

FUNDAMENTALS OF CIVIL ENGINEERING

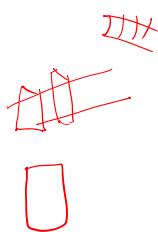
CE 1102

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Constitutive Laws of Materials

Some Common Engineering Materials

- Mild steel
- Tor steel
- Copper
- Brass
- Cement
- Rubber
- Aluminium



$$\sigma = F/A \quad \epsilon = \Delta/L$$

Constitutive Laws of Materials

- Material properties link stresses and strains
- The link between material properties and the stresses and strains are described in constitutive equations
- Material properties are determined through laboratory testing (eg: Tensile test, ~~Torsion test, Hardness Test~~)

Material Properties

- Some common material properties are,

Material Property	Notation	Units
Modulus of Elasticity (Young's Modulus or Elastic modulus)	E	Units of stress – Pa (N/mm ²), MPa, GPa
Poisson's Ratio	v	dimensionless
Modulus of rigidity (Shear Modulus)	G	Units of stress – Pa (N/mm ²), MPa, GPa
Bulk modulus	K	Units of stress – Pa (N/mm ²), MPa, GPa
Coefficient of thermal expansion	α	1/degree (1/°C)

Modulus of Elasticity (E)

- In 1676, Robert Hooke discovered the relationship between stress and strain which is named as **Hooke's Law**

$$\sigma = E\epsilon$$

Hand-drawn sketch of Hooke's Law showing stress (σ) proportional to strain (ϵ) with a spring diagram.

- The parameter which connects stress and strain in the above equation (E) is called Modulus of Elasticity, Young's Modulus or Elastic modulus (E)
- Modulus of Elasticity of a material can be found using the tensile test

Example 3.1

A metal wire is 2.5 mm in diameter and 2 m long. A force of 12 N is applied to it and it stretches 0.3 mm. Assume the material is elastic. Determine stress and the strain in the wire.

Also, find the Young's Modulus of the material.

Answer:

Stress = Force/ cross sectional area

Strain = change in length / initial length

E = stress/ strain = 16.2 GPa

$$\sigma = \frac{F}{A} = \frac{12 \text{ N}}{\pi (2.5)^2} =$$

$$\epsilon = \frac{\Delta L}{L} = \frac{0.3}{2}$$

$$\begin{aligned}\sigma &= 2.46 \text{ MPa} \\ \epsilon &= 0.15 \times 10^{-3} \\ \sigma &= E \epsilon \\ E &= \frac{\sigma}{\epsilon} \\ &= \frac{2.46}{0.15 \times 10^{-3}} \\ &= \end{aligned}$$

Example 3.2

A steel column is 3 m long and has a diameter of 0.4 m. It carries a load of 50 MN. Given that the modulus of elasticity is 200 GPa, calculate the compressive stress and strain of the column. Determine how much the column compressed.

Answer:

Stress = Load/ Area = 398 MPa

Strain = Stress / E = 0.002

Change of length = Strain x initial length = 6 mm

Example 3.3

A steel bar is stressed to 280 MPa. The modulus of elasticity is 205 GPa. The bar is 80 mm diameter and 240 mm long.

Determine the strain and the force applied to the bar.

Answer:

Strain = Stress/ E = 0.00136

Force = Stress x Cross sectional area = 1.407 MN

Example 3.4

A circular metal column is to support a load of 500 Tonnes and it must not compress more than 0.1 mm. The modulus of elasticity is 210 GPa. The column is 2 m long. Calculate the cross sectional area and the diameter.

Answer:

