What is a Robot?

Virtually anything that operates with some degree of autonomy, usually under computer control, has at some point been called Robot.

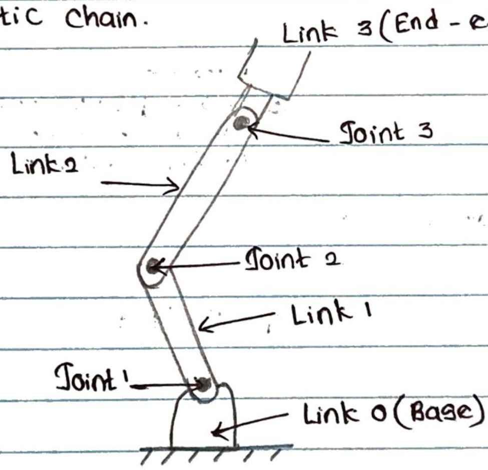
Mechanical arm operating under computer controls.

Advantages : Labour Cost ↓
Precision and productivity ↑
Flexibility ↑
Repetitive or Hazardous jobs are done by Robots.

CNC was developed due to high precision required in the machining of certain items.

Components and the structures of Robots.

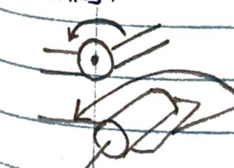
Robot Manipulators are composed of links connected by joints into kinematic chain.

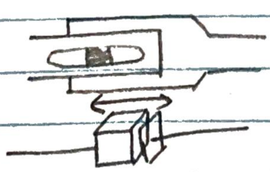


Kinematics is a branch of classical mechanics that describes the motion of points, bodies and systems of bodies without considering the forces that caused the motion.

Joint

- Rotary (R) (revolute): a hinge and allows rotation between two links.
- linear (P) (prismatic): allows linear relative motion between two links.





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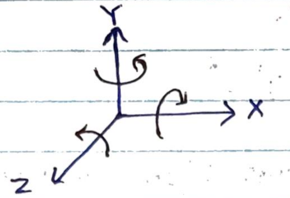
Degree of Freedom and ~~Woke~~ Workspace.

The no. of joints determines the degrees-of-freedom (DOF) of the manipulator.
A Manipulator should process at least 6 independent DOF.

- 3 for positioning.
- 3 for orientation. (Rotation around X, Y, Z axes seperately)



- To reach around or behind obstacles require more tha 6 DOF.
- A Manipulator which has more than 6 links is referred to as a kinematically redundant manipulator.

Workspace of a manipulator is the total volume swept out by the end effector as the manipulator execute all possible motions.

Workspace

- Reachable
  - Entire ~~workspace~~ set of points that manipulator can reach.
- dextrous.
  - those points that manipulator can reach with an arbitary orientation of the end effector.

Workspace depends on,
-
-
- Type of joints

Classification of Robots.

- power source
- way of joints are actuated (hydrolic, pneumatics, Electrically)
- their geometry
- Kinematic structure
- intended application area
- Method of control.

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Power Source

Hydraulically, Electrically or pneumatically powered.

Power Source

Hydraulic
- speed of response and torque producing capability is high.
- Used for lifting heavy loads.
- drawbacks are,-
  - leak hydraulic fluids.
  - require more equipment.

Electrica
- Driven by AC or DC motors.
- Cheaper, cleaner and quieter.

pneumatic
- inexpensive and simple.
- Cannot be controlled precisely.

Robots (Assembly)

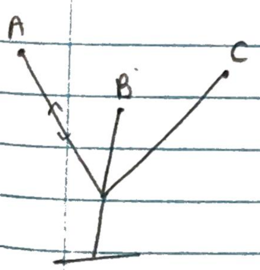
Assembly
- Small
- Electrically driven.

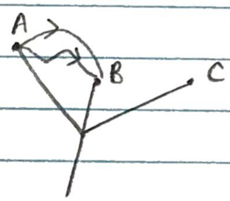
Non-assembly
- Welding.
- Machine loading and unloading.
- Spray painting.
- Material handling.

Robots (Control)

Servo
- point to point Robot.
- Continous path robots.

non-servo





~~Geometry~~

- Point to point robots are severely limited in their range of applications.
- In continous path robots, the entire path of the end-effector can be controlled.

No: ....

Geometry

Articulate Configuration (RRR)
* Has 2/more rotary joints, Usually 6 DOF
* specially like human arm - shoulder, elbow, wrist

* Workspace is shaped like a part of sphere.
* Rotary, Pivot(lb), prismatic joints.

Spherical Configuration (RRP)

SCARA Configuration (RRP)
Fast, precise horizontal movements, two rotary, one prismatic, workspace is shaped like a doughnut slice.

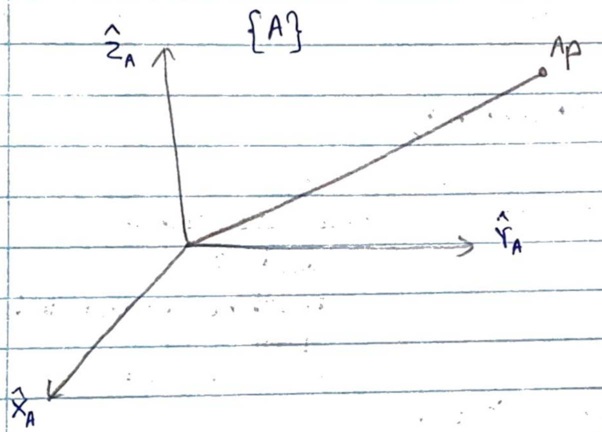
* workspace is shaped as a cylinder.
* 2 prismatic, one rotary joints.

Cylindrical Configutation (RPP)

Cartesia Configuration (PPP)
* Motion is purely linear along 3 cartesian planes.
* workspace is shaped as a rectangular box.

Five geometric Types.
Each of these five configurations are serial link robots.

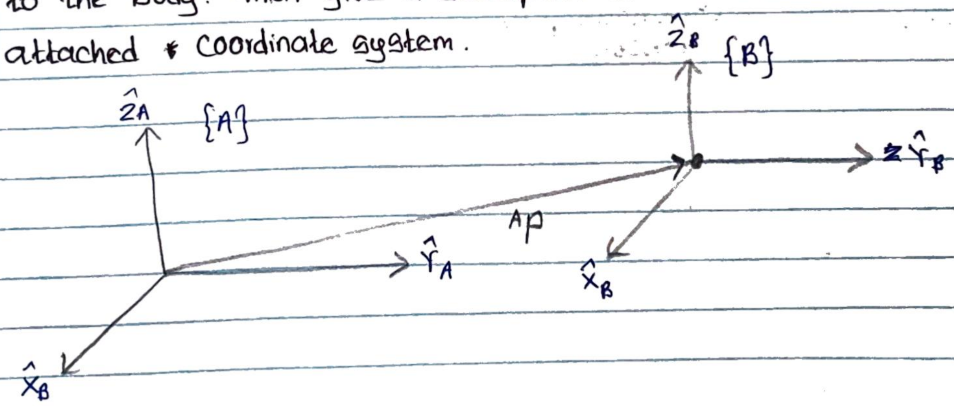
Description of a Position



$$Ap = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

Description of an orientation.

To decribe orientation of a body we attach coordinate system to the body. Then give a description of orientation w.r.t. the attached coordinate system.



when written in terms of coordinate system {A},

$${}^{A}X_B, \quad {}^{A}Y_B, \quad {}^{A}Z_B$$

$$\begin{bmatrix} X_B \cdot X_A\, r_{11} \\ X_B \cdot Y_A\, r_{21} \\ X_B \cdot Z_A\, r_{31} \end{bmatrix} \begin{bmatrix} Y_B \cdot X_A\, r_{12} \\ Y_B \cdot Y_A\, r_{22} \\ Y_B \cdot Z_A\, r_{32} \end{bmatrix} \begin{bmatrix} Z_B \cdot X_A \\ Z_B \cdot Y_A \\ Z_B \cdot Z_A \end{bmatrix} \begin{matrix} r_{13} \\ r_{23} \\ r_{33} \end{matrix}$$

~~rotation Matrix.~~ each are unit vectors, when stack up togethers rotation Matrix.

Rotation Matrix = ${}^A_B R$ (describes $\{B\}$ relative to $\{A\}$).

$$
{}^B_A R = \begin{bmatrix} X_A \cdot X_B & Y_A \cdot X_B & Z_A \cdot X_B \\ X_A \cdot Y_B & Y_A \cdot Y_B & Z_A \cdot Y_B \\ X_A \cdot Z_B & Y_A \cdot Z_B & Z_A \cdot Z_B \end{bmatrix}.
$$

$$
\boxed{{}^B_A R = {}^A_B R^T}
$$

$$
{}^A_B R^T \cdot {}^A_B R = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
$$

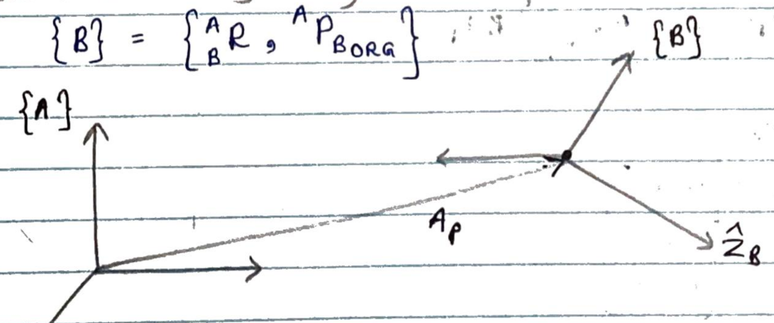
$$\Downarrow$$

$$
{}^A_B R = {}^B_A R^{-1} = {}^B_A R^T.
$$

Description of a Frame.

The point whose position we will describe, is chosen as the origin of the body-attached frame. (මෙතන Ap කියන point එක තමයි $\{B\}$ කියන frame එකේ origin එක.)

$$
\{B\} = \{{}^A_B R, {}^A P_{BORG}\}
$$



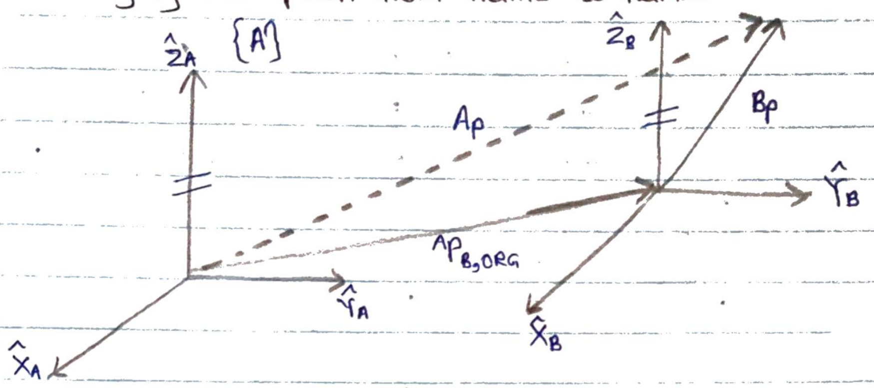
Locating an object in position and Orientation.

Frame representing both position and Orientation.

* Frame is a local coordination system that helps describe position and orientation of a robot.

Mapping

Changing description from frame to frame.



Here no any rotation is involved.

Mapping ~~involves~~ involving ~~rotated~~ translated frames.

