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GENERAL SIR JOHN KOTELAWALA DEFENCE UNIVERSITY

Faculty of Engineering

Department of Electrical, Electronic and Telecommunication Engineering

BSc Engineering Degree

Semester 5 Examination Intake 36 (ET/MC) – April 2021

Semester 5 Repeat Examination Intake 35 (ET/MC)

ET3142– DIGITAL SIGNAL PROCESSING

Time allowed: 3 hours 19th April 2021

ADDITIONAL MATERIAL PROVIDED

Page 5 - 7 USEFUL FORMULAE

INSTRUCTIONS TO CANDIDATES

- This paper contains **five** questions and answer **all** the questions on answer booklets.
- This paper contains 7 pages with the cover page.
- This is a closed book examination.
- Those sitting for this examination will be allowed to use a non-programmable calculator.
- This examination accounts for 70% of the module assessment. A total maximum mark obtainable is 100. The marks assigned for each questions and parts thereof are indicated in square brackets.
- If you have any doubt as to the interpretation of the wordings of the question, make your own decision, but clearly state it on the script.
- Assume any reasonable values for any data not given in or provided with the question paper, clearly make such assumptions made in the script.
- All examinations are conducted under the rules and regulations of the KDU.

$x(n) = x_a(nT_s)$

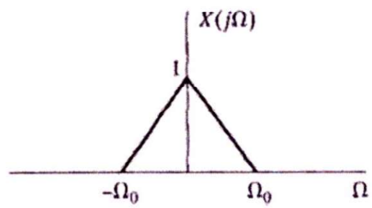
Question 01 **[20 marks]**

1) The continues-time signal

$$x_c(t) = \sin(2\pi(100)t)$$

Is sampled with sampling period $T = {}^{1}/_{400}$ second to obtain a discrete time signal $x[n]$. What is the resulting signal $x[n]$.

2) A signal has the Fourier transform given as following diagram. What is the minimum sampling rate such that no aliasing occurs in the sampled signal?



$\Omega_s = 2\Omega_0$

Question 02 **[20 marks]**

Design a low-pass Butterworth filter using the bilinear transformation method for satisfying the following constraints:

Passband: 0 - 400 Hz
Passband ripple: 2 dB A_p
Sampling frequency: 10 kHz → T
Stopband: 2.1 – 4 kHz
Stopband attenuation: 20 dB A_s

Question 03 **[20 marks]**

Given a sequence $x(n) = \{2,1,2,1,2,1,2,1\}$, determine the 8-point DFT of $X(k)$ by DIT FFT algorithm.

Question 04 **[20 marks]**

Design an ideal low-pass filter with a frequency response

$$H_d\left(e^{j\omega}\right)=\begin{cases} e^{j2\omega}, -\frac{\pi}{4}\leq\omega\leq\frac{\pi}{4} \\ 0, \quad \frac{\pi}{4}\leq|\omega|\leq\pi \end{cases}$$

with the window fuction

$$W(n)=\begin{cases} 1, & 0\leq n\leq 4 \\ 0, & otherwise \end{cases}$$

Question 05 **[20 marks]**

Determine the system function $H(z)$ of the lowest order Chebyshev IIR digital filter with the following specifications:

4 dB ripple in passband $0 \leq \omega \leq 0.4\pi$

30 dB attenuation in stopband $0.5\pi \leq \omega \leq \pi$

(Hint: $T = 1$ and bilinear transformation is used)

USEFUL FORMULAE

Fourier Transform Theorems

Sequence $x[n]$ $y[n]$	Fourier Transform $X(e^{j\omega})$ $Y(e^{j\omega})$
1. $ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
2. $x[n - n_d]$ (n_d an integer)	$e^{-j\omega n_d} X(e^{j\omega})$
3. $e^{j\omega_0 n} x[n]$	$X(e^{j(\omega-\omega_0)})$
4. $x[-n]$	$X(e^{-j\omega})$ $X^*(e^{j\omega})$ if $x[n]$ real.
5. $nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
6. $x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
7. $x[n]y[n]$	$\dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$

Discrete Fourier Transform

DFT of $x[n]$,

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\Omega_0 kn}, 0 \le k \le N-1, \Omega_0 = \frac{2\pi}{N}$$

DFT of ideal frequency response

$$H_d\left(e^{j\omega}\right) = \sum_{-\infty}^{\infty} h_d[n]e^{-j\omega n}$$

Inverse Discrete Fourier transform (IDFT) of ideal frequency response

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d\left(e^{j\omega}\right)e^{j\omega n}d\omega$$

