Ex find $(1-i)^{1/5}$.

$$(1-i) = \sqrt{(1)^2+(-1)^2}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)$$

$$= \sqrt{2}\left[\cos(-\pi/4) + i\sin(-\pi/4)\right]$$

$$(1-i)^{1/5} = \left\{\sqrt{2}\left[\cos(-\pi/4 + i\sin(-\pi/4))\right]\right\}^{1/5}$$

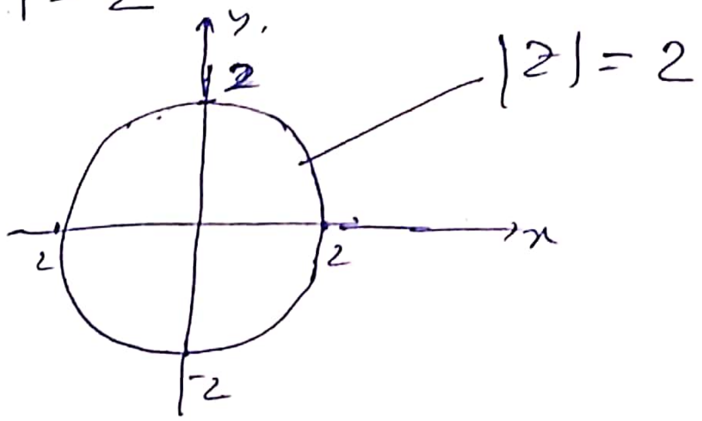
$$z_k = (\sqrt{2})^{1/5}\left(\cos(-\pi/4) + i\sin(-\pi/4)\right)^{1/5}$$

$k = 0, 1, 2 \ldots, 4.$

$$z_k = (\sqrt{2})^{1/5}\, e^{i\left(\frac{-\pi/4 + 2k\pi}{5}\right)} \quad ; k = 0, 1, 3 \ldots 4.$$

