ME 1012: Applied Mechanics

# Friction: Problems


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# Problems: 6.1

- And effort of 200 N is required just to move a certain body up on an inclined plane of angle 15º the force acting parallel to the plane. If the angle of inclination of the plane is made 20º the effort required, again applied parallel to the plane, is found to be 230 N. Find the weight of the body and the coefficient of friction.

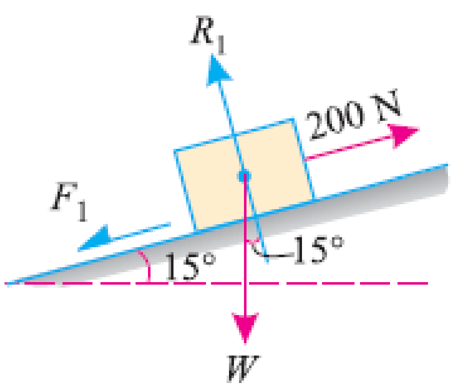
**Solution.** Given: First case : When effort ($P_1$) = 200 N, then angle of inclination ($\alpha_1$) = 15° and second case : When effort ($P_2$) = 230 N, then angle of inclination ($\alpha_2$) = 20°.

Let $\mu$ = Coefficient of friction,

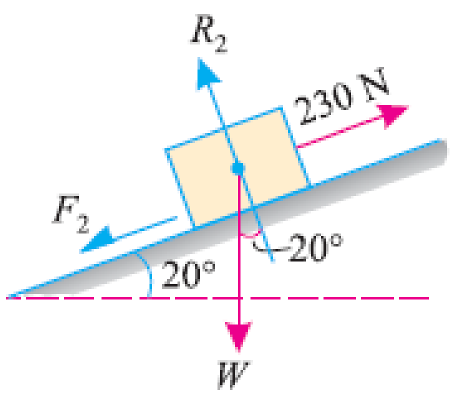
$W$ = Weight of the body,

$R$ = Normal reaction, and

$F$ = Force of friction.




(a) Body lying at 15°




(b) Body lying at 20°

# Solution: 6.1

First of all, consider the body lying on a plane inclined at an angle of 15° with the horizontal and subjected to an effort of 200 N as shown in Fig. 8.12 (*a*).

Resolving the forces at right angles to the plane,

$$R_1 = W \cos 15° \qquad \text{...}(i)$$

and now resolving the forces along the plane,

$$200 = F_1 + W \sin 15° = \mu.R_1 + W \sin 15° \qquad \text{...}(\because\ F = \mu.R)$$

$$= \mu\ W \cos 15° + W \sin 15° \qquad \text{...}(\because\ R_1 = W \cos 15°)$$

$$= W(\mu \cos 15° + \sin 15°) \qquad \text{...}(ii)$$

Now consider the body lying on a plane inclined at an angle of 20° with the horizontal and subjected to an effort of 230 N shown in Fig. 8.12 (*b*).

Resolving the forces at right angles to the plane,

$$R_2 = W \cos 20° \qquad \text{...}(iii)$$

and now resolving the forces along the plane,

$$230 = F_2 + W \sin 20° = \mu\ R_2 + W \sin 20° \qquad \text{...}(\because\ F = \mu.R)$$

$$= \mu\ W \cos 20° + W \sin 20° \qquad \text{...}(\because\ R_2 = W \cos 20°)$$

$$= W(\mu \cos 20° + \sin 20°) \qquad \text{...}(iv)$$

# Solution: 6.1 (cont…)

*Coefficient of friction*

Dividing equation (*iv*) by (*ii*),

$$\frac{230}{200} = \frac{W\,(\mu\ \cos 20^\circ + \sin 20^\circ)}{W\,(\mu\ \cos 15^\circ + \sin 20^\circ)}$$

$$230\ \mu \cos 15^\circ + 230 \sin 15^\circ = 200\ \mu \cos 20^\circ + 200 \sin 20^\circ$$

$$230\ \mu \cos 15^\circ - 200\ \mu \cos 20^\circ = 200 \sin 20^\circ - 230 \sin 15^\circ$$

$$\mu\,(230 \cos 15^\circ - 200 \cos 20^\circ) = 200 \sin 20^\circ - 230 \sin 15^\circ$$

$$\therefore\ \mu = \frac{200 \sin\ 20^\circ - 230 \sin 15^\circ}{230 \cos 15^\circ - 200 \cos 20^\circ} = \frac{(200 \times 0.3420) - (230 \times 0.2588)}{(230 \times 0.9659) - (200 \times 0.9397)} = 0.259 \qquad \textbf{Ans.}$$

