• **EE1102 – Fundamentals of Electrical Engineering**

6.0 Alternating Current Theory

Prof J. Rohan Lucas

Department of Electrical, Electronic & Telecommunication Engineering

General Sir John Kotelawala Defence University

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Alternating Current Theory *(8 hrs)*

KDU – Prepared by Prof J R Lucas/2021

Learning Outcome:

Perform AC circuit calculations using phasor and complex notation.

- **Syllabus:**

➢ Sinusoidal Waveform,

➢ Phasor and Complex representation, Phasor Diagrams.

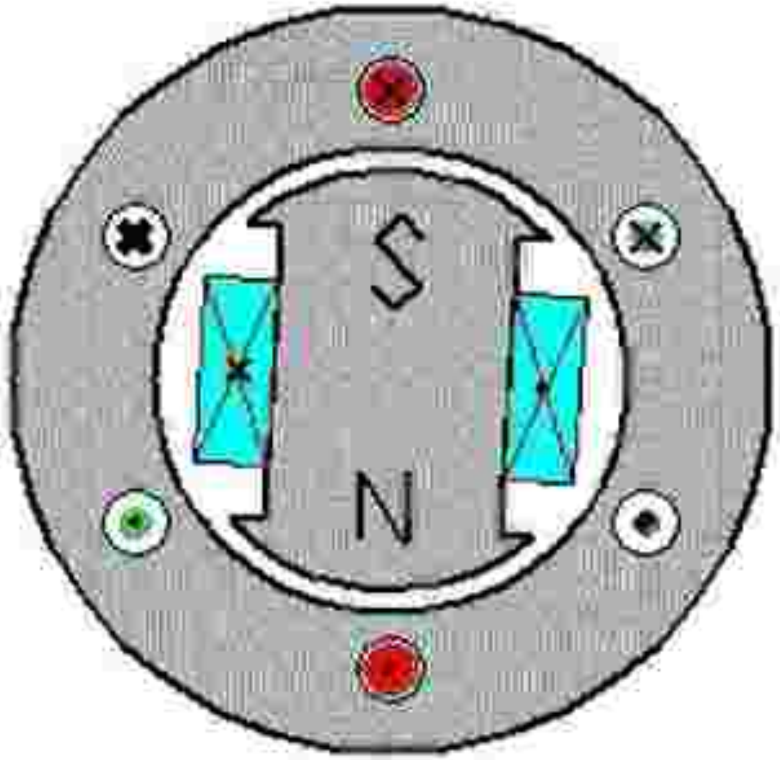
➢ Impedance and Admittance.

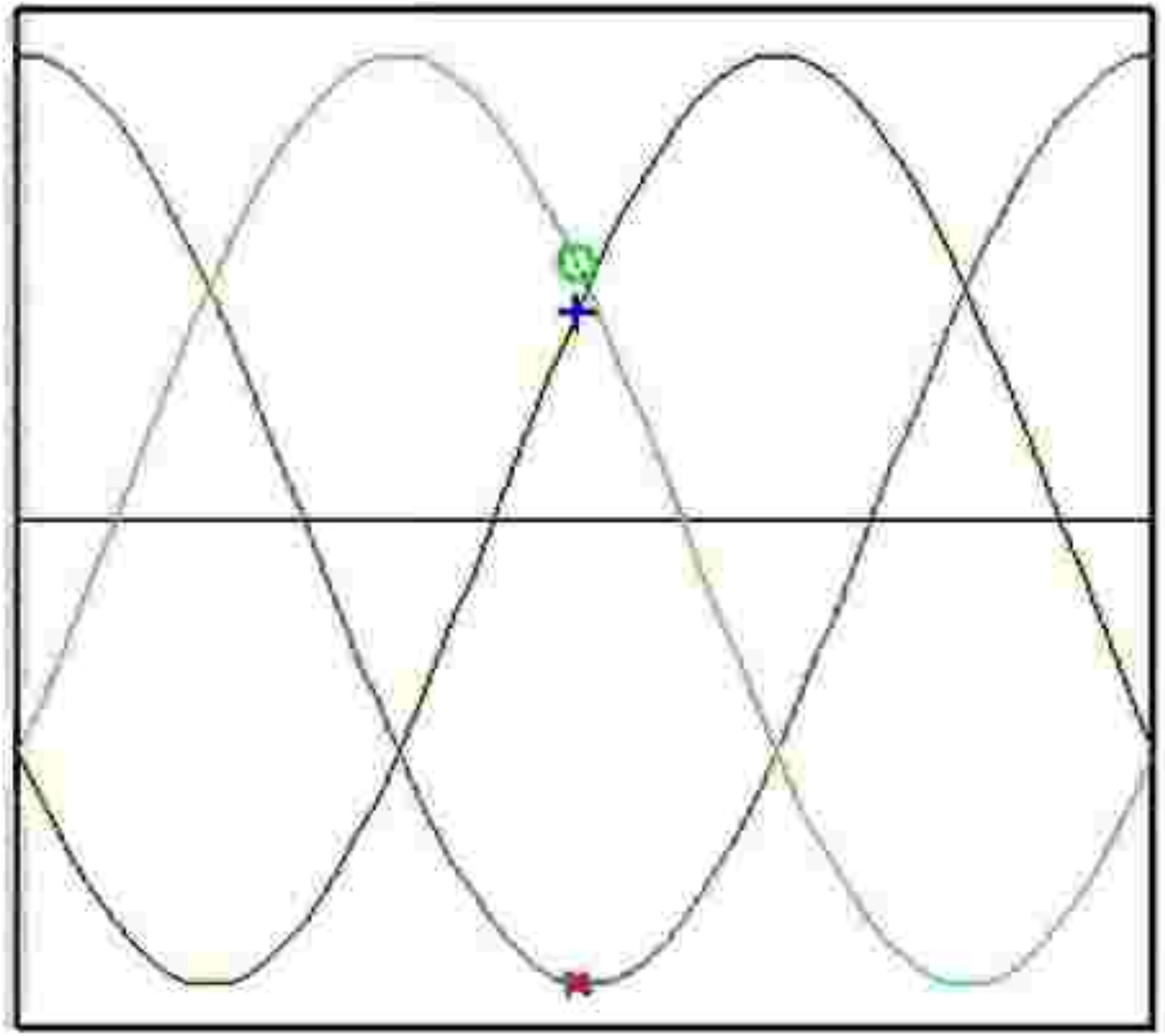
➢ Power and Power Factor, Reactive Power.

➢ Calculations for Simple R-L-C Circuits using Phasor-diagrams and complex notation.

Some advantages of sinusoidal waveform for electrical power

a. Sinusoidally varying voltages are easily generated by rotating machines





b. **Differentiation** of a Sinusoidal waveform produces a sinusoidal waveform of the same frequency

$$\frac{d\ sin(\omega t+\phi)}{dt}=\omega\cdot cos(\omega t+\phi)=\omega\cdot sin(\omega t+\phi+\pi/2\Big)$$

Integration of a sinusoidal waveform also produces a sinusoidal waveform of the same frequency

$$\int \sin(\omega t+\phi)\cdot dt=-\frac{\cos(\omega t+\phi)}{\omega}=\frac{1}{\omega}\sin(\omega t+\phi-\pi/2\Big)$$

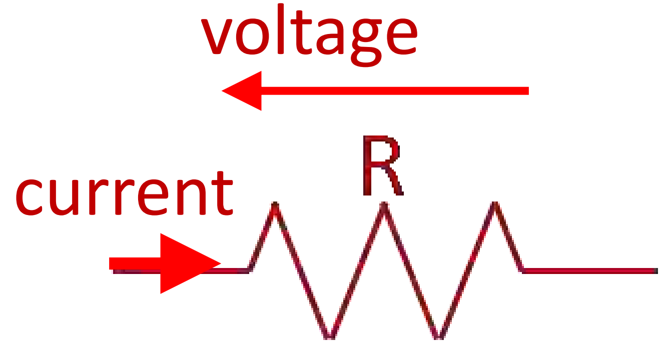
➢Thus differentiation and integration of a sinusoidal waveform produces waveform of same frequency, differing only in magnitude, and phase angle by +/– 90^0.

c. When a sinusoidal current is passed through, or voltage applied across. a resistor, inductor or a capacitor

- ➢ a sinusoidal voltage (or current) waveform is obtained
- ➢ same frequency, differing only in magnitude and phase angle.

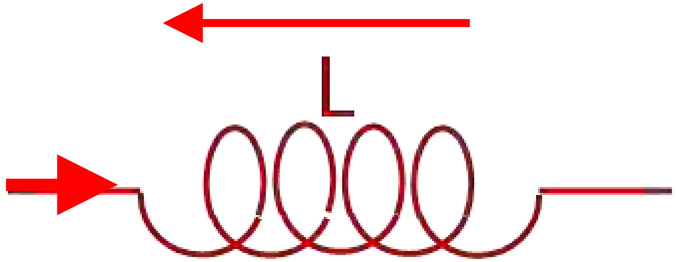
If $i(t) = I_m \sin(\omega t+\phi)$,

for a resistor,



$$\boldsymbol{v(t) = R.i(t) = R.I_m \sin(\omega t+\phi) = V_m \sin(\omega t+\phi)}$$

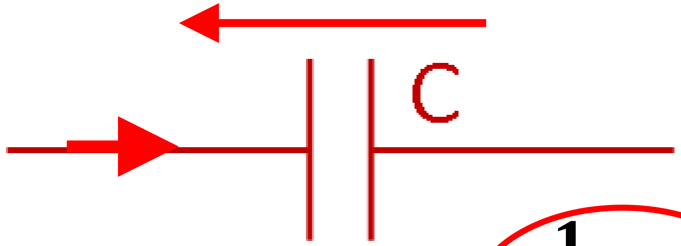
- ➢ magnitude changed by R but no phase shift



*for an **inductor**,*

$$v(t) = L.\frac{di}{dt} = L.\frac{d}{dt}(I_m \sin(\omega t + \phi)) = L.\omega.I_m \cos(\omega t + \phi) = L.\omega.I_m \sin(\omega t + \phi + \pi/2)$$

- magnitude changed by Lω and phase angle by π/2



*for a **capacitor**,*

$$v(t) = \frac{1}{C}.\int i \cdot dt = \frac{1}{C}\int I_m \sin(\omega t + \phi) \cdot dt = \frac{-1}{C \cdot \omega}.I_m \cos(\omega t + \phi) = \frac{1}{C \cdot \omega}.I_m \sin(\omega t + \phi - \pi/2)$$

- magnitude changed by 1/Cω and phase angle by –π/2

*Thus for a **resistor**, **inductor** and **capacitor***

- Waveform remains sinusoidal and frequency unchanged.

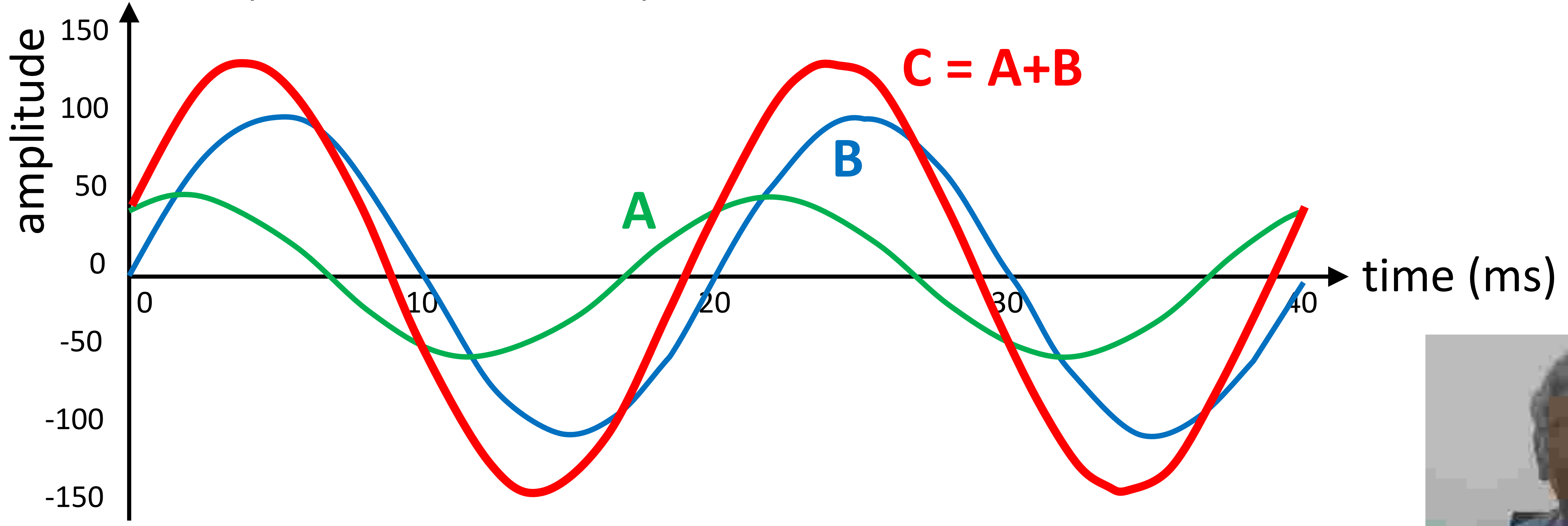
d. Sinusoidal waveforms (sinusoid) remain unaltered in shape when other sinusoidal waveforms of same frequency but different magnitude and phase are added.

$A \sin(\omega t+\alpha) + B \sin(\omega t+\beta)$

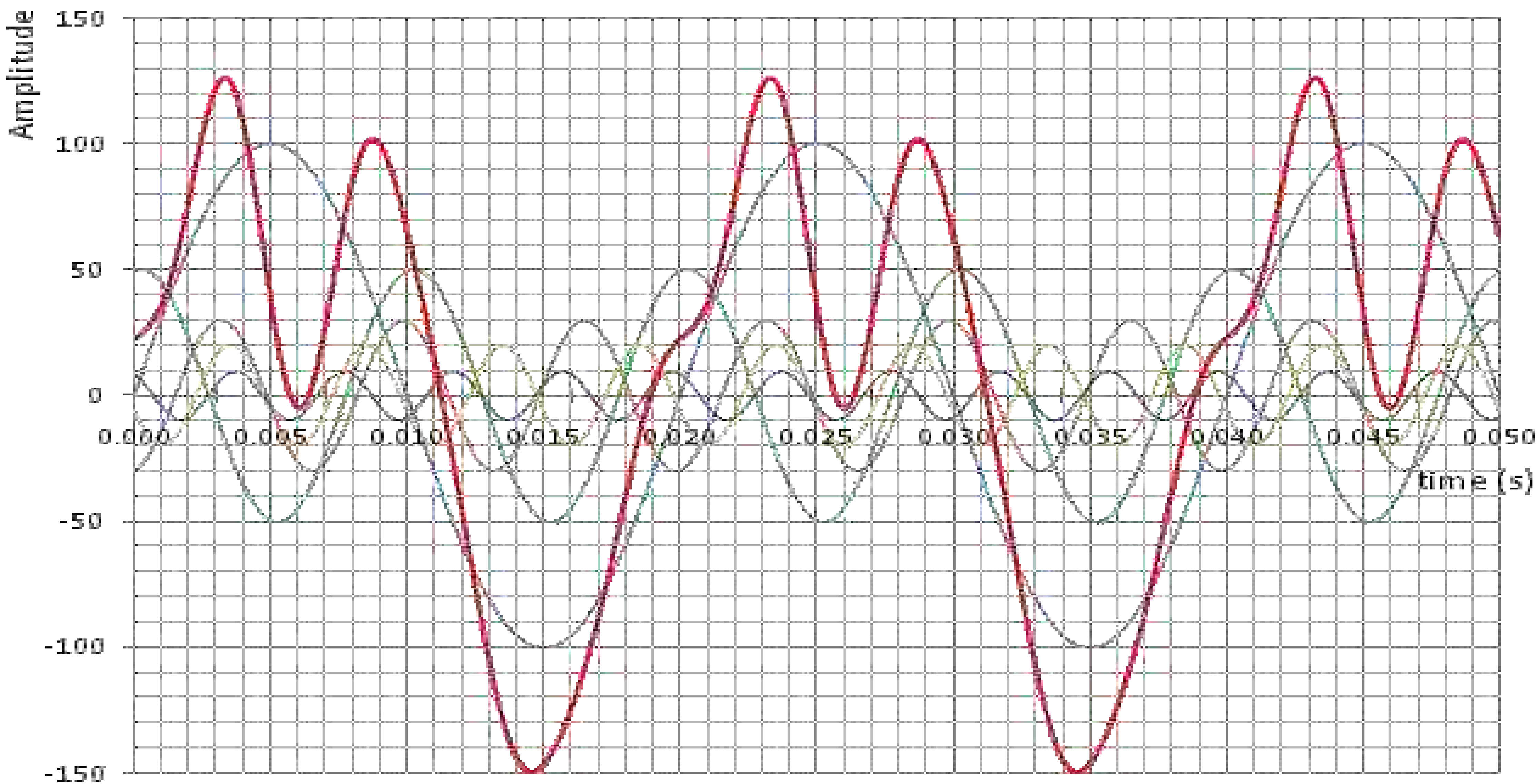
$= A \sin \omega t.\cos \alpha + A \cos \omega t.\sin \alpha + B \sin \omega t.\ os\ \beta + B \cos \omega t.\sin \beta$

$= (A.\cos \alpha + B.\cos \beta) \sin \omega t + (A.\sin \alpha + B.\sin \beta) \cos \omega t$

$= C \sin(\omega t + \phi)$, where C and ϕ are constants from trigonometry.

