Effect of Modulation Index on AM Envelope

Let $m(t) = A_m \sin \omega_m t$ and $c(t) = A_c \cos \omega_c t$

For single tone sinusoidal signal expression for AM is given by,

$$s(t) = A_c\left[1 + m_a \cancel{\cos}(2\pi f_m t)\right]\cos(2\pi f_c t)$$

sin

(i) For $m_a = 0$, $s(t) = A_c \cos \omega_c t$

(ii) For $m_a = 0.3$, $s(t) = A_c[1 + 0.3 \sin \omega_m t]$

$$s(t)_{\max} = 1.3 A_c, \quad s(t)_{\min} = 0.7 A_c$$

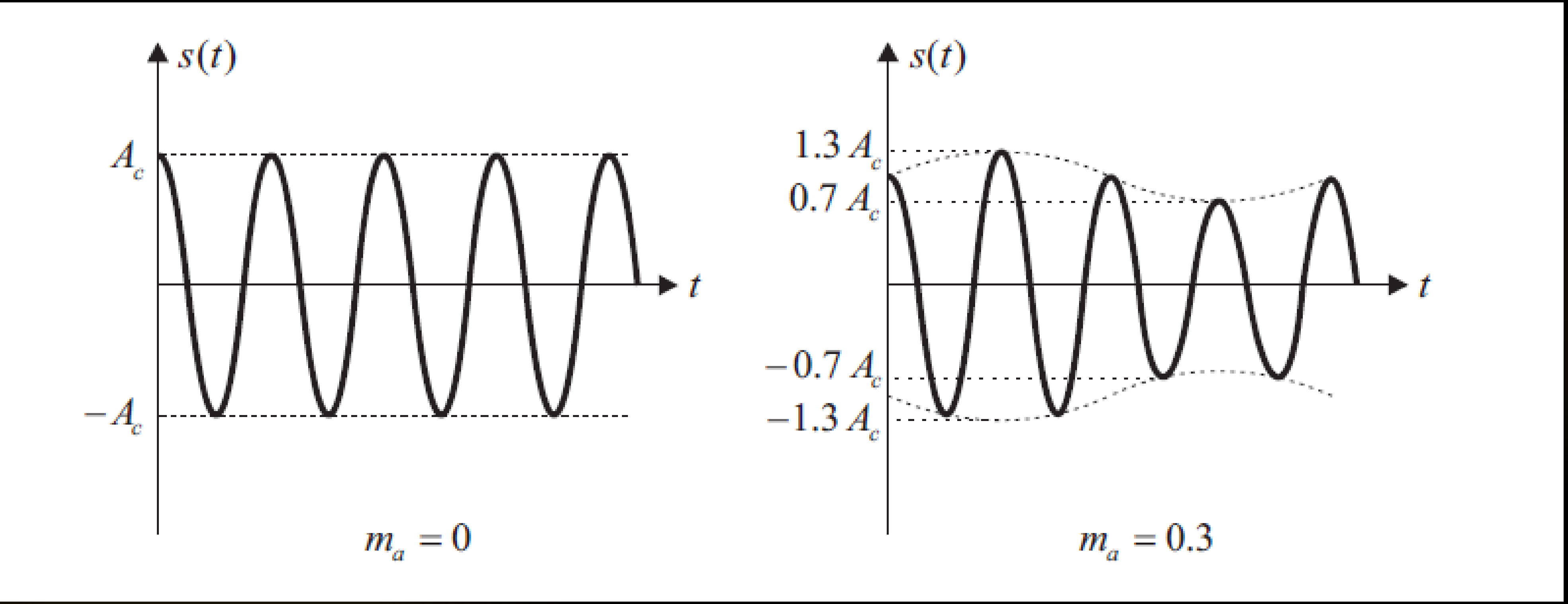
(iii) For $m_a = 0.8$, $s(t) = A_c[1 + 0.8 \sin \omega_m t]$