Sketch of some equation on an Argand diagram

① $|z| = 2$

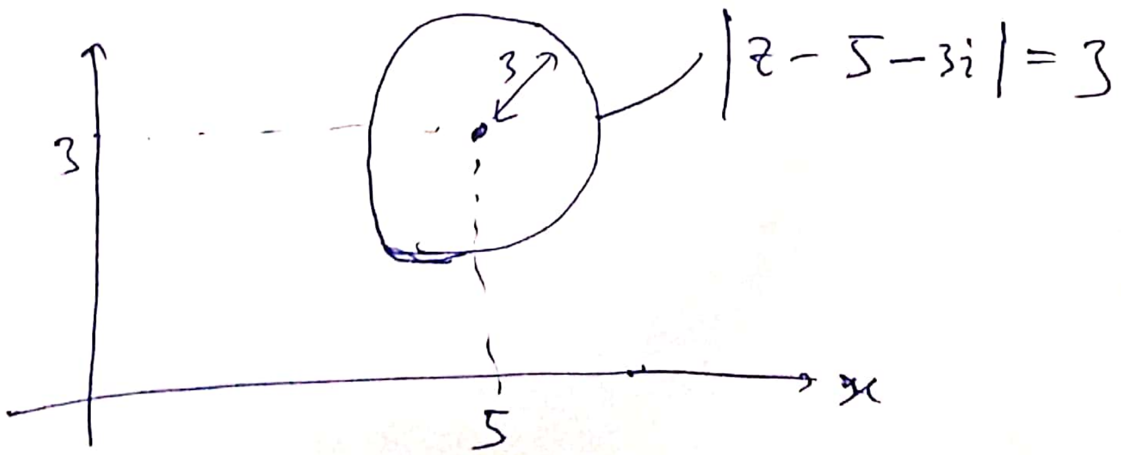


If $z = x + iy$, then

$|z| = 2$

$\sqrt{x^2+y^2} = 2 \implies \boxed{x^2+y^2 = 2^2}$

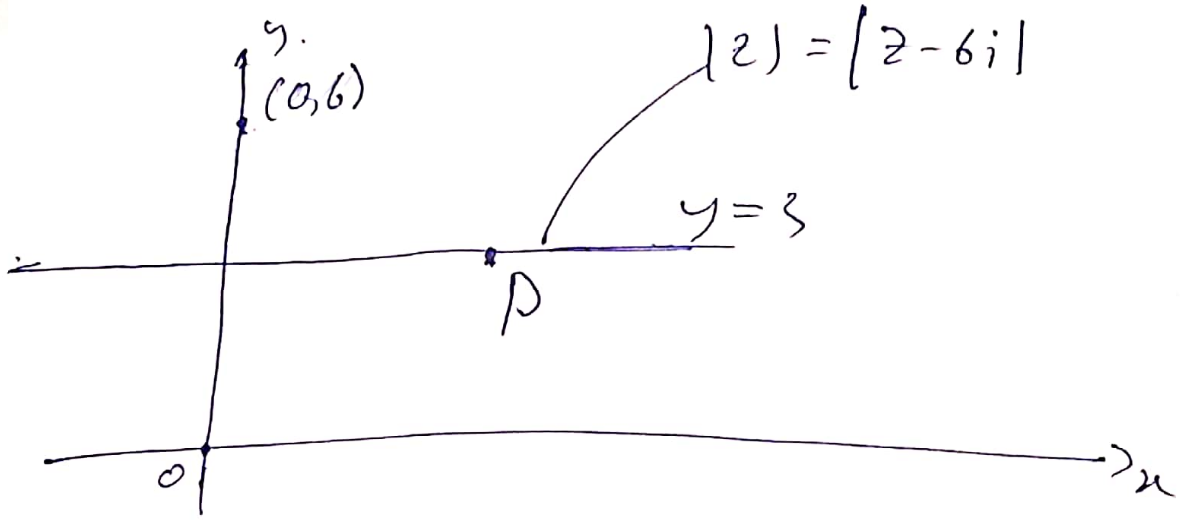
② $|z - 5 - 3i| = 3$



In general $|z-z_1| = r$ is represented by a circle center (x_1, y_1) with radius r, where

$z_1 = x_1 + iy_1$

(*) $|z| = |z-6i|$



$|z|$ represents the distance from the origin to P. $|z-6i|$ represents the distance from the point (0,6) to P. As $|z| = |z-6i|$, then P is the point which are equidistant from the point (0,0) and (0,6).

which has equation $y=3$.

Note:

A locus of points is a set of points which obey a particular rule.

Ex: If $|z-3| = |z+i|$, use an algebraic method to find a Cartesian equation of locus of z.

