Lecture 04
Bandpass Transmission and Modulation

M-ary (Multi level) Signalling Scheme

- BASK, BFSK, BPSK and DPSK are binary modulation techniques.
- They are represented using only two symbols consisting of single bit either '1' or '0'.
- Two amplitudes, two frequencies or two phases are used to represent two symbols respectively.
- The maximum bandwidth of BASK is $2f_b$, BFSK is $4f_b$, BPSK is $2f_b$ and DPSK is f_b

- To reduce the bandwidth requirement, two are more bits are combined to form one symbol.
- Thus the number of symbols is always greater than two, given by,

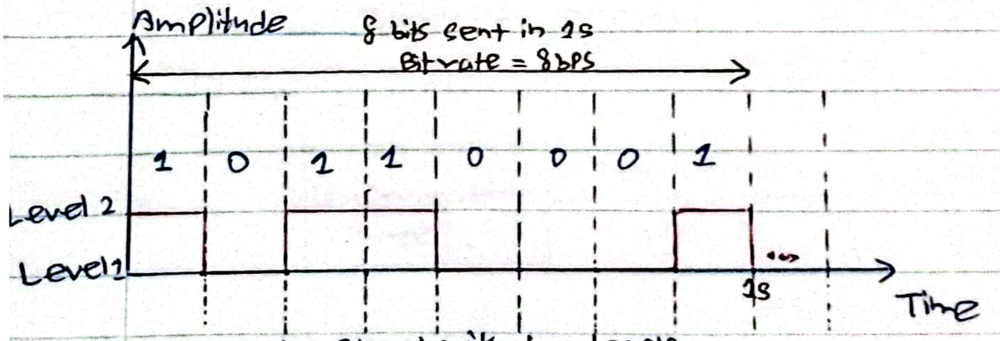
$$M = 2^k$$

where,

k = number of bits combined $(n = 2, 3, 4, \ldots)$

M = Total number of symbols or levels.

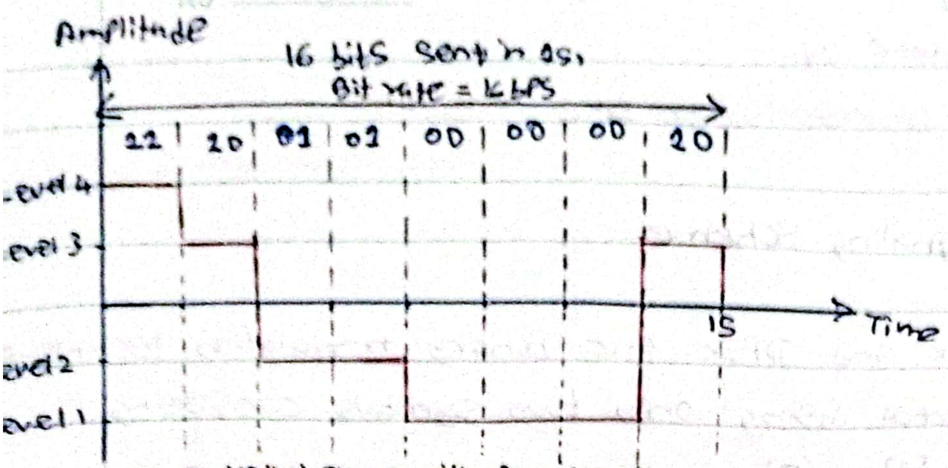
- The signal representation which uses the combination of two or more bits to represent a single symbol is known as M-ary (Multi-level) Signaling Scheme.



a. A digital signal with two levels

Binary: Levels = M = 2

Figure 1: Binary and M-ary Signal Representation



b. A digital signal with four levels

M-ary: Levels = M = 4

- M-level signal is used to modulate the carrier, then this technique is known as M-ary Digital Modulation Technique.
- In this, one of the M possible symbol is transmitted during each symbol duration, T_s, given by,

$$T_S = T_b \log_2 M = n T_b \qquad \text{where } T_b = \text{one bit duration}$$

- Based on the type of modulation used, they are classified as
  1. M-ary ASK
  2. M-ary FSK
  3. M-ary PSK
  4. M-ary Amplitude and phase shift keying (QAM)

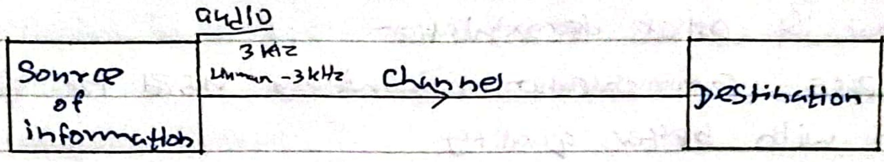
BaseBand Communication

BaseBand Signal:- [original message signals]

• Signals containing the original information to be transmitted baseband signals (or) modulating signals (or) information signals

"Base band" → designates the band of frequencies representing the signal.

→ low frequency signals



If the baseband signal is transmitted directly, then it is called baseband communication [Direct communication]

Drawbacks:

It is preferred at low frequencies

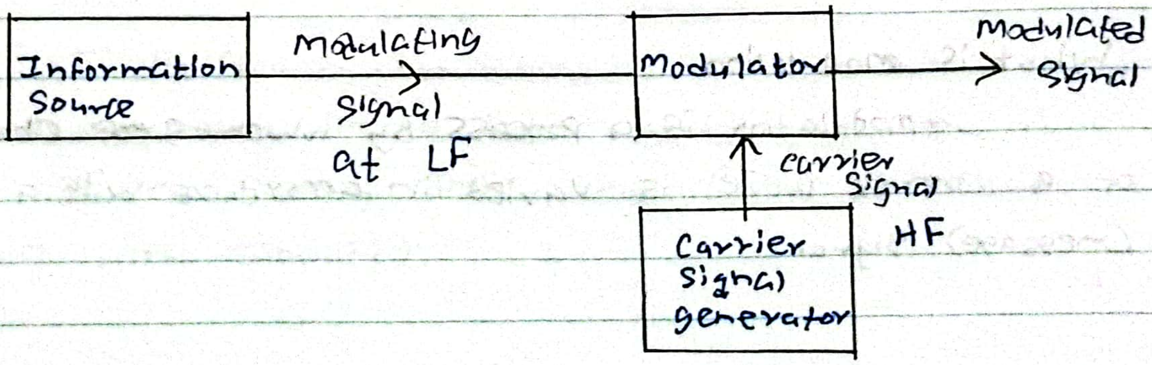
used for short distances

Interference occurs

Poor quality of reception

Band Pass Communication

→ In Band pass communication, the high frequency signal [carrier signal] is used to modulate the low frequency information signal.



