ME 1012: Applied Mechanics

Equilibrium of Rigid Bodies:

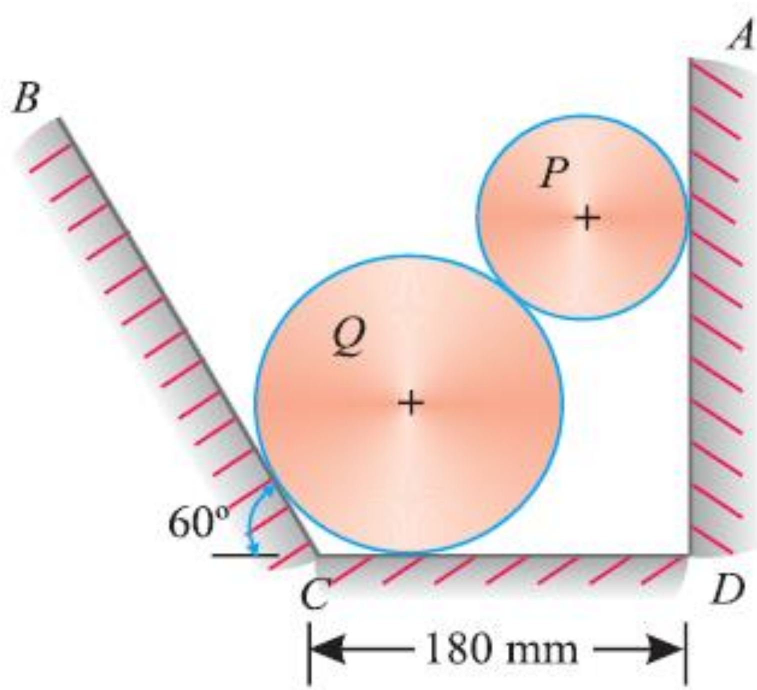
Q & A

Department of Mechanical Engineering
General Sir John Kotelawala Defence University

Problems: 4.5

- Two cylinders P and Q rest in a channel as shown in Figure. The cylinder P has diameter of 100 mm and weighs 200 N, whereas the cylinder Q has diameter of 180 mm and weighs 500 N.

  If the bottom width of the box is 180 mm, with one side vertical and the other inclined at 60◦, determine the pressures at all the four points of contact.



Solution: 4.5

Solution. Given : Diameter of cylinder $P = 100$ mm ; Weight of cylinder $P = 200$ N ; Diameter of cylinder $Q = 180$ mm ; Weight of cylinder $Q = 500$ N and width of channel = 180 mm.

First of all, consider the equilibrium of the cylinder P. It is in equilibrium under the action of the following three forces which must pass through A *i.e.*, the centre of the cylinder P as shown in Fig. 5.14 (*a*).

1. Weight of the cylinder (200 N) acting downwards.
2. Reaction (R_1) of the cylinder P at the vertical side.
3. Reaction (R_2) of the cylinder P at the point of contact with the cylinder Q.

From the geometry of the figure, we find that

$$ED = \text{Radius of cylinder } P = \frac{100}{2} = 50 \text{ mm}$$

Similarly $$BF = \text{Radius of cylinder } Q = \frac{180}{2} = 90 \text{ mm}$$

and $$\angle BCF = 60^\circ$$

$$\therefore \quad CF = BF \cot 60^\circ = 90 \times 0.577 = 52 \text{ mm}$$

$$\therefore \quad FE = BG = 180 - (52 + 50) = 78 \text{ mm}$$

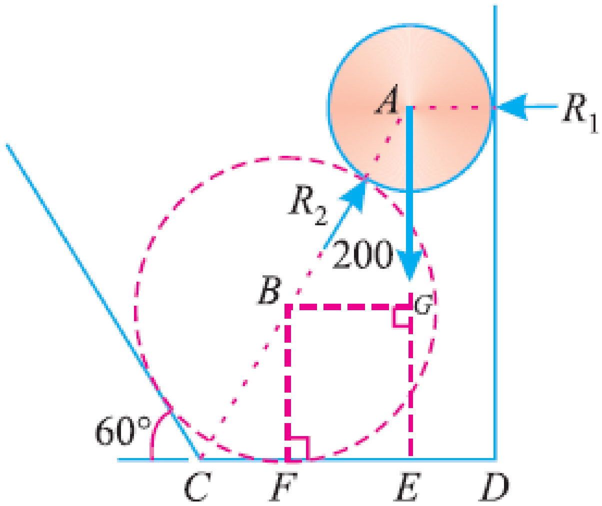
and $$AB = 50 + 90 = 140 \text{ mm}$$

$$\therefore \quad \cos \angle ABG = \frac{BG}{AB} = \frac{78}{140} = 0.5571$$

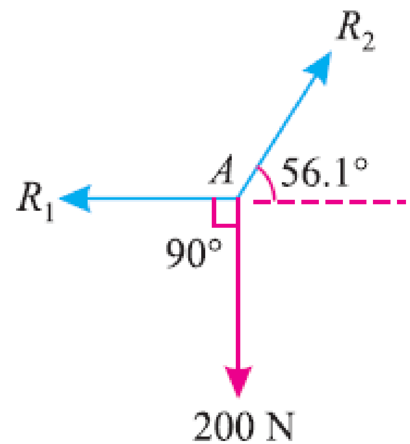
or $$\angle ABG = 56.1^\circ$$

Solution: 4.5

The system of forces at A is shown in Fig. 5.14 (b).



(a) Free body diagram



(b) Force diagram

Fig. 5.14.