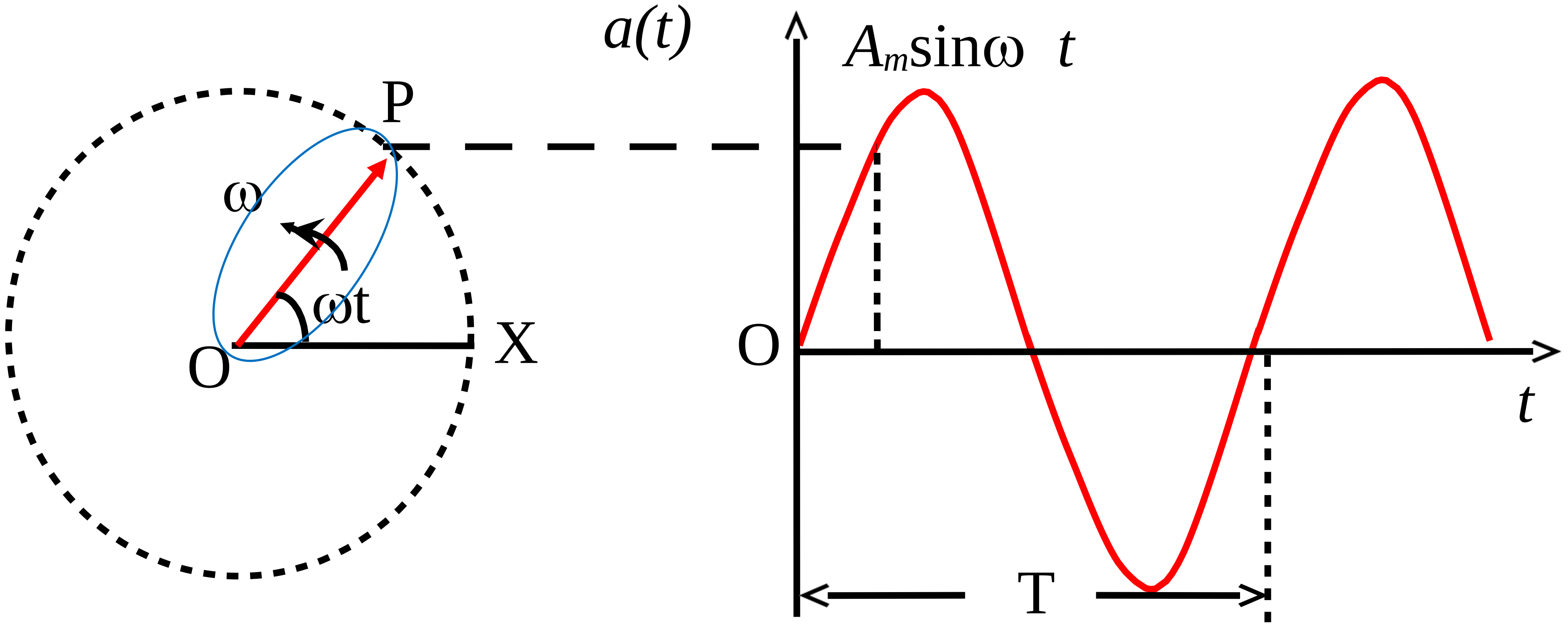


e. Periodic, but non-sinusoidal waveforms can be broken to its fundamental/harmonics



f. Sinusoids can be represented by projections of a rotating phasor.

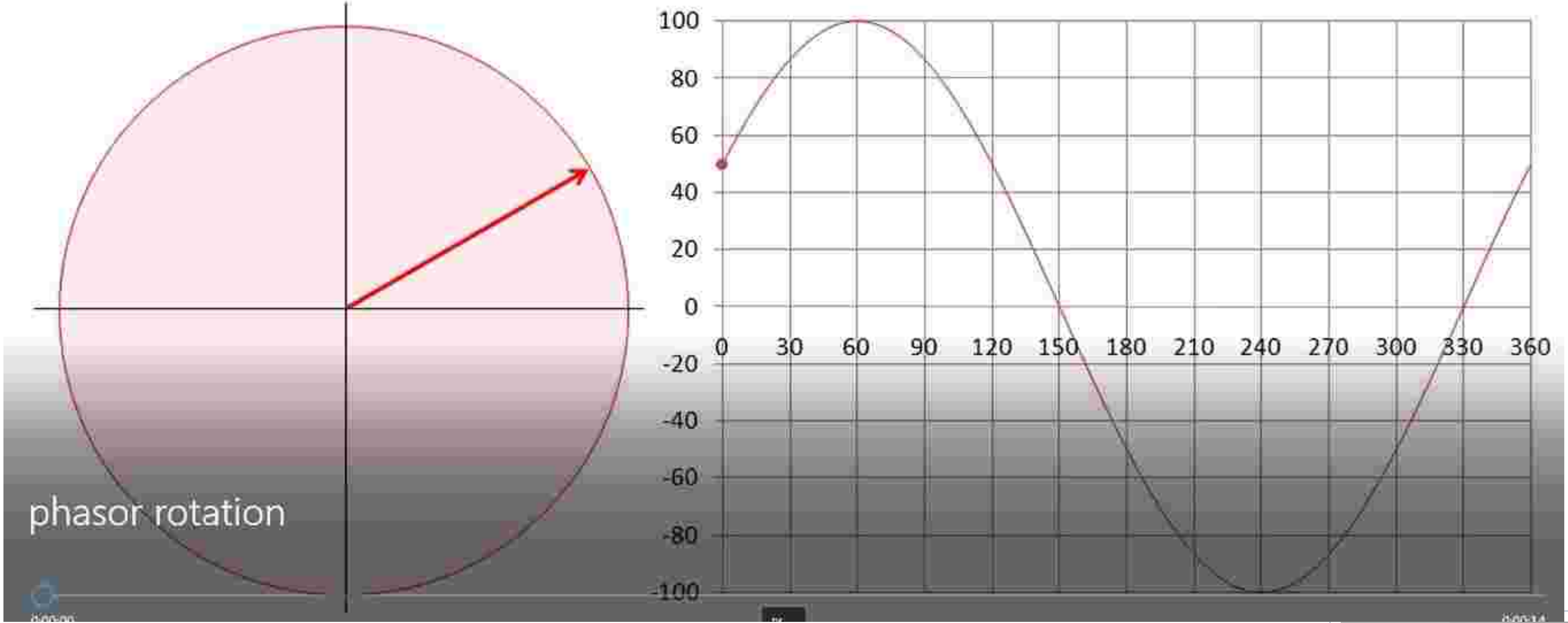
Phasor Representation of Sinusoids



sin θ can be written in terms of exponentials and complex numbers.

$$e^{j\theta} = \cos\theta + j\sin\theta \quad \text{or} \quad e^{j\omega t} = \cos\omega t + j\sin\omega t$$

For line OP (length A_m) in horizontal direction OX at time t=0.


100
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20
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-20
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-60
-80
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60
90
120
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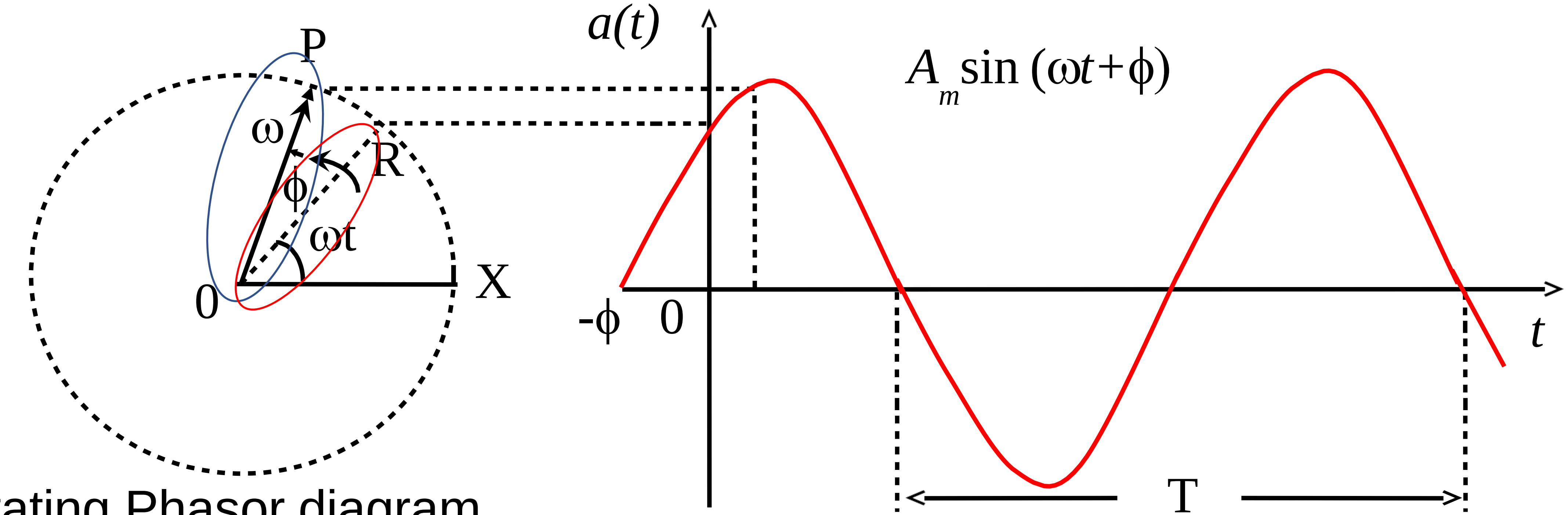
180
210
240
270
300
330
360
phasor rotation

If OP rotates at angular velocity $\boldsymbol{\omega}$, then in time $\boldsymbol{t}$ it corresponds to an angle of $\boldsymbol{\omega t}$.



Rotating Phasor diagram

A ***phasor*** is similar to a vector,

– but does not have a physical direction in space but a *phase* angle.

Projection of rotating phasor OP on the

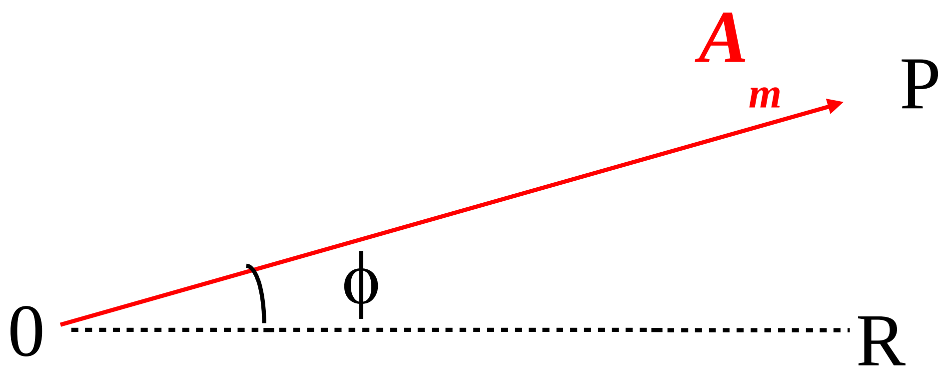
y-axis would correspond to OP sin (ωt+ϕ) or $\boldsymbol{A_m \sin}\ (\omega t+\phi)$

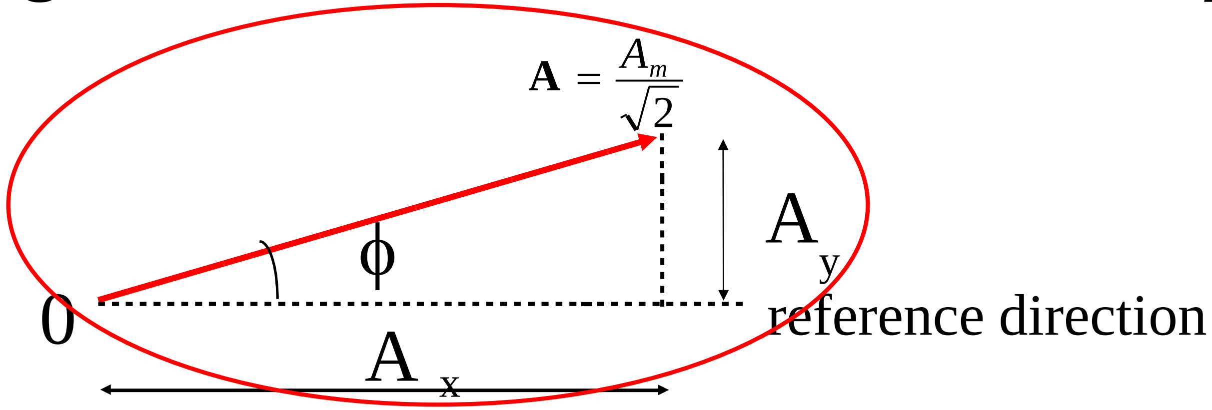
x-axis would correspond to $\boldsymbol{A_m \cos}\ (\omega t+\phi)$

Sinusoid is a projection on a given direction of complex exponential $e^{j\omega t+\phi}$

+ϕ.

Usual to draw *Phasor diagram* using *rms A* of sinusoid, rather than with *peak* A_m.





Phasor diagram

Unless otherwise specified, rms value is drawn on phasor diagram.

Values on phasor diagram are no longer time variables.

Phasor **A**

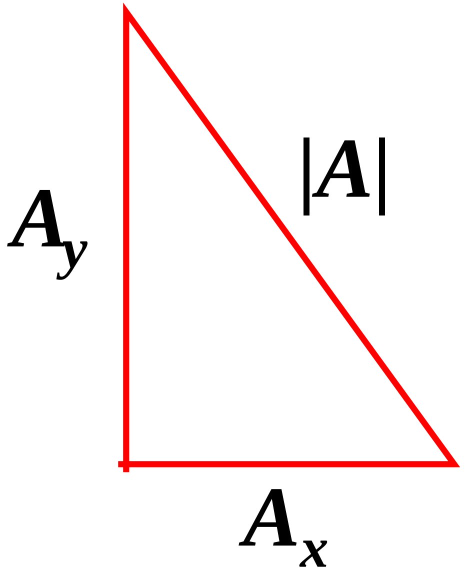
– characterised by magnitude $|\mathbf{A}|$
  and its phase angle ϕ (polar co-ordinates)
– commonly written as $|\mathbf{A}|\angle\phi$
– also characterised by cartesian co-ordinates $\mathbf{A_x}$ and $\mathbf{A_y}$
– written using complex numbers as $\mathbf{A} = \mathbf{A_x} + \mathbf{j}\,\mathbf{A_y}$.

In electrical engineering, letter j is used for complex operator because symbol i or I is reserved for electric current.

j corresponds to an anticlockwise rotation of 90°.

Also $j = \sqrt{-1}$

$$|A| = \sqrt{A_x^2 + A_y^2} \qquad \tan\phi = \frac{A_y}{A_x} \quad \text{or} \quad \phi = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



Also, $A_x = |A| \cos\phi$, $\quad A_y = |A| \sin\phi$

and $|A|\, e^{j\phi} = |A| \cos\phi + j|A| \sin\phi = A_x + j\,A_y$

If period of a sinusoidal waveform is **T**, with angular frequency ω

– corresponding angle of period would be ω**T**

– corresponds to one complete cycle or 2 π radians or 360^0.

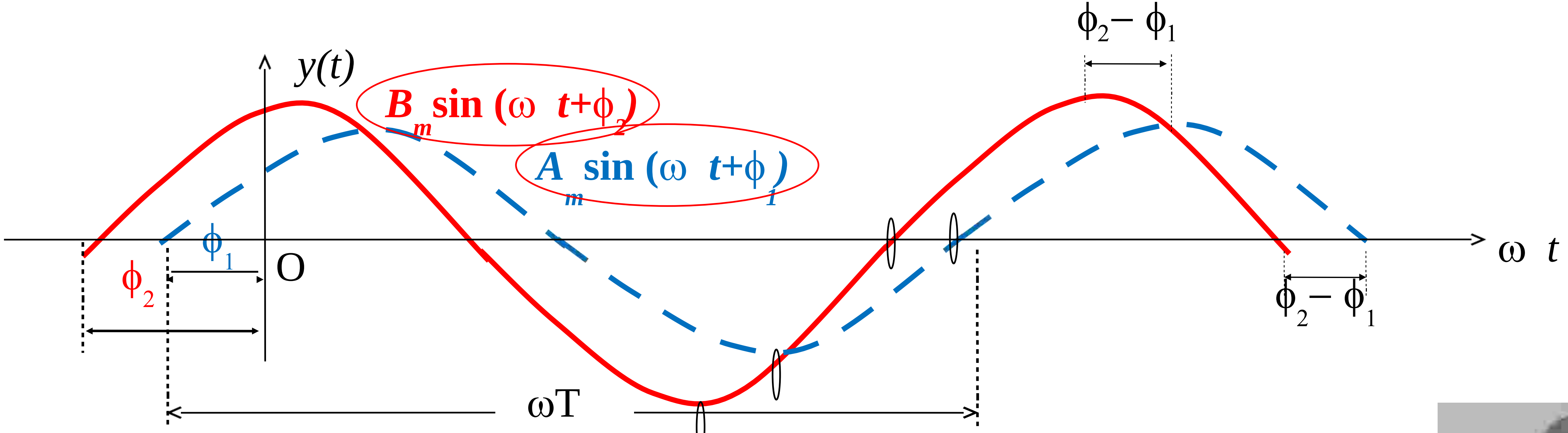
$\therefore \omega T = 2\pi$

Phase difference

With more than one phasor rotating at same angular frequency, there is no relative motion.

If reference phasor OR is fixed in direction,

then all other phasors would be fixed at a relative position.



Projection on the y-axis would yield the corresponding waveforms $A_m \sin(\omega t+\phi_1)$ and $B_m \sin(\omega t+\phi_2)$.

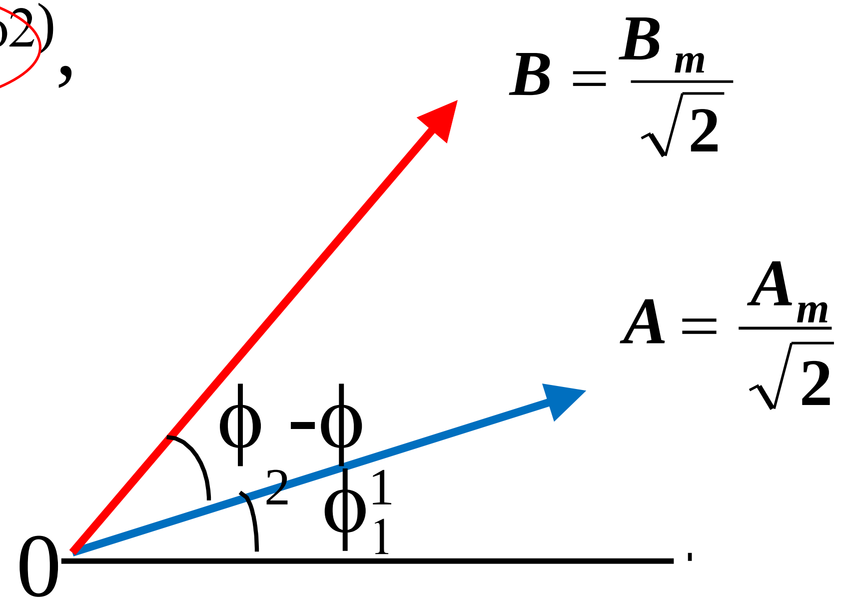
Two waveforms having different amplitudes and different phase angles can be represented by rotating phasors $A_m\, e^{j(\omega t+\phi 1)}$ and $B_m\, e^{j(\omega t+\phi 2)}$, or by normal phasor diagram with complex ***A*** and ***B*** or polar co-ordinates $|\mathbf{A}|\angle\phi_1$ and $|\mathbf{B}|\angle\phi_2$.

positive peak A_m occurs $(\phi_2 - \phi)$ after positive peak B_m

zero of a(t) occurs $(\phi_2 - \phi_1)$ after matching zero of b(t).

