General Sir John Kotelawala Defence University
Faculty of Engineering
Department of Mathematics

Mathematics - MA 1103

Tutorial 03 - Matrices, Determinants, & Eigen values

Year: 2021 Intake: 38 - 03$^{\text{rd}}$ Batch Semester: 01

Learning Outcomes Covered: LO2

Name of the Instructor Prepared: Ashani AG

Questions to be discussed:

(1)(d), (2)(a)(i),(iii), (2)(b), (3)(i)(a), (4)(c), (5)(b), (6)(a)(ii), (6)(b)(i)

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1. Given matrices A, and B, prove the following properties.
   (a) $(A^T)^T = A$
   (b) $(A+B)^T = A^T + B^T$
   (c) For a scalar c, $(cA)^T = cA^T$
   (d) $(AB)^T = B^T A^T$

2. (a) Find the inverse (A^{-1}) of the following matrices A using Gauss - Jordan Elimination and Adjoint Matrix.

   i. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 5 & 8 \end{pmatrix}$

   ii. $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 9 & 3 \\ 1 & 4 & 2 \end{pmatrix}$

   iii. $\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$

   iv. $\begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$

   (b) Find $(2A)^{-1}$ and $(A^T)^{-1}$ of the above matrices A in (02)(a).

3. Evaluate the determinant of the following matrices
   (a) along the first row
   (b) along the third column

   i. $\begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 0 \\ 2 & 0 & 1 & 3 \\ 1 & 1 & 1 & 0 \end{pmatrix}$

   ii. $\begin{pmatrix} 3 & 2 & 5 & 7 \\ -1 & -4 & -3 & 0 \\ 6 & 4 & 2 & -1 \\ 2 & -1 & 0 & 3 \end{pmatrix}$

4. (a) Prove that $\begin{vmatrix} x & l & m & 1 \\ \alpha & x & n & 1 \\ \alpha & \beta & x & 1 \\ \alpha & \beta & \gamma & 1 \end{vmatrix} = (x-\alpha)(x-\beta)(x-\gamma)$

(b) Using properties of determinants, solve for x : $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

(c) Show that $x = -(a+b+c)$ is one root of the equation $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$

and solve the equation completely.

(d) If $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0,$ prove that $abc = 1$

(e) Prove that $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$

5. (a) Express the following system of equations in matrix form and solve them by Gauss - Jordan elimination.

$$2x_1 + x_2 + 2x_3 + x_4 = 6$$
$$6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$$
$$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$$
$$2x_1 + 2x_2 - x_3 + x_4 = 10$$

(b) The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number we get 12. Use determinants to find the numbers.

(c) Using inverse of matrices, solve the following system of linear equations.

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6$$

6. (a) Find the eigen values and the eigen space for the following matrices.

i. $\begin{pmatrix} 3 & -3 & 3 \\ 2 & 1 & 1 \\ 1 & 5 & 6 \end{pmatrix}$

iii. $\begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}$

ii. $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

iv. $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$

(b) i. The matrix A is defined as $A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{pmatrix}$

Find the eigen values of $3A^3 + 5A^2 - 6A + 2I$.

ii. The matrix A is defined as $A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$

Find the eigen values of $\frac{1}{4}A^2 + A - \frac{5}{4}I$.