*Weight of the body*

Substituting the value of μ in equation (*ii*),

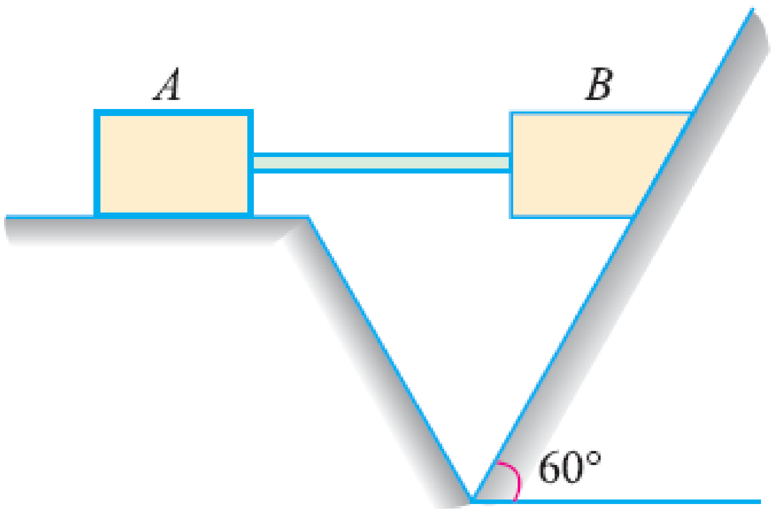
$$200 = W\,(0.259 \cos 15^\circ + \sin 15^\circ)$$

$$= W\,(0.259 \times 0.9659 + 0.2588) = 0.509\ W$$

$$\therefore \qquad W = \frac{200}{0.509} = 392.9\ \text{N} \qquad \textbf{Ans.}$$

# Problem: 6.2

- Two blocks A and B, connected by a horizontal rod and frictionless hinges are supported on two rough planes as shown in Figure.




- The coefficient of friction are 0.3 between block A and the horizontal surface, and 0.4 between block B and the inclined surface. If the block B weighs 100 N, what is the smallest weight of block A, that will hold the system in equilibrium?

# Solution: 6.2

**Solution.** Given: Coefficient of friction between block A and horizontal surface $(\mu_A) = 0.3$; Coefficient of friction between block $B$ and inclined surface $(\mu_B) = 0.4$ and weight of block $B$ $(W_B) = 100$ N.

Let $\quad W_A$ = Smallest weight of block $A$.

We know that force of friction of block $A$, which is acting horizontally on the block $B$,

$$P = \mu_A W_A = 0.3 \times W_A = 0.3\, W_A$$

and angle of friction of block $B$

$$\tan \phi = \mu_B = 0.4 \qquad \text{or} \qquad \phi = 21.8^\circ$$

We also know that the smallest force, which will hold the system in equilibrium (or will prevent the block $B$ from sliding downwards),

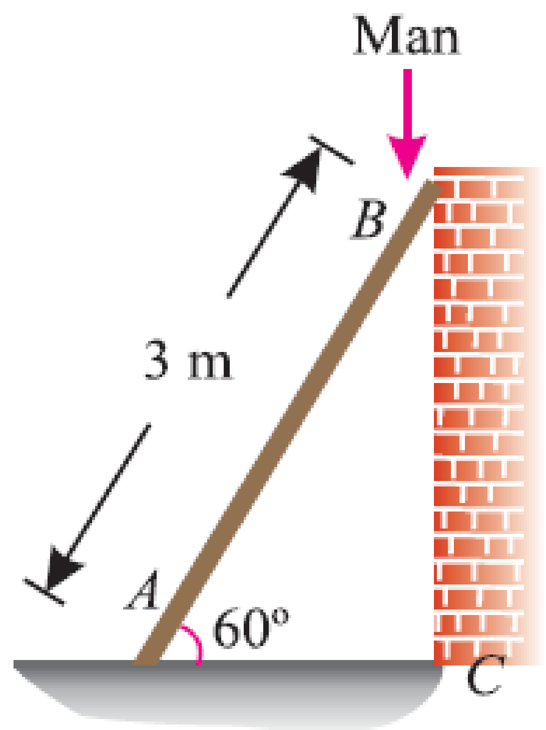
$$P = W_B \tan(\alpha - \phi) = 100 \tan(60^\circ - 21.8^\circ)$$

