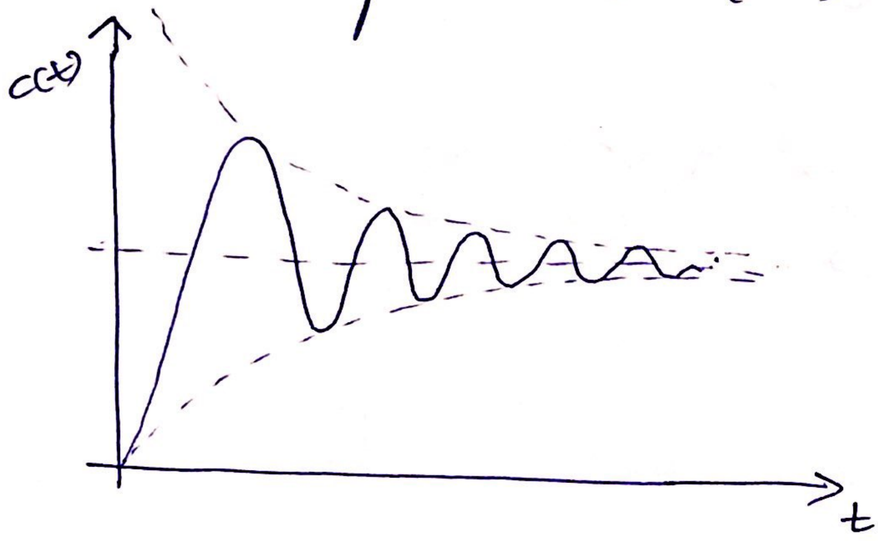
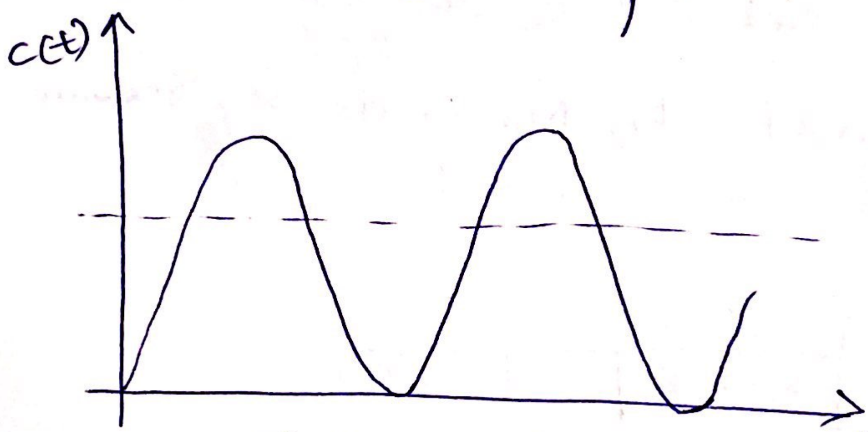
(a) Underdamped Case ($0 < \xi < 1$)

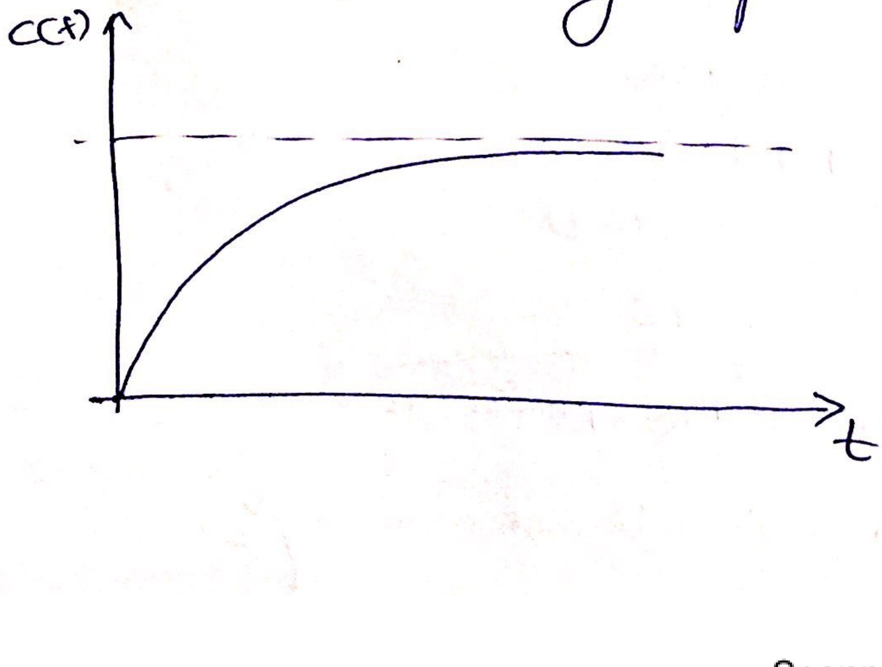


(b) when $\xi = 0$, undamped.