Both frames has the same ~~origin~~ orientation. ~~as~~ {B} differs from {A} only by translation.

$$^A p = {}^B p + {}^A P_{B,ORG}$$

$^A P_{B,ORG}$ = Vector that locates the origin of {B} relative to {A}.

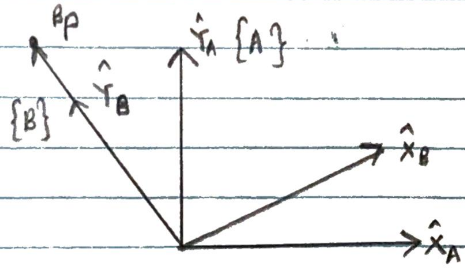
Mapping involving rotated frames.

$$^A p_x = \left[{}^A \hat{X}_B\right]^T {}^B p$$

Positive angles are occured in anti-clockwise rotations.

Example:

$$^B p = \begin{bmatrix} 0.0 \\ 2.0 \\ 0.0 \end{bmatrix}$$



$$^A_B R = \left[{}^A \hat{X}_B \quad {}^A \hat{Y}_B \quad {}^A \hat{Z}_B\right]$$

Unit vectors of {B} written in {A}. (columns)

$$\begin{array}{c|c} & \begin{array}{ccc} x_B & y_B & z_B \end{array} \\ \begin{array}{c} x_A \\ y_A \\ z_A \end{array} & = \begin{bmatrix} \hat{X}_B \cdot \hat{X}_A & \hat{Y}_B \, \hat{X}_A & \hat{Z}_B \, \hat{X}_A \\ \hat{X}_B \, \hat{Y}_A & \hat{Y}_B \, \hat{Y}_A & \hat{Z}_B \, \hat{Y}_A \\ \hat{X}_B \, \hat{Z}_A & \hat{Y}_B \, \hat{Z}_A & \hat{Z}_B \, \hat{Z}_A \end{bmatrix} \end{array}$$

Unit vectors of {A} written in {B}. (rows)

$$= \begin{bmatrix} \cos 30^\circ & \cos 120^\circ & \cos 90^\circ \\ \cos 60^\circ & \cos 30^\circ & \cos 90^\circ \\ \cos 90^\circ & \cos 90^\circ & \cos 90^\circ \end{bmatrix} = \begin{bmatrix} 0.86 & -0.5 & 0 \\ 0.5 & 0.86 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^{A}p = {}^{A}_{B}R \cdot {}^{B}p$$

$$= \begin{bmatrix} 0.86 & -0.5 & 0 \\ 0.5 & 0.86 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1.72 \\ 0 \end{bmatrix}$$

Mapping involving General frames

(General frames involve both rotational and translational mappings).

To do this change ^{B}p to 1st its description relative to a intermediate frame has the same orientation as $\{A\}$, whose origin coincident with origin of $\{B\}$.

$$\begin{bmatrix} ^{A}p \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{B,ORG} \\ 0\,0\,0 & 1 \end{bmatrix} \begin{bmatrix} ^{B}p \\ 1 \end{bmatrix}.$$

$$^{A}p = {}^{A}_{B}R \; {}^{B}p + {}^{A}P_{B\,ORG}$$

$$^{A}p = {}^{A}_{B}T \; {}^{B}p$$

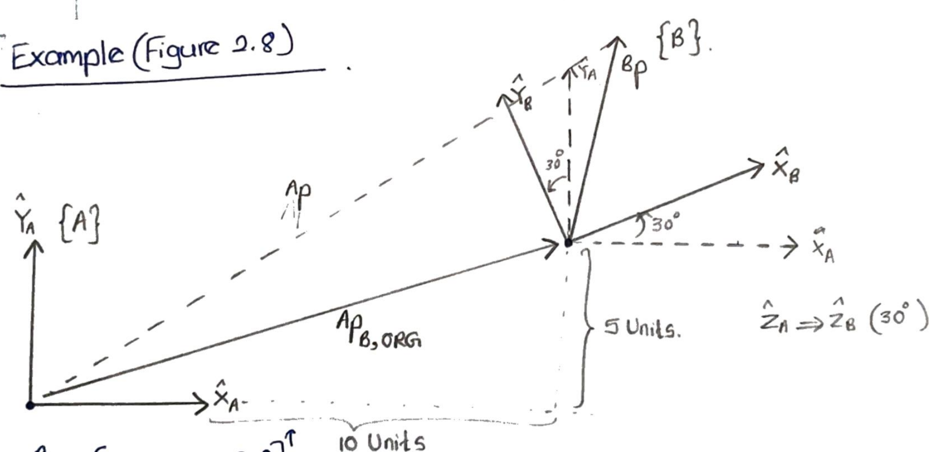
$$I_p = {}^{A}_{B}R \cdot {}^{B}p \;\Big\}\; \text{Intermediate Matrix.}$$

$^{A}_{B}$

	Rotation	Translation
${}^{A}_{B}T =$	${}^{A}_{B}R$	${}^{A}P_{BORG}$
	000	1

Example (Figure 2.8).

Example (Figure 2.8)



$$^{B}p = [3.0 \;\; 7.0 \;\; 0.0]^{T}$$

$$^{A}p = {}^{A}T_{B} \cdot {}^{B}p.$$

$$^{A}T_{B} = \left[\begin{array}{c|c} ^{A}R_{B} & ^{A}P_{B,ORG} \\ \hline 000 & 1 \end{array}\right]$$

$$^{A}R_{B} = \begin{bmatrix} \hat{X}_B\hat{X}_A & \hat{Y}_B\hat{X}_A & \hat{Z}_B\hat{X}_A \\ \hat{X}_B\hat{Y}_A & \hat{Y}_B\hat{Y}_A & \hat{Z}_B\hat{Y}_A \\ \hat{X}_B\hat{Z}_A & \hat{X}_B\hat{Z}_A & \hat{Z}_B\hat{Z}_A \end{bmatrix} = \begin{bmatrix} \cos 30^\circ & \cos 120^\circ & \cos 90^\circ \\ \cos 60^\circ & \cos 30^\circ & \cos 90^\circ \\ \cos 90^\circ & \cos 90^\circ & \cos 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^{A}P_{B,ORG} = \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}$$

$$^{A}T_{B} = \begin{bmatrix} 0.866 & -0.5 & 0 & 10 \\ 0.5 & 0.866 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$^{A}p = \begin{bmatrix} 0.866 & -0.5 & 0 & 10 \\ 0.5 & 0.866 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \\ 0 \\ 1 \end{bmatrix}$$

$${}^{B}P = [3.0 \;\; 7.0 \;\; 0.0]^T$$

$${}^{A}P = {}^{A}T_B \cdot {}^{B}P.$$

$${}^{A}T_B = \left[\begin{array}{c|c} {}^{A}R_B & {}^{A}P_{B,ORG} \\ \hline 000 & 1 \end{array}\right]$$

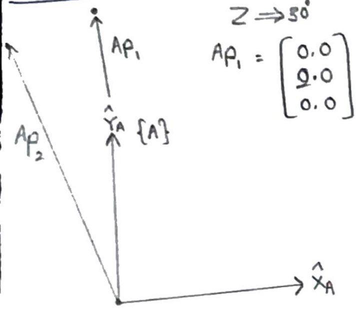
$${}^{A}R_B = \begin{bmatrix} \hat{X}_B \hat{X}_A & \hat{Y}_B \hat{X}_A & \hat{Z}_B \hat{X}_A \\ \hat{X}_B \hat{Y}_A & \hat{Y}_B \hat{Y}_A & \hat{Z}_B \hat{Y}_A \\ \hat{X}_B \hat{Z}_A & \hat{X}_B \hat{Z}_A & \hat{Z}_B \hat{Z}_A \end{bmatrix} = \begin{bmatrix} \cos 30^\circ & \cos 120^\circ & \cos 90^\circ \\ \cos 60^\circ & \cos 30^\circ & \cos 90^\circ \\ \cos 90^\circ & \cos 90^\circ & \cos 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad {}^{A}P_{B,ORG} = \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}$$

$${}^{A}T_B = \begin{bmatrix} 0.866 & -0.5 & 0 & 10 \\ 0.5 & 0.866 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad {}^{A}P = \begin{bmatrix} 0.866 & -0.5 & 0 & 10 \\ 0.5 & 0.866 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \\ 0 \\ 1 \end{bmatrix}$$

$${}^{A}P = \begin{bmatrix} 9.09 \\ 12.56 \\ 0 \end{bmatrix} //$$

Example (Figure 2.10)



$Z \Rightarrow 30^\circ$

$${}^{A}P_1 = \begin{bmatrix} 0.0 \\ 2.0 \\ 0.0 \end{bmatrix}$$

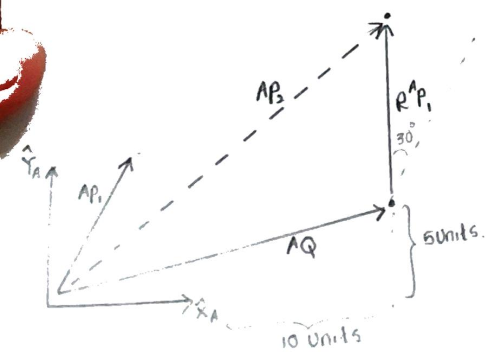
$$R_z(30) = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$${}^{A}P_2 = R_z(30) \cdot {}^{A}P_1$$

$$= \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1.73 \\ 0 \end{bmatrix}.$$

Example ~~(2.11)~~ (Figure 2.11).



$$T = \begin{bmatrix} 0.866 & -0.5 & 0 & 10 \\ 0.5 & 0.866 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}P_1 = \begin{bmatrix} 3 \\ 7 \\ 0 \end{bmatrix}.$$

$${}^{A}P_2 = T \cdot {}^{A}P_1$$

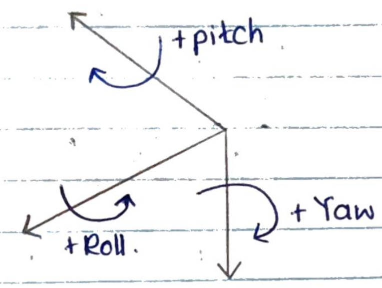
$$= \begin{bmatrix} 0.866 & -0.5 & 0 & 10 \\ 0.5 & 0.866 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \\ 0 \\ 1 \end{bmatrix}$$

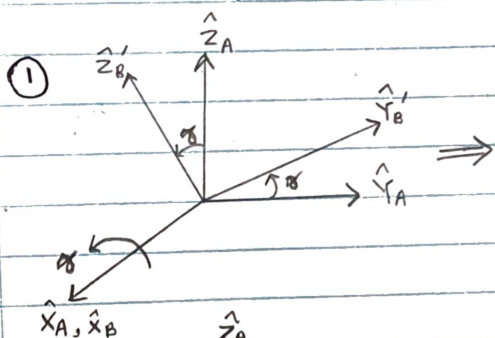
$$= \begin{bmatrix} 9.098 \\ 12.562 \end{bmatrix}$$

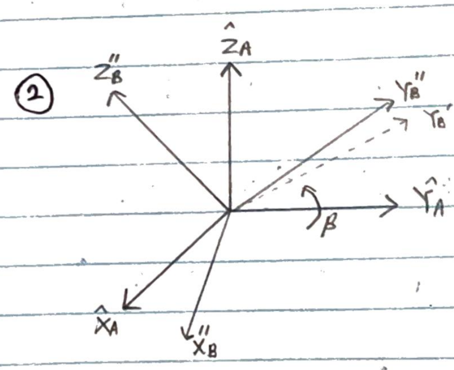
Fixed angles

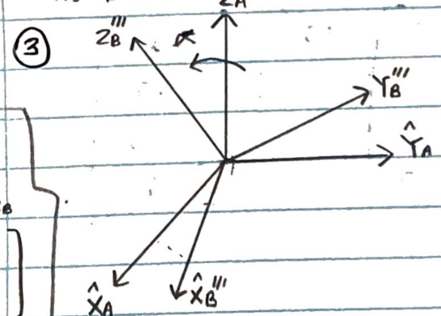
$\cancel{{}^A_B R_{XYZ}(\gamma, \beta, \alpha) = c\alpha c\beta \quad c\alpha s\beta}$

1. Rotate $\{B\}$ first about $\hat{X}_A$ by an angle = ~~roo~~ roll
2. Rotate $\{B\}$ about $\hat{Y}_A$ by an angle β = pitch
3. Rotate $\{B\}$ about $\hat{Z}_A$ by an angle = Yaw









$$
{}^A_B R \quad
\begin{array}{c|c}
 & X_B \; Y_B \; Z_B \\
\begin{array}{c} X_A \\ Y_A \\ Z_A \end{array} & \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}
\end{array}
$$

Rotations are performed in the order of $R_X(\gamma)$, $R_Y(\beta)$, $R_Z(\alpha)$.

