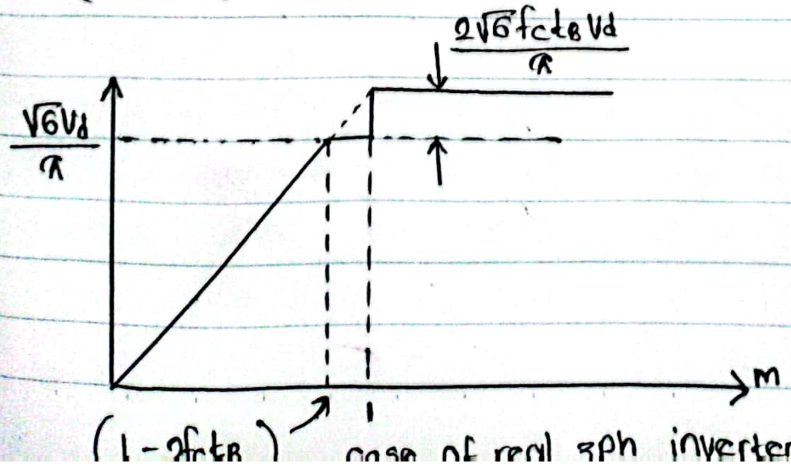
$$\frac{\sqrt{6}\, m V_d}{\pi} = \frac{\sqrt{6}(1-2f_c t_B) V_d}{\pi} \;;\; 3\text{ph. Inverter.}$$

$\therefore$ Step change of $(V_{o,fund})_{rms}$ due to sudden dissapearence of t_B OFF-pulse =

$$\begin{cases} \dfrac{\sqrt{8} V_d}{\pi} - \dfrac{\sqrt{8}(1-2f_c t_B) V_d}{\pi} = \dfrac{2\sqrt{8} f_c t_B V_d}{\pi} \;;\; 1\text{ph} \\[2ex] \dfrac{\sqrt{6} V_d}{\pi} - \dfrac{\sqrt{6}(1-2f_c t_B) V_d}{\pi} = \dfrac{2\sqrt{6} f_c t_B V_d}{\pi} \;;\; 3\text{ph} \end{cases}$$



Case of real 1-ph inverter.



Case of real 3-ph inverter.

case of real 3ph inverter

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Ex: A 3-ph. inverter uses a blanking time of 2 μs between upper and lower IGBTs in each inverter leg to avoid shoot-through faults. The inverter is operated with square-wave-PWM with a carrier frequency of 12kHz. Input DC voltage is 650V.

Determine,

(i) $(V_{line,fund})_{rms}$ after overmodulation applied to the PWM control.

(ii) Step change in the $(V_{line,fund})_{rms}$ that occurs when overmodulation is done.

Ans: $t_B = 2\,\mu s$

$f_c = 12\,kHz$

$V_d = 650V.$

(i) $$(V_{line,fund})_{rms} = \frac{\sqrt{6}\,V_d}{\pi}$$

$$= \frac{\sqrt{6} \times 650}{\pi}$$

$$= 506.8V \ rms$$

(ii) $$\Delta (V_{line,fund})_{rms} = \frac{2\sqrt{6}\,f_c\,t_B\,V_d}{\pi}$$

$$= \frac{2\sqrt{6}\,(12\times10^3)(2\times10^{-6})\,650}{\pi}$$

$$= 24.3V \ rms$$

(C) Sinusoidal PWM

This is, perhaps, the widely applied PWM in majority applications. It uses an analog modulator, which is available as a single integrated circuit from almost all semiconductor manufacturers.

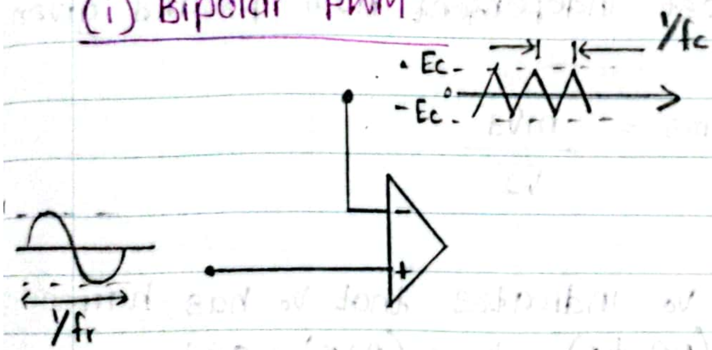
This PWM ~~offers~~ gives lowest content of harmonic in the output voltage, which is very attractive. Therefore, filtering is quite easier to seperate the fundamental. Voltage controlability is inherent.

(i) 1-phase sinusoidal PWM

This has 2 options.

(i) Bipolar PWM.
(ii) Unipolar PWM.

(i) Bipolar PWM



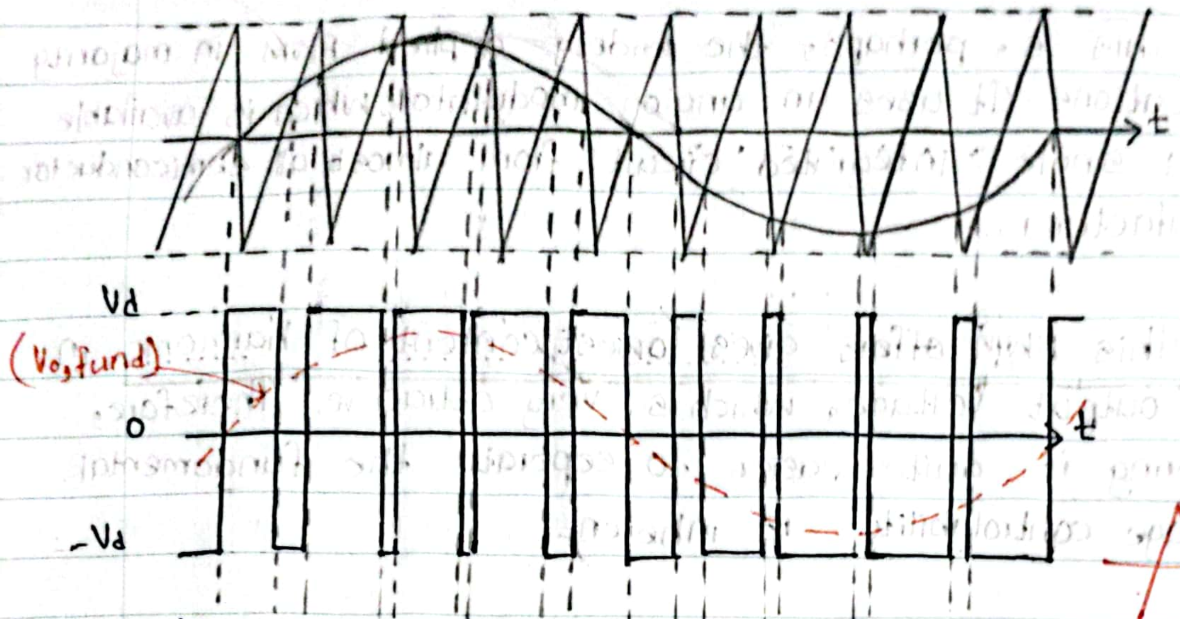
E_c and f_c - fixed
E_r and f_r - Variable
f_c will be switching frequency.
f_r will be output fund. frequency.

$$m = \frac{E_r}{E_c} \quad ; 0 \leq E_r \leq E_c$$

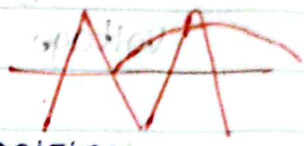
$$\therefore m \approx 0 \sim 1$$

$$P = \frac{f_c}{f_r}$$; P should be an odd integer of a higher value.

V_c and V_r are synchronized in such a way that the positive-going zero-crossing of V_r coincides with the negative-going zero crossing of V_c.



Example illustration $p=9$ and $m=0.75$.



Odd p and negative - zero - crossing synchronizing produce half - wave and quarter wave symmetric V_o waveform. The no. of voltage pulses for a cycle of V_o is p.