$$s(t)_{\max} = 1.8 A_c, \quad s(t)_{\min} = 0.2 A_c$$

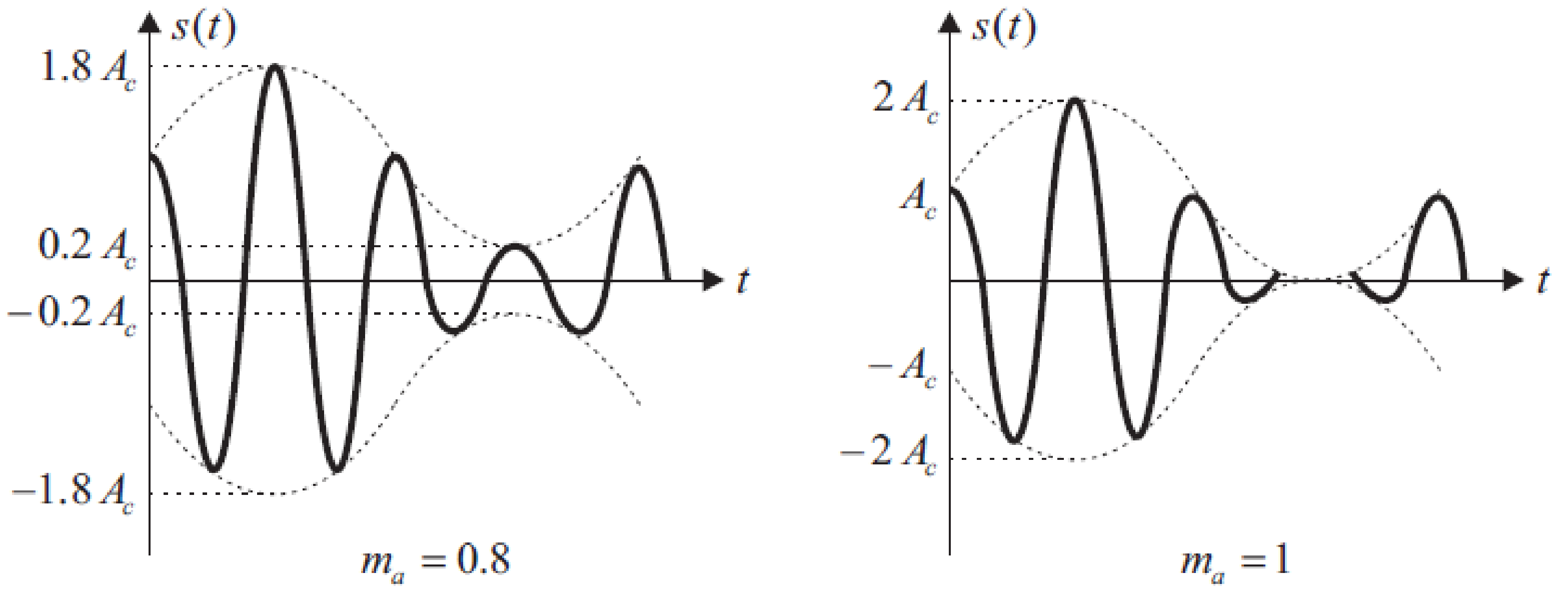
(iv) For $m_a = 1$, $s(t) = \cancel{2} A_c[1 + \sin \omega_m t]$

$$s(t)_{\max} = 2 A_c, \quad s(t)_{\min} = 0$$

Effect of Modulation Index on AM Envelope



Effect of Modulation Index on AM Envelope



- For $m_a = 1$, the maximum signal voltage of the modulated AM wave varies between zero and twice the maximum amplitude of unmodulated carrier signal.
- For $m_a = 0$, the original unmodulated carrier signal is the resultant waveform because $A_m = 0$ means absence of modulating signal.

TEST 1

Q.1 Which of the following is the most correct?

(A) A_m should be greater than A_c.

(B) A_c should be greater than A_m.

(C) A_m should be equal to or less than A_c.

(D) A_c must always equal A_m.

Q.2 Which of the following is NOT another name for modulation index?