Thus waveform **b(t)** ***leads*** waveform **a(t)** by $(\phi_2 - \phi_1)$.

or waveform **a(t)** ***lags*** the waveform **b(t)** by phase angle $(\phi_2 - \phi_1)$



[*Note:* Only angles less than 180° is used to specify lead or lag]

lead and **lag** can be simply defined referring to phasor diagram.

B* leads *A by angle $(\phi_2 - \phi_1)$ or

A* lags *B by angle $(\phi_2 - \phi_1)$.

An exercise for you to try out before you proceed

- The voltage applied to an equipment and current through it are given as the instantaneous waveforms
- $v(t) = 325 \sin (315\, t + 15^o)$ V, and $i(t) = 7.5 \sin (315t - 30^o)$ A

a) What are the peak values and the rms values of the voltage and current?
b) What are the angular frequency and the normal frequency of the voltage?
c) Does the current have the same frequency as the voltage?
d) What are the phase angles of the voltage and the current relative to a reference waveform sin 315t ?
e) Write down the voltage and current in complex polar form.
f) Write down the voltage and current in complex cartesian form.
g) Does the current lead or lag the voltage, and by how much?
h) Can you obtain this from the time variables or phasors or both?

Answers to the exercise

- *v(t) = 325 sin (315 t + 15^o^)* V, and *i(t) = 7.5 sin (315t – 30^o^)* A

a) Peak value of voltage = ***325V***, and of current = ***7.5A***
rms values of voltage = $325/\sqrt{2}$ = ***229.8V*** and of current = $7.5/\sqrt{2}$ = ***5.30A***

b) Angular frequency ω = ***315 rad/s*** and frequency f = $315/2\pi$ = ***50.13 Hz***

c) Does the current have the same frequency as the voltage? ***Yes***

d) Relative phase angles of voltage = $\boldsymbol{15^o}$ and current = $\boldsymbol{-30^o}$

e) In complex polar form: voltage = $\boldsymbol{229.8\angle 15^o V}$, current = $\boldsymbol{5.30 \angle -30^o A}$

f) In complex cartesian: voltage = $229.8\ (\cos 15^o + j\sin 15^o)$ = $\boldsymbol{221.98 + j59.48V}$
current = $5.30\ (\cos(-30^o) + j\sin(-30^o))$ = $\boldsymbol{4.59 + j2.65A}$

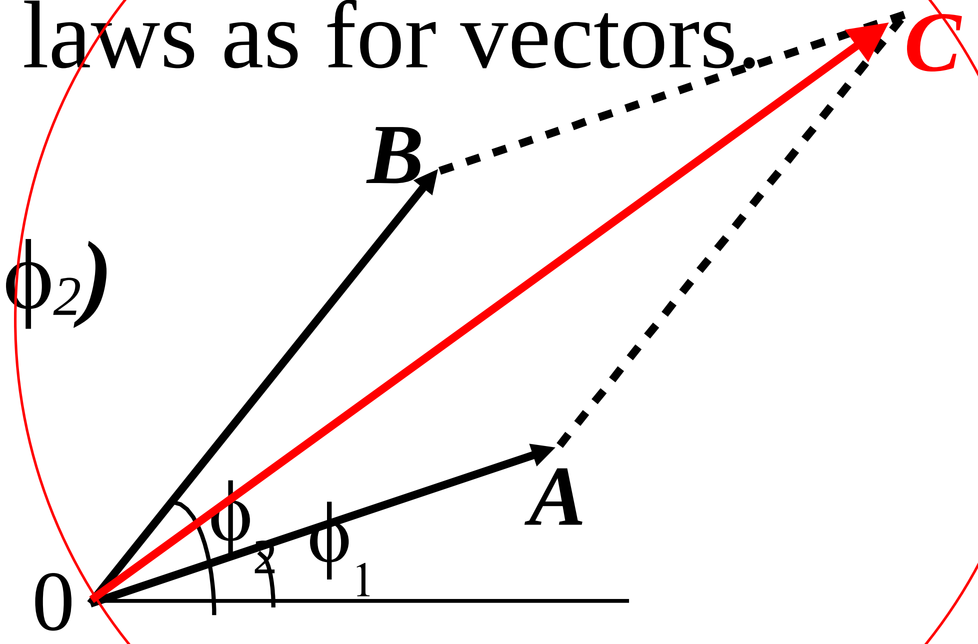
g) Current lags the voltage by $\boldsymbol{15^o - (-30^o) = 45^o}$

h) The ***phase difference*** can be obtained from either the *equations*

- *of the time waveforms*, or from the *polar form of the phasor.*

Addition and subtraction of phasors

Done using same parallelogram and triangle laws as for vectors.



Addition of phasors ***A*** and ***B***

$$\boldsymbol{A}+\boldsymbol{B}=(A\cos\phi_1+jA\sin\phi_1)+(B\cos\phi_2+jB\sin\phi_2)$$

$$=(A\cos\phi_1+B\cos\phi_2)+j(A\sin\phi_1+B\sin\phi_2)$$

$$=C_x+jC_y=\left|C\right|\angle\phi_c=\boldsymbol{C}$$

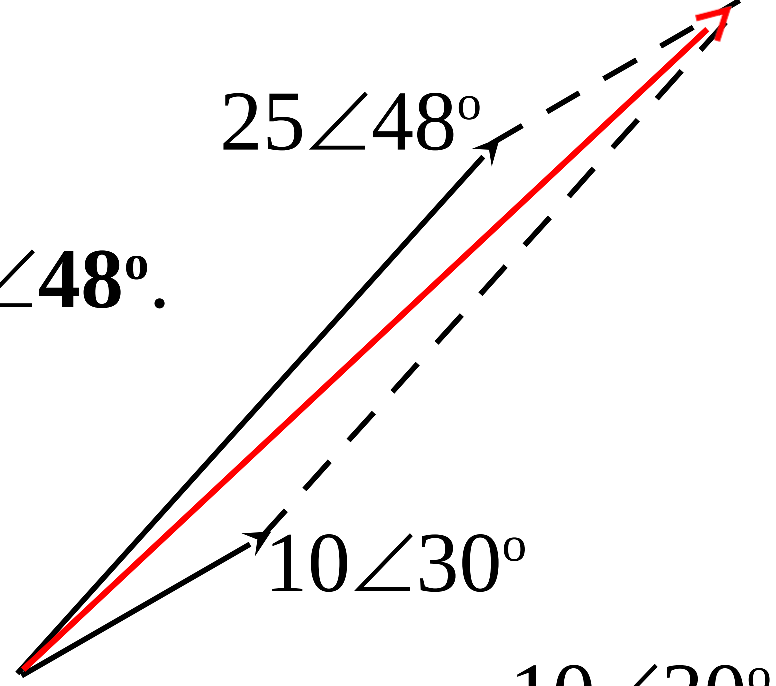
where

$$\left|C\right|=\sqrt{C_x^2+C_y^2}=\sqrt{(A\cos\phi_1+B\cos\phi_2)^2+(A\sin\phi_1+B\sin\phi_2)^2}$$

$$\phi_c=\tan^{-1}\left(C_y/C_x\right)=\tan^{-1}\left((A\sin\phi_1+B\sin\phi_2)/(A\cos\phi_1+B\cos\phi_2)\right)$$

Example

Find addition and subtraction of $\mathbf{10\angle 30^o}$ and $\mathbf{25\angle 48^o}$.



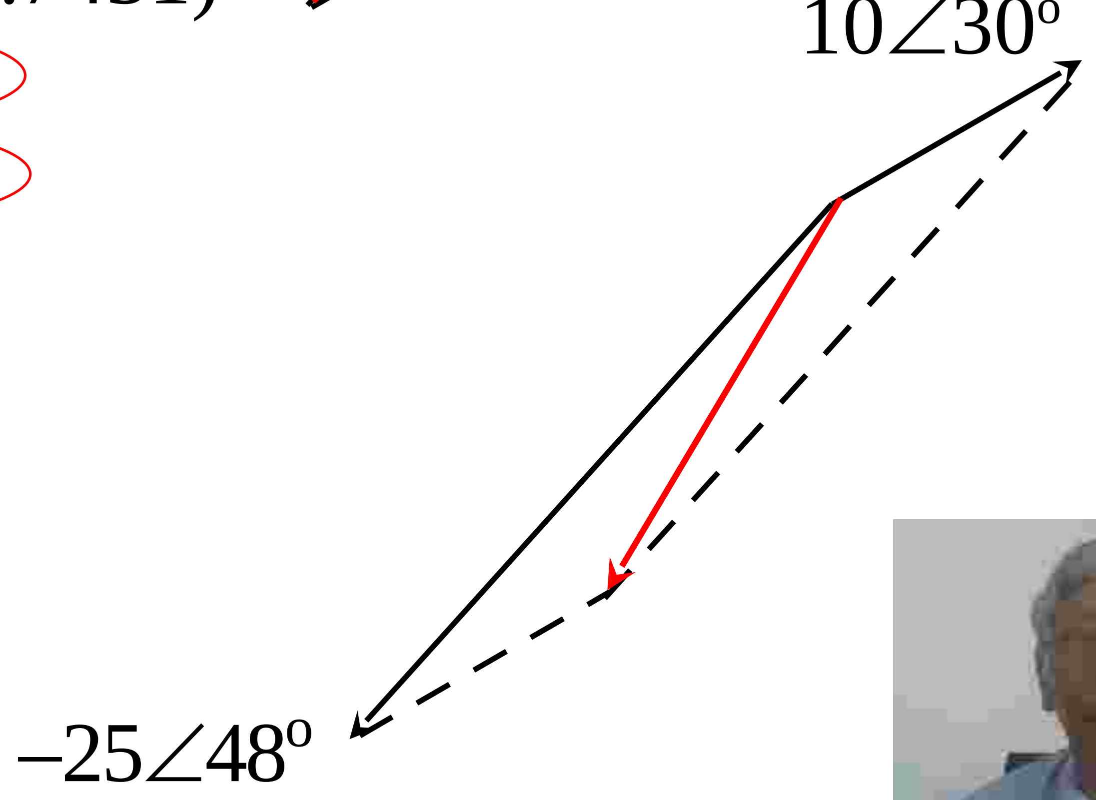
Addition

$\mathbf{10\angle 30^o + 25\angle 48^o}$

$= 10(0.8660+j0.5000) + 25(0.6691+j\,0.7431)$

$= (8.660 + 16.728) + j\,(5.000 + 18.577)$

$= 25.388 + j\,23.577 = \mathbf{34.647\angle 42.9^o}$



Subtraction

$\mathbf{10\angle 30^o - 25\angle 48^o}$

$= (8.660 - 16.728) + j\,(5.000 - 18.577)$

$= -8.068 - j\,13.577 = \mathbf{15.793\angle 239.3^o}$

Multiplication and division of phasors

Most easily done using polar form of complex numbers.

Multiplication of phasor ***A*** and phasor ***B***

$$\boldsymbol{A} * \boldsymbol{B} = |\boldsymbol{A}|\angle\phi_1 * |\boldsymbol{B}|\angle\phi_2 = |\boldsymbol{A}|\, e^{j\phi 1} * |\boldsymbol{B}|\, e^{j\phi 2}$$

$$= |\boldsymbol{A}| * |\boldsymbol{B}|\, e^{j(\phi 1+\ \phi 2)} = |\boldsymbol{A}| * |\boldsymbol{B}| \angle(\phi_1 + \phi_2) = |\boldsymbol{C}|\angle\phi_c$$

where $|\boldsymbol{C}| = |\boldsymbol{A}| * |\boldsymbol{B}|$ and $\phi_c = \phi_1 + \phi_2$

Division of phasor ***A*** by phasor ***B*:** Similarly $|\boldsymbol{C}| = |\boldsymbol{A}| / |\boldsymbol{B}|$ and $\phi_c = \phi_1 - \phi_2$

Thus, for addition/subtraction, use cartesian, and for multiplication/division use polar.

Example

Muliply and divide complex numbers $10\angle 30^o$ and $25\angle 48^o$

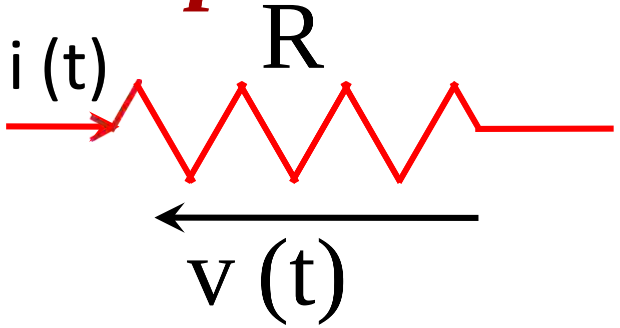
Multiplication = $10\angle 30^o * 25\angle 48^o = 250\angle 78^o$

Division = $10\angle 30^o \div 25\angle 48^o = 0.4\angle -18^o$

Currents and voltages in simple circuit elements

Resistor

$\boldsymbol{v(t) = R.i(t)}$



for a sinusoid,

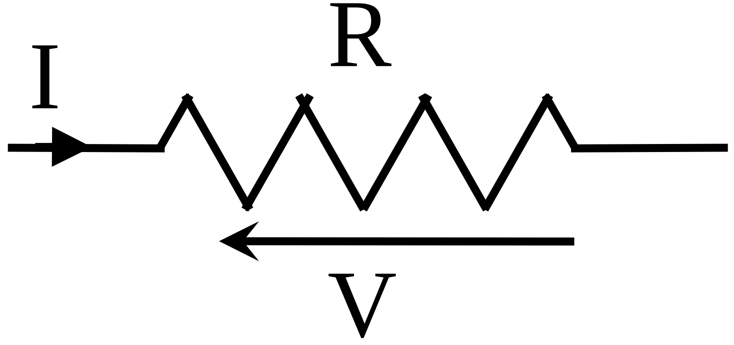
$i(t) = I_m \cos(\omega t+\theta)$

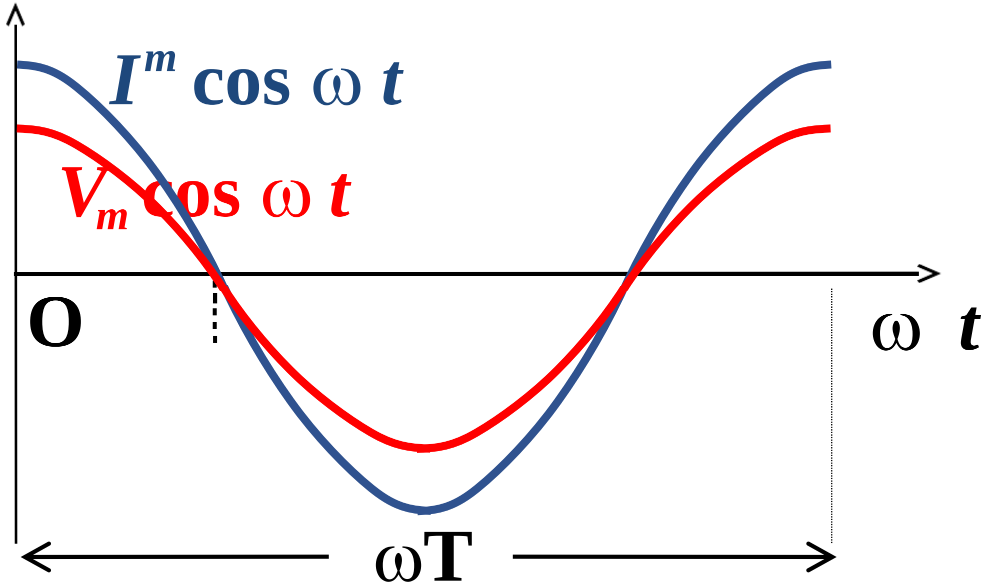
$v(t) = \mathbf{R} \,.\, I_m \cos(\omega t+\theta) = V_m \cos(\omega t+\theta)$

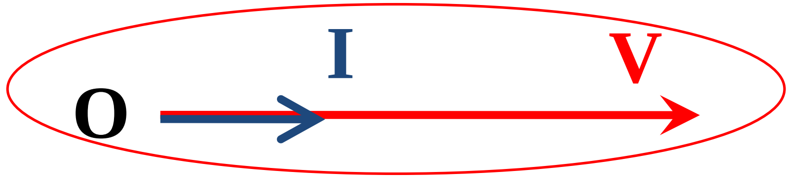
$\therefore \boldsymbol{V_m = R.I_m}$ and

$V_m/\sqrt{2} = R.I_m/\sqrt{2}$

i.e. $\boldsymbol{V = R \,.\, I}$







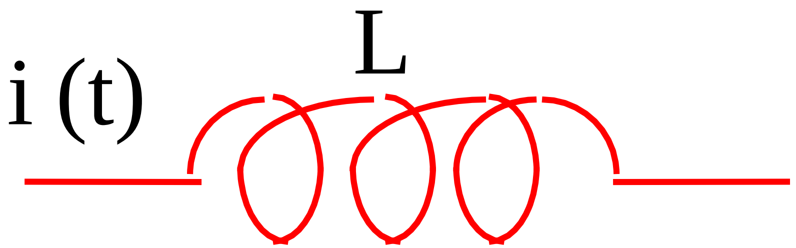
Phasor diagram

V and ***I*** are rms values

No phase angle change has occurred in the resistor .

