

$$\sin\theta = \frac{\sin\theta}{\theta}$$

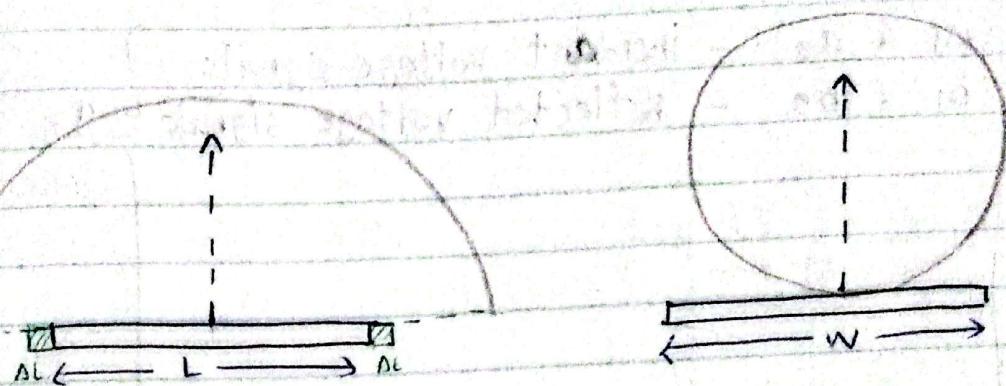
$$Z_{IN} = \frac{90}{\epsilon_r - 1} \left(\frac{\epsilon_r L}{W} \right)^2$$

$$Z_{IN} = \frac{90}{4.2 - 1} \left(\frac{4.2 \times 5.9}{8.6} \right)^2$$

$$= 28.125 \times 8.29$$

$$= 233.233.15 \Omega$$

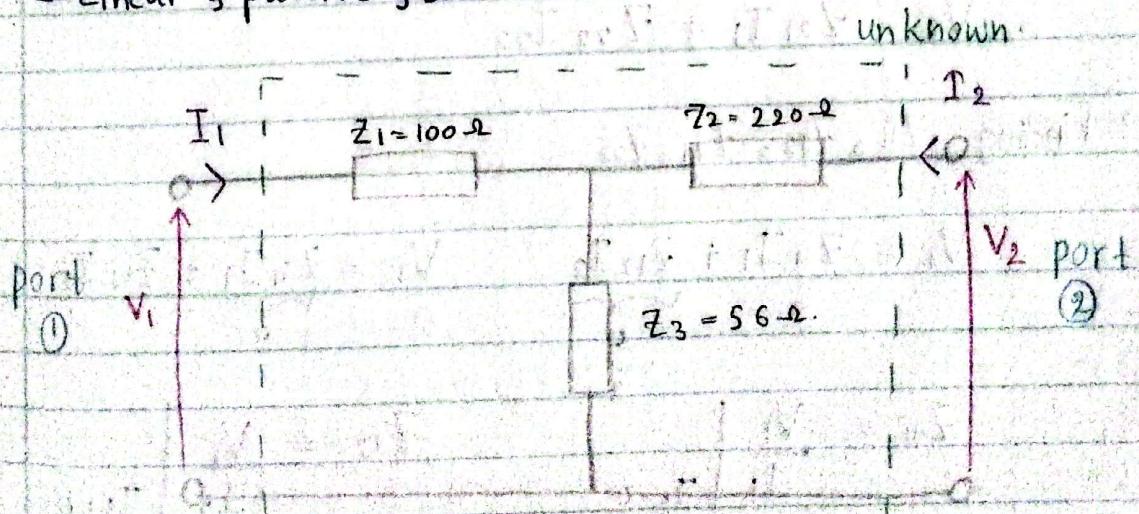
radiation pattern



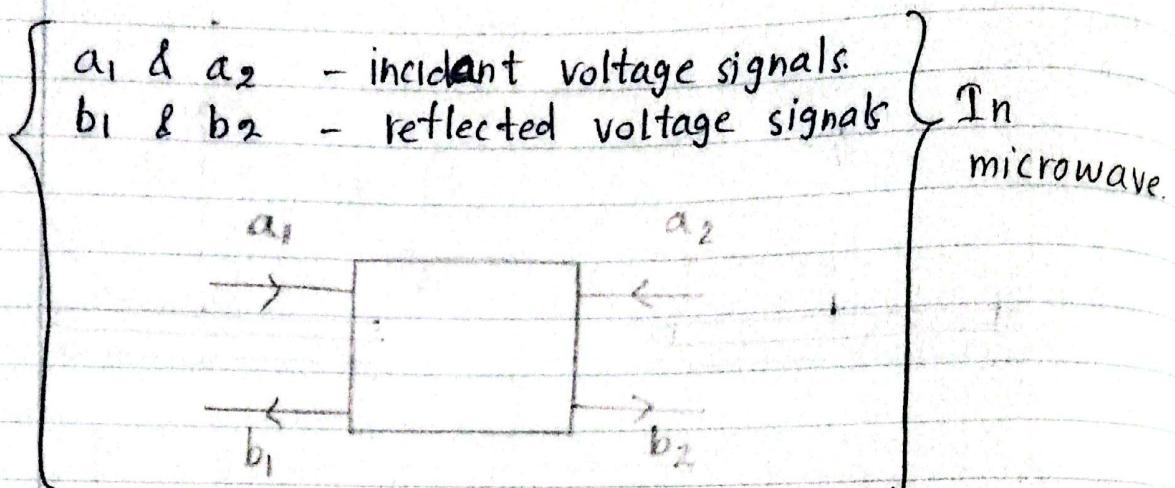
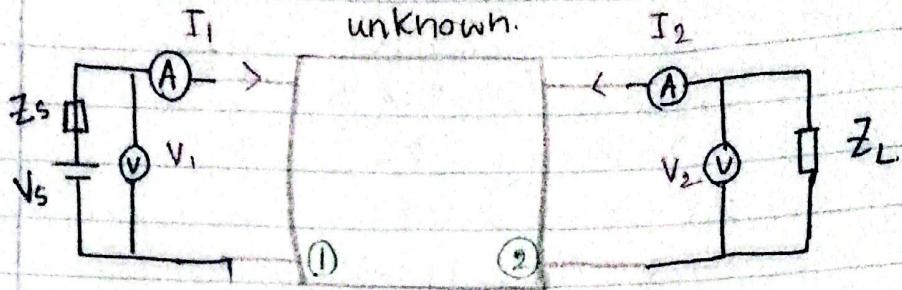
Two - Port Networks.

Method of analysis and representation of a circuit.

- Linear, passive, bilateral.



No. Date: ...
 V_1, V_2 & I_1, I_2 measurable, 4 variables.