or $\quad 0.3\, W_A = 100 \tan 38.2^\circ = 100 \times 0.7869 = 78.69$

$$\therefore \quad W_A = \frac{78.69}{0.3} = 262.3 \text{ N} \qquad \textbf{Ans.}$$

# Problems: 6.3

- A uniform ladder 3 m long weights 200 N. It is placed against a wall making an angle of 60 with the floor as shown in Figure.
- The coefficient of friction between the wall and the ladder is 0.25 and that between the floor and ladder is 0.35. The ladder, in addition to its own weight, has to support a man of 1000 N at its top at B. Calculate:




i. The horizontal force **P** to be applied to ladder at the floor level to prevent slipping
ii. If the force P is not applied, what should be the minimum inclination of the ladder with the horizontal, so that there is no slipping of it with the man at its top.

# Solution: 6.3

**Solution.** Given: Length of the ladder ($l$) = 3 μ ; Weight of the ladder ($W$) = 200 N; Coefficient of friction between the wall and the ladder ($\mu_w$) = 0.25 and coefficient of friction between the floor and ladder ($\mu_f$) = 0.35.

The forces acting in both the cases are shown in Fig. 9.6 (*a*) and (*b*).

First of all, cosider the ladder inclined at an angle of 60° and subjected to a horizontal force ($P$) at the floor as shown in Fig. 9.6 (*a*).

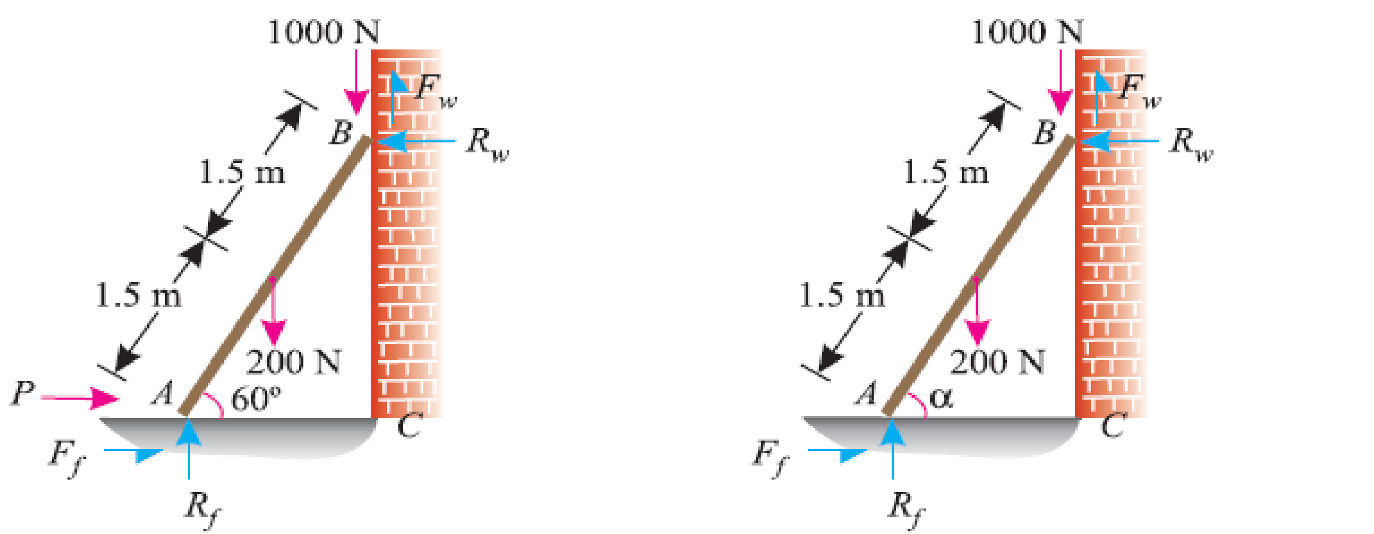



**Fig. 9.6.**

(*i*) *Horizontal force* ($P$) *applied to the ladder at floor level to prevent slipping*