(A) Modulation reciprocal

(B) Modulation factor

(C) Degree of modulation

(D) Modulation coefficient

Q.3 The degree or depth of modulation expressed as a percentage is computed using the expression

(A) $2A_m$ (B) $\frac{100}{m_a}$

(C) $\frac{m_a}{100}$ (D) $100\% \times m_a$

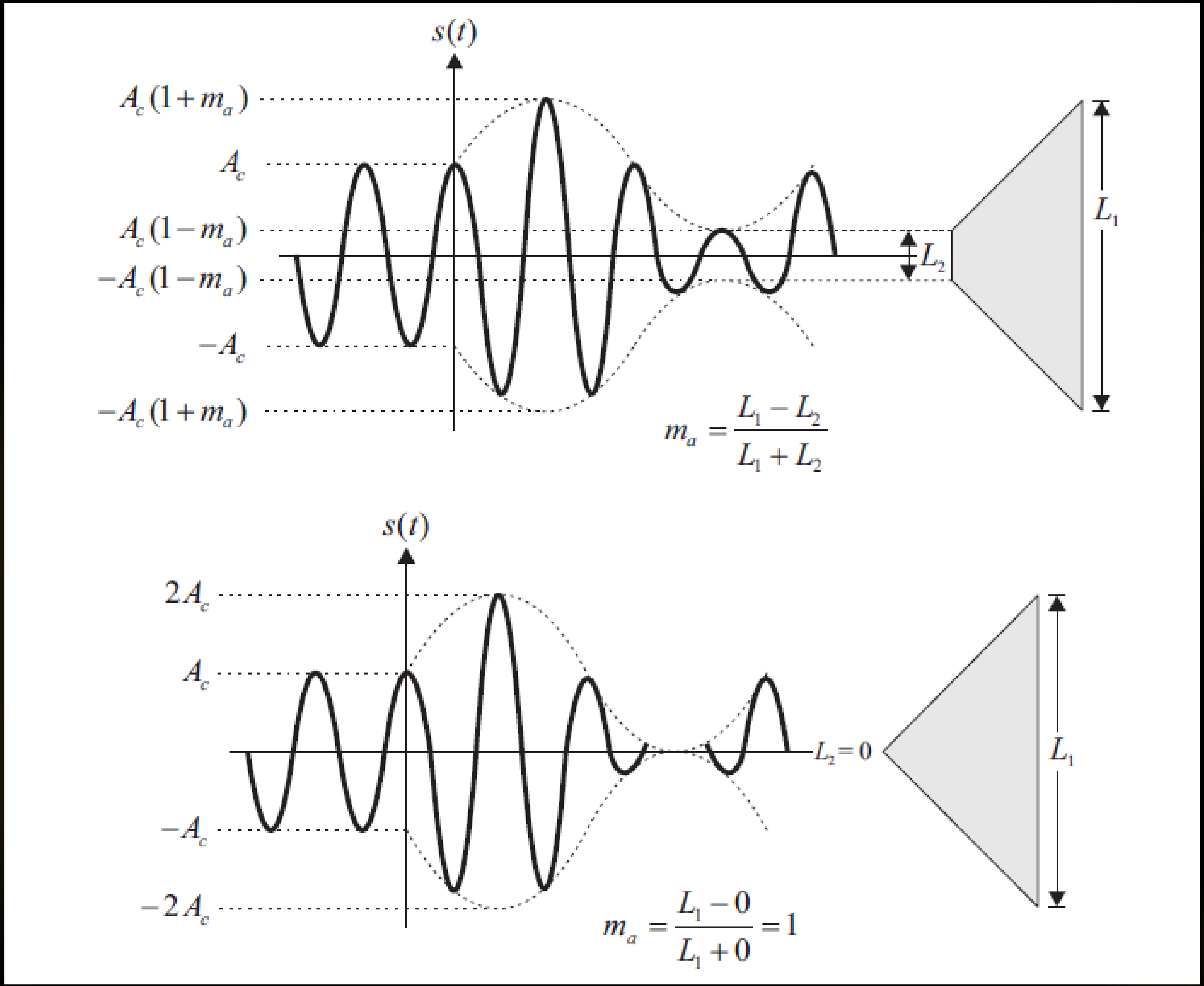
Q.4 An AM wave displayed on an oscilloscope has values of $E_{\max} = 3.8$ and $E_{\min} = 1.5$ as read from the graticule. The percentage of modulation is ________%.