Derivation of the equivalant rotational Matrix,

$$
{}^A_B R_{XYZ}(\gamma, \beta, \alpha) = R_Z(\alpha) R_Y(\beta) R_X(\gamma).
$$

$$
= \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix}.
$$

$$
= \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}
$$

Manipulator Kinematics

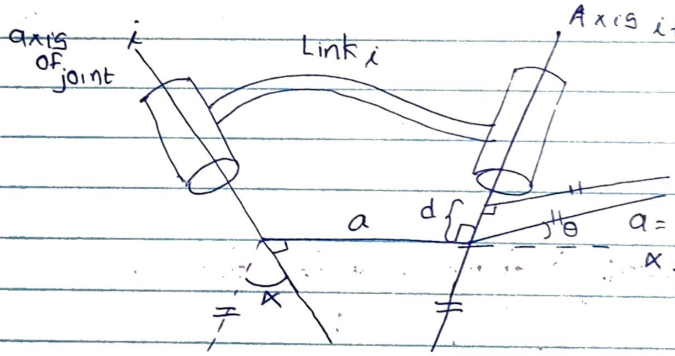
Manipulator may be thought of as a set of 'bodies' connected in a chain by joints. These bodies are called links.

Lower pair is used to describe the connection between a pair of bodies. When the relative motion is characterized by two surfaces sliding over one another.

The six possible lower-pair joints,

- Spherical
- Revolute
- Planar
- Prismatic.
- Cylindrical.
- Screw.

The kinematic function of a link is to maintain a fixed relationship between the two joint axes it supports.



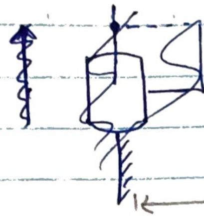
a = link length.
α = link twist
(∡ betⁿ the axes)
} fixed parameters.

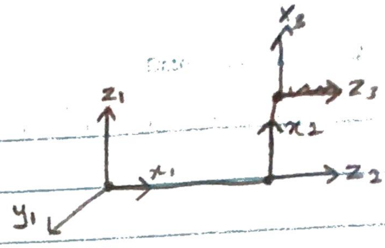
d_i = link offset - Variable in linear joint, $d \longrightarrow$ constant. (0)

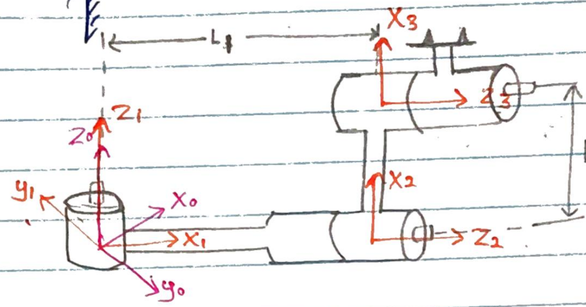
θ = joint angle - Variable in revolute joint. , $\theta \longrightarrow$ constant (0).

* z axis of the frame should always align with the axis of the joint.

* The z axis of the end tool should be as same as the joint before it.







a_{i-1} = the distance from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured along $\hat{X}_i$.

α_{i-1} = the angle from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured about $\hat{X}_i$.

d_i = the distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured along $\hat{Z}_i$

θ_i = the angle from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured about $\hat{Z}_i$

Derivation of link transformations.

$a_i, \alpha_i, d_i, \theta_i$, Only one from these are varible all ~~the~~ others are fixed by mechanical design.

a_i, d_i – linear / prismatic

α_i, θ_i – Revolutional.

* Representing point in frame i in the frame $i-1$

$$^{i-1}P = \underbrace{\left(\underset{\alpha_i}{^{i-1}_{R}T}\ \underset{a_i}{^{R}_{Q}T}\ \underset{\theta_i}{^{Q}_{P}T}\ \underset{d_i}{^{P}_{i}T} \right)}_{^{i-1}_{i}T} {}^{i}P$$

(Refer the figure 3.15 in P-75)

$$^{i-1}_{i}T = \underbrace{R_X(\alpha_{i-1})}_{\text{Rotational}}\ \underbrace{D_X(a_{i-1})}_{\text{translation (By distance)}}\ R_X(\theta_i)\ D_X(\theta_{i-1})$$

$$\begin{array}{ccc} i & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{array} \qquad \begin{array}{ccc} c & -s & 0 \\ s & c & 0 \\ 0 & & c\,i \end{array}$$

$$
{}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

$$
{}^1_2T = 1
$$

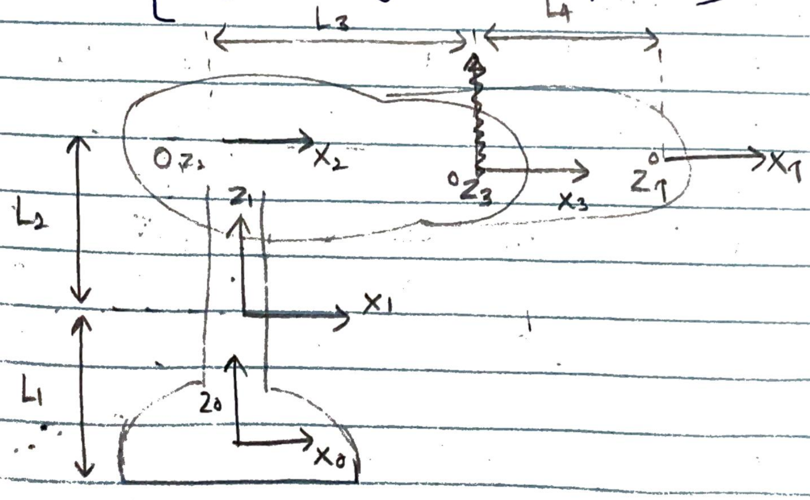
Example 3.~~2~~4

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	~~90°~~ 0	0	L_1	θ_1
2	90° ~~0~~	~~L_2~~ 0	~~0~~ L_2	θ_2
3	0	L_3	0	θ_3
4	0	L_4	0	0

$$
{}^0_1T = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \\ & & & \\ & & & \end{bmatrix}
$$

$$
{}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_3 \cancel{\cos c\theta_2} \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



stationary fram

$$ {}^{S}_{T}T = {}^{0}_{1}T \; {}^{1}_{2}T \; {}^{2}_{3}T \; {}^{3}_{4}T $$

Transformational frame.

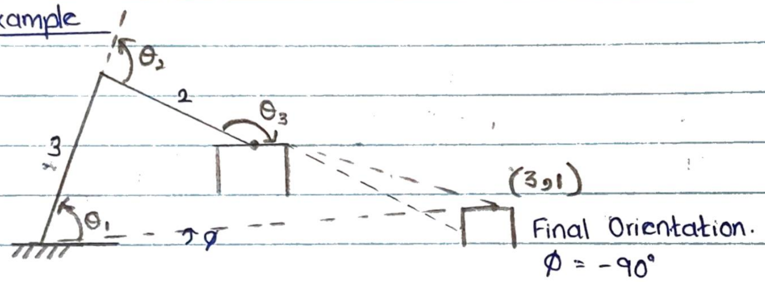
Inverse Kinematics.

Finding the joint angle by the given positions and orientation.

Dextrous space - The reachable workspace that the robot can achieve in any orientation. (At least one Orientation)

$$C_{123} = \cos(\theta_1+\theta_2+\theta_3)$$
$$C_{12} = \cos(\theta_1+\theta_2)$$

Example



$$\begin{bmatrix} R(\theta_1) & & 0 \\ & & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R(\theta_2) & & 3 \\ & & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R(\theta_3) & & 2 \\ & & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta_1) = \begin{bmatrix} C\theta_1 & -S\theta_1 \\ S\theta_1 & C\theta_1 \end{bmatrix}$$ Common matrix for all Rotational matrices.

$$R(\theta_2) = \begin{bmatrix} C\theta_2 & -S\theta_2 \\ S\theta_2 & C\theta_2 \end{bmatrix} \qquad R(\theta_3) = \begin{bmatrix} C\theta_3 & -S\theta_3 \\ S\theta_3 & C\theta_3 \end{bmatrix}$$

Final Rotational matrix,

$$\begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} C\theta_2 & -S\theta_2 & 3 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} C\theta_3 & -S\theta_3 & 2 \\ S\theta_3 & C\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_1 . C\theta_2 + -S\theta_1 . S\theta_2 & -C\theta_1 . S\theta_2 \; - S\theta_1 C\theta_2 & 3C\theta_1 \\ S\theta_1 C\theta_2 + C\theta_1 . S\theta_2 & -S\theta_1 S\theta_2 + C\theta_1 C\theta_2 & 3S\theta_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 2 \\ S\theta_3 & C\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} C_1C_2 - S_1S_2 & -C_1S_2 - S_1C_2 & 3C_1 \\ S_1C_2 + C_1S_2 & -S_1S_2 + C_1C_2 & 3S_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 2 \\ S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\cancel{C_3(C_1C_2 - S_1S_2) + -S_3(C_1S_2 + S_1C_2) + 0}$

$$= \begin{bmatrix} C_3(C_1C_2 - S_1S_2) - S_3(C_1S_2 + S_1C_2) & -S_3(C_1C_2 - S_1S_2) - C_3(C_1S_2 + S_1C_2) & 2(C_1C_2 - S_1S_2) + 3C_1 \\ C_3(S_1C_2 + C_1S_2) - S_3(-S_1S_2 + C_1C_2) & -S_3(S_1C_2 + C_1S_2) + C_3(-S_1S_2 + C_1C_2) & 2(S_1C_2 + C_1S_2) + 3S_1 \\ 0 & 0 & 1 \end{bmatrix}$$

=

= Answer

$$\begin{bmatrix} C_{123} & -\sin_{23} & 3C_1 + 2C_{12} \\ S_{123} & C_{123} & 3S_1 + 2S_{12} \\ 0 & 0 & 1 \end{bmatrix} \leftarrow$$ RRR Planer Manipulator.

