29/08/2024

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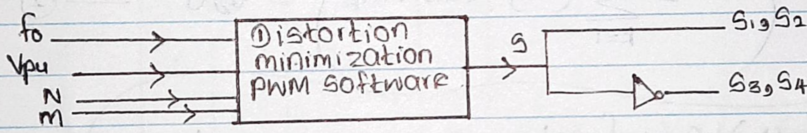
(f) Distortion Minimization PWM

This PWM has some similarities with the harmonic elemination PWM. ~~(HEM)~~ (HEP)

- In ~~HEM~~ (HEP), we remove a selected set of harmonics from the output voltage. In distortion minimization PWM we minimize the total harmonic distortion due to all harmonics upto a selected upper order in the output voltage.

For most practical applications the dist. min. PWM works better than the HEP.

(1) 1-phase PWM

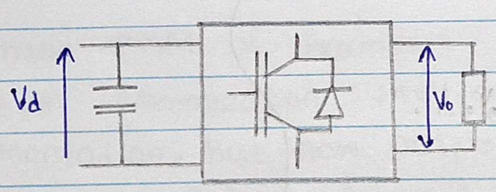


f_o = Fund. frequency of V_o, required.

V_{pu} = Magnitude of V_o, required.

N = Upper ~~harmonic minimization~~ order of harmonics included in the harmonic minimization.

m = no. of switching points per quater cycle of V_o, chosen.



$$V_o = \sum b_n \sin(n\omega t)$$

$$b_n = \begin{cases} \dfrac{4V_d}{n\pi}\left(1 + 2\sum_{k=1}^{m}(-1)^k \cos(n\alpha_k)\right); & \text{for EVEN } m \\ \dfrac{-4V_d}{n\pi}\left(1 + 2\sum_{k=1}^{m}(-1)^k \cos(n\alpha_k)\right); & \text{for ODD } m \end{cases}$$

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Total rms value of harmonics up to N^{th} order,

$$V_{harmonics} = \sqrt{\sum_{n=3}^{N} \left(\frac{b_n}{\sqrt{2}}\right)^2}$$

$$= \sqrt{\sum_{n=3}^{N} \frac{1}{2}\left(\frac{4V_d}{n\pi}\left(1+2\sum_{k=1}^{m}(-1)^k \cos n\alpha_k\right)\right)^2} \quad \text{— } ①$$

Considering the fundamental component,

$$V_{pu} \cdot \left(\frac{4V_d}{\pi}\right) = b_1 = \frac{4V_d}{\pi}\left(1+2\sum_{k=1}^{m}(-1)^k \cos n\alpha_k\right);$$

(+) for even m.
(−) for odd m.

$$\therefore V_{pu} \mp \left(1+2\sum_{k=1}^{m}(-1)^k \cos n\alpha_k\right) = 0 \quad \text{— } ②$$

$z = f(x, y) \leftarrow$ function to be minimized

$g(x, y) = 0 \leftarrow$ constraint.

How do we get z_{min}?

$F = f(x, y) + \lambda\, g(x, y)$

$$\left.\begin{aligned} \frac{\partial F}{\partial x} &= 0 \quad \text{— } ① \\ \frac{\partial F}{\partial y} &= 0 \quad \text{— } ② \\ \frac{\partial F}{\partial \lambda} &= 0 \quad \text{— } ③ \end{aligned}\right\} x, y, \lambda.$$

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Equation (1) is the function to be minimized. eg
Equation (2) is the constraint.
Using standard mathematical software, we can find $\alpha_1, \alpha_2, \alpha_3, \ldots$ and α_k that make $V_{harmonic}$ minimum.

We repeat there calculations for different m and N combinations, and store $\alpha_1, \alpha_2, \ldots \alpha_m$ data in lookup tables.

