
Chapter - 01

Differentiation and Integration

Independent Study Material No. 03

Differentiation Rules

1. **Derivative of a Constant Function:**

$$\frac{d}{dx}(c) = 0$$

2. **The Power Rule:** If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

3. **The Constant Multiple Rule:** If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

4. **The Sum Rule:** If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

5. **The Difference Rule:** If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

6. **Derivative of the Natural Exponential Function:**

$$\frac{d}{dx}(e^x) = e^x$$

7. **The Product Rule:** If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

8. **The Quotient Rule:** If f and g are both differentiable, then

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

9. **Derivatives of Trigonometric Functions:**

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

10. **The Chain Rule:** If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and F' is given by the product

$$F'(x) = f'(g(x)) \cdot g'(x)$$

If $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}.$$

11. **The Power Rule Combined with the Chain Rule:** If n is any real number and $u = g(x)$ is differentiable, then

$$\frac{d}{dx}(u^n) = nu^{n-1} \frac{du}{dx}$$

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} g'(x).$$

Practice Problems: Differentiate the following functions.

$$f(x) = (3x^2 - 5x)e^x$$

$$y = \frac{x}{e^x}$$

$$g(x) = \frac{1 + 2x}{3 - 4x}$$

$$H(u) = (u - \sqrt{u})(u + \sqrt{u})$$

$$J(v) = (v^3 - 2v)(v^{-4} + v^{-2})$$

$$F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4} \right) (y + 5y^3)$$

$$f(z) = (1 - e^z)(z + e^z)$$

$$y = \frac{x^2 + 1}{x^3 - 1}$$

$$y = \frac{t^3 + 3t}{t^2 - 4t + 3}$$

$$y = e^p(p + p\sqrt{p})$$

$$y = \frac{s - \sqrt{s}}{s^2}$$

$$y = \frac{\sqrt{x}}{2 + x}$$

$$y = \frac{1}{t^3 + 2t^2 - 1}$$

$$h(r) = \frac{ae^r}{b + e^r}$$

$$y = (z^2 + e^z)\sqrt{z}$$

$$V(t) = \frac{4 + t}{te^t} \quad f(t) = \frac{\sqrt[3]{t}}{t - 3}$$

$$F(t) = \frac{At}{Bt^2 + Ct^3} \quad f(x) = \frac{x^2 e^x}{x^2 + e^x}$$

$$f(x) = \frac{ax + b}{cx + d} \quad f(x) = \frac{x}{x + \frac{c}{x}}$$

1–16 Differentiate.

1. $f(x) = x^2 \sin x$

3. $f(x) = e^x \cos x$

5. $y = \sec \theta \tan \theta$

7. $y = c \cos t + t^2 \sin t$

9. $y = \frac{x}{2 - \tan x}$

11. $f(\theta) = \frac{\sin \theta}{1 + \cos \theta}$

13. $y = \frac{t \sin t}{1 + t}$

15. $f(\theta) = \theta \cos \theta \sin \theta$

2. $f(x) = x \cos x + 2 \tan x$

4. $y = 2 \sec x - \csc x$

6. $g(\theta) = e^\theta (\tan \theta - \theta)$

8. $f(t) = \frac{\cot t}{e^t}$

10. $y = \sin \theta \cos \theta$

12. $y = \frac{\cos x}{1 - \sin x}$

14. $y = \frac{\sin t}{1 + \tan t}$

16. $f(t) = te^t \cot t$

1. $F(x) = (5x^6 + 2x^3)^4$

2. $f(x) = \sqrt{5x + 1}$

3. $f(\theta) = \cos(\theta^2)$

4. $y = x^2 e^{-3x}$

5. $f(t) = e^{at} \sin bt$

6. $f(x) = (2x - 3)^4 (x^2 + x + 1)^5$

7. $g(x) = (x^2 + 1)^3 (x^2 + 2)^6$

8. $h(t) = (t + 1)^{2/3} (2t^2 - 1)^3$

9. $F(t) = (3t - 1)^4 (2t + 1)^{-3}$

10. $y = \sqrt{\frac{x}{x + 1}}$

11. $y = e^{\tan \theta}$

12. $g(u) = \left(\frac{u^3 - 1}{u^3 + 1} \right)^8$

13. $r(t) = 10^{2\sqrt{t}}$

14. $H(r) = \frac{(r^2 - 1)^3}{(2r + 1)^5}$