The purpose of modulation is to convert the message into a suitable signal in order to achieve long distance communication.

modulated signal → Demodulator → Information signal

Receiver

- At the receiver side, the modulated signal is received.
- The process of extracting a Information modulating signal from the modulated signal is called demodulation.
- Band pass communication is widely used for long distance communication with better quality.

Baseband communication	Bandpass communication
No modulation & demodulation techniques are used	Modulation & demodulation processes are involved
Short distances	Long distance communication
LF	HF
Poor quality of reception	Better quality

Amplitude shift keying (ASK)
Phase shift keying (PSK)
Frequency shift keying (FSK)

$\Big\}$ Digital Modulation Techniques

What is modulation

- modulation is a process by which some characteristics of a carrier wave is varied in accordance with a modulating (message) signal

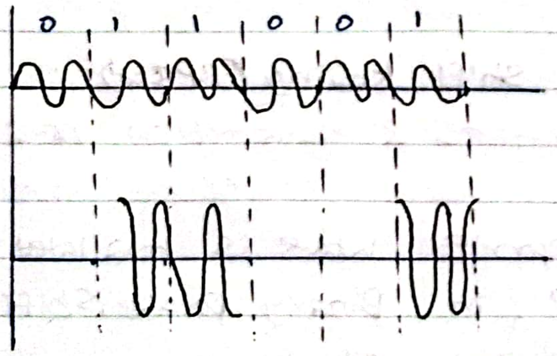
Digital modulation → It is a special kind of modulation, where the message signal (modulating signal) is of digital in nature (Binary or m-ary encoded version), if the carrier wave ~~is in the analog nature~~ to be modulated is of usually sinusoidal (analog) (having fixed frequency) nature

In digital modulation switching / keying of the amplitude, frequency or phase-off of the carrier wave is done

The ASK, PSK & FSK are ~~analogue~~ analogous to AM, PM, & FM respectively. The difference (ASK, PSK & FSK) the modulating signal is of digital nature while in AM, FM & PM the modulating signal is of analogue nature.

Amplitude shift keying (ASK)

In ASK, the amplitude of the carrier wave is changed (switched) according to the digital input signal (modulating signal) Therefore, ASK is analogous to AM (analog modulation)



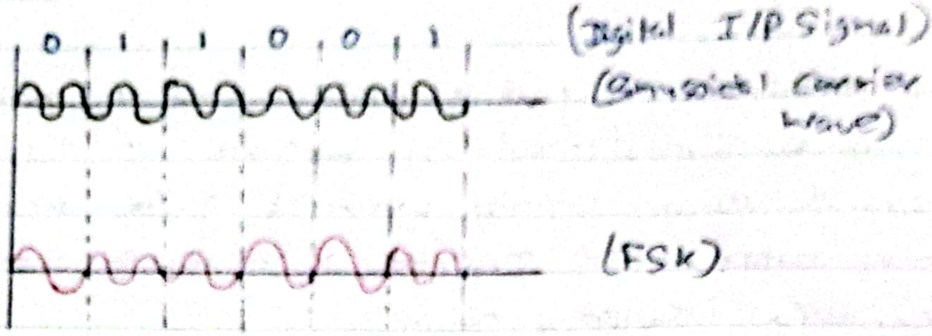
(Digital I/P signal)

(Sinusoidal carrier wave)

(ASK)

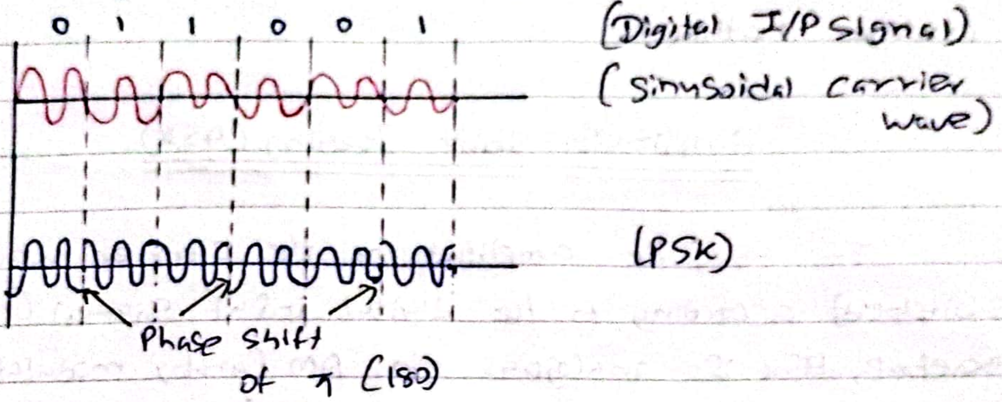
Frequency shift keying (FSK)

• If the frequency of sinusoidal carrier wave is varied (switched) depending on the digital input signal, then it is known as the Frequency shift keying (FSK). It is analogous to FM (Analog freq modulation)



Phase shift keying (PSK)

In phase shift keying, phase of the carrier wave (analog) is switched as per the input ~~signal~~ digital signal. This is analogous to PM (analog phase modulation)



Binary Phase shift keying (BPSK)

(Equations & Mathematical Analysis)

- The phase of the carrier wave is modulated by the binary symbols '1' & '0', in Binary phase shift keying (BPSK)
- If carrier is given as

$$s(t) = A\cos(2\pi f_c t)$$

- Here, A is peak amplitude of the sinusoidal carrier
- For standard 1Ω load resistor, power descipated is given as

$$\therefore P = \frac{V_{rms}^2}{R} = \left(\frac{V_{max}}{\sqrt{2}}\right)^2 / R = \left(\frac{A}{\sqrt{2}}\right)^2 / 1 = \frac{A^2}{2}$$

$$\therefore \frac{A^2}{2} = P \Rightarrow A = \sqrt{2P}$$

∵ We know that in PSK, phase of the carrier is changed by $\pi (180^\circ)$ when the symbol changes ('1' to '0' or '0' to '1')

Therefore for ex
If input changes from 1 to 0 $(1 \to 0)$
then if for symbol '1' we have

$\rightarrow S_1(t) = \sqrt{2P} \cos(2\pi f_c t)$ --- (i)

then for next symbol '0' we have

$\rightarrow S_2(t) = \sqrt{2P} \cos(2\pi f_c t + \pi)$ --- (ii)

or $S_2(t) = -\sqrt{2P} \cos(2\pi f_c t)$ --- (iii)

$[\because \cos(\pi + \theta) = -\cos(\theta)]$

Now using eq^{n}s (i) and (iii)
By combining eq^{n}s (i) & (iii) we can define BPSK signal as →

$$s(t) = b(t)\sqrt{2P} \cos(2\pi f_c t)$$

Here
$b(t) = +1$ when transmitting '1'

$= -1$ when transmitting '0' is to be transmitted.

