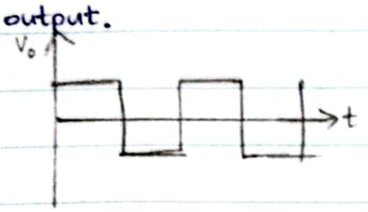
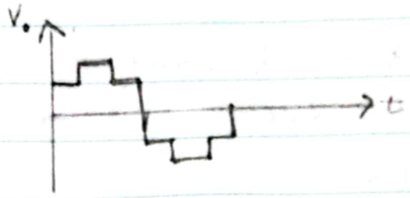
Squre-wave DC to AC Inverters.

(a) Introduction

Squre-wave inverters produce squre-wave AC voltage as the output.





There are many applications that accept squre-wave voltage eg: lighting, heating, motor control etc.

SQ.wave inverters are energy efficient as their switching losses are lower due to the lower swiching frequency used. Often, the switching frequency is same as the fundamental frequency of V_o.

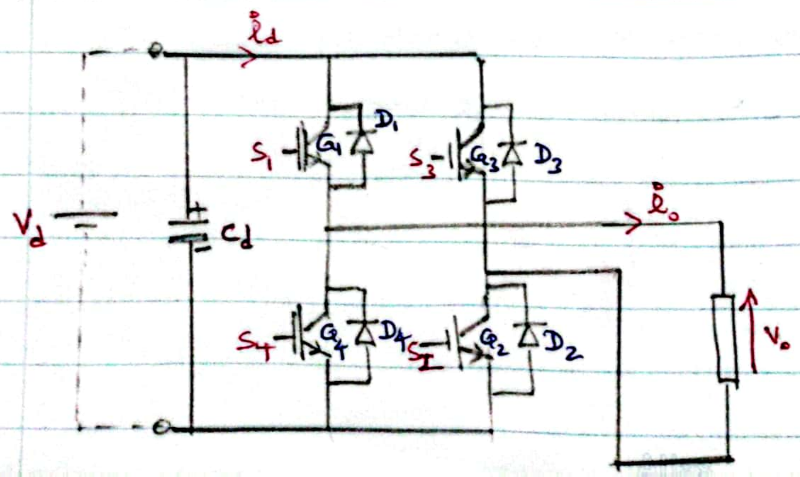
We have two basic categories.

i) 2-level inverters
ii) Multilevel inverter

The 2-level inverter is the conventional or classical inverter. Multilevel inverter is the new arrival and that provides ~~that~~ ~~p~~ better performances.

(a) 2-level SQ-Wave inverter

i) 1-Phase inverter.:



Modern inverters use "self switching" power semiconductor devices, such as IGBTs and MOSFETs. Thyristor option has now gone to the history.

$G_1 \sim G_4$: IGBT (Insulated gate bipolar transistor) switches

$D_1 \sim D_4$: Fast-recovery power diodes

$S_1 \sim S_4$: binary ~~on/~~ ON/OFF Switching signal output

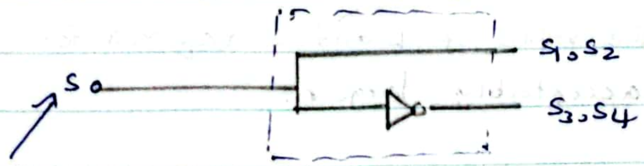
C_d : DC side capacitors

Note : ~~Switching signal S is~~ Each IGBT is provided with a "gate drive circuit", that connects the switching signal input S to the IGBT gate G. This circuit provides the necessary isolation and IGBT compatible gate voltage to the device. The IGBT and its companion diode are non-separable. The IGBT facilitates conductive positive current and the diode, negative current. Whichever the one conducts, the voltage across the branch is zero. Also, each switch is supported with appropriate protection against abnormal conditions.

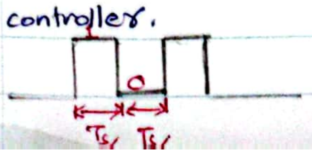
To obtain ~~the st~~ 1 ph. output, we use the following switching sequence.

