ANGLE MODULATION

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- Angle Modulation is the process in which the **frequency or the phase** of the carrier signal varies according to the message signal.

- The standard equation of the angle modulated wave is

$$s(t) = A_c \cos \theta_i(t)$$

Where,

- **Ac** is the amplitude of the modulated wave, which is the **same as the amplitude of the carrier signal**
- **θi(t)** is the angle of the modulated wave

WHY ANGLE MODULATION??

- It provides **better noise discrimination against noise and interference** than Amplitude Modulation.
- Provides us with a practical means of **exchanging channel bandwidth** for improved noise performances which is not possible in Amplitude modulation.

Note:

- Improvement in noise performance in angle modulation is achieved at the cost of **increased system complexity in both the transmitter and the receiver**.

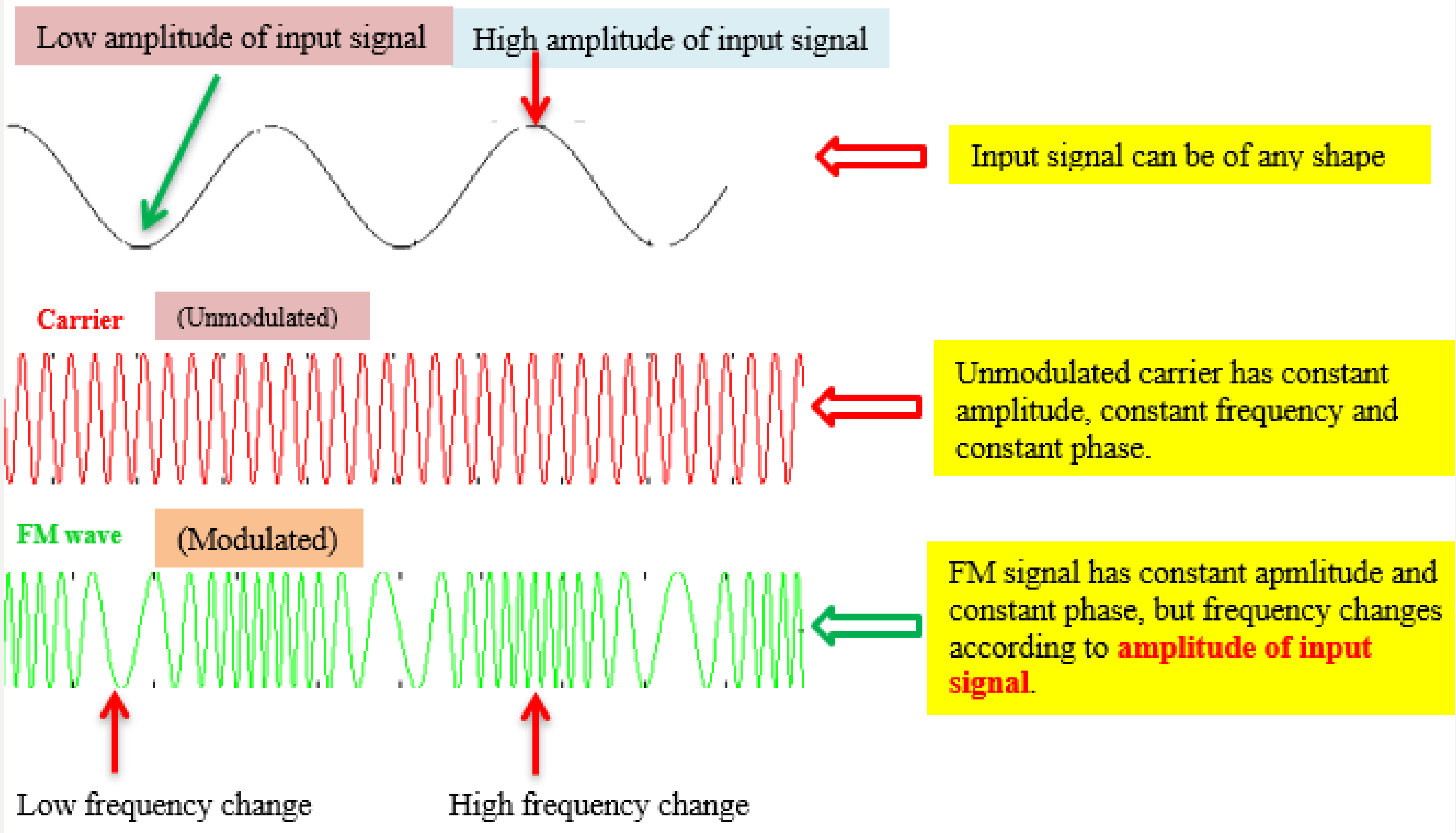
Angle modulation is further divided into frequency modulation and phase modulation.

- **Frequency Modulation** is the process of **varying the frequency of the carrier signal l**inearly with the message signal.