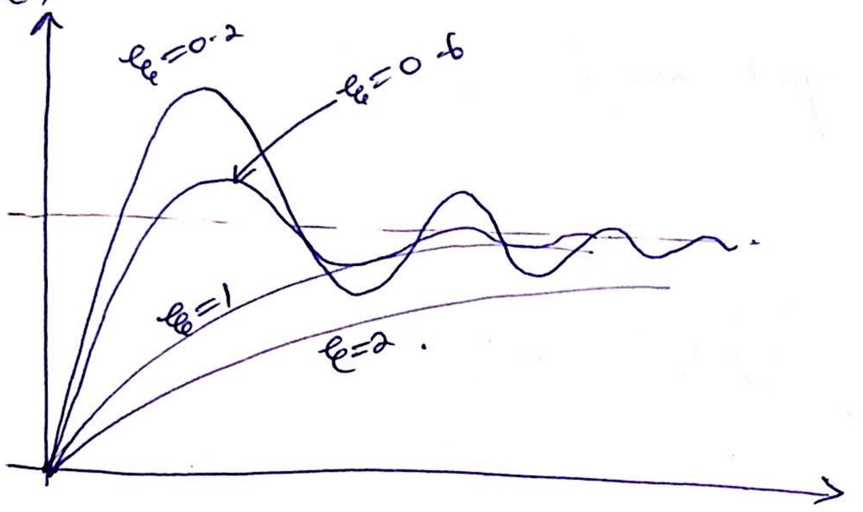


Sustained Oscillations.

(c) when $\xi = 1$, Critically damped.

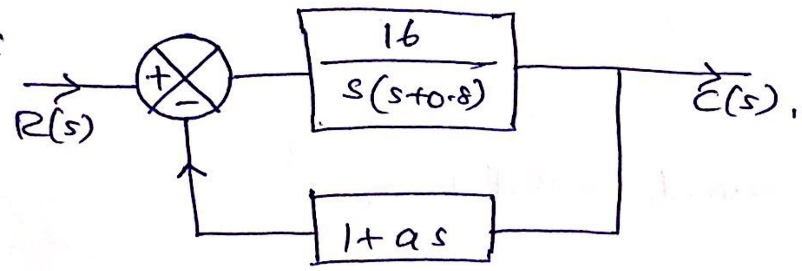


(d)



Example: Consider the system shown below. Determine the value of 'a' such that the damping ratio is 0.5. Obtain the values of t_r, M_p in its step response.

Ans::



$$\frac{C(s)}{R(s)} = \frac{\dfrac{16}{s(s+0.8)}}{1+\dfrac{16(1+as)}{s(s+0.8)}}$$

$$= \frac{16}{s^2+(0.8+16a)s+16}.$$

Compare with the characteristic eq^n,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0.$$

$$\omega_n^2 = 16, \quad \omega_n = 4 \text{ rad s}^{-1}.$$

$$2\xi\omega_n = 0.8 + 16a$$

$$2 \times 0.5 \times 4 = 0.8 + 16a$$

$$\therefore a = 0.2.$$

$$t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}.$$

$$= \frac{\pi - \tan^{-1}\frac{\sqrt{1-(0.5)^2}}{0.5}}{4\sqrt{1-(0.5)^2}}$$

$$= 0.605 \text{ s}.$$

$$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100 = e^{\frac{-0.5\pi}{\sqrt{1-0.5^2}}} \times 100$$