Load = 500 x 1000 x 9.81 N

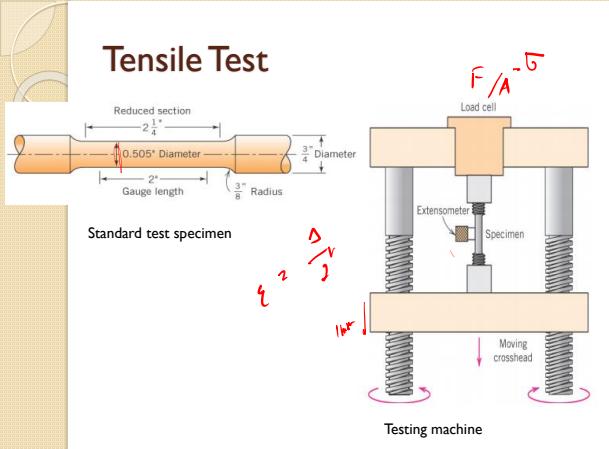
Strain = 0.1 mm / 2000 mm

Stress = E x Strain

Cross sectional area = Load / Stress = 0.467 m²

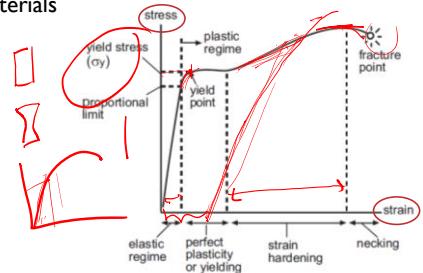
Diameter = 0.771 m

Tensile Test



Tensile Test

- Tensile test is used to get the stress strain curves for materials



Example stress strain curve

Stress Strain Diagram

- Four different regions in a stress strain curve
 - Elastic**
 - Stress is proportional to strain (linear)
 - Hooke's law is applicable
 - Material is elastic (i.e. deformations are recoverable)
 - Yielding**
 - Plastic deformations start to occur (deformations are not fully recoverable)
 - Large strains occur for small increment of stress

Stress Strain Diagram (contd.)

- Hardening**
 - Unloading result in an elastic curve (stress is linearly proportional to the strain and has the same slope as the elastic portion of the stress strain diagram)
 - Repeat in load follow the same unloading curve but will reach a higher yielding point indicating that the strain hardening improves the yield strength
- Necking**
 - Rapid reduction of cross section occurs over a small length just before fracture



Tensile Test (Contd.)

- Some important points in the stress strain curve,
 - Proportional limit – the limit at which the linear nature of the graph ceases
 - Yield stress – also called the elastic limit, at which the elastic nature of the material ceases
(In most cases it is assumed that proportional limit and the elastic limit are the same)
 - Ultimate stress – beyond which the necking occurs
 - Fracture stress – at which the specimen fails. This has no practical value

Conventional(Engineering) Vs True Stress Strain Diagrams

Engineering strain = change in length/ original length

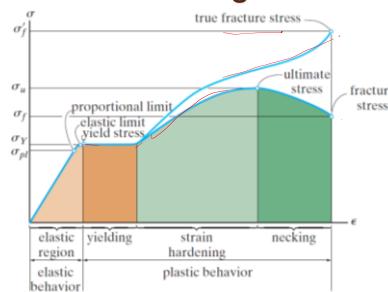
Engineering stress = Load/ original cross sectional area

True strain = change in length/ length at that time

True stress = Load/ cross sectional area at that time

As Engineers, we use the **Engineering** stress & strain definitions

Conventional (Engineering) Vs True Stress Strain Diagrams



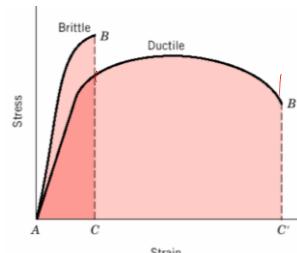
Stress strain curves for two definitions are slightly different

Ductile Vs Brittle Materials

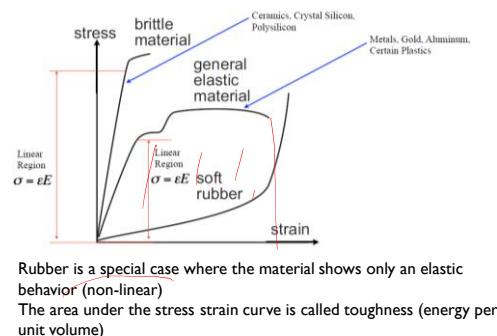
- Depending on the ability to undergo plastic deformation before the fracture, materials can be classified into two types as brittle and ductile
- Brittle materials do not have yielding