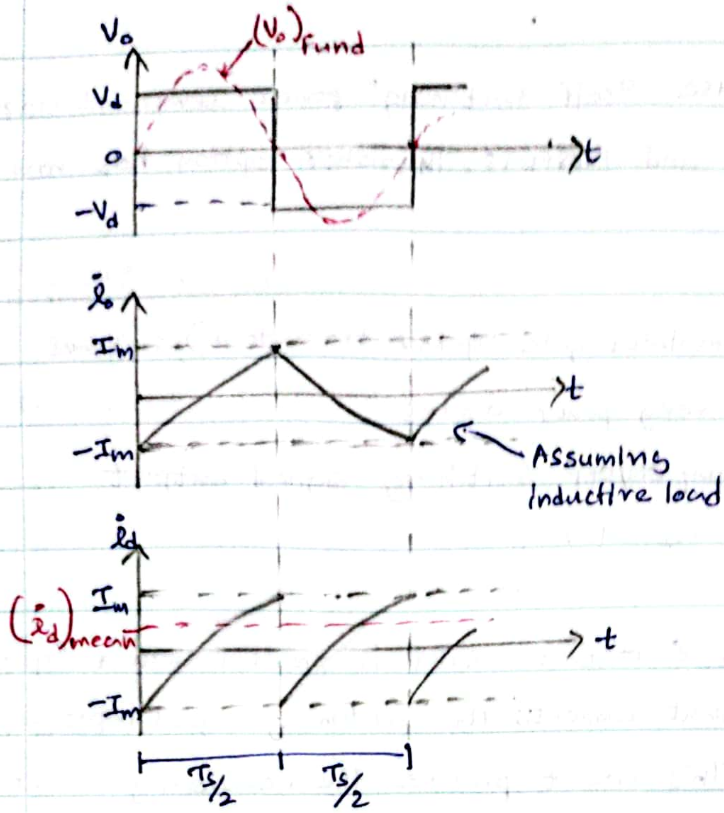
G_1 & G_2 ON	G_3 & G_4 ON	G_1 & Q_2 ON	...
$T_s/2$	$T_s/2$		

T_s = switching cycle time



Main switching signal generated by the inverter controller.





steady state operating waveforms

$$i_d = \begin{cases} i_o, & \text{if } s=1 \\ -i_o, & \text{if } s=0 \end{cases}$$

- We observe that i_d goes negative for brief intervals during the cycle. Input capacitor C_d absorbs this negative current, and this C_d is a must especially when V_d is obtained with a diode or thyristor converter (which cannot carry reverse current)

- V_o from a sq-wave inverter is used as it is without filtering it to separate the fundamental. On the one hand, the filtering attenuates the fundamental itself heavily and on the otherhand the value of L and C required for the filtering are unacceptably larger.

mathematically,

$$V_o = \frac{4V_d}{\pi}\left(\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\cdot\sin(5\omega t) + \right.$$

$$(V_{o,fund})_{rms} = \frac{\left(\frac{4V_d}{\pi}\right)}{\sqrt{2}} = \frac{\sqrt{8}\,V_d}{\pi}$$

order of harmonics

$$\left(V_{o,nth}\right)_{rms} = \frac{\left(V_{o,Fund}\right)_{rms}}{n}$$

Ex: A square-wave, 1 ph. inverter serves an R-L load with $R = 12\Omega$ and $L = 96\ mH$. Input DC voltage is $350\ V$. Inverter switching frequency is $50\ Hz$

Determine,

i) RMS values of the fundemental, and the three lowest order harmonics of the output voltage.

ii) RMS values of the fundemental and the three lowest order harmonics of the load current.

iii) Peak value of current through an IGBT and a Diode

iv) Mean value of input DC current.

ANS :

$R = 12\Omega$

$L = 96\ mH$

$V_d = 350\ V$

$f_s = 50\ Hz$

i) $\left(V_{o,fund}\right)_{rms} = \frac{\sqrt{8}\,V_d}{\pi} = \frac{\sqrt{8}\times 350}{\pi}\ V = 315.1\ V$