Binary Frequency Shift Keying (BFSK)

• Frequency of the carrier signal is shifted according to the binary symbols (0s and 1s), in Binary Frequency Shift keying (BFSK). There is no change in the phase of the carrier.

• Since we have 2 binary symbols 0 & 1, so we have 2 different frequency signals.

• Let's represent these frequencies as high & low (differing by Ω)

If the carrier wave is given as

$$S(t) = A\cos(2\pi f_c t)$$

- Here A is peak volume of the sinusoidal carrier wave
- For a standard 1Ω load resistor the power dissipated would be

$$\left[\because P = \frac{V_{rms}^2}{R} = \left(\frac{V_{max}}{\sqrt{2}}\right)^2\Big/1 = \left(\frac{A}{\sqrt{2}}\right)^2 = \frac{A^2}{2}\right]$$

$$\therefore P = \frac{A^2}{2} \Rightarrow A = \sqrt{2P}$$

eq^n (i) $S_H(t) = \sqrt{2P_S}\cos(2\pi f_c + \Omega)t$; $b(t) = '1'$

eq^n (ii) $S_L(t) = \sqrt{2P_S}\cos(2\pi f_c - \Omega)t$; $b(t) = '0'$

[Increment & Decrement in frequency by Ω]

By combining eq^n (i) & (ii) we get

$$\boxed{S(t) = \sqrt{2P_S}\cos[(2\pi f_c + d(t)\,\Omega)t]}$$

$[\because \omega = 2\pi f]$

$\therefore f_H \rightarrow f_c + \frac{\Omega}{2\pi}$; For transmitting symbol '1'

$f_L \rightarrow f_c - \frac{\Omega}{2\pi}$; for transmitting symbol '0'.

DPSK (Differential phase shift keying)

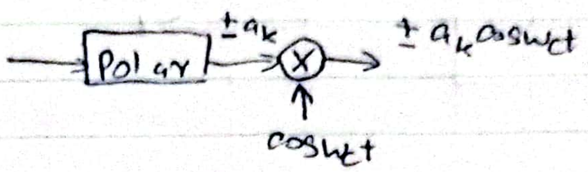
Outlines

- Basics of DPSK
- DPSK transmitter
- DPSK waveform generation
- DPSK Receiver
- Advantages of DPSK
- Disadvantages of DPSK

Basics of DPSK

- It is not possible to have non coherent detection of PSK
- to detect non-coherent detection of phase we use DPSK
- It reduces cost of circuit.

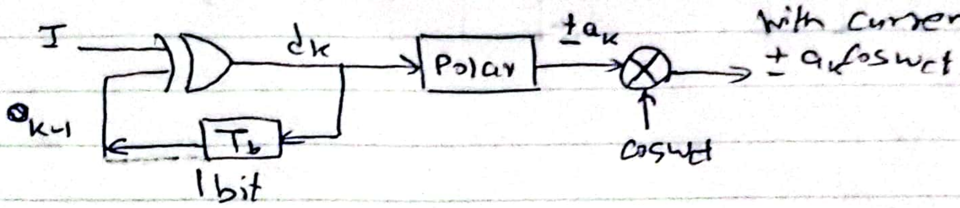
DPSK transmitter



PSK

In DPSK

XORed with 1 bit delayed output that will give phase change with current and previous bit



