Particular integral when $g(x) = x^m$, where m is a positive integer

In this case the tentative method is to expand $\dfrac{1}{F(D)}$ in a series of ascending powers of D.

$$\frac{1}{1+D} = 1 - D + D^2 - D^3 + D^4 - \ldots$$

$$\frac{1}{1-D} = 1 + D + D^2 + D^3 + D^4 + \ldots$$

Example (27)

Find the general solution of the differential equation:

$$(D^2 + 4)y = x^2$$

Example (28)

Find the general solution of the differential equation:

$$(D^2 - 4D + 3)y = x^3$$

Example (29)

Find the general solution of the differential equation:

$$D^2(D^2 + 4)y = 96x^2$$

Particular Integrals in other simple cases

We shall now give some typical examples of the evaluation of particular integrals in simple cases which have not been dealt with in the preceding sections.

Example (30)

Find the general solution of the differential equation:

$$(D^2 - 5D + 6)y = e^{2x}x^3$$

Example (31)

Find the general solution of the differential equation:

$$(D^2 - 6D + 13)y = 8e^{3x} \sin 3x$$

Additional Examples

1. $(D^4 - D^3 - 9D^2 - 11D - 4)y = 0$
2. $(D^3 + 4D)y = 0$
3. $(D^4 + D^3 + 2D^2 - D + 3)y = 0$
4. $(D^2 - 2D + 5)^2 y = 0$
5. $(D^2 - 3D + 2)y = e^{5x}$
6. $(D^3 - 5D^2 + 7D - 3)y = e^{2x} \cosh x$
7. $(D^2 + 4)y = x^3$
8. $(D^2 + 8D + 25)y = 50x$

4.5 Series Solutions

Series Solutions

Many differential equations can not be solved in terms of finite combinations of simple familiar functions.

Hence, we use the method of power series, that is we look for a solution of the form

$$y = f(x) = \sum_{n=0}^{\infty} c_n x^n$$

Example (32)

Solve the differential equation

$$y'' + y = 0$$

Example (33)

Solve the differential equation

$$y'' - 2xy' + y = 0$$

4.6 Solving ODEs by Laplace Transformation

Solving ODEs by Laplace Transformation

Recall:

Let $\mathcal{L}\{f(t)\} = F(s)$, then

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

and

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - f^{(n-1)}(0)$$

Example (34)

Solve

$$y'' + y' + y = 0$$

with $y(0) = 0$ and $y'(0) = 3$.

Example (35)

Solve

$$y'' + y' + y = 0$$

with $y(0) = 2$ and $y'(0) = 0$.

Example (36)

Solve

$$y'' + y' + y = 5$$

with $y(0) = 0$ and $y'(0) = 0$.

In Class Assignment

Solve

$$y'' + 2y' + 3y = \sin 5t$$

with $y(0) = 0$ and $y'(0) = 0$.