$$X[\omega] = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \qquad \omega \in [-\pi, \pi] \qquad \left[k\frac{2\pi}{N} = \omega \right]$$

DTFT

$$= \sum_{n=0}^{N-1} x[n]e^{-j\omega n}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-jk\frac{2\pi}{N}n} \qquad k = 0, 1, ..., N-1$$
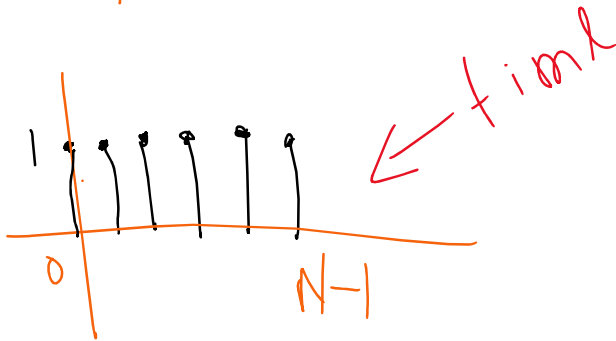
DFT

$$X[k] = X\left(k\frac{2\pi}{N} \right)$$

DFT

DFT is the DTFT of discrete time finite-length signal evaluate at N equally space points with $\left[\omega = k\frac{2\pi}{N} \right] \quad k = 0, 1, 2, ..N-1$

Example 1 : (DFT)

$\delta\langle n\rangle$

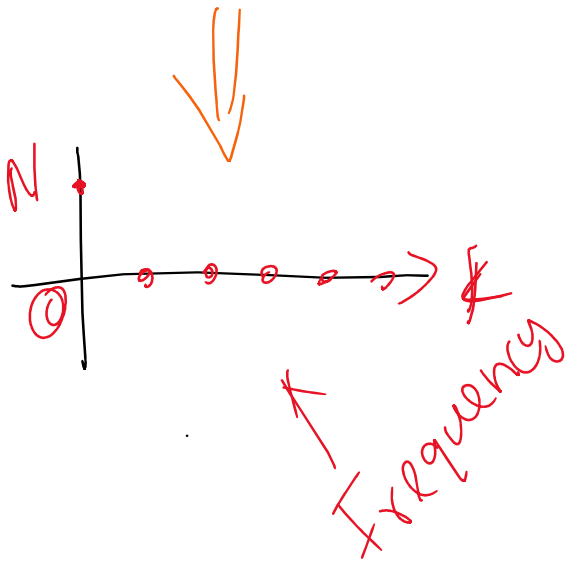$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$
$$= W_N^{0k} = 1 \qquad \forall k$$

← time

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$
$$= \sum_{n=0}^{N-1} W_N^{nk}$$
$$= W_N^0 + W_N^k + W_N^{2k} + \cdots + W_N^{(N-1)k}$$
$$= \frac{1-\left(W_N^k\right)^N}{1-W_N^k}$$

$$\begin{cases} X[k] = N & \text{If } k = 0 \\ \quad 0 & \text{If } k \neq 0 \end{cases}$$

$\rightarrow k$

Frequency