Applying Lami's equation at A,

$$\frac{R_1}{\sin(90° + 56.1°)} = \frac{R_2}{\sin 90°} = \frac{200}{\sin(180° - 56.1°)}$$

$$\frac{R_1}{\cos 56.1°} = \frac{R_2}{1} = \frac{200}{\sin 56.1°}$$

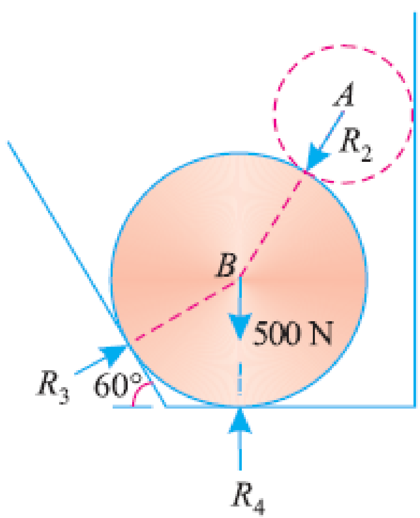
$$\therefore \qquad R_1 = \frac{200 \cos 56.1^\circ}{\sin 56.1^\circ} = \frac{200 \times 0.5571}{0.830} = 134.2 \text{ N} \quad \textbf{Ans.}$$

and

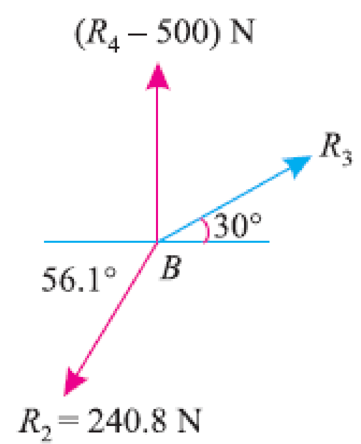
$$R_2 = \frac{200}{\sin 56.1^\circ} = \frac{200}{0.8300} = 240.8 \text{ N} \quad \textbf{Ans.}$$

Now consider the equilibriXum of the cylinder Q. It is in equilibrium under the action of the following four forces, which must pass through the centre of the cylinder as shown in Fig. 5.15 (*a*).

1. Weight of the cylinder Q (500 N) acting downwards.
2. Reaction R_2 equal to 240.8 N of the cylinder P on cylinder Q.
3. Reaction R_3 of the cylinder Q on the inclined surface.
4. Reaction R_4 of the cylinder Q on the base of the channel.



(*a*) Free body diagram



(*b*) Force diagram

Fig. 5.15.

A little consideration will show, that the weight of the cylinder Q is acting downwards and the reaction R_4 is acting upwards. Moreover, their lines of action also coincide with each other.

$\therefore$ Net downward force = $(R_4 - 500)$ N

The system of forces is shown in Fig. 5.15 (*b*).

Applying Lami's equation at B,

$$\frac{R_3}{\sin(90^\circ + 56.1^\circ)} = \frac{240.8}{\sin 60^\circ} = \frac{R_4 - 500}{\sin(180^\circ + 30^\circ - 56.1^\circ)}$$

$$\frac{R_3}{\cos 56.1^\circ} = \frac{240.8}{\sin 60^\circ} = \frac{R_4 - 500}{\sin 26.1^\circ}$$

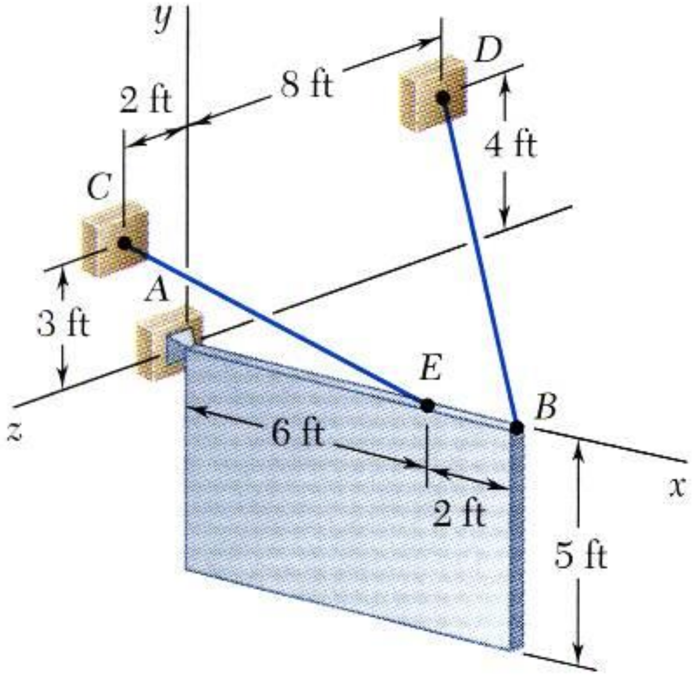
$$\therefore \quad R_3 = \frac{240.8 \times \cos 56.1^\circ}{\sin 60^\circ} = \frac{240.8 \times 0.5577}{0.866} = 155 \text{ N} \quad \textbf{Ans.}$$

and

$$R_4 - 500 = \frac{240.8 \times \sin 26.1^\circ}{\sin 60^\circ} = \frac{240.8 \times 399}{0.866} = 122.3 \text{ N}$$

$$\therefore \quad R_4 = 122.3 + 500 = 622.3 \text{ N} \quad \textbf{Ans.}$$

Problems: 4.6



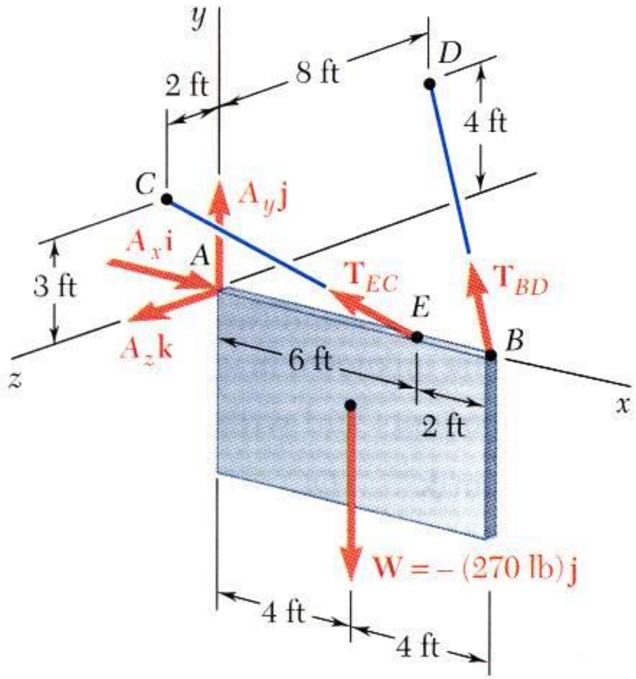
A sign of uniform density weighs 270 lb and is supported by a ball-and-socket joint at *A* and by two cables.