Two port network description.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

→ Finding $Z_{11}, Z_{12}, Z_{21}, Z_{22}$.

$$V_1 = Z_{11} I_1 + \underbrace{Z_{12} I_2}_0$$

$$V_1 = \underbrace{Z_{11} I_1}_0 + Z_{12} I_2$$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

$$= 156 \Omega$$

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

$$= 56 \Omega$$

$$V_2 = \underbrace{Z_{21} I_1}_0 + Z_{22} I_2 \quad V_1 = Z_{21} I_1 + \underbrace{Z_{22} I_2}_0$$

$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

$$= 276 \Omega$$

$$= 5G \rightarrow$$

$$\cancel{\frac{V_1}{I_1}} = Z_{11} I_1 + Z_{12} I_2$$

$$= Z_{11} + Z_{12} \frac{I_2}{I_1}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\frac{V_2}{I_1} = Z_{21} + Z_{22} \frac{I_2}{I_1}$$

$$Z_{12} A_I = \frac{V_1}{I_1} - Z_{11}$$

$$Z_{22} A_I = \frac{V_2}{I_1} - Z_{21}$$

$$-\frac{Z_{12} Z_{21}}{Z_{22} + Z_L} = \frac{V_1}{I_1} - Z_{11}$$

$$V_2 = Z_L (-I_2)$$

$$V_1 = Z_{11} I_1 - \left(\frac{Z_{12} Z_{21}}{Z_{22} + Z_L} \right) I_1 \quad Z_{22} \frac{I_2}{I_1} = -\frac{Z_L I_2}{I_1} - Z_{21}$$

$$A_V = \frac{V_2}{V_1}$$

$$= \frac{Z_L (-I_2)}{Z_{11} I_1 - \left(\frac{Z_{12} Z_{21}}{Z_{22} + Z_L} \right) I_1}$$

$$\frac{I_2}{I_1} (Z_{22} + Z_L) = -Z_{21}$$

$$A_I = -\frac{Z_{21}}{Z_{22} + Z_L}$$

$$= -\frac{Z_L A_I}{Z_{11} + Z_{12} A_I}$$

$$A_I = \frac{-5G}{276 + 1000}$$

$$= -\frac{5G}{1276}$$

$$= -0.044$$

$$= -\frac{Z_L A_I}{Z_{11} + Z_{12} A_I}$$

$$= \frac{1 + Z_L Z_{21}}{Z_{11} (Z_{22} + Z_L) - Z_{12} Z_L}$$

$$\frac{V_2}{V_1} = \frac{Z_{21} I_1 + Z_{22} I_2}{Z_{11} I_1 + Z_{12} I_2}$$

$$= \frac{Z_{21} + Z_{22} A_I}{Z_{11} + Z_{12} A_I}$$

$$A_V = \frac{Z_{22} Z_L}{Z_{11} Z_{22} - Z_{12} Z_{21} + Z_{11} Z_L}$$

$$A_V = 0.281$$

$$A_S = \frac{V_2}{V_S}$$

$$V_S = Z_S I_1 + V_1$$

$$V_S - Z_S I_1 = V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_S = (Z_{11} + Z_S) I_1 + Z_{12} I_2$$

$$A_S = \frac{V_2}{V_S} \\ = \frac{Z_{21} I_1 + Z_{22} I_2}{(Z_{11} + Z_S) I_1 + Z_{12} I_2}$$

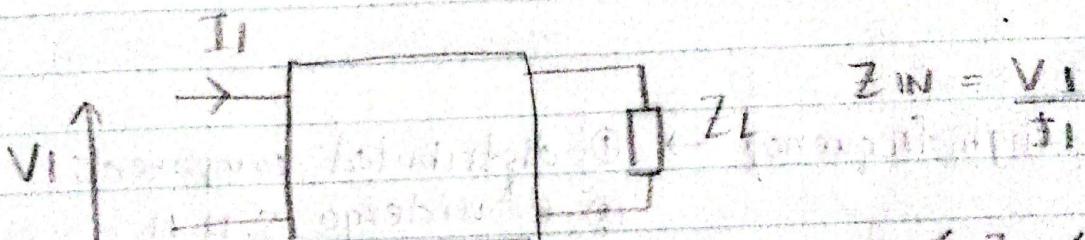
$$= \frac{Z_{21} + Z_{22} A_I}{(Z_{11} + Z_S) + Z_{12} A_I} \\ = \frac{Z_{21} + Z_{22} \left(-\frac{Z_{21}}{Z_{22} + Z_L} \right)}{(Z_{11} + Z_S) + Z_{12} \left(-\frac{Z_{21}}{Z_{22} + Z_L} \right)}$$

$$A_S = \frac{Z_{21}Z_{22} + Z_{21}Z_L - Z_{22}Z_{21}}{Z_S Z_{11} Z_{22} + Z_S Z_L Z_{11} - Z_{21}Z_{12}}$$

$$A_S = \frac{Z_{21} Z_L}{(Z_{11} + Z_S)(Z_{22} + Z_L) - Z_{12}Z_{21}}$$

$$Z_S = 100 \Omega \quad Z_L = 1k\Omega \quad A_V = 0.283$$

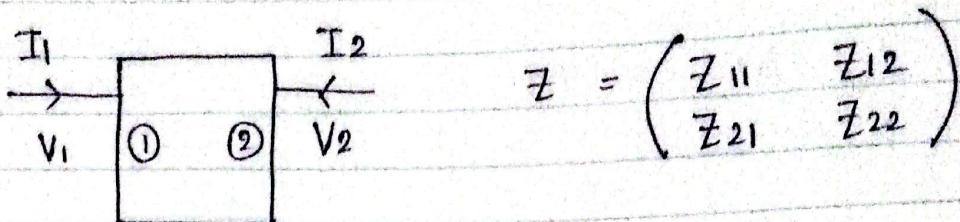
$$A_S = 0.173$$



Scatter Parameters / S-parameters

Two-port networks ← low frequency circuit theory

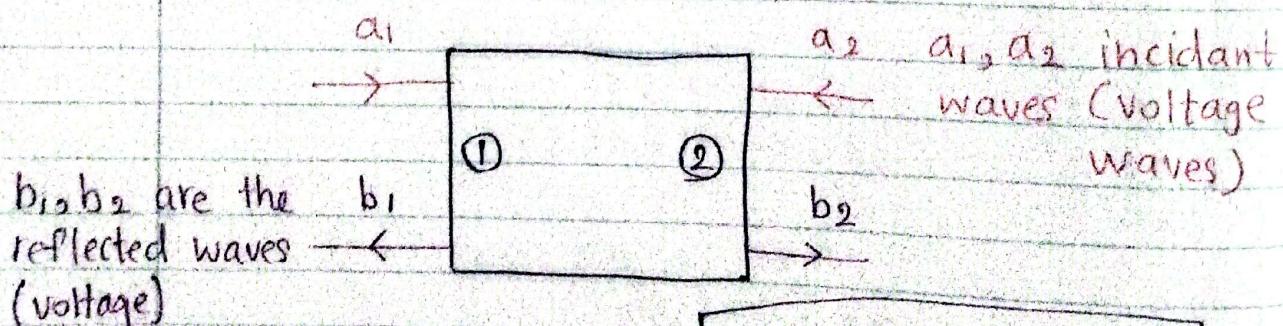
Scatter parameters ← same concept but adapted for high frequency use.



High frequency →

- ① distributed components
- ② e.^h undergo S.H.M
- ③ measuring power is convenient at high freq

(i) VSWR
 (ii) Network Analyser (better we measure reflected power)



$$A \equiv a$$

$$B \equiv b$$

$$V = A e^{\beta z} + B e^{-\beta z}$$

$$I = \frac{A}{Z_0} e^{\beta z} - \frac{B}{Z_0} e^{-\beta z}$$

← I_L

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \longrightarrow \begin{array}{l} S - \text{parameter matrix} \\ \downarrow \\ \text{description of the system} \end{array}$$

The scatter parameters are obtained as ratios when one port is matched ($a_1=0$ or $a_2=0$)!