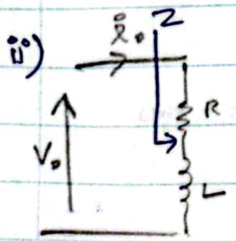
Three lowest order harmonics are $= 3^{rd}, 5^{th}, 7^{th}$

$$\left(V_{o,3rd}\right)_{rms} = \frac{315.1}{3}\ V = 105.0\ V$$

$$\left(V_{o,5th}\right)_{rms} = \frac{315.1\ V}{5} = 63.0\ V$$

$$\left(V_{o,7th}\right)_{rms} = \frac{315.1\ V}{7} = 45.0\ V$$

ii)



$\omega \equiv \omega_s = 2\pi f_s = 2\pi\times 50 = 100\pi\ rad/s$

$R = 12\Omega$

$L = 0.096\ H$

$$Z = \begin{cases} \sqrt{R^2+\omega^2L^2} = 32.46\,\Omega, \text{ for fundamental} \\ \sqrt{R^2+(3\omega)^2L^2} = 91.27\,\Omega, \text{ for } 3^{rd} \text{ harmonic} \\ \sqrt{R^2+(5\omega)^2L^2} = 151.27\,\Omega, \text{ for } 5^{th} \text{ harmonic} \\ \sqrt{R^2+(7\omega)^2L^2} = 211.46\,\Omega, \text{ for } 7^{th} \text{ harmonic} \end{cases}$$

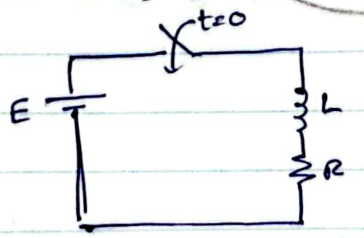
$$\therefore (i_{o,fund})_{rms} = \frac{315.1}{32.46} \text{ A} = 9.71 \text{ A}$$

$$(i_{o,3^{rd}})_{rms} = \frac{105.0}{91.27} \text{ A} = 1.15 \text{ A}$$

$$(i_{o,5^{th}})_{rms} = \frac{63.0}{151.27} \text{ A} = 0.42 \text{ A}$$

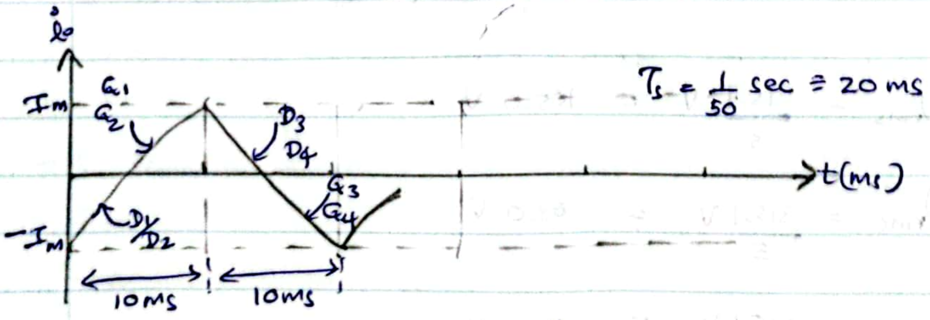
$$(i_{o,7^{th}})_{rms} = \frac{45.0}{211.46} \text{ A} = 0.21 \text{ A}$$

iii) Inductive charging



$\tau = \frac{L}{R}$ = time constant

$$i(t) = \frac{E}{R}\left(1 - e^{-t/\tau}\right) + i(0)\cdot e^{-t/\tau}$$



$T_s = \frac{1}{50}$ sec = 20 ms

Using standard inductor-charging current equation on step-voltage input,

$$I_m = \frac{V_d}{R}\left(1 - e^{-\frac{T_s/2}{\tau}}\right) + (-I_m)e^{-\frac{T_s/2}{\tau}}; \quad \tau = \frac{L}{R} = \frac{96}{12} \text{ ms} = 8\text{ms}$$