- **Phase Modulation** is the process **of varying the phase of the carrier signal** linearly with the message signal.

FREQUENCY MODULATION

- In amplitude modulation, the **amplitude of the carrier signal varies**.
- Whereas, in **Frequency Modulation (FM)**, the frequency of the carrier signal varies **in accordance with the instantaneous amplitude of the modulating signal**.



- The frequency of the **modulated wave increases**, when the amplitude of the modulating or **message signal increases**. Similarly, the frequency of the modulated wave decreases, when the amplitude of the modulating signal decreases.
- The frequency of the modulated wave **remains constant** and it is equal to the frequency of the carrier signal, when the **amplitude of the modulating signal is zero**.

MATHEMATICAL REPRESENTATION

The equation for instantaneous frequency f_i in FM modulation is

$$f_i = f_c + k_f m\left(t\right)$$

Where,

f_c is the carrier frequency

k_t is the frequency sensitivity - Measured in Hertz per Volt

$m\left(t\right)$ is the message signal

We know the relationship between angular frequency ω_i and angle $\theta_i(t)$ as

$$\omega_i = \frac{d\theta_i(t)}{dt}$$

$$\Rightarrow 2\pi f_i = \frac{d\theta_i(t)}{dt}$$

$$\Rightarrow \theta_i(t) = 2\pi \int_0^t f_i dt$$

Substitute, f_i value in the above equation.

$$\theta_i(t) = 2\pi \int_0^t (f_c + k_f m(t))\, dt$$

$$\Rightarrow \theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt$$

Substitute, $\theta_i(t)$ value in the standard equation of angle modulated wave.

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right)$$

This is the **equation of FM wave**.

If the modulating signal is $m(t) = A_m \cos(2\pi f_m t)$, then the equation of FM wave will be

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

Where,

$$\beta = \textbf{\textit{modulation index}} = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

The difference between FM modulated frequency (instantaneous frequency) and normal carrier frequency is termed as **Frequency Deviation**. It is denoted by Δf, which is equal to the product of k_f and A_m.

FM can be divided into **Narrowband FM** and **Wideband FM** based on the values of modulation index β.

Narrow band FM **β is very small** compared to one radian.
Wide band FM **β is large** compared to one radian

Problem 1

A sinusoidal modulating waveform of amplitude 5 V and a frequency of 2 KHz is applied to FM generator, which has a frequency sensitivity of 40 Hz/volt. Calculate the frequency deviation, modulation index, and bandwidth.

Problem 2

An FM wave is given by $s(t) = 20\cos\left(8\pi \times 10^6 t + 9\sin\left(2\pi \times 10^3 t\right)\right)$. Calculate the frequency deviation, bandwidth, and power of FM wave.

PROPERTIES OF ANGLE MODULATED SIGNAL

1. **Constancy of transmitted power:**

Let us the Angle modulated wave equations as: $s(t) = A_c \cos\theta_i(t)$

FM wave : $s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right)$

PM Wave : $s(t) = A_c \cos(2\pi f_c t + k_p m(t))$

You can see that the Amplitude of the FM and PM is maintained **at a constant value equals to the carrier Amplitude Ac at all time t,** irrespectively the sensitivity factors Kf and Kp. Therefore, **regardless of the message Signal - m(t),**

Average transmitted power is give $S_T = \frac{1}{2} A_c^2$

- Assume that Load resistance (RL) is 1 Ohms.

2. Non-linearity of the modulation process

Both PM and FM waves are **violating the principle of superposition.**

.

Assume message signal m3(t) is made up of two different components m1(t) and m2(t) as :

m3(t)=m1(t)+m2(t)

Let: S3(t), S1(t) and S2(t) denotes the PM waves produced by the m3(t), m1(t) and m2(t)

$$\begin{aligned} s_1(t) &= A_c \cos\left[2\pi f_c t + k_p m_1(t)\right] \\ s_2(t) &= A_c \cos\left[2\pi f_c t + k_p m_2(t)\right] \end{aligned}$$

$$\begin{aligned} s_3(t) &= A_c \cos\left[2\pi f_c t + k_p(m_1(t) + m_2(t))\right] \\ &\neq s_1(t) + s_2(t) \end{aligned}$$

Same can be applied to **FM also.** Therefore, **angle modulation** is non-linear.

3. Irregularity of zero Crossing

Zero crossing is defined as instants of time, at which waveform changes **amplitude from positive to negative** or vice versa.