Q.5 To achieve 85% modulation of a carrier of $A_c = 40\,\text{V}$, a modulating signal of $A_m =$ ________ is needed.

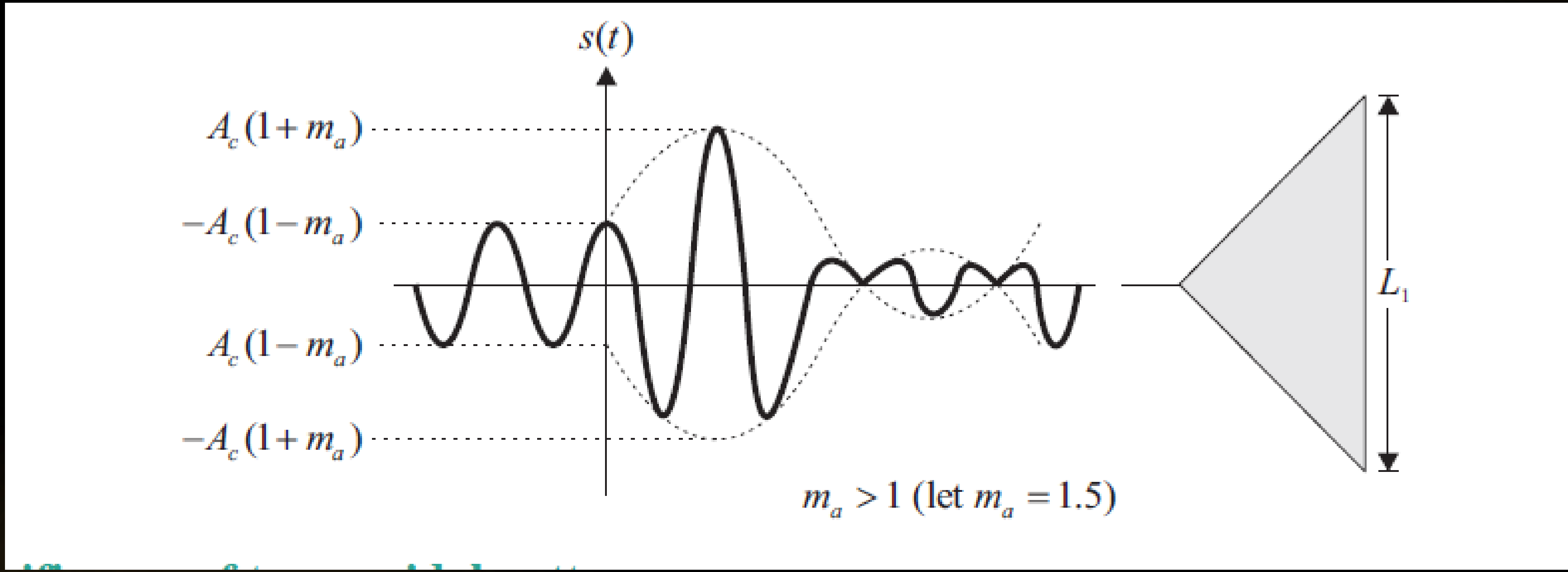
Trapezoidal Pattern

- Definition : **Trapezoidal pattern is a display on a standard oscilloscope** used for measurement of the modulation characteristics (modulation index, percent modulation, coefficient of modulation, and modulation symmetry) of amplitude-modulated waveform.

Trapezoidal Pattern



Trapezoidal Pattern



- Trapezoidal pattern is quite useful when the modulating signal is a non-periodic signal such as speech waveform which contains complex sinusoidal waveforms
- AM transmitter modulation characteristics such as modulation symmetry and modulation index can be easily and accurately interpreted by observing trapezoidal patterns of the modulated signal on the standard oscilloscope display