$C\phi = C(\theta_1 + \theta_2 + \theta_3)$

$-90 = \theta_1 + \theta_2 + \theta_3$ — (1)

$3 = 3C(\theta_1) + 2C(\theta_1 + \theta_2)$ — (2)

$1 = 3S(\theta_1) + 2S(\theta_1 + \theta_2)$ — (3)

$(2)^2 + (3)^2 \Longrightarrow$

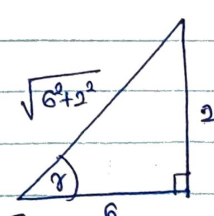
$(3 - 3\cos\theta_1)^2 + (1 - 3S\theta_1)^2 = 2^2C^2(\theta_1 + \theta_2) + 2^2S^2(\theta_1 + \theta_2)$

$9 - 18C_1 + 9C_1^2 + 1 - 6S_1 + 9S_1^2 = 4(1)$

$18C\theta_1 + 6S\theta_1 = 15$

$\dfrac{18C\theta_1}{\sqrt{18^2+6^2}} + \dfrac{6S\theta_1}{\sqrt{18^2+6^2}} = \dfrac{15}{\sqrt{18^2+6^2}}$

$\dfrac{6C\theta_1}{2\sqrt{10}} + \dfrac{2S\theta_1}{2\sqrt{10}} = \dfrac{3}{2\sqrt{10}}$



$\cancel{3C\theta_1}$

$\cos\gamma\, C\theta_1 + S\gamma\, S\theta_1 = \dfrac{5}{\sqrt{6^2+2^2}}$

$\cos(\theta_1 - \gamma) = \pm\ 0.79$

$\theta_1 - \gamma = \cos^{-1}(\pm 0.79)$; $\gamma = 18.43$

~~On~~ Considering (+),

$\cancel{\theta_1 - 18.43 = 37.81}$

$\cancel{\theta_1 = 56.24^\circ}$

~~Considering (-)~~

$\cancel{\theta_1 - 18.43 = 142.18}$

$\cancel{\theta_1 = 160.61^\circ}$

$\theta_1 = \gamma \pm \cos^{-1}(0.79)$

$= 18.43 \pm 37.81$

(+), $\theta_1 = 56.24^\circ$ (-), $\theta_1 = -19.38^\circ$

$3 - 3C(\theta_1) = 2C(\theta_1+\theta_2)$ — (A)

$1 - 3S(\theta_1) = 2S(\theta_1+\theta_2)$ — (B)

(A)/(B),

$$\tan(\theta_1+\theta_2) = \frac{1-3\sin(\theta_1)}{3-3\cos(\theta_1)}$$

considering $\theta_1 = 56.24^\circ$

$$\tan(56.24+\theta_2) = \frac{1-3\sin(56.24^\circ)}{3-3\cos(56.24)}$$

$56.24+\theta_2 = \tan^{-1}(-1.121)$

$\theta_2 = -104.51^\circ$

Considering $\theta_1 = -19.38^\circ$

$$\tan(-19.38^\circ+\theta_2) = \frac{1-3\sin(-19.38^\circ)}{3+3\cos(19.38^\circ)}$$

$\theta_2 = 104.51^\circ$.

Considering $\theta_1 = 56.24^\circ$, $\theta_2 = -104.51^\circ$

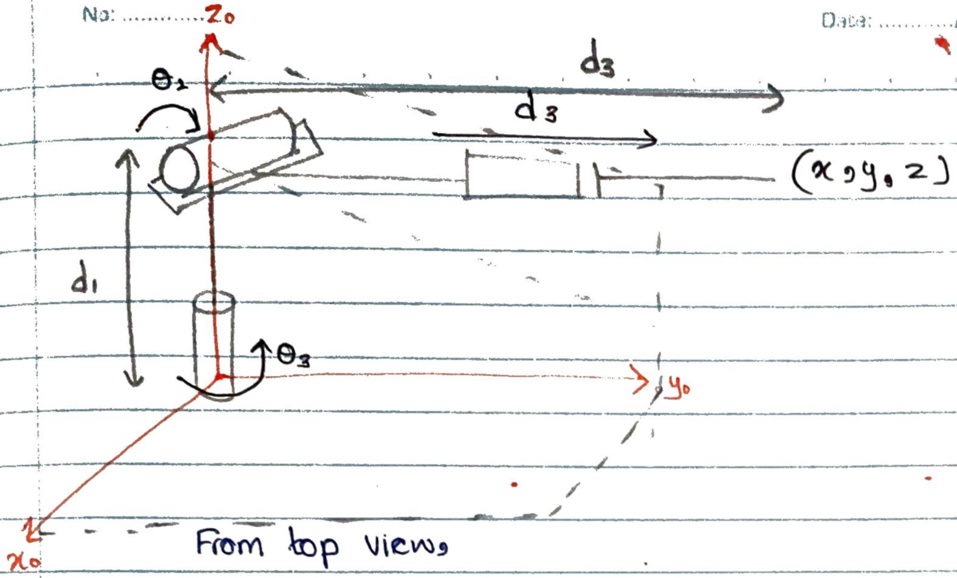
$\theta_3 = -90^\circ - 56.24^\circ + 104.51^\circ$

$= -41.73^\circ$

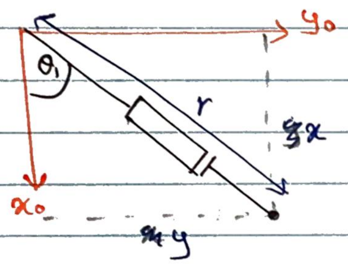
considering $\theta_1 = -19.38^\circ$, $\theta_2 = 104.51^\circ$

$\theta_3 = -175.13^\circ$

Geomatrical approach



From top view,

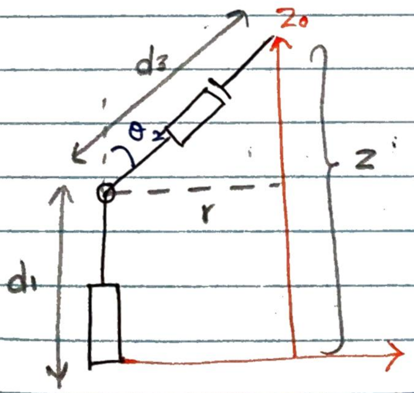


$$\tan\theta_1 = \cancel{\frac{y}{x}} \quad \frac{y}{x}$$

$$\theta_1 = \mathrm{Atan2}\left(\frac{y}{x}\right)$$

$$r = \sqrt{x^2 + y^2}$$

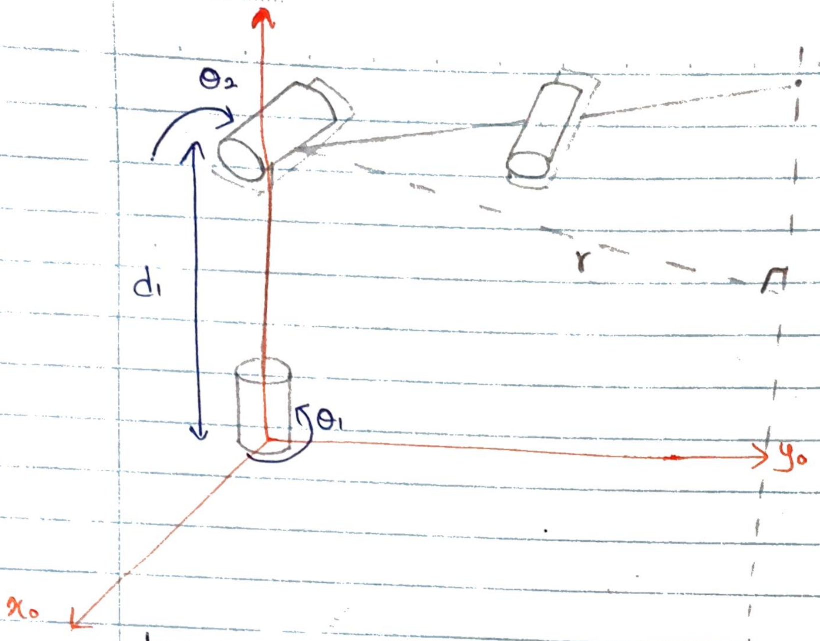
From side view,



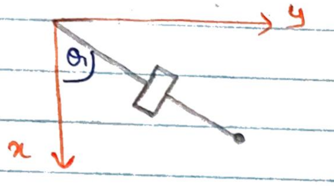
$$\tan\theta_2 = \left(\frac{r}{z - d_1}\right)$$

$$\theta_2 = \mathrm{Atan2}\left(\frac{r}{z - d_1}\right).$$

$$d_3 = \sqrt{r^2 + (z - d_1)^2}$$
$$= \sqrt{x^2 + y^2 + (z - d_1)^2}$$

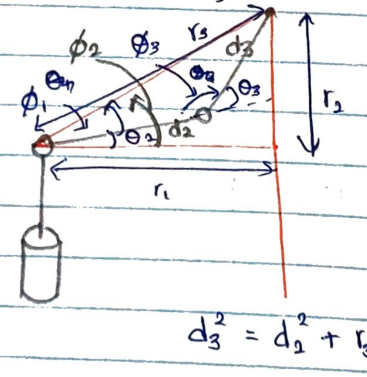


top view,



$$\theta_1 = A\tan 2\left(\frac{y}{x}\right).$$

From side view,



$$\theta_2 = \phi_2 - \phi_1$$

$$\phi_2 = \tan^{-1}\left(\frac{r_2}{r_1}\right)$$

$$r_2 = Z - d$$

$$r_1 = \sqrt{x^2+y^2}$$

$$d_3^2 = d_2^2 + r_3^2 - 2d_2 r_3 \cos\phi_1$$

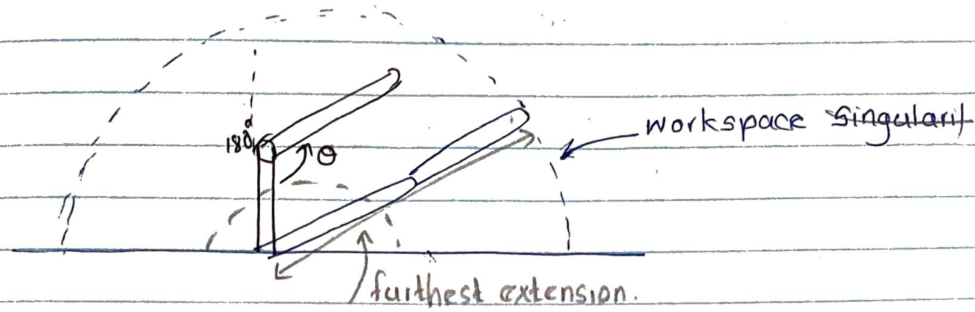
$$\phi_1 = \cos^{-1}\left(\frac{d_2^2 - d_3^2 + r_3^2}{2d_2 r_3}\right) \qquad r_3 = \sqrt{r_1^2 + r_2^2}$$

$$\theta_3 = 180^\circ - \phi_3$$

$$\theta_2 = \phi_2 - \phi_1$$

$$\phi_3 = \cos^{-1}\left(\frac{d_2^2 + d_3^2 - r_3^2}{2d_2 d_3}\right)$$

Kinematics Singularities and Jacobian.



In Jacobian matrix,

If $\det[J] = 0 \longrightarrow$ Singularity exist

$\det[J] \neq 0 \longrightarrow$ Singularity does not exist.