The irregularity of zero-crossings in angle-modulation wave is attributed to the nonlinear character of the modulation process

As shown on the graph there is no perfect regularity of modulated wave (right side graph)

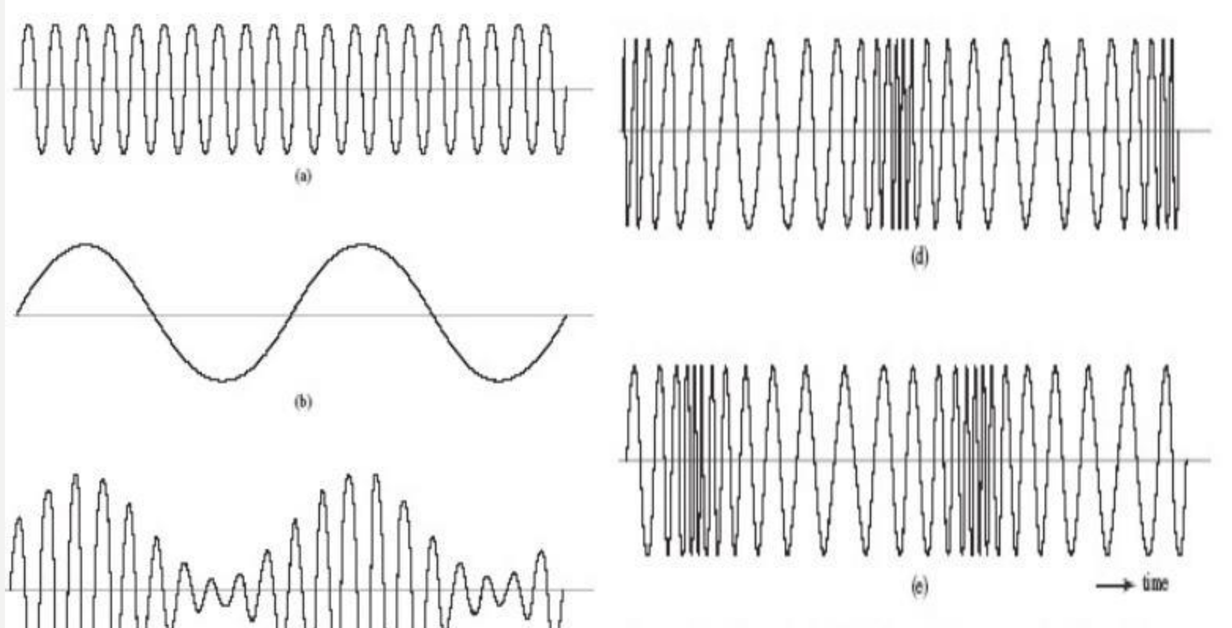
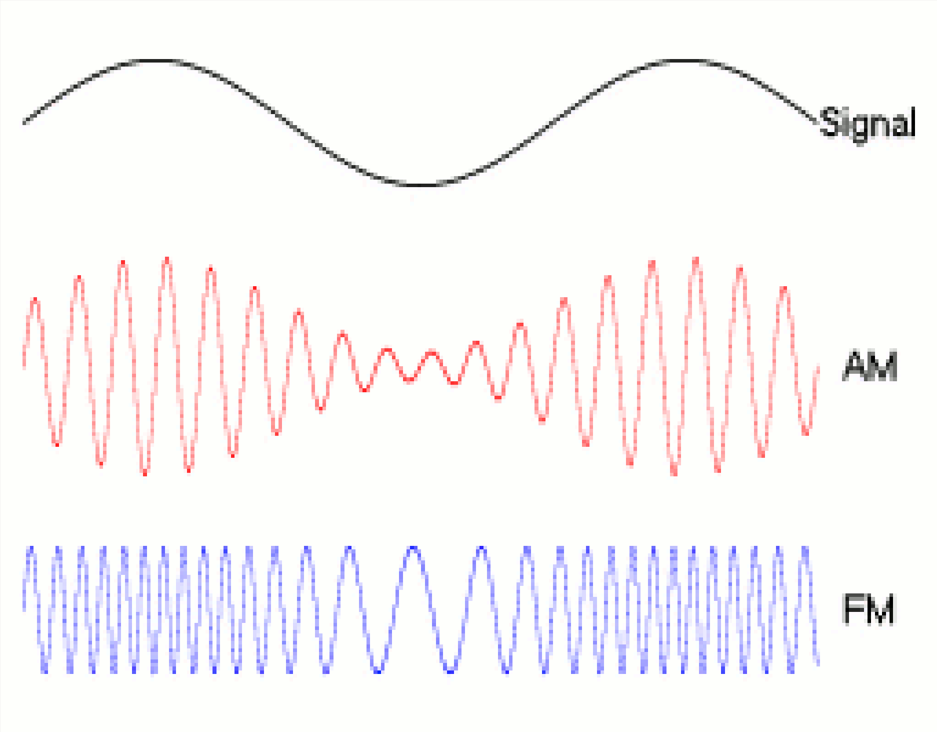


FIGURE 4.1 Illustration of AM, PM, and FM waves produced by a single tone. (a) Carrier wave. (b) Sinusoidal modulating signal. (c) Amplitude-modulated signal. (d) Phase-modulated signal. (e) Frequency modulated signal.