When p is higher (eg: $p > 21$), we can show that $(V_{o,fund})_{rms}$ is almost independent from p, and given by,

$$\boxed{(V_{o,fund})_{rms} = \frac{mV_d}{\sqrt{2}}}$$

Fourier analysis of V_o indicates that V_o has harmonics of the order of $(np \pm r)$ where $(n+r) = \text{odd}$

n and r are positive integers.

eg: $n=1$; p, $p \pm 2$, $p \pm 4$, $p \pm 6$, $p \pm 8$ ....

$n=2$; $2p \pm 1$, $2p \pm 3$, $2p \pm 5$, $2p \pm 7$ ...

$n=3$; $3p$, $3p \pm 2$, $3p \pm 4$, $3p \pm 6$, $3p \pm 8$, ...

$n=4$; $4p \pm 1$, $4p \pm 3$, $4p \pm 5$, $4p \pm 7$, ...

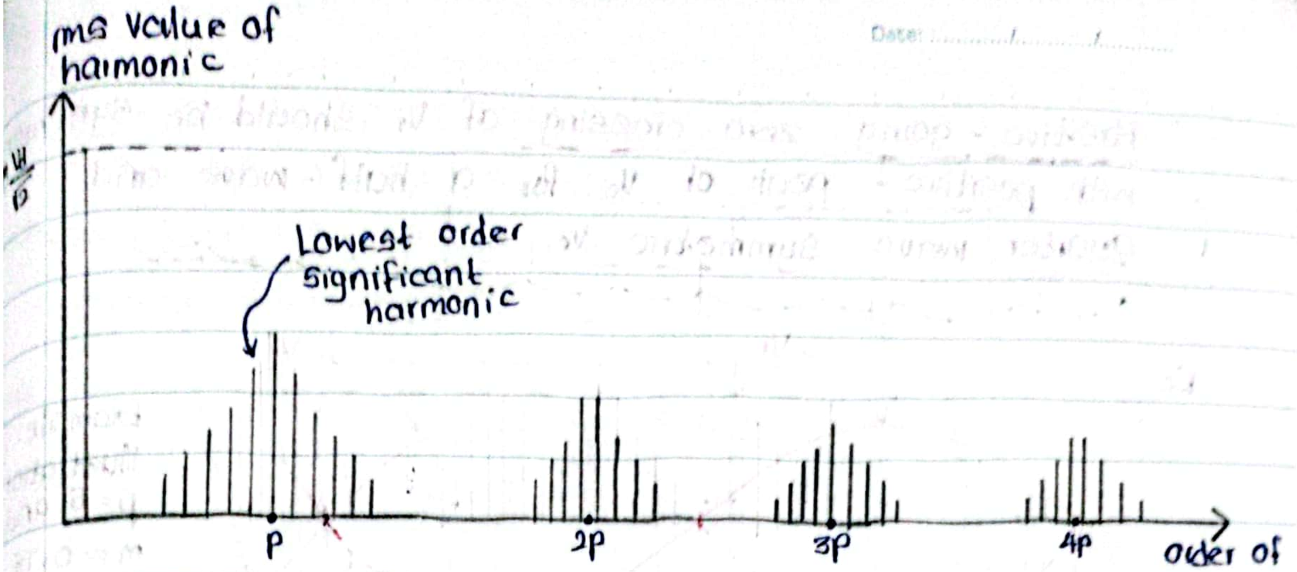
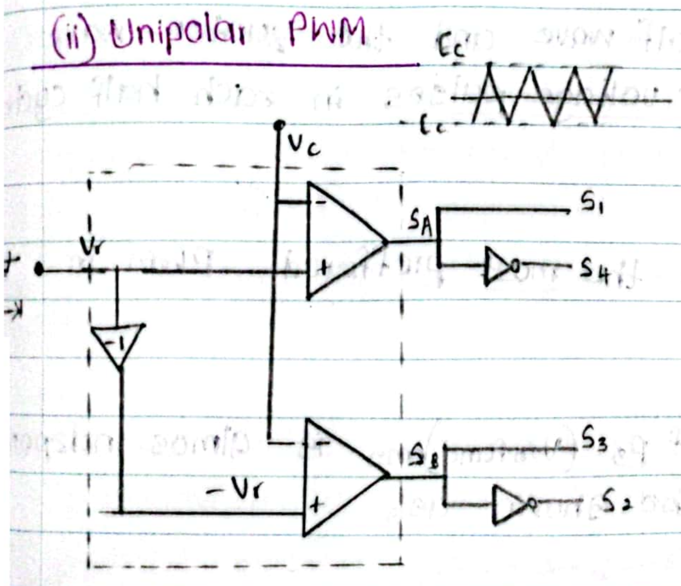


Illustration of the spectrum of harmonics in V_o.

A harmonic becomes significant, when its order is low and the rms value is high.

Lowest order significant harmonic in V_o $= p-2$

(ii) Unipolar PWM

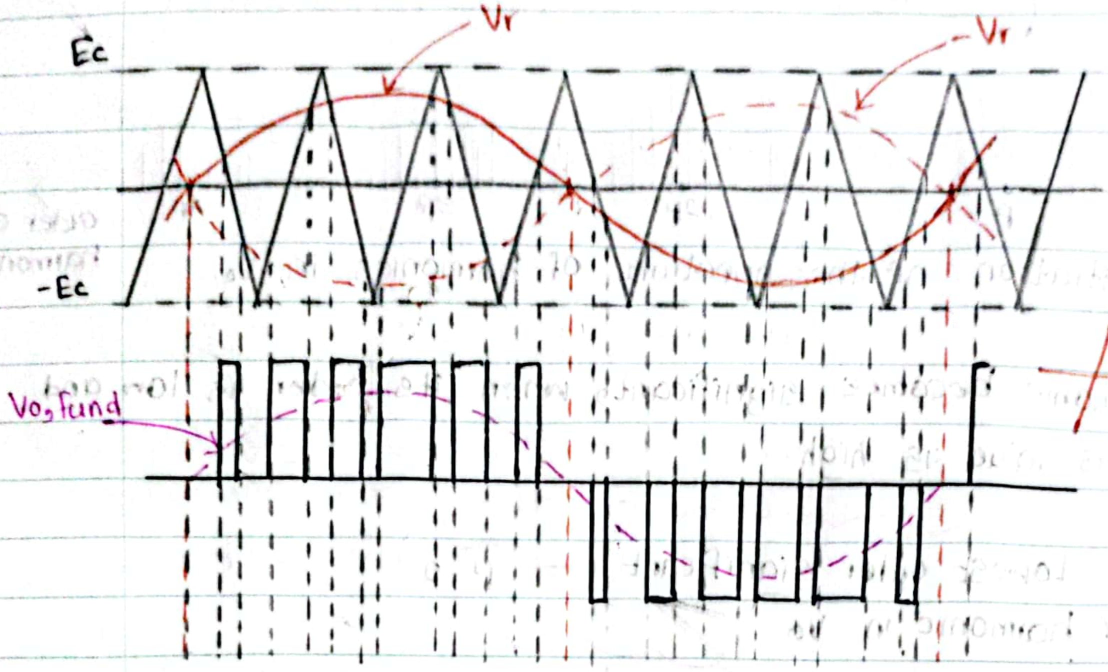


E_c and f_c - fixed
E_r and f_r - variable
f_c will be switching frequency.
f_r will be output fund. frequency.

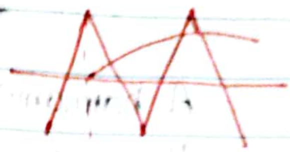
$m = \dfrac{E_r}{E_c}$; $0 \leq E_r \leq E_c$
$\therefore m = 0 \sim 1$

$p = \dfrac{f_c}{f_r}$; p should be an EVEN integer of a higher value for lesser content of harmonics in V_o.

Positive - going zero - crossing of V_r should be synchronized with positive - peak of V_c for a half - wave and quarter wave symmetric V_o.



Example illustration for $p = 6$ and $m \approx 0.75$.



- V_o is both the half wave and the quater - wave symmetric. No. of voltage pulses in each half cycle is equal to p.

Unipolar PWM is the most preffered PWM in 1- ph. inverter.