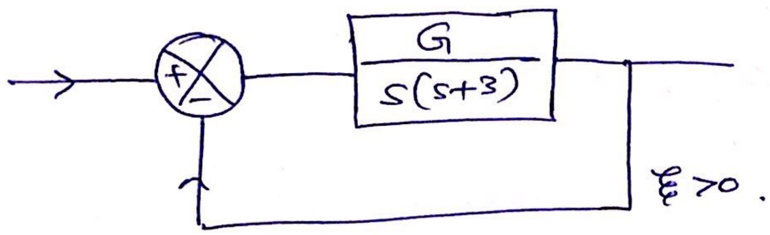
$$= 16.31\%.$$

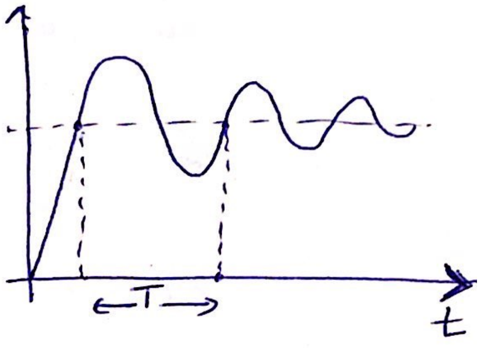
<u>example</u>: A block diagram of a feedback system is shown below. ~~Find~~

(a) Find closed loop transfer f^n.

(b) Find minimum value of G

<u>Example</u>: The block diagram of a feedback system is shaw below.





(a) Find closed loop transfer function.

(b) Find the minimum value of G for which the step response of the system would exhibit an overshoot as shown in the graph. (Assume that $\xi = 0.6$)

(c) For G equals to twice the minimum value, Find the time response 'T' indicated in the graph.

<u>Ans</u>:

(a) Closed loop transfer function.

$$\frac{C(s)}{R(s)} = \frac{G/s(s+3)}{1 + G/s(s+3)}$$

$$= \frac{G}{s^2+3s+G}.$$

(b) characteristic eq^n: $s^2+3s+G = 0$.

Compare with the $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$.

$\omega_n^2 = G, \Rightarrow \omega_n = \sqrt{G}$.

$2\xi\omega_n = 3$.

but $\xi = 0.6$

$\therefore 2 \times 0.6 \sqrt{G} = 3$.

$G = 6.25$.

$G' = 2G = 2 \times 6.25 = 12.5$

$\omega_n = \sqrt{12.5} = 3.53 \text{ rad s}^{-1}$.

$$\xi = \frac{3}{2\omega_n} = \frac{3}{2 \times 3.53} = 0.424.$$

$\omega_d = \omega_n \sqrt{1-\xi^2} = 3.53\sqrt{1-0.424^2}$

$= 3.197$.

$$\frac{2\pi}{T} = 3.197,$$

$$T = \frac{2\pi}{3.197}$$

$= 1.96 \text{ s}$.

Example: The open loop transfer function of a servo system with unity feedback is given by

$$G(s) = \frac{10}{(s+2)(s+5)}$$

(a) Determine the damping ratio, undamped natural frequency of oscillation.

(b) What is the percentage overshoot of the response to a unit step input.

Ans:

$$G(s) = \frac{10}{(s+2)(s+5)}$$

$$H(s) = 1$$

Characteristic eqⁿ,

$$1 + G(s)H(s) = 0.$$

$$1 + \frac{10}{(s+2)(s+5)} = 0$$

$$s^2 + 7s + 20 = 0.$$

Compare with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0.$

$$\omega_n^2 = 20.$$

$$\omega_n = 2\sqrt{5} = 4.472 \text{ rad s}^{-1}.$$

$$2\xi\omega_n = 7$$

$$2\xi \times 4.472 = 7,$$

$$\xi = 0.7826$$

$$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%$$

$$= 1.92\%.$$

Example: The open loop transfer function of a unity feedback system is given by,

$$G(s) = \frac{k}{s(1+sT)}.$$

where k, T are positive constants. By what factor should the amplifier gain be reduced so that the peak overshoot of unit step response of the system is reduced from 75% to 25%.