Determine the tension in each cable and the reaction at *A*.

SOLUTION:

- Create a free-body diagram for the sign.
- Apply the conditions for static equilibrium to develop equations for the unknown reactions.

Solution: 4.6



- Create a free-body diagram for the sign.

  Since there are only 5 unknowns, the sign is partially constrain. It is free to rotate about the *x* axis. It is, however, in equilibrium for the given loading.

$$\vec{T}_{BD} = T_{BD}\frac{\vec{r}_D - \vec{r}_B}{\left|\vec{r}_D - \vec{r}_B\right|}$$

$$= T_{BD}\frac{-8\vec{i} + 4\vec{j} - 8\vec{k}}{12}$$

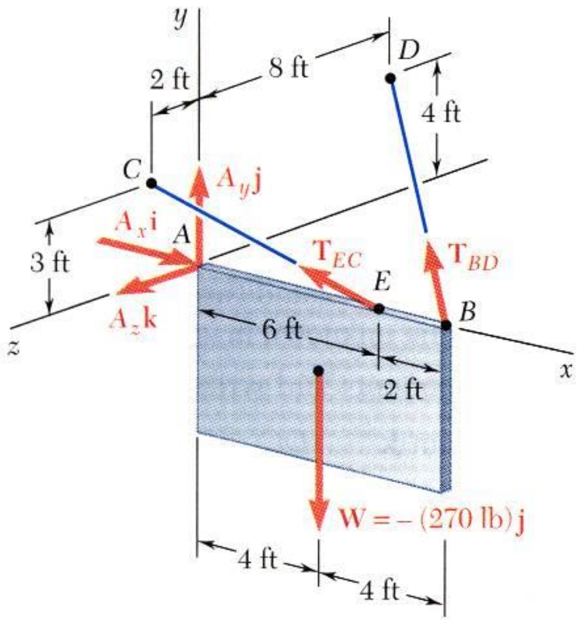
$$= T_{BD}\left(-\tfrac{2}{3}\vec{i} + \tfrac{1}{3}\vec{j} - \tfrac{2}{3}\vec{k}\right)$$

$$\vec{T}_{EC} = T_{EC}\frac{\vec{r}_C - \vec{r}_E}{\left|\vec{r}_C - \vec{r}_E\right|}$$

$$= T_{EC}\frac{-6\vec{i} + 3\vec{j} + 2\vec{k}}{7}$$

$$= T_{EC}\left(-\tfrac{6}{7}\vec{i} + \tfrac{3}{7}\vec{j} + \tfrac{2}{7}\vec{k}\right)$$

Solution: 4.6



- Apply the conditions for static equilibrium to develop equations for the unknown reactions.

$$\sum \vec{F} = \vec{A} + \vec{T}_{BD} + \vec{T}_{EC} - (270\,\text{lb})\vec{j} = 0$$

$$\vec{i}: \quad A_x - \tfrac{2}{3}T_{BD} - \tfrac{6}{7}T_{EC} = 0$$

$$\vec{j}: \quad A_y + \tfrac{1}{3}T_{BD} + \tfrac{3}{7}T_{EC} - 270\,\text{lb} = 0$$

$$\vec{k}: \quad A_z - \tfrac{2}{3}T_{BD} + \tfrac{2}{7}T_{EC} = 0$$

$$\sum \vec{M}_A = \vec{r}_B \times \vec{T}_{BD} + \vec{r}_E \times \vec{T}_{EC} + (4\,\text{ft})\vec{i} \times (-270\,\text{lb})\vec{j} = 0$$

$$\vec{j}: \quad 5.333T_{BD} - 1.714T_{EC} = 0$$

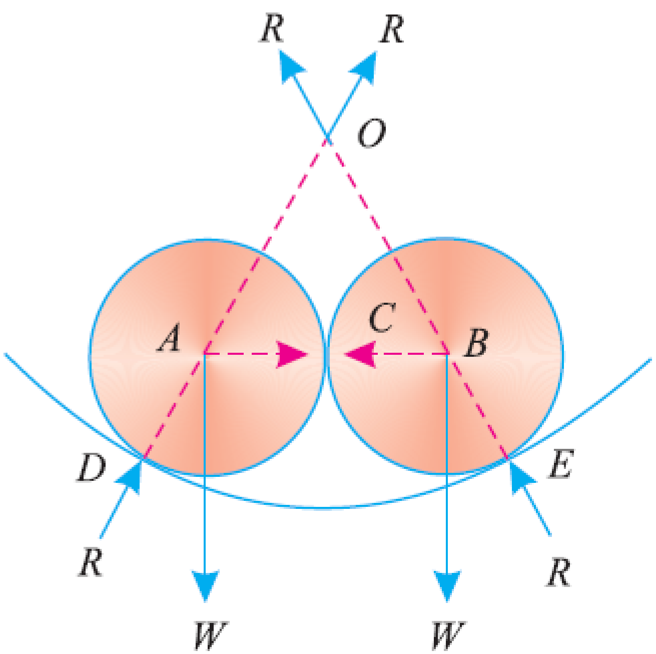
$$\vec{k}: \quad 2.667T_{BD} + 2.571T_{EC} - 1080\,\text{lb} = 0$$

Solve the 5 equations for the 5 unknowns,

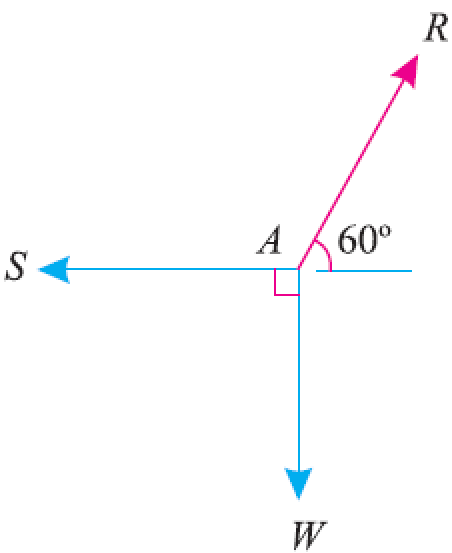
$$\boxed{\begin{array}{l} T_{BD} = 101.3\,\text{lb} \quad T_{EC} = 315\,\text{lb} \\ \vec{A} = (338\,\text{lb})\vec{i} + (101.2\,\text{lb})\vec{j} - (22.5\,\text{lb})\vec{k} \end{array}}$$

Problems: 4.7

- Two equal heavy spheres of 50 mm radius are in equilibrium within a smooth cup of 150 mm radius. Show that the reaction between the cup of one sphere is double than that between the two spheres.



