Scatter Parameters

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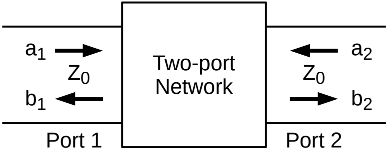
Outline

Introduction

Introduction

- Scatter parameters are an extention of two-port network theory to microwave frequencies
- Instead of inputs and outputs in terms of voltages or currents all inputs and outputs are in terms of *incident and reflected voltage waves*
- Instead of short circuit or open circuit tests, tests are done for matched ports

Two-Port Network Model



- All ports are matched with characteristic impedance Z_0
- Incident waves are denoted by a_i and reflections by b_i
  - The letter denotes the amplitude of each wave
- The incident waves and reflections are related by

$$
\begin{aligned}
b_1 &= s_{11}a_1 + s_{12}a_2 \\
b_2 &= s_{21}a_1 + s_{22}a_2
\end{aligned}
$$

Scatter Parameters

- The scatter parameters are obtained as ratios when one port is matched (i.e., $a_1 = 0$ or $a_2 = 0$)
- Port reflection coefficients

$$
\begin{aligned}
s_{11} &= \left.\frac{b_1}{a_1}\right|_{a_2=0} \\
s_{22} &= \left.\frac{b_2}{a_2}\right|_{a_1=0}
\end{aligned}
$$

  - s_{11} is the RC of Port 1 when Port 2 is matched
  - s_{22} is the RC of Port 2 when Port 1 is matched

Scatter Parameters (Contd..)

- Port transmission coefficients

$$s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$
$$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

  - s_{12} is the TC of the wave from Port 2 to matched Port 1
  - s_{21} is the TC of the wave from Port 1 to matched Port 2
- If the network is lossless $|s_{11}|^2 + |s_{21}|^2 = |s_{12}|^2 + |s_{22}|^2 = 1$

Scatter Parameters (Contd..)

- Insertion loss in dB

$$IL = -20 \log_{10} |s_{21}|$$

- Return loss in dB

$$RL = -20 \log_{10} |s_{11}|$$

- VSWR

$$VSWR = \frac{1 + |s_{11}|}{1 - |s_{11}|}$$

Exercise

For the following two scatter matrices determine

$$A: \quad \begin{pmatrix} 0.1 + j0.4 & 0.6 + j0.2 \\ 0.5 + j0.6 & 0.2 + j0.3 \end{pmatrix}$$

$$B: \quad \begin{pmatrix} 0.1 + j0.2 & 0.7 + j0.3 \\ 0.6 + j0.7681146 & 0.2 + j0.6164414 \end{pmatrix}$$

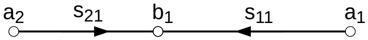
1. Whether the circuit is lossless
2. The insertion loss in dB
3. The reflection loss in dB
4. The VSWR

Signal Flow Graphs

Signal Flow Graphs

- Signal flow graphs are a graph theoretic method of representing microwave frequency circuits
- Consists of
  1. Vertices that represent variables i.e., nodes
  2. Directed edges that represent signal paths i.e., branches
- Can be conveniently converted into transfer functions using Mason's gain formula

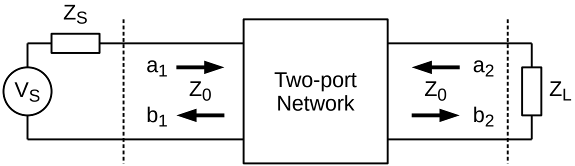
Basic Signal Flow Model



- Nodes represent variables
  - Inputs (independant variables) flow into a node
  - Outputs (dependant variables) flow out
  - Multiple edges entering a node represent a sum
- This results in

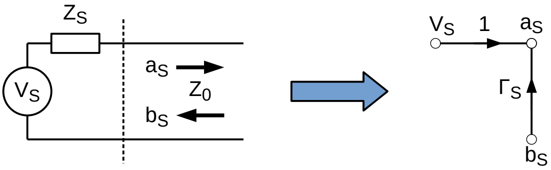
$$b_1 = s_{11}a_1 + s_{12}a_2$$

Terminated Networks



- The basic terminated network consists of a source and load connected to the two-port network
- Take the source voltage as v_S, reflection coefficient Γ_S and load reflection coefficient as Γ_L

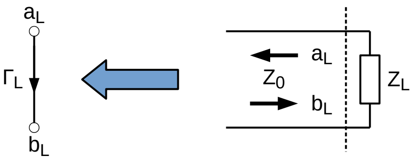
Source Model



- A source adds an exclusive input to the system v_S
- It also consists of the wave reflected back into it b_S
- Therefore, the effective input to the system is a_S

$$a_S = v_S + \Gamma_S b_S$$

Load Model

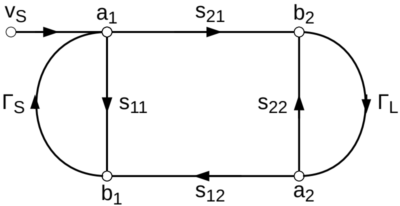


- Every load adds only a reflection
- This results in

$$b_L = \Gamma_L a_L$$

Terminated Network Model

- The network reduces to the following signal flow graph



- This can now be analyzed using Mason's gain formula
  - Need to identify paths and loops w.r.t. the input and output