For higher values of p, $(V_{o,fund})_{rms}$ is almos independent from p, and can be shown as,

$$(V_{o,fund})_{rms} \approx \frac{mV_d}{\sqrt{2}}$$

The orders of harmonic present in V_o are $(2np \pm r)$, where $(2n+r)$ = ODD.

$n=1$; $2p \pm 1$, $2p \pm 3$, $2p \pm 5$, $2p \pm 7$ .....

$n=2$; $4p \pm 1$, $4p \pm 3$, $4p \pm 5$, ...

$n=3$; $6p \pm 1$, $6p \pm 3$, $6p \pm 5$, ...

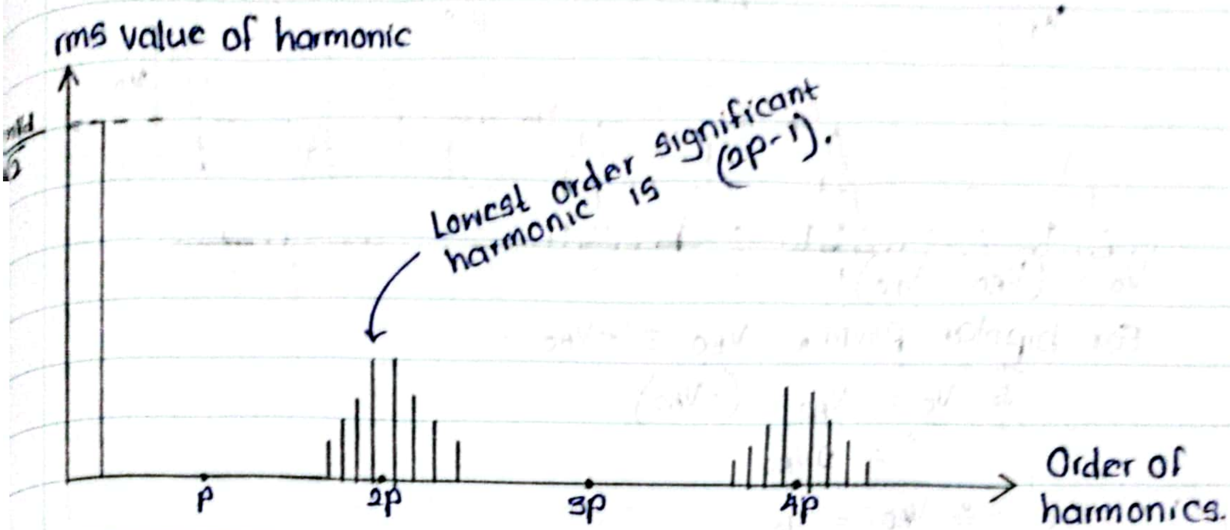


Illustration of the spectrum of harmonics in V_o. (Observe that the content of harmonics in V_o is much lower than that in the bipolar PWM V_o).

$$\boxed{\text{Lowest order (significant harmonic)} = 2p-1}$$

Note: It must be noted that, if an ODD p used, the content of harmonics would be serve as that produced by the bipolar PWM.

EVEN $\rightarrow p = 46$; $2 \times 46 - 1 = 91$

ODD $\rightarrow p = 45$; $45 - 2 = 43$

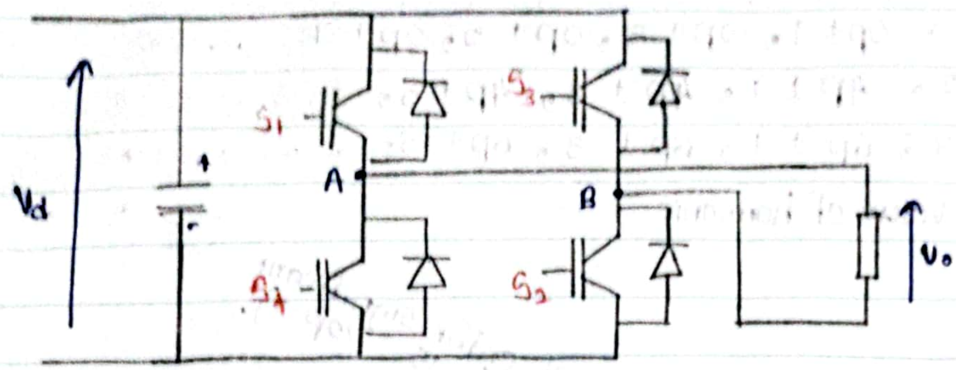
Q: Output voltage V_o of a bipolar PWM operator sinusoidal 1-ph inverter is given as $(np \pm r)$, where n and r are positive integers with $(n+r) =$ ODD.

Show that the output voltage V_o of unipolar PWM operated sinusoidal, 1-phase is inverter is,

(1) ~~2p~~ $(2np \pm r)$ when $p =$ EVEN, in which $(2n+r) =$ ODD

(ii) $(np \pm r)$ when $p =$ ODD, in which $(n+r) =$ ODD.

Ans:



$V_o = (V_{AO} - V_{BO})$

For bipolar PWM, $V_{BO} \equiv -V_{AO}$

$\therefore V_o = V_{AO} - (-V_{AO})$

$= 2V_{AO}$

$\therefore V_{AO} = \frac{V_o}{2}$

$\therefore$ All harmonics present in V_o are present in V_{AO} too, ~~i.e~~ which means $(np \pm r)$ order.

(i) For a unipolar PWM too, V_{AO} has $(np \pm r)$ order harmonics.

Let

$(V_{AO})_Q = V_{mQ} \cdot \sin Q\omega_r t \leftarrow$ ω_r is fund. freq. of V_o.
Q is order of harmonics $(Q = np \pm r)$,
V_{mQ} is peak value of Q^{th} order harmonic in V_{AO}.

$$(V_o)_Q = (V_{AO})_Q + (V_{BO})_Q$$
$$= V_{mQ}\left[(\sin Q\omega_r t) - \sin Q(\omega_r t - 180^\circ)\right]$$
$$= 2V_{mQ} \cos Q(\omega_r t - 90^\circ) \cdot \underbrace{\sin 90^\circ Q}_{}$$

This term goes to zero when Q is EVEN.

$\therefore (V_o)_Q$ will not exists for EVEN Q.

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(i) P = Even. — ①
$Q = np \pm r$, where $(n+r)$ = ODD. — ②

Q = ODD only — ③

To meet ①, ② and ③ n must be EVEN only.

$\therefore Q = np \pm r$; n = EVEN
$n+r$ = ODD.
$\equiv (2np \pm r)$; $2n+r$ = ODD.

$\therefore V_o$ will only have harmonics of the order of $(2np \pm r)$, where $(2n+r)$ = ODD.//

(ii) P = ODD — ①
$Q = np \pm r$ — ② ; $(n+r)$ = ODD.
Q = ODD only — ③

①, ② and ③ are satisfied for all n.
All of $(np \pm r)$ order harmonics will be present in V_o. //

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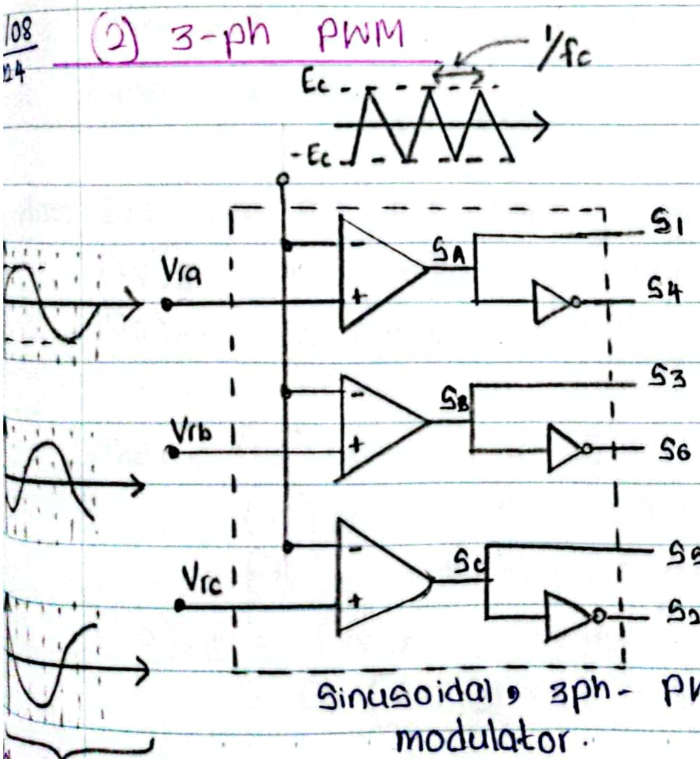
(2) 3-ph PWM



Sinusoidal, 3ph- PWM modulator.