Linearity:

$$\begin{cases} \alpha x[n] \longleftrightarrow \alpha X[k] \\ x_1[n] + x_2[n] \longleftrightarrow X_1[k] + X_2[k] \end{cases}$$

Time Shift:

$$x[((n-n_0))_N] \longleftrightarrow e^{-j\Omega_0 n_0 k} X[k]$$

Frequency Shift:

$$e^{jin} x[n] \longleftrightarrow X[((k-i))_N]$$

Convolution:

$$x_1[n] * x_2[n] = \sum_{m=0}^{N-1} x_1[((n-m))_N] x_2[m]$$

$$x_1[n] * x_2[n] \longleftrightarrow X_1[k] X_2[k]$$

Multiplication:

$$X_1[k] * X_2[k] = \sum_{i=0}^{N-1} X_1[((k-i))_N] X_2[i]$$

$$x_1[n] x_2[n] \longleftrightarrow \frac{1}{N} X_1[k] * X_2[k]$$

Time Differencing:

$$x[n] - x[((n-1))_N] \longleftrightarrow \left(1 - e^{-j\Omega_0 k}\right) X[k]$$

Accumulation:

$$\sum_{m=0}^{n} x[m] \longleftrightarrow \frac{1}{1 - e^{-j\Omega_0 k}} X[k] \quad \text{(only for } X[0] = 0\text{)}$$

IIR Filter – Properties

A_1 = Gain at a passband frequency ω_1

A_2 = Gain at a passband frequency ω_2

Ω_1 = Analog frequency corresponding to ω_1

Ω_2 = Analog frequency corresponding to ω_2

$\Omega_1 = \frac{2}{T}\tan\frac{\omega_1}{2}$, $\Omega_2 = \frac{2}{T}\tan\frac{\omega_2}{2}$ where T is the time.

Low-Pass Digital Butterworth

The order N of the filter,

$$N \geq \frac{1}{2} \frac{\log\left\{\left[\frac{1}{A_2^2} - 1\right] \Big/ \left[\frac{1}{A_1^2} - 1\right]\right\}}{\log\frac{\Omega_2}{\Omega_1}}$$

Cutoff Frequency, $\Omega_c = \dfrac{\Omega_1}{\left[\dfrac{1}{A_1^2}-1\right]^{-1/2N}}$

Transfer Function,

when the order N is even for unity dc gain filter, $H_a(s)=\prod_{k=1}^{N/2}\dfrac{\Omega_c^2}{s^2+b_k\Omega_c s+\Omega_c^2}$.

when the order N is odd for unity dc gain filter, $H_a(s)=\dfrac{\Omega_c}{s+\Omega_c}\prod_{k=1}^{N-1/2}\dfrac{\Omega_c^2}{s^2+b_k\Omega_c s+\Omega_c^2}$.

The coefficient of b_k is given by $b_k = 2\sin\left[\dfrac{(2k-1)\pi}{2N}\right]$

Chebyshev Filter

Attenuation Constant $\varepsilon = \left[\dfrac{1}{A_1^2}-1\right]^{\frac{1}{2}}$

The order N of the filter,

$$N \geq \left\{\frac{\cosh^{-1}\left\{\dfrac{1}{\varepsilon}\left[\dfrac{1}{A_2^2}-1\right]^{1/2}\right\}}{\cosh^{-1}\left\{\dfrac{\Omega_2}{\Omega_1}\right\}}\right.$$

Cutoff Frequency, $\Omega_c = \dfrac{\Omega_1}{\left[\dfrac{1}{A_1^2}-1\right]^{1/2N}}$

Transfer Function,

when the order N is even, $H_a(s)=\prod_{k=1}^{N/2}\dfrac{B_k\Omega_c^2}{s^2+b_k\Omega_c s+c_k\Omega_c^2}$.

when the order N is odd, $H_a(s)=\dfrac{B_0\Omega_c}{s+c_0\Omega_c}\prod_{k=1}^{N-1/2}\dfrac{B_k\Omega_c^2}{s^2+b_k\Omega_c s+c_k\Omega_c^2}$.

where

$$b_k = 2y_N\sin\left[\frac{(2k-1)\pi}{2N}\right]$$

$$c_k = y_N^2\cos^2\left[\frac{(2k-1)\pi}{2N}\right]$$

$$c_0 = y_N$$

$$y_N = \frac{1}{2}\left\{\left[\left(\frac{1}{\varepsilon^2}+1\right)^{\frac{1}{2}}+\frac{1}{\varepsilon}\right]^{\frac{1}{N}}-\left[\left(\frac{1}{\varepsilon^2}+1\right)^{\frac{1}{2}}+\frac{1}{\varepsilon}\right]^{\frac{-1}{N}}\right\}$$