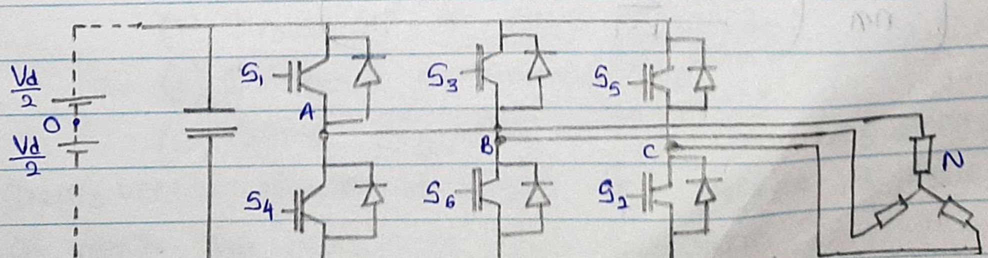
V_{pu}	α_1	α_2	α_3	....	α_m
0.00	✓	✓	✓	....	✓
0.05	✓	✓	✓	....	✓
0.10	✓	✓	✓	....	✓
⋮	⋮	⋮	⋮	⋮	⋮
0.95	✓	✓	✓	....	✓
1.00	✓	✓	✓	....	✓

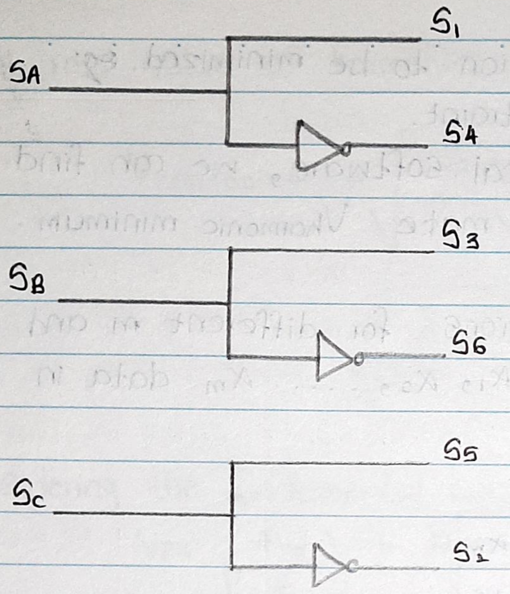
Lookup table for one set of m and N.

Therefore, the implementation of the PWM, is same as that of the Harmonic Elimination PWM.

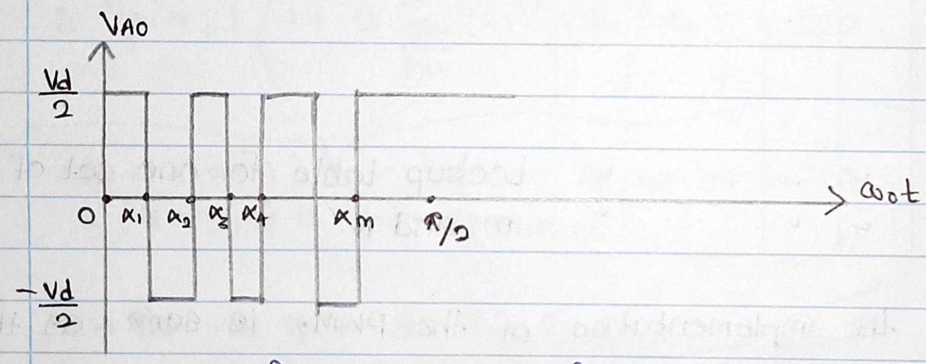
Note

3-phase PWM of harmonic Elimination PWM and Distortion Minimization PWM are similar to their 1-phase implementation, but now output three switching signals S_A, S_B and S_C. Infact S_B and S_C are deduced from S_A, according to 3-phase symmetry.





It is the pole - voltage V_{AO} that represents switching signal S_A in the 3ph. inverter.



