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Mathematics - MA 1103
Tutorial 07 - Probability & Statistics

Year: 2021 Intake: 38 - 03rd Batch Semester: 01
Learning Outcomes Covered: LO4, LO5

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(1) The chance that a bit transmitted through a digital transmission channel is received in error is 0.1. Also, assume that the transmission trials are independent. Let X be the number of bits in error in the next four bits transmitted.

(a) Determine $P(X = 2)$.

(b) Find μ_X and σ_X.

(2) Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant.

(a) Find the probability that in the next 18 samples, exactly 2 contain the pollutant.

(b) Determine the probability that at least four samples contain the pollutant.

(c) Determine the probability that $3 \leq X < 7$.

(d) Find μ_X and σ_X.

(3) A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%.

(a) The inspector randomly picks 20 items from a shipment. What is the probability that there will be atleast one defective item among these 20?

(b) Suppose that the retailer receives 10 shipments in a month and the inspector randomly tests 20 devices per shipment. What is the probability that there will be exactly 3 shipments each containing at least one defective device among the 20 that are selected and tested from the shipment?

(4) (a) The probability that a wafer contains a large particle of contamination is 0.01. If it is assumed that the wafers are independent, what is the probability that exactly 125 wafers need to be analyzed when a large particle is detected?

(b) For a certain manufacturing process, it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found?

(c) At a busy time, a telephone exchange is very near capacity, so callers have difficulty placing their calls. It may be of interest to know the number of attempts necessary in order to make a connection. Suppose that we let $p = 0.05$ be the probability of a connection during a busy time. Find the probability that 5 attempts are necessary for a successful call.

(5) (a) For the case of the thin copper wire, suppose that the number of flaws follows a Poisson distribution with a mean of 2.3 flaws per millimeter.

i. Determine the probability of exactly two flaws in 1 millimeter of wire.

ii. Determine the probability of 10 flaws in 5 millimeters of wire.

iii. Determine the probability of at least one flaw in 2 millimeters of wire.

(b) During a laboratory experiment, the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is the probability that 6 particles enter the counter in a given millisecond?

(c) 10 is the average number of oil tankers arriving each day at a certain port. The facilities at the port can handle at most 15 tankers per day. What is the probability that on a given day tankers have to be turned away?

(6) (a) A random variable X has a continuous uniform distribution on $[4.9, 5.1]$. The probability density function of X is $f(x) = 5, 4.9 \leq x \geq 5.1$. What is the probability that a measurement of current is between 4.95 and 5.0 mA?

(b) Suppose that a large conference room at a certain company can be reserved for no more than 4 hours. Both long and short conferences occur quite often. In fact, it can be assumed that the length X of a conference has a uniform distribution on the interval [0, 4].

i. What is the probability density function?

ii. What is the probability that any given conference lasts at least 3 hours?

(7) Suppose that a system contains a certain type of component whose time, in years, to failure is given by T. The random variable T is modeled nicely by the exponential distribution with mean time to failure 5. If 5 of these components are installed in different systems, what is the probability that at least 2 are still functioning at the end of 8 years?

(8) The probability that a regularly scheduled flight departs on time is $P(D) = 0.83$; the probability that it arrives on time is $P(A) = 0.82$; and the probability that it departs and arrives on time is $P(D \cap A) = 0.78$. Find the probability that a plane

(a) arrives on time, given that it departed on time,

(b) departed on time, given that it has arrived on time

(9) (a) Suppose that we have a fuse box containing 20 fuses, of which 5 are defective. If 2 fuses are selected at random and removed from the box in succession without replacing the first, what is the probability that both fuses are defective?

(b) One bag contains 4 white balls and 3 black balls, and a second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?

(c) A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building, find the probability that both the ambulance and the fire engine will be available, assuming they operate independently.

(10) In a certain assembly plant, three machines, B_1, B_2, and B_3, make $30\%, 45\%$, and 25%, respectively, of the products. It is known from past experience that $2\%, 3\%$, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

(11) A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for $30\%, 20\%$, and 50% of the products, respectively. The defect rate is different for the three procedures as follows:

$$P(D|P_1) = 0.01, \quad P(D|P_2) = 0.03, \quad P(D|P_3) = 0.02$$

where $P(D|P_j)$ is the probability of a defective product, given plan j. If a random product was observed and found to be defective, which plan was most likely used and thus responsible?