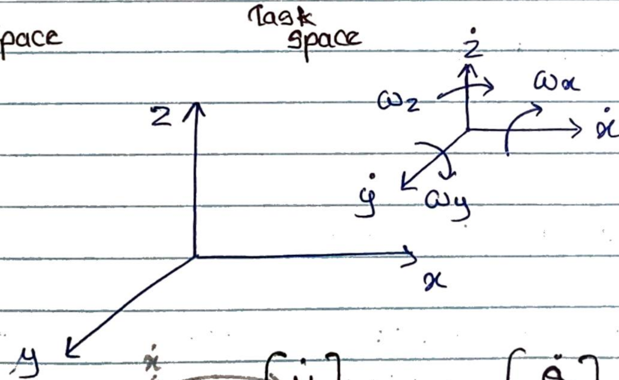
Jacobian Matrix :

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} \underset{\text{Inverse}}{\overset{\text{forward Kinematic}}{\rightleftarrows}} \begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \end{bmatrix}$$

Joint space   Task space

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \end{bmatrix} \xleftrightarrow{\text{Jacobian Matrix}} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Relationship betwee joint space ~~bet~~ with task space velocity.



$\dot{x}, \dot{y}, \dot{z} \rightarrow$ $\omega_x, \omega_y, \omega_z \rightarrow$

$$\begin{bmatrix} V \\ \dot{\omega} \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

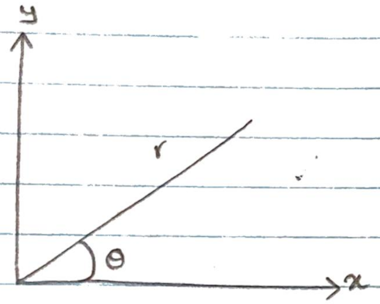
$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = J^{-1} \begin{bmatrix} V \\ \dot{\omega} \end{bmatrix} \quad ; \quad J^{-1} = \begin{bmatrix} \frac{-\sin\theta}{r} & \frac{\cos\theta}{r} \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$M^{-1} = \frac{1}{\det(M)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\frac{dx}{dt} = \frac{\partial (r\cos\theta)}{\partial r} \cdot \frac{dr}{dt} + \frac{\partial (r\cos\theta)}{\partial \theta} \cdot \frac{d\theta}{dt} \Rightarrow \dot{x} = \cos\theta\,\dot{r} - r\sin\theta\,\dot{\theta}$$

$$\frac{dy}{dt} = \frac{\partial (r\sin\theta)}{\partial r} \cdot \frac{dr}{dt} + \frac{\partial (r\sin\theta)}{\partial \theta} \cdot \frac{d\theta}{dt} \Rightarrow \dot{y} = \sin\theta\,\dot{r} + r\cos\theta\,\dot{\theta}$$



$x = r\cos\theta$

$y = r\sin\theta$

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial r} \\ \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial r} \end{bmatrix} \begin{bmatrix} \frac{d\theta}{dt} \\ \frac{d\theta}{dt} \end{bmatrix}.$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} -r\sin\theta & \cos\theta \\ r\cos\theta & \sin\theta \end{bmatrix}}_{\text{Jacobian Matrix} = J} \begin{bmatrix} \dot{\theta} \\ \dot{r} \end{bmatrix}$$

$$\det[J] = -r\sin^2\theta - r\cos^2\theta$$
$$= -r(1)$$

∴ If r is 0, a singularity exist.

01) Partial Derivation Method. → DH Table → Transformation Matrix

02) Velocity propagation Method.

$${}^{0}_{E}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdots$$

$$\begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} \quad {}^{0}_{E}T = \begin{bmatrix} R & P \\ 0\;0\;0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} = {}^{0}J \begin{bmatrix} \theta_1 \\ \theta_n \end{bmatrix}$$

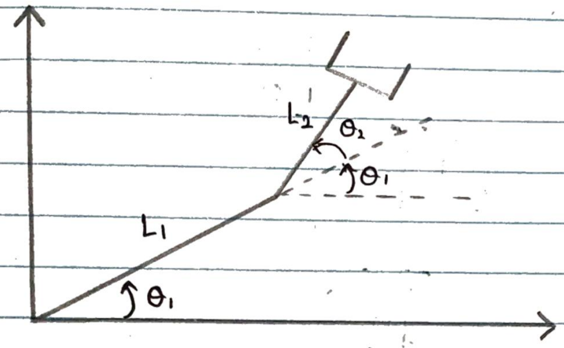
Jacobian in 0^{th} frame.

No: .... Date: ..../......./.......

2. Velocity Propagation Method.

$${}^{E}V = {}^{E}J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}.$$

Jacobian in End effector.



$${}^{0}_{2}T = {}^{0}_{1}T \cdot {}^{1}_{2}T$$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1				
2				

$$T_1^0 = \begin{bmatrix} C_1 & -S_1 & 0 & C_1L_1 \\ S_1 & C_1 & 0 & S_1L_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_2^1 = \begin{bmatrix} C_2 & -S_2 & 0 & C_2L_2 \\ S_2 & C_2 & 0 & S_2L_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^2 = \begin{bmatrix} C_{12} & -S_{12} & 0 & C_1L_1 + L_2C_{12} \\ S_{12} & C_{12} & 0 & S_1L_1 + L_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\dot{X} = (-S_1L_1 - L_2S_{12})\dot{\theta}_1 - L_2S_{12}\dot{\theta}_2$$

$$\dot{Y} = (C_1L_1 + L_2C_{12})\dot{\theta}_1 + L_2C_{12}\dot{\theta}_2$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} -S_1L_1 - L_2S_{12} & -L_2S_{12} \\ C_1L_1 + L_2C_{12} & L_2C_{12} \end{bmatrix}}_{J} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}.$$

$${}^0V = {}^0J\,[\dot{\theta}] \qquad {}^2J = {}^2_0R\,{}^0J$$

Give the J matrix 2 frame,

$${}^2V = {}^2J\,[\dot{\theta}] \qquad {}^0_2R = \begin{bmatrix} C_{12} & -S_{12} & 0 \\ S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^2_0R = {}^0_2R^{-1} \Rightarrow \left[{}^0_2R\right]^T$$

$${}^2J = \begin{bmatrix} -S_1L_1 - L_2S_{12} & -L_2S_{12} & 0 \\ C_1L_1 + L_2C_{12} & L_2C_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{12} & -S_{12} & 0 \\ S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} \quad \end{bmatrix}$$

$$\det[{}^0J] = (-S_1L_1 - L_2S_{12})(L_2C_{12}) + L_2S_{12}(C_1L_1 + L_2C_{12})$$

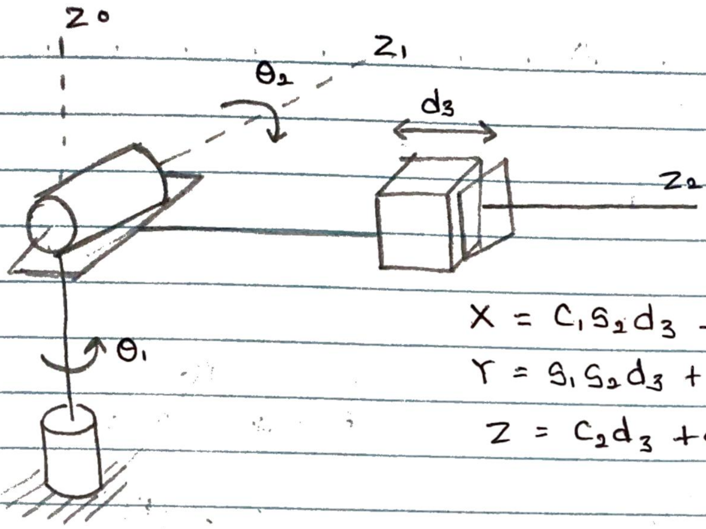
$$0 = -L_1L_2S_1C_{12} - L_2^2S_{12}C_{12} + L_1L_2S_{12}C_1 + L_2^2S_{12}C_{12}$$

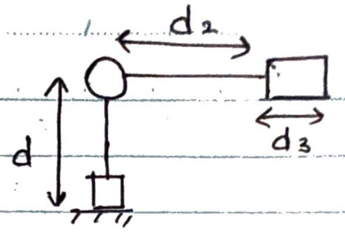
$$= L_1L_2(-S_1C_{12} + S_{12}C_1)$$

$$= L_1L_2[\sin(\theta_1+\theta_2)\cos\theta_1 - \sin\theta_1\cos(\theta_1+\theta_2)]$$

$$= L_1L_2[\sin(\theta_1+\theta_2-\theta_1)]$$

$$= L_1L_2S\theta_2$$





$$X = C_1 S_2 d_3 - S_1 d_2$$
$$Y = S_1 S_2 d_3 + C_1 d_2$$
$$Z = C_2 d_3 + d_1$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \quad \begin{bmatrix} \frac{dx}{d\theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial d_3} \\ \frac{dy}{d\theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial d_3} \\ \frac{dz}{d\theta_1} & \frac{\partial z}{\partial \theta_2} & \frac{\partial z}{\partial d_3} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

$$J = \begin{bmatrix} -S_1 S_2 d_3 - C_1 d_2 & C_1 C_2 d_3 & C_1 S_2 \\ C_1 S_2 d_3 - S_1 d_2 & S_1 C_2 d_3 & S_1 S_2 \\ 0 & -S_2 d_3 & C_2 \end{bmatrix}.$$

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a(ei - hf) - b(di - fg) + c(dh - eg)$$

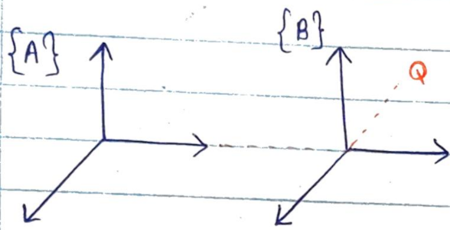
$$\det[J] = (-S_1 S_2 d_3 - C_1 d_3)(S_1 C_2^2 d_3 + S_1 S_2^2 d_3) - C_1 C_2 d_3 (C_1 C_2 S_2 d_3 - S_1 C_2 d_2)$$
$$+ C_1 S_2 (-C_1 S_2^2 d_3^2).$$

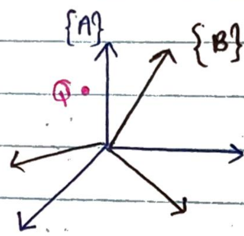
To find out Jacobian,

1) Partial derivation Method.
2) Velocity Propagation Method.

Velocity Propagation Method

Linear Velocity | Angular Velocity.





- A is fixed frame
- B has the same orientation of A.

${}^{B}P_Q \, , \, {}^{B}V_Q \Longrightarrow {}^{A}V_Q$?

$${}^{A}V_Q = {}^{A}V_B + {}^{A}_{B}R \, {}^{B}V_Q$$

Angular velocity of B w.r.t to A

$${}^{A}V_Q = {}^{A}_{B}R \, {}^{B}V_Q + {}^{A}_{B}\omega \times {}^{A}_{B}R \cdot Q^{B}$$

cross product

$${}^{1}V_2 = {}^{1}_{2}R \times {}^{2}V_2$$

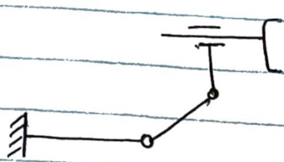
$${}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R \left({}^{i}\omega_i\right) + \dot{\theta}_{i+1} \, {}^{i+1}\hat{Z}_{i+1}$$

Will be given in question paper

$${}^{i+1}V_{i+1} = {}^{i+1}_{i}R \left({}^{i}V_i + {}^{i}\omega_i \times {}^{i}P_{i+1}\right) + \dot{d}_{i+1} \, {}^{i+1}\hat{Z}_{i+1}$$

$$\dot{\theta}_{i+1} \hat{Z} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix} \qquad \dot{d}_{i+1} \hat{Z} = \begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix}$$

Example



a) linear and angular velocities at last frame.
b) Jacobian at the last frame.
c) Jacobian at the base frame.
d) Equation for singularities.