Brittle
glass, concrete,
cast iron

Ductile
aluminium, steel,
copper



Stress Strain Curves

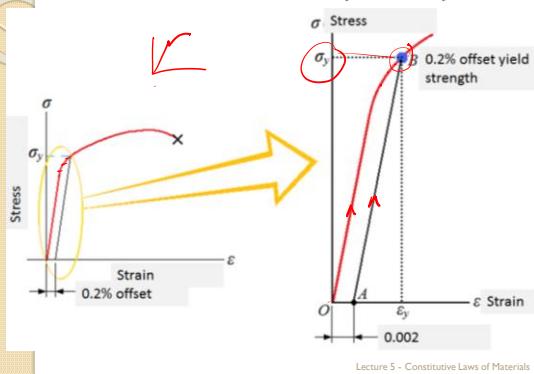


Offset Yield Point

- There are some materials for which yield point cannot be identified clearly from the stress strain diagrams
- In such cases offset yield point will be used to define the yield strength
- Usually this yield strength is measured at 0.1% or 0.2% offset

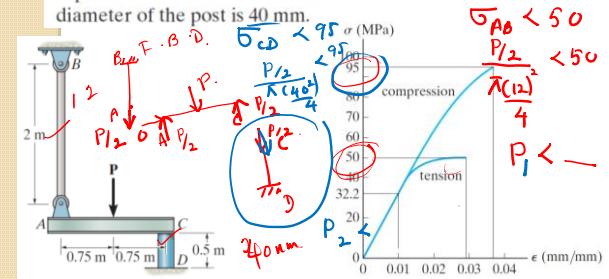
Lecture 5 - Constitutive Laws of Materials

Offset Yield Point (contd.)



Example 5.11

The stress-strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD made from this material, determine the largest load P that can be applied to the beam before it ruptures. The diameter of the strut is 12 mm and the diameter of the post is 40 mm.



Answer

FBD of the beam :
 $R_1 = R_2 = P/2$

FBD of AB
 R_1

FBD of CD
 R_1

AB under tension, CD under compression

Max² load for AB = $50 \text{ MPa} \times \pi (0.006)^2$
 $R_1 = P/2 < 50 \times 10^6 \times \pi (0.006)^2$
 $P < 11.309 \text{ kN}$

Max² load for CD = $95 \times 10^6 \times \pi (0.02)^2$
 $R_2 = \frac{P}{2} < 95 \times 10^6 \times \pi (0.02)^2$
 $P < 238.76 \text{ kN}$

Largest load P = 11.309 kN before rupture

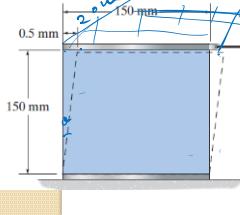
Shear Modulus G

- Hooke's Law is applicable to shear stress and strain as well. $\tau = G\gamma$
- The proportional constant between shear stress and shear strain is called "Shear Modulus" and denoted by "G"
- G is also called "Modulus of Rigidity"
- G has the same units as shear stress
- Torsion test can be used to find the shear modulus of a material

Example

(Mechanics of Materials, RC Hibbeler)

A 20 mm wide block is firmly bonded to rigid plates at its top and bottom. When the force P is applied the block deforms to the shape shown by the dashed line. Determine the magnitude of P . Shear modulus of the material is 26 GPa.



Answer:

$$\text{Shear strain} = \text{change of angle} = \theta$$

$$\theta = \tan^{-1}\left(\frac{0.5}{150}\right) \text{ radian}$$

$$\text{Shear stress} = \text{shear strain} \times G$$

$$\text{Area} = 150 \times 20 \text{ mm}^2$$

$$\text{Force} = \text{stress} \times \text{Area} = 260 \text{ kN}$$

Poisson's Ratio (ν)

When a strain is applied to a body in one direction (axial strain), that body will strain in other two directions (lateral strain). This is called Poisson's effect.



Poisson's Ratio (contd.)

- In 1800s, French Scientist Poisson discovered that the ratio between lateral strain and axial strain is a constant within the elastic range.



- This constant is called Poisson's ratio.

$$\text{Poisson's Ratio} = -\frac{\text{(Lateral strain)}}{\text{(Axial strain)}}$$

$$= -\frac{(-0.1)}{0.2} = (+0.5)$$

Negative sign is used to keep Poisson's Ratio positive, because elongation in axial direction causes contraction in lateral direction and vice versa.

Poisson's Ratio (Contd.)

- For most engineering materials, Poisson's ratio is within 0.25 – 0.33
- The maximum value of Poisson's ratio for engineering materials is 0.5 which is the value for rubber
- Due to the Poisson's effect, strain in one direction is contributed by both the stress in that direction and in lateral directions.