V_{AO} for the case of an EVEN m

$$V_{AO} = \sum_{n=1,3,5} b_n \sin(n\omega t)$$

$$b_n = \begin{cases} \dfrac{2V_d}{n\pi}\left(1 + 2\sum_{k=1}^{m}(-1)^k \cos n\alpha_k\right); & \text{for EVEN } m \\ \dfrac{-2V_d}{n\pi}\left(1 + 2\sum_{k=1}^{m}(-1)^k \cos n\alpha_k\right); & \text{for ODD } m. \end{cases}$$

1ph : 5 nos harm
$3^{rd}, 5^{th}, 7^{th}, 9^{th}, 11^{th}$

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3ph : 5 nos
$5^{th}, 7^{th}, 11^{th}, 13^{th}, 17^{th}$.

We must use this b_n expressions when computing $\alpha_1, \alpha_2 \ldots$ and α_m for S_A of a 3-phase inverter.

Another important difference is the orders of harmonics to be eliminated (or to be considered for minimization). The 3ph. inverter does not produce triplan order harmonics (i.e 3, 9, 15, 21 ...) in the output voltage, so we do not need to consider these.

eg: If we want to remove 5 nos of harmonics in the harmonic elemination PWM, then we select,

1-phase : $3^{rd}, 5^{th}, 7^{th}, 9^{th}, 11^{th}$

3-phase : $5^{th}, 7^{th}, 11^{th}, 13^{th}$ and 17^{th}

If we want to minimize harmonics up to N^{th} order in the distortion minimization PWM, then,

1-phase : $V_{harmonics} = \sqrt{\sum_{n=1,3,5\ldots}^{N} \left(\frac{b_n}{\sqrt{2}}\right)^2}$

3-phase : $V_{harmonic} = \sqrt{\sum_{n=5,7,11\ldots}^{N} \left(\frac{b_n}{\sqrt{2}}\right)^2}$

(g) Voltage Vector PWM.

"Vector" is a mathematical tool used to model 3 phase quantities, such as voltage, current, magnetic flux, etc. The speciality of vector is that it handles any 3 ph. variation, irrespective of whether it is,

(i) Sinusoidal or non sinusoidal.
(ii) Balanced or unbalanced.
(iii) Same frequency or different frequencies.

Thus, vector is an ideal tool to analyse 3 phase inverters, of which the output is non-sinusoidal (i.e pulsed output)

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(i) Definition

$V_a(t)$ = instantaneous voltage of phase - a
$V_b(t)$ = instantaneous voltage of phase - b
$V_c(t)$ = instantaneous voltage of phase - c

Voltage vector $\bar{V}(t)$ is defined as,

$$\bar{V}(t) = V_a(t) + \underline{a}\, V_b(t) + \underline{a}^2 V_c(t)$$

$a = 1\angle 120^\circ$
$= \cos 120^\circ + j \sin 120^\circ$

For vector definition to hold, the summation $\sum V_a(t) + V_b(t) + V_c(t)$ must be zero, for all t.

Real part of a vector is called α - component
Imaginery part of a vector is called β - component.

Ex: At an instant of time the voltages of phases a, b and c are given as 420V, -100V. and -320V, respectively. Find the magnitude and the direction of voltage vector of this instant. Show your result on an α-β plane.

Ans:

$V_a = 420V$
$V_b = -100V$
$V_c = -320V$

$\bar{V} = V_a + \underline{a} V_b + \underline{a}^2 V_c$
$= 420 + (1\angle 120^\circ)(-100) + (1\angle 120^\circ)^2(-320)$
$= 420 - 100\angle 120^\circ - 320\angle 240^\circ$
$= 718.6\angle 21.2^\circ$ V //

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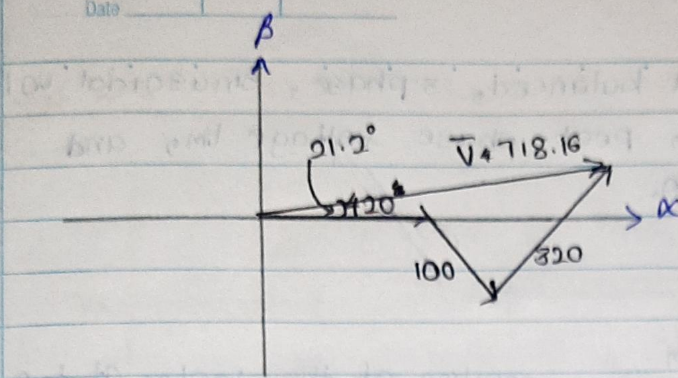