$$I_m = \frac{350}{12}\left(1 - e^{-10/8}\right) - I_m e^{-10/8}$$

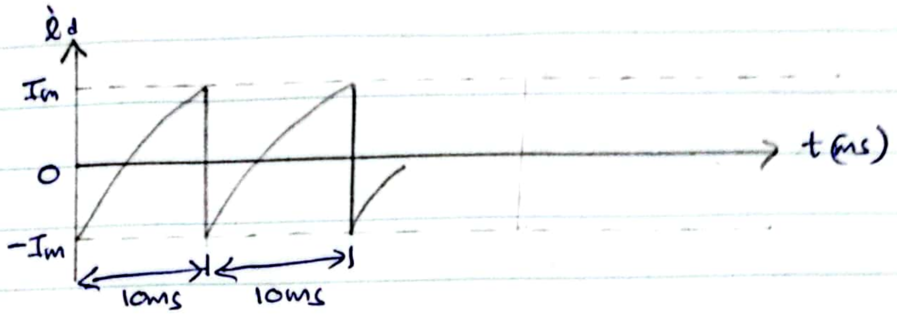
$$I_m = \frac{350/12\left(1 - e^{-10/8}\right)}{\left(1 + e^{-10/8}\right)} \text{ A} = 16.17 \text{ A}$$

No : ....   Date: ..../..../....

$\therefore$ Peak current carried by an IGBT $\equiv I_m = 16.17\ A$

$\therefore$ Peak current carried by a diode $\equiv I_m\ 16.17\ A$

iv)



For the first 10ms interval,

in ms

$$i_d = \frac{350}{12}\left(1 - e^{-t/8}\right) - 16.7 \cdot e^{-t/8}\ A$$

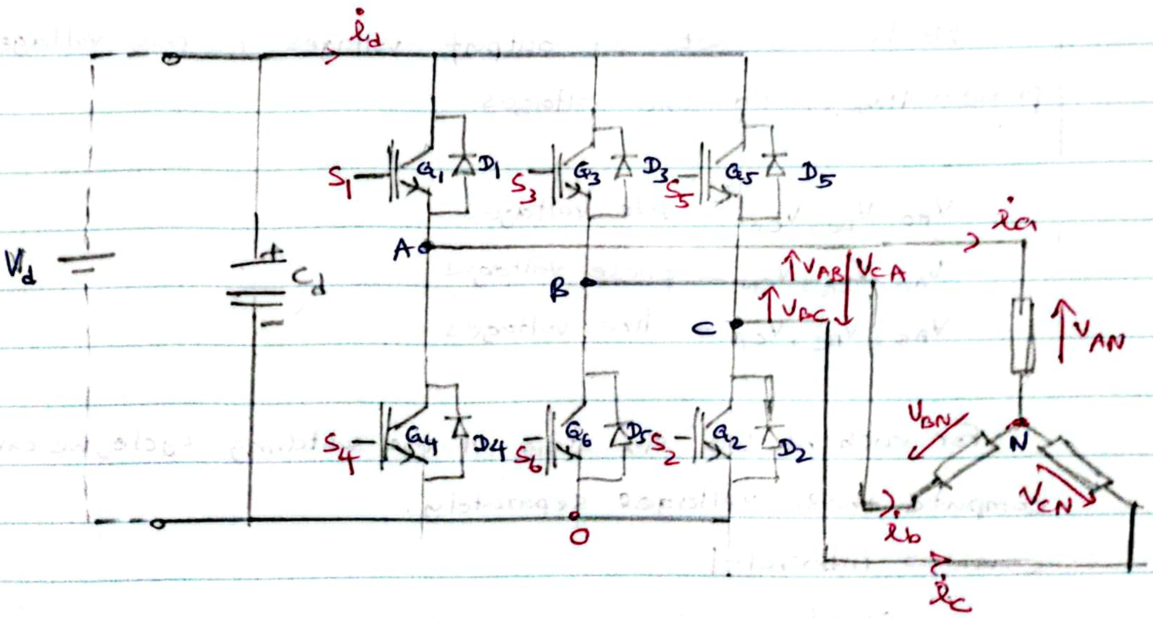
$$= \left(29.17 - 45.34\, e^{-t/8}\right)\ A$$

$$(i_d)_{mean} = \frac{\int_0^{10} i_d\, dt}{10} = \frac{1}{10}\int_0^{10} 29.17 - 45.34 \cdot e^{-t/8} \cdot dt$$