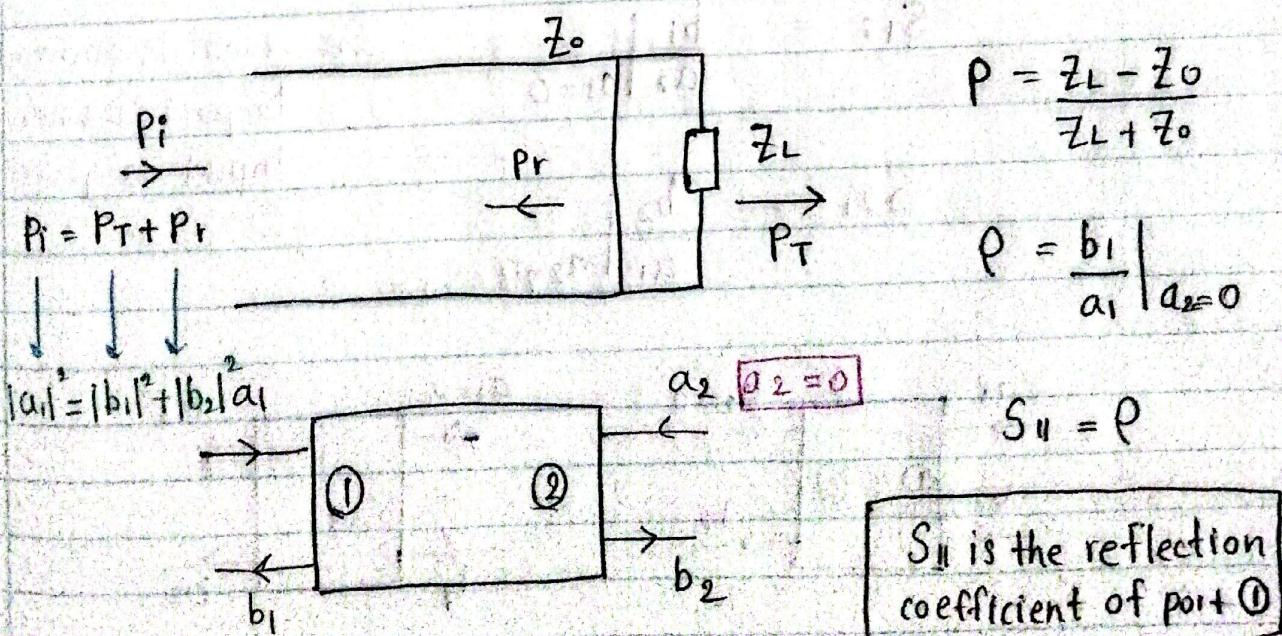
* Port reflection coefficients

reflection coefficient of port ②

$$S_{11} = \frac{b_1}{a_1} \Big|_{a_2=0}$$

Here by making $a_2=0$ or $a_1=0$ we make the matching circuit.

$$\rightarrow S_{22} = \frac{b_2}{a_2} \Big|_{a_1=0}$$

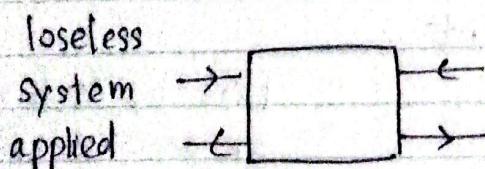


$$|a_{11}|^2 = |b_2|^2 + |S_{11}|^2 |a_{11}|^2$$

$$\begin{aligned} |b_2|^2 &= |a_{11}|^2 - |S_{11}|^2 |a_{11}|^2 \\ &= (1 - |S_{11}|^2) |a_{11}|^2 \end{aligned}$$

$$\rho_1 = S_{11} = \frac{b_1}{a_1} \Big|_{a_2=0} \Rightarrow |S_{11}| = \left| \frac{b_1}{a_1} \right|$$

$$P_t = (1 - |\rho_1|^2) P_i$$



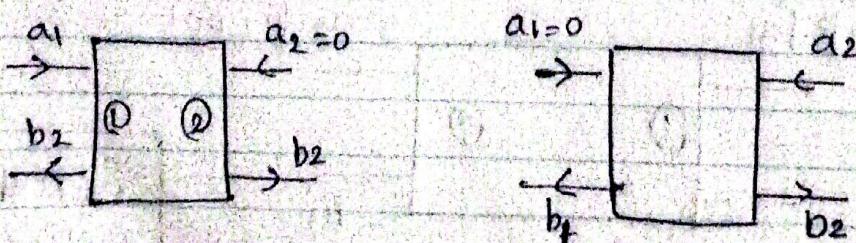
* port reflection coefficients.

S_{21} - what is transmitted to port ② when an input is given port ①

$$S_{12} = \frac{b_1}{a_2} \Big|_{a_1=0}$$

S_{12} - What is transmitted to port ① when an input is given port ②

$$S_{21} = \frac{b_2}{a_1} \Big|_{a_2=0}$$



$$|a_{11}|^2 = |b_1|^2 + |b_2|^2$$

lossless

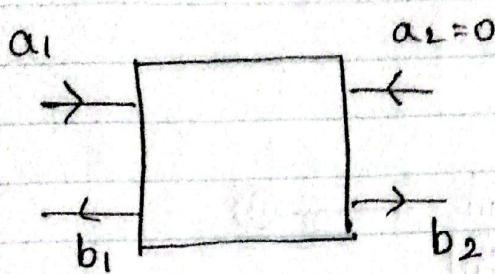
$$|a_{21}|^2 = |b_2|^2 + |b_1|^2$$

lossless

$$\frac{b_1}{a_1} = S_{11}$$

$$|S_{11}|^2 = \frac{|b_1|^2}{|a_1|^2} \quad |b_2|^2 = (1 - |S_{11}|^2) |a_1|^2$$

$$|b_1|^2 = (1 - |S_{22}|^2) |a_2|^2$$



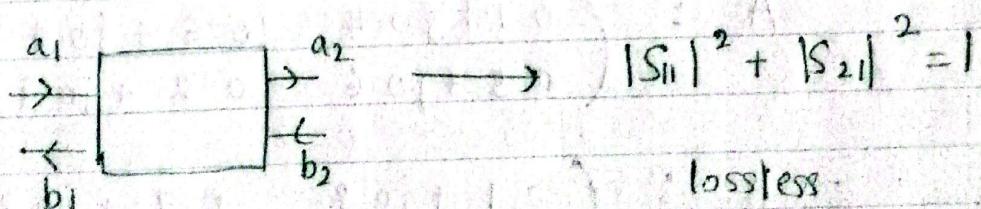
$$|a_1|^2 = |b_1|^2 + |b_2|^2$$

$$|b_2|^2 = (1 - |S_{11}|^2) |a_2|^2 \quad \text{--- } ①$$

$$|b_2|^2 = |S_{21}| |a_1|^2$$

$$|S_{21}|^2 |a_1|^2 = (1 - |S_{11}|^2) |a_1|^2$$

$$|S_{11}|^2 + |S_{21}|^2 = 1 \quad \leftarrow \text{N.W. is lossless.}$$



$$|S_{11}|^2 + |S_{21}|^2 = 1$$

lossless

→ Insertion loss in dB

$$IL = -20 \log_{10} |S_{21}|$$

→ Return loss in dB

$$RL = -20 \log_{10} |S_{11}|$$

→ VSWR

$$VSWR = \frac{1 + |S_{11}|}{1 - |S_{11}|} \quad \text{--- } ①$$

$$VSWR = \frac{1 + |S_{22}|}{1 - |S_{22}|} \quad \text{--- } ②.$$

Exercise

For the following two scatter matrices determine

$$A : \begin{pmatrix} 0.1 + j0.4 & 0.6 + j0.2 \\ 0.5 + j0.6 & 0.2 + j0.3 \end{pmatrix}$$

$$B : \begin{pmatrix} 0.1 + j0.2 & 0.7 + j0.3 \\ 0.6 + j0.7681146 & 0.2 + j0.6164414 \end{pmatrix}$$

- 1) Whether the circuit is lossless
- 2) I.L. in dB
- 3) R.L. in dB
- 4) VSWR

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

1) A

$$|S_{11}|^2 + |S_{21}|^2$$

$$\begin{aligned} &= (0.1)^2 + (0.4)^2 + (0.5)^2 + (0.6)^2 \\ &= 0.01 + 0.16 + 0.25 + 0.36 \\ &= 0.78 // \end{aligned}$$