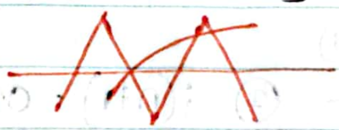
balanced, 3ph reference sinusoidal signals.

E_c and f_c are fixed.
E_r and f_r are variable.
f_c will be switching - frequency f_s.
E_r is variable between $0 \sim E_c$.
f_r will be fund. frequency of output voltage.

$$m = \frac{E_r}{E_c} \quad ; 0 \leq m \leq 1$$

$$p = \frac{f_c}{f_r}$$ p should be odd, a multiple of 3, and large.

- Reference signals and career signals are synchronized, such that the positive-going zero-crossing of reference signal coincides with the negative-going zero-crossing of the ~~career~~ Carrier.



Ex: construct ~~sketch~~ V_{AB} and V_{AN} waveforms for a 3-phase sinusoidal PWM with $p=9$ and $m \approx 0.75$. Assume default synchronizing between the reference signals and the carrier signal:

We will observe that the V_{AB} waveform has,

- p + no. of voltage pulses in each half cycle.
- Both half wave and quater wave symmetry.
- Positive only pulses in the positive half cycle, and negative only pulses in the negative half cycle.

We can show, for larger values of p, that the rms values of the fund. component of output voltage is almost independent from p, and,

$$(V_{line, fund})_{rms} = m\left(\frac{\sqrt{6}\,V_d}{4}\right).$$

Also, we can show that the order of harmonics present in the output line as well as phase voltages are $(np \pm r)$ except tripplan orders $(\times 3)$, ~~in which~~ where $(n+r)$ = odd.

Ex: pole-voltage due to sinusoidal PWM contains harmonics of the order of $(np \pm r)$, where $(n+r)$ = ODD and p is carrier ratio. Show that the line-line voltage due to 3-ph sinusoidal PWM will have harmonics of the order $(np \pm r)$, without three times orders.

Ans: ~~Let the qth~~ ~~Qth order~~ Let $(V_Q)_{AO}$ = Q^{th} order harmonic of V_{AO}

$(V_Q)_{BO}$ = Q^{th} order harmonic of V_{BO}

$(V_Q)_{CO}$ = Q^{th} order harmonic of V_{CO}.

Then, $(V_Q)_{AO} = E_Q \sin Q\omega t$

$(V_Q)_{BO} = E_Q \sin Q(\omega t - 120^\circ)$

$\cancel{(V_Q)_{CO} = E_Q \sin Q(\omega t - 240^\circ)}$

$(V_Q)_{AB} = (V_Q)_{AO} - (V_Q)_{BO}$

$= E_Q \left[\sin Q\omega t - \sin Q(\omega t - 120^\circ)\right]$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right).$$

$$\therefore (V_Q)_{AB} = 2 \cdot \cancel{\sin}\overset{\cos Q}{}(\cancel{Q}\omega t - 60^\circ) . \sin 60^\circ Q . E_Q$$

$$\cancel{= \sqrt{3} E_Q \sin (Q\omega t - 60^\circ)}$$

$$= \underbrace{2E_Q \sin(60^\circ Q)}\cos Q(\omega t - 60^\circ)$$

This amplitude goes to zero,
when $60^\circ Q$ = multiple of 180°,
i.e Q = multiple of 3.

This proves that line voltage V_{AB} that have harmonics of order of multiple of 3, but all other Q of, $(np \pm r)$.

(i) Voltage Utilization

"Voltage Utilization" is a performance index for an inverter, used to compare between different control.

$$\boxed{\text{Voltage Utilization} = \frac{\left(\begin{array}{l}\text{Highest fund. rms voltage of the output} \\ \text{with a given control.}\end{array}\right)}{\left(\begin{array}{l}\text{fund. rms voltage due to square} \\ \text{wave control}\end{array}\right)}}$$

~~Sinusoidal~~ For sinusoidal PWM, *

$$\text{voltage Utilization} = \frac{(V_{AB}, \text{fund})_{rms, max}}{\frac{\sqrt{6}V_d}{\pi}}$$

$$= \frac{\left(m\left(\frac{\sqrt{6}V_d}{4}\right)\right)_{m=1}}{\left(\frac{\sqrt{6}V_d}{\pi}\right)} \times 100\%$$

* m=1 is the highest.

$$= \frac{\pi}{4} \times 100\%$$

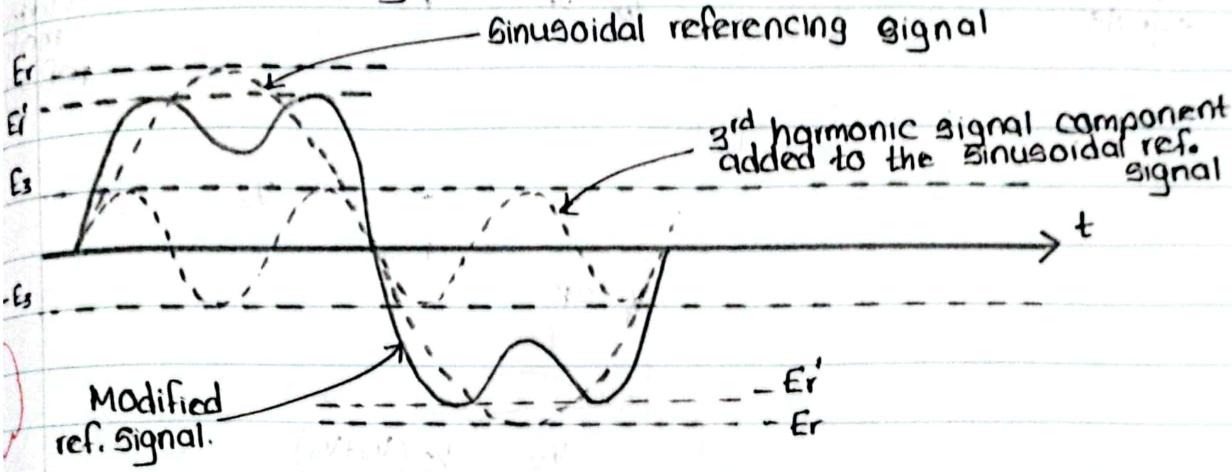
$$= 78.5\%$$

//

- This reduced voltage utilization, demands correspondingly a higher input voltage to deliver the same $(V_{line}, fund)_{rms}$). and In some cases, this will increase the inverter cost.

If the increase of the cost of sinusoidal PWM inverter option is likely significant, we can consider other options the options of raising the voltage utilization, without increasing V_d. We have 2 basic options.

- Mixing of a 3rd harmonic component with reference signals.
- Mixing of self-generate offset voltage component with reference signals.



We can increase E_r' up to E_c (carrier amplitude) without causing overmodulation. ~~With a proper choise of E_3, we can achieve E_r.~~ Value of E_r at this condition depends on the choise of E_3, and we can show that E_r can be increased upto about $1.15\,E_r'$ with an $E_3 \approx 0.2 E_r$.