Frequency Domain Representation of AM wave

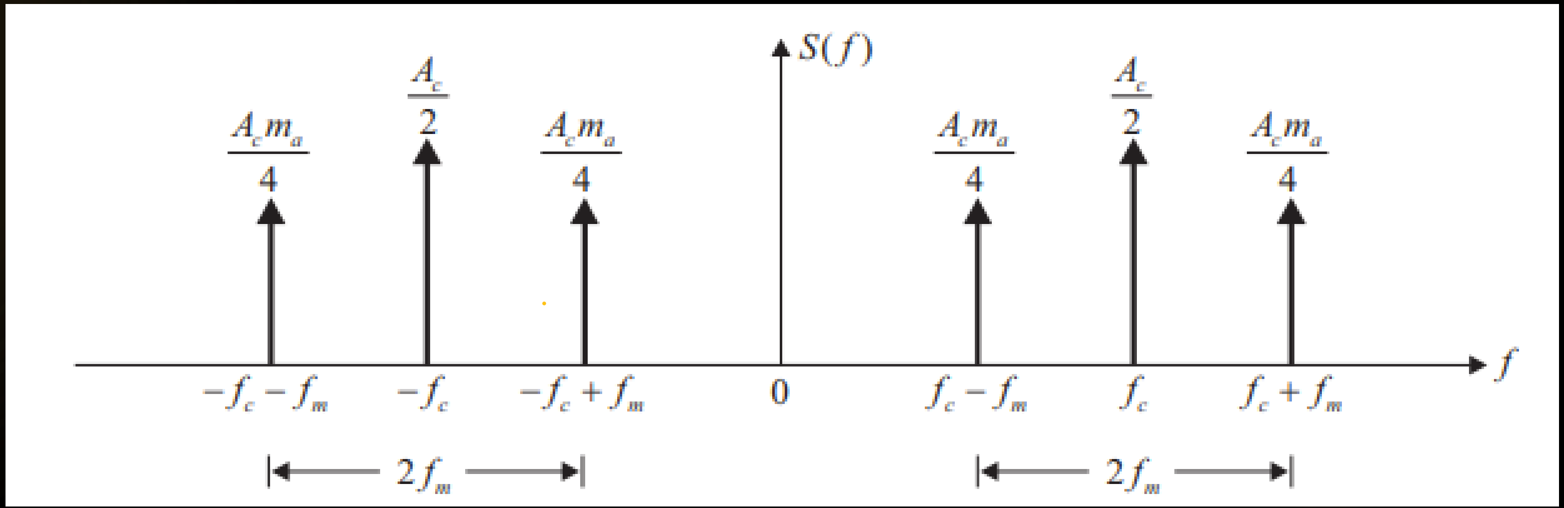
- We Can write S(t) As: **$S(t)=Ac[1+\mu\cos(2\pi fmt)]\cos(2\pi fct)$**

$\Rightarrow S(t)=A_c\cos(2\pi f_c t)+A_c\mu\cos(2\pi f_c t)\cos(2\pi f_m t)$

$\Rightarrow S(t)=A_c\cos(2\pi f_c t)+(1/2)A_c\mu\cos[2\pi(f_c+f_m)t]+A_c\mu(1/2)\cos[2\pi(f_c-f_m)t]$

- We need to take Furrier Transformation :

$A_c\cos(2\pi f_c t) \xleftrightarrow{FT} (A_c/2)[\delta(f-f_c)+\delta(f+f_c)]$; Do the same for each components



Bandwidth of AM Wave

- Bandwidth (BW) is the **difference between the highest and lowest frequencies** of the signal. Mathematically, we can write it as

$$BW = f_{max} - f_{min}$$

- Hence, the amplitude modulated wave has three frequencies. Those are **carrier frequency- f_c**, **upper sideband frequency $f_c + f_m$** and **lower sideband frequency $f_c - f_m$**

  Here,

  $f_{max} = f_c + f_m$ and $f_{min} = f_c - f_m$

Substitute, f_{max} and f_{min} values in bandwidth formula.

$BW = f_c + f_m - (f_c - f_m)$

$\Rightarrow BW = 2f_m$

Thus, it can be said that the **bandwidth required for amplitude modulated wave is twice the maximum frequency of the modulating signal**.

Q-1 : Bandwidth of AM wave is 20KHz, and highest frequency present is 800 KHz. Find out the **carrier frequency** used?