B

$$|S_{11}|^2 + |S_{21}|^2$$

$$\begin{aligned} &= (0.1)^2 + (0.2)^2 + (0.6)^2 + (0.7681146)^2 \\ &= 0.01 + 0.04 + 0.36 + 0.59 \\ &= 1 // \end{aligned}$$

$$2) A \quad IL_A = -20 \log_{10} |S_{21}|$$

$$\begin{aligned} &= -20 \log_{10} 0.78 \\ &= 2.14 \text{ dB} // \end{aligned}$$

$$\begin{aligned} RL_A &= -20 \log_{10} |S_{21}| \\ &= -20 \log_{10} 0.974 \\ &= 0.22 \text{ dB} // \end{aligned}$$

$$3) \quad RL_A = -20 \log_{10} |S_{11}|$$

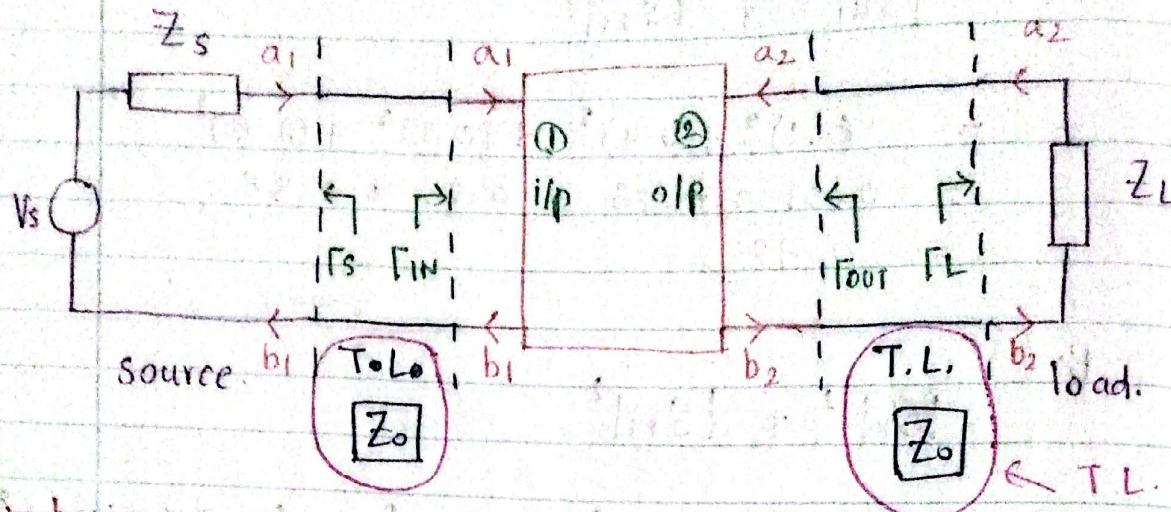
$$\begin{aligned} &= -20 \log_{10} 0.17041 \\ &= -15.39 \text{ dB} // \quad = 7.74 \text{ dB} // \end{aligned}$$

$$RL_B = -20 \log_{10} |S_{11}|$$

$$= -20 \log_{10} 0.22 = 13.19 \text{ dB} //$$

4) $VSWRA = 1.7 \text{ dB}$
 $VSWRB = 1.288 \text{ dB}$

Terminated Network



Analysis
- in terms of
the mismatch
with respect
to the TL

- Network
- passive
- Active i.e. an amplifier

is the
back bone
of signal flow

- ① Identify the possible mismatches.
- ② Define a reflection coefficient for each mismatch.

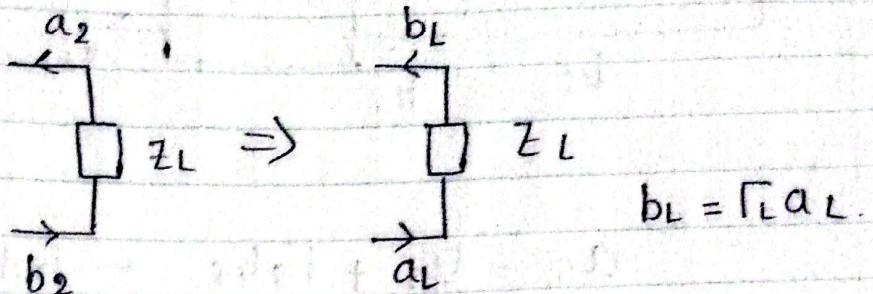
now all $a_1, a_2, b_1, b_2 \neq 0$.

$$b_1 = S_{11}a_1 + S_{12}a_2 \quad (1) \quad a_2 = \Gamma_L b_2 \quad (3)$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \quad (2)$$

$$\left. \begin{array}{l} b_1 = S_{11}a_1 + S_{12}\Gamma_L b_2 \\ b_2 = S_{21}a_1 + S_{22}\Gamma_L b_2 \end{array} \right\} \quad b_2 = \frac{S_{21}a_1}{1 - S_{22}\Gamma_L}$$

$$b_1 = \left[S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \right] a_1$$



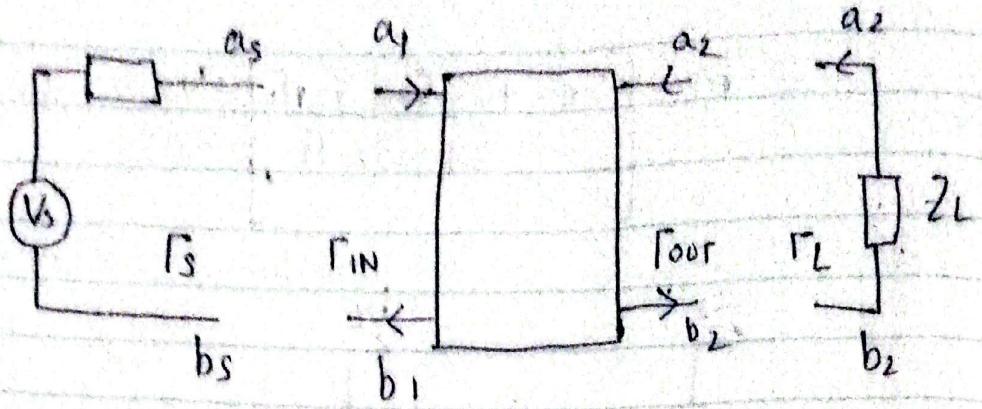
$a_L = b_2$
 $b_L = a_2$

$$\frac{b_1}{a_1} = \Gamma_{IN} \quad \frac{b_1}{a_1} = S_{11} \quad (\text{if } a_2 = 0)$$

$$\Gamma_{IN} - \frac{b_1}{a_1} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