$$|z-3| = |z+i|$$

$$|x+iy-3| = |x+iy+i|$$

$$\cancel{|x+iy-3|^2 = x+iy}$$

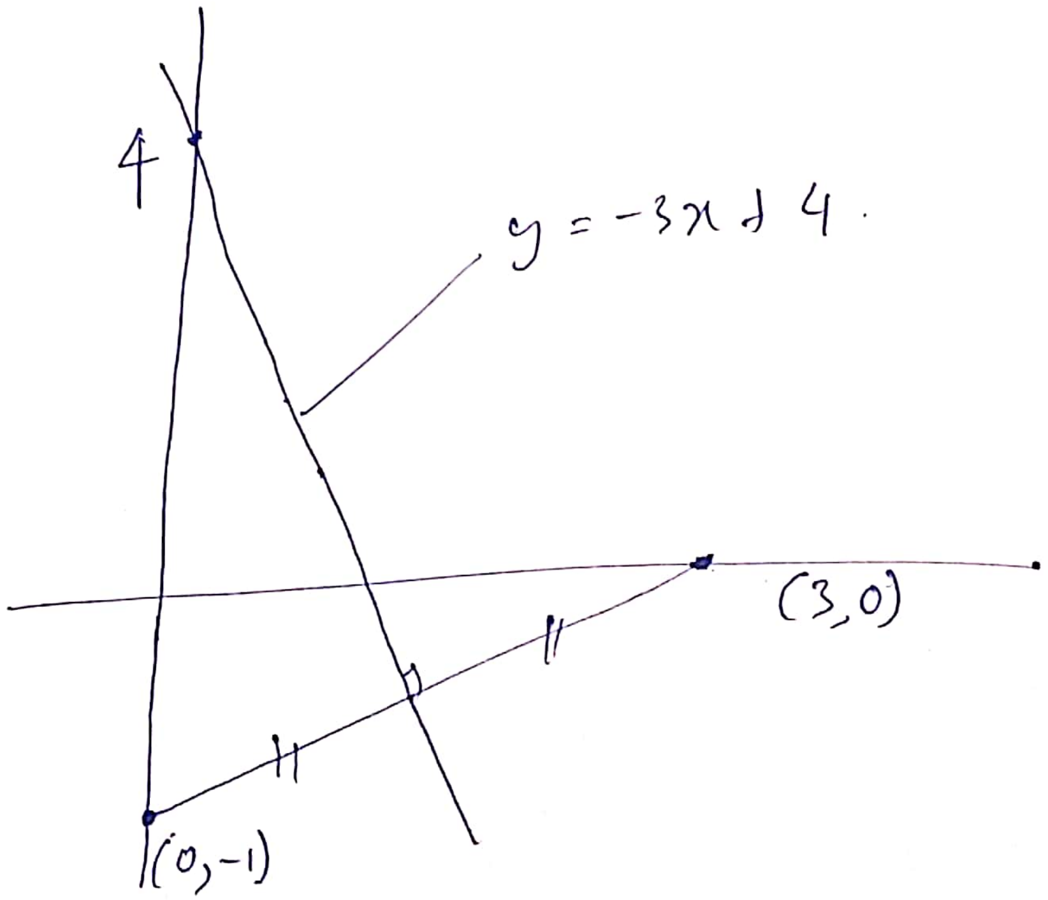
$$|(x-3)+iy|^2 = |x+i(y+1)|^2$$

$$(x-3)^2 + y^2 = x^2 + (y+1)^2$$

$$2y = -6x + 8$$

Hence the Cartesian equation of the locus of z is

$$y = -3x + 4$$



Note:
It follows that
$|z - z_1| = |z - z_2|$ is represented

by a perpendicular bisector of the line segment joining the points z_1 to z_2.

* If $|z-6| = 2|z+6-9i|$, use algebra to show that the locus of z is a circle.