XOR will be applied the modulo-2 addition

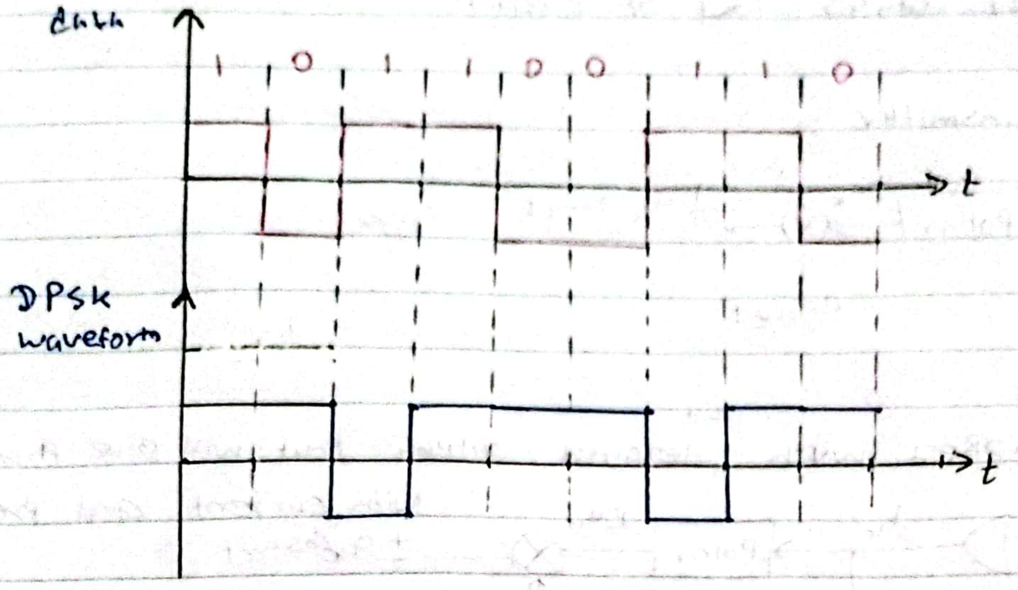
then $d_k = \theta_k - \theta_{k-1}$

θ_k	θ_{k-1}	d_k
0	0	0
0	1	1
1	0	1
1	1	0

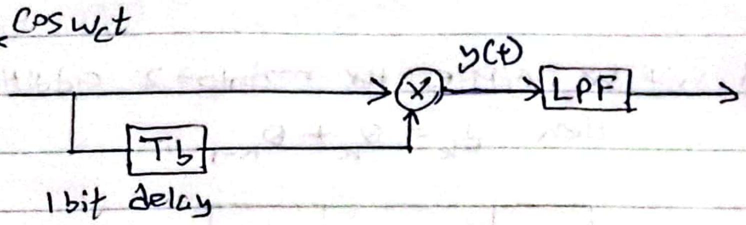
- This difference will give a phase in between current and previous bit. When we give it to polar signal. It will ~~g~~ be generating $\pm a_k$ amplitude when you multiply with high carrier frequency signal. It will be generating $\pm a_k \cos w_c t$

DPSK waveforms

- If next data is 1, then change polarity of O/P
- If next data is 0, then do not change polarity of



DPSK Receiver



$$z(t) = \frac{a_k^2}{2} \rightarrow \text{bit } 1$$
$$= -\frac{a_k^2}{2} \rightarrow \text{bit } 0$$

- Case-I [Same polarity of input & delayed signal]

$$y(t) = (a_k \cos w_c t)(a_k \cos w_c t) = a_k^2 \cos^2 w_c t$$
$$= \frac{a_k^2}{2}[1 + \cos 2w_c t] = \frac{a_k^2}{2} + \frac{a_k^2}{2}\cos 2w_c t \xrightarrow{LPF} = \frac{a_k^2}{2}$$

- Case-II [Opposite polarity of Input & delayed signal]

$$y(t) = (a_k \cos w_c t)(-a_k \cos w_c t) = -a_k^2 \cos^2 w_c t$$
$$= -\frac{a_k^2}{2} - \frac{a_k^2}{2}\cos 2w_c t$$
$$\xrightarrow{LPF} = -\frac{a_k^2}{2}$$

Advantages of DPSK

- Non coherent detection is possible
- Cost is less
- Circuit complexity is less

Drawbacks of DPSK

- Noise

Signal Energy per Bit

* Sinusoidal carrier:

$$y(t) = A_c \cos(2\pi f_c t)$$

amplitude

* Power of a sinusoid:

$$\frac{A_c^2}{2} \text{ - OR - } (A_{c\,RMS})^2 = \left(\frac{A_c}{\sqrt{2}}\right)^2 = \frac{A_c^2}{2}$$

* Energy over ~~a~~ one bit interval T_b:

unit energy = power × time = $\frac{A_c^2}{2} \times T_b = 1$

$$\Rightarrow \boxed{A_c = \sqrt{\frac{2}{T_b}}}$$

Why do this?

"Signal strength" expressed as energy per bit E_b

Binary phase shift keying (coherent BPSK)

- In BPSK, phase of the sinusoidal carrier is changed according to the data bit to be transmitted.
- Also, a bipolar NRZ signal is used to represent the digital data coming from the digital source.
- PSK is now widely used in both military and commercial communications systems.

Expression for BPSK

- In a binary phase shift keying (BPSK), the binary symbols '1' and '0' modulate the phase of the carrier.
- Let us assume that the carrier is given as,

$$s(t) = A\cos(\omega_c t) = A\cos(2\pi f_c t)$$

- Here 'A' represent peak value of sinusoidal carrier.
- For the standard $1\,\Omega$ load resistor, the power dissipated would be,

$$P = \frac{A^2}{2} \Longrightarrow A = \sqrt{2P}$$

~~Let us consider, for example~~

- Now, when the symbol is changed, then the phase of the carrier will also be changed by an amount of 180 degrees (i.e, π radians)
- Let us consider, for example,
  - For symbol '1', we have $s_1(t) = \sqrt{2P}\cos(2\pi f_c t)$ — ①
  - For symbol '0', we have $s_2(t) = \sqrt{2P}\cos(2\pi f_c t + \pi)$

- Now, because $\cos(\pi + \theta) = -\cos(\theta)$, therefore, the last equation can be written as,

$$s_2(t) = -A\cos(\omega_c t) = -\sqrt{2P}\cos(2\pi f_c t) \quad \text{—②}$$

- With the above equations ① & ②, we can define BPSK signal combinely as,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_c t)$$

we have, $P = \frac{E_b}{T_b}$

E_b - bit energy signal

T_b - bit duration

$$\text{Where, } b(t) = \begin{cases} +1 & \text{when binary '1' is to be transmitted} \\ -1 & \text{when binary '0' is to be transmitted.} \end{cases}$$

Table. shows input digital and corresponding bipolar NRZ signal

S.No	Input digital signal	Bipolar NRZ signal b(t)	BPSK output signal
1	Binary 0	$b(t) = -1$	$-\sqrt{2P}\cos\omega_c t$
2	Binary 1	$b(t) = +1$	$+\sqrt{2P}\cos\omega_c t$

Signal constellation of Coherent Binary PSK consists of two message points

Geometrical Representation for BPSK signals

- We know that BPSK signal is expressed as,

$$S(t) = b(t)\sqrt{2P}\cos(2\pi f_c t) = b(t)\sqrt{2\frac{E_b}{T_b}}\cos(2\pi f_c t)$$

- Let us rearrange the last equation as,

$$S(t) = b(t)\sqrt{E_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) = b(t)\sqrt{E_b}\,\phi_1(t)$$

$$\Longrightarrow S(t) = \pm\sqrt{E_b}\,\phi_1(t) \quad \text{—(1)}$$

- Where, $\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)$; which is orthogonal carrier signal or basis function.

- Here, $b(t)$ is simply ± 1. Thus, on the single axis of $\phi_1(t)$, there will be two points.