Power dissipated in ***R*** is ***V . I***

$\boldsymbol{P = R \,.\, I^2}$



Inductor

$$v(t) = L\frac{di(t)}{dt}$$

for a sinusoid, $i(t) = I_m \cos(\omega t+\theta)$

$$\therefore \quad v(t) = \boldsymbol{L.\frac{d}{dt} I_m \cos(\omega t+\theta)}$$

$$= \; -L.\omega.I_m \sin(\omega t+\theta)$$

$$= \; L.\ \omega.I_m \cos(\omega t+\theta+\pi/2)$$

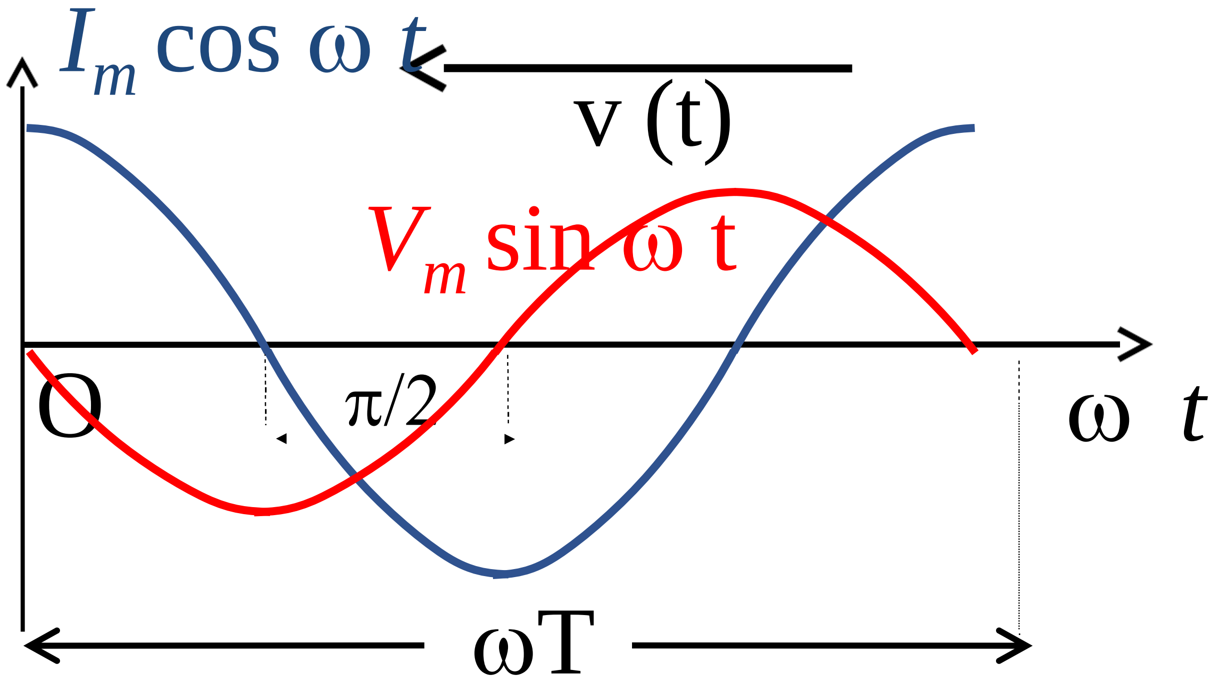
$$= V_m \cos(\omega t+\theta+\pi/2)$$

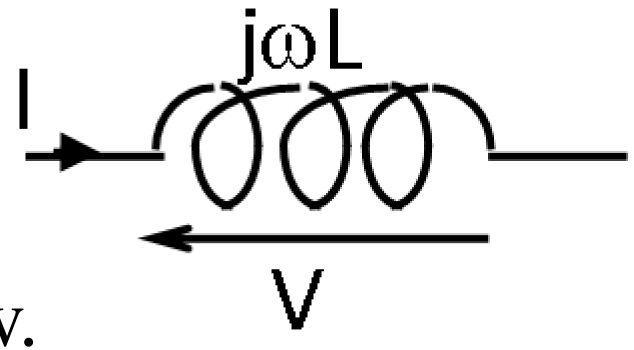
$\therefore \boldsymbol{V_m = \omega L.I_m}$ and $V_m/\sqrt{2} = \omega \boldsymbol{L}.I_m/\sqrt{2}$

rms voltage is related to ***rms*** current by the multiplying factor ω***L.***

Voltage leads current by 90^o or $\pi/2$ radians.

Current lags voltage by 90^o for an inductor.





$\boldsymbol{V = j\omega L.I}$ **or** $\boldsymbol{V = \omega\ L.I\angle 90^{o}}$

Impedance Z of inductance is thus $j\omega\ L$.

$V = Z\ .\ I$ corresponds to the generalised form of Ohm's Law.

Power dissipation in pure inductor is zero, but product $V\ .\ I$ is not zero.

Example:

An inductor of value 100 mH is supplied from a 230V, 50 Hz alternating supply. Taking supply voltage as reference, determine magnitude and phase angle of the current flowing into the circuit.

Answer:

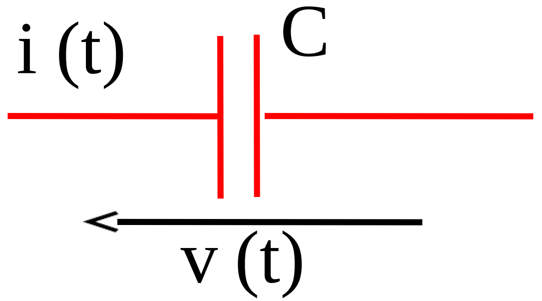
Impedance = $j\ \omega\ L = j\ 2\pi\times 50\times 100\times 10^{-3}$

$= j31.416 = 31.416\angle 90^{0}\ \Omega$

$$\therefore\quad I = \frac{230\angle 0^{0}}{31.416\angle 90^{0}} = 7.32\angle -90^{0}\ A$$

Capacitor

$$v(t) = \frac{1}{C}\int i(t).dt$$

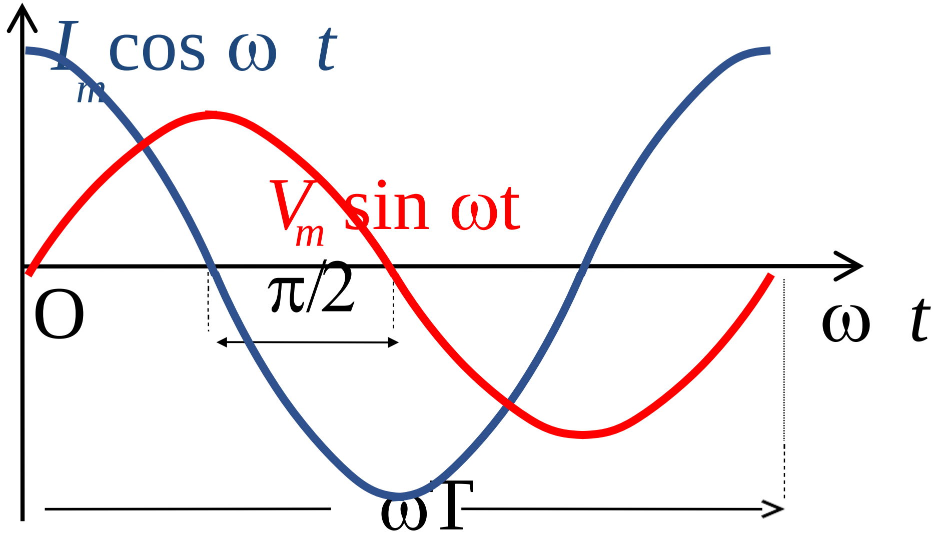


for a sinusoid $i(t) = I_m \cos(\omega t+\theta)$

$$v(t) = \frac{1}{C}\int I_m \cos(\omega t + \theta).dt = \frac{1}{C\omega} I_m \sin(\omega t+\theta)$$

$$= \frac{1}{C\omega} I_m \cos(\omega t+\theta-\pi/2) = V_m \cos(\omega t+\theta-\pi/2)$$

$$\therefore V_m = \frac{1}{\omega C}.I_m$$ and $V_m/\sqrt{2} = \frac{1}{\omega C}.I_m/\sqrt{2}$



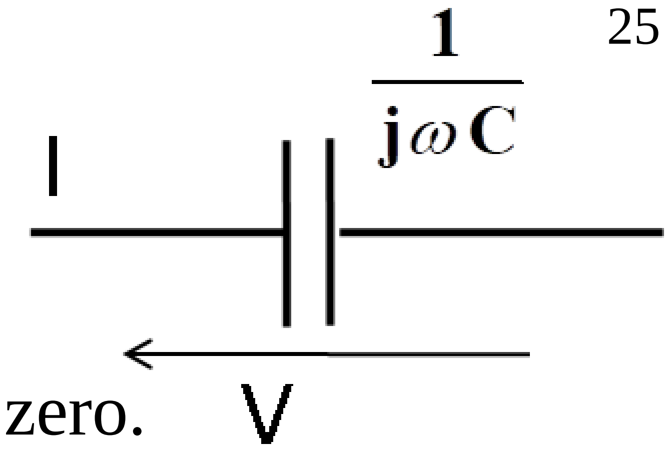
rms voltage is related to ***rms*** current by factor.

Voltage waveform lags current by 90° or π/2 rad

Current waveform leads voltage waveform by 90°.

Phasor diagram

Usual to write relationship as $\boldsymbol{V} = 1/j\omega C\,.\boldsymbol{I}$ or $\boldsymbol{V} = 1/\omega C.\boldsymbol{I}\angle{-90^{o}}$



$\boldsymbol{V} = \boldsymbol{Z}\,.\,\boldsymbol{I}$ corresponds to the generalised form of Ohm's Law.

Impedance Z of capacitance is thus defined as $1/j\omega C$

Power dissipation in a pure capacitor is zero. But product $V.I$ is not zero.

Example:

A capacitor of value 10 μF is supplied from a 230V, 50 Hz alternating supply. Taking the supply voltage as reference, determine the magnitude and phase angle of the current flowing into the circuit.

Answer:

Impedance $= 1/j\omega C = 1/(j2\pi\times 50\times 10\times 10^{-6}) = -\,j318.31$

$= 318.31\angle{-90^{0}}\ \Omega$

$$\therefore \boldsymbol{I} = \frac{\mathbf{230\angle 0^{0}}}{\mathbf{318.31\angle{-90^{0}}}} = \mathbf{0.722\angle 90^{0}}\ \mathrm{A}$$

Impedance and Admittance in an a.c. circuit

Impedance $\boldsymbol{Z}$ defines relation between complex *rms voltage* and *rms current*.

Admittance $\boldsymbol{Y}$ is inverse of impedance $\boldsymbol{Z}$.

$$\boldsymbol{V} = \boldsymbol{Z} \,.\, \boldsymbol{I}, \qquad \boldsymbol{I} = \boldsymbol{Y} \,.\, \boldsymbol{V}$$

where $\boldsymbol{Z} = \boldsymbol{R} + \boldsymbol{j}\,\boldsymbol{X},$ and $\boldsymbol{Y} = \boldsymbol{G} + \boldsymbol{j}\,\boldsymbol{B}$

Usual to express $\boldsymbol{Z}$ in cartesian form in terms of $\boldsymbol{R}$ and $\boldsymbol{X}$, and $\boldsymbol{Y}$ in terms of $\boldsymbol{G}$ and $\boldsymbol{B}$.

Real part of impedance $\boldsymbol{Z}$ is resistive - usually denoted by $\boldsymbol{R}$.

Imaginary part of impedance $\boldsymbol{Z}$ is a *reactance* - denoted by $\boldsymbol{X}$. Pure inductor/capacitor has reactance only and not resistive part, while a pure resistor has only resistance and not a reactive part.

$\mathbf{Z = R + j\,0}$ for a resistor,

$\mathbf{Z = 0 + j\omega L}$ for an inductor, and

$\mathbf{Z} = \dfrac{\mathbf{1}}{\boldsymbol{j\omega C}} = \mathbf{0} - \boldsymbol{j}\dfrac{\mathbf{1}}{\boldsymbol{\omega C}}$ for capacitor.

Real part of admittance Y is a *conductance* and denoted by G.

Imaginary part of admittance Y is a *susceptance* and denoted by B.

Relationships exist between components of Z and components of Y

$$\boldsymbol{G + j\,B = Y = \frac{1}{Z} = \frac{1}{R + j\,X} = \frac{R - j\,X}{R^2 + X^2}}$$

so that $\boldsymbol{G = \dfrac{R}{R^2 + X^2}}$, and $\boldsymbol{B = \dfrac{-X}{R^2 + X^2}}$

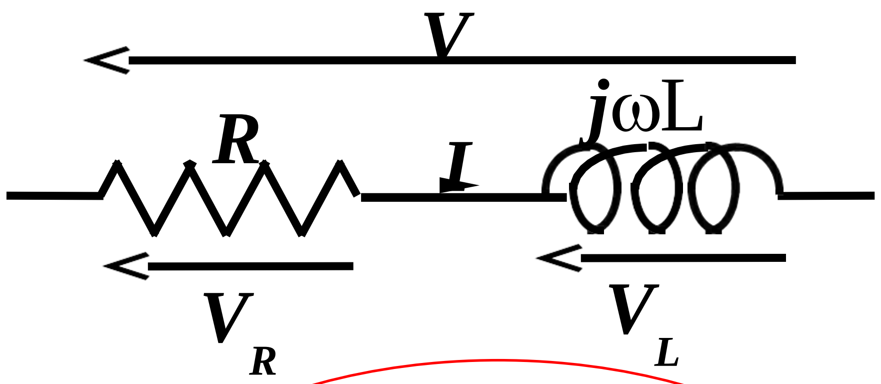
Reverse process can be also done if necessary.

G does not correspond to inverse of resistance **R.**

Effective value of G is influenced by **X** as well.

Simple Series Circuits

For single elements ***R***, ***L*** and ***C*** angle difference between voltage and current is either 0°, or ± 90°. Situation changes with more than one component.



R-L series circuit

Considering current **I** as reference

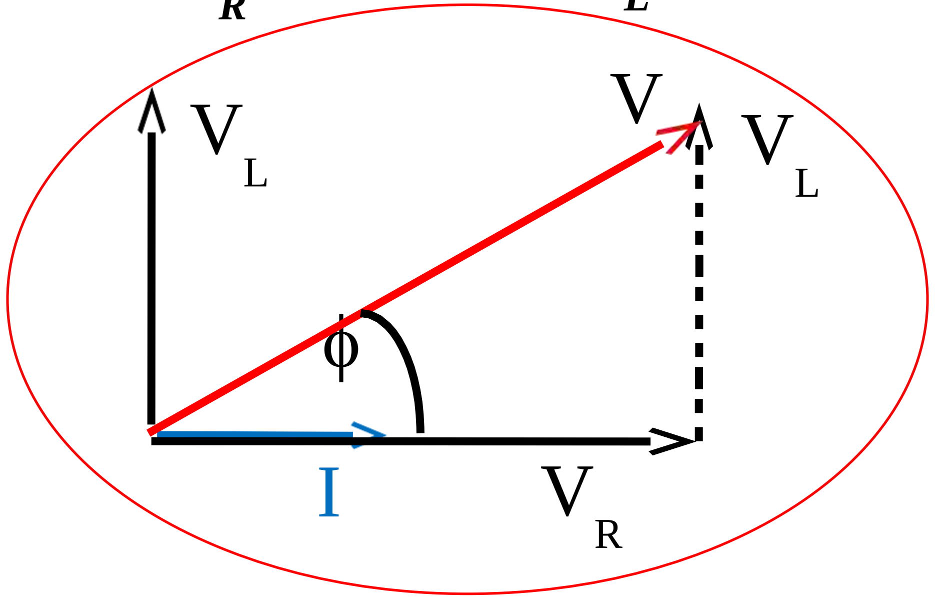
$V_R = R.I$, $V_L = j\omega L.I$, and $V = V_R + V_L$

$\therefore$ $V = (R + j\omega L).I$ so that

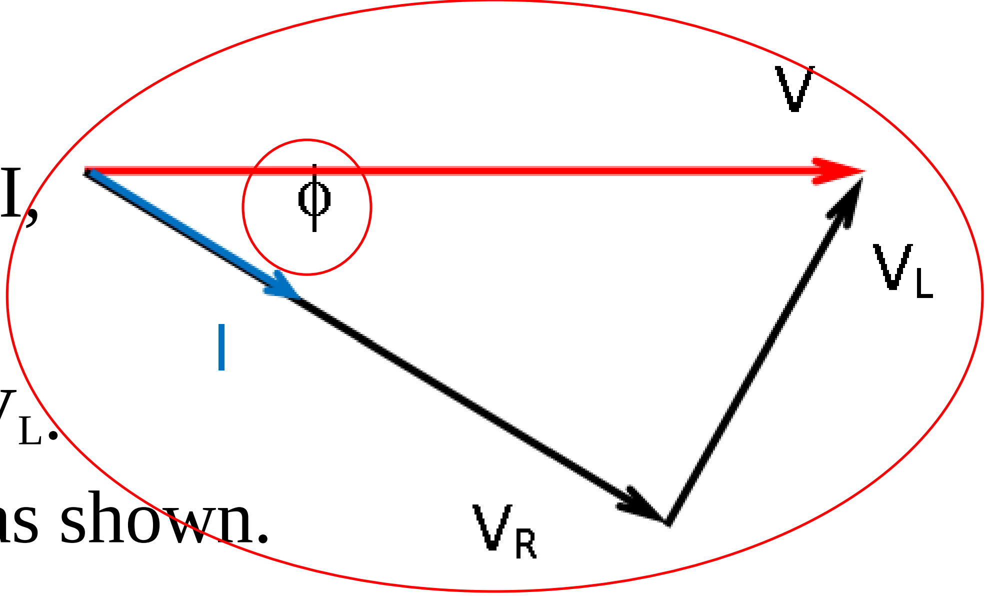
total series impedance $Z = R + j\omega L$

Phasor diagram is drawn with ***I*** as reference.

[i.e. ***I*** is drawn along the x-axis direction].



Select I as reference, as it goes to both R and L making drawing phasor diagram easier.



In the diagram, voltage across resistor $\mathbf{V_R}$ is in phase with I, whereas voltage across inductor $\mathbf{V_L}$ is 90^o leading I.