Relationship between the Material Constants

$$E = 2G(1+\nu)$$



$$G = E/(2(1+\nu)), \quad \nu = E/(2G) - 1$$

$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} \quad \nu = E \varepsilon \quad \Delta z, \Delta y$

Example 3.5 $0.3 = -\frac{\varepsilon L}{8 \times 10^{-5}}$

A bar made of steel has dimensions as shown below. Assuming that the material behaves elastically, find the change in length and in the dimensions of cross section after applying the load as shown. Young's modulus and Poisson's ratio of steel can be taken as 200 GPa and 0.3 respectively.

Given: $E = 200 \times 10^9 \text{ N/mm}^2$, $\nu = 0.3$, $P = 80 \text{ kN}$, $L = 1.5 \text{ m}$, $b = 50 \text{ mm}$, $t = 100 \text{ mm}$.

Calculation:

$$\nu = -\frac{\Delta z}{\varepsilon_{\text{axial}}} \quad \Delta z = -\nu \varepsilon_{\text{axial}} L$$

$$\varepsilon_{\text{axial}} = \frac{P}{E b t} = \frac{80 \times 10^3}{200 \times 10^9 \times 50 \times 100} = 8 \times 10^{-5} \text{ mm/mm}$$

$$\Delta z = -0.3 \times 8 \times 10^{-5} \times 1500 = -1.2 \times 10^{-3} \text{ m} = -1.2 \text{ mm}$$

$$\Delta y = -2.4 \times 10^{-5} \times 1500 = -3.6 \times 10^{-3} \text{ m} = -3.6 \text{ mm}$$

$$\Delta z = -2.4 \times 10^{-5} \times 1500 = -3.6 \times 10^{-3} \text{ m} = -3.6 \text{ mm}$$

Answer

$$\epsilon_{\text{Axial}} = \frac{\Delta z}{105} = \frac{\epsilon}{E} = \frac{F \cdot \frac{1}{A}}{E} = \frac{F \cdot \frac{1}{200 \times 10^{-3}}}{200 \times 10^9} = 1.5 \times 10^{-5}$$

$$\Delta z = \frac{80 \times 10^3}{0.1 \times 0.05} \times \frac{1}{200 \times 10^9} \times 1.5 = 0.12 \text{ mm} = \text{change in length}$$

$$\epsilon_{\text{lateral}} = -0.3 \times \epsilon_{\text{Axial}} = -0.3 \times 1.5 \times 10^{-5} = -4.5 \times 10^{-6}$$

$$\epsilon_{\text{lateral}} = \frac{\Delta x}{0.1} = \frac{\Delta y}{0.05}$$

$$\Delta x = -2.4 \times 10^{-3} \text{ mm } \left. \begin{array}{l} \text{dimensions of} \\ \text{the cross section} \end{array} \right\}$$

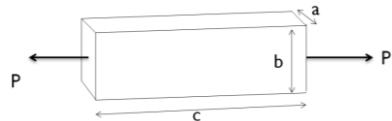
$$\Delta y = -1.2 \times 10^{-3} \text{ mm } \left. \begin{array}{l} \text{dimensions of} \\ \text{the cross section} \end{array} \right\}$$

Example 3.6

The box given in figure below is made of a material with Young's modulus E and Poisson's ratio ν . An axial load of P is applied to the box as shown.

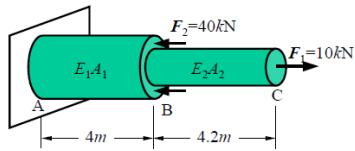
Determine the new volume of the box under the applied load.

Prove that the new volume (after applying the loads) and the previous volume (before applying the loads) is not equal.



Example 3.7

The composite bar shown in the figure is made of two segments, AB and BC having cross sectional areas of 200 and 100 mm^2 respectively. Their Young's moduli are 100 and 210 GPa respectively. Find the total displacement at the right end.



Assignment 2

- Depending on the geometry, connections and load transfer mechanism, structures can be divided into several types. Name such types of structures.
- Give two famous examples for each type of structures.
- Write a brief description (about 100 words) for two of the examples mentioned above.

Assignment (contd.)

Note

- Submit on or before 17th June 2022 to Instructor in Civil Engineering Department (3rd floor).
- 50% of the marks will be deducted for late submissions without any valid reason.
- No cover page is needed. Mention your name and index number in the top right hand corner. Mention the heading as "Structural Engineering Assignment 2"