${}^1_2R = \left[{}^2_1R\right]^T$ ${}^1_2P = $

$${}^0_1T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1_2T = \begin{bmatrix} \boxed{\begin{matrix} -S_2 & -C_2 & 0 \\ 0 & 0 & -1 \\ C_2 & -S_2 & 0 \end{matrix}} & \boxed{\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}$$

$${}^2_3T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Cross Product

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$a \times b = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix}$$

1) $${}^0\omega_0 = {}^0V_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$i=0$,

$${}^1\omega_1 = {}^1_0R\left({}^0\omega_0\right) + \dot\theta_1\hat{Z}$$

$${}^1V_1 = {}^1_0R\left({}^0V_0 + {}^0\omega_0 \times {}^0P_1\right) + \dot{d}_1\hat{Z}$$

$${}^1V_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad {}^1\omega_1 = \dot\theta_1\hat{Z} = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix}.$$

$i=1$,

$${}^2\omega_2 = {}^2_1R\left({}^1\omega_1\right) + \dot\theta_2\hat{Z}$$

$$= {}^2_1R\begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix}.$$

$$= \begin{bmatrix} -S_2 & 0 & C_2 \\ -C_2 & 0 & -S_2 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix}$$

$$= \begin{bmatrix} C_2\dot\theta_1 \\ -S_2\dot\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix}.$$

$$= \begin{bmatrix} C_2\dot\theta_1 \\ -S_2\dot\theta_1 \\ \dot\theta_2 \end{bmatrix}.$$

$$ {}^2V_2 = {}^2_1R\left({}^1V_1 + {}^1\omega_1 \times {}^1P_2\right) + \dot{d}_2 \hat{Z} $$

$$ = \begin{bmatrix} -S_2 & 0 & C_2 \\ -C_2 & 0 & -S_2 \\ 0 & -1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) + 0 $$

$$ = \begin{bmatrix} -S_2 & 0 & C_2 \\ -C_2 & 0 & -S_2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

$$ = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} // $$

$$ {}^3\omega_3 = {}^3_2R\left({}^2\omega_2\right) + 0 $$

$$ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} C_2\dot{\theta}_1 \\ -S_2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} $$

$$ = \begin{bmatrix} C_2\dot{\theta}_1 \\ \dot{\theta}_2 \\ \end{bmatrix} // $$

$0 - \dot{\theta}_2 d_3$

$0 - 0$

$C_2\dot{\theta}_1 d_3 - 0$

$$ {}^3V_3 = {}^3_2R\left({}^2V_2 + {}^2\omega_2 \times {}^2P_3\right) + \dot{d}_3 \hat{Z} $$

$$ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & \end{bmatrix} \left(\begin{bmatrix} C_2\dot{\theta}_1 \\ -S_2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \times \begin{bmatrix} 0 \\ -d_3 \\ 0 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 0 \\ \dot{d}_3 \end{bmatrix} $$

$$ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & \end{bmatrix} \begin{bmatrix} d_3\dot{\theta}_2 \\ 0 \\ -C_2\dot{\theta}_1 d_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{d}_3 \end{bmatrix} $$

$$ = \begin{bmatrix} d_3\dot{\theta}_2 \\ -C_2\dot{\theta}_1 d_3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ d_3 \end{bmatrix} $$

$$ = \begin{bmatrix} d_3\dot{\theta}_2 \\ -C_2\dot{\theta}_1 d_3 \\ \dot{d}_3 \end{bmatrix} // $$

2) ^{3}J = Jacobian at the last f

$$\begin{bmatrix} d_3\dot{\theta}_2 \\ -d_3C_2\dot{\theta}_1 \\ \dot{d}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & d_3 & 0 \\ -d_3C_2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{J_3} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix}$$

3) $^{0}J = {}^{0}_{3}R\ {}^{3}J$ To find ${}^{0}_{3}R$ we have to find,

$^{3}J = {}^{3}_{0}R\ {}^{0}J.$ ${}^{0}_{3}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T$

$${}^{0}_{3}T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -S_2 & -C_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ C_2 & -S_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -S_2C_1 & -C_1C_2 & S_1 & 0 \\ -S_2S_1 & -S_1C_2 & -C_1 & 0 \\ C_2 & -S_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -S_2C_1 & S_1 & C_1C_2 & C_1C_2d_3 \\ -S_2S_1 & C_1 & S_1C_2 & S_1C_2d_3 \\ C_2 & 0 & S_2 & S_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$${}^{0}J = \begin{bmatrix} -S_2C_1 & S_1 & C_1C_2 \\ -S_2S_1 & C_1 & S_1C_2 \\ C_2 & 0 & S_1 \end{bmatrix} \cancel{\begin{bmatrix} d_3\dot{\theta}_2 \\ -d_3C_2\dot{\theta}_1 \\ \dot{d}_3 \end{bmatrix}} \begin{bmatrix} 0 & d_3 & 0 \\ -d_3C_2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

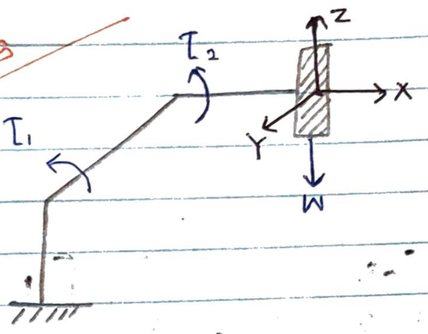
$$= \begin{bmatrix} -S_1d_3C_2 & -S_2C_1d_3 & C_1C_2 \\ -C_1d_3C_2 & -S_2S_1d_3 & S_1C_2 \\ 0 & C_2d_3 & S_1 \end{bmatrix}$$

4) Equation for singularities.

$$\det({}^{0}J) = -d_3C_1S_2(-d_3S_1S_2^2 - d_3S_1C_2^2) - d_3(0 + d_3C_1C_2S_2) + C_1C_2(-d_3^2C_1C_2^2)$$

$$d_3^2C_1S_1S_2 - d_3^2C_1^2C_2S_2^2 - d_3^2C_1^2C_2^3 + d_3^2C_1(S_1S_2 - C_1C_2S_2^2 - C_1C_2^3) = 0$$

Torques and Forces



How to use the Jacobian to find torques and forces.

Assume self weight of Robot = 0.

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ F_1 \\ F_2 \\ \vdots \end{bmatrix} = [{}^{0}J]^T \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}.$$

Torques $\longrightarrow$ Revolute

Forces $\longrightarrow$ Prismatic.

$${}^{0}v_1 = \dot{d}_1\hat{z} = \begin{bmatrix} 0 \\ 0 \\ \dot{d}_1 \end{bmatrix} \qquad {}^{0}\omega_1 = \dot{\theta}\hat{z} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

38- Part 3

b) ii) ${}^{0}\omega_0 = {}^{0}v_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$

$i = 0$

$${}^{1}\omega_1 = {}^{1}_{0}R({}^{0}\omega_0) + \dot{\theta}_1\,{}^{1}\hat{z}_1$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \cancel{\dot{\theta}_1}\,0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ \cancel{\dot{\theta}_1}\,0 \end{bmatrix}$$

$${}^{1}v_1 = {}^{1}_{0}R(\cancel{{}^{0}v_0} + \cancel{{}^{0}\omega_0} \times {}^{0}P_1) + \dot{d}_1 z^0$$

$$= \begin{bmatrix} 0 \\ 0 \\ \dot{d}_1 \end{bmatrix}$$

$$ {}^{0}_{1}V = {}^{0}_{1}R \times V'_1 \qquad \cancel{{}^{0}_{1}\omega = {}^{0}_{1}R \times} $$

$$ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

(iv) $i = 1$,

$$ {}^{2}\omega_2 = {}^{2}_{1}R\,({}^{1}\omega_1) + \dot{\theta}_2 \hat{Z} $$

$$ = \begin{bmatrix} C_2 & S_2 & 0 \\ -S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} $$

$$ = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} $$

$$ = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} // $$

$$ {}^{2}V_2 = {}^{2}_{1}R\,({}^{1}V_1 + {}^{1}\omega_1 \times {}^{1}P_2) + \dot{d}_2 \hat{Z} $$

$$ = \begin{bmatrix} C_2 & S_2 & 0 \\ -S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ \dot{d}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

$$ = \begin{bmatrix} C_2 & S_2 & 0 \\ -S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{d}_1 \end{bmatrix} $$

$$ = \begin{bmatrix} 0 \\ 0 \\ \dot{d}_1 \end{bmatrix} // $$

(vi) $i = 2$,

$$ {}^{3}\omega_3 = {}^{3}_{2}R\,({}^{2}\omega_2) + \dot{\theta}_3 \hat{Z} $$

$$ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

$$ = \begin{bmatrix} 0 \\ -\dot{\theta}_2 \\ 0 \end{bmatrix} $$

$$
{}^3_3V = {}^3_2R\left({}^2_2V + {}^2_2\omega \times {}^2_3P\right) + \dot{d}_3\hat{Z}
$$

$$
= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\left(\begin{bmatrix} 0 \\ 0 \\ \dot{d}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} \times \begin{bmatrix} 0 \\ d_3 \\ 0 \end{bmatrix}\right) + \begin{bmatrix} 0 \\ 0 \\ \dot{d}_3 \end{bmatrix}
$$

$$
= \begin{bmatrix} -d_3\theta_2 \\ -d_1 \\ d_3 \end{bmatrix}.
$$

(vii)
$$
\begin{bmatrix} -d_3\theta_2 \\ -d_1 \\ d_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -d_3 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{{}^3_0J} \begin{bmatrix} \dot{d}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix}
$$

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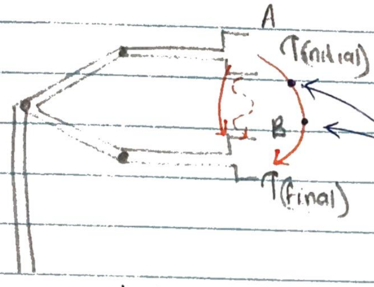
Trajectory Generation

Trajectory refers to a time history of position, velocity and acceleration for each DOF.



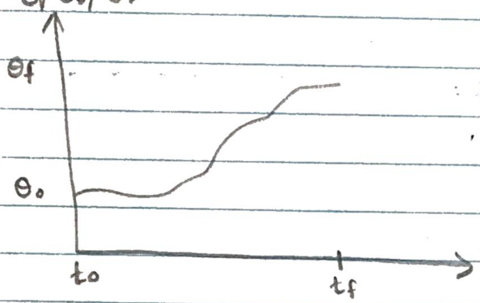
θ_1	θ_2	position
$\dot{\theta}_1$	$\dot{\theta}_2$	velocity
$\ddot{\theta}_1$	$\ddot{\theta}_2$	Acceleration.

Via points.

A → B speed, time, refresh rate.

" If the refresh rate is 20Hz that means for $\frac{1}{20}$ s the position, velocity and acceleration refreshed.

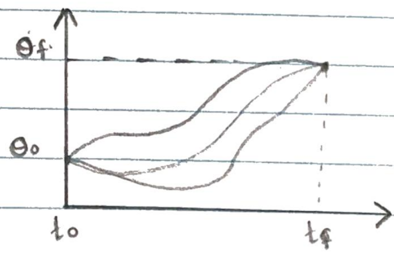
$\dot{\theta}/\dot{\theta}_1/\ddot{\theta}_2$



Graph which shows the motion ~~conversion~~ of θ_0 to θ_1.

