

Mathematics - MA 1103
Tutorial 01 - Vectors

Year: 2021

Intake: 38 - 03rd Batch

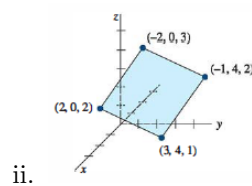
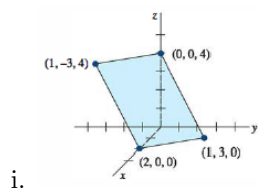
Semester: 01

Learning Outcomes Covered: LO2

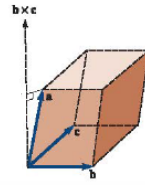
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- The points A and B have coordinates $(2, 4, 1)$ and $(3, 2, -1)$ respectively. The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OB}$, where O is the origin.
 - Find the vectors:
 - \overrightarrow{OC}
 - \overrightarrow{AB}
 - Show that the distance between the points A and C is 5
 - Find the size of $\angle BAC$, giving your answer to the nearest degree
 - The point $P(\alpha, \beta, \gamma)$ is such that BP is perpendicular to AC . Show that $4\alpha - 3\gamma = 15$.
 - The points A, B and C have coordinates $(3, -2, 4)$, $(5, 4, 0)$, and $(11, 6, -4)$ respectively.
 - Find the vector \overrightarrow{BA}
 - Show that the size of $\angle ABC$ is $\cos^{-1}(-\frac{5}{7})$
 - The line l has the equation $\overrightarrow{r}(t) = \langle 8, -3, 2 \rangle + t\langle 1, 3, -2 \rangle$
 - Verify that C lies on l .
 - Show that AB is parallel to l .
 - The Quadrilateral $ABCD$ is a parallelogram. Find the coordinates of D .
 - The quadrilateral $ABCD$ has vertices $A(2, 1, 3)$, $B(6, 5, 3)$, $C(6, 1, -1)$, and $D(2, -3, -1)$. The line l_1 has the vector equation $\overrightarrow{r}(t) = \langle 6, 1, -1 \rangle + t\langle 1, 1, 0 \rangle$.
 - Find the vector \overrightarrow{AB}
 - Show that AB is parallel to l_1 .
 - Verify that D lies on l_1 .
 - The line l_2 passes through $D(2, -3, -1)$, and $M(4, 1, 1)$.
 - Find the vector equation of l_2 .
 - Find the angle between l_2 and AC .

4. (a) Let
- $$l_1 : \overrightarrow{r(t_1)} = \langle -1, 1, 1 \rangle + t_1 \langle 2, -2, 4 \rangle$$
- $$l_2 : \overrightarrow{r(t_2)} = \langle 1, 0, 3 \rangle + t_2 \langle -1, 1, -2 \rangle$$
- $$l_3 : \overrightarrow{r(t_3)} = \langle 1, -1, 4 \rangle + t_3 \langle 2, -1, 3 \rangle$$
- Show that l_2 and l_3 intersect and find the angle between them.
 - Show that l_1 and l_3 are skew (neither parallel nor intersecting).
- (b) The lines l_1 and l_2 have equations $\overrightarrow{r(t)} = \langle 8, 6, -9 \rangle + t \langle 3, -3, -1 \rangle$ and $\overrightarrow{r(\lambda)} = \langle -4, 0, 11 \rangle + \lambda \langle 1, 2, -3 \rangle$ respectively.
- Show that l_1 and l_2 are perpendicular.
 - Show that l_1 and l_2 intersect and find the coordinates of the point of intersection P .
 - The point $A(-4, 0, 11)$ lies on l_2 . The point B on l_1 is such that $AP = BP$. Find the length of AB .
5. (a) Find the Cartesian equation of the plane passing through the points P, Q , and R below:
- $P(1, -1, 4)$, $Q(2, 7, -1)$, and $R(5, 0, -1)$
 - $P(1, 0, 2)$, $Q(-3, 5, 0)$, and $R(6, -4, 2)$
- (b) Find the point of intersection and the angle between the following line(l) and the plane(p).
- $l : x = -1 - 2t; \quad y = 5; \quad z = 1 + t$
 $p : 4x + z - 2 = 0$
 - $l : \overrightarrow{r(t)} = \langle 1, -2, -1 \rangle + t \langle 4, 5, 6 \rangle$
 $p : x + 2y + 3z = 5$
6. (a) Find the line of intersection and acute angle between the following planes.
- $\vec{r} \cdot \langle 3, 0, -1 \rangle = 2$ and $\vec{r} \cdot \langle 1, 2, 5 \rangle = -1$
 - $3x - y + 2z - 4 = 0$ and $2x - y + 4z - 3 = 0$
- (b)
 - Find the distance of the plane $3x + 2y - 4z = 8$ from the Origin and the unit vector perpendicular to the plane.
 - Find the perpendicular distance from the point $P(3, 5, 2)$ to the plane $3x - 2y + z = 4$.
7. (a) Find the equation of the line that passes through the point $(-2, 3, 1)$ and is perpendicular to the plane $2x + 3y + z - 3 = 0$ also find the point of intersection and the distance from the point to the plane.
- (b) Convert the Cartesian equation of the plane $x - 2y + 2z = 5$ into
- a vector equation of the form $\vec{r} \cdot \hat{n} = D$, where \hat{n} is a unit vector.
 - vector form
 - State the perpendicular distance of the plane from the origin.
8. (a) Find the area of the triangle determined by the points P_1, P_2 and P_3 below:
- $P_1(1, 1, 1)$, $P_2(2, 3, 4)$, and $P_3(3, 0, -1)$
 - $P_1(0, 0, 0)$, $P_2(0, 1, 2)$, and $P_3(2, 2, 0)$
- (b) Verify that the following quadrilateral is a parallelogram, and then find the area of the parallelogram.



(c) Consider the parallelepiped given below:



Find the volume of the parallelepiped when \vec{a} , \vec{b} , and \vec{c} are given as follows:

- i. $\vec{a} = \langle 7, 1, 9 \rangle$, $\vec{b} = \langle 1, 2, 3 \rangle$, and $\vec{c} = \langle 3, 0, 6 \rangle$
- ii. $\vec{a} = \langle 3, 4, 5 \rangle$, $\vec{b} = \langle 5, -1, 1 \rangle$, and $\vec{c} = \langle -2, 3, 4 \rangle$