(a)



(b)

Solution: 4.7

The two spheres with centres A and B, lying in equilibrium, in the cup with O as centre are shown in Fig. 5.11 (a). Let the two spheres touch each other at C and touch the cup at D and E respectively.

Let R = Reactions between the spheres and cup, and

S = Reaction between the two spheres at C.

From the geometry of the figure, we find that OD = 150 mm and AD = 50 mm. Therefore OA = 100 mm. Similarly OB = 100 mm. We also find that AB = 100 mm. Therefore OAB is an equilateral triangle. The system of forces at A is shown in Fig. 5.11 (b).

Applying Lami's equation at A,

$$\frac{R}{\sin 90^\circ} = \frac{W}{\sin 120^\circ} = \frac{S}{\sin 150^\circ}$$

$$\frac{R}{1} = \frac{W}{\sin 60^\circ} = \frac{S}{\sin 30^\circ}$$

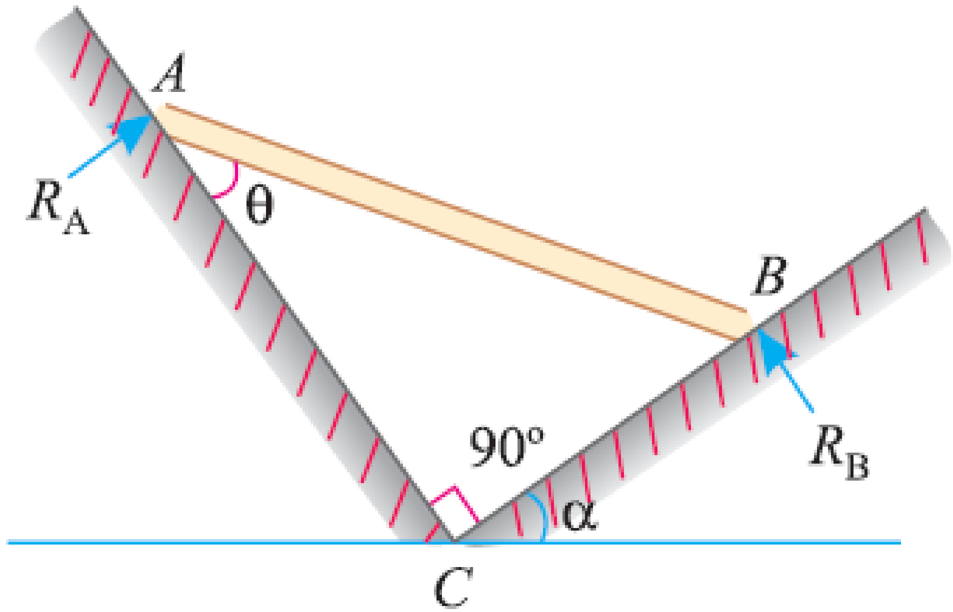
$$\therefore \quad R = \frac{S}{\sin 30^\circ} = \frac{S}{0.5} = 2S$$

Hence the reaction between the cup and the sphere is double than that between the two spheres. **Ans.**

Problems: 4.8

- A uniform rod AB remains in equilibrium position resting a smooth inclined planes AC and BC, which are at an angle of 90 as shown in figure given below.

  If the plane BC makes an angle of α with the horizontal, then what is the inclination θ of the rod AB with the plane AC.



Solution: 4.8

Solution. The rod is in equilibrium under the action of the following three forces,

1. Weight of the rod acting vertically through the mid-point G of the rod AB.
2. Reaction R_A at A normal to the plane AC, and
3. Reaction R_B at B normal to the plane BC.

Let these three forces meet at point D as shown in fig. 5.20

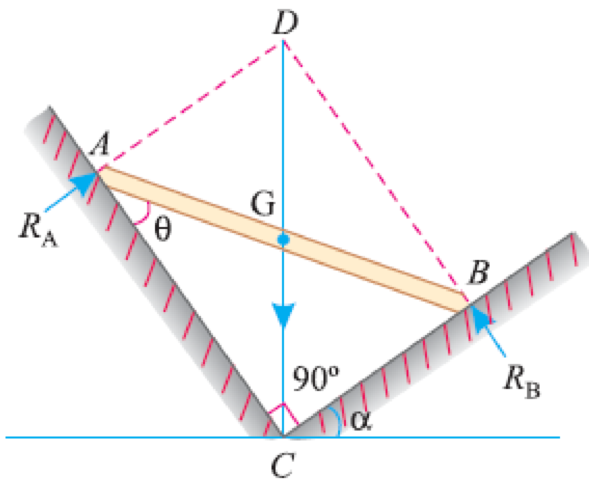


Fig. 5.20.