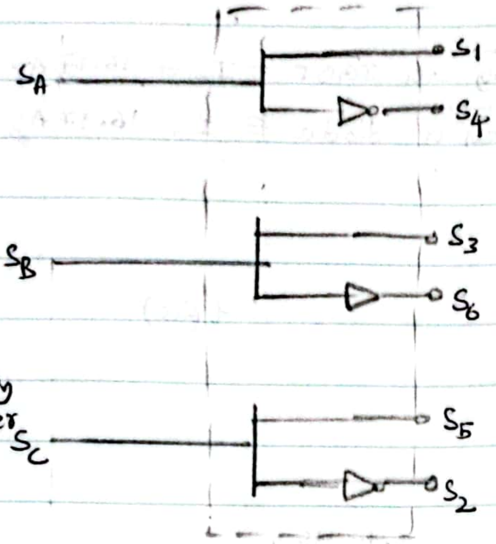
$$= \frac{1}{10}\left(29.17 \times 10 + 45.34 \times 8\left(e^{-10/8} - e^{0}\right)\right)\ A$$

$$= 3.29\ A$$

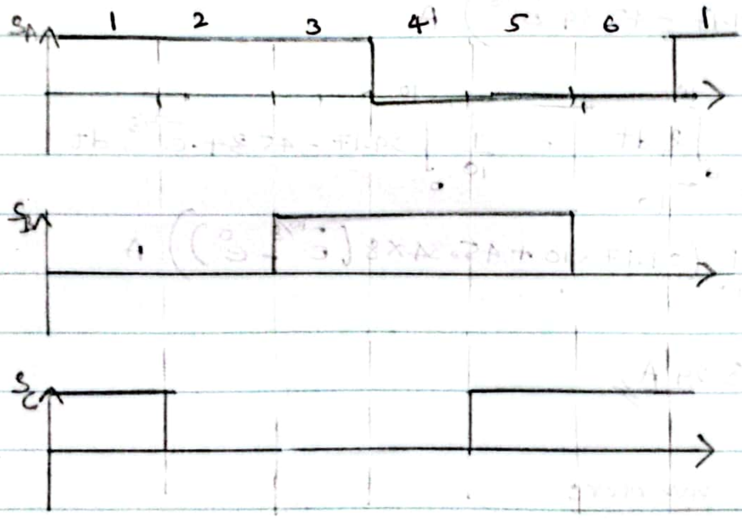
(ii) 3 phase inverter:



Three main switching signals generated by the inverter controller



We use the following S_A, S_B, S_C to produce balanced, 3 phase squre-wave voltage output.



Each interval is $T/6$

We have 3 sets of output voltages, i.e. pole voltages, phase voltages, and line Voltages

V_{AO}, V_{BO}, V_{CO} = pole voltages

V_{AN}, V_{BN}, V_{CN} = phase voltages

V_{AB}, V_{BC}, V_{CA} = line Voltages

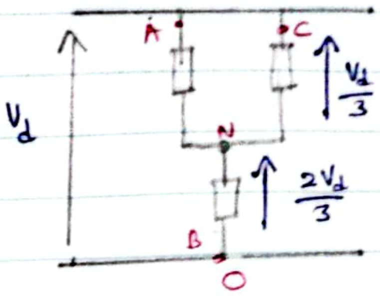
For each of the six steps of the switching cycle, we can compute these voltages separately.