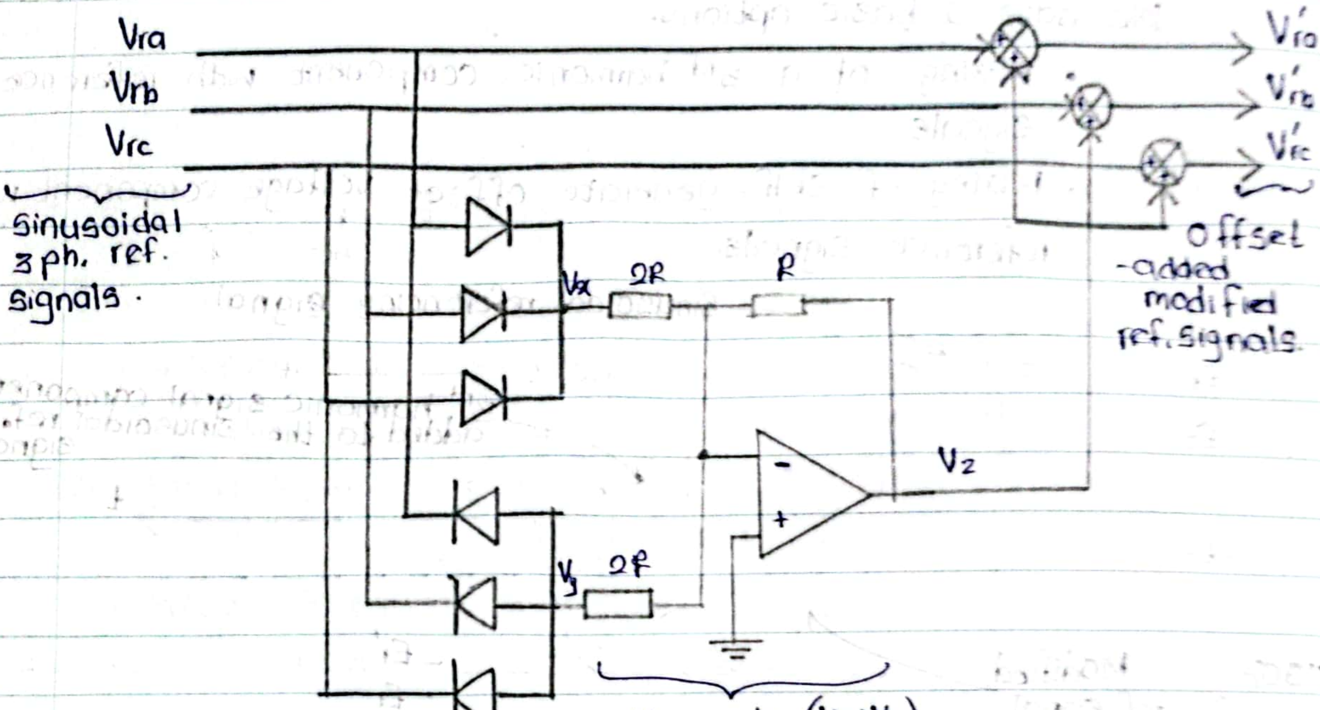
$$\therefore (m)_{max} = \frac{E_r}{E_c} = \frac{E_r}{E_r'} = 1.15$$

Ex: $V = a \sin \omega t + b \sin 3\omega t$

(i) Find an expression for V_{pk}, in terms of a and b.

(ii) Derive a relationship between a and b that makes V_{pk} of its minimum.

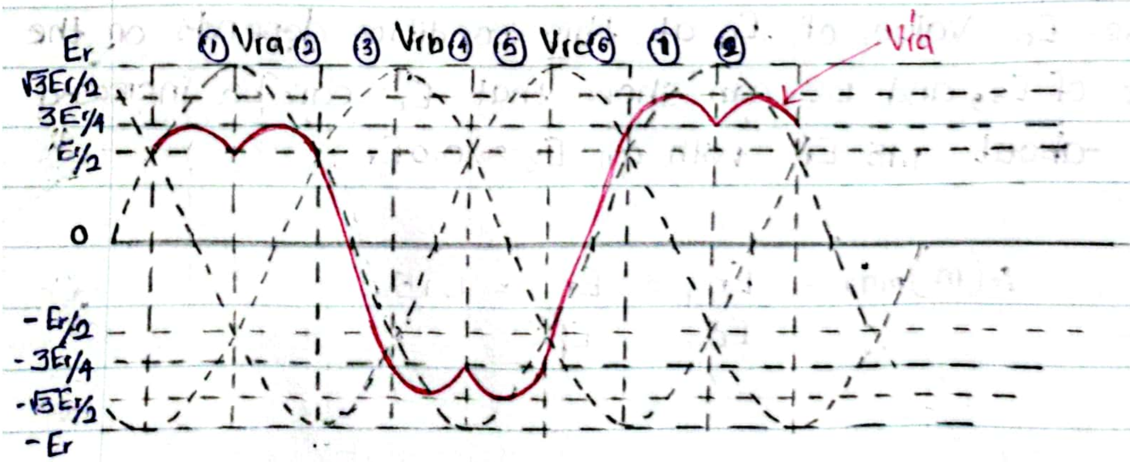
(iii) Hence, show that the $(V_{pk})_{min}$ occurs at $b \approx 0.29$, resulting $(V_{pk})_{min}$ = ~~0.869~~ 0.87.



$$\underbrace{\quad}_{V_2 = -\frac{1}{2}(V_x + V_y)}$$

V_x = uppermost envelope of V_{ra}, V_{rb} and V_{rc}

V_y = lower-most envelope of V_{ra}, V_{rb} and V_{rc}.



$$V'_{ra} = V_{ra} - \frac{1}{2}(V_x + V_y)$$

Interval No.	V_α	V_y	V'_{ra}
1	V_{ra}	V_{rb}	$\frac{1}{2}(V_{ra}-V_{rb})$
2	V_{ra}	V_{rc}	$\frac{1}{2}(V_{ra}-V_{rc})$
3	V_{rb}	V_{rc}	$\frac{3}{2}V_{ra}$
4	V_{rb}	V_{ra}	$\frac{1}{2}(V_{ra}-V_{rb})$
5	V_{rc}	V_{ra}	$\frac{1}{2}(V_{ra}-V_{rc})$
6	V_{rc}	V_{rb}	$\frac{3}{2}V_{ra}$

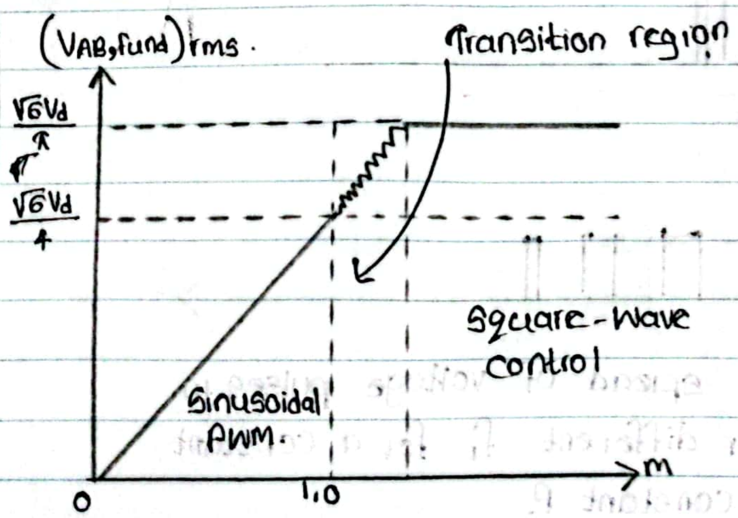
$$V'_{ra} = V_{ra} - \frac{1}{2}(V_{ra}+V_{rb})$$
$$= \frac{1}{2}(V_{ra}-V_{rb})$$

$$V_{rc} = \frac{1}{2}\underbrace{(V_{rb}+V_{rc})}_{-V_{ra}} = \frac{3}{2}V_{ra}$$

$$\therefore (m)_{max} = \frac{E_r}{E_c}$$
$$= \frac{E_r}{\left(\frac{\sqrt{3}E_r}{2}\right)}$$; Because we can take $E_c = \sqrt{3}E_r/2$ without over modulation.
$$= 1.15$$

(ii) Overmodulation

Overmodulation is a strategy to transfer control from sinusoidal PWM to square-wave control. Unlike in square-wave-PWM, the sinusoidal PWM does not produce single large-step of voltage (due to the influence of blanking time adopted in the inverter design) but multiple steps of smaller sites.



~~This transition is~~ Both the 3-phase and 1-phase sinusoidal PWM inverters produce nearly smooth transition between sinusoidal PWM and square-wave control.

(iii) Gear Changing

Geer-changing comes in the context of voltage-frequency control.

To control voltage, we vary m.
To control frequency, we vary f_r.