- From figure, it is obvious that the distance between the two points is, $d = \sqrt{E_b} - (-\sqrt{E_b}) = 2\sqrt{E_b}$

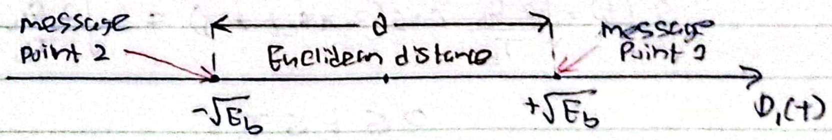
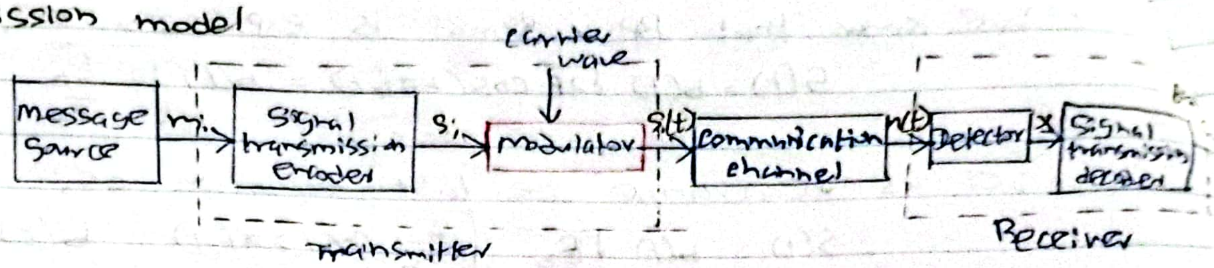


Fig. Geometrical representation of BPSK signal

Passband Transmission model

Transmission model



Modulator generates distinct signals $s_i(t)$, $i = 1, \dots M$ of duration T seconds

Channel in bandpass transmission model

1. Linear with bandwidth that accommodates modulated signal $s_i(t)$ with negligible distortion.

2. Channel noise, $w(t)$, is AWGN with PSD of $N_0/2$

Binary Phase-shift keying (BPSK): The signals representing binary symbols 1 and 0 are

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

$$0 \le t \le T_b$$

- E_b: transmitted signal energy per bit
- $f_c = n_c/T_b$ n_c is an integer (integer number of cycles during T_b)
- $s_1(t)$ and $s_2(t)$ are called antipodal signals

To show that $\phi_1(t)$ has a unit energy

$$E_1 = \int_{-\infty}^{\infty} \phi_1^2(t)\,dt = \frac{2}{T_b}\int_0^{T_b} \cos^2(2\pi f_c t)\,dt$$

$$= \frac{2}{T_b}\int_0^{T_b} \frac{1}{2}\,dt + \frac{2}{T_b}\int_0^{T_B} \frac{\cos(4\pi f_c t)}{2}\,dt$$

$$= 1 + \frac{1}{T_b}\int_0^{T_b} \cos(4\pi f_c t)\,dt = 1$$

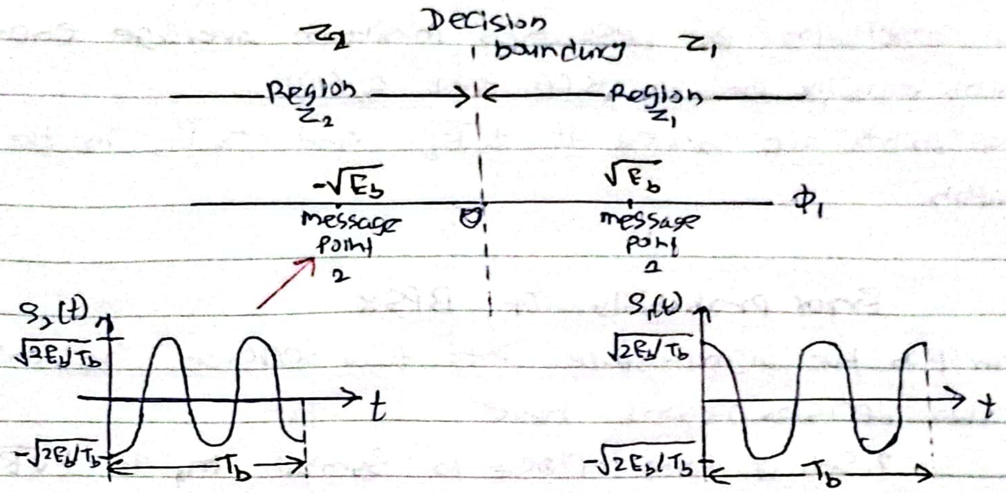
where $f_c = n_c/T_b$ $\sin(4\pi f_c T_b) = \sin(4\pi n_c) = 0$

Signal constellation, T_x and R_x for coherent PSK

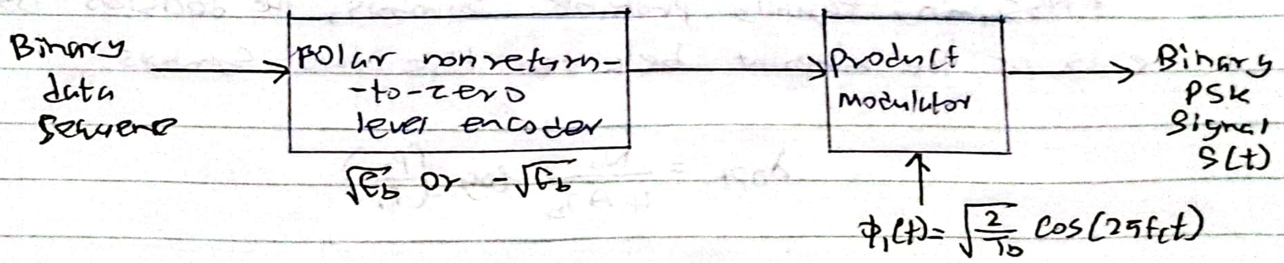
The Signals $S_1(t)$ and $S_2(t)$ in terms of $\phi_1(t)$

$$S_1(t) = \sqrt{E_b}\,\phi_1(t), \quad 0 \le t \le T_b$$
$$S_2(t) = -\sqrt{E_b}\,\phi_1(t), \quad 0 \le t \le T_b$$