Consider interval-1

$S_A = 1$

$S_B = 0$

$S_C = 1$



$\therefore V_{AO} = V_d$ $\quad V_{AN} = V_d/3$ $\quad V_{AB} = V_d$

$V_{BO} = 0$ $\quad V_{BN} = \frac{-2V_d}{3}$ $\quad V_{BC} = -V_d$

$V_{CO} = V_d$ $\quad V_{CA} = 0$

$V_{CN} = \frac{V_d}{3}$

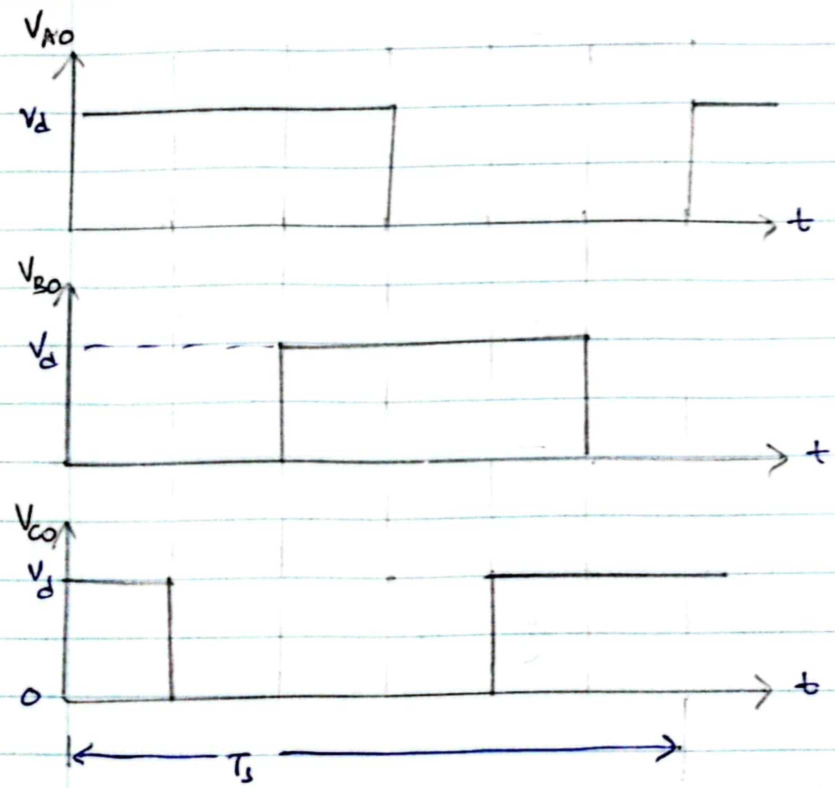
Equivalant circuit during interval-1

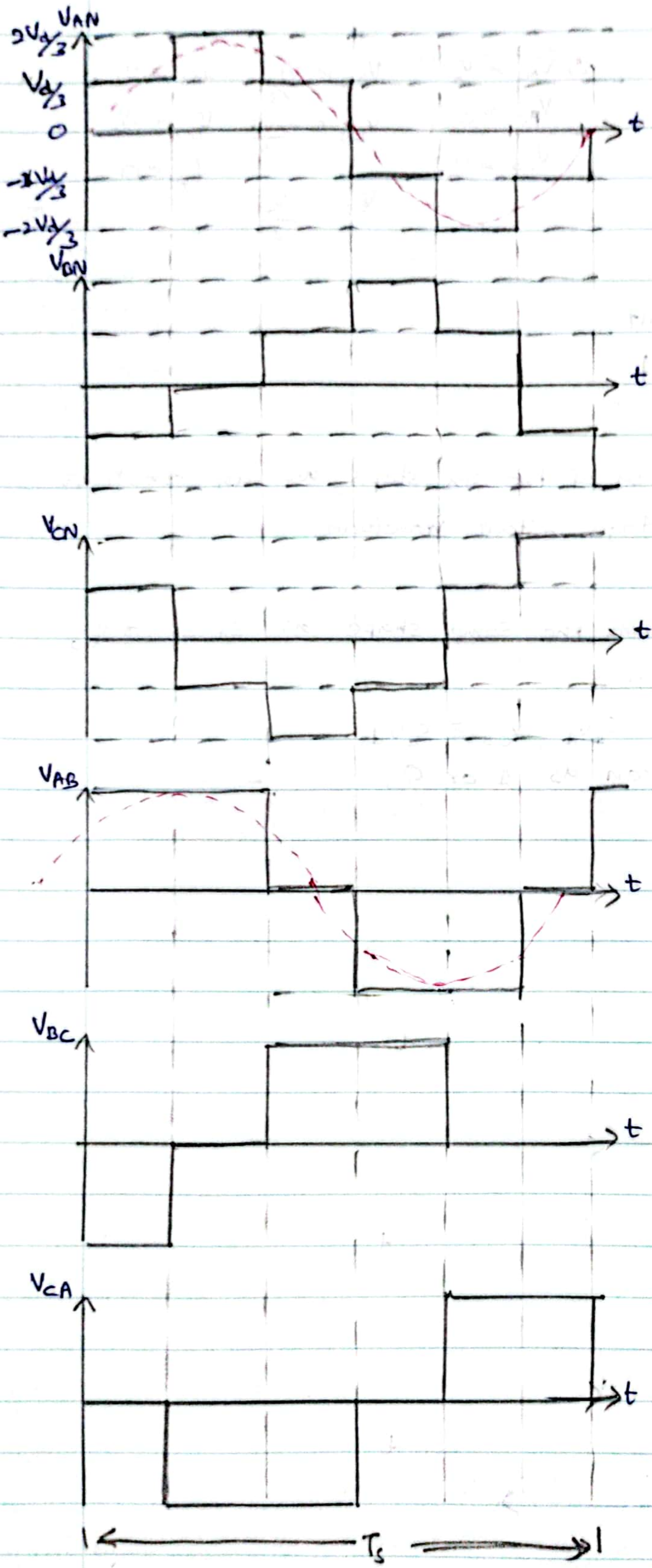
· After computing for each of the six steps, we can construct pole, phase, and line voltage output waveforms.