$$|z-6| = 2|z+6-9i|$$

$$|x+iy-6| = 2|x+iy+6-9i|$$

$$|(x-6)+iy| = 2|(x+6)+i(y-9)|$$

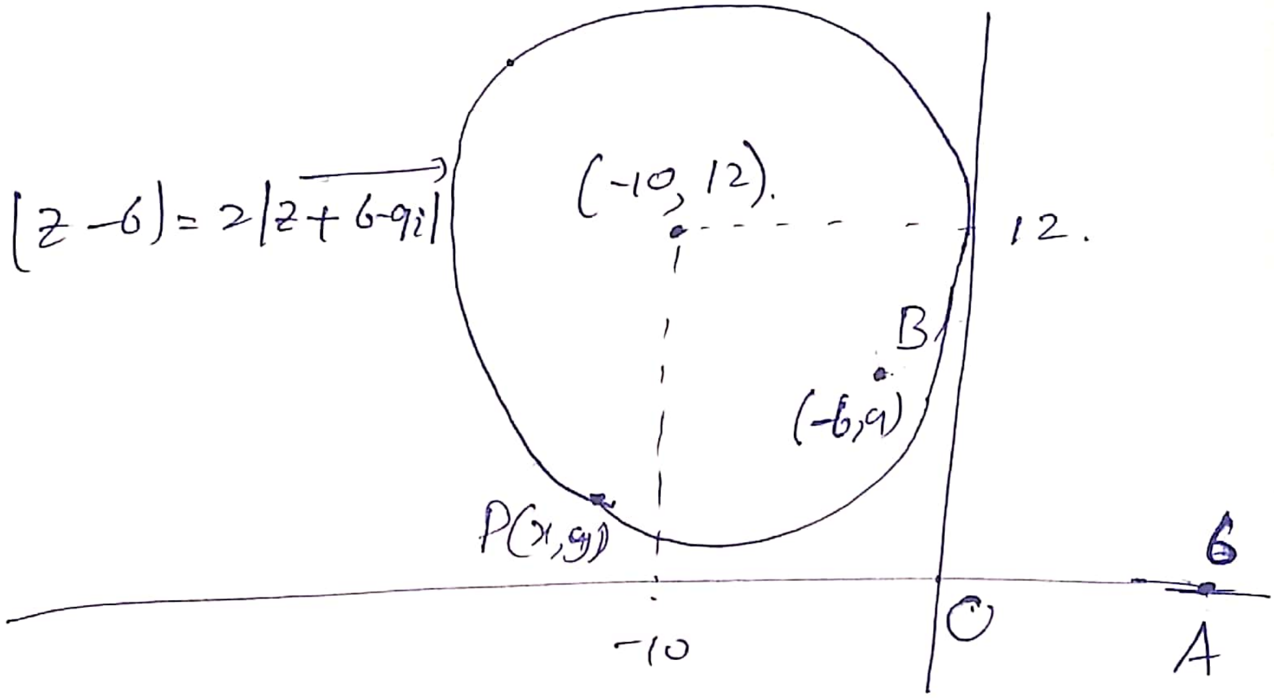
$$(x-6)^2+y^2 = 4[(x+6)^2+(y-9)^2]$$

$$x^2+20x+y^2-24y+144=0$$

$$(x+10)^2-100+(y-12)^2-144+144=0$$

$$(x+10)^2 + (y-12)^2 = 100$$

Hence the ~~focus~~ locus of z is a circle center $(-10,12)$, radius 10.



$|z-6|$ represents the distance from the point $A=(6,0)$ to $P=(x,y)$. $|z+6-9i| = |z-(-6+9i)|$ represent the distance from the point $B=(-6,9)$ to $P=(x,y)$.

$|z+6-9i| = 2|z+6-9i|$ gives

$AB = 2BP$. This means that

P is the locus of points where the distance AP is twice the distance BP.

However, from the outset that the locus of points is a circle.

<u>Note</u>

If $|z - z_1| = \lambda |z - z_2|$, where $\lambda > 0$, $\lambda \neq 1$, then it may be more appropriate to apply an algebraic method to find the locus of points z, represented by above equation.