if the load will perfectly match,

$$\Gamma_{IN} = S_{11}$$



$$a_s = V_s + r_s b_s \leftarrow \text{isolated source}$$

↓

connect to port ①

$$a_1 = V_s + r_s b_1$$

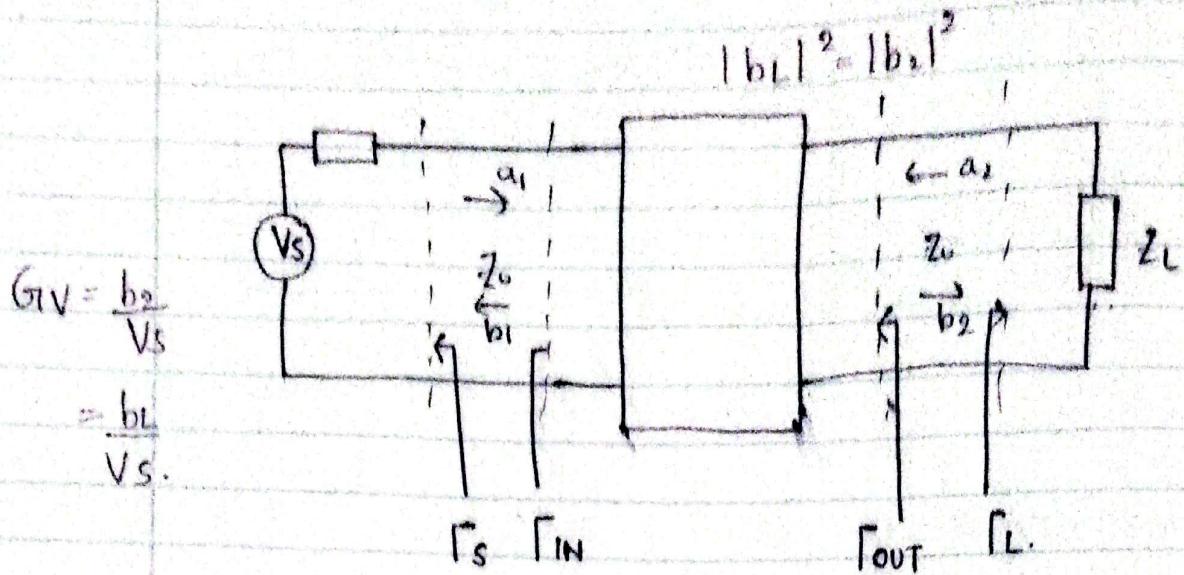
$$\frac{b_1}{a_1} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$a_1 = V_s + r_s \left(S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \right) a_1$$

$$V_s = \left[1 - r_s \left(S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \right) \right] a_1$$

$$= \left[\frac{1 - S_{22} \Gamma_L - S_{11} r_s + S_{11} S_{22} r_s \Gamma_L + -S_{12} S_{21} \Gamma_L r_s}{1 - S_{22} \Gamma_L} \right] a_1$$

$$G_{IV} = \frac{b_2}{V_s} = \frac{S_{21} a_1 + S_{22} a_2}{V_s}$$

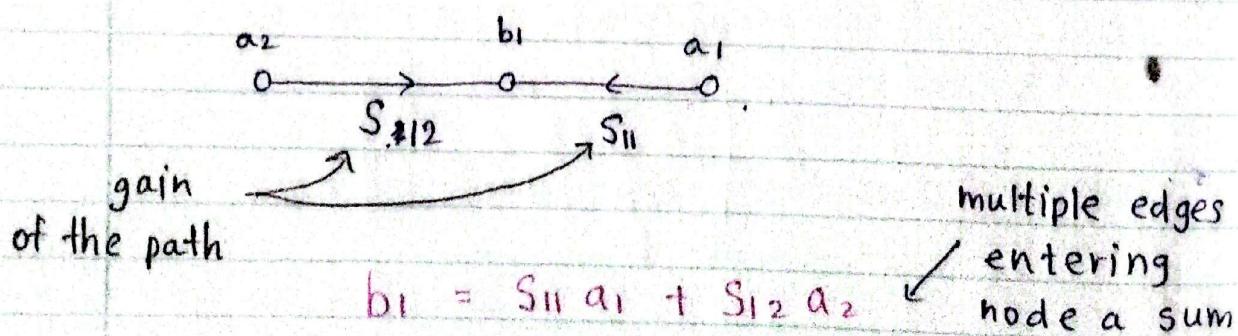


Basic signal flow model.

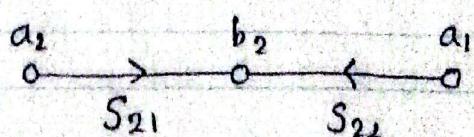
$o \rightarrow \text{node} \rightarrow \text{variable}$

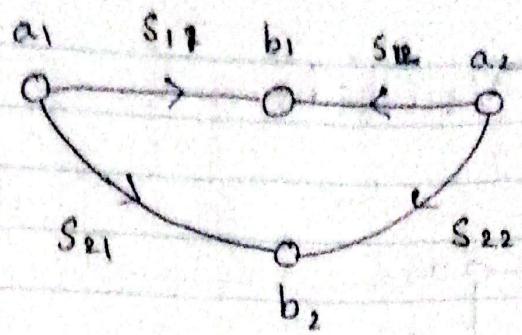
* $o \rightarrow$ independent variable. [input]
can change as we like

* $o \leftarrow$ dependent [output]



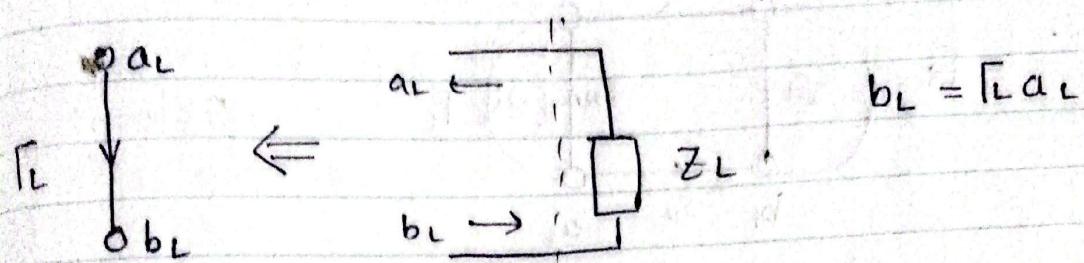
$$b_2 = S_{21} a_1 + S_{22} a_2$$



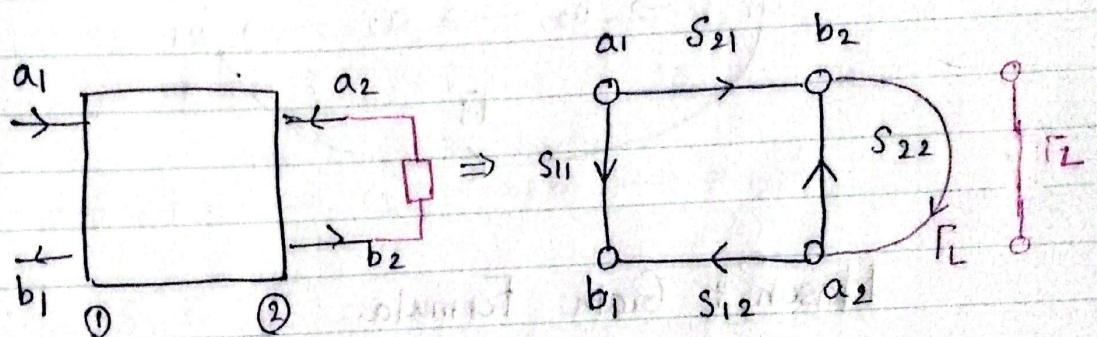


a - input / incident

b - output / reflection.



$$b_L = r_L a_L$$



$$b_2 = a_1$$

$$a_2 = u_L$$

when we
connect the

Signal into the load $\rightarrow b_2$

Signal out of the load $= a_2$

it creates a loop between a_2 and b_2 .