Total voltage **V** is obtained by phasor addition of V_R and V_L.

If total V is taken as reference, diagram would just rotate as shown.

In new diagram, I lags V by same angle as V was seen to be leading I.

V_L is drawn from end of V_R rather than from origin for ease of obtaining resultant V from triangular law.

In R-L circuit, current lags voltage by angle less than 90^o and circuit is inductive.

Note that the power dissipation can only occur in resistance in the circuit and is equal to $\boldsymbol{R.I^2}$ and that this is not equal to product $\boldsymbol{V.I}$ for the circuit.

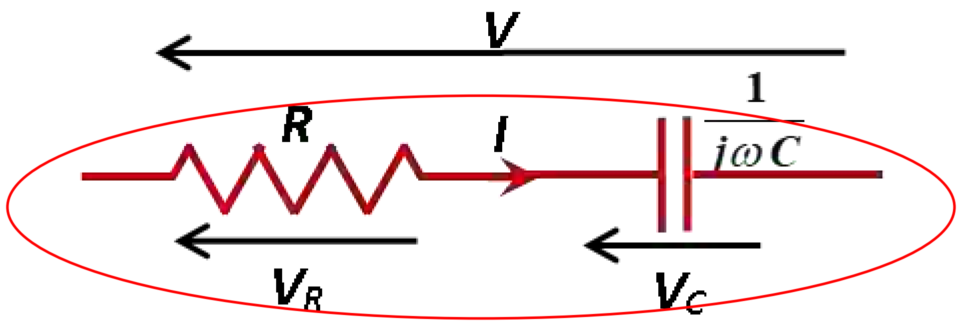
R-C series circuit

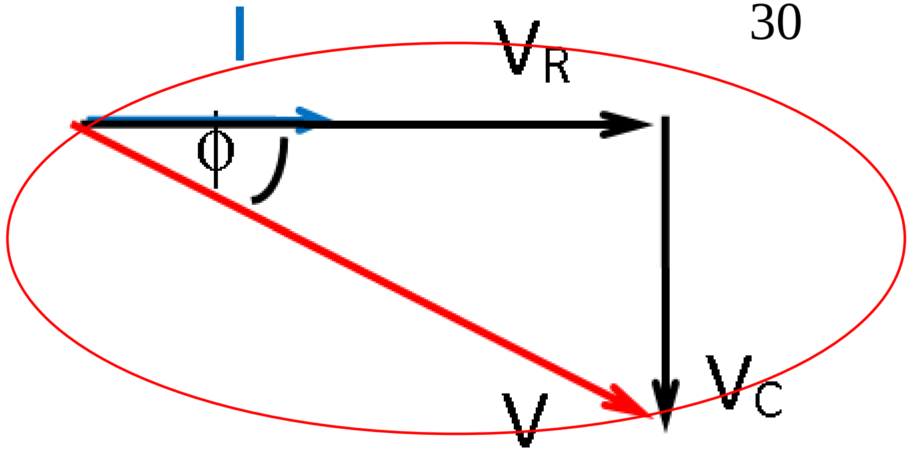
$V_R = R.I,$

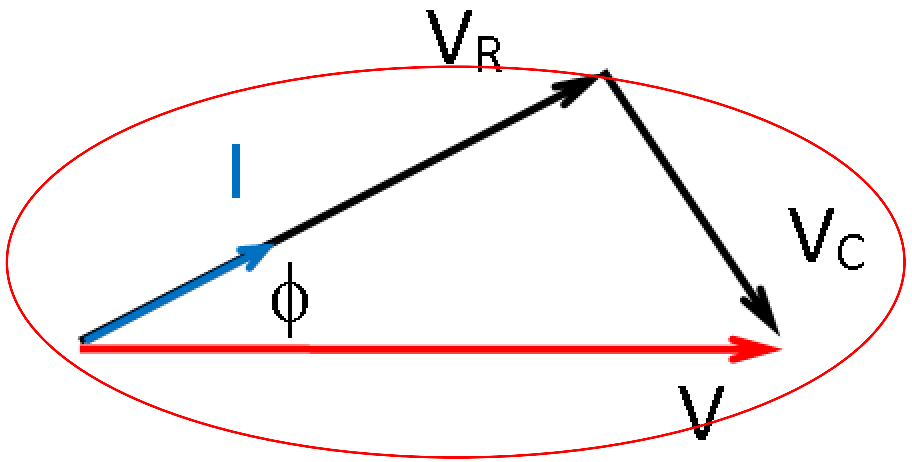
$$V_C = \frac{1}{j\omega C} I = -j\frac{1}{\omega C}.I$$

and $V = V_R + V_C$

$$\therefore V = (R + \frac{1}{j\omega C}).I \text{ so that } Z = R + \frac{1}{j\omega C}$$





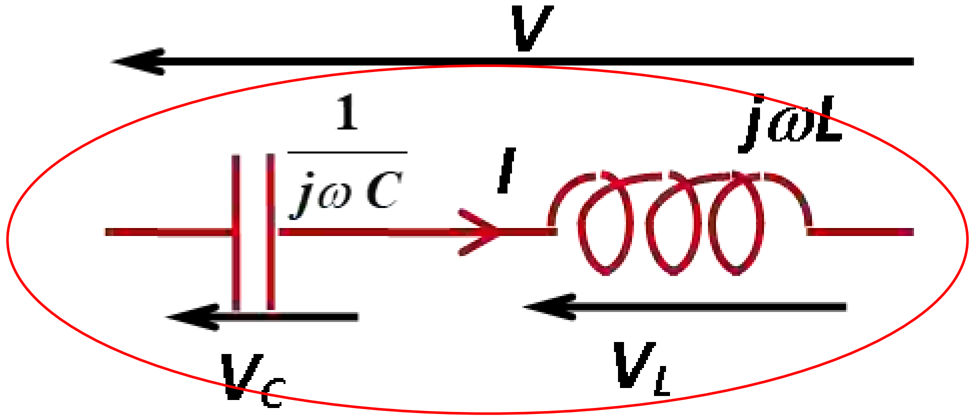


Phasor diagram has been first drawn with **I** as reference and then with **V** as reference.

In R-C circuit, current leads voltage by angle less than 90° and circuit is capacitive.

Power dissipation only occurs in resistance = $\boldsymbol{R.I^2}$ and is not equal to product $\boldsymbol{V.I}$.

L-C series circuit

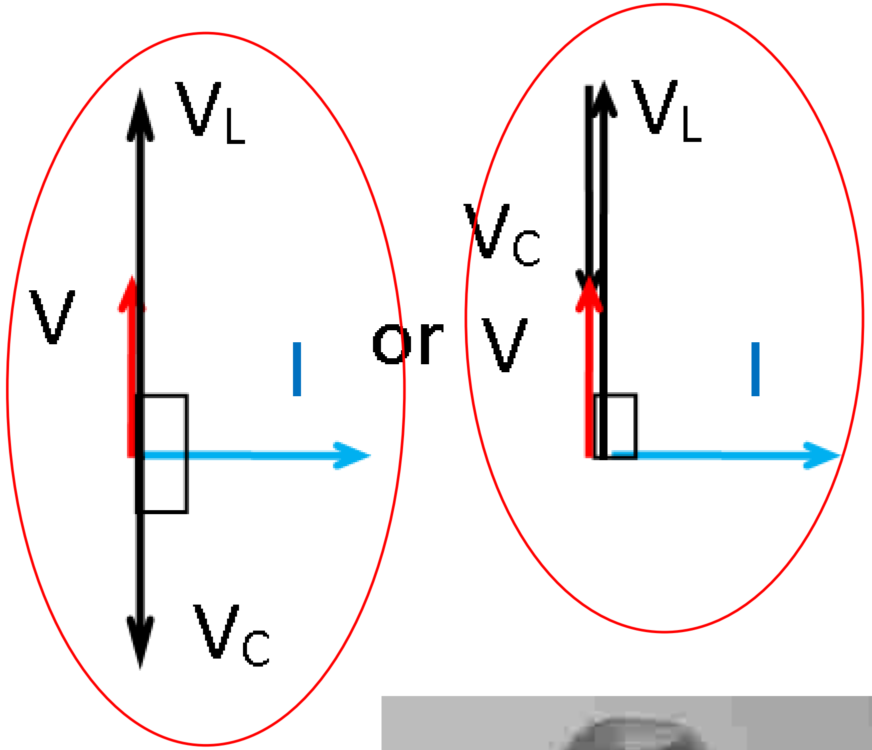


$$V_L = j\omega L.I, \quad V_C = \frac{1}{j\omega C}.I = -j\frac{1}{\omega C}$$

and $V = V_L + V_C$

$$\therefore \quad V = (j\omega L + \frac{1}{j\omega C}).I$$

so that $Z = j\omega L + \dfrac{1}{j\omega C} = j\omega L - j\dfrac{1}{\omega C}$



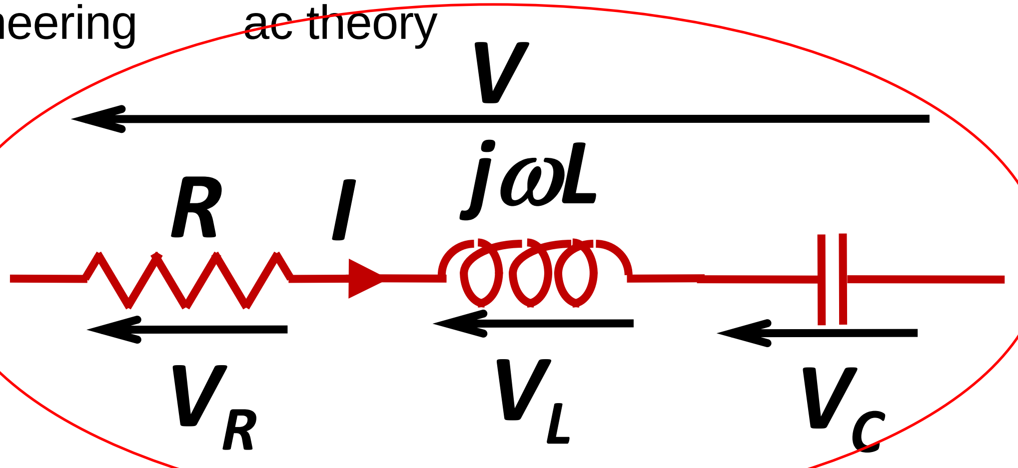
Resultant voltage could be either up or down.

When $\omega L = \dfrac{1}{\omega C}$ total impedance becomes zero, so that circuit current is theoretically infinity (series resonance).

In pure L-C circuit, current either lags or leads voltage by 90^o and resultant circuit is either purely inductive or capacitive.

No power dissipation occurs but the product ***V.I*** is non zero.

R-L-C series circuit



$V_R = R.I,$

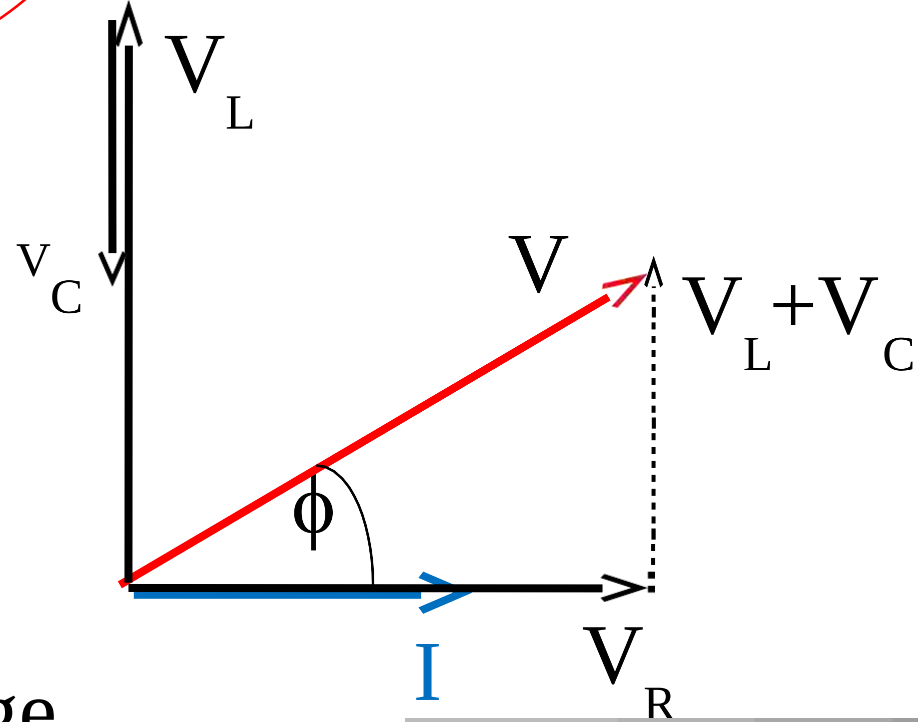
$V_L = j\omega L.I,$

$V_C = \dfrac{1}{j\omega C}.I$

and $V = V_R + V_L + V_C = (R + j\omega L + \dfrac{1}{j\omega C}).I$

Total $Z = R + j\omega L + \dfrac{1}{j\omega C} = R + j(\omega L - \dfrac{1}{\omega C})$

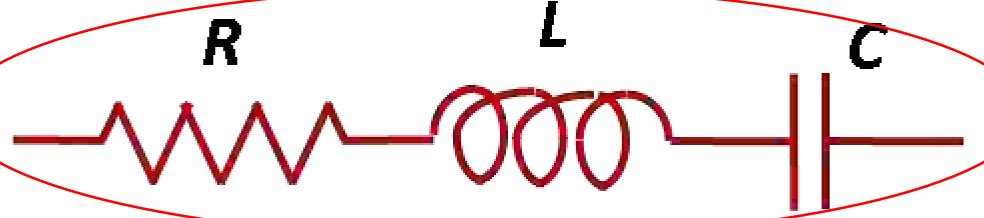
$|Z| = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$ has minimum at $\omega L = \dfrac{1}{\omega C}$



In R-L-C circuit, current can either lag or lead voltage between –90° and 90° and circuit is inductive or capacitive

Power dissipation can only occur in resistance as $R . I^2$.

Example:



An R-L-C series circuit, with R = 10 Ω, L = 100 mH, C = 10 μF is supplied from a 230 V, 50 Hz supply.

Determine the current, the voltage across each element and the power loss.

Answer:

Impedance of circuit $= 10 + j2\pi\times50\times100\times10^{-3} + 1/(\, j2\pi\times50\times10\times10^{-6})$ Ω

$= 10 + j\,31.416 - j\,318.31 = 10 - j\,286.9 = 287.1\angle{-88.0^0}\,\Omega$

Current through circuit $= \dfrac{230\angle 0^0}{287.1\angle -88.0^0} = 0.801\angle 88.0^0\ A$

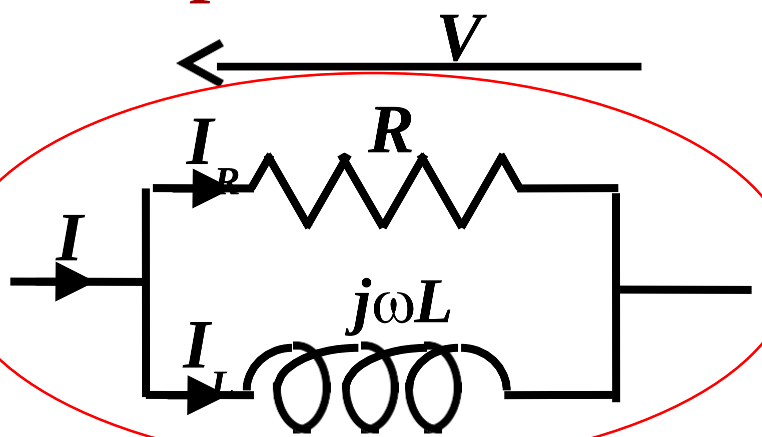
Voltage across R $= 0.801\angle 88.0^0\times 10 = 8.01\angle 88.0^0$ V

Voltage across L $= 0.801\angle 88.0^0\times 31.416\angle 90^0 = 25.17\angle 178.0^0$ V

Voltage across C $= 0.801\angle 88.0^0\times 318.31\angle{-90^0} = 255.0\angle{-2.0^0}$ V