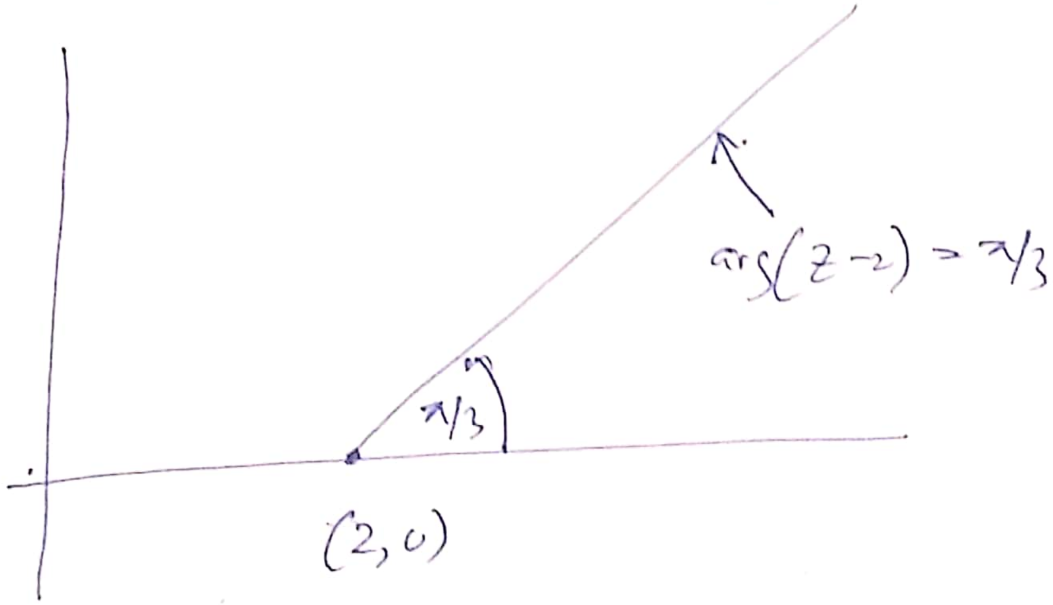
(v) $\arg(z-2) = \pi/3$

$\arg(x+iy-2) = \pi/3$

$\arg[(x-2)+iy] = \pi/3$

$\Rightarrow \dfrac{y}{x-2} = \tan(\pi/3)$

$\Rightarrow y = \sqrt{3}(x-2)$; when; $y \geq 0$, $x \geq 2$



Note:

Locus is a half line,

this equation is restricted for $x \geq 2$, $y \geq 0$

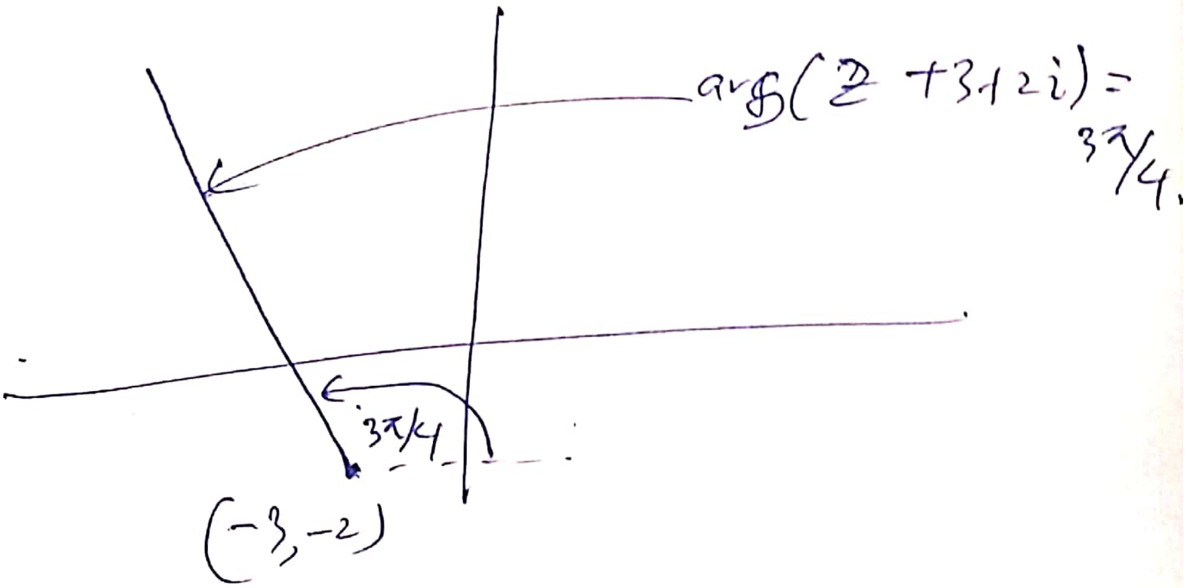
(e) $\arg(z+3+2i) = 3\pi/4$.

$\arg(x+iy+3+2i) = 3\pi/4$

$\Rightarrow \arg[(x+3)+i(y+2)] = 3\pi/4$.

$$\frac{y+2}{x+3} = \tan(3\pi/4)$$

$\Rightarrow y = -x-5$.



Note:

The locus is a half-plane, this equation is restricted for $x<-3$, $y \geq -2$.

It follows that,
$\arg(z - z_1) = \theta$ is represented by a half-line from the fixed point z_1 making an angle θ with a line from fixed point z_1 parallel to the real axis.

(ii) Shade the region $|z-4-2i| \leq 2$.

$|z-4-2i| = 2$ give a circle with radius 2 centred at $(4,2)$. So $|z-4-2i| \leq 2$ represents the region on the inside of this circle.