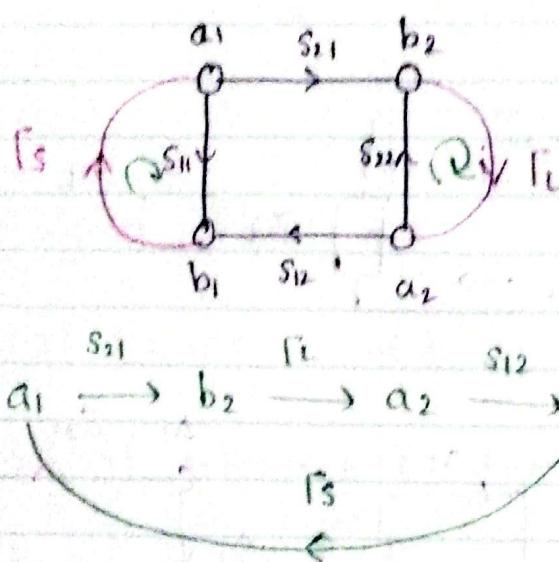
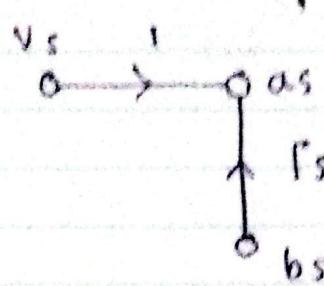
$$as = V_s + f_s b_s$$

incident
signal

$$as \equiv a_1$$

$$b_s \equiv b_1$$

reflected signal



3 loops

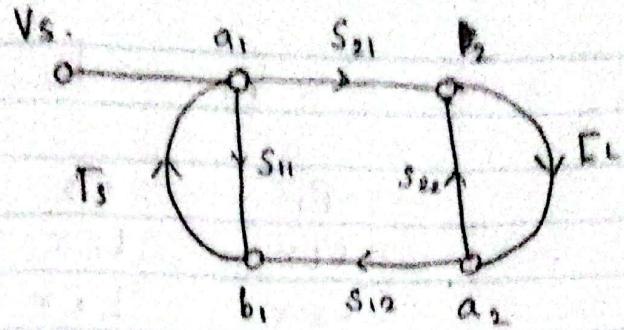
Mason's Gain Formula.

$$G_i = \frac{\sum_i P_i \Delta_i}{\Delta}$$

$P_i \rightarrow$ total gain of path i

$\Delta \rightarrow$ determinant of the adjacency matrix of the graph

$\Delta_i \rightarrow$ determinant of the adjacency matrix of the nodes of the path P_i



In the identified path it should not have repeated edges/nodes.

1. Vs to b₂

P₁, Vs → a₁ → b₂

2. Vs to b₁

P₁, Vs → a₁ → b₁

P₂, Vs → a₁ → b₂ → a₂ → b₂

L₁ = a₁ → b₂ → a₂ → b₁ → a₁

$$L_1(G) = S_{21} \Gamma_L S_{12} \Gamma_S$$

L₂ = a₁ → b₁ → a₁

$$L_2(G) = S_{11} \Gamma_S$$

L₃ = b₂ → a₂ → b₂

$$F \cdot L_3(G) = \Gamma_L S_{22}$$

$$V_s \rightarrow b_2, \quad V_s \rightarrow b_1$$

P
L₁ X
L₂ X
L₃ X

P₁
L₁ X..
L₂ X
L₃ ✓

P₂
L₁ X
L₂ X
L₃ X

det of

the entire
signal flow

graph

$$\Delta = 1 - \sum L(1) + \sum L(2) - \sum L(3)$$

$\Delta = 1 - \text{sum of all loop gain} + \text{sum of product of gains of non-touching loops two at a time}$

$$\Delta = 1 - (S_{11}\Gamma_s + S_{22}\Gamma_L + S_{12}S_{21}\Gamma_s\Gamma_L) + S_{11}S_{22}\Gamma_L\Gamma_s$$

$$P(G) = S_{21}$$

$$P_1(G) = S_{11}$$

$$P_2(G) = S_{21}\Gamma_L S_{12}$$

P₁ → L₁, L₂ touch
L₃ don't touch

$$\Delta_1 = 1 - S_{22}\Delta_L$$

P₂ → all touch.

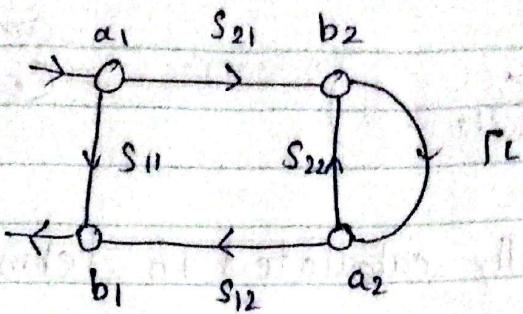
$$\Delta_2 = 1 - 0$$

$$G = \frac{S_{11}\Delta_1 + S_{21}\Gamma_L S_{12}\Delta_2}{\Delta}$$

$$G = \frac{S_{11}(1 - S_{22}\Delta_L) + S_{12}\Gamma_L S_{21}}{\Delta}$$

here onto the
sum of the
gains of the
non-touching
loop

Exercise.



$$\textcircled{1} \quad \frac{b_1}{V_s}$$

$$P_1 \quad a_1 \xrightarrow{S_{11}} b_1$$

$$P_2 \quad a_1 \xrightarrow{S_{21}} b_2 \xrightarrow{S_{22}} a_2 \xrightarrow{S_{12}} b_1$$

$$\textcircled{2} \quad \frac{b_1}{a_1}$$

$$\textcircled{3} \quad \frac{b_2}{a_2}$$

$$\Delta = 1 - \Delta^*$$

$$\Delta = 1 - \Gamma_L S_{22}$$

$$\Delta_1 = 1 - S_{22} \Gamma_L$$

$$\Delta_2 = 1 - 0 = 1$$

$$\Gamma_{IN} = \frac{b_1}{a_1}$$

$$\Gamma_{OUT} = \frac{b_2}{a_2}$$

$$G = \frac{b_1}{a_1} = \frac{S_{11} \Delta_1 + S_{21} S_{12} \Gamma_L \Delta_2}{\Delta}$$

$$= \frac{S_{11} (1 - S_{22} \Gamma_L) + S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$= S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

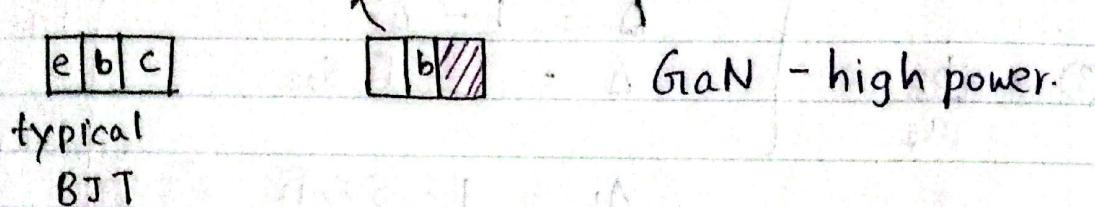
Microwave Active Devices

amplification
(energy input)

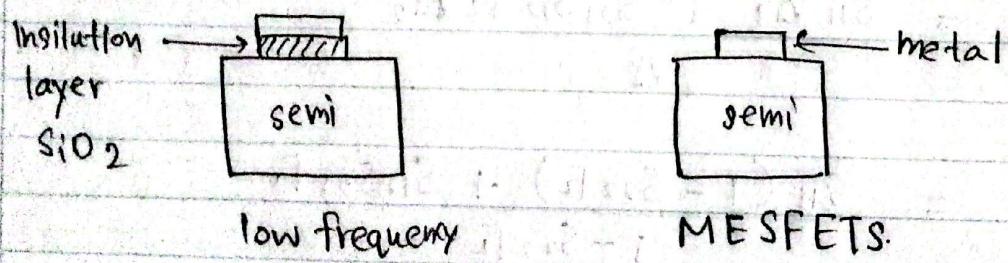
- * Typically are unilateral devices ($S_{12} \approx 0$)
- * Gains are typically calculated in terms of power