Note : V_{AO}, V_{BO} + V_{CO} have the same shape as S_A, S_B, + S_C, respectively, because

$V_{AO} \equiv S_A V_d$, $V_{BO} \equiv S_B V_d$, $V_{CO} \equiv S_C V_d$

when s is taken as 1 or 0.





VAN
2Vd/3
Vd/3
0
-Vd/3
-2Vd/3
t
VBN
t
VCN
t
VAB
t
VBC
t
VCA
t
Ts

Mathematically,

$$V_{AN} = \frac{2V_d}{\pi}\left(\sin(\omega t) + \frac{1}{5}\sin(5\omega t) + \frac{1}{7}\sin(7\omega t) + \ldots\right)$$

$$V_{AB} = \frac{2\sqrt{3}V_d}{\pi}\left(\sin(\omega t+30^\circ) - \frac{1}{5}\sin 5(\omega t+30^\circ) - \frac{1}{7}\sin(\omega t+30^\circ) + \frac{1}{11}\sin(\omega t+30^\circ) + \ldots\right)$$

$$\therefore (V_{AN,fund})_{rms} = \frac{\left(2V_d/\pi\right)}{\sqrt{2}} = \frac{\sqrt{2}V_d}{\pi}$$

$$\therefore (V_{AB,fund})_{rms} = \frac{\left(\frac{2\sqrt{3}V_d}{\pi}\right)}{\sqrt{2}} = \frac{\sqrt{6}V_d}{\pi}$$

Order of harmonics in eithe V_{AN} or V_{AB} is $(6k \pm 1)$, $k = 1, 2, 3 \ldots$

$$(V_{AN,n^{th}})_{rms} = \frac{(V_{AN,fund})_{rms}}{n}$$

$$(V_{AB,n^{th}})_{rms} = \frac{(V_{AB,fund})_{rms}}{n}$$

Further,

$$V_{AO} = S_A V_d$$
$$V_{BO} = S_B V_d$$
$$V_{CO} = S_C V_d$$

$$\begin{bmatrix} V_{AO} \\ V_{BO} \\ V_{CO} \end{bmatrix} = V_d \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_A \\ S_B \\ S_C \end{bmatrix}$$

$$V_{AB} = V_{AO} - V_{BO}$$
$$V_{BC} = V_{BO} - V_{CO}$$
$$V_{CA} = V_{CO} - V_{AO}$$

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = V_d \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_A \\ S_B \\ S_C \end{bmatrix}$$

$$V_{AN} = V_{AO} - V_{NO}$$
$$V_{BN} = V_{BO} - V_{NO}$$
$$V_{CN} = V_{CO} - V_{NO}$$

$$\therefore (V_{AN} + V_{BN} + V_{CN}) = (V_{AO} + V_{BO} + V_{CO}) - 3V_{NO}$$

$$\therefore V_{NO} = \frac{1}{3}(V_{AO} + V_{BO} + V_{CO})$$

$$\therefore V_{AN} = V_{AO} - \frac{1}{3}(V_{AO} + V_{BO} + V_{CO}) \equiv \frac{1}{3}(2V_{AO} - V_{BO} - V_{CO})$$

similarly

$$V_{BN} = \frac{1}{3}(-V_{AO} + 2V_{BO} - V_{CO})$$

$$V_{CN} = \frac{1}{3}(-V_{AO} - V_{BO} + 2V_{CO})$$

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \frac{V_d}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} S_A \\ S_B \\ S_C \end{bmatrix}$$