Q-2 : Consider the AM Signal $S(t) = [1+m(t)]\cos(2\pi f_c t)$,It is given that the **BW of real Law Pass Massage Signal is 2 KHz**. Find the BW of AM wave?

<u>Power Calculations of AM Wave</u>

- Consider the following equation of amplitude modulated wave **(Single Tone).**

$S(t)=A_c[1+\mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$ or $S(t)=A_c[1+K_a m(t)]\cos(2\pi f_c t)$

$$S(t)=A_c\cos(2\pi f_c t)+(1/2)A_c\mu\cos[2\pi(f_c+f_m)t]+(1/2)A_c\mu\cos[2\pi(f_c-f_m)t]$$

- Power of AM wave is equal to the sum of powers of <u>**carrier**</u>, <u>**upper sideband**</u>, and <u>**lower sideband**</u> frequency components.

$$P_t=P_c+P_{USB}+P_{LSB}$$

- In general, the power of a signal is given as the ratio of square of RMS value of signal voltage and the load resistance. We know that the standard formula for power is:

$$\text{Signal Power} = \frac{(\text{RMS value of signal voltage})^2}{\text{Load Resistance}}$$

$$P = I_{rms}^2 R = V_{rms}^2 / R$$

Where,

v_{rms} is the RMS [Root Mean Square] value of cos signal.

First, let us find the **powers of the carrier**, the upper and lower sideband one by one.

The peak amplitude of the carrier term in AM signal is the **same as that of un modulated carrier signal**. Therefore, the carrier power is given as:

Carrier power

$$P_c = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2}{2R}$$

If the value of R is not given, assume as R = 1

Sideband power :

The average upper sideband power is given by,

$$P_{USB} = \frac{\left(\frac{A_c}{2\sqrt{2}} m_a\right)^2}{R} = \frac{m_a^2 A_c^2}{8R} = \frac{m_a^2}{4}\frac{A_c^2}{2R} = \frac{m_a^2}{4} P_c$$

The average lower sideband power is given by,

$$P_{LSB} = \frac{\left(\frac{A_c}{2\sqrt{2}} m_a\right)^2}{R} = \frac{m_a^2 A_c^2}{8R} = \frac{m_a^2}{4}\frac{A_c^2}{2R} = \frac{m_a^2}{4} P_c$$

The average sideband power is given by,

$$P_{USB} = P_{LSB} = \frac{m_a^2 P_c}{4}$$

Total sideband power,

$$P_{SB} = P_{USB} + P_{LSB} = \frac{m_a^2}{4} P_c + \frac{m_a^2}{4} P_c = \frac{m_a^2}{2} P_c$$

Total power :

$$P_T = P_c + P_{SB} = P_c + \frac{m_a^2}{2} P_c$$

Total transmitted power, $P_T = P_c \left[1 + \frac{m_a^2}{2}\right]$

- **Transmission efficiency of AM Wave**
- Transmission efficiency = $\dfrac{\text{Total Side Band Power}}{\text{Total Transmitted Power}}$

$$\eta = \frac{P_{LSB} + P_{USB}}{P_t} = \frac{\left[\frac{m^2}{4}P_c + \frac{m^2}{4}P_c\right]}{\left[1+\frac{m^2}{2}\right]P_c}$$

or,

$$\eta = \frac{\frac{m^2}{2}}{1+\frac{m^2}{2}} = \frac{m^2}{2+m^2}$$

The percent transmission efficien

$$\eta = \frac{m^2}{2+m^2} \text{ x } 100\%$$

- **Q-1**: For an Amplitude Modulated Signal, Which carrier power is 500W ,and the Modulation index is 0.75, Calculate the total power ,and Power of each side Bands?
- **Q-2** : **Due to an AM**, by a sinusoidal wave , if the total current in the Antenna increases from 4 A to 4.8 A, Find the Depth of the modulation in percentage? **(Assignment 1-Qu 1)**
- **Q-3** :

A modulating signal $m(t) = 10\cos(2\pi \times 10^3 t)$ is amplitude modulated with a carrier signal $c(t) = 50\cos(2\pi \times 10^5 t)$. Find the modulation index, the carrier power, and the power required for transmitting AM wave.