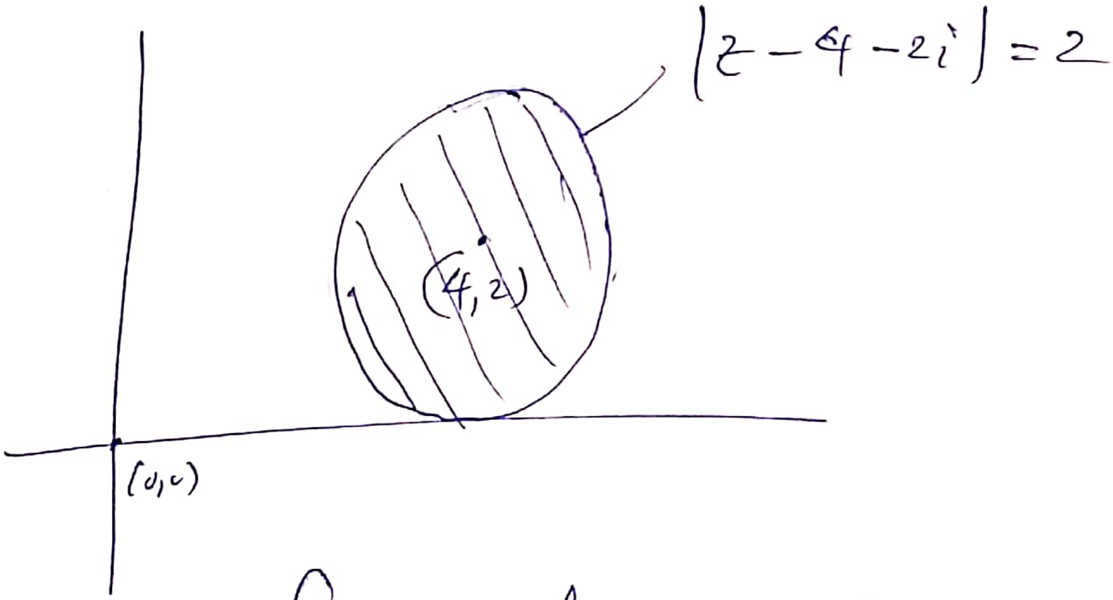
Or we can take a particular point on the complex plane and check whether it satisfies the inequality.

Ex take (0,0). Then

$|0-4-2i| \leq 2$

$\sqrt{4^2+2^2} \leq 2$

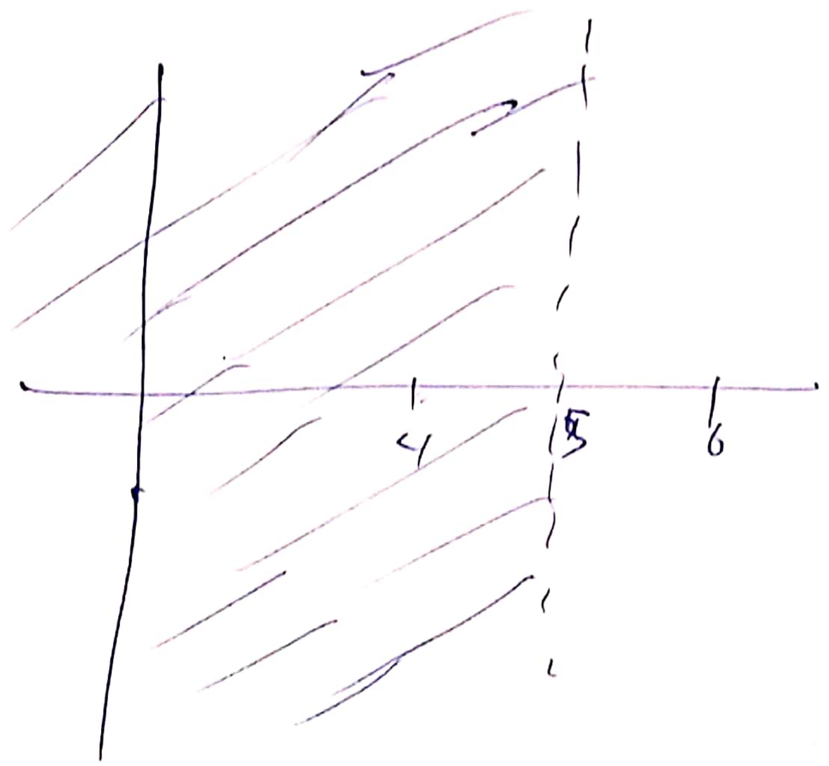
$\sqrt{20} \leq 2$ (this is not true).