Paths and Loops

- Take input v_S and output b_2
- Paths from v_S to b_2
  - Only $\mathcal{P}_1 : v_S \to a_1 \to b_2$ with gain s_{21}
- Loop gains
  - $\mathcal{L}_1 : a_1 \to b_2 \to a_2 \to b_1 \to a_1$ with gain $s_{12}\Gamma_L s_{21}\Gamma_S$
  - $\mathcal{L}_2 : a_1 \to b_1 \to a_1$ with gain $s_{11}\Gamma_S$
  - $\mathcal{L}_3 : a_2 \to b_2 \to a_2$ with gain $s_{22}\Gamma_L$
- All loops touch $\mathcal{P}_1$

Mason's Gain Formula

- General formula

$$G = \frac{\Sigma_i P_i \Delta_i}{\Delta}$$

where

- P_i is the total gain of path i
- Δ is the determinant of the adjacency matrix of the graph
- Δ_i is the determinant of the adjacency matrix of the nodes of path P_i

Mason's Gain Formula (Contd..)

- A first order loop is defined as any loop within an adjacency matrix M
  - M can be the entire graph or consist of all nodes of a path
  - A second order loop is the product gains of any two *non-touching* first order loops within M
  - A third order loop is the product gains of any three *non-touching* first order loops within M
  - An n-order loop is the product gains of any n *non-touching* first order loops within M
- Usually loop order rarely exceeds three

Mason's Gain Formula (Contd..)

- Therefore,

$$\begin{aligned}
\Delta &= 1 - \underbrace{\Sigma L(1)}_{\text{Sum of gain of first order loops}} + \underbrace{\Sigma L(2)}_{\text{Sum of gain of second order loops}} \\
&\quad - \underbrace{\Sigma L(3)}_{\text{Sum of gain of third order loops}} + \cdots + (-1)^n \underbrace{\Sigma L(n)}_{\text{Sum of gain of first order loops}} + \ldots \\
&= 1 - (\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3) + \mathcal{L}_2\mathcal{L}_3 \\
&= 1 - (s_{12}s_{21}\Gamma_L\Gamma_S + s_{11}\Gamma_S + s_{22}\Gamma_L) + s_{11}s_{22}\Gamma_L\Gamma_S
\end{aligned}$$

Mason's Gain Formula (Contd..)

- In the case of Δ_i the sums $\Sigma L(n)$ consit of the loops that do not touch the path $\mathcal{P}_1$
- Since $\mathcal{P}_1$ has no non-touching loops $\Delta_1 = 1$
- Therefore,

$$G = \frac{b_2}{v_S} = \frac{s_{21}}{1 - (s_{12}s_{21}\Gamma_L\Gamma_S + s_{11}\Gamma_S + s_{22}\Gamma_L) + s_{11}s_{22}\Gamma_L\Gamma_S}$$

- When rearranged

$$\frac{b_2}{v_S} = \frac{s_{21}}{(1 - s_{11}\Gamma_S)(1 - s_{22}\Gamma_L) - s_{12}s_{21}\Gamma_L\Gamma_S}$$

Exercise

Using Mason's gain formula obtain the following transfer functions

1. $\frac{b_1}{v_s}$
2. $\frac{b_1}{a_1}$
3. $\frac{b_2}{a_2}$

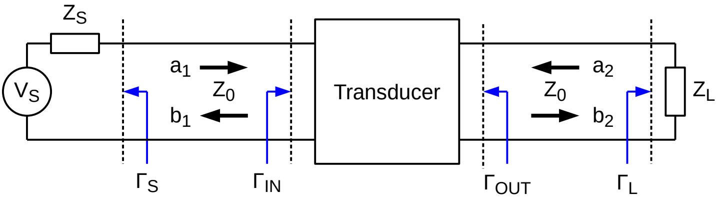
Verify the result of $\frac{b_1}{a_1}$ using the scatter matrix

Amplifier Analysis

Microwave Active Devices

- Microwave frequency active devices
  - Si Bipolar Junction Transistors up to 4 GHz
  - GaAs Heterojunction BJTs
  - GaAs Metal Semiconductor Field Effect Transistors
  - GaAs High Electron Mobility Devices
- Typically are unilateral devices i.e., $s_{12} \approx 0$
- Gains are typically calculated in terms of power

The Amplifier Network



- An amplifier network requires a further two parameters
  - The input reflection coefficient Γ_{IN}
  - The output reflection coefficient Γ_{OUT}
- Necessary for both cascading and matching to a standard 50Ω transmission line

Input and Output Reflection Coefficient

- By applying Mason's gain formula

$$\begin{aligned}\Gamma_{IN} &= s_{11} + \frac{s_{21}s_{12}\Gamma_L}{1 - s_{22}\Gamma_L} \\ \Gamma_{OUT} &= s_{22} + \frac{s_{21}s_{12}\Gamma_S}{1 - s_{11}\Gamma_S}\end{aligned}$$

Power Gain

- The most conservative power gain of a microwave circuit is the transducer power gain given by

$$G_T = (1 - |\Gamma_S|^2) \left| \frac{b_2}{v_S} \right|^2 (1 - |\Gamma_L|^2)$$

- Which results in

$$G_T = \frac{(1 - |\Gamma_S|^2)|s_{21}|^2(1 - |\Gamma_L|^2)}{|(1 - s_{11}\Gamma_S)(1 - s_{22}\Gamma_L) - s_{12}s_{21}\Gamma_L\Gamma_S|^2}$$

Power Gain (Contd..)