Four constraints for $\theta(t)$ for smooth motion

1) $\theta(t_0) = \theta_0$
2) $\theta(t_f) = \theta_f$ } angle
3) $\dot{\theta}(t_0) = 0$
4) $\dot{\theta}(t_f) = 0$ } velocity.



Several possible path shapes for a single joint.

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t$$

Angle

When $t = 0$,

$\theta_0 = a_0$

When $t = t_f$

$\theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3$

$\theta_f = \theta_0 + a_2 t_f^2 + a_3 t_f^3$

Velocity

when $t = 0$,

$0 = a_1$

when $t = t_f$

$0 = a_1 + 2a_2 t_f + 3a_3 t_f^3$

$0 = t_f(2a_2 + 3a_3 t_f)$

by solving above equations,

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0)$$

$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0)$$

Example 7.1

$\theta_0 = 15$, $\theta_f = 75^\circ$, $t_0 = 0$, $t_f = 3s$

$a_0 = \theta_0 = 15$

$a_1 = 0$

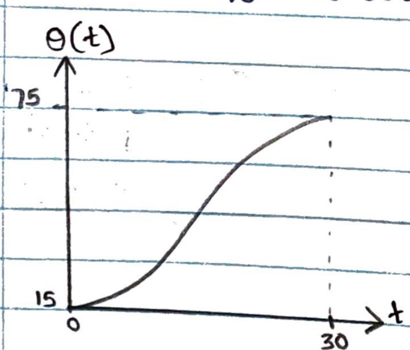
$$a_2 = \frac{3}{3^2}(75-15) = 20$$

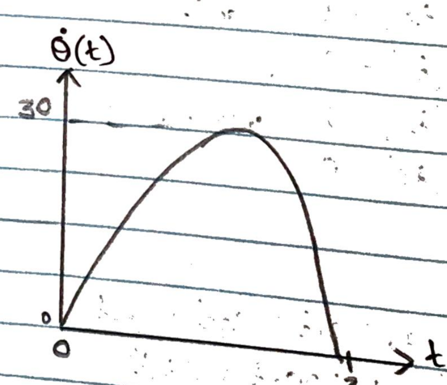
$$a_3 = \frac{-2}{3^3}(75-15) = -4.44$$

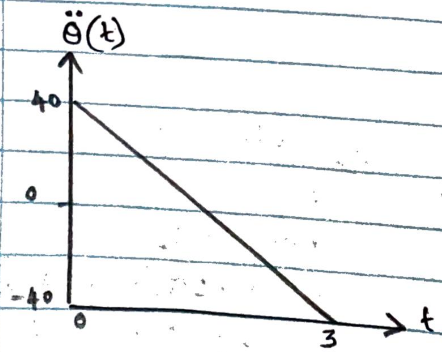
$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$
$$= 15 + 20t^2 - 4.44t^3$$

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$
$$= 40t - 13.33t^2$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3$$
$$= 40 - 26.66t$$







When there are via points.

V ← (via point)

A $\qquad\qquad$ $\dot{\theta} \neq 0$ $\qquad\qquad$ B.

$\dot{\theta} = 0$ $\qquad\qquad\qquad\qquad$ $\dot{\theta} = 0$

$t = 0$ $\qquad\qquad$ $t = t_f$

$\dot{\theta}(t_f) = \dot{\theta}_f$

If desired velocities of the joints at the via points are known,

$$\dot{\theta}(0) = \dot{\theta}_0$$
$$\dot{\theta}(t_f) = \dot{\theta}_f$$

$$\theta_0 = a_0$$
$$\theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3$$
$$\dot{\theta}_0 = a_1$$
$$\dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 t_f^2$$

$$a_0 = \theta_0$$
$$a_1 = \dot{\theta}_0$$
$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f$$
$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0) + \frac{1}{t_f^2}(\dot{\theta}_f + \dot{\theta}_0)$$

There are several ways in which the desired velocity at the via points might be specified,

1. User can choose velocities at via points.
2. System automatically choose velocity.
3. System choose velocity such a way that acceleration ($\ddot{\theta}$) constant at via point.

Example

$\theta_0 = 5^\circ$
$\theta_v = 15^\circ$ $\Big\}$ 1s
$\theta_g = 40^\circ$ $\Big\}$ 1s

$\dot{\theta}_v = 17.5$ degrees/sec

Sketch the position, velocity and acceleration graph.

For $\theta_0 \rightarrow \theta_v$,

$a_0 = 5$

$a_1 = 17.5$

$$a_2 = \frac{3}{1^2}(15-5) - \frac{2}{1} \times 17.5 - \frac{1}{1} \times 17.5$$

$$= -22.5$$

$$a_3 = \frac{-2}{1^3}(15-5) + \frac{1}{1^2}(15+5)$$

$$= 0$$

For $\theta_0 \longrightarrow \theta_v$,

$a_0 = 5$

$a_1 = 0$

$$a_2 = \frac{3}{1}(15-5) - 2\times 0 - \frac{1}{1} \times 17.5$$

$$= 12.5$$

$$a_3 = \frac{-2}{1^3}(15-5) + \frac{1}{1^2}(\cancel{15}\,17.5 + \cancel{5}\,0)$$

$$= \cancel{0}\ -20 + 17.5$$

$$= -2.5$$

$\theta(t) = 5 + 12.5t^2 - 2.5t^3$

$\dot{\theta}(t) = 25t - 7.5t^2$

$\ddot{\theta}(t) = 25 \cancel{t} - 15t$

For $\theta_v \longrightarrow \theta_f$,

$a_0 = 15$

$a_1 = 17.5$

$$a_2 = \frac{3}{1}(40-15) - \frac{2}{1} \times 17.5 - \frac{1}{1} \times 0$$

$$= 40$$

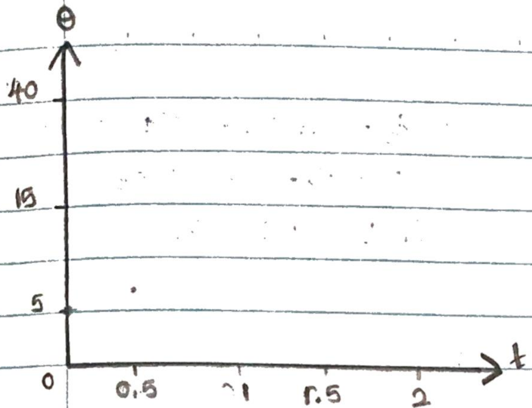
$$a_3 = \frac{-2}{1^3}(40-15) + \frac{1}{1^2}(0+17.5)$$

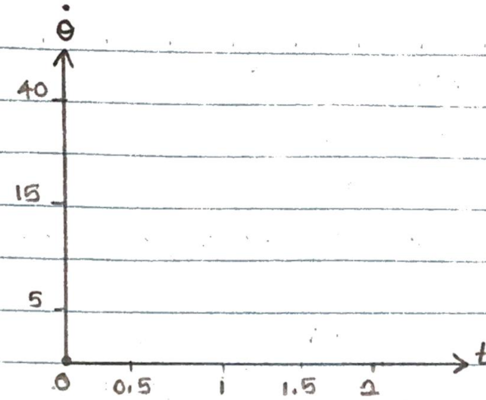
$$= -32.5$$

$\theta(t) = 15 + 17.5t + 40t^2 - 32.5t^3$

$\dot{\theta}(t) = 17.5 + 80t - 97.5t^2$

$\ddot{\theta}(t) = 80 - 195t$





Example

$\theta_0 = 0$

At $t = 2s$, $\theta_1 = 20^\circ$, $\dot{\theta}_1 = 10^\circ/s$

At $t = 4s$, $\theta_2 = 50^\circ$, $\dot{\theta}_2 = 5^\circ/s$

$\theta_f = 70^\circ$, $t_f = 6s$, $\theta_f = 0$

For $\theta_0 \longrightarrow \theta_1$

$a_0 = 0$

$a_1 = 0$

$$a_2 = \frac{3}{2^2}(20-0) - \frac{2}{2} \times 0 - \frac{1}{2} \times 10$$

$$= 10$$

$$a_3 = \frac{-2}{2^3}(20-0) + \frac{1}{2^2}(10+0)$$

$$= -2.5.$$

$\theta(t) = 10t^2 - 2.5t^3$

$\dot{\theta}(t) = 20t - 7.5t^2$

$\ddot{\theta}(t) = 20 - 15t$

For $\theta_1 \longrightarrow \theta_2$

$a_0 = 20$

$a_1 = 10$

$a_2 = \frac{3}{2^2}(50-20) - \frac{2}{2} \times 10 - \frac{1}{2} \times 5$

$= 10$

$a_3 = \frac{-2}{2^3}(50-20) + \frac{1}{2^2}(5+10)$

$= -3.75$

$\theta(t) = 20 + 10t + 10t^2 - 3.75t^3$

$\dot{\theta}(t) = 10 + 20t - 7.5t^2$

$\ddot{\theta}(t) = 20 - 15t$

For $\theta_2 \longrightarrow \theta_f$

$a_0 = 50$

$a_1 = 5$

$a_2 = \frac{3}{2^2}(70-50) - \frac{2}{2} \times 5 - \frac{1}{2} \times 0$

$= 10$

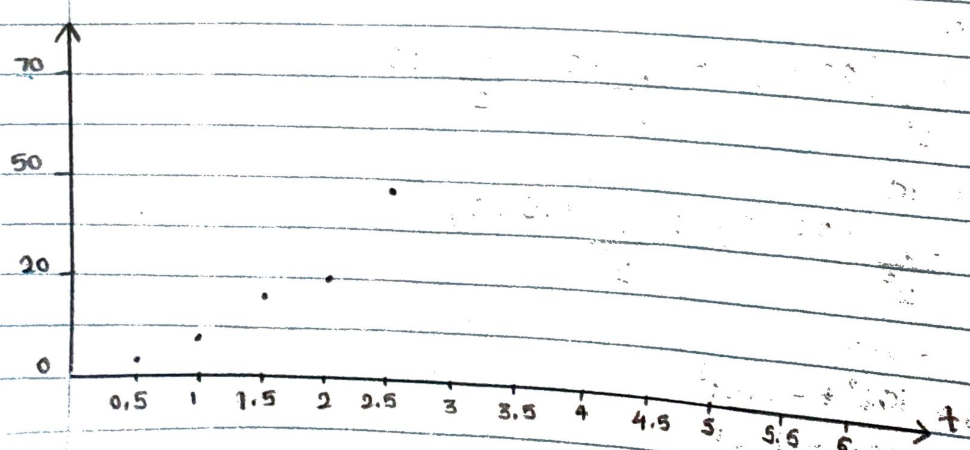
$a_3 = \frac{-2}{2^3}(70-50) + \frac{1}{2^2}(5+0)$

$= -3.75$

$\theta(t) = 50 + 5t + 10t^2 - 3.75t^3$

$\dot{\theta}(t) = 5 + 20t - 7.5t^2$

$\ddot{\theta}(t) = 20 - 15t$



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Sensors

Main objective is to enable robots to work in non-structural and random environments.

