Module 9

General Sir John Kotelawala Defense University
Department of Mathematics

MA 3102 APPLIED STATISTICS

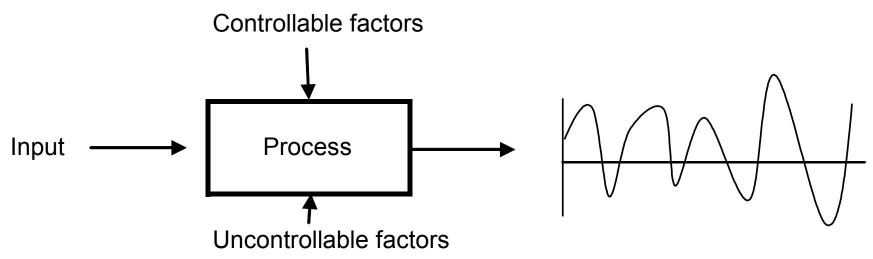
9.0 INTRODUCTION TO QUALITY CONTROL

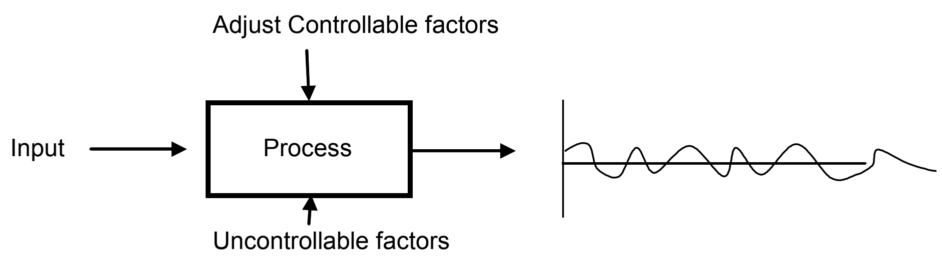
9.1 Dimensions of Quality (Harvard Business Review)

1. Performance: Will the product do the intended job?
2. Reliability: How often does the product fail?
3. Durability: How long does the product last?
4. Serviceability: How easy is it to repair the product?
5. Aesthetics: What does the product look like?
6. Features: What does the product do?
7. Perceived Quality: What is the reputation of the company or its product?
8. Conformance to standards: Is the product made exactly as the designer intended?

Definition 1: Quality is inversely proportional to variability

Definition 2: Quality improvement is the reduction of variability in process and products.





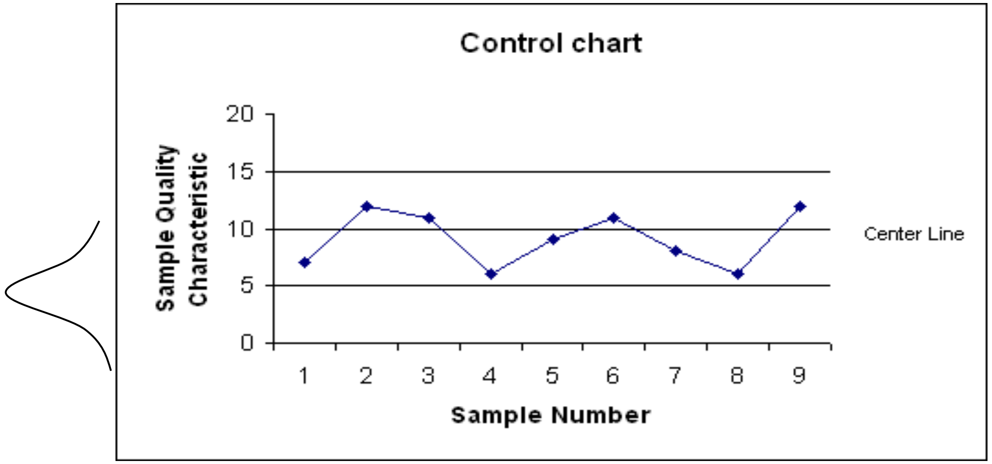
9.2 Statistical Process Control (SPC)

SPC is an effort to identify and eliminate noises, wastes and inefficiencies in a process to reduce the variation in the function of the product.

Tools of SPC (Magnificent Seven)

1. Histogram or Stem & leaf display
2. Check sheet
3. Pareto chart
4. Cause and effect diagram
5. Defect concentration diagram
6. Scatter plot
7. Control Chart

9.3 Control Chart



Control chart is a very useful process monitoring technique. When unusual source of variability (assignable causes) a will plot outside the control limits. This is a signal that some investigation of the process should be made and correc unusual sources of variability taken. Systematic use of a control chart is an excellent way to reduce variability.