Since AD is perpendicular to AC and BD is perpendicular to BC, therefore AD is parallel to BC and BD is parallel to AC.

and $\angle ADB = 90°$

The figure $ADBC$ is a rectangle whose diagonal DGC is vertical

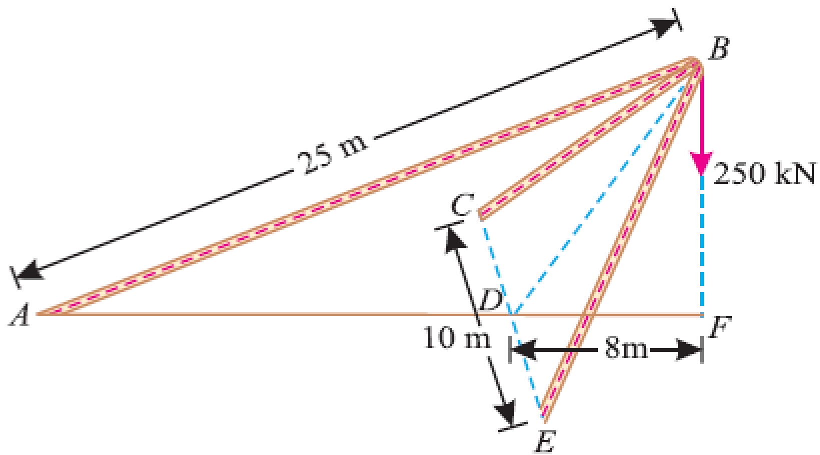
$$GA = GC$$

$$\angle GAC = \angle GCA$$

$\therefore$ $\theta = \alpha$ **Ans.**

Problems: 4.9

- Figure shows a shear leg crane lifting a load of 250 kN.



The legs BC and BE are 20 m long and 10 m apart at the base. The back stay AB is 25 m long. If all the members are pin-jointed at A, C and E, at the same level, find the forces in all the three members of the crane.

Solution: 4.9

Solution. Given : Weight at $B = 250$ kN

Let P = Force in each members BC and BE, and

T = Force in the member AB.

From the geometry of the figure, we find that the points $ABDF$ lie in one vertical plane, in which $\angle AFB$ is a right angle. Moreover, the points $BCDE$ also lie in one plane, in which $\angle BDC$ and $\angle BDE$ are also right angles and D is in the mid point of C and E.

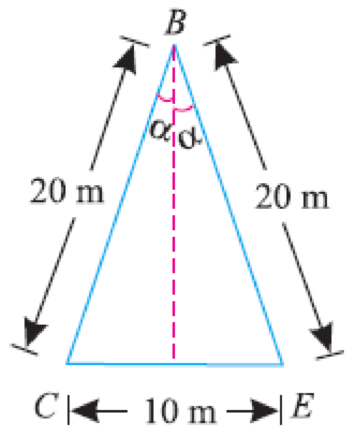


Fig. 5.24.

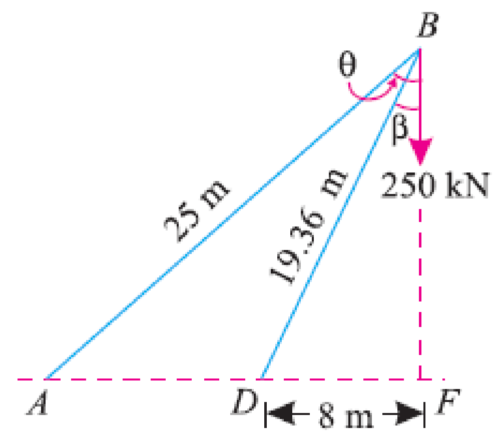


Fig. 5.25.

First of all, draw the isosceles triangle BCE with BC and BE each equal to 20 m and CE equal to 10 m with D as mid point of C and E as shown in Fig. 5.24.

Now in triangle BCD, we find that

$$\sin \alpha = \frac{5}{20} = 0.25 \quad \text{or} \quad \alpha = 14.5°$$

and
$$BD = \sqrt{(20)^2 - (5)^2} = 19.36 \text{ m}$$

Solution: 4.9

Now draw the triangle *ABF* with *DF* equal to 8 m, *AFB* equal to 90°, *DB* equal to 19.36 m and *AB* equal to 25 m as shown in Fig. 5.25.

From the geometry of the triangle *BDF*, we find that

$$\sin\beta = \frac{DF}{BD} = \frac{8}{19.36} = 0.4132 \quad \text{or} \quad \beta = 24.4°$$

and

$$BF = \sqrt{(19.36)^2 - (8)^2} = 17.63 \text{ m}$$

From the geometry of the triangle *ABF*, we also find that

$$\cos\angle ABF = \frac{BF}{AB} = \frac{17.63}{25} = 0.7052 \quad \text{or} \quad \angle ABF = 45.1°$$

$$\therefore \quad \theta = 45.1° - 24.4° = 20.7°$$

We know that resultant of the forces in members *BC* and *BE* (acting along *BD*)

$$R = 2P\cos\alpha = 2P\cos 14.5°$$

$$= 2P \times 0.9680 = 1.936\,P$$

The system of forces acting at *B* is shown in Fig 5.26.

Applying Lami's equation at *B*,

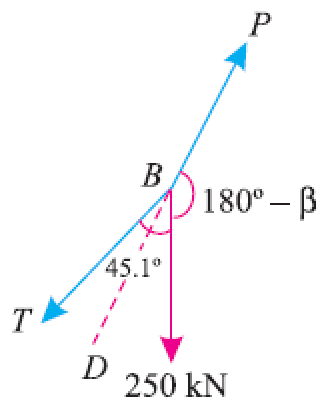


Fig. 5.26.

$$\frac{T}{\sin(180° - 24.4°)} = \frac{1.936\,P}{\sin 45.1°} = \frac{250}{\sin(180° - 20.7°)}$$

$$\frac{T}{\sin 24.4°} = \frac{1.936\,P}{\sin 45.1°} = \frac{250}{\sin 20.7°}$$

$$\therefore \quad T = \frac{250 \times \sin 24.4°}{\sin 20.7°} = \frac{250 \times 0.4131}{0.3543} = 291.5 \text{ kN} \quad \textbf{Ans.}$$