Resolving the forces horizontally,

$$P + F_f = R_w \quad \text{... (i)}$$

and now resolving the forces vertically,

$$R_f + F_w = 1000 + 200 = 1200 \text{ N} \quad \text{...(ii)}$$

# Solution: 6.3 

Taking moments about $A$ and equating the same,

$$(200 \times 1.5 \cos 60°) + 1000 \times 3 \cos 60°)$$
$$= (F_w \times 3 \cos 60°) + (R_w \times 3 \sin 60°)$$

Dividing both sides by the cos 60°,

$$300 + 3000 = (3 \times F_w) + (3 \times R_w \tan 60°)$$

$$\therefore \quad 1100 = F_w + R_w \tan 60° \quad ...(iii)$$

We know that $F_w = \mu_w \times R_w = 0.25\, R_w$ $\quad ...(\because \mu_w = 0.25)$

Substituting this value of $F_w$ in equation $(iii)$,

$$1100 = (0.25\, R_w) + (R_w \tan 60°) = R_w\,(0.25 + 1.732) = R_w \times 1.982$$

$$\therefore \quad R_w = \frac{1100}{1.982} = 555 \text{ N}$$

and $$F_w = 0.25\, R_w = 0.25 \times 555 = 138.7 \text{ N}$$

Now substituting the value of $F_w$ in equation $(ii)$,

$$R_f + 138.7 = 1200$$

$$\therefore \quad R_f = 1200 - 138.7 = 1061.3 \text{ N}$$

and $$F_f = \mu_f R_f = 0.35 \times 1061.3 = 371.5 \text{ N}$$

Now substituting the value of $F_f$ in equation $(i)$,

$$P + 371.5 = 555$$

$$\therefore \quad P = 555 - 371.5 = 183.5 \text{ N} \quad \textbf{Ans.}$$

# Solution: 6.3 cont…

*(ii) Inclination of the ladder with the horizontal for no slipping*

Now consider the ladder inclined at angle ($\alpha$) and without any horizontal force acting at the floor as shown in Fig. 9.6 (*b*).

Resolving the forces horizontally,

$$R_w = F_f = \mu_f \times R_f = 0.35 \times R_f \qquad \text{...}(iv)$$

and now resolving the forces vertically,

$$R_f + F_w = 1000 + 200 = 1200 \text{ N}$$

We know that $\quad F_w = \mu_w \times R_w = 0.25(0.35\,R_f) = 0.09\,R_f \qquad \text{...}(\because R_w = 0.35\,R_f)$

or $\quad R_f + 0.09\,R_f = 1200$

$$\therefore \qquad R_f = \frac{1200}{1.09} = 1101 \text{ N}$$

and $\quad R_w = 0.35 R_f = 0.35 \times 1101 = 385.4 \text{ N}$

Similarly $\quad F_w = 0.09\,R_f = 0.09 \times 1101 = 99.1 \text{ N}$

Taking moments about *A* and equating the same,

$$(1000 \times 3 \cos \alpha) + (200 \times 1.5 \cos \alpha)$$
$$= (F_w \times 3 \cos \alpha) + (R_w \times 3 \sin \alpha)$$