- Alternatively in terms of Γ_{IN} or Γ_{OUT}

$$\begin{aligned} G_T &= \frac{(1-|\Gamma_S|^2)|s_{21}|^2(1-|\Gamma_L|^2)}{|(1-\Gamma_{IN}\Gamma_S)(1-s_{22}\Gamma_L)|^2} \\ &= \frac{(1-|\Gamma_S|^2)|s_{21}|^2(1-|\Gamma_L|^2)}{|(1-s_{11}\Gamma_S)(1-\Gamma_{OUT}\Gamma_L)|^2} \end{aligned}$$

- By using the unilateral approximation this further simplifies to

$$G_T \approx G_{UT} = \frac{(1-|\Gamma_S|^2)|s_{21}|^2(1-|\Gamma_L|^2)}{|(1-s_{11}\Gamma_S)(1-s_{22}\Gamma_L)|^2}$$

Exercise

The 50Ω normalized scatter parameters for the GaAs FPD6836P70 pHEMT manufactured by Qorvo, Inc. at 10 GHz are given by

$$S = \begin{pmatrix} 0.488\angle 103.4^\circ & 0.073\angle 0.9^\circ \\ 3.339\angle -17.3^\circ & 0.355\angle -124.8^\circ \end{pmatrix}$$

If the transistor is connected to a source with an impedance of 40Ω and a load with an impedance of 60Ω, determine

1. The transducer power gain
2. The unilateral tranducer power gain

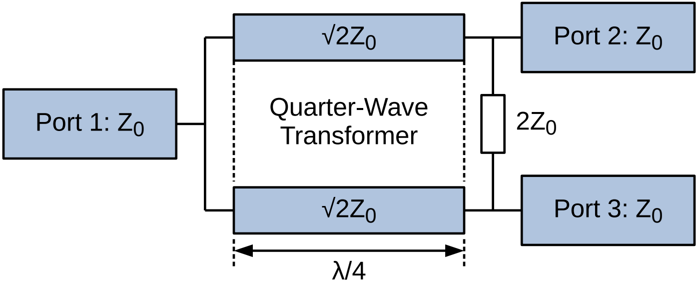
N-Port Devices

N-Port Devices

- Some microwave devices have more than two ports
  - Directional couplers can have three or four ports
  - Power dividers can have three ports
- In which case the two port scatter matrix has to be extended to N ports

$$\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} & \cdots & s_{1N} \\ s_{21} & s_{22} & \cdots & s_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ s_{N1} & s_{N2} & \cdots & s_{NN} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix}$$

Wilkinson Power Divider



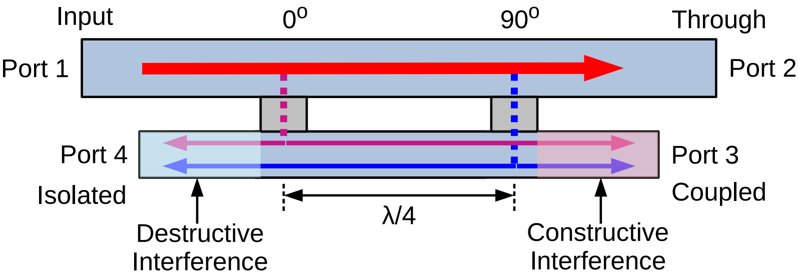
Wilkinson Power Divider (Contd..)

- The basic Wilkinson power divider uses the quarter wave transformer principle to divide the power of port 1 equally among port 2 and port 3
- Has a scatter matrix of

$$S = \frac{-j}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

- Using the same principle can divide power asymmetrically as well

Directional Couplers



Directional Couplers (Contd..)

- When a signal propagates from port 1 to port 2 the additional $\lambda/2$ distance travelled by the blue signal results in a phase shift of π at port 4
  - Destructive interference
  - The opposite takes place when a signal propagates from port 1 to port 2 i.e., constructive interference
- Has a scatter matrix of

$$S = \frac{-1}{\sqrt{2}} \begin{pmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{pmatrix}$$

Conclusion

Conclusion

- Scatter parameters are an extention of two-port network theory to microwave frequencies
  - Use incident and reflected waves which can be conveniently measured in terms of powers
- The basic description of s-parameters is the scatter matrix
  - Can be used for both passive and active semiconductor devices
  - Can also be extended to multiple port elements
- Practical measurements of s-parameters can be taken using
  - A network analyzer
  - A VSWR meter in limited cases