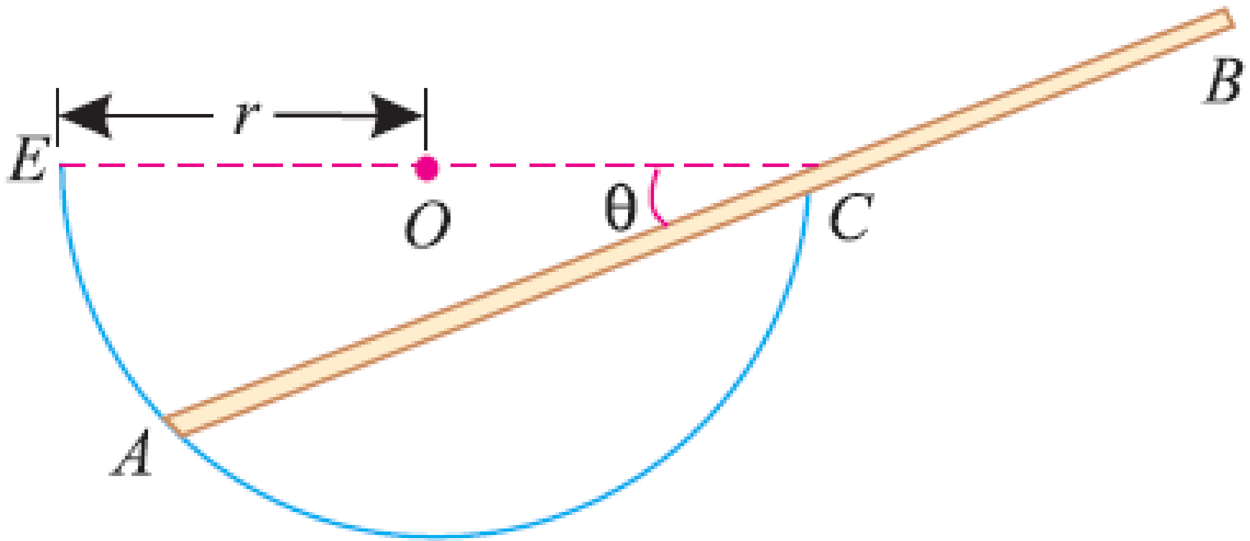
and

$$P = \frac{250 \times \sin 45.1°}{1.936 \times \sin 20.7°} = \frac{250 \times 0.7090}{1.936 \times 0.3543} = 258.4 \text{ kN} \quad \textbf{Ans.}$$

Problems: 4.10

- A uniform rod AB of length 3r remains in equilibrium on a hemispherical bowl of radius r as shown in the figure.

  Ignoring friction find the inclination of the rod (θ) with the horizontal.



Solution: 4.10

Solution. Given : Length of the rod $AB = 3r$ and radius of hemispherical ball $= r$

The rod is in equilibrium under the action of the following three forces as shown in Fig. 5.22.

1. Weight of the rod (W) acting vertically downwords through the mid-point G of the rod AB
2. Reaction at A acting in the direction AO
3. Reaction at C acting at the right angle to AB

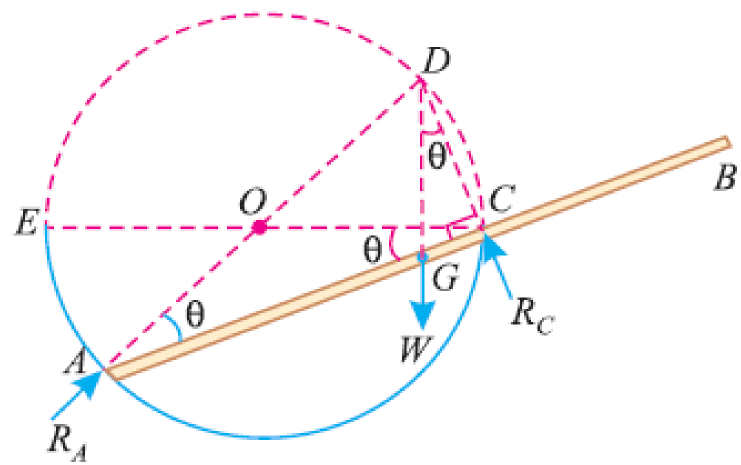


Fig. 5.22.

From the geometry of the figure we know that

$$AD = 2r$$

$$AC = AD \cos\theta = 2r \cos\theta$$

$$CD = AD \sin\theta = 2r \sin\theta$$

$$AG = GB = 1.5r$$

$$GC = AC - AG = 2r \cos\theta - 1.5r$$

Solution: 4.10 cont…

From the geometry of the figure we also find that

$$\angle GDC = \theta$$

$$\therefore \qquad \tan\theta = \tan\angle GDC = \frac{GC}{CD} = \frac{2r\cos\theta - 1.5r}{2r\sin\theta} = \frac{r(2\cos\theta - 1.5)}{2r\sin\theta}$$

$$\text{or} \qquad \frac{\sin\theta}{\cos\theta} = \frac{2\cos\theta - 1.5}{2\sin\theta}$$