Human	Robot
eye-sight	RGB sight / Infrared
ear - hear	Outside 20-20kHz
smell	chemical analysis.
Taste	chemical analysis.
Touch	Ultrasonic range

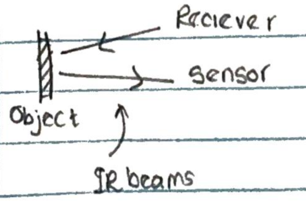
Classification of sensors

- Internal vs external state.
- Active vs non.active.
- Contact vs non contact,
- Visual vs non visual.

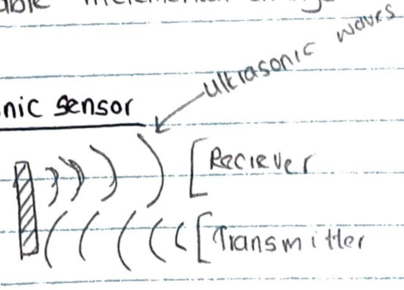
Sensor selection

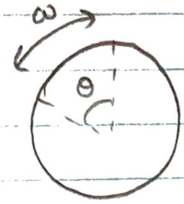
- Range
- Response
- Accuracy
- Sensitivity
- Linearity - (0.1 → 0.5 change එක මුළු range එකේම එකම ගානට වෙනවා.
- Repeatability
- Resolution. - smallest detectable incremental change.

IR sensor



Ultrasonic sensor





θ = Rotational angle.

ω = Rotational speed.

E

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Q.2) a)

	Axis(i)	a_{i-1}	α_{i-1}	d_i	θ_i
$0 \to$	1	0	0	d_1	0
$1 \to$	2	N	-90	0	θ_1
$2 \to$	3	0	-90	d_3	0
$3 \to$	4	0	0	0	θ_2

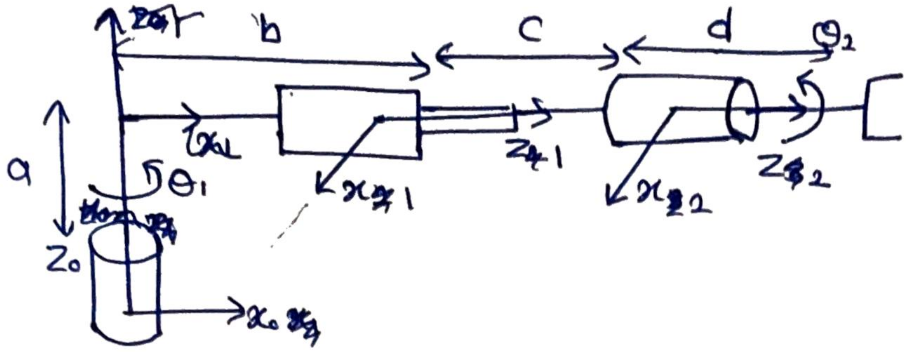
$$ {}^{0}_{1}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}_{2}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & N \\ 0 & 0 & 1 & 0 \\ -s\theta_1 & -c\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{3}_{4}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{4}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & N \\ 0 & 0 & 1 & 0 \\ -s\theta_1 & -c\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

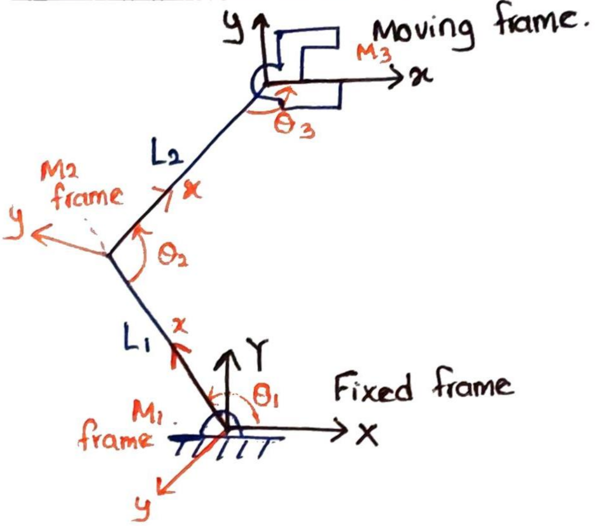
$$ = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & N \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = c\theta_1 c\theta_2 \quad -c\theta_1 s\theta_2 \quad -1 \quad N $$



Axis	a_{i-1}	α_{i-1}	d_i	θ_i
$0 \to 1$	0	-90°	a	θ_1
$1 \to 2$	0	0	b	0
$2 \to 3$	0	0	d	θ_2

Forward Kinematics. (2D)

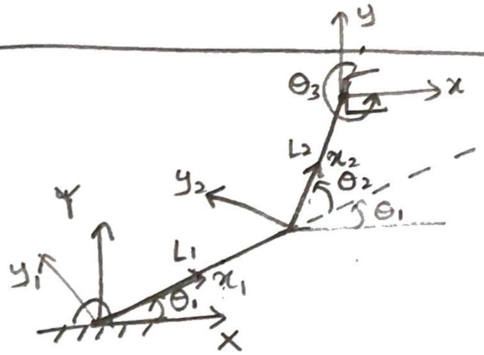


When an object's position is given w.r.t end effector frame ($\vec{x}$) we have to give that position w.r.t the Fixed frame. ($\vec{X}$)

$$\vec{x} \longrightarrow \overline{X}$$

Steps

1) Define and draw the fixed frame and end effector frame.
2) Draw a frame at each joint with it's X-axis parallel to the next link.
3) Mark the rotation angle.
   * ↺ is positive.
   * rotation angle is going to be w.r.t the previous frame.



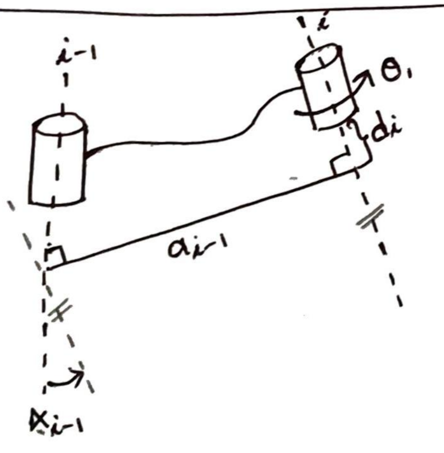
$x_1 = L_1 \cos\theta_1 \qquad x_2 = L_2 \cos(\theta_1+\theta_2)$

$y_1 = L_1 \sin\theta_1 \qquad y_2 = L_2 \sin(\theta_1+\theta_2)$

$$X = L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2)$$

$$Y = L_1 \sin\theta_1 + L_2 \sin(\theta_1+\theta_2)$$

D-H Tables



a_{i-1} = Distance from z_i to z_{i+1} measured along x_i

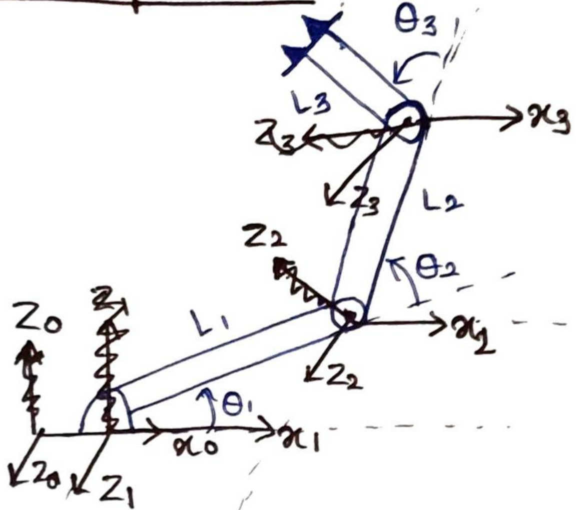
α_{i-1} = Angle from z_i to z_{i+1} measured about x_i

d_i = Distance from x_{i-1} to x_i measured along z_i

θ_i = Angle from x_{i-1} to x_i measured about z_i.

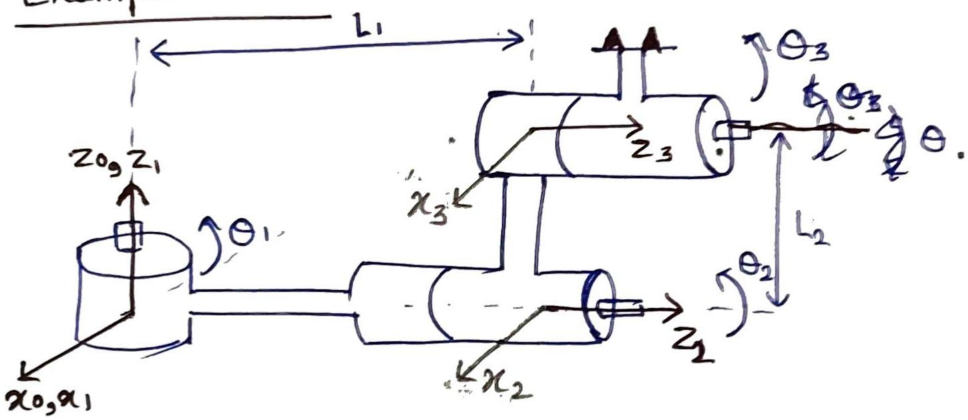
Axis (i)	a_{i-1}	α_{i-1}	d_i	θ_i

Example 3.3.



Axis(i)	a_{i-1}	α_{i-1}	d_i	θ_i
$0 \rightarrow 1$	0	0	0	θ_1
$1 \rightarrow 2$	L_1	0	0	θ_2
$2 \rightarrow 3$	L_2	0	0	θ_3

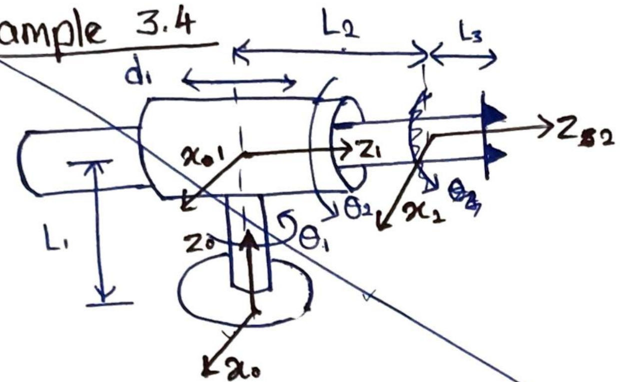
Example 3.5



Axis (i)	a_{i-1}	α_{i-1}	d_i	θ_i
$0 \rightarrow 1$	0	0	0	θ_1
$1 \rightarrow 2$	0	-90	L_1	θ_2
$2 \rightarrow 3$	0	0	L_2	θ_3

$$
{}^{2}_{3}T = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & L_2 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

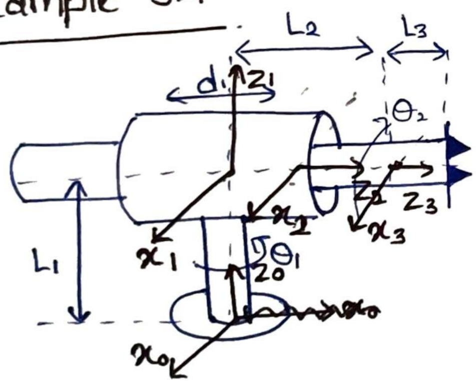
Example 3.4



DH Table

	Axis (i)	a_{i-1}	α_{i-1}	d_i	θ_i
0 →	1	0	90	L_1	θ_1
1 →	2	L_1	0	d_1	0
2 →	3	L_2	0	0	θ_2
3 →	4				

Example 3.4



	Axis(i)	a_{i-1}	α_{i-1}	d_i	θ_i
0 →	1	0	0	0	θ_1
1 →	2	0	90	L_2	0
2 →	3	0	0	L_3	θ_2

Ex