Random Variables & Distributions

Random Signals & Processes
Lecture 3
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Poisson Distribution

- A RV X is called a Poisson RV with parameter λ (> 0) if its pmf is given by

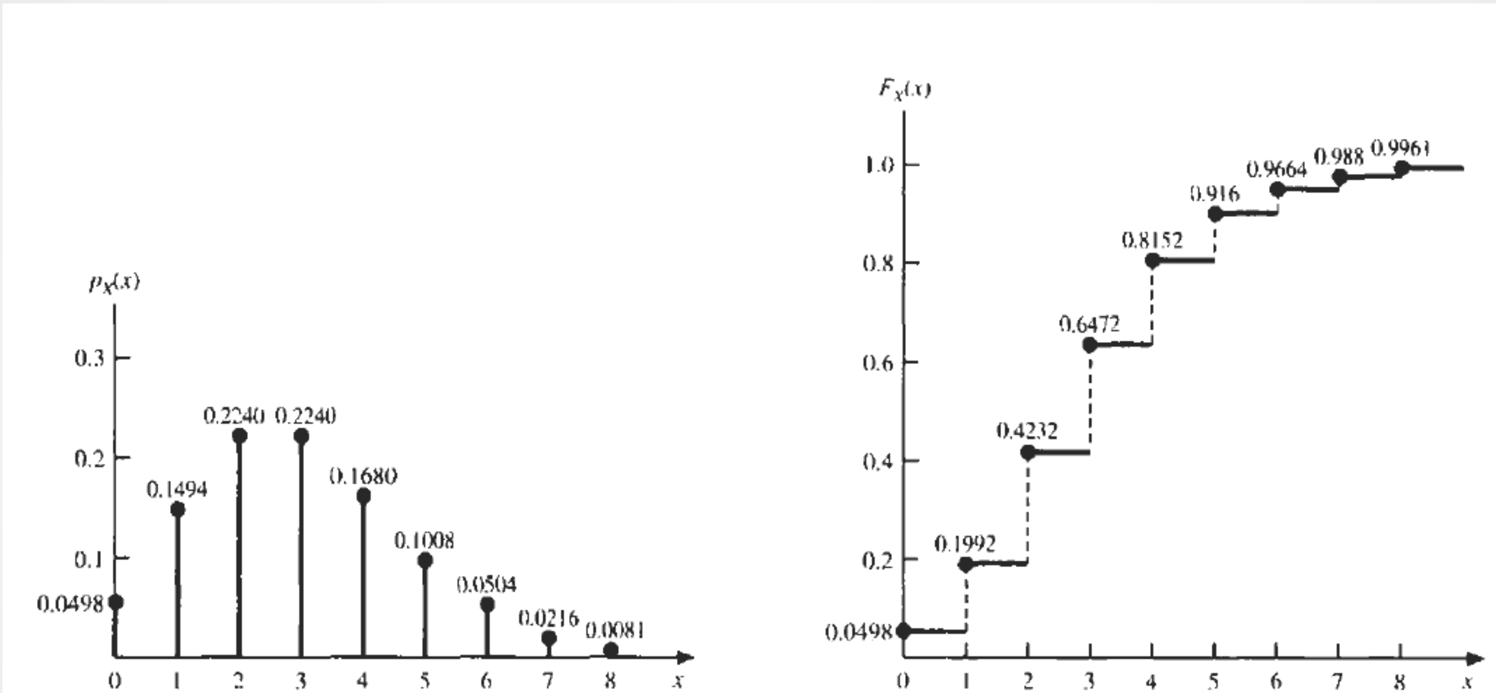
$$p_X(k) = P(X = k) = e^{-\lambda}\frac{\lambda^k}{k!} \qquad k = 0, 1, \ldots$$

The corresponding cdf of X

$$F_X(x) = e^{-\lambda}\sum_{k=0}^{n}\frac{\lambda^k}{k!} \qquad n \le x < n + 1$$

Example

Poisson Distribution with λ=3.



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CDF

Mean μ_x= E(X)= λ

Variance σ_x^2= Var(X)= λ

Examples for Poisson Distribution

1. The number of telephone calls arriving at a switching center during various intervals of time.

2. The number of misprints on a page of a book.

3. The number of customers entering a bank during various intervals of time.

- Poisson Distribution can be used as an approximation for a binomial RV with parameters (n, p) when n is large and p is small enough so that np is of a moderate size.