$$\left| \frac{b_2}{V_s} \right|^2$$

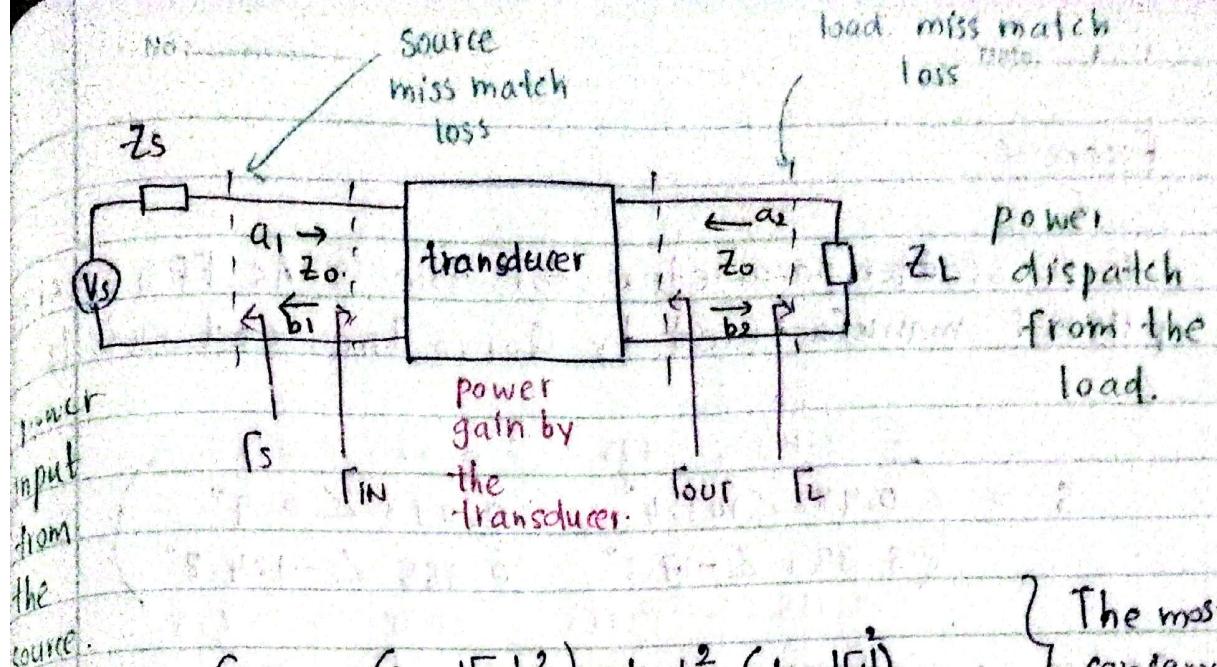
- * MW frequency active devices (npn)
 - Si Bipolar Junction Transistors up to 4GHz
 - GaAs Heterojunction BJTs



- GaAs Metal Semiconductor Field Effect Transistors



- GaAs High Electron Mobility Devices.



$$G_T = \left(1 - |\Gamma_S|^2\right) \left| \frac{b_S}{V_s} \right|^2 \left(1 - |\Gamma_L|^2\right)$$

SOURCE missmatch gain trans. LOAD missmatch

The most conservative power gain of a MW circuit is the transducer power gain given by,

$$G_T = \left| \Gamma_S \right|^2$$

$$G_T = \frac{\left(1 - |\Gamma_S|^2\right) |S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{\left(1 \left(1 - S_{11}\Gamma_S\right) \left(1 - S_{22}\Gamma_L\right) - S_{12}S_{21}\Gamma_L\Gamma_S\right)^2}$$

$$\Gamma_S = 0 \quad \Gamma_L = 0 \quad G_T = |S_{21}|^2$$

maximum possible gain

$S_{12} \approx 0 \leftarrow$ unilateral approximation

$$G_T \approx G_{UT} = \frac{\left(1 - |\Gamma_S|^2\right) |S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{\left(1 \left(1 - S_{11}\Gamma_S\right) \left(1 - S_{22}\Gamma_L\right)\right)^2}$$

Exercise.

so-a scatter parameter for the Grahs FPD6236P pHEMT manufactured by Qorvo, Inc. at 10 GHz

$$S = \begin{pmatrix} -0.113 & 0.478 & 0.073 & 0.001 \\ 0.488 \angle 103.4^\circ & 0.073 \angle 0.9^\circ & & \\ 3.339 \angle -17.3^\circ & 0.385 \angle -124.8^\circ & & \\ 3.188 & -0.993 & -1.915 & -2.735 \end{pmatrix}$$

source impedance of 40Ω Γ_s
load " " 60Ω Γ_L

- ① The transducer power gain
- ② The unilateral transducer power gain

$$\Gamma_s = \frac{50 - 40}{50 + 40} = \frac{1}{9}$$

$$\Gamma_L = \frac{60 - 50}{60 + 50} = \frac{1}{11}$$

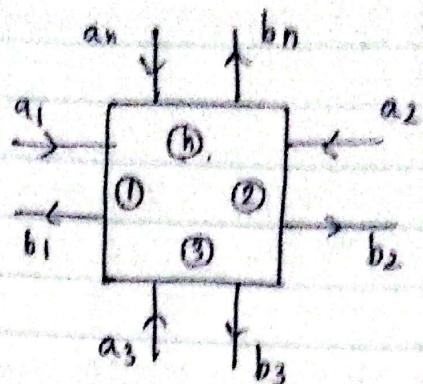
- ① $S_{12} \neq 0$

$$G_{IT} = \frac{(1 - |\Gamma_s|^2) |S_{21}|^2 (1 - |\Gamma_L|^2)}{(1 - S_{11}\Gamma_s)(1 - S_{22}\Gamma_L) - S_{12}S_{21}|\Gamma_L\Gamma_s|^2}$$

=

N-Port Devices

+ MW device which have more than two ports



reflection for each port.

$$S_{ij} = T_x \text{ coefficient from port } j \text{ to port } i$$

$$\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & \dots & S_{1N} \\ S_{21} & S_{22} & \dots & S_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1} & S_{N2} & \dots & S_{NN} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix}$$