4. Visualization difficulty of message waveform

The **difficulty in visualizing the message waveform in angle-modulated waves** is also attributed to the nonlinear character of angle-modulated waves

Where in case of Amplitude Modulation, we can see the shape of the message wave form at the envelop of the modulated signal.



NARROWBAND FM

Following are the features of Narrowband FM.

- This frequency modulation has a **small bandwidth** when compared to wideband FM.
- The **modulation index β is small, i.e., less than 1**.
- Its spectrum consists of the **carrier,** the **upper sideband** and **the lower sideband.**
- This is used in **mobile communications** such as police wireless, ambulances, taxicabs, etc.

- Consider the standard FM equation;

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

- Expanding this equation using trigonometric identity we get:

$$s(t) = A_c \cos(2\pi f_c t) \cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t) \sin[\beta \sin(2\pi f_m t)]$$

Assuming that the modulation index β is small compared to one radian, we may use the following approximations:

$$\cos[\beta \sin(2\pi f_m t)] \simeq 1$$

and

$$\sin[\beta \sin(2\pi f_m t)] \simeq \beta \sin(2\pi f_m t)$$

Hence, Equation (2.34) simplifies to

$$s(t) \simeq A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

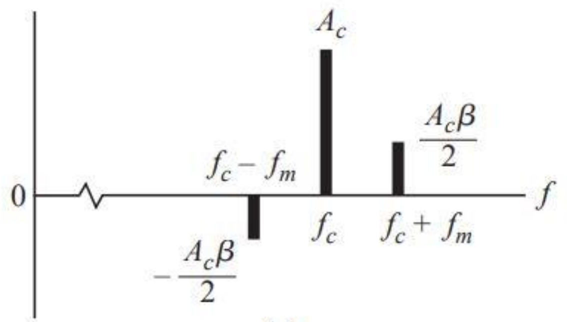
- We can expand this equation again as:

$$s(t) \simeq A_c \cos(2\pi f_c t) + \frac{1}{2}\beta A_c\{\cos[2\pi(f_c + f_m)t] - \cos[2\pi(f_c - f_m)t]\}$$

This expression is somewhat similar to the corresponding one defining an AM signal, which is as follows:

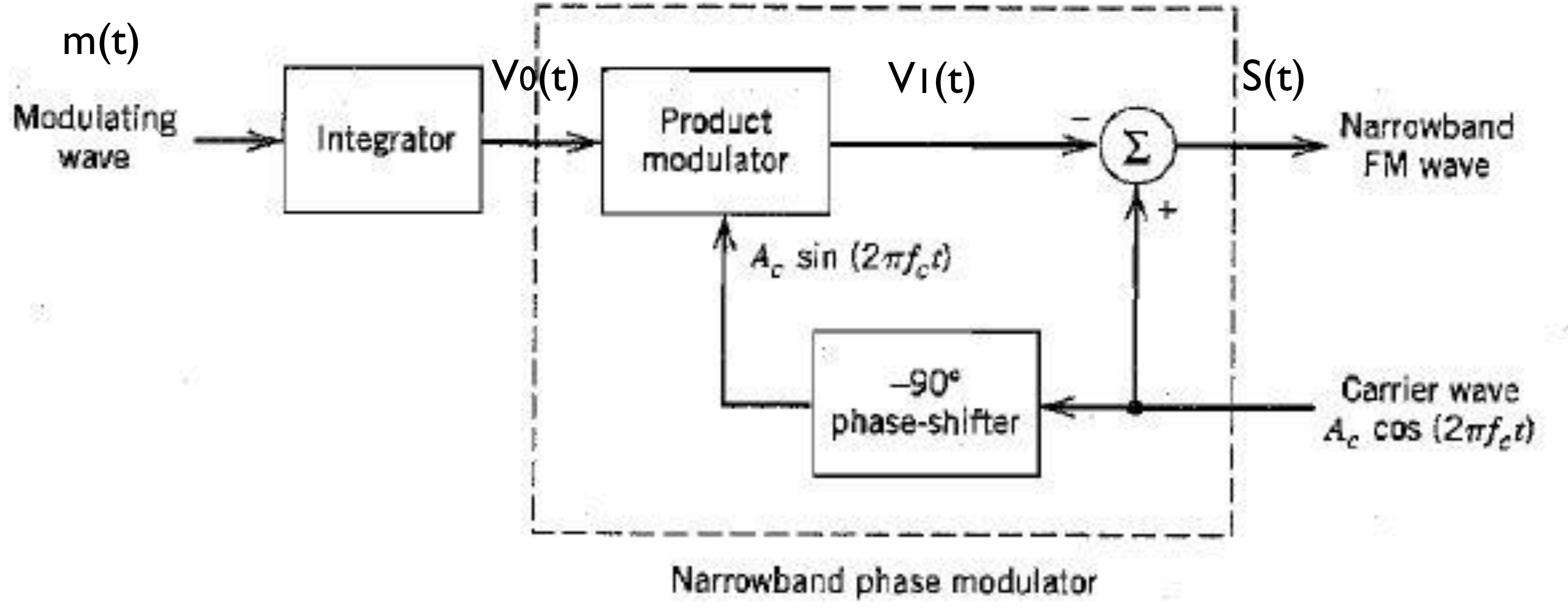
$$s_{\mathrm{AM}}(t) = A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c\{\cos[2\pi(f_c + f_m)t] + \cos[2\pi(f_c - f_m)t]\} \quad (2.37)$$