Mean and the Variance

$$\mu_X = E(X) = \frac{a+b}{2}$$

$$\sigma_X{}^2 = \mathrm{Var}(X) = \frac{(b-a)^2}{12}$$

Uniform Distribution

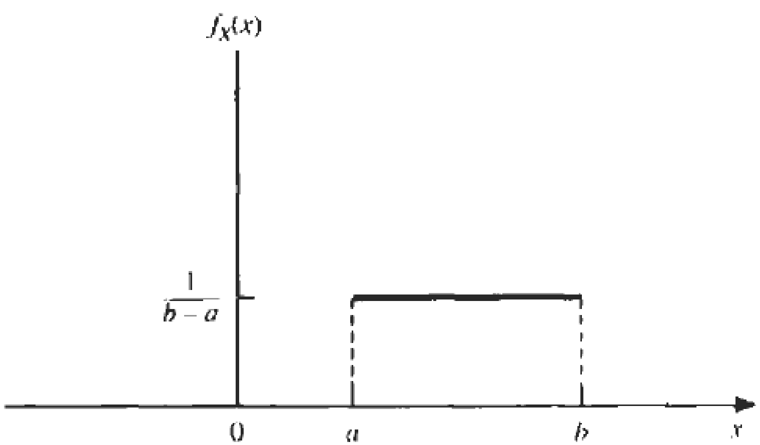
- A RV X is called a uniform RV over (a, b) if its pdf is given by

$$f_X(x) = \begin{cases} \dfrac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

- CDF is as follows

$$F_X(x) = \begin{cases} 0 & x \le a \\ \dfrac{x-a}{b-a} & a < x < b \\ 1 & x \ge b \end{cases}$$

Uniform Distribution over (a,b)



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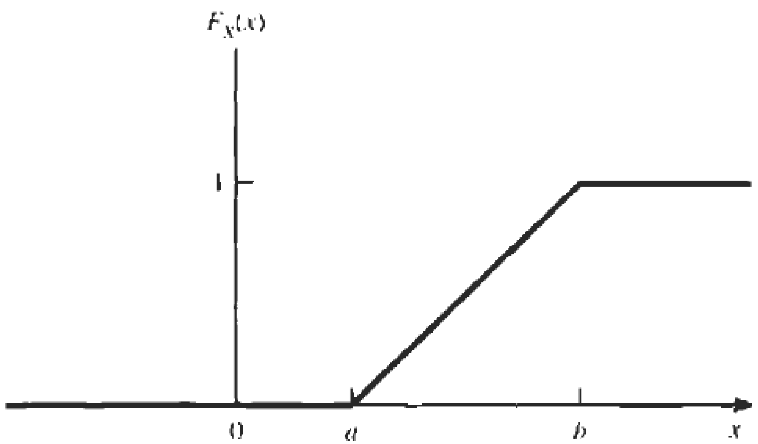


CDF

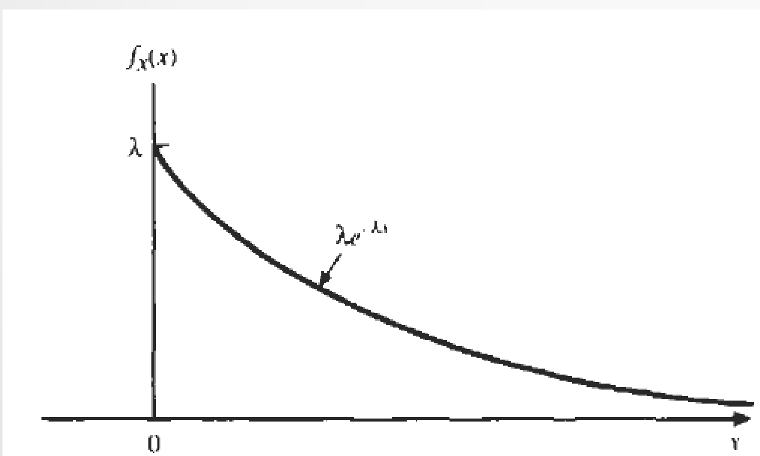
Exponential Distribution

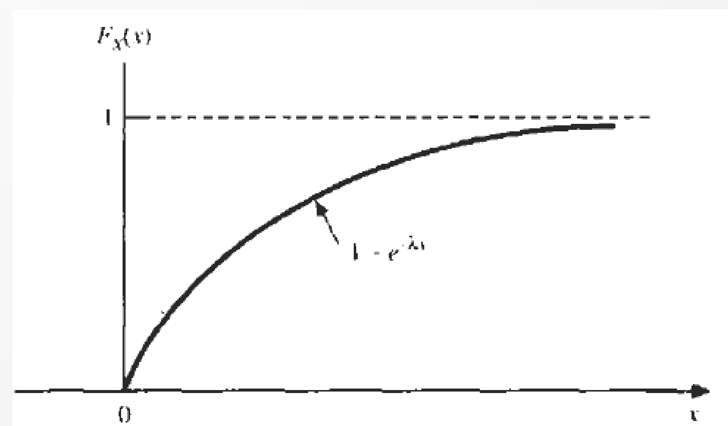
- A RV X is called an exponential RV with parameter λ (>O) if its pdf is given by