Ans:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}.$$

$$= \frac{\frac{k}{s(1+sT)}}{1+\frac{k}{s(1+sT)}\times 1} = \frac{k}{s^2T+s+k}.$$

$$= \frac{K/T}{s^2 + \frac{1}{T}s + K/T}$$

Characteristic eqn : $s^2 + \frac{1}{T}s + \frac{K}{T} = 0$

Compare with : $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$.

$$2\xi\omega_n = \frac{1}{T}, \quad \therefore \xi = \frac{1}{2T\omega_n}.$$

$$\omega_n^2 = \frac{K}{T}, \quad \therefore \omega_n = \sqrt{\frac{K}{T}}$$

$$\therefore \xi = \frac{1}{2T\sqrt{\frac{K}{T}}} = \frac{1}{2\sqrt{KT}}$$

$$M_{P_1} = e^{\frac{-\pi\xi_1}{\sqrt{1-\xi_1^2}}} = 0.75.$$

$$M_{P_2} = e^{\frac{-\pi\xi_2}{\sqrt{1-\xi_2^2}}} = 0.25.$$

$$e^{\frac{-\pi\xi_1}{\sqrt{1-\xi_1^2}}} = 0.75, \quad \therefore \frac{\pi\xi_1}{\sqrt{1-\xi_1^2}} = 0.287. \quad \text{—(1)}$$

$$e^{\frac{-\pi\xi_2}{\sqrt{1-\xi_2^2}}} = 0.25, \quad \frac{\pi\xi_2}{\sqrt{1-\xi_2^2}} = 1.386. \quad \text{—(2)}$$

$$(1)/(2), \quad \frac{\pi\xi_1}{\sqrt{1-\xi_1^2}} \times \frac{\sqrt{1-\xi_2^2}}{\pi\xi_2} = \frac{0.287}{1.386} = 0.207.$$

$$\frac{\zeta_1}{\zeta_2}\frac{\sqrt{1-\zeta_2^2}}{\sqrt{1-\zeta_1^2}} = 0.207.$$

$$\left(\frac{1}{2\sqrt{K_1T}} \times 2\sqrt{K_2T}\right)\frac{\sqrt{1-\left(\frac{1}{2\sqrt{K_2T}}\right)^2}}{\sqrt{1-\left(\frac{1}{2\sqrt{K_1T}}\right)^2}} = 0.207,$$

$$\Rightarrow \quad K_1 = 20K_2,$$

~~Ques~~ Example: A thermometer requires 1 minute to indicate 98% of the response to a step input. Assume the thermometer to be a first order system, find the time constant.

Ans:

$$c(t) = 1 - e^{-t/T}.$$

T: time constant.

$t = 1$ minute $= 60$ s.

$c(t) = 0.98.$

$$0.98 = 1 - e^{-60/T}$$

$$\therefore T = 15.33 \text{ s}.$$

Example: The open loop transfer function of a unity feeback system is given by

$$G(s) = \frac{K}{s(1+sT)}$$

By what factor the amplifier gain K should be multiplied so that the damping ratio is increased from 0.3 to 0.9.

Ans:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{K}{s(1+sT)}}{1+\frac{K}{s(1+sT)}\times 1}$$

$$\frac{C(s)}{R(s)} = \frac{K}{s^2T+s+K} = \frac{K/T}{s^2+\frac{1}{T}s+K/T}.$$

characteristic eq^n,

$$s^2+\frac{1}{T}s+\frac{K}{T} = 0,$$

Comparing with $s^2+2\xi\omega_n s+\omega_n^2 = 0.$

$$2\xi\omega_n = \frac{1}{T}.$$

$$\omega_n^2 = K/T \quad \therefore \quad \omega_n = \sqrt{\frac{K}{T}}.$$

$$2\xi\sqrt{\frac{K}{T}} = \frac{1}{T}.$$

$$\xi = \frac{1}{2T}\sqrt{\frac{T}{K}} = \frac{1}{2\sqrt{KT}}.$$

damping ratio $\xi_1 = 0.3$, $\xi_2 = 0.9$.

$$\xi_1 = \frac{1}{2\sqrt{K_1 T}}$$

$$\xi_2 = \frac{1}{2\sqrt{K_2 T}}.$$

$$\frac{\xi_1}{\xi_2} = \frac{1}{2\sqrt{K_1 T}} \times 2\sqrt{K_2 T} = \frac{\sqrt{K_2}}{\sqrt{K_1}}$$

$$\therefore \quad \frac{K_2}{K_1} = \left(\frac{\xi_1}{\xi_2}\right)^2.$$

$$= \left(\frac{0.3}{0.9}\right)^2 = 1/9.$$

$$\therefore \; 9K_2 = K_1$$

$$\Rightarrow K_1 = 9K_2$$

Hence at ~~ξ~~ gain K_1 at which $\xi = 0.3$ should be multiply by $1/9$ to increase the damping ratio from 0.3 to 0.9.