And Find out the Transmission Efficiency?

Q-4 :

The equation of amplitude wave is given by $s(t) = 20\left[1 + 0.8\cos(2\pi \times 10^3 t)\right]\cos(4\pi \times 10^5 t)$. Find the carrier power, the total sideband power, and the band width of AM wave.

Exam Questions (Intake 36) Class Room

A Sinusoidal carrier frequency of 1.2MHz is amplitude modulated by a sinusoidal voltage of frequency 20KHz , resulting in maximum and minimum modulated carrier amplitude of 110V & 90V respectively. Find out followings:

i. Frequency of lower and upper side bands

ii. Un-modulated carrier amplitude

iii. Modulation index

iv. Maximum Amplitude of each side bands

Exam Questions (Intake 37) :Home Work

A 350W, 1.2 MHz carrier is amplitude-modulated with a sinusoidal message signal of 2.7kHz. The depth of modulation is 65%.

find out followings:

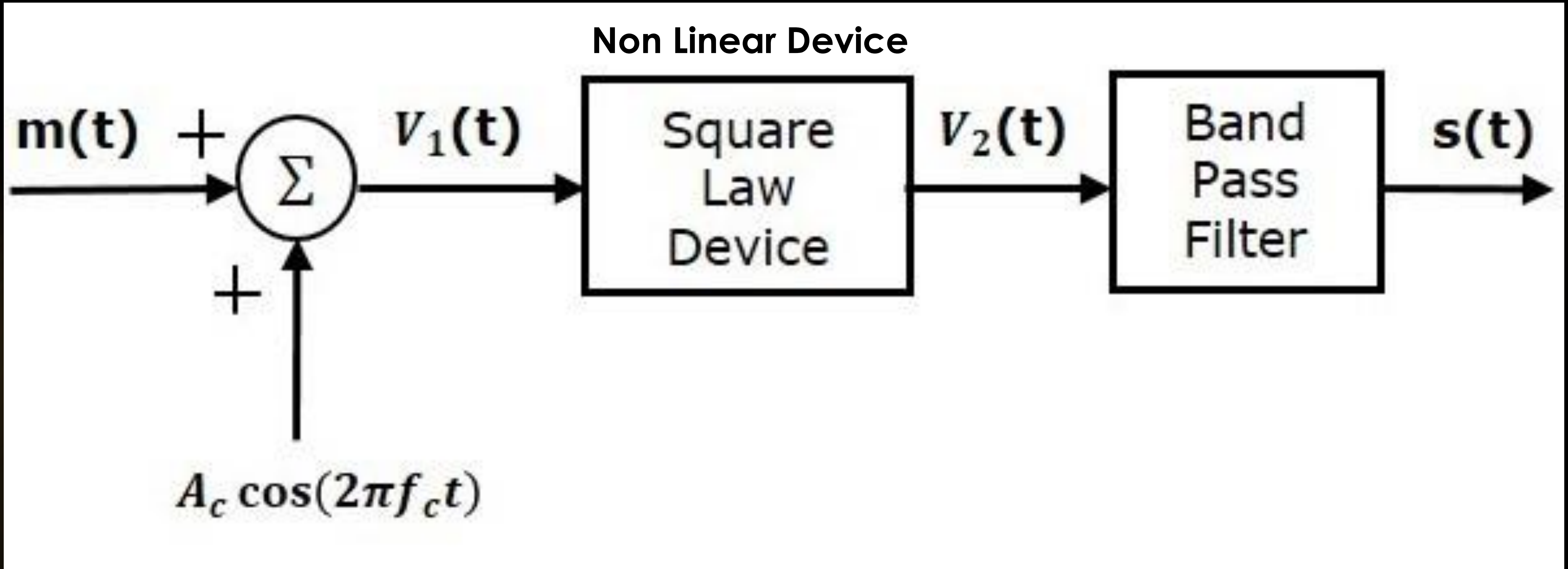
i. frequency of lower and upper sidebands.
ii. bandwidth.
iii. power in sidebands.
iv. total power in modulated wave. `