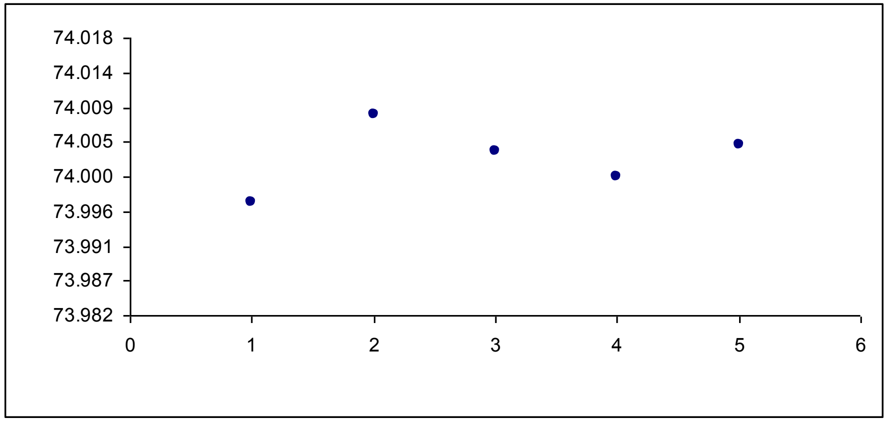
9.4 Shewhart Theory of Variability

- In any production process, regardless of how well designed or carefully maintained it is, certain amount of ***inherent or natural variability*** will always exist.
- This ***inherent variability or background noise*** is the cumulative effect of many small, essentially unavoidable causes.
- A process that is operating with only ***chance causes of variation*** present is said to be ***in statistical control.***
- Other causes of variation are known as ***assignable causes***. Such variability is generally large when compared with the background noise.
- A process that is operating in the presence of assignable causes is said to ***be out of control.***

Example for a Control Chart

Consider the piston ring example. The target mean of the inside ring diameter is 74 mm. The standard deviation of the ring diameter can be estimated from past data as 0.01 mm. Set up the 3-sigma control limits.

Sample #	I	II	III	IV	V	Mean
1	73.997	74.002	73.984	73.992	74.008	**73.997**
2	73.988	74.024	74.021	74.005	74.002	**74.008**
3	73.992	74.007	74.015	73.989	74.014	**74.003**
4	73.995	74.006	73.994	74.000	74.005	**74.000**
5	74.008	73.995	74.009	74.005	74.004	**74.004**



a. Examples: Set up the X bar R control chart control chart following data given below.

Sample No	Observation							
	I	II	III	IV	V			
1	10	11	10	11	13			
2	12	12	10	15	10			
3	9	9	8	10	10			
4	12	11	10	10	12			
5	12	12	11	14	11			
6	9	10	12	8	11			
7	14	14	12	12	13			
8	10	12	10	11	12			
9	10	9	11	10	10			
10	12	10	10	12	11			

Control limits for the $\bar{x}$ chart can be determined as follows:

$$UCL = \mu + 3\sigma/\sqrt{n} \qquad CL = \mu \qquad LCL = \mu - 3\sigma/\sqrt{n}$$

μ is usually unknown and has to be estimated:

$$\hat{\mu} = \bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \dots\dots + \bar{x}_m}{m}$$

σ is also usually unknown and can be estimated using

$$\hat{\sigma} = \frac{\overline{R}}{d_2} \qquad \overline{R} = \frac{R_1 + R_2 + \ldots\ldots + R_m}{m} \quad where \quad R_1 = x_{1\max} - x_{1\min}$$

Where $R_1, R_2 \ldots R_m$ are the ranges of the m samples

$$UCL = \overline{\overline{x}} + \frac{3}{d_2\sqrt{n}}\overline{R} = \overline{\overline{x}} + A_2\overline{R}$$