$$2\sin^2\theta = 2\cos^2\theta - 1.5\cos\theta$$

$$2(\cos^2\theta - \sin^2\theta) = 1.5\cos\theta$$

$$2(2\cos^2\theta - 1) = 1.5\cos\theta$$

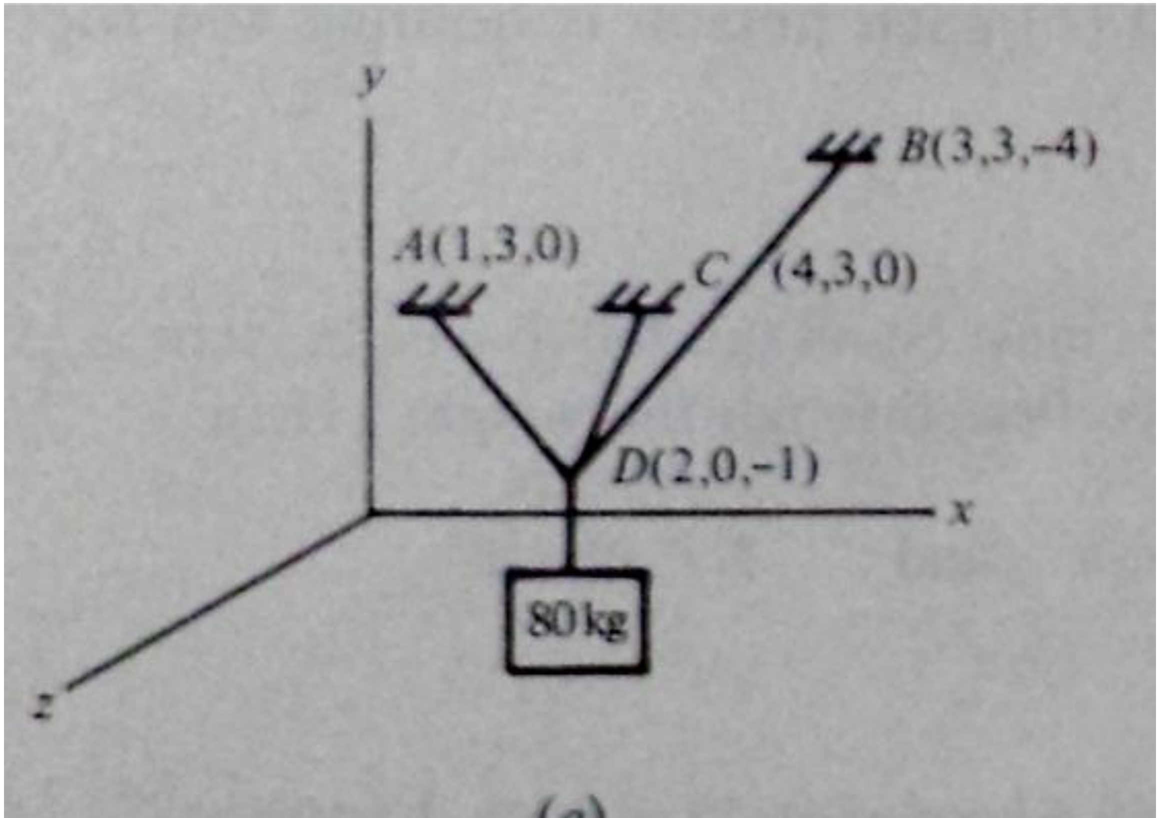
$$4\cos^2\theta - 1.5\cos\theta - 2 = 0$$

Solving it as a quardrotic equation,

$$\cos\theta = \frac{1.5 + \sqrt{2.25 + 32}}{8} = 0.9 \quad \text{or} \quad \theta = 25.8° \quad \textbf{Ans.}$$

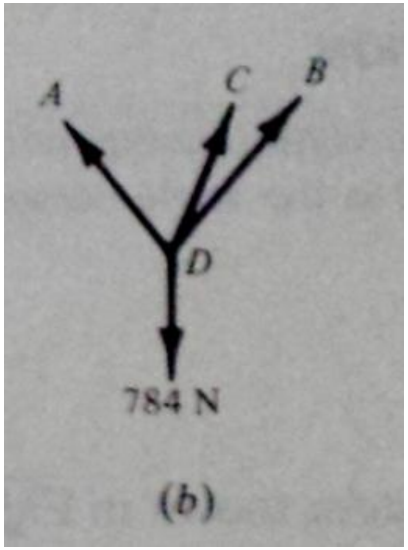
Problems: 4.11

- The 80 kg mass in Figure supported by three wires concurrent at D (2,0,-1). The wires are attached to points A (1,3,0), B (3,3,-4), and C(4,3,0). The coordinates are in meters. Determine the tension in the wire attached to C.



(a)

Solution: 4.11



The tension in the wire attached from D to C is most easily found by setting the sum of the moments about the line AB equal to zero. In this equation the moments of the forces in DA and DB will be zero because these forces intersect AB. The only forces with moments about AB will be the force in DC and gravity force (80 x 9.8 N in the negative j direction)

Tension in DC:

$$C = C\frac{(4-2)i + (3-0)j + [0-(-1)]k}{\sqrt{(2)^2 + (3)^2 + (+1)^2}} = C\frac{2i + 3j + k}{\sqrt{14}}$$

To find sum of moments about the line AB, we need to find the unit vector along the line. Hence,

$$e_{AB} = C\frac{(3-1)i + (3-3)j + (-4-0)k}{\sqrt{20}} = \frac{i - 2k}{\sqrt{5}}$$

Solution: 4.11 cont…

The moments of the tension C and gravity force can be found about any point on the line AB. Let us choose point A. The position vector for both forces as chosen here will be from A to D. Hence,

$$r_{AD} = (2-1)i + (0-3)j + (-1-0)k = i - 3j - k$$

The sum of the moments of the two forces will be

$$\sum r\,x\,F = \left\|\begin{bmatrix} i & j & k \\ +1 & -3 & -1 \\ 0 & -784 & 0 \end{bmatrix}\right\| + \frac{C}{\sqrt{14}}\left\|\begin{bmatrix} i & j & k \\ +1 & -3 & -1 \\ +2 & +3 & +1 \end{bmatrix}\right\|$$