Some applications (eg: 3ph. induction motor speed control) need inverter to deliver output voltage and frequency with the ratio V/f kept at a constant. This is ~~ca~~ known as "V to f control". This delivered by ensuring the ratio m/f_r constant.

Illustration of the spread of voltage pulses in the half-cycle for different f_r for a constant V/f control, with constant P.

- As evident from the waveforms, the voltage pulses in the waveform spread far apart at lower f_r frequencies (which is highly undesirable), when the carrier ratio p remains constant.

- Remedy is increase p, when f_r is lowered, so that the no. of pulses in the half cycle is increased, giving a better spread of pulses over the half cycle.

eg:

f (Hz)	p
5 - 15	459
16 - 30	409
31 - 60	387
61 - 100	315
101 - 175	171
above 175	117

An example choice of p for different range of f_r in a V/f control.

- This change of carrier ratio is known as gear changing.

Gear changing = process of increasing carrier ratio p when lowering f_r in a constant V/f control of an inverter in order to improve the output voltage waveform with lesser harmonics.

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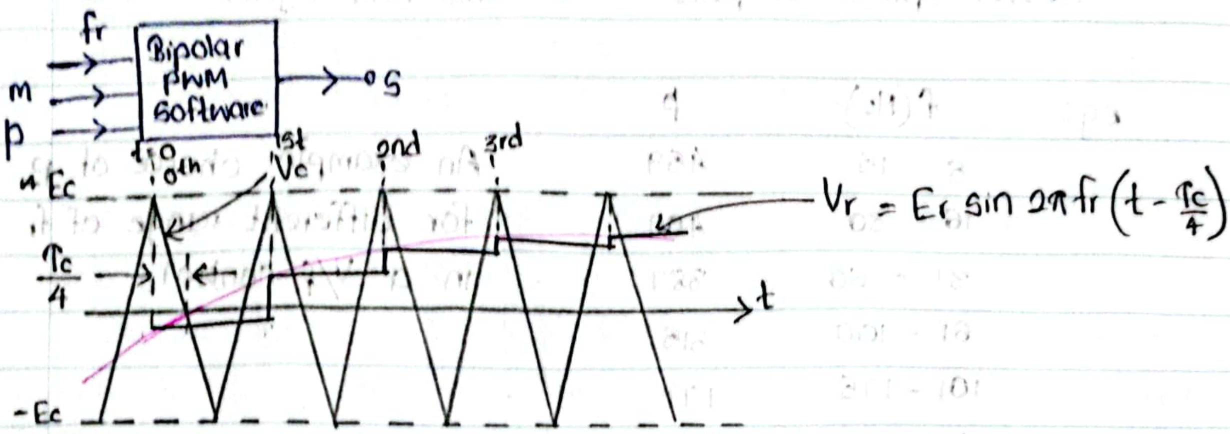
d) Regular Sampled PWM

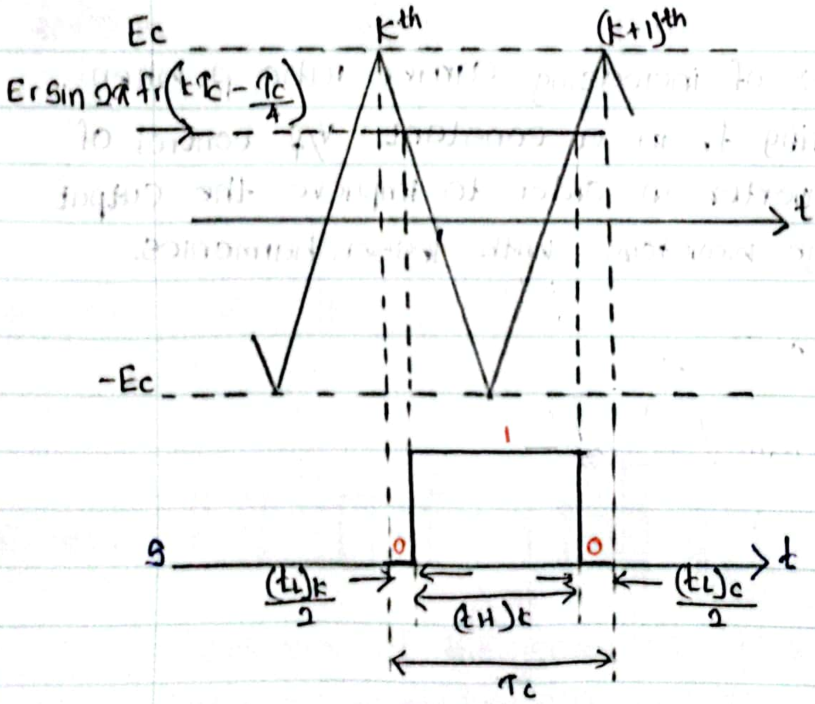
This is the software implementable version of sinusoidal PWM.

(1) 1-phase PWM.

(i) Bipolar PWM.



In regular sampled PWM, we treat V_r signal to be sampled at the instants of positive peak of V_c.



In the interval occuring after the k^{th} sampling instant,

$$S = \begin{cases} 0, & \text{for initial } t_L/2 \\ 1, & \text{for next } t_H \\ 0, & \text{for last } \frac{t_L}{2} \end{cases}$$

Date: ..../........../..........

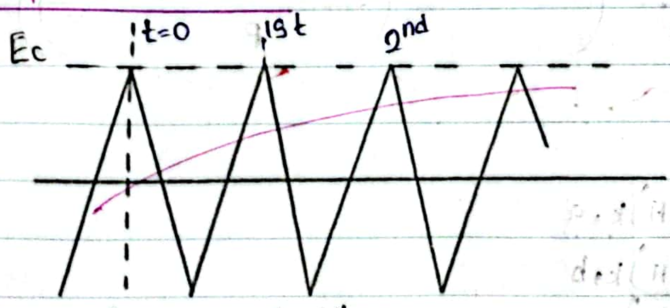
$$(t_H)_k = \frac{E_c + E_r \sin 2\pi f_r \left(kT_c - T_c/4\right) \times T_c}{2E_c}$$

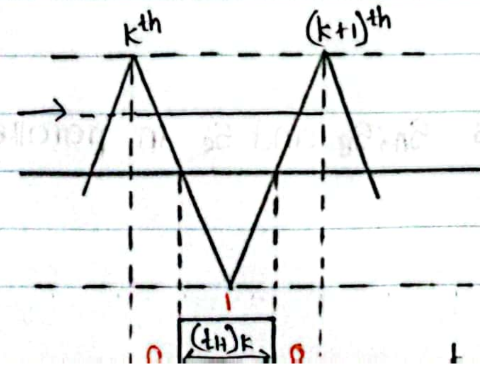
$$= \frac{1}{2pf_r}\left(1 + m \sin \frac{2\pi\,(k - 0.25)}{p}\right) \quad ; \; m = \frac{E_r}{E_c}, \; p = \frac{f_c}{f_r}, \; T_c = \frac{1}{f_c}$$

1. START
2. INPUT : p, f_r, m
3. $T = \dfrac{1}{pf_r}$
4. ~~Loo~~ Loop
   - Loop : $k = 0$ to p
     - $T_H = \dfrac{T}{2}\left[1 + m \sin \dfrac{2\pi\,(k - 0.25)}{p}\right]$.
     - OUTPUT : $s = 0$
     - WAIT : $\frac{1}{2}(T - T_H)$
     - OUTPUT : $s = 1$
     - WAIT : t_H
     - OUTPUT : $s = 0$
     - WAIT : $\frac{1}{2}(T - T_H)$
   - LOOP END.

   Loop END.

(ii) Unipolar PWM





$$(t_H)_k = \frac{E_c + E_r \sin(2\pi f_r k T_c) \times T_c}{2E_c}$$

$$= \frac{1}{2pf_r}\left(1 + m \sin\left(2\pi k/p\right)\right)$$

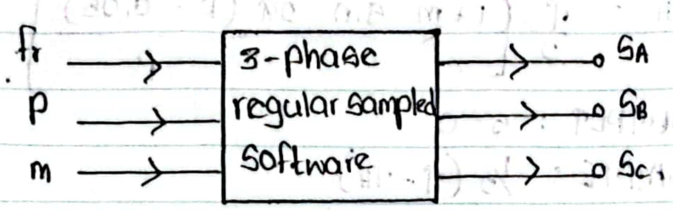
$$(t_L)_k = T_c - (t_H)_k$$

Based on these t_H and t_L expressions, we implement unipolar PWM.