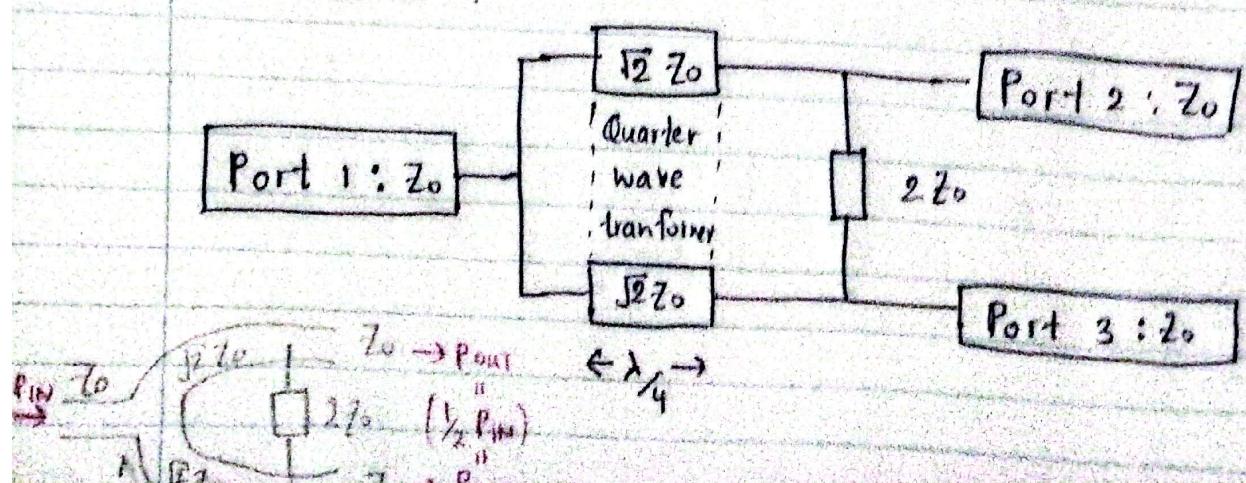
$\uparrow T_x \text{ coefficient}$

i - o/p $\sum j - i/p$

$$i \neq j$$

$$S_{ii} \neq S_{ji}$$

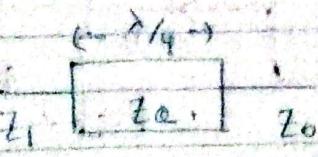
Winkinson Power Divider



Scatter matrix

Quarter Wave Transformer

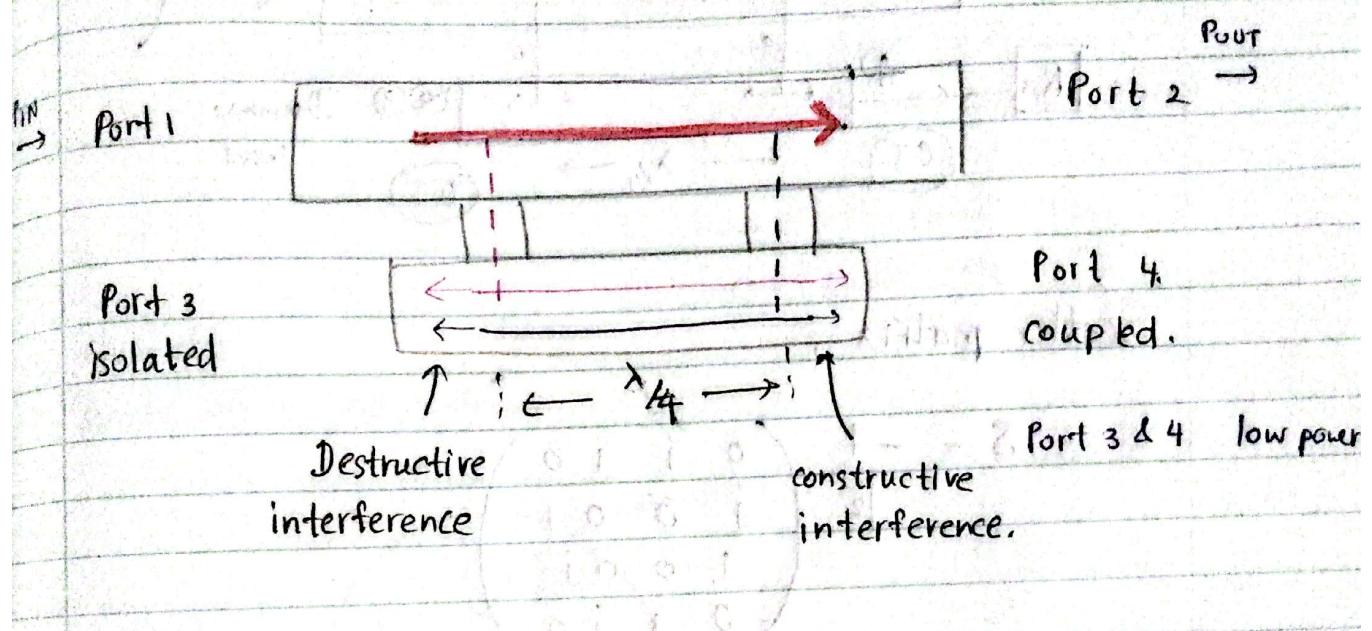
$$S = -\frac{j}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$



$$Z_L Z_0 = (Z_0)^2$$

Directional couplers.

Port 1 & 2 high power



Isolation of high power and low power sections
ex - radar systems

common antenna.

Tx - many kW or even MWatts

Rx - μW, nW, pW

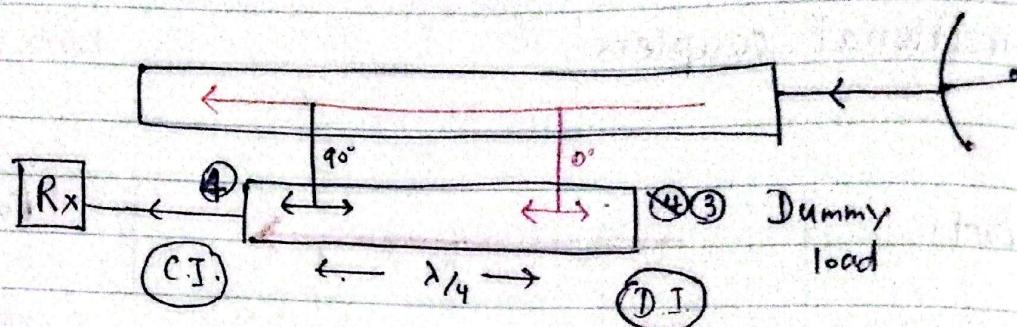
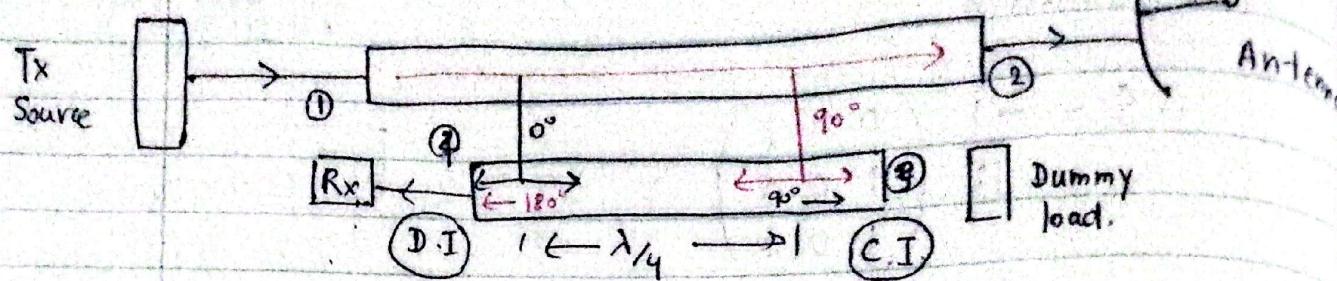
10^{-6}

10^{-9}

10^{-12}

-sensitive

MW tube.



Scatter matrix,

$$S = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{pmatrix}$$

Microwave Tubes

microwave tubes \rightarrow oscillators, magnetron \rightarrow microwave
 \rightarrow amplifiers

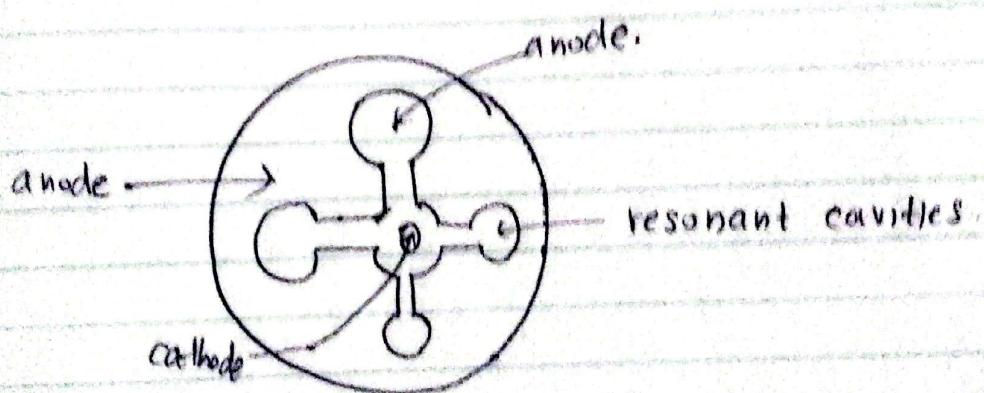
- 1. MW tubes are vacuum tubes.
- 2. Use thermionic emission.

$\exists \parallel e$, 1. large E field for
heater accelerating e^+
(provides the amplification).

2. Large H field for
focus/guide the e^+

schematic

Anode of a magnetron.



Schematic of a Klystron - Amplifier.

- low power input & high power output.
- two resonant cavities.
- amplification by a modulated e^+ beam
- klystron have a very small bandwidth.