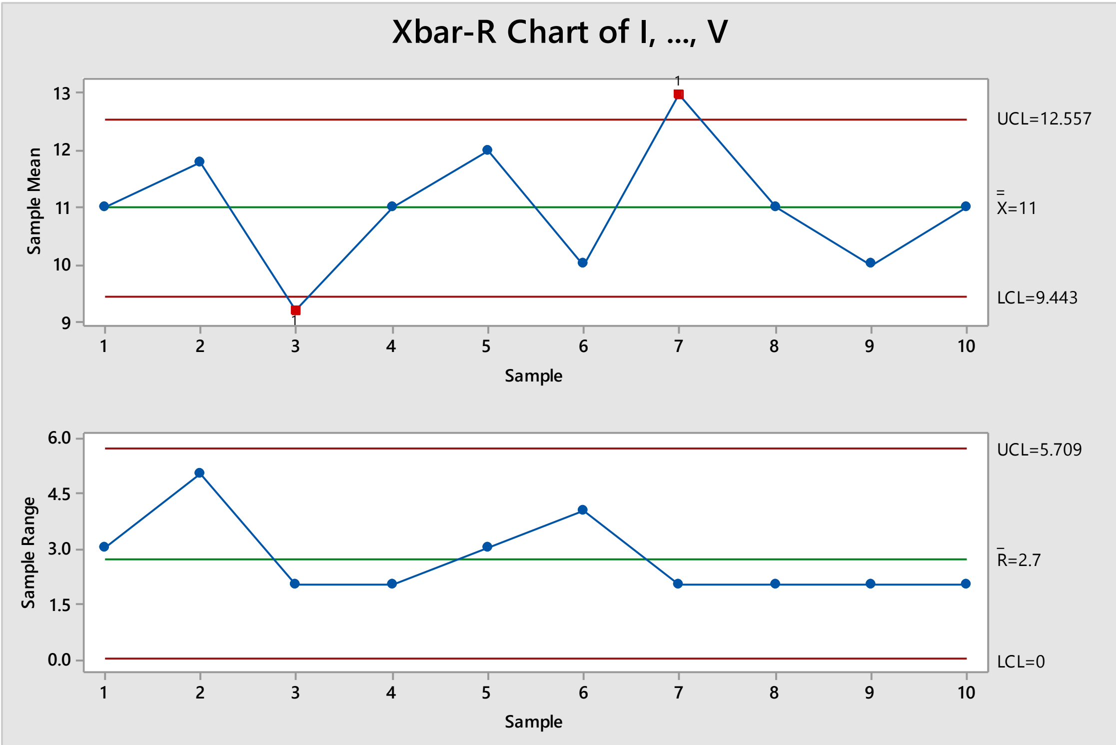
$LCL =$

$CL=$

Control Limits for R chart

$UCL = D_4 \overline{R}$ $\quad CL = \overline{R}$ $\quad LCL = D_3 \overline{R}$

Note: When setting up $\bar{x}$ bar and R control charts, it is better to begin with R chart because the control limits on the $\bar{x}$ bar chart depend on the process variability. Unless process variability is in control, these limits will not have much meaning.



Note:

The R chart measure the within sample variability (instantaneous process variability at a given time) and the $\bar{x}$ chart monitors between sample variability (variability in process over time).

Observations in Sample, n	Chart for Averages			Chart for Standard Deviations						Chart for Ranges						
	Factors for Control Limits			Factors for Center Line		Factors for Control Limits				Factors for Center Line		Factors for Control Limits				
	A	A_2	A_3	c_4	$1/c_4$	B_3	B_4	B_5	B_6	d_2	$1/d_2$	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.575
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.115
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.04230	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608

(*continued overleaf*)

Exercise

1. A high-voltage power supply should have a nominal output voltage of 350 V. A sample of four units is selected each day and tested for process-control purposes. The data shown in Table 6E.2 give the difference between the observed reading on each unit and the nominal voltage times ten; that is, xi = (observed voltage on unit i - 350)10

(a) Set up and R charts on this process. Is the process in statistical control?

TABLE 6E.2
Voltage Data for Exercise 6.2.

Sample Number	x_1	x_2	x_3	x_4
1	6	9	10	15
2	10	4	6	11
3	7	8	10	5
4	8	9	6	13
5	9	10	7	13
6	12	11	10	10
7	16	10	8	9
8	7	5	10	4
9	9	7	8	12
10	15	16	10	13
11	8	12	14	16
12	6	13	9	11
13	16	9	13	15
14	7	13	10	12
15	11	7	10	16
16	15	10	11	14
17	9	8	12	10
18	15	7	10	11
19	8	6	9	12
20	13	14	11	15

9.5 Control Charts for Attributes

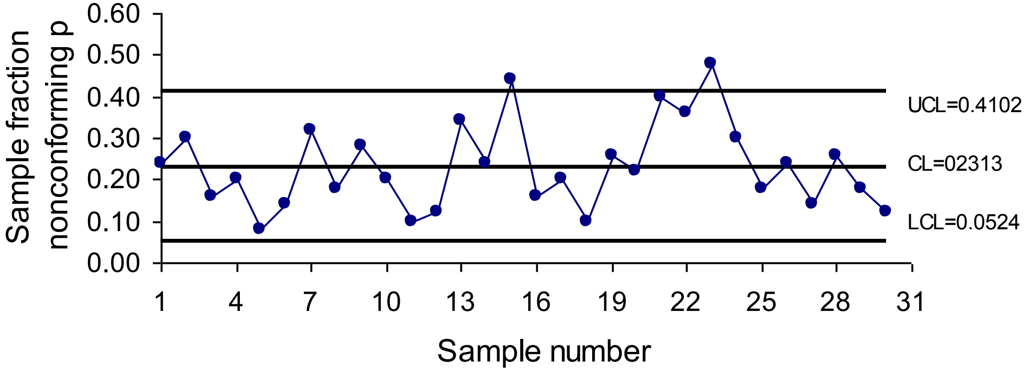
Example:

Frozen orange juice concentrate is packed in 2kg cardboard cans. These cans are formed on a machine by rotating them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine whether, when filled, it could possibly leak either on the side joint or around the bottom joint.