where μ is the modulation factor of the AM signal. Comparing Equations we see that in the case of sinusoidal modulation, the basic difference between an AM signal and a narrowband FM signal is that the algebraic sign of the lower side frequency in the narrowband FM is reversed. Thus, a narrowband FM signal requires essentially the same transmission bandwidth (i.e., $2f_m$) as the AM signal.



(a)

GENERATION OF NBFM



Block diagram of a method for generating a narrowband FM signal.

- Here, the integrator is used to **integrate the modulating signal m(t)** and the carrier wave is spitted to two parts by a phase shifter.

- Let the modulating signal be, $\mathbf{m(t)=A_m cos(2\pi f_m t)}$

  carrier signal be, $\mathbf{c(t)=A_c cos(2\pi f_c t)}$
- Then we get $\mathbf{V_0(t) = \beta Sin(2\pi f_m t)}$
- The Output of Product Modulator is V1(t)
- V1(t) = $\mathbf{V_0(t).A_c Sin(2\pi f_c t)}$

  $\mathbf{= \beta Sin(2\pi f_m t)\, A_c Sin(2\pi f_c t)}$

  $= \beta\, \mathbf{A_c Sin(2\pi f_m t)\; Sin(2\pi f_c t)}$

  As the output of the summer block we get

  $\mathbf{S(t) = C(t) - V1(t)}$

  $\mathbf{= A_c cos(2\pi f_c t) - \beta\, A_c Sin(2\pi f_c t)\; Sin(2\pi f_m t)}$

This is the equation which is received for the NBFM wave.

WIDEBAND FM

Following are the features of Wideband FM.

- This frequency modulation has infinite bandwidth.
- The modulation index **β is large, i.e., higher than 1.**
- Its spectrum consists of **a carrier and infinite number of sidebands**, which are located around it.
- This is used in **entertainment, broadcasting applications such as FM radio, TV,** etc

wide-band FM wave – WBFM

- **Modulating index – β >>1**

- The standard equation of the FM wave is;

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

- This FM wave is produced by a sinusoidal modulating wave m(t), **that is a periodic function of time t,** only when fc = n.fm (integral multiple of modulating frequency)

- Note that, here fc is large enough to compared to BW of FM Wave.

the **FM modulated wave is calculated in complex mode**:

- It can be written as: (Real part of the complex quantity)

$$s(t) = \Re\left[A_c \exp[j2\pi f_c t + j\beta \sin(2\pi f_m t)]\right] = \Re[\tilde{s}(t)\exp(j2\pi f_c t)]$$

- Where $\tilde{s}(t) = A_c \exp\left[j\beta \sin(2\pi f_m t)\right]$ is **called complex envelop** .

- Note that the **complex envelope is a periodic function of time** with a fundamental frequency equals to the f_m, which means

$$\tilde{s}(t) = \tilde{s}(t + kT_m) = \tilde{s}(t + \frac{k}{f_m})$$

where $T_m = 1/f_m$

- Then we can rewrite

$$
\begin{aligned}
\tilde{s}(t) &= \tilde{s}(t + k/f_m) \\
&= A_c \exp[j\beta \sin(2\pi f_m(t + k/f_m))] \\
&= A_c \exp[j\beta \sin(2\pi f_m t + 2k\pi)] \\
&= A_c \exp[j\beta \sin(2\pi f_m t)]
\end{aligned}
$$

- We can **Expand above equation** as per the **complex Fourier series** form:

$$
\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_m t)
$$

- Where the **complex Fourier Coefficient Cn** is defined as:

$$
\begin{aligned}
c_n &= f_m \int_{-1/(2f_m)}^{1/(2f_m)} \tilde{s}(t) \exp(-j2\pi n f_m t)\, dt \\
&= f_m A_c \int_{-1/(2f_m)}^{1/(2f_m)} \exp[j\beta \sin(2\pi f_m t) - j2\pi n f_m t]\, dt
\end{aligned}
$$

- Define the new variable: $x = 2\pi f_m t$

  Then we can rewrite

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp[j(\beta \sin x - nx)]\, dx$$

- nth order Bessel function of the first kind and argument β

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp[j(\beta \sin x - nx)]\, dx$$