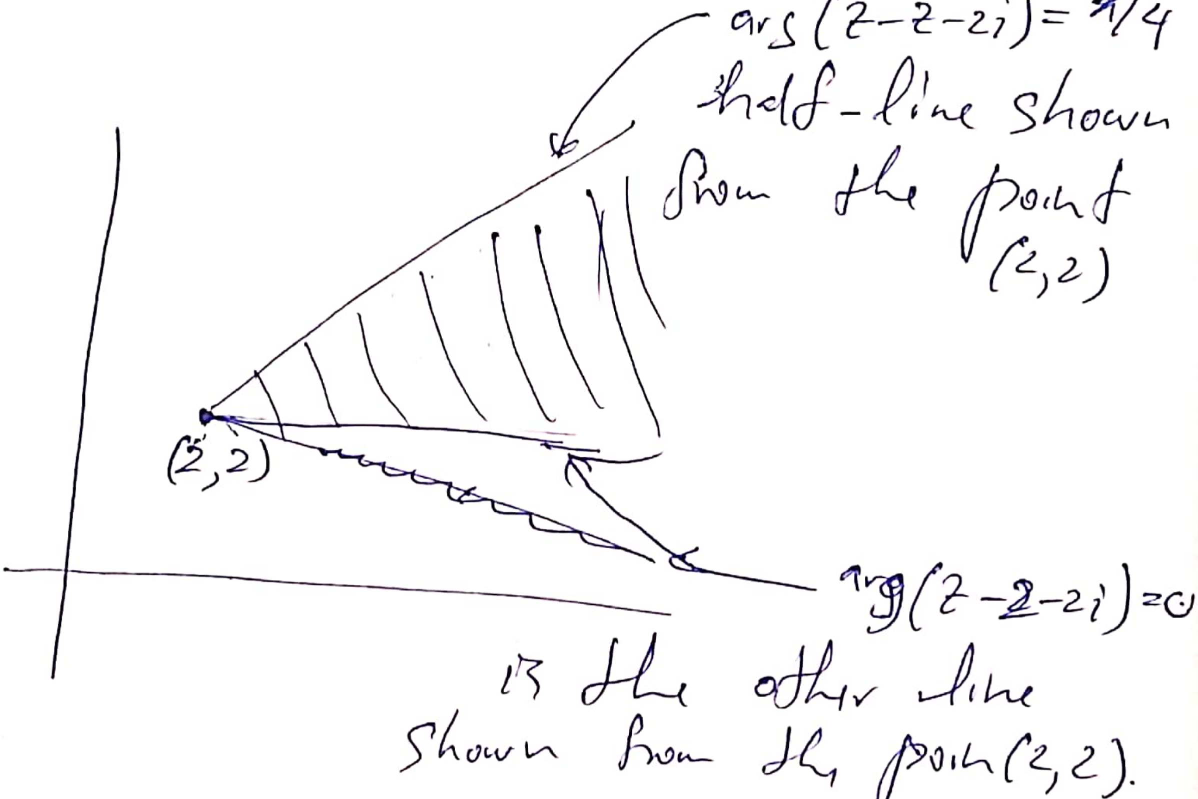
Therefore the point (0,0) not in the required region.

(f) $|z-4| < |z-6|$

$|z-4| = |z-6|$ is represented by the line $x=5$ which is the perpendicular bisector of the line segment joining $(4,0)$ to $(6,0)$. It is clear that the origin $(0,0)$ satisfies the inequality. Hence shade the side on the origin.



(vi) $0 \le \arg(z-2-2i) \le \pi/4$



$\therefore$ $0 \le \arg(z-2-2i) \le \pi/4$ is represented by the region in between and include these two half-lines.