To establish the p control chart, 30 samples sample of n=50 cans each were selected at half –hour intervals over a three shift period in which the machine was in continuous operation. Obtained data is given in Table below. Set up a p chart for the frozen orange juice cans data.

Sample Number	# of Nonconforming cans (D_i)	Sample fraction Nonconforming (p_i)
1	12	0.24
2	15	0.30
3	8	0.16
4	10	0.20
5	4	0.08
6	7	0.14
7	16	0.32
8	9	0.18
9	14	0.28
10	10	0.20
11	5	0.10
12	6	0.12
13	17	0.34
14	12	0.24
15	22	0.44
16	8	0.16
17	10	0.20
18	5	0.10
19	13	0.26
20	11	0.22
21	20	0.40
22	18	0.36
23	24	0.48
24	15	0.30
25	9	0.18
26	12	0.24
27	7	0.14
28	13	0.26
29	9	0.18
30	6	0.12

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.2313 + 3\sqrt{\frac{0.2313(1-0.2313)}{50}} = 0.4106 \qquad CL = \overline{p} = 0.2313 \quad LCL = 0.0524$$



Fraction non conforming control chart for the orange juice data

The np chart

It is possible t base a control chart on the number of nonconforming, rather than the fraction nonconforming.

3σ approach:

$UCL = np + 3\sqrt{np(1-p)}$ $\qquad CL = np$ $\qquad LCL = np - 3\sqrt{np(1-p)}$

Note: for unknown p, p can be replaced by $\bar{p}$.

<u>9.6 Sampling Plans</u>

This chapter presents lot-by-lot acceptance-sampling plans for attributes.

Acceptance-Sampling

- Acceptance sampling is concerned with inspection and decision making regarding products, one of the oldest aspects of quality assurance.
- In the 1930s and 1940s, acceptance sampling was one of the major components of the field of statistical quality control, and was used primarily for incoming or receiving inspection. In more recent years, it has become typical to work with suppliers to improve their process performance through the use of SPC and designed experiments, and not to rely as much on acceptance sampling as a primary quality assurance tool.
- A typical application of acceptance sampling is as follows: A company receives a shipment of product from a supplier. This product is often a component or raw material used in the company's manufacturing process. A sample is taken from the lot, and some quality characteristic of the units in the sample is inspected. On the basis of the information in this sample, a decision is made regarding **lot disposition.**
- Usually, this decision is either to accept or to reject the lot. Sometimes we refer to this decision as **<u>lot sentencing.</u>**
- Accepted lots are put into production; rejected lots may be returned to the supplier or may be subjected to some other **lot disposition action.**
- It is the purpose of acceptance sampling to sentence lots, not to estimate the lot quality.
- Generally, there are three approaches to lot sentencing:
    - **a.** accept with no inspection;
    - **b.** 100% inspection—that is, inspect every item in the lot, removing all defective1 units found (defectives may be returned to the supplier, reworked, replaced with known good items, or discarded)

c. acceptance sampling.

Acceptance sampling is most likely to be useful in situations like testing is destructive, cost of 100% inspection is extremely high and many items to be inspected

Advantages of Sampling

When acceptance sampling is contrasted with 100% inspection, it has the following advantages:

a. **L**ess expensive
b. **L**ess handling of the product, hence reduced damage.
c. **A**pplicable to destructive testing.
d. Fewer personnel are involved in inspection activities.
e. It often greatly reduces the amount of inspection error.
f. The rejection of entire lots as opposed to the simple return of defectives often provides a stronger motivation to the supplier for quality improvements.

Disadvantages Sampling

1. There are risks of accepting “bad” lots and rejecting “good” lots.

2. Less information is usually generated about the product or about the process that manufactured the product.

3. Acceptance sampling requires planning and documentation of the acceptance-sampling procedure whereas 100% inspection does not.

Sampling Plans

a. **Single-sampling plan:** It is a lot-sentencing procedure in which one sample of n units is selected at random from the lot, and the disposition of the lot is determined based on the information contained in that sample. Select n items at random from the lot. If there are c or fewer defectives in the sample, accept the lot, and if there are more than c defective items in the sample, reject the lot.

b. **Double-sampling plans**. Following an initial sample, a decision based on the information in that sample is made either to (1) accept the lot, (2) reject the lot, or (3) take a second sample. If the second sample is taken, the information from both the first and second sample is combined in order to reach a decision whether to accept or reject the lot.