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & x < 0 \end{cases}$$

- CDF

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$





Mean
μ_x=E(X) = 1/λ

Variance
σ_x^2=E(X) = 1/λ^2

- Property of the exponential distribution is its "memoryless" property.
- If the lifetime of an item is exponentially distributed, then an item which has been in use for some hours is as good as a new item with regard to the amount of time remaining until the item fails.

Normal (or Gaussian) Distribution

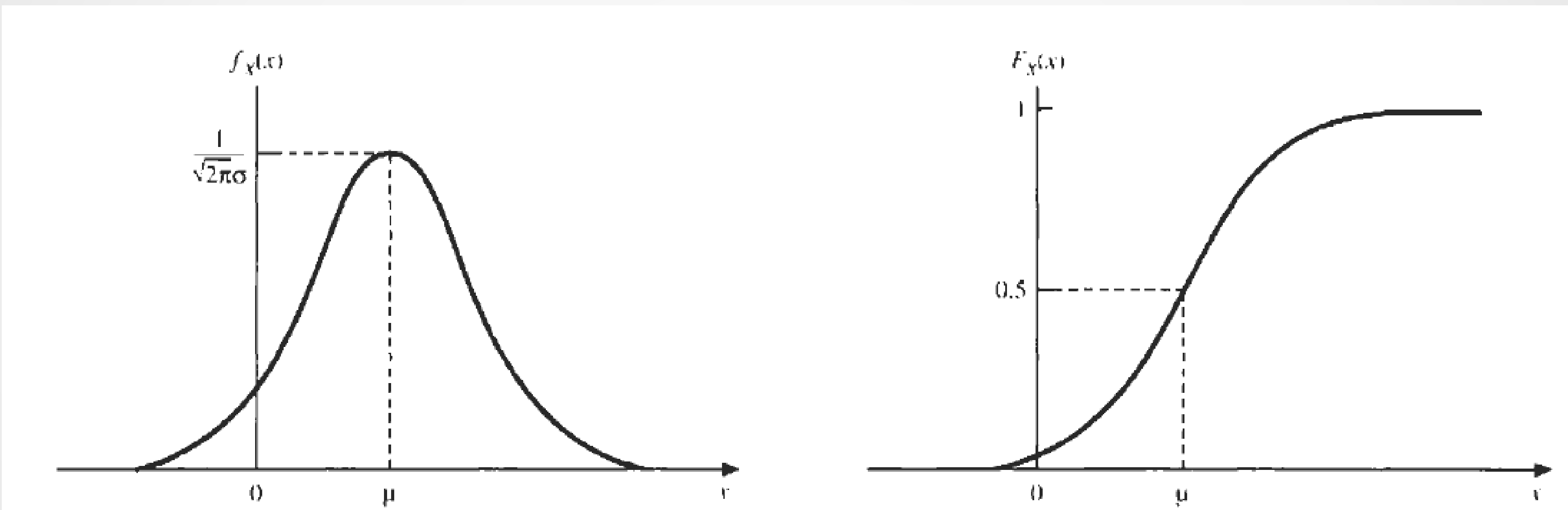
- A RV X is called a normal (or gaussian) RV if its pdf is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/(2\sigma^2)}$$

- CDF

$$F_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{x} e^{-(\xi-\mu)^2/(2\sigma^2)}\, d\xi = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(x-\mu)/\sigma} e^{-\xi^2/2}\, d\xi$$

Normal (or Gaussian) Distribution



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CDF

Mean

$\mu_x = E(X) = \mu$

Variance

$\sigma_x^2 = E(X) = \sigma^2$

Normal Distribution

- Many naturally occurring random phenomena are approximately normal – $N(\mu,\sigma^2)$.
- Another reason for the importance of the normal RV is a remarkable theorem called the central limit theorem.
- This theorem states that the sum of a large number of independent r.v.'s, under certain conditions, can be approximated by a normal RV