General Sir John Kotelawala Defence University
Faculty of Engineering
Department of Mathematics

Mathematics - MA 1103
Tutorial 04 - Roots of Polynomials

Year: 2021 | Intake: 38 - 03$^{\text{rd}}$ Batch | Semester: 01

Learning Outcomes Covered: LO1

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1. The roots of the quadratic equation $x^2 + 2x - 4 = 0$ are denoted by α and β.

   Find the quadratic equation, with integer coefficients, whose roots are

   $$\alpha^4 + \frac{1}{\beta^2} \quad \text{and} \quad \beta^4 + \frac{1}{\alpha^2}$$

   .

2. $$x^2 - 4\sqrt{2}kx + 2k^4 - 1 = 0$$

   The two roots of the above quadratic equation, where k is a positive constant, are denoted by α and β.

   Given further that $\alpha^2 + \beta^2 = 66$, determine the exact value of $\alpha^3 + \beta^3$.

3. The roots of the quadratic equation

   $$x^2 - 3x + 4 = 0$$

   are denoted by α and β.

   Find the quadratic equation, with integer coefficients, whose roots are

   $$\alpha^3 - \beta \quad \text{and} \quad \beta^3 - \alpha$$

   .

4. $$\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}, \qquad x \neq -p\,;\, x \neq -q$$

   The roots of the above quadratic equation, where p, q, and r are non zero constants, are equal in magnitude but opposite in sign.

   Show that the product of these roots is

   $$-\frac{1}{2}[p^2 + q^2].$$

5. The quadratic equation $ax^2 + bx + 1 = 0, \quad a \neq 0$ where a and b are constants, has roots α and β.

Find in terms of α and β, the roots of the equation

$$x^2 + (b^3 - 3ab)x + a^3 = 0.$$

6.

$$x^3 - 6x^2 + 4x + 12 = 0$$

The roots of the above cubic are denoted by α, β and γ.

Find the values of

(a) $\alpha + \beta + \gamma$.
(b) $\alpha^2 + \beta^2 + \gamma^2$.
(c) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma.}$

7. The roots of the quadratic equation

$$2x^2 - 5x + 8 = 0,$$

are denoted by α and β.

Determine the cubic equation, with integer coefficients, whose three roots are

$$\alpha^2\beta, \quad \alpha\beta^2, \quad \text{and} \quad \alpha\beta.$$

8. The roots of the cubic equation

$$ax^3 + bx^2 + cx + d = 0,$$

where $a, \quad b, \quad c,$ and d are non zero constants, are the first three terms of a geometric sequence with common ration 2.

Show clearly that

$$4bc = 49ad$$

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