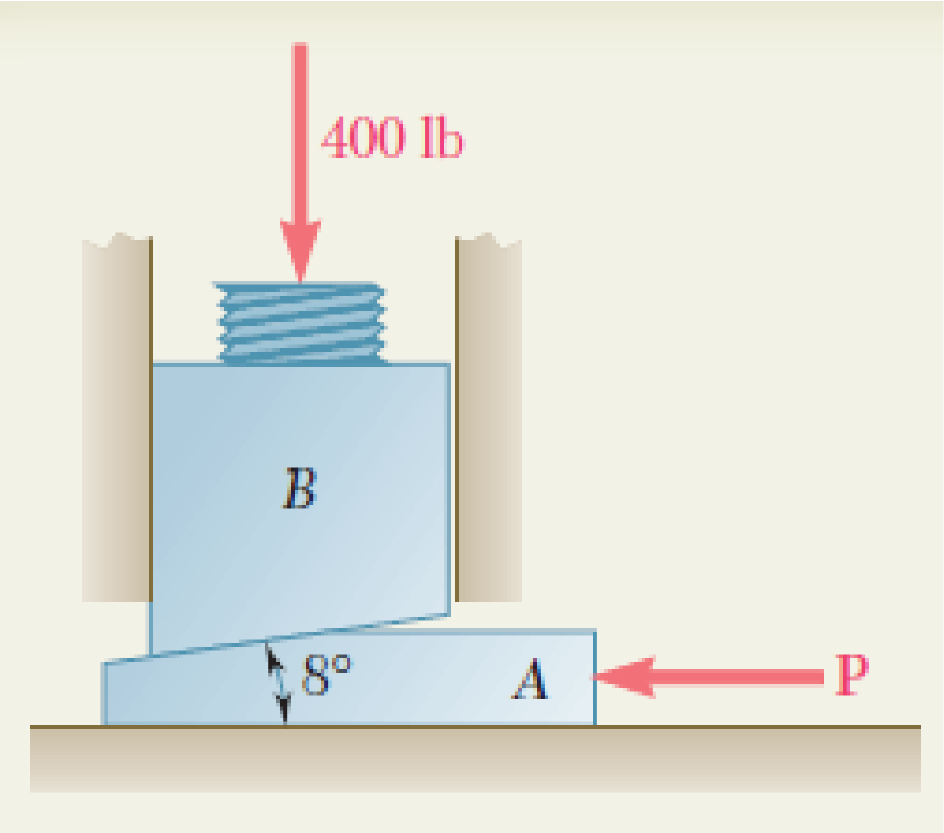
Dividing both sides by 3 cos $\alpha$,

$$1000 + 100 = F_w + R_w \tan \alpha$$
$$1100 = 99.1 + 385.4 \tan \alpha$$
$$385.4 \tan \alpha = 1100 - 99.1 = 1000.9$$

$$\therefore \qquad \tan \alpha = \frac{1000.9}{385.4} = 2.5970 \qquad \text{or} \qquad \alpha = 68.9° \qquad \textbf{Ans.}$$