; Complex notation of the Bessel functi

- Accordingly

$$c_n = A_c J_n(\beta)$$

  which gives

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j2\pi n f_m t)$$

- Then the FM wave can be written as

$$
\begin{aligned}
s(t) &= \Re[\tilde{s}(t)\exp(j2\pi f_c t)] \\
&= \Re\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\exp[j2\pi(f_c + n f_m)t]\right] \\
&= A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + n f_m)t]
\end{aligned}
$$

This is the desired form of the furrier series representation of the Single Tone FM Signal For an arbitrary value of modulation index

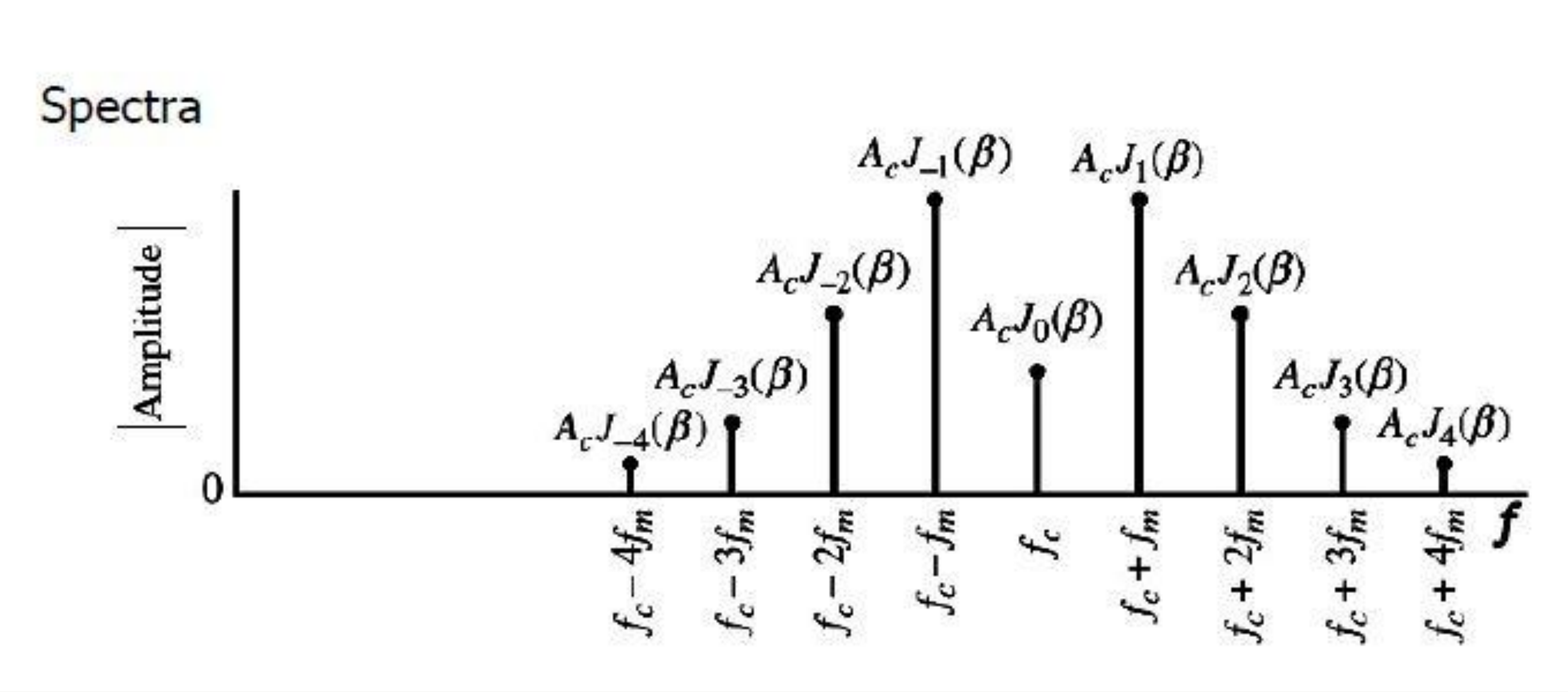
Discrete Spectrum of FM is obtained by taking **Furrier Transformation** of both sides:

$$S(f) = \frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)\right]$$

- Above equation shows an **infinite number of delta functions**

spaced at fc +- nfm

- For n= +- 0, +-1, +-2,…
- Note: FM is composed of a carrier with a set of side band frequencies spaced symmetrically, on either side of the carrier at a frequency separation of fm, 2fm, 3fm, 4fm,…



- The variation of Bessel function determines **the Amplitude of the varies sideband components** of the WBFM.