(2) Voltage vector for a balanced, 3-phase, sinusoidal voltage.

$$V_a = V_m \cos(\omega t + \theta_0)$$
$$V_b = V_m \cos(\omega t + \theta_0 - 120^\circ)$$
$$V_c = V_m \cos(\omega t + \theta_0 - 240^\circ)$$; θ_0 is same initial phase angle.

$$\bar{V} = (V_m \cos(\omega t + \theta_0)) + (1\angle 120^\circ)(V_m \cos(\omega t + \theta_0 - 120^\circ)) + (1\angle 120^\circ)^2 (V_m \cos(\omega t + \theta_0 - 240^\circ)).$$

$$= V_m \cos(\omega t + \theta_0) + (\cos 120^\circ + j \sin 120^\circ) V_m \cos(\omega t + \theta_0 - 120^\circ) + (\cos 240^\circ + j \sin 240^\circ) V_m \cos(\omega t + \theta_0 - 240^\circ)$$

$$= V_m (\cos(\omega t + \theta_0) + \cos 120^\circ \cos(\omega t + \theta_0 - 120^\circ) + \cos 240^\circ \cdot \cos(\omega t + \theta_0 - 240^\circ)) + j V_m (\sin 120^\circ \cdot \cos(\omega t + \theta_0 - 120^\circ) + \sin 240^\circ \cdot \cos(\omega t + \theta_0 - 240^\circ)).$$

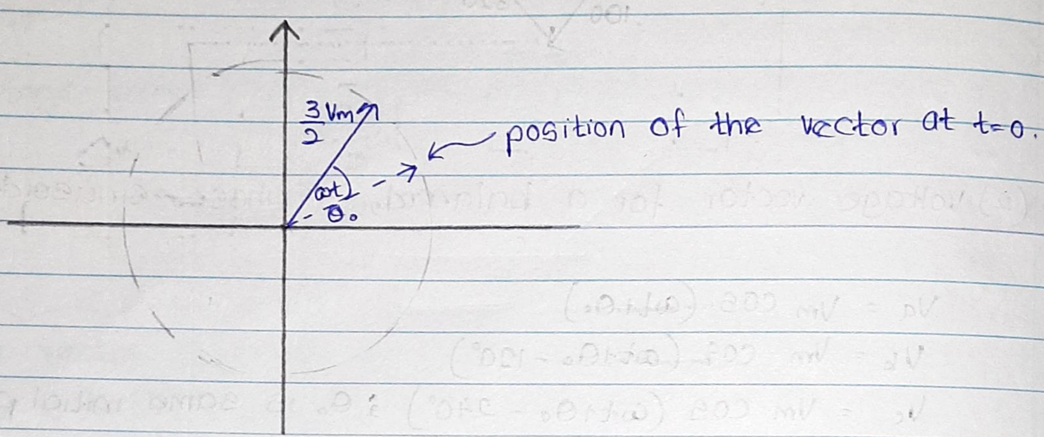
$$= \frac{V_m}{2}\left(3\cos(\omega t + \theta_0) + \underbrace{[\cos(\omega t + \theta_0 - 120^\circ) + \cos(\omega t + \theta_0 - 240^\circ)]}_{=0}\right) + \frac{jV_m}{2}\left(3\sin(\omega t + \theta_0) - \underbrace{[\sin(\omega t + \theta_0) + \sin(\omega t + \theta_0 - 120^\circ) + \sin(\omega t + \theta_0 - 240^\circ)]}_{=0}\right).$$

$$= \frac{3V_m}{2}[\cos(\omega t + \theta_0) + j \sin(\omega t + \theta_0)] \;//$$

$$= \frac{3V_m}{2} \angle(\omega t + \theta_0).$$

$$\bar{V} = \frac{3}{2} V_m \angle \omega t + \theta_0 = \frac{3}{2} V_m [\cos(\omega t + \theta_0) + j \sin(\omega t + \theta_0)].$$

Voltage vector for a balanced 3phase sinusoidal voltage of ang. fr. ω, Voltage vector for a balanced, 3 phase, sinusoidal voltage of ang. frequency ω, peak-phase voltage V_m, and initial phase angle θ_o.