$$= -1\,x\,784i - 784k + \frac{C}{\sqrt{14}}(0i - 3j + 9k)$$

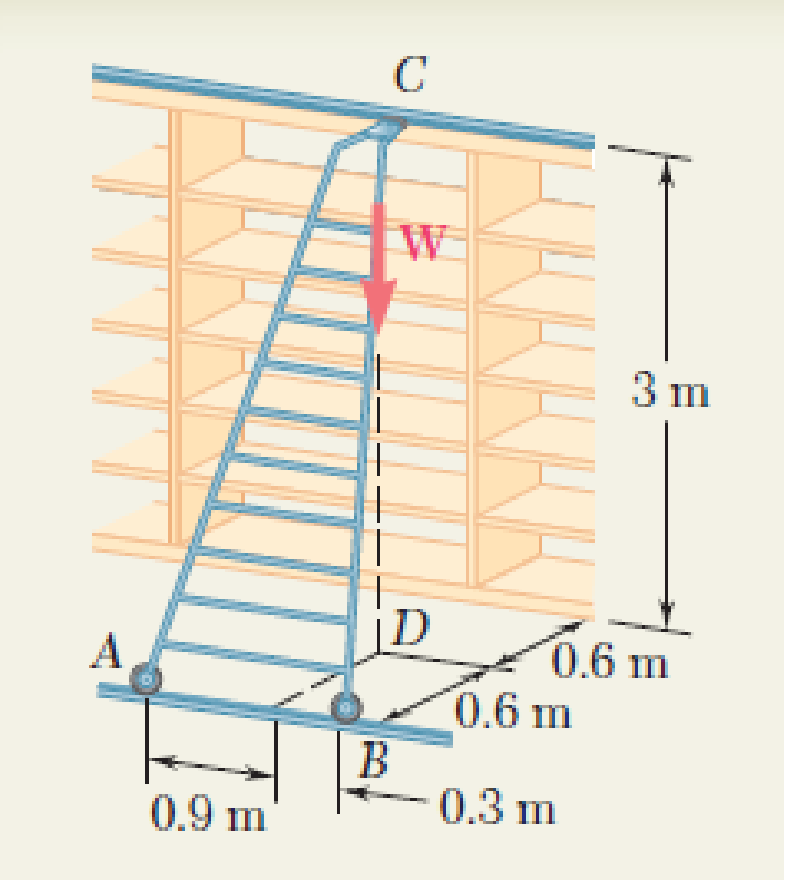
Finally,

$$e_{AB}\,.\sum r\,x\,F = 0$$

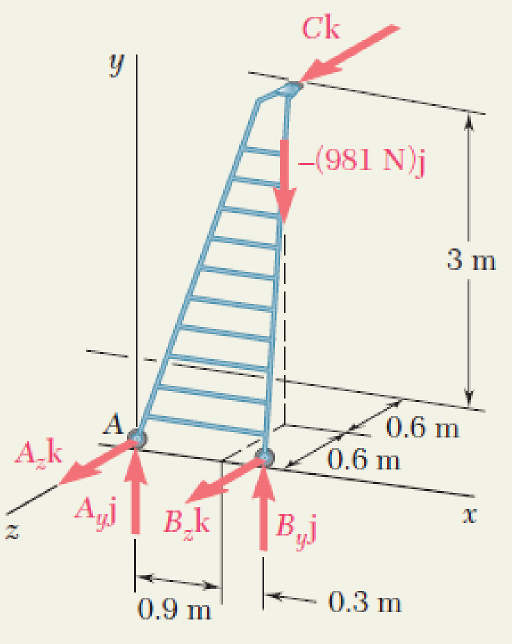
This yields C = 163 N.

Problems: 4.12

- A 20-kg ladder used to reach high shelves in a storeroom is supported by two flanged wheels A and B mounted on a rail and by an unflanged wheel C resting against a rail fixed to the wall. An 80-kg man stands on the ladder and leans to the right. The line of action of the combined weight W of the man and ladder intersects the floor at point D. Determine the reaction at A, B, and C.



Solution: 4.12



Free-Body Diagram. A free-body diagram of the ladder is drawn. The forces involved are the combined weight of the man and ladder,

$$\mathbf{W} = -mg\mathbf{j} = -(80 \text{ kg} + 20 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(981 \text{ N})\mathbf{j}$$

and five unknown reaction components, two at each flanged wheel and one at the unflanged wheel. The ladder is thus only partially constrained; it is free to roll along the rails. It is, however, in equilibrium under the given load since the equation $\Sigma F_x = 0$ is satisfied.

Equilibrium Equations. We express that the forces acting on the ladder form a system equivalent to zero:

$$\Sigma\mathbf{F} = 0: \quad A_y\mathbf{j} + A_z\mathbf{k} + B_y\mathbf{j} + B_z\mathbf{k} - (981 \text{ N})\mathbf{j} + C\mathbf{k} = 0$$

$$(A_y + B_y - 981 \text{ N})\mathbf{j} + (A_z + B_z + C)\mathbf{k} = 0 \qquad (1)$$

$$\Sigma\mathbf{M}_A = \Sigma(\mathbf{r} \times \mathbf{F}) = 0: \quad 1.2\mathbf{i} \times (B_y\mathbf{j} + B_z\mathbf{k}) + (0.9\mathbf{i} - 0.6\mathbf{k}) \times (-981\mathbf{j}) + (0.6\mathbf{i} + 3\mathbf{j} - 1.2\mathbf{k}) \times C\mathbf{k} = 0$$

Computing the vector products, we have†

$$1.2B_y\mathbf{k} - 1.2B_z\mathbf{j} - 882.9\mathbf{k} - 588.6\mathbf{i} - 0.6C\mathbf{j} + 3C\mathbf{i} = 0$$

$$(3C - 588.6)\mathbf{i} - (1.2B_z + 0.6C)\mathbf{j} + (1.2B_y - 882.9)\mathbf{k} = 0 \qquad (2)$$

Solution: 4.12 cont...

Setting the coefficients of **i**, **j**, **k** equal to zero in Eq. (2), we obtain the following three scalar equations, which express that the sum of the moments about each coordinate axis must be zero:

$$\begin{aligned} 3C - 588.6 &= 0 & C &= +196.2 \text{ N} \\ 1.2B_z + 0.6C &= 0 & B_z &= -98.1 \text{ N} \\ 1.2B_y - 882.9 &= 0 & B_y &= +736 \text{ N} \end{aligned}$$

The reactions at B and C are therefore

$$\mathbf{B} = +(736 \text{ N})\mathbf{j} - (98.1 \text{ N})\mathbf{k} \qquad \mathbf{C} = +(196.2 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

Setting the coefficients of **j** and **k** equal to zero in Eq. (1), we obtain two scalar equations expressing that the sums of the components in the y and z directions are zero. Substituting for B_y, B_z, and C the values obtained above, we write

$$\begin{aligned} A_y + B_y - 981 &= 0 & A_y + 736 - 981 &= 0 & A_y &= +245 \text{ N} \\ A_z + B_z + C &= 0 & A_z - 98.1 + 196.2 &= 0 & A_z &= -98.1 \text{ N} \end{aligned}$$

We conclude that the reaction at A is $\mathbf{A} = +(245 \text{ N})\mathbf{j} - (98.1 \text{ N})\mathbf{k} \quad \blacktriangleleft$

†The moments in this sample problem and in Sample Probs. 4.8 and 4.9 can also be expressed in the form of determinants (see Sample Prob. 3.10).