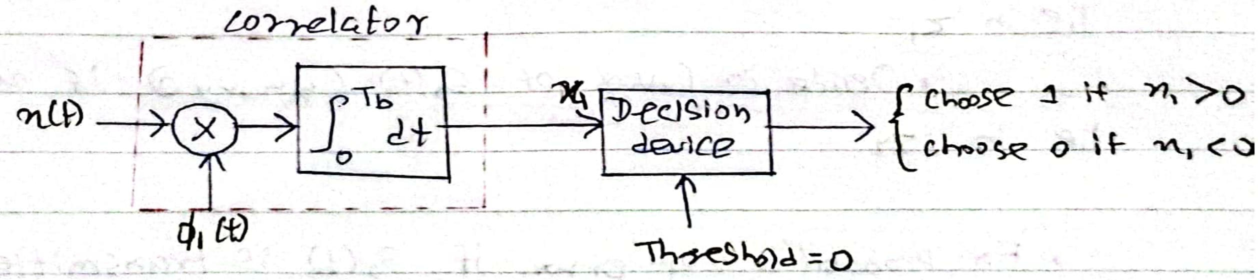
Signal constellation



(a) binary PSK transmitter



(b) Coherent binary PSK receiver



- Signal-space is one dimensional ($N=1$), and number messages $M=2$

$$s_{11} = \int_0^{T_b} s_1(t)\,\phi_1(t)\,dt = +\sqrt{E_b}$$

$$s_{21} = \int_0^{T_b} s_2(t)\,\phi_1(t)\,dt = -\sqrt{E_b}$$

- Signal constellation for BPSK has minimum average energy (assuming equally probably $s_1(t)$ and $s_2(t)$)
- Signal points are located at $+\sqrt{E_b}$ and $-\sqrt{E_b}$ in the signal constellation

Error probability of BPSK

- Partition the signal-space into two decision regions (z_1 and z_2)
- Two decision regions have
  1. Set of points close to symbol m_1 at $+\sqrt{E_b}$
  2. Set of points close to symbol m_2 at $-\sqrt{E_b}$

- Assuming equally probable symbols, the decision boundary line is at the midpoint between the two symbols.

$$\boxed{\lambda_{opt} = \frac{N_0}{4AT_b}\log\left(\frac{P_0}{P_1}\right)}$$

Decision rule:

- Decide in favor of $s_1(t)$ (binary 1) if received signal lie in z_1
- Decide in favor of $s_2(t)$ (binary 0) if received signal lie in z_2

- For probability of error if $s_2(t)$ is transmitted, consider the region

$$z_1 : 0 < x_1 < \infty$$

- where

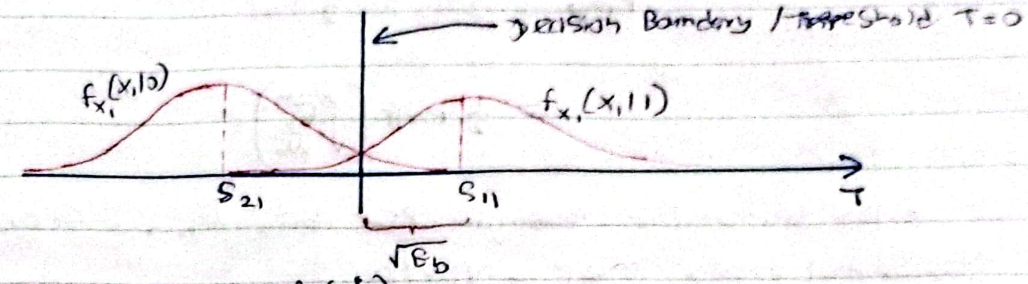
$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt$$

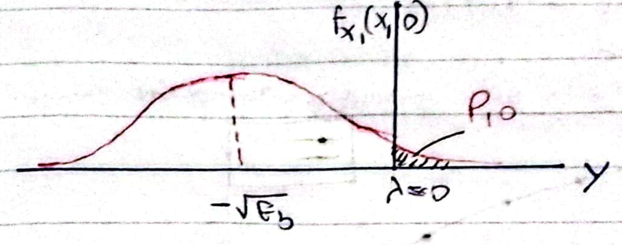
CamScanner

Conditional probability density function of x_1 given $S_2(0)$ is sent

$$f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - s_{21})^2\right]$$

$$= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right]$$





Conditional probability of error given symbol 0 is sent,

$$P_{10} = \int_0^\infty f_{x_1}(x_1|0)\, dx_1$$

$$= \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1$$

Let $z = \frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b})$, then

$$P_{10} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp[-z^2]\, dz = \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]$$

Where $\mathrm{erfc}(x)$ is the complementary error function

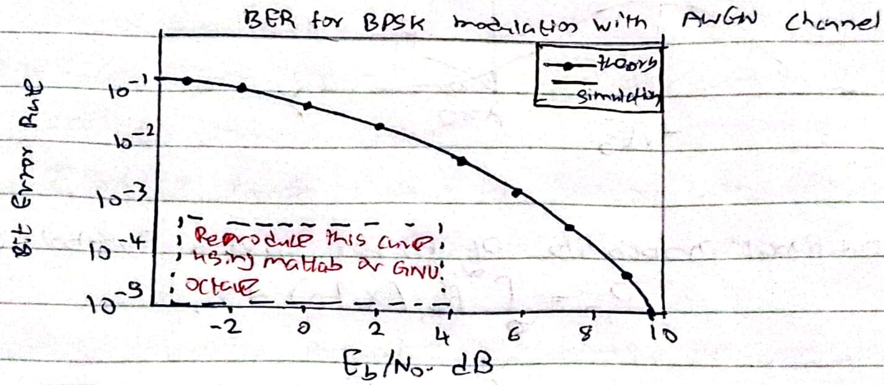
- Similarly, conditional error probability if $s_1(t)$ is transmi (binary 1) can be evaluated
  - Due to symmetry with respect to origin, $P_{01} = P_{10}$
  - Average symbol error probability (or bit error rate BER) is

$$P_e = \frac{1}{2}P_{10} + \frac{1}{2}P_{01}$$

$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

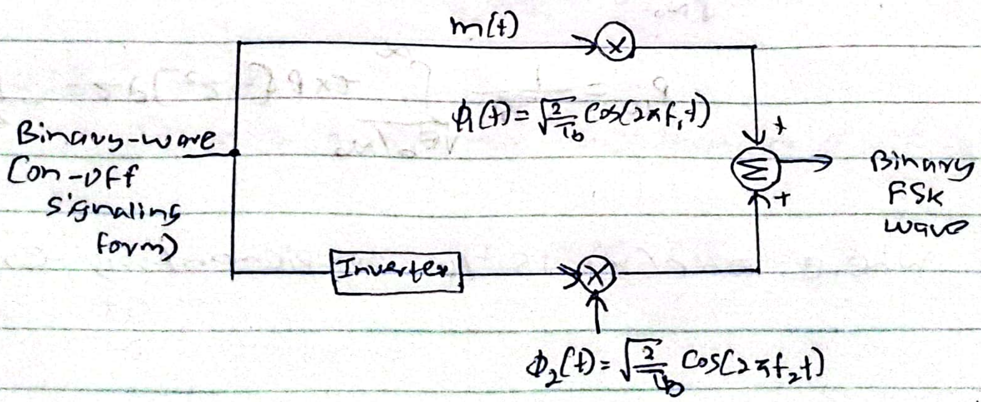
- As transmitted energy per bit, E_b, increases for specific N_0, signal points move apart.

