

DFT:

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} \quad k = 0, 1, \dots, N-1$$

$$X[0] = x[0]W_N^0 + x[1]W_N^0 + x[2]W_N^0 + \dots + x[N-1]W_N^0$$

$$X[1] = x[0]W_N^0 + x[1]W_N^1 + x[2]W_N^2 + \dots + x[N-1]W_N^{N-1}$$

$$X[2] = x[0]W_N^0 + x[1]W_N^2 + x[2]W_N^4 + \dots + x[N-1]W_N^{2(N-1)}$$

$\vdots$

$$X[N-1] = x[0]W_N^0 + x[1]W_N^{N-1} + x[2]W_N^{2(N-1)} + \dots + x[N-1]W_N^{(N-1)^2}$$

Linear Algebra

$$\begin{matrix} \begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix} \\ N \times 1 \end{matrix} = \begin{matrix} \begin{bmatrix} W_N^0 & W_N^0 & \dots & W_N^0 \\ W_N^0 & W_N^1 & \dots & W_N^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ W_N^0 & W_N^{(N-1)} & \dots & W_N^{(N-1)^2} \end{bmatrix} \\ N \times N \end{matrix} \begin{matrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} \\ N \times 1 \end{matrix} \Rightarrow X = Fx$$

$F$  is orthogonal

columns all have the length of  $N$

perpendicular

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Handwritten diagram illustrating the DFT equation  $X[k] = \sum_n x[n] W_N^{kn}$ . It shows a vertical vector of 1s (labeled  $N \times 1$ ) being multiplied by a horizontal vector of 1s (labeled  $x[0]$ ), then a horizontal vector of 1s (labeled  $x[1]$ ), and finally a horizontal vector of 1s (labeled  $x[n]$ ). The result is a horizontal vector of 1s (labeled  $X[k]$ ).