BESSEL FUNCTION

$$J_p(x) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{x}{2}\right)^{2k+p}}{k!\,(k+p)!}$$

where:

$J_p(x) =$ Magnitude of the frequency component

$p =$ Side frequency number (not to be confused with sidebands)

$x =$ Modulation index

As a point of interest, Bessel's functions are a solution to the following equation:

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - p^2\right) = 0$$

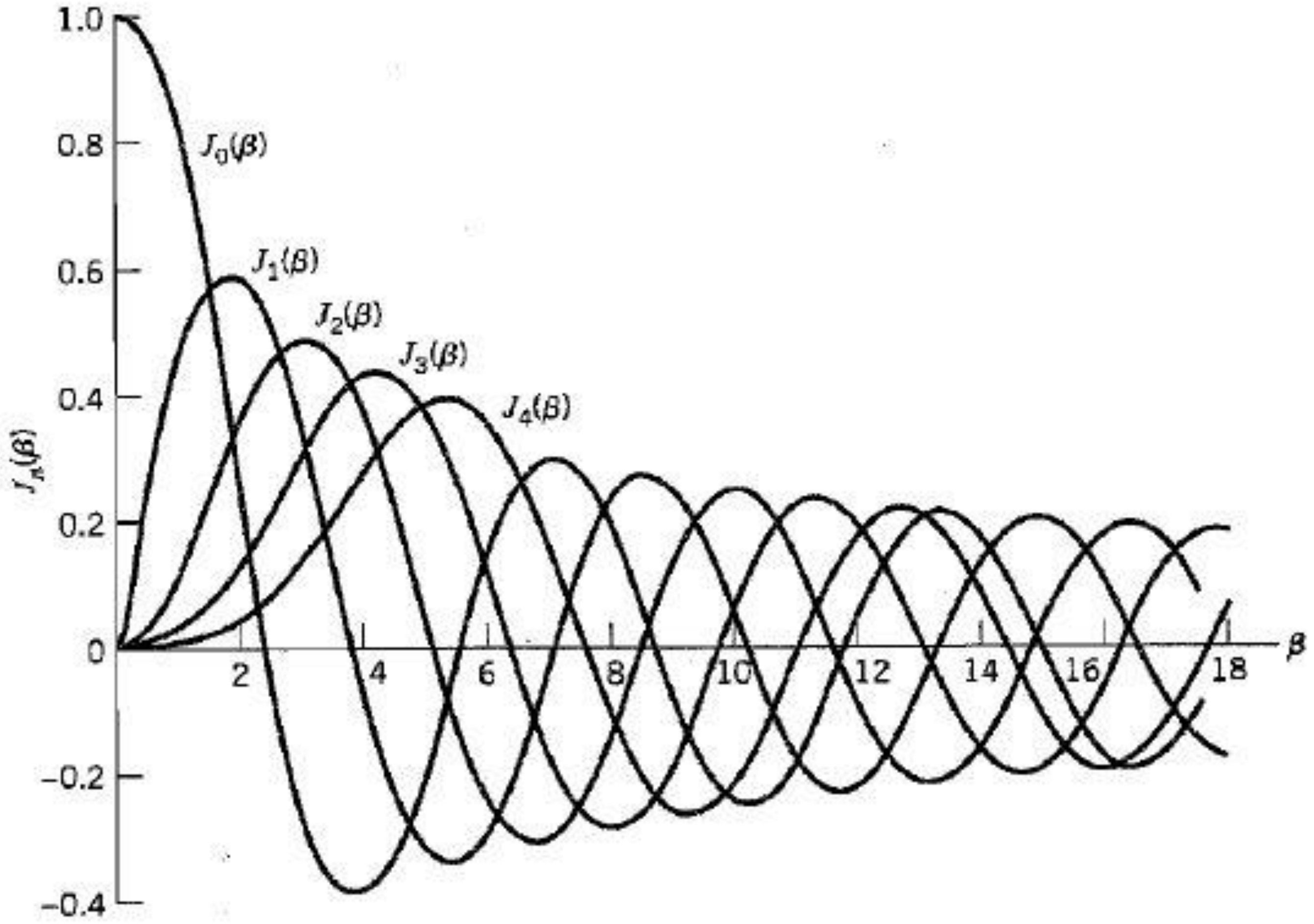
$$J_0(\beta) = \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\beta\right)^{2m}}{m!\,m!} = 1 - \frac{\beta^2}{1!\,1!\,2^2} + \frac{\beta^4}{2!\,2!\,2^4} - \frac{\beta^6}{3!\,3!\,2^6} + \cdots$$

$$J_1(\beta) = \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\beta\right)^{1+2m}}{m!\,(1+m)!} = \frac{\beta}{2} - \frac{\beta^3}{1!\,2!\,2^3} + \frac{\beta^5}{2!\,3!\,2^5} - \cdots$$

$$J_2(\beta) = \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\beta\right)^{2+2m}}{m!\,(2+m)!} = \frac{\beta^2}{2!\,2^2} - \frac{\beta^4}{1!\,3!\,2^4} + \frac{\beta^6}{2!\,4!\,2^6} - \cdots$$