P_e is reduced.



Coherent Binary FSK

Transmitter



Binary FSK transmitter

- The input binary sequence is represented in its on-off form
- Symbol 1 → constant amplitude of $\sqrt{E_b}$ volts
- Symbol 0 → zero volts
- By using an inverter in the lower channel, we make sure that when we have symbol 1 at the input, the oscillator with f_1 frequency in the upper channel is switched on while the oscillator with frequency f_2 in the lower channel is switched off, with the result that frequency f_1 is transmitted.

- Conversely, when we have symbol 0 at the input, the oscillator in the upper channel is switched off, and the oscillator in the lower channel is switched on, with the result that frequency f_2 is transmitted

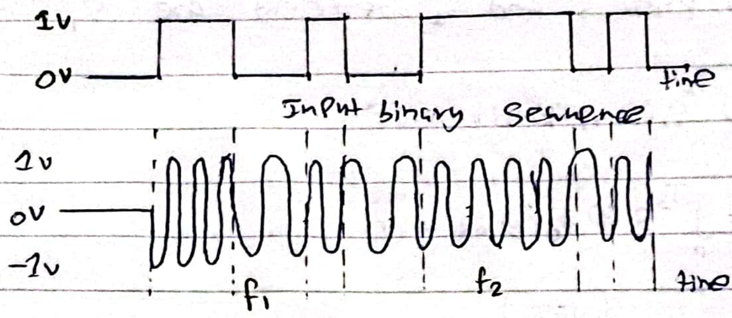
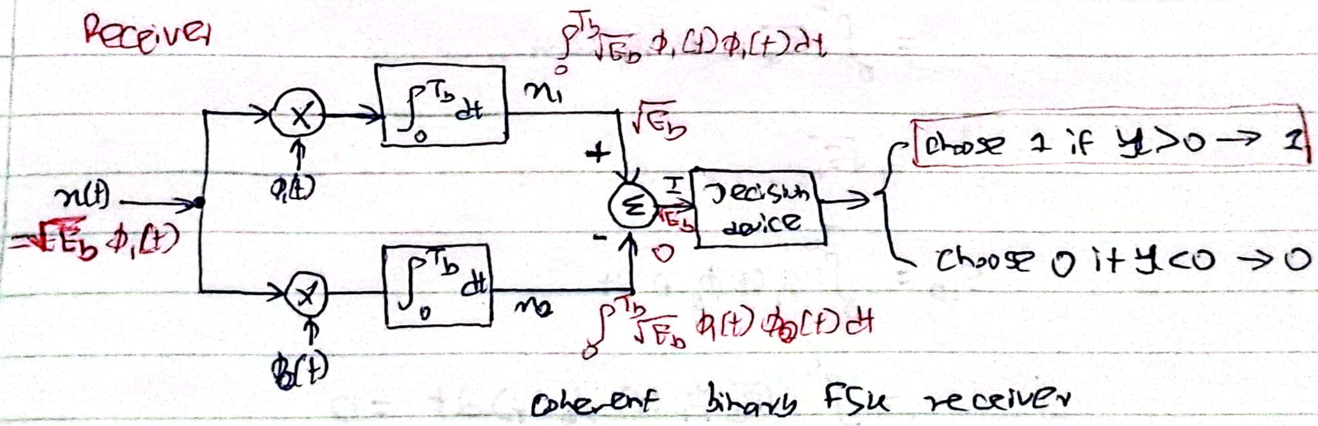


Figure 2 ~~figure 2~~: Coherent binary FSK waveform

Receiver

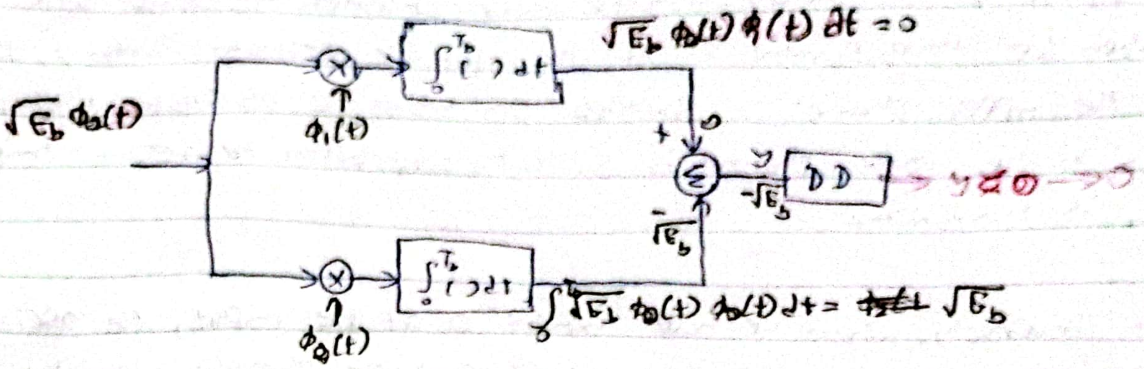


Coherent binary FSK receiver

Sent $\sqrt{E_b}$ means, I have sent H
when it indicates H it gives $\sqrt{E_b}$

$$\int_0^{T_b} \sqrt{E_b}\,\phi_1(t)\,\phi_1(t)\,dt = \sqrt{E_b}$$

$$\int_0^{T_b} \sqrt{E_b}\,\phi_1(t)\,\phi_2(t)\,dt = 0$$



Signal space diagram :-

Two msg point S_1 and S_2 coefficient are S_{11}, and S_{12} are computed.