Simple Parallel Circuits

R-L parallel Circuit

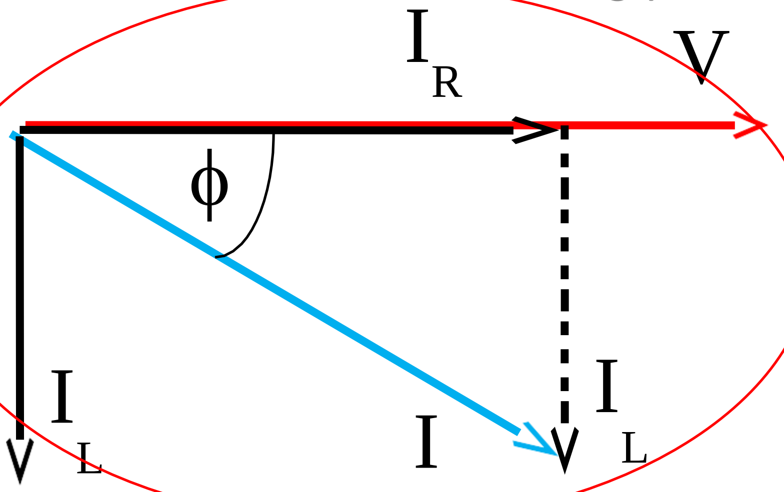


consider **V** as reference

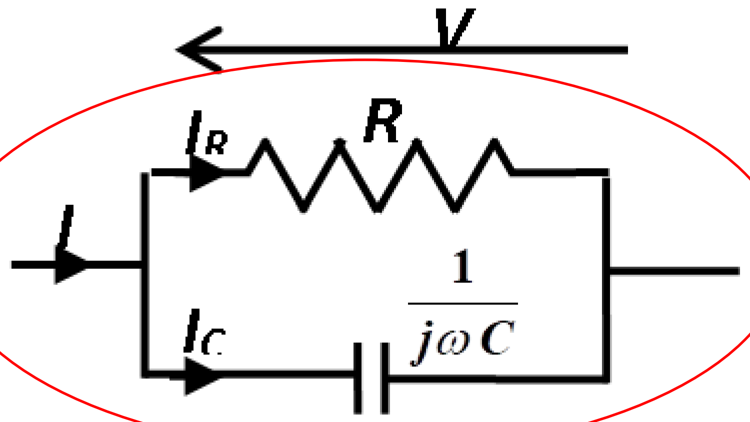
$V = R.I_R$, $V = j\omega L.I_L$, $I = I_R + I_L$

$$\therefore\ I = \frac{V}{R} + \frac{V}{j\omega L}$$

$$\therefore \text{total } Y = \frac{1}{R} + \frac{1}{j\omega L}$$



R-C parallel Circuit

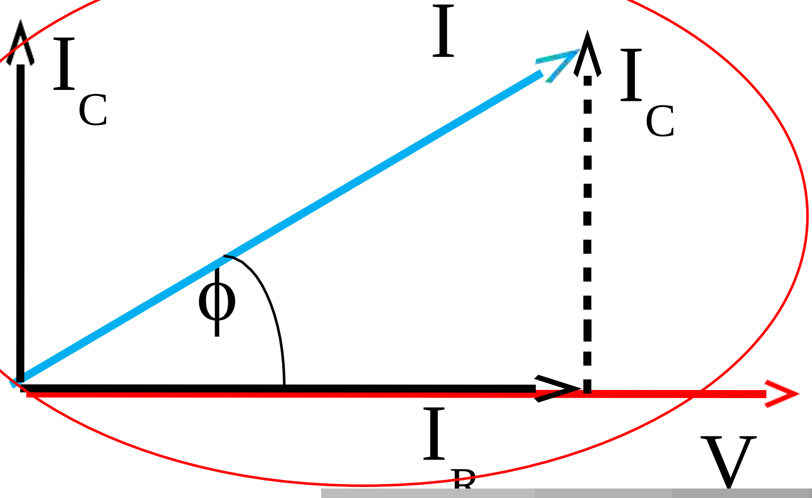


consider **V** as reference

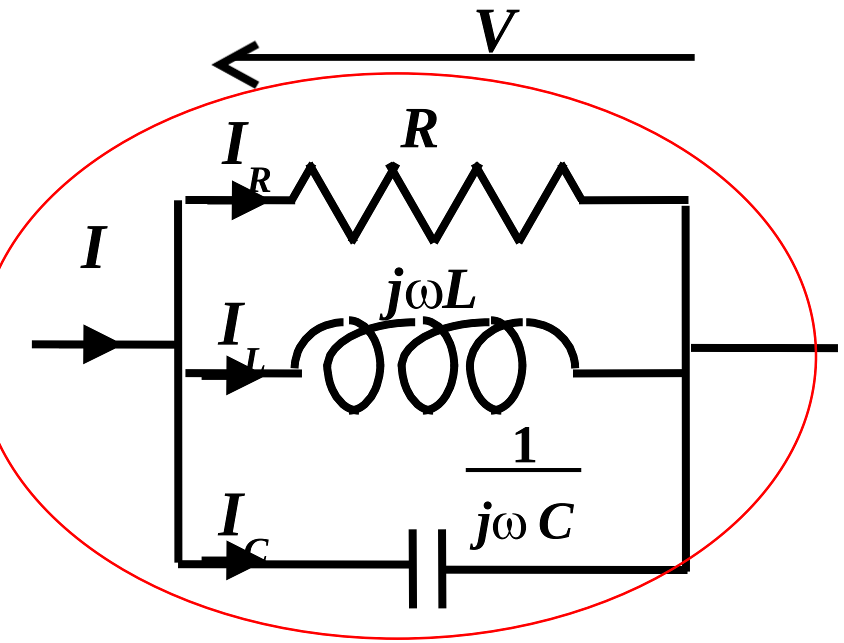
$V = R.I_R$, $I_C = j\omega C.V$, $I = I_R + I_C$

$$\therefore\ I = \frac{V}{R} + V.j\omega C$$

$$\therefore \text{total } Y = \frac{1}{R} + j\omega C$$



R-L-C parallel Circuit



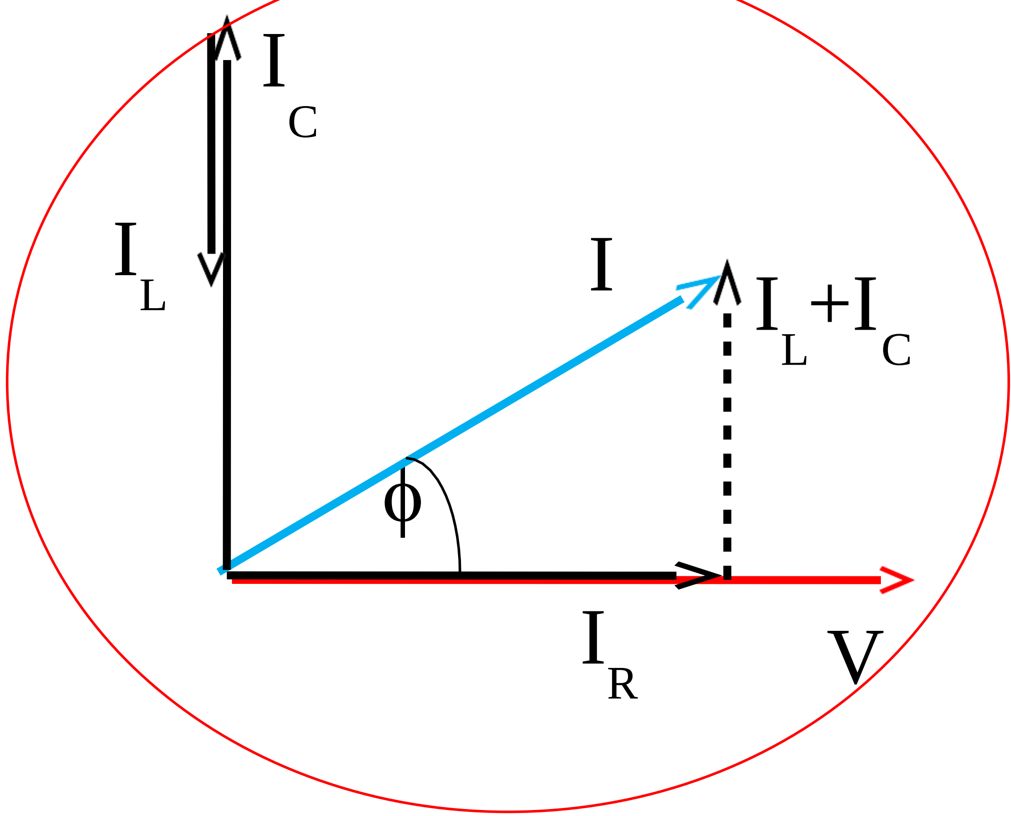
consider **V** as reference

$V = R.\,I_R$, $V = j\omega L.I_L$,

$I_C = j\omega C.V$ and $I = I_R + I_L + I_C$

$$I = \frac{V}{R} + \frac{V}{j\omega L} + V.j\omega C$$

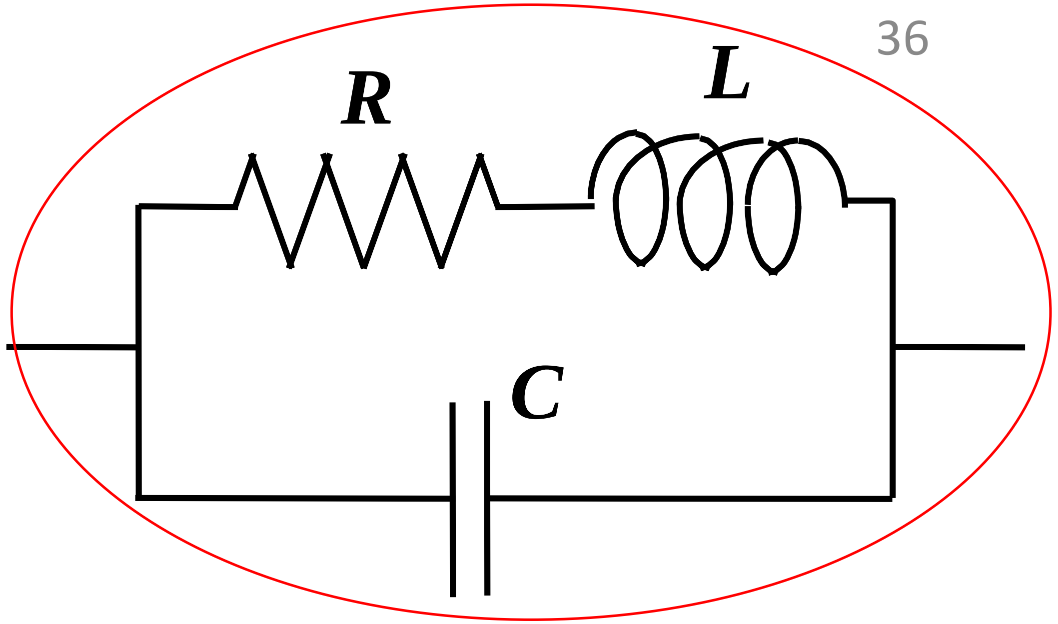
$$\text{total } Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$



As in series circuit, shunt resonance at $\frac{1}{\omega L} = \omega C$ giving a minimum value of shunt admittance.

Power loss can only occur in resistive element and that product ***V.I*** is not usually equal to power loss.

Example for you to try out before I explain:



A circuit consists of a series combination of a resistor R = 10 Ω and an inductor L = 100 mH, connected in parallel with capacitor C = 10 μF.

If it is supplied from a 230 V, 50 Hz supply, calculate

(a) the total impedance of the circuit,

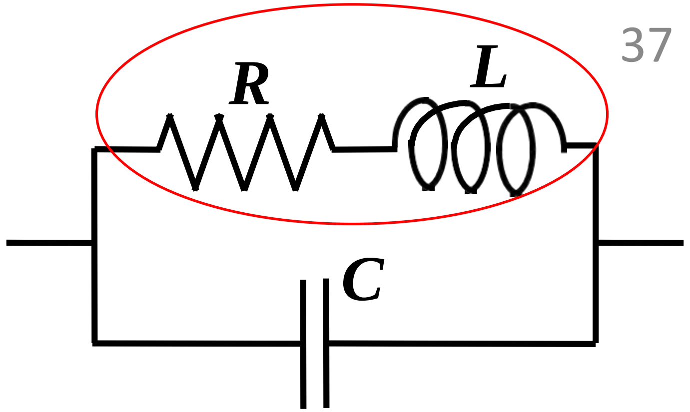
(b) the current supplied from the source,

(c) the current in each of the branches

(d) the voltage across each of the elements of the circuit, and

(e) power loss in the circuit.

Answer:



R = 10 Ω, $X_L = 2\pi\times50\times100\times10^{-3} = 31.416\ \Omega$,

$X_C = 1/(2\pi\times50\times10\times10^{-6}) = 318.31\ \Omega$

Z of R and L in series = $(10 + j\,31.416) = 32.97\angle72.34^0\ \Omega$

Z of 'R series L' in parallel with C = $\dfrac{32.97\angle72.34^0 \times 318.31\angle-90^0}{32.97\angle72.34^0 + 318.31\angle-90^0}$

$$= \frac{10494.7\angle-17.66^0}{10+j31.416-j318.31} = \frac{10494.7\angle-17.66^0}{10-j286.89} = \frac{10494.7\angle-17.66^0}{287.09\angle-88.0^0} = 36.56\angle70.34^0\ \Omega$$

Current supplied from circuit = $\dfrac{230\angle0^0}{36.56\angle70.34^0} = 6.291\angle-70.34^0\ A$

Currents in parallel circuit divide itself inversely as the impedances

$$I_{RL} = \frac{Z_C}{Z_{RL}+Z_C} I = \frac{318.31\angle-90^0}{10-j286.89} \times 6.291\angle-70.34^0$$

$$= 6.975\angle-72.34^0\ A$$

$I_C = 6.291\angle{-70.34^0} - 6.975\angle{-72.34^0} = (2.116 - j5.924) - (2.116 - j6.646)$

i.e. $I_C = j\,0.722 = 0.722\angle 90.0^0$ A

voltages across the elements are thus given by

$V_R = 6.974\angle{-72.34^0} \times 10 = 69.74\square{-72.34^0}$ V

$V_L = 6.974\angle{-72.34^0} \times 31.416\angle 90^0 = 219.1\angle 17.64^0$ V

$V_C = 0.722\angle 90.0^0 \times 318.31\angle{-90^0} = 229.8\angle 0.0^0$ V

(which must equal input voltage $230\angle 0^0$ V)

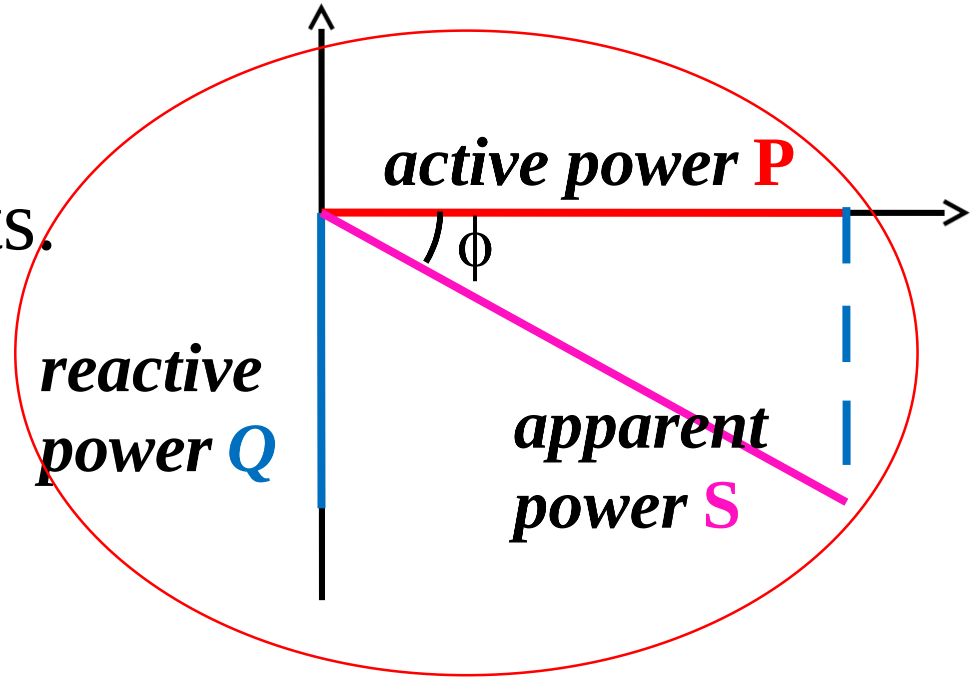
Power loss in the circuit = $|\,I\,|^2$ R = $6.975^2 \times 10$ = 486.5W

volt×ampere product of circuit = 230×6.291 =1446.9VA

ratio of **power loss** to **volt×ampere** product = 486.5/1446.9 = **0.336**

cosine of angle difference between total voltage and current = **cos 70.34^0** = **0.336**

Power and Power Factor



In a.c. circuit, power loss occurs only in resistive parts.

Power is not equal to product ***V .I*** .

Purely inductive/capacitive parts do not have any active power associated with them.

Define product of ***V.I*** as ***apparent power S***.

unit of ***Apparent power*** - volt-ampere (VA) and not the watt (W).

unit of ***active power*** - watt (W) used only for ***P*** (commonly called ***power***)

Define a new term called the ***reactive power Q*** for the reactance ***X***.

$P = S\cos\phi$, $Q = S\sin\phi$, $S^2 = P^2 + Q^2$

$$\cos\phi = \frac{\text{active power}}{\text{apparent power}}$$ = power factor (for sinusoidal waveform)

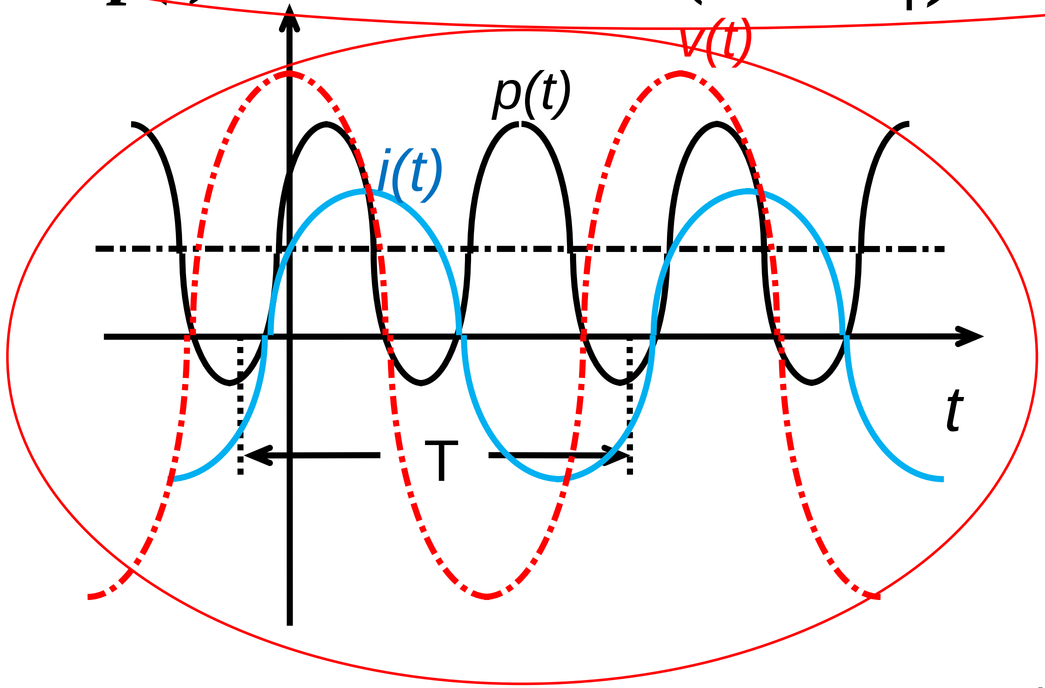
Instantaneous power $p(t) = v(t) \,.\, i(t)$