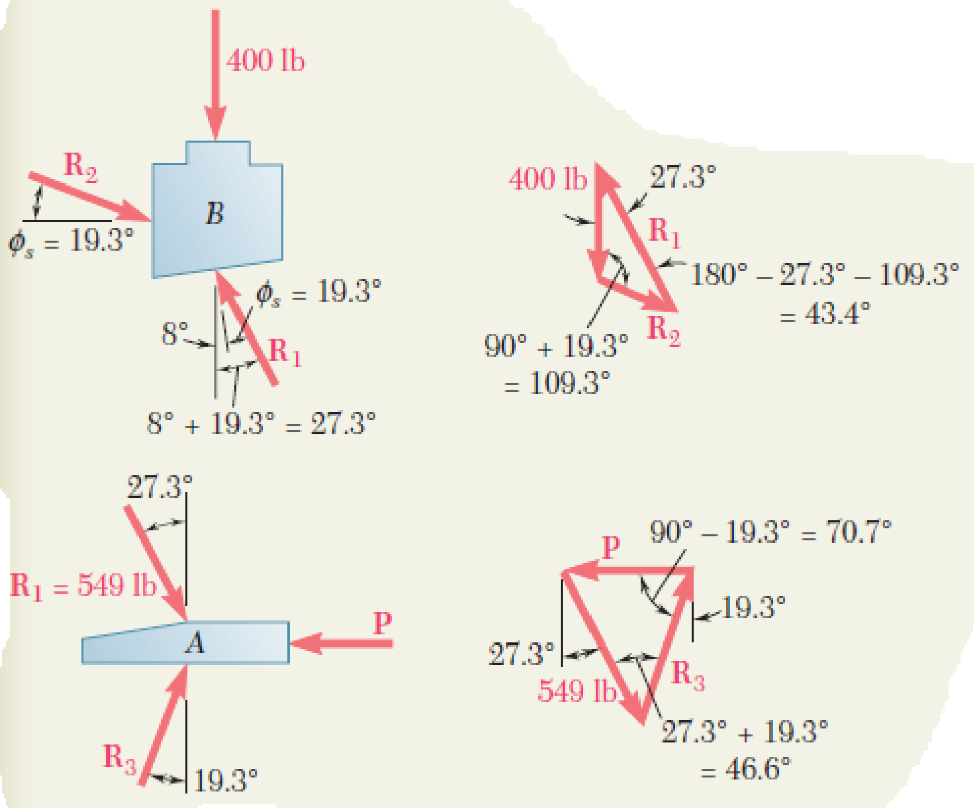
# Problems: 6.4

- The position of the machine block B is adjusted by moving the wedge A. Knowing that the coefficient of static friction is 0.35 between all surfaces of contact, determine the force P required (a) to raise block B, (b) to lower block B.



# Solution: 6.4

For each part, the free-body diagrams of block $B$ and wedge $A$ are drawn, together with the corresponding force triangles, and the law of sines is used to find the desired forces. We note that since $\mu_s = 0.35$, the angle of friction is