This shows us that the voltage vector for a balanced, 3ph. sinusoidal voltages rotate at uniform angular velocity at ω in ccw direction with a constant amplitude of $3/2\, V_m$.

Ex: $V_a = V_m \sin \omega t$

$V_b = V_m \sin(\omega t - 120°)$

$V_c = V_m \sin(\omega t - 240°)$

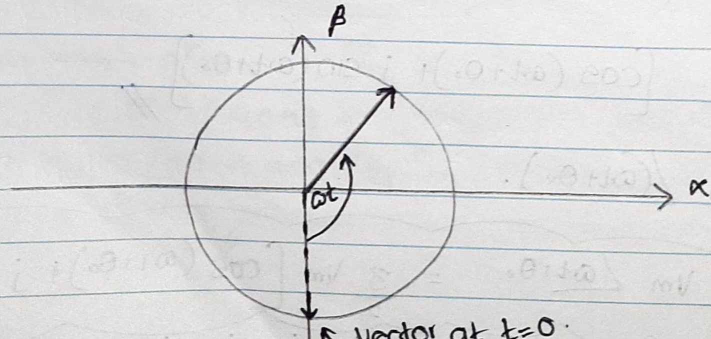
Illustrate $\bar{V}$ an α, β plane.

Ans:

$V_a = V_m \sin \omega t = V_m \cos(\omega t - 90°)$

$V_b = V_m \cos(\omega t - 90° - 120°)$

$V_c = V_m \cos(\omega t - 90° - 240°)$



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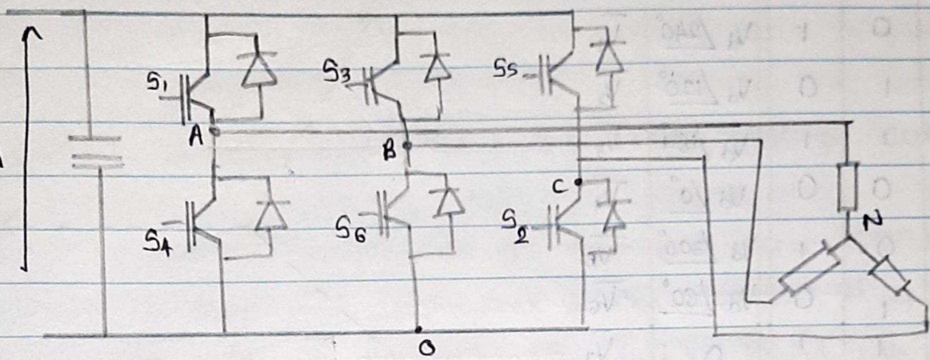
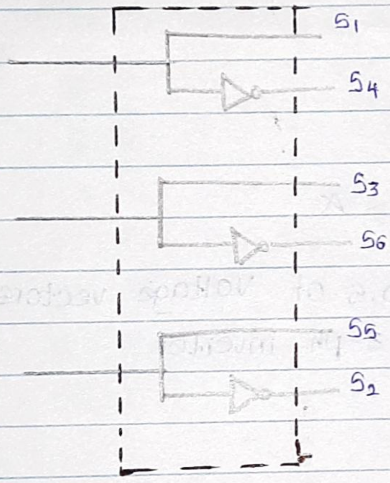
(3) Voltage vectors produced 3ph. Inverter.





output phase voltages:

V_{AN}, V_{BN}, V_{CN}.