(2) 3-phase PWM

In 3-phase sinusoidal PWM, we synchronize zero-crossing of V_r with the positive going zero crossing of V_c.

Let $(t_H)_{k,a}$, $(t_H)_{k,b}$, $(t_H)_{k,c}$ be high durations of S_A, S_B, S_C, after the k^{th} sampling instant.



$$(t_H)_{k,a} = \frac{1}{2pf_r}\left(1 + m \sin \frac{2\pi(k-0.25)}{p}\right)$$

$$(t_H)_{k,b} = \frac{1}{2pf_r}\left(1 + m \sin\left(\frac{2\pi(k-0.25)}{p} - \frac{2\pi}{3}\right)\right)$$

$$(t_H)_{k,c} = \frac{1}{2pf_r}\left(1 + m \sin\left(\frac{2\pi(k-0.25)}{p} - \frac{4\pi}{3}\right)\right)$$

Then,

$$(t_L)_{k,a} = T_c - (t_H)_{k,a}$$
$$(t_L)_{k,b} = T_c - (t_H)_{k,b}$$
$$(t_L)_{k,c} = T_c - (t_H)_{k,c}.$$

Our programme should process S_A, S_B and S_C in parallel, and output in parallel.

Example

Regular sampled PWM is implemented ~~on~~ a 3-phase VSI with $f_r = 50Hz$, $P = 135$ and $m = 0.8$. You may assume that the positive-going zero-crossing of each reference signal is coinsided with a negative-going zero-crossing of the carrier. Compute timing of S_A, S_B and S_C in the sampling interval following the 75th sampling instant.

Ans:

$f_r = 50Hz$

$P = 135$

$m = 0.8$

Std. Synchronizing

$k = 75$.

$$(t_H)_a = \frac{1}{2pf_r}\left(1 + m \sin\left[\frac{2\pi(k - 0.25)}{P}\right]\right)$$

$$= \frac{1}{2 \times 135 \times 50}\left(1 + 0.8 \sin\left[\frac{360^\circ(75 - 0.25)}{135}\right]\right)$$

$$= 5.445 \times 10^{-5}\ s$$

$$= 54.45\ \mu s.$$

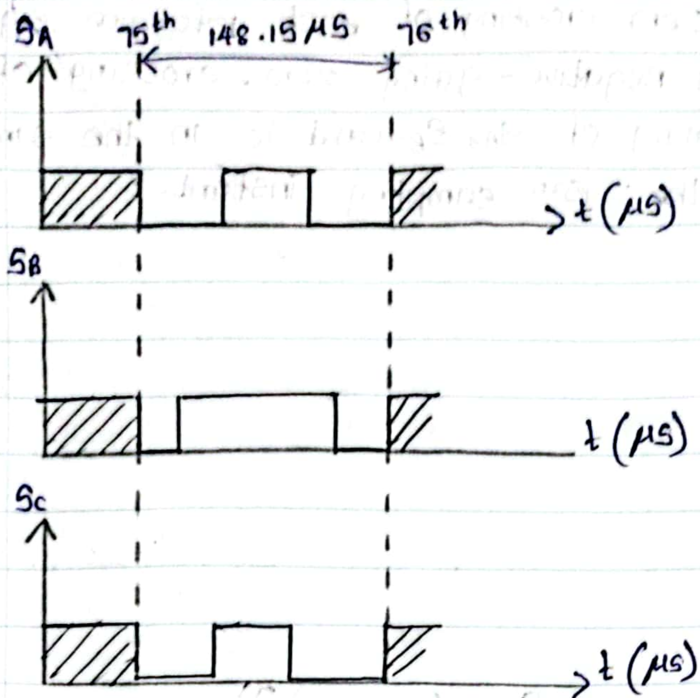
$$(t_H)_b = \frac{1}{2 \times 135 \times 50}\left(1 + 0.8 \sin\left[\frac{360^\circ(75 - 0.25)}{135}\right] - 120^\circ\right)$$

$$= 132.31\ \mu s$$

$$(t_H)_c = \frac{1}{2 \times 135 \times 50}\left(1 + 0.8 \sin\left[\frac{360^\circ(75 - 0.25)}{135}\right] - 120^\circ\right)$$

$$= 35.46\ \mu s.$$

$$T_c = \frac{1}{pf_r} = \frac{1}{135 \times 50} s = 148.15\ \mu s.$$

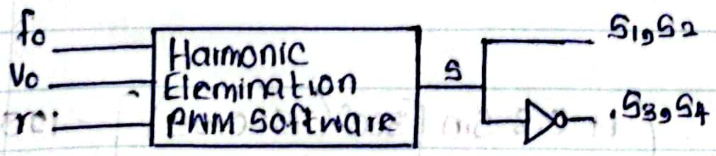


(c) Harmonic Elimination PWM.

This is a software implemented PWM, that does not follow conventional carrier and reference approach.

Here, we specify which orders of harmonics should be absent in the output, ~~die~~ directly. The software programme then produces switching signals to achieve that.

(i) 1- phase PWM.



f_o = fundamental frequency of desired V_o.

V_{rms} = per unit value of the desired $(V_o)_{fund}$.

r = no. of harmonics to be eliminated from V_o.

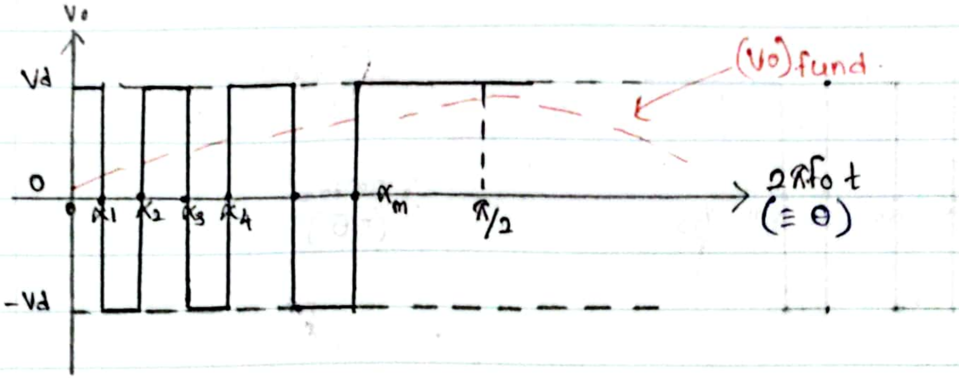
Note: Base value for per unit voltage is the output voltage ~~is the o~~ obtained by the same inverter with square wave control

When we specify r no. of harmonics to be eliminated, it means the r no. of lowest order harmonics

In order to eliminate r no. of harmonics, and set the fundamental output voltage at the desired V_{pu}, we need $(r+1)$ no. of switching points per quarter cycle of s.

Case 1 : $(r+1)$ is EVEN

Let $(r+1) = m$, and $\alpha_1, \alpha_2, \alpha_3$ and $\alpha_n \ldots\ldots \alpha_m$ be the m switching angles of s, in the 1st quater cycle.



