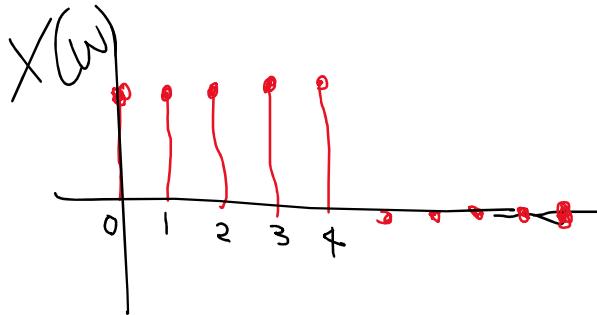


Example : DTFT



$$\begin{aligned}
 X(\omega) &= \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \\
 &= \sum_{n=0}^{5} e^{-j\omega n} \Rightarrow \frac{1 - e^{-5j\omega}}{1 - e^{-j\omega}} \Rightarrow \frac{e^{-5j\omega/2} \left[e^{5j\omega/2} - e^{-5j\omega/2} \right]}{e^{-j\omega/2} \left[e^{j\omega/2} - e^{-j\omega/2} \right]} \\
 &= e^{-2j\omega} \frac{2j \sin\left(\frac{5\omega}{2}\right)}{2j \sin\left(\frac{\omega}{2}\right)} \Rightarrow e^{-2j\omega} \frac{\sin\left(\frac{5\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}
 \end{aligned}$$

Let's take DFT of length of 10

$$\begin{aligned}
 X[k] &= \sum_{n=0}^{9} x[n] e^{-j\frac{2\pi}{10}kn} \\
 &= \sum_{n=0}^{4} e^{-j\frac{2\pi}{10}kn} = \sum_{n=0}^{4} W_{10}^{kn} \\
 &= \frac{1 - W_{10}^{5k}}{1 - W_{10}} \Rightarrow \frac{1 - e^{-j\frac{10\pi}{10}k}}{1 - e^{-j\frac{2\pi}{10}k}} \\
 &= \frac{\left[e^{-j\frac{\pi}{10}k} - e^{-j\frac{9\pi}{10}k} \right] \left[e^{-j\frac{\pi}{10}k} \right]}{\left[e^{-j\frac{\pi}{10}k} \right] \left[e^{j\frac{\pi}{10}k} - e^{-j\frac{\pi}{10}k} \right]} = e^{-j\frac{4\pi}{10}k} \frac{\sin\left(k\pi/2\right)}{\sin\left(\pi/2\right)}
 \end{aligned}$$

From DTFT to DFT :

$$\text{Let's take } \omega = \frac{2\pi k}{10} = e^{2j\frac{2\pi k}{10}} \frac{\sin\left(\frac{\pi k}{2}\right)}{\sin\left(\frac{\pi k}{10}\right)}$$

So what I wanted to show is when I sample DTFT I get DFT values.