- **Exam Questions (Intake 38) :Home Work (Assignment 1 – Q 2)**

- Let $c(t) = A_c cos(2\pi f_c t)$ and $m(t) = cos(2\pi f_m t)$. It is given that $f_c \gg 5\ f_m$.
- The signal c(t) + m(t) is applied to the input of a non-linear device, whose output $v_o\ (t)$ is related to the input $v_o\ (t)$ as $v_o\ (t)$ =$a. v_i\ (t) + b. v_i^2(t)$ where a and b are positive constants. The output of the non-linear device is passed through an ideal band-pass filter with center frequency f_c and bandwidth of $3\ f_m$ to produce an amplitude-modulated (AM) wave.
- If it is desired to have the total sideband power of the AM wave to be half of the carrier power, show that a/b is equal to 2.

Square Law Modulator

- Following is the block diagram of the square law modulator



Let the modulating and carrier signals be denoted as $m(t)$ and $A\cos(2\pi f_c t)$ respectively. These two signals are applied as inputs to the summer (adder) block. This summer block produces an output, which is the addition of the modulating and the carrier signal. Mathematically, we can write it as

$$V_1 t = m(t) + A_c \cos(2\pi f_c t)$$

This signal $V_1 t$ is applied as an input to a nonlinear device like diode. The characteristics of the diode are closely related to square law.

$$V_2 t = k_1 V_1(t) + k_2 V_1^2(t) \qquad \text{(Equation 1)}$$

Where, k_1 and k_2 are constants.

Substitute $V_1(t)$ in Equation 1

$$V_2(t) = k_1 [m(t) + A_c \cos(2\pi f_c t)] + k_2 [m(t) + A_c \cos(2\pi f_c t)]^2$$

$$\Rightarrow V_2(t) = k_1 m(t) + k_1 A_c \cos(2\pi f_c t) + k_2 m^2(t) + k_2 A_c^2 \cos^2(2\pi f_c t) + 2k_2 m(t) A_c \cos(2\pi f_c t)$$

$$\Rightarrow V_2(t) = k_1 m(t) + k_2 m^2(t) + k_2 A_c^2 \cos^2(2\pi f_c t) + k_1 A_c \left[1 + \left(\frac{2k_2}{k_1}\right) m(t)\right] \cos(2\pi f_c t)$$

The last term of the above equation represents the desired AM wave and the first three terms of the above equation are unwanted. So, with the help of band pass filter, we can pass only AM wave and eliminate the first three terms.

Therefore, the output of square law modulator is

$$s(t) = k_1 A_c \left[1 + \left(\frac{2k_2}{k_1}\right) m(t)\right] \cos(2\pi f_c t)$$

The standard equation of AM wave is

$$s(t) = A_c \left[1 + k_a m(t)\right] \cos(2\pi f_c t)$$

Limitations of Amplitude Modulation

- **Amplitude Modulation is wasteful of power.**
  - The carrier wave **c(t) is completely independent from the information bearing signal m(t)**. The transmission of carrier wave therefor represents a **waste of power**, which mean that in Amplitude Modulation only a fraction of the total transmitted power is actually affected by m(t).
- **Amplitude Modulation is wasteful of Bandwidth.**
  - The upper and lower side bands of AM wave are uniquely related to each other by virtue of their symmetry about the carrier frequency.
  - Hence, given the magnitude and phase spectra of either sideband, we can uniquely determine the other.

- This means that insofar as the **transmission of information is concerned, only one sideband is necessary**, and the communication channel needs to provide only the same bandwidth as the baseband signal.
- In light with of this observation, **amplitude modulation is wasteful of bandwidth as it requires a transmission bandwidth equal to twice the message bandwidth**
- **To overcome these limitations**, we must make certain modifications: **suppressed the carrier and modify the sidebands of the AM Wave.**
- These modifications naturally result in **increased system complexity**. In effect we trade system complexity for improved use of communication resources.
  The basis of this trade off is linear modulation.
- In a strict sense, full amplitude modulation does not qualify as linear modulation because of the presence of the carrier wave.