$$\bar{V} = V_{AN} + \underline{a}\, V_{BN} + \underline{a}^2\, V_{CN}$$

$$= (V_{AO} - V_{NO}) + \underline{a}\,(V_{BO} - V_{NO}) + \underline{a}^2\,(V_{CO} - V_{NO})$$

$$= V_{AO} + \underline{a}\, V_{BO} + \underline{a}^2\, V_{CO} - \underbrace{(1 + \underline{a} + \underline{a}^2)}_{0} V_{NO}$$

$$= V_{AO} + \underline{a}\, V_{BO} + \underline{a}^2\, V_{CO}$$

$$V_{AO} = S_A V_d$$

$$V_{BO} = S_B V_d$$

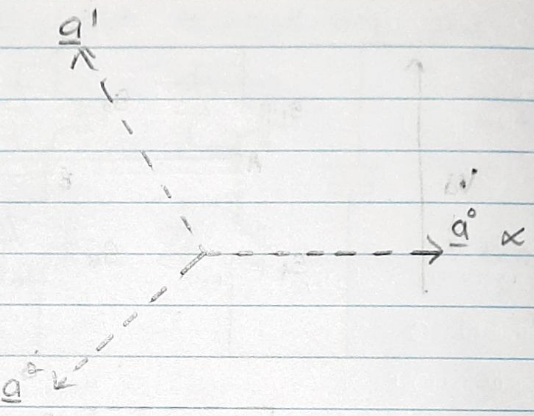
$$V_{CO} = S_C V_d$$

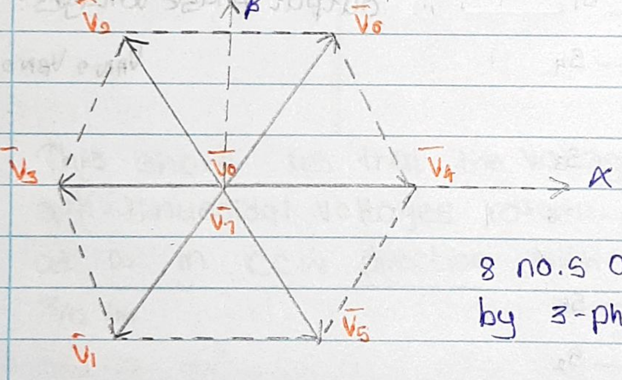
; S_A, S_B, S_C each 1 or 0.

$$\bar{V} = S_A V_d + \underline{a}\,(S_B V_d) + \underline{a}^2\,(S_C V_d)$$

$$= (S_A + \underline{a}\, S_B + \underline{a}^2\, S_C) V_d$$

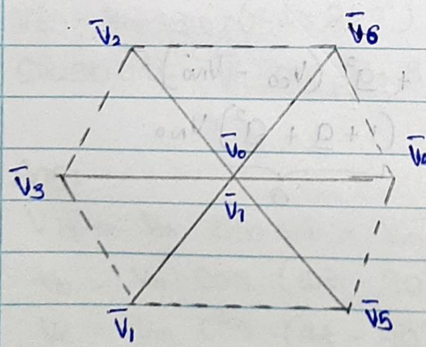
S_A	S_B	S_C	$\bar{V}$	
0	0	0	0	$\bar{V}_0$
0	0	1	$V_d \angle 240^\circ$	$\bar{V}_1$
0	1	0	$V_d \angle 120^\circ$	$\bar{V}_2$
0	1	1	$V_d \angle 180^\circ$	$\bar{V}_3$
1	0	0	$V_d \angle 0^\circ$	$\bar{V}_4$
1	0	1	$V_d \angle 300^\circ$	$\bar{V}_5$
1	1	0	$V_d \angle 60^\circ$	$\bar{V}_6$
1	1	1	0	$\bar{V}_7$





8 no.s of voltage vectors produced by 3-ph inverter.

(a) Implementation of Voltage vector PWM.



The 8 voltage vectors ~~with~~ represented complete output capability ~~as~~ of the inverter.
So, we must be able to use ~~the~~ these vectors to explain any control ~~whether~~ method used with a 3-phase inverter.