$$\phi_s = \tan^{-1} 0.35 = 19.3°$$

## a. Force P to Raise Block

***Free Body: Block B***

$$\frac{R_1}{\sin 109.3°} = \frac{400 \text{ lb}}{\sin 43.4°}$$

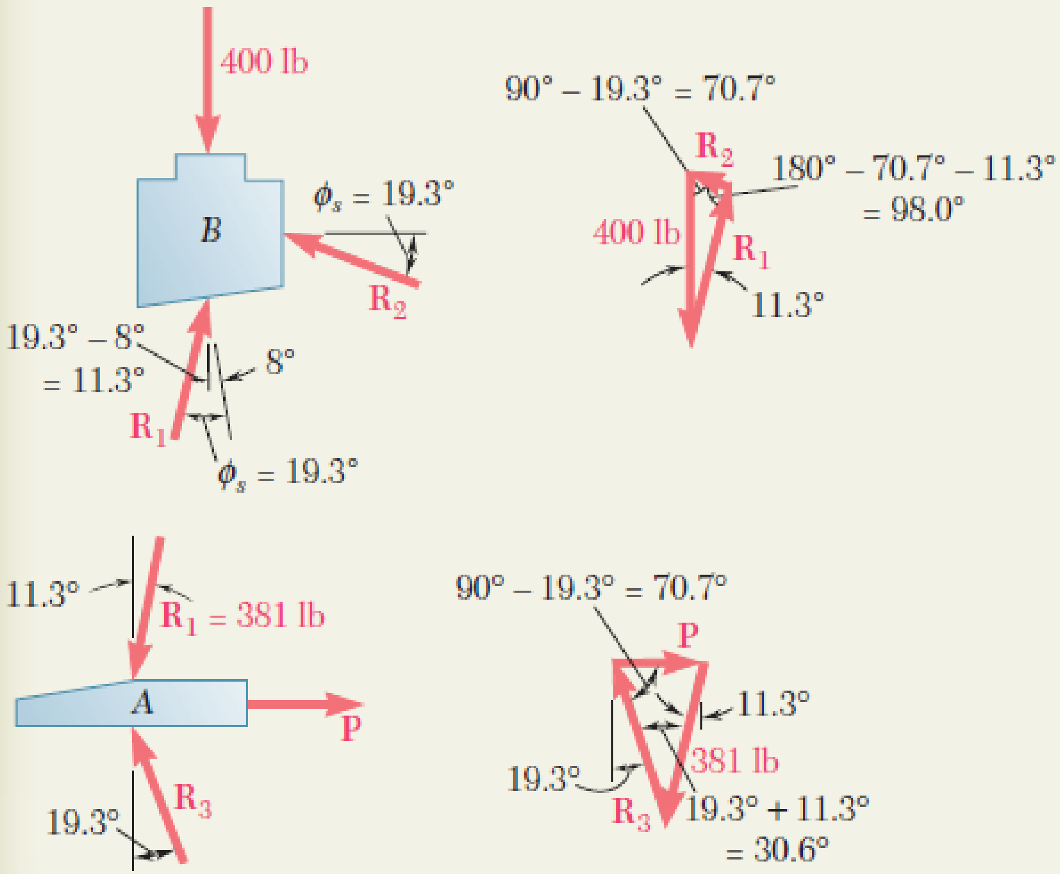
$$R_1 = 549 \text{ lb}$$

***Free Body: Wedge A***

$$\frac{P}{\sin 46.6°} = \frac{549 \text{ lb}}{\sin 70.7°}$$

$$P = 423 \text{ lb} \qquad \mathbf{P} = 423 \text{ lb} \leftarrow$$

# Solution: 6.4 (cont…)




**b. Force P to Lower Block**

*Free Body: Block B*

$$\frac{R_1}{\sin 70.7°} = \frac{400 \text{ lb}}{\sin 98.0°}$$

$$R_1 = 381 \text{ lb}$$

*Free Body: Wedge A*

$$\frac{P}{\sin 30.6°} = \frac{381 \text{ lb}}{\sin 70.7°}$$

$$P = 206 \text{ lb} \qquad \mathbf{P} = 206 \text{ lb} \rightarrow$$

# Problems: 6.5

- The screw of a jack is square threaded with two threads in a centimeter. The outer diameter of the screw is 5 cm. If the coefficient of friction is 0.1, calculate the force required to be applied at the end of the lever, which is 70 cm long (a) to lift a load of 4 kN, and (b) to lower it.

**Solution.** Given: Outer diameter of the screw ($D$) = 5 cm; Coefficient of friction ($\mu$) = 0.1 = $\tan \phi$; Length of the lever ($l$) = 70 cm and load to be lifted ($W$) = 4 kN = 4000 N.

We know that as there are two threads in a cm, (*i.e.* $n = 2$) therefore pitch of the screw,

$$p = 1/2 = 0.5 \text{ cm}$$

and internal diameter of the screw,

$$= 5 - (2 \times 0.5) = 4 \text{ cm}$$

$\therefore$ Mean diameter of the screw,

$$d = \frac{5+4}{2} = 4.5 \text{ cm}$$

Let $\alpha$ = Helix angle.

We know that $\tan \alpha = \dfrac{p}{\pi d} = \dfrac{0.5}{\pi \times 4.5} = 0.0353$

# Solution: 6.5

(*i*) *Force required at the end of 70 cm long lever to lift the load*

Let $P_1$ = Force required at the end of the lever to lift the load.

We know that the force required to be applied at the mean radius to lift the load,

$$P = W \tan(\alpha + \phi) = W \times \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}$$

$$= 4000 \times \frac{0.0353 + 0.1}{1 - 0.0353 \times 0.1} = 543.1 \text{ N}$$

Now the force required at the end of the lever may be found out from the relation,

$$P_1 \times 70 = P \times \frac{d}{2} = 543.1 \times \frac{4.5}{2} = 1222$$

$$\therefore \quad P_1 = \frac{1222}{7.0} = 17.5 \text{ N} \qquad \textbf{Ans.}$$

# Solution: 6.5 (cont…)

*(ii) Force required at the end of 70 cm long lever to lower the load*

Let $P_2$ = Force required at the end of the lever to lower the load.

We know that the force required at the mean radius to lower the load,

$$P = W\tan(\phi - \alpha) = 4000 \times \frac{\tan\phi - \tan\alpha}{1 + \tan\phi\tan\alpha}$$

$$= 4000 \times \frac{0.1 - 0.0353}{1 + 0.1 \times 0.0353} = 257.9 \text{ N}$$

Now the force required at the end of the lever may be found out from the relation:

$$P_2 \times 70 = P \times \frac{d}{2} = 257.9 \times \frac{4.5}{2} = 580.3$$

$$\therefore \quad P_2 = \frac{580.3}{70} = 8.3 \text{ N} \qquad \textbf{Ans.}$$