Vo is half wave and Quater-wave symmetric According to Fourier expansion,

$$V_o = \sum_{n=1,3,5\ldots} b_n \sin n\theta \quad \text{—} (1) \quad ; \theta = 2\pi f_o t.$$

$$b_n = \frac{8}{2\pi} \int_0^{\pi/2} V_o \sin n\theta \, d\theta \quad \text{—} (2)$$

$$(2) ; b_n = \frac{8}{2\pi} \left(\int_0^{\alpha_1} V_d \sin n\theta \, d\theta + \int_{\alpha_1}^{\alpha_2} -V_d \sin n\theta \, d\theta + \int_{\alpha_2}^{\alpha_3} V_d \sin n\theta \, d\theta \ldots\ldots + \int^{\pi/2} V_d \sin n\theta \, d\theta \right)$$

$$= \frac{4V_d}{n\pi}\left((\cos 0^\circ - \cos n\alpha_1) - (\cos n\alpha_1 - \cos n\alpha_2) + \dots + (\cos n\alpha_m - \underbrace{\cos n\pi/2}) \right)$$

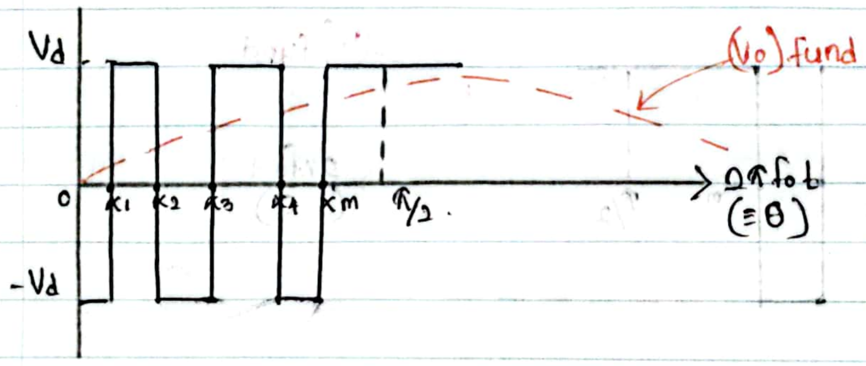
← Always 0, because n is ODD only.

$$= \frac{4V_d}{n\pi}\left(1 - 2\cos n\alpha_1 + 2\cos n\alpha_2 - 2\cos n\alpha_3 + \dots \; 2\cos n\alpha_m \right)$$

$$\boxed{b_n = \frac{4V_d}{n\pi}\left(1 + 2\sum_{k=1}^{m} (-1)^k \cos n\alpha_k \right)}$$

Case 2: $(r+1)$ is ODD

In order to avoid un-intended 180° phase-shifts of V_o waveform, for ODD valued $(r+1)$, we start switching signal with $s=1$ in the initial $0-\alpha_1$ interval.



For this case we obtain,

$$V_o = \sum_{n=1,3,5\dots} b_n \sin n\theta$$

$$b_n = -\frac{4V_d}{n\pi}\left(1 + 2\sum_{k=1}^{m} (-1)^k \cos n\alpha_k \right)$$

In this PWM, first we need to compute $\alpha_1, \alpha_2, \alpha_3 \dots$ and α_m to suit the required r and V_{pu}. Then, we construct switching signal s.

Generally, computation of αs is done offline, during the development stage. The computed switching angle are kept in memory as lookup tables and retrieved ~~duri~~ online during the inverter running.

Solving ① ~ ⑥, we obtain, $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_6$.

Implementation.

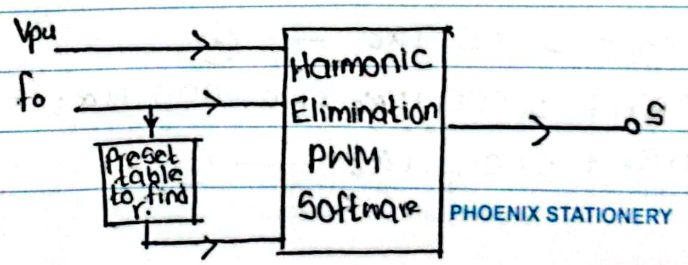
In the development stage, we compute switching angles for ~~the~~ different r and different V_{pu}, and store these in lookup tables. There is one table for each r.

V_{pu}	α_1	α_2	α_3	α_4	α_5	α_6
0.00	✓	✓	✓	✓	✓	✓
0.05	✓	✓		✓	✓	✓
0.10	✓	✓	✓	✓	✓	✓
0.15	✓	✓	✓	✓	✓	✓
0.20	✓	✓	✓	✓	✓	✓
⋮						
0.95	✓	✓	✓	✓	✓	✓
1.00	✓	✓	✓	✓	✓	✓

Example lookup table for r = 5.

Note : We should decide on the resolution of V_{pu}, depending on the required control resolution of the two output voltage. For example, in a case of $V_d = 320V$, 0.05 pu resolution allows an output voltage control in $0.05 \left(\frac{\sqrt{8} \times 320}{\pi} \right)$ Vrms steps.

Which r value to be used for the generation of S, can be allowed to be determined by the disired f_o itself. For a lower f_o value r should be higher, and for a higher f_o r ~~cz~~ can be smaller.



PHOENIX STATIONERY

Ex: 1-phase harmonic elimination PWM is to be implemented to eliminate 3^{rd}, 5^{th}, 7^{th}, 9^{th} and 11^{th} harmonics from V_o, and give the $(V_o)_{fund}$ of 0.7 pu. Write algebraic equations for solving for the required switching angles.

Ans: $r = 5$, $V_{pu} = 0.7$ pu

~~This case is a case of~~ $m = (r+1) = (5+1) = 6$; this is a case of even m.

Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ and α_6 be switching angles.

$$b_n = \frac{4V_d}{n\pi}\left(1 + 2\sum_{k=1}^{m}(-1)^k \cos n\alpha_k\right)$$

To meet specifications,

$$b_1 = 0.7\left(\frac{4V_d}{\pi}\right)$$

Notice that $\frac{4V_d}{\pi}$ is the peak value of V_o with square-wave control.

$$\left.\begin{aligned} b_3 &= 0 \\ b_5 &= 0 \\ b_7 &= 0 \\ b_9 &= 0 \\ b_{11} &= 0 \end{aligned}\right\}$$

as we have to eliminate others from the output voltage waveform.

Thus we set ~~$+ 2\cos\alpha_1, +$~~

$$0.7 = 1 - 2\cos\alpha_1 + 2\cos\alpha_2 - 2\cos\alpha_3 + 2\cos\alpha_4 - 2\cos\alpha_5 + 2\cos\alpha_6 \quad (1)$$

$$0 = 1 - 2\cos 3\alpha_1 + 2\cos 3\alpha_2 - 2\cos 3\alpha_3 + 2\cos 3\alpha_4 - 2\cos 3\alpha_5 + 2\cos 3\alpha_6 \quad (2)$$

$$0 = 1 - 2\cos 5\alpha_1 + 2\cos 5\alpha_2 - 2\cos 5\alpha_3 + 2\cos 5\alpha_4 - 2\cos 5\alpha_5 + 2\cos 5\alpha_6 \quad (3)$$

$$0 = 1 - 2\cos 7\alpha_1 + 2\cos 7\alpha_2 - 2\cos 7\alpha_3 + 2\cos 7\alpha_4 - 2\cos 7\alpha_5 + 2\cos 7\alpha_6 \quad (4)$$

$$0 = 1 - 2\cos 9\alpha_1 + 2\cos 9\alpha_2 - 2\cos 9\alpha_3 + 2\cos 9\alpha_4 - 2\cos 9\alpha_5 + 2\cos 9\alpha_6 \quad (5)$$

$$0 = 1 - 2\cos 11\alpha_1 + 2\cos 11\alpha_2 - 2\cos 11\alpha_3 + 2\cos 11\alpha_4 - 2\cos 11\alpha_5 + 2\cos 11\alpha_6 \quad (6)$$

1. START
2. INPUT : f_0, V_{pu}
3. REAR r from pre-set table corresponding to f_0.
4. Access r- table and read $\alpha_1, \alpha_2 \ldots$ and α_{r+1}, corresponding to V_{pu}.
5. If (r+1) is EVEN:

   $s = 1$ for $t = 0$ to $\dfrac{\alpha_1}{2\pi f_0}$

   $s = 0$ for $t = \left(\dfrac{\alpha_1}{2\pi f_0}\right)$ or $\left(\dfrac{\alpha_2}{2\pi f_0}\right)$.

   ⋮

   ELSE, $s = 0$ for $t = 0$ to $\left(\dfrac{\alpha_1}{2\pi f_0}\right)$

   $s = 1$ for $t = \left(\dfrac{\alpha_1}{2\pi f_0}\right)$ to $\left(\dfrac{\alpha_2}{2\pi f_0}\right)$

   ⋮

Note : Direct data of α is used to construct s for 0° to 90°. Thereafter from 90°-180° then 180°-270° and finally 270°-360° s is assembled using symmetry relationships