If $v(t) = V_m \cos \omega t$ and $i(t) = I_m \cos(\omega t - \phi)$,

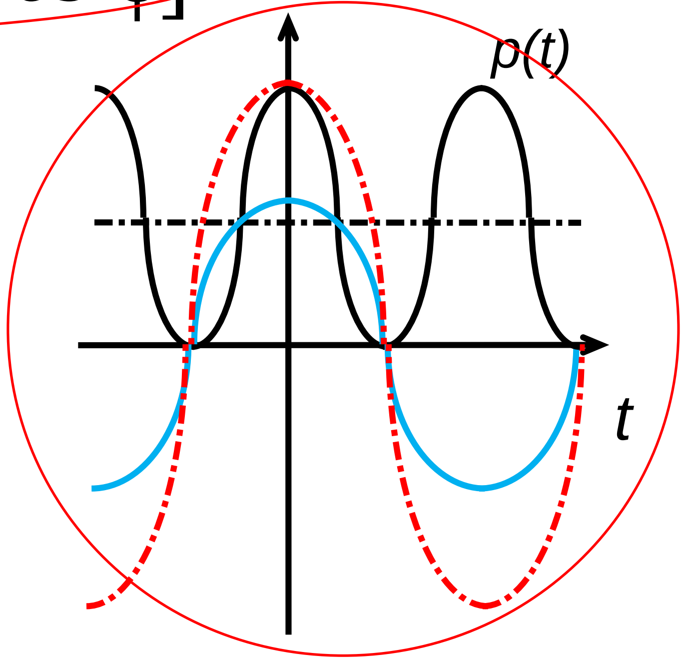
with V as reference and I lagging V by a phase angle ϕ, then

$$p(t) = V_m \cos\omega t \,.\, I_m \cos(\omega t - \phi) = V_m I_m \,.\, \tfrac{1}{2} \,.\, 2 \cos\omega t \,.\, \cos(\omega t - \phi)$$

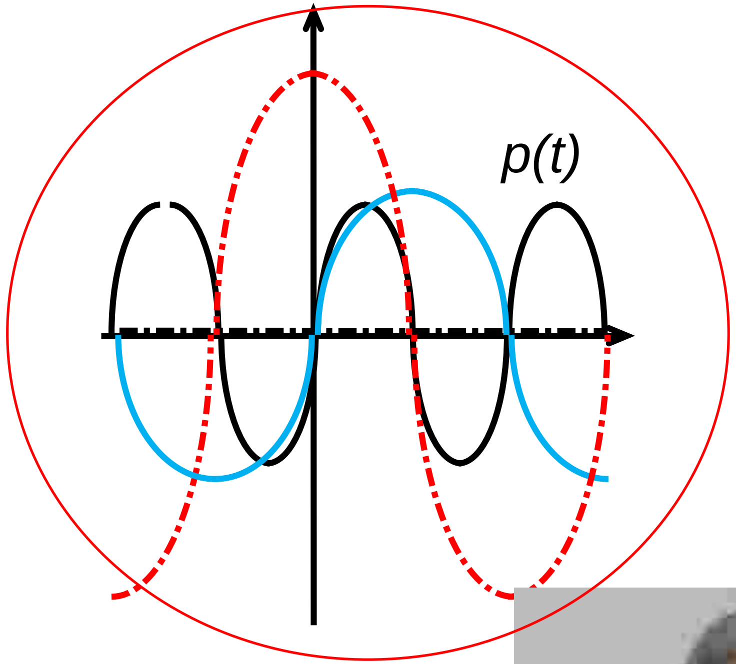
$$p(t) = \tfrac{1}{2}\; V_m I_m [\cos(2\omega t - \phi) + \cos\phi]$$



current lagging voltage by angle ϕ



inphase



quadrature

Waveform of p(t) has a sinusoidal component and a constant component.

Average value of active power $\boldsymbol{P}$ is given by constant $\frac{1}{2} \boldsymbol{V_m I_m \cos\phi}$

active power $\boldsymbol{P} = \frac{1}{2} \boldsymbol{V_m I_m \cos\phi} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos\phi = \boldsymbol{V \cdot I \cos\phi}$

$\frac{\text{active power}}{\text{apparent power}}$ is defined as ***power factor***

Thus $\boldsymbol{\cos\phi}$ is equal to the power factor for sinusoidal V and I.

for ***resistor***, $\phi = 0^o \rightarrow \boldsymbol{P = V \cdot I}$ and $\boldsymbol{p.f. = 1}$

for ***inductor***, $\phi = 90^o$ lagging (i.e. current lags voltage by 90^o) $\rightarrow \boldsymbol{P = 0}$ and $\boldsymbol{p.f. = 0}$

for ***capacitor***, $\phi = 90^o$ leading (i.e. current leads voltage by 90^o) $\rightarrow \boldsymbol{P = 0}$ and $\boldsymbol{p.f. = 0}$

For combinations of resistor, inductor and capacitor, P is between V.I and 0

For an inductor or capacitor, ***V. I*** exists although $\boldsymbol{P = 0}$.

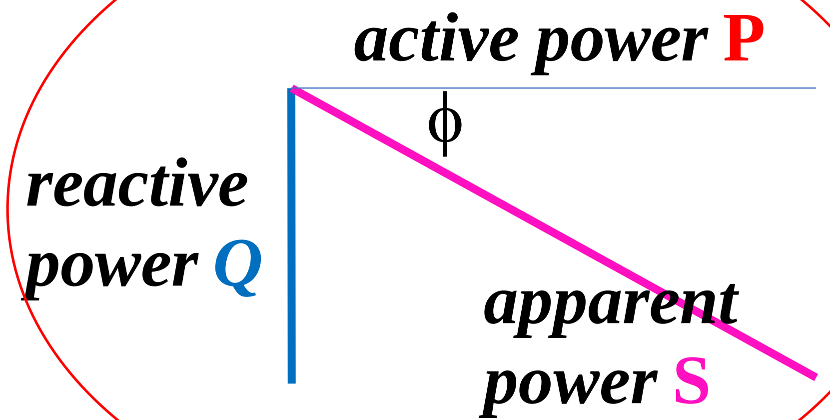
For pure L or C the product ***V. I*** is defined as ***reactive power Q***.

Occurs when ***voltage*** and ***current*** are ***quadrature*** (90^o out of phase).

Reactive power is usually defined from $P^2 + Q^2 = S^2$

For sinusoidal waveforms, it becomes the product of quadrature components of voltage and current

***reactive power** $Q = V.\, I \sin \phi$*



Define

– inductive reactive power when current is lagging the voltage

–capacitive reactive power when current is leading the voltage.

Inductive and capacitive reactive power have opposite signs.

Reactive power does not consume active energy but reduces power factor below unity.

For power factor below unity for same power transfer P,

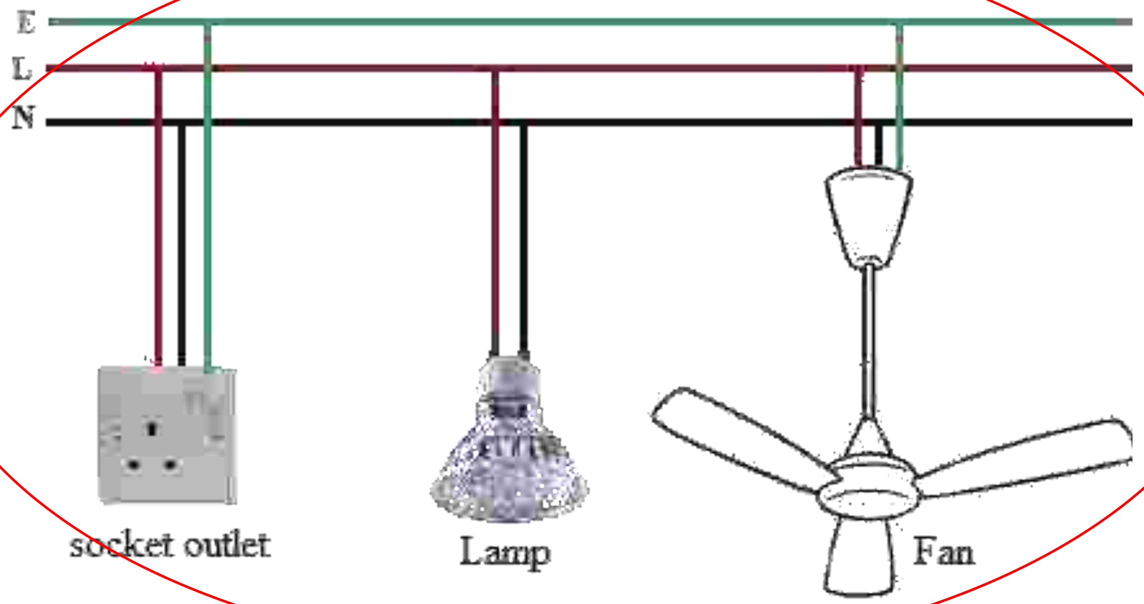
– current required becomes larger ($P \propto I$)

– power losses becomes still larger (power loss $\propto I^2$).

Supply authorities encourage industries to improve power factors close to unity.

Single Phase Power and Three Phase Power

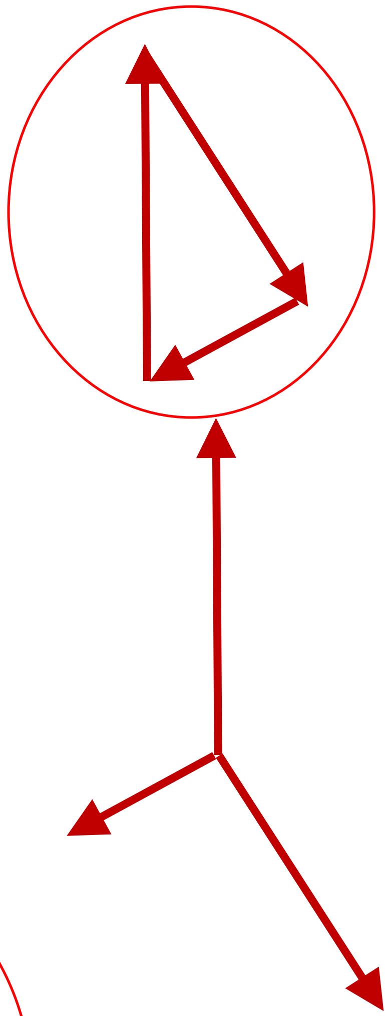
To send power using single phase ac, 2 wires required (L and N).
They may have an earth wire (which does not carry any current) for protection.



You may also have seen that distribution lines usually have 4 wires.
What are these 4 wires ?

It is the three phase wires and the neutral wire.

Three Phase Power and Balanced Three Phase



Now how does three phase power make our transmission more efficient ?
You know that if we have three equal forces at angles of 120^o to each other, using the triangular law we get an equilateral triangle where the resultant force is zero.
Similarly in 3-phase, except that there is phasors instead of vectors.

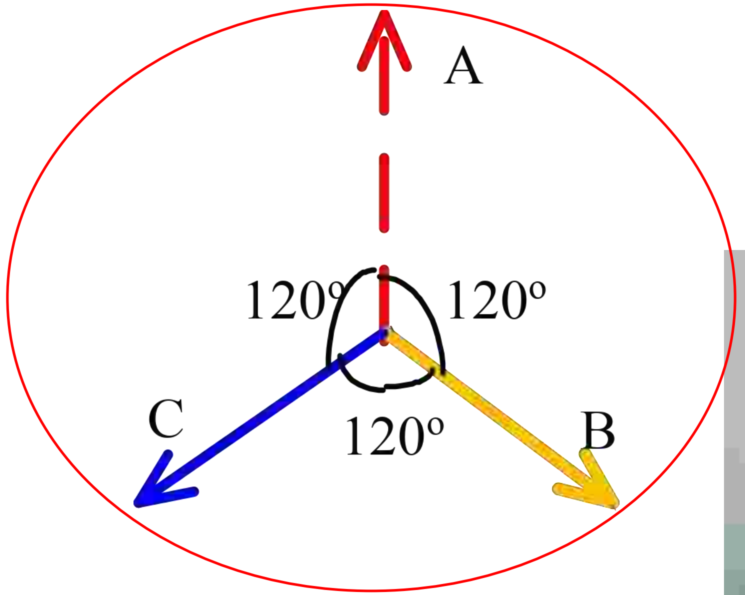
That is, we have 3 voltages (or currents) which are equal in magnitude but differing in phase angle by 120^o from each other.

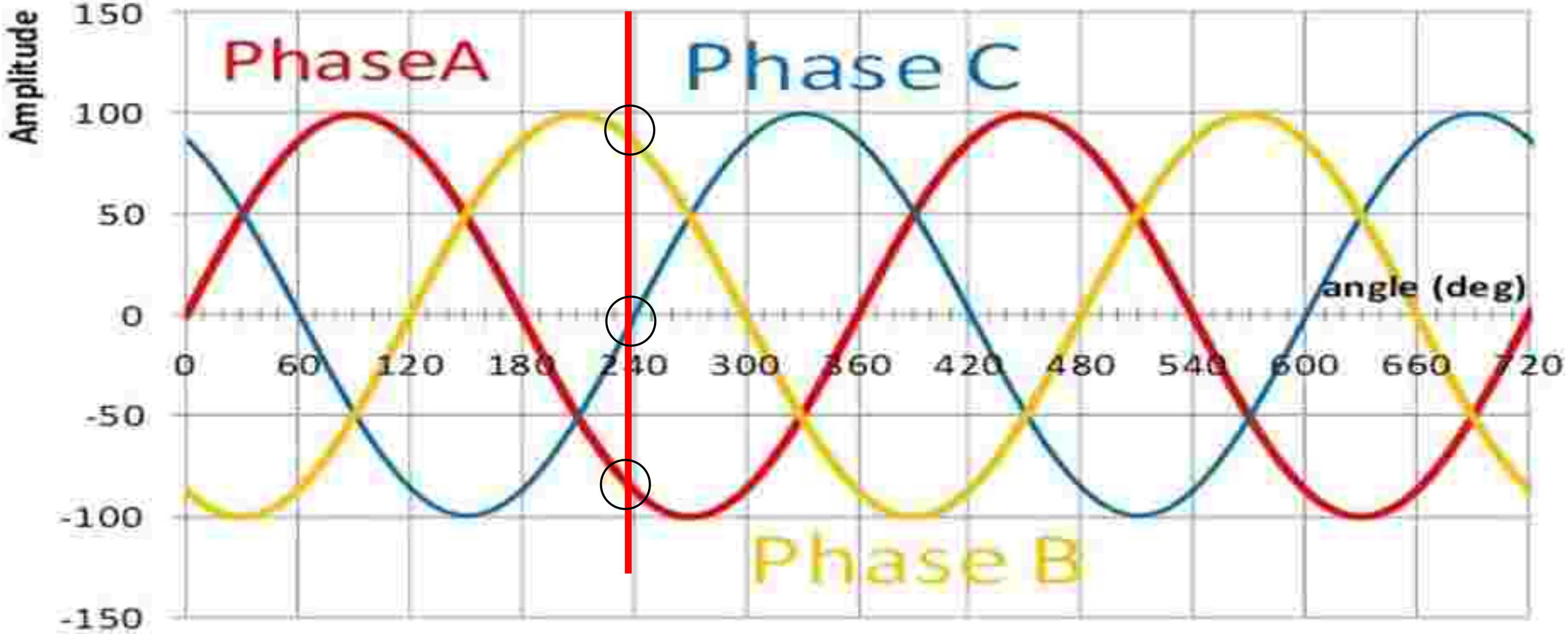
$$v_A(t) = V_m \cos \omega t,$$

$$v_B(t) = V_m \cos (\omega t - 2\pi/3)$$

$$v_C(t) = V_m \cos (\omega t + 2\pi/3)$$

Phase A is taken as reference and drawn vertical (or horizontal)





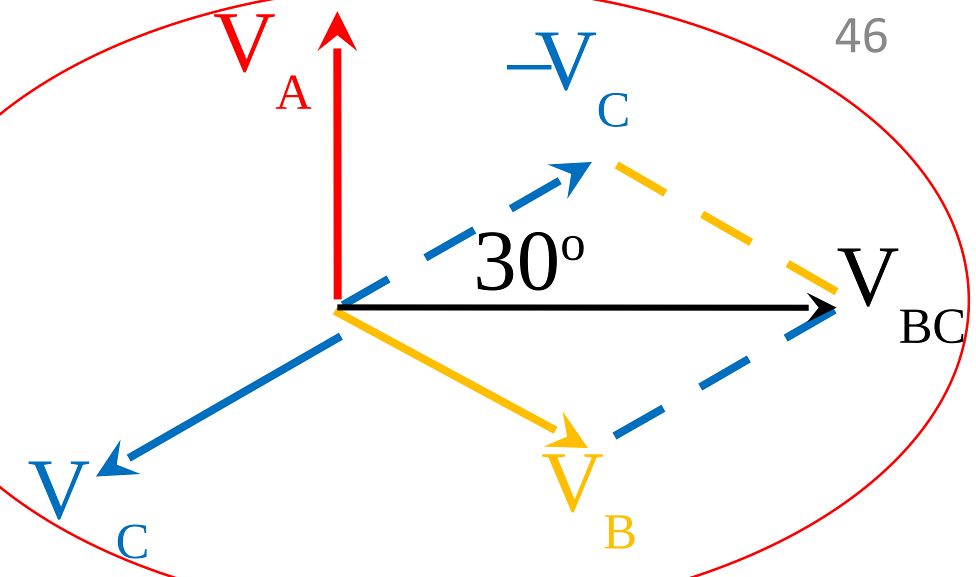
The addition of the three balanced waveforms at any instant is zero.

This is true for balanced three phase currents as well.

Thus for a balanced three phase system of currents, their addition would become zero, and no neutral wire would be required.

However in practice, three phase currents are never perfectly balanced and the neutral wire would carry the unbalance.

Phase voltage and Line voltage



r.m.s.voltages V_A, V_B, and V_C

– are with respect to neutral,

– they are normally called ***phase voltages***.

Voltage between any two phase-wires is ***line voltage.***

Line voltage is √3 times (i.e. 2 cos 30°) phase voltage for a balanced system.

In 3-phase system, it is the "***r.m.s. line voltage***" that is specified.

For example in the domestic supply,

– single phase voltage is 230 V and three phase voltage is 400 V (~ 230 × √3).

If V_p is phase voltage magnitude

$V_A = V_p \angle 0$, $V_B = V_p \angle -2\pi/3$, $V_C = V_p \angle 2\pi/3$.