$$S_{11} = \int_0^{T_b} S_1(t)\,\phi_1(t)\,dt = \sqrt{E_b}$$

$$S_{12} = \int_0^{T_b} S_1(t)\,\phi_2(t)\,dt = 0$$

$$S_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}$$

$$S_{11} = \int_0^{T_b} S_1(t)\,\phi_1(t)\,dt$$

$$= \int_0^{T_b} \sqrt{E_b}\,\phi_1(t)\,\phi_1(t)\,dt$$

$$= \sqrt{E_b}$$

$$S_{12} = \int_0^{T_b} S_1(t)\,\phi_2(t)\,dt$$

$$= \int_0^{T_b} \sqrt{E_b}\,\phi_1(t)\,\phi_2(t)\,dt = 0$$

$$S_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}$$

$$S_{01} = \int_0^{T_b} s_0(t)\,\phi_1(t)\,dt$$

$$= \int_0^{T_b} \sqrt{E_b}\,\phi_0(t)\,\phi_1(t)\,dt$$

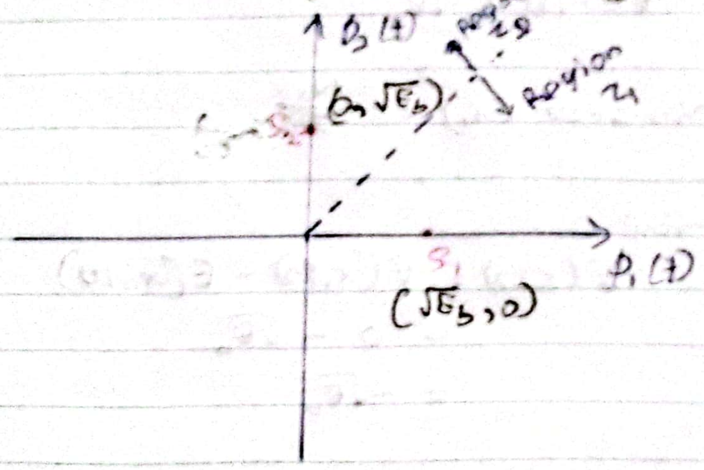
$$= 0$$

$$S_{00} = \int_0^{T_b} s_0(t)\,\phi_0(t)\,dt$$

$$= \int_0^{T_b} \sqrt{E_b}\,\phi_0(t)\,\phi_0(t)\,dt$$

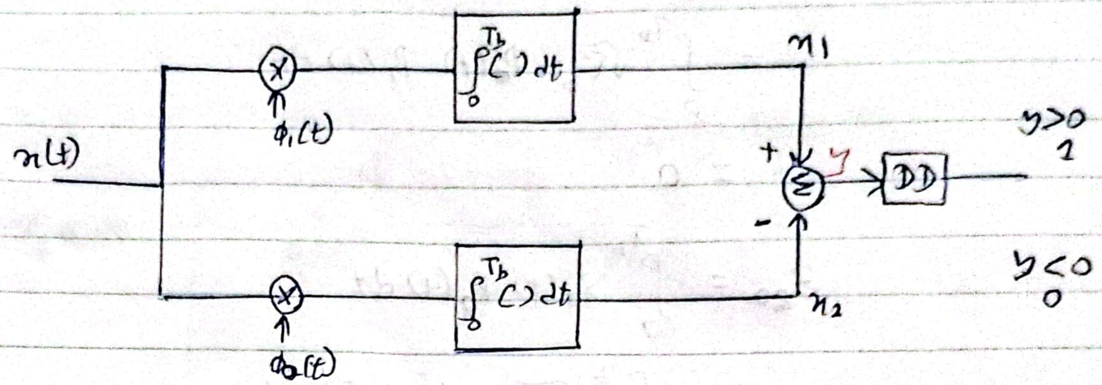
$$= \sqrt{E_b}$$

$$s_0 = \begin{bmatrix} S_{01} \\ S_{00} \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$



- Anything in Region z_1 is symbol "1"
- Anything in Region z_0 is symbol "0"
- on decision boundary → random guess in favor of symbol "0" or "1"

Probablity of error of FSK.



$$y = x_1 - x_2$$

Symbol 1

$$E(Y|1) = E(X_1|1) - E(X_2|1)$$
$$= \sqrt{E_b} - 0$$
$$E(Y|1) = \sqrt{E_b}$$

$$\text{Variance } (x_1) = \frac{N_0}{2}$$

Symbol 0

$$E(Y|0) = E(X_1|0) - E(X_2|0)$$
$$= 0 - \sqrt{E_b}$$
$$= -\sqrt{E_b}$$

$$\text{variance } (x_2) = \frac{N_0}{2}$$

$$\text{Variance of } (Y) = \text{Variance of } (X_1) + \text{variance of } (X_2)$$
$$= \frac{N_0}{2} + \frac{N_0}{2}$$
$$= N_0$$

Suppose symbol 0 was sent

$$f_Y(y|0) = \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{(y+\sqrt{E_b})^2}{2N_0}\right]$$

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$\eta_p > \eta_0$ ~~⅔~~ $y > 0$; corresponds to the carrier making a decision in favour of symbol 1 conditional probability of error given that symbol 0 was sent,

$$P_{10} = P(y > 0 \mid \text{symbol } 0 \text{ was sent})$$

$$= \int_0^\infty f_y(y|0)\,dy$$

$$= \frac{1}{\sqrt{2\pi N_0}} \int_0^\infty \exp\left[-\frac{(y + \sqrt{E_b})^2}{2N_0}\right] dy$$

$$\frac{y + \sqrt{E_b}}{\sqrt{N_0}} = z$$

$$y = z\sqrt{N_0} - E_b$$

$$dy = \sqrt{N_0}\,dz$$

$y \to \infty,$ $\quad z \to \infty$

$y \Rightarrow 0$ $\quad z = \sqrt{\frac{E_b}{N_0}}$

$$P_{10} = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp\left(-\frac{z^2}{2}\right) dz$$

$$P_{10} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_e = P_{10} P_0 + P_{01} P_1$$

$$= \frac{1}{2} Q\left(\sqrt{\frac{E_b}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$\boxed{P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

Power spectral density of FSK

$$x(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t \pm \frac{\pi t}{T_b}\right)$$