- Special Points:
- For small values of the **β**
  - **J0(β) = 1**
  - **J1(β) = β/2**
  - **Jn(β)** = 0 :for n>1

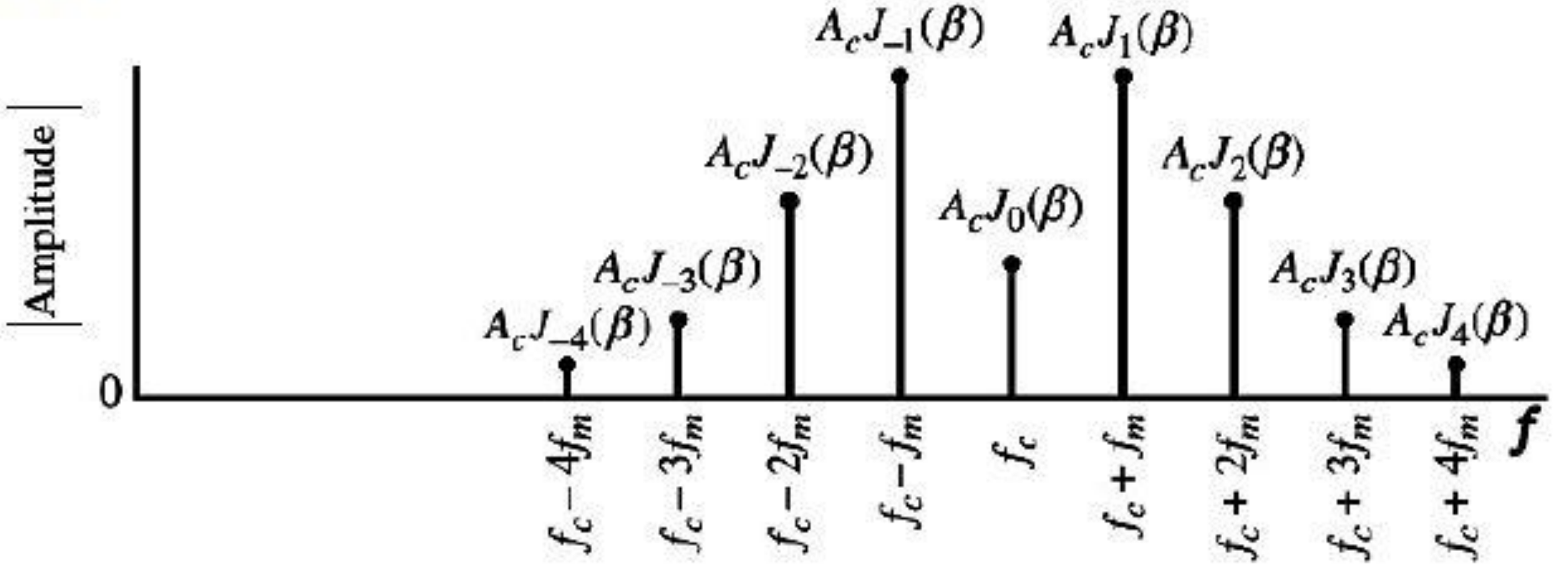
we have plotted the Bessel function $J_n(\beta)$ versus the modulation index β for different positive integer values of n. We can develop further insight into the behavior



Plots of Bessel functions of the first kind for varying order.

$$
\begin{aligned}
s(t) &= A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t \\
&= A_c J_0(\beta)\cos(\omega_c t) \\
&\quad + A_c J_1(\beta)\cos(\omega_c + \omega_m)t + A_c J_{-1}(\beta)\cos(\omega_c - \omega_m)t \\
&\quad + A_c J_2(\beta)\cos(\omega_c + 2\omega_m)t + A_c J_{-2}(\beta)\cos(\omega_c - 2\omega_m)t \\
&\quad + A_c J_3(\beta)\cos(\omega_c + 3\omega_m)t + A_c J_{-3}(\beta)\cos(\omega_c - 3\omega_m)t \\
&\quad + \cdots\cdots
\end{aligned}
$$

Spectra



$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$$

- When β=0 → $J_0(0) = 1$ and all other J_n are zero
  ⇒Only the carrier (no modulation, no sidebands)
- The spectrum consists of carrier-frequency component plus an infinite number of sideband components at frequencies $\omega_c \pm n\omega_m$ (n=1,2,3,...).
- The relative amplitudes of the spectral lines depend on the value of $J_n(\beta)$, and the value of $J_n(\beta)$ becomes very small for large values of n.
- The number of significant spectral lines (that is, having appreciable relative amplitude) is a function of the modulation index β.
  With β <<1, only J_0 and J_1 are significant, so the spectrum will consist of carrier and two sideband lines.
  But if β>>1, there will be many sideband lines.

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