Line voltage magnitude will be $\sqrt{3} \times V_p = V_L$ and

$V_{AB} = V_L \angle \pi/6$, $V_{BC} = V_L \angle(-2\pi/3+\pi/6) = V_L \angle -\pi/2$,

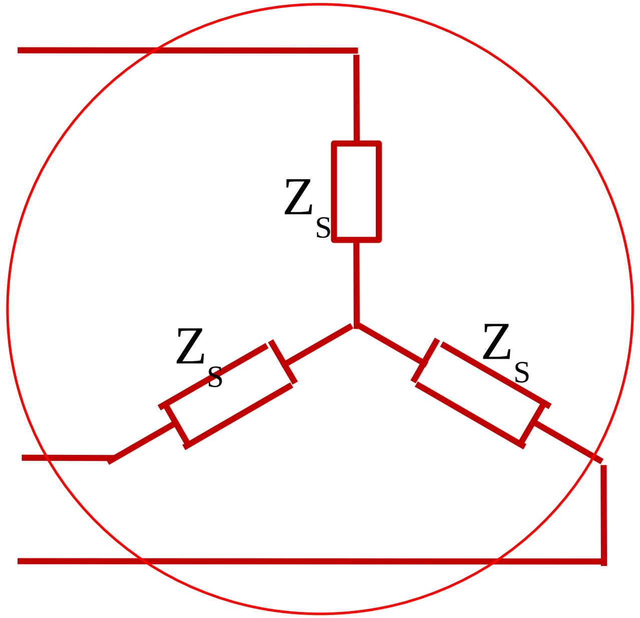
$V_{CA} = V_L \angle(2\pi/3+\pi/6) = V_L \angle 5\pi/6$.

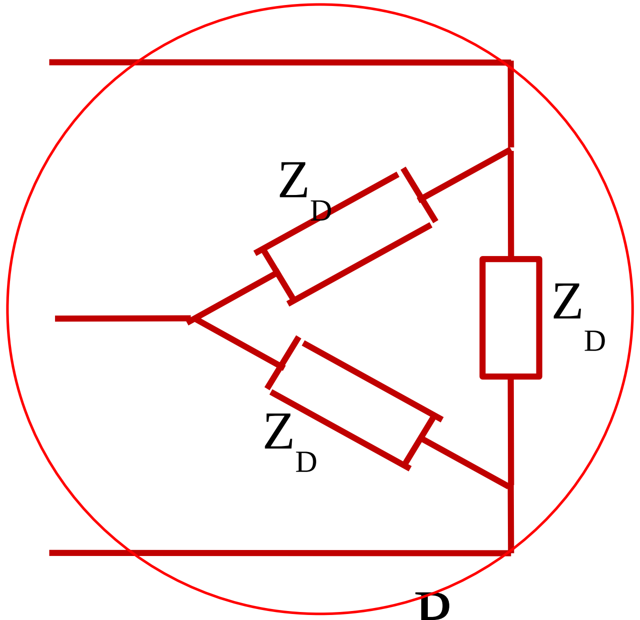
Balanced loads

May be connected either in star or delta.

A balanced star connected load can be converted to an equivalent delta and the other way round.

$Z_D = 3\,Z_S$



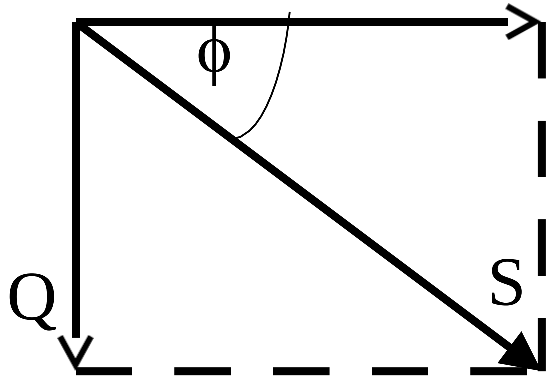


It can be shown that for a balanced load (star or delta connected),

total 3-phase active power $P = \sqrt{3}\, V_L\, I_L \cos\phi$

where $\cos\phi$ is the power factor of the balanced load,

V_L is the line voltage and I_L is the line current.



Reactive power in a three phase circuit is similarly defined as

$Q = \sqrt{3}\, V_L\, I_L \sin\phi$

and apparent power in a three phase circuit is defined as $S = \sqrt{3}\, V_L\, I_L$

In both single phase as well as three phase circuit,

$S^2 = P^2 + Q^2$ and $\tan\phi = Q/P$

Example

A 3 phase, 400V, 50 Hz supply feeds a 3 phase, 12 kW balanced load at a power factor of 0.8 lagging.

Determine the line current in magnitude and phase relative to supply voltage, apparent power and reactive power drawn.

Answer

$P = 12$ kW, $V_L = 400$V, $\cos\phi = 0.8$ lag,

$P = \sqrt{3}\, V_L\, I_L \cos\phi$, i.e. $12000 = \sqrt{3} \times 400 \times I_L \times 0.8$

$\therefore\ I_L = 21.65$ A, $\phi = \cos^{-1}(0.8) = 36.87^o$.

i.e. $I_L = 21.65 \angle{-36.87^o}$ A.

Apparent power $= \sqrt{3} V_L I_L = \sqrt{3} \times 400 \times 21.65 = 15000 = 15$ kVA

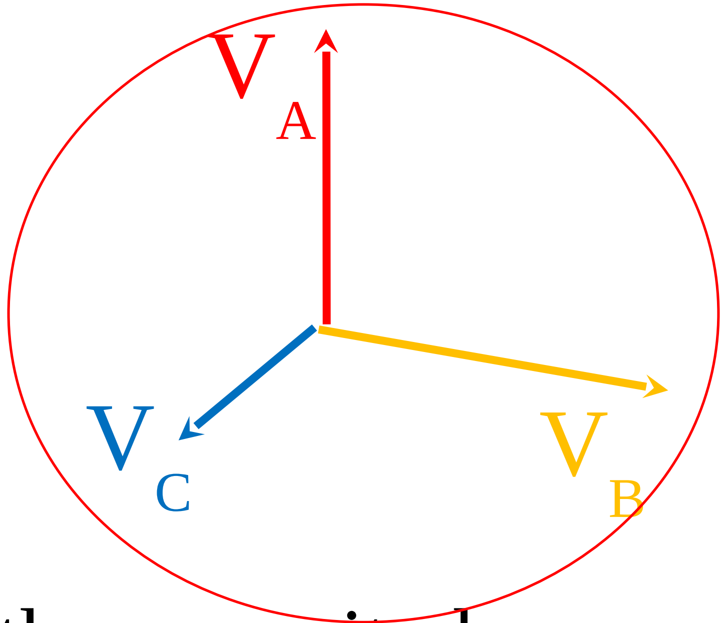
Reactive power $= \sqrt{3} V_L I_L \sin\phi = \sqrt{3} \times 400 \times 21.65 \times \sin(36.87^o)$

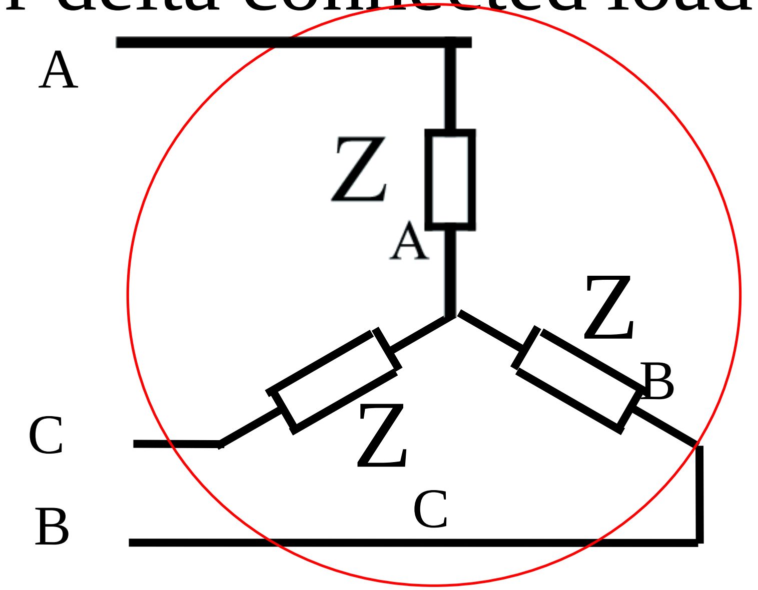
$= 9000 = 9$ kvar

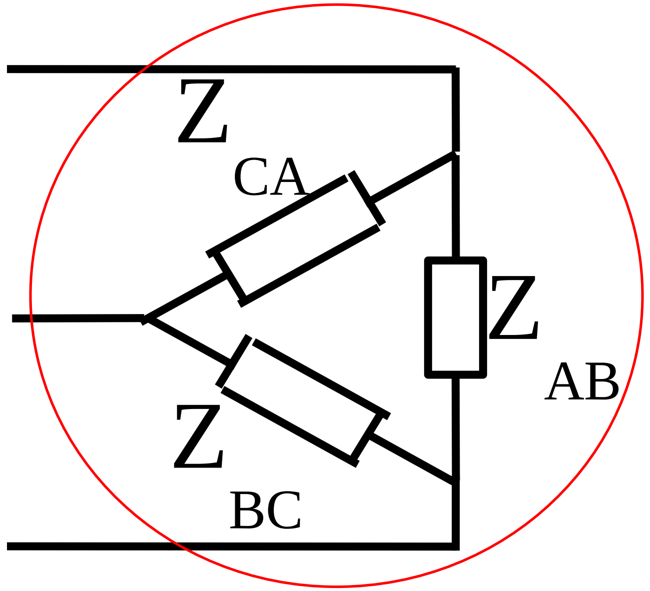
[Alternatively $Q = 15000 \sin 36.87^o = 15000 \times 0.6 = 9000 = 9$ kvar]

Unbalanced three phase systems

Consists of either balanced and unbalanced, star or delta connected sources and an unbalanced star connected or delta connected load.







Either magnitudes are unequal or phase angles are unequal or both.

Such systems cannot be solved using one phase of the system, but could be solved using the normal network theorems.

Power factor correction

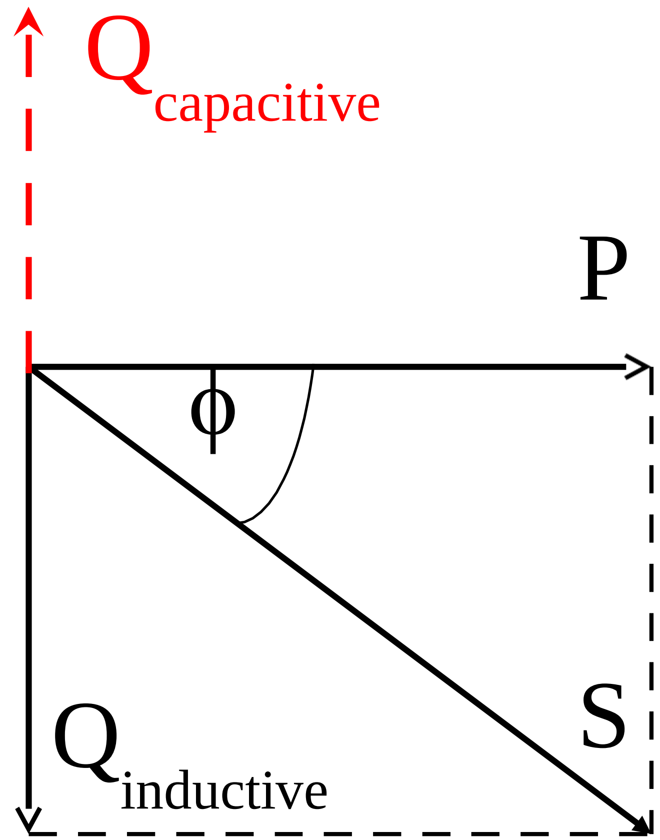
Most practical loads are inductive in nature

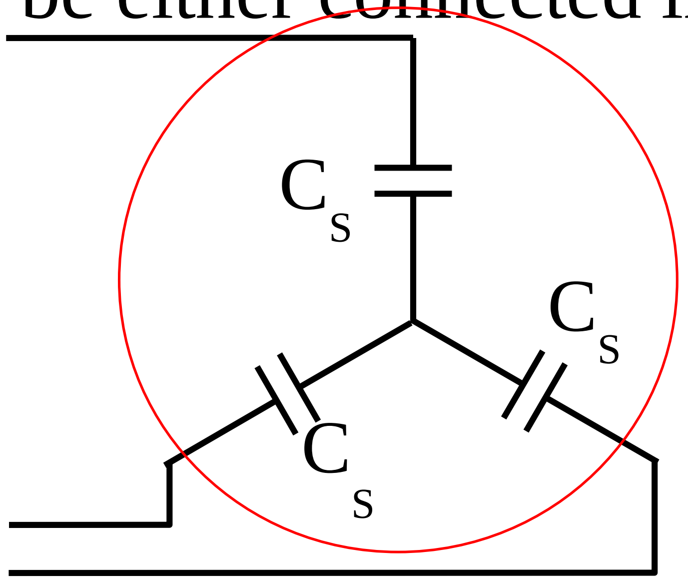
– they have coils rather than capacitors

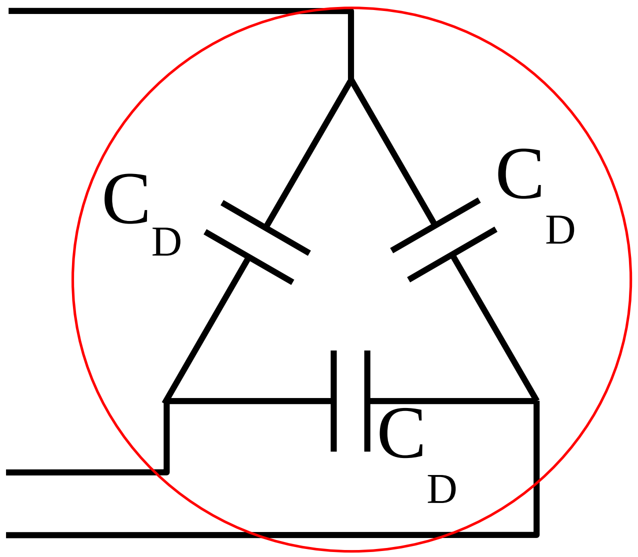
Pure capacitor affects only reactive power and not active power.

Thus power factor is improved by using capacitors across load to partially cancel out inductive reactive power of load.

Capacitors may be either connected in star or delta.





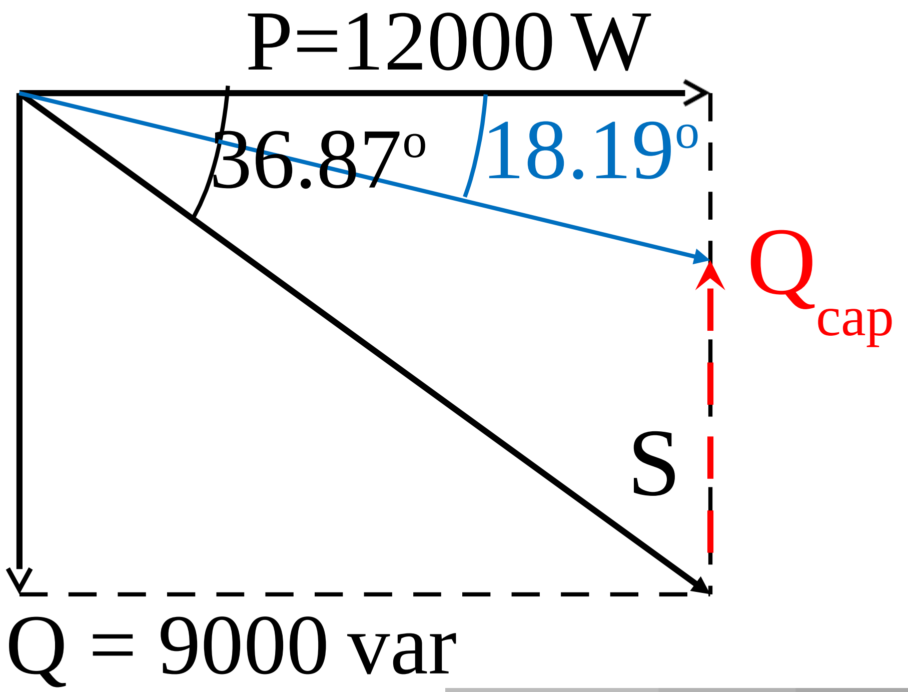


Example

A 3-phase load has P = 12000 W, Q = 9000 var (p.f. = 0.8 lag, – 36.87^o).

Delta connected capacitor bank is used to improve power factor to 0.95 lag.

What is the value of the capacitors required.



New power factor = 0.95 lag.

$\therefore$ new power factor angle = $\cos^{-1}(0.95) = 18.19^o$

Active power is unchanged by capacitor bank

$\therefore$ P = 12000 W

$\therefore$ new reactive power = P tan ϕ

= $12000 \times \tan(18.19^o)$ = 3.944 kvar

$\therefore$ reactive power supplied from capacitor bank

= 9000 - 3944 = 5056 = 5.056 kvar.

Each of 3 capacitors in delta bank supplies = 5.056/3 = 1.685 kvar.

Since connected in delta, each would get a voltage of 400 V.

- $\therefore V_L^2\, C\, \omega = 400^2 \times C \times 2\,\pi \times 50 = 1685$ var
- $\therefore C = 33.5 \times 10^{-6} = 33.5\ \mu F$.
- If capacitors had been connected in star,
- then reactive power would still have been the same,

but voltage across each would be $400/\sqrt{3}$ and

- value of each capacitor C_S would be
  - $V_p^{\,2}\, C_S\, \omega = (400/\sqrt{3})^2 \times C_S \times 2\,\pi \times 50 = 1685$
- $\therefore C_S = 100.6 \times 10^{-6} = 100.6\ \mu F$.

Recap of ac theory

- Some advantages of the Sinusoidal Waveform
- Phasor and Complex representation of Sinusoids.
- Impedance and Admittance in an ac circuit
- Calculations for Simple R -L-C Circuits using Phasor-diagrams and complex notation.
- Power and Power Factor, Reactive Power and Power factor correction

END OF PRESENTATION