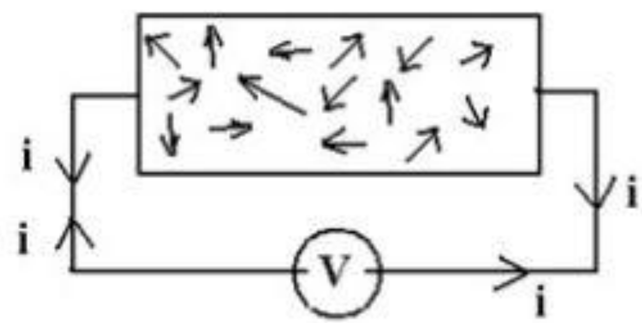
Thermal noise

- Thermal noise is the result of the **random motion of charged particles** (Usually electrons) in a conducting medium **such as a resistor**.
- This kind of noise is generated by all resistances (Eg. A resister, Semiconductor, real part of the impedance, cable.)



- When the **temperature increases the movement of free electrons increases** and the current flows through the conductor.

- Experimental results (By Johnson)and theoretical studies by (Nyquist) gives the **mean square noise**

$$\boxed{\bar{V}^{2} = 4kTBR\ (volt^{2})}$$

Where k = Boltzmann's constant = 1.38 x 10^{-23} Joules per K
T = absolute temperature (Kelvin)
B = bandwidth noise measured in (Hz)
R = resistance (ohms)

- **Example 1.**

One operational amplifier with a frequency range of (18-20) MHz has input resistance 10 k Ω. Calculate noise voltage at the input if the amplifier operate at ambient temperature of 27 ^{0}C.

- $P_n = kTf$
  - Where: The noise power, Pn (in watts)
  - k - is Boltzmann's constant in joules per kelvin, (1.380 649 × 10^{-23} J·K^{-1})
  - T - is the conductor temperature in kelvins,
  - f - is the bandwidth in hertz.

Example 2:

- Calculate the thermal noise power available from any resistor at room temperature (290 K) for a bandwidth of 1MHz. Calculate also the corresponding noise voltage, given that R = 100 Ω.

Shot Noise

- Shot noise is **random fluctuation** that accompanies any **direct current crossing potential barrier**.
- The effect occurs because the carriers (**electrons and holes in semiconductors**) do not cross the barrier **simultaneously** but rather with **random distribution in the timing of each carrier**, which gives rise to random component of current **superimpose on the steady current.**
- Although it is always present, **shot noise is not normally observed during measurement of direct current,** because it is small compared to the DC value; however, it does contribute significantly to the **noise in amplifier circuits**.

Shot Noise Equation

$$I_s = (2eI_dB)^{1/2}$$

Where: I_s = shot noise current

e = electronic charge (1.6×10 coulomb)

I_d = dark leakage current (A)

B = bandwidth of system (Hertz)

Example:

- Calculate the shot noise component of the current present on the direct current of 1mA flowing across a semiconductor junction, given that the effective noise bandwidth is 1 MHz.

White Noise process

- A very **commonly-used random process** is white noise. White noise is often used to **model the thermal** noise in electronic systems. By definition, **the random process X(t) is called white noise if $S_X(f)$ (PSD**: power Spectral Density) **is constant for all frequencies**. By convention, the **constant is usually denoted** by $N_0/2$.
- The random process X(t) is called a **white noise** process

$$S_X(f) = \frac{N_0}{2}, \quad \text{for all } f.$$

N_0 - noise power density

- Before going any further, let's calculate the **expected power in X(t)**. We have

$$\begin{aligned} E[X(t)^2] &= \int_{-\infty}^{\infty} S_X(f)\,df \\ &= \int_{-\infty}^{\infty} \frac{N_0}{2}\,df = \infty. \end{aligned}$$

- Thus, **white noise,** as defined above, has **infinite power**! In reality, white noise is in fact an **approximation to the noise that is observed in real systems**. To better understand the idea, consider the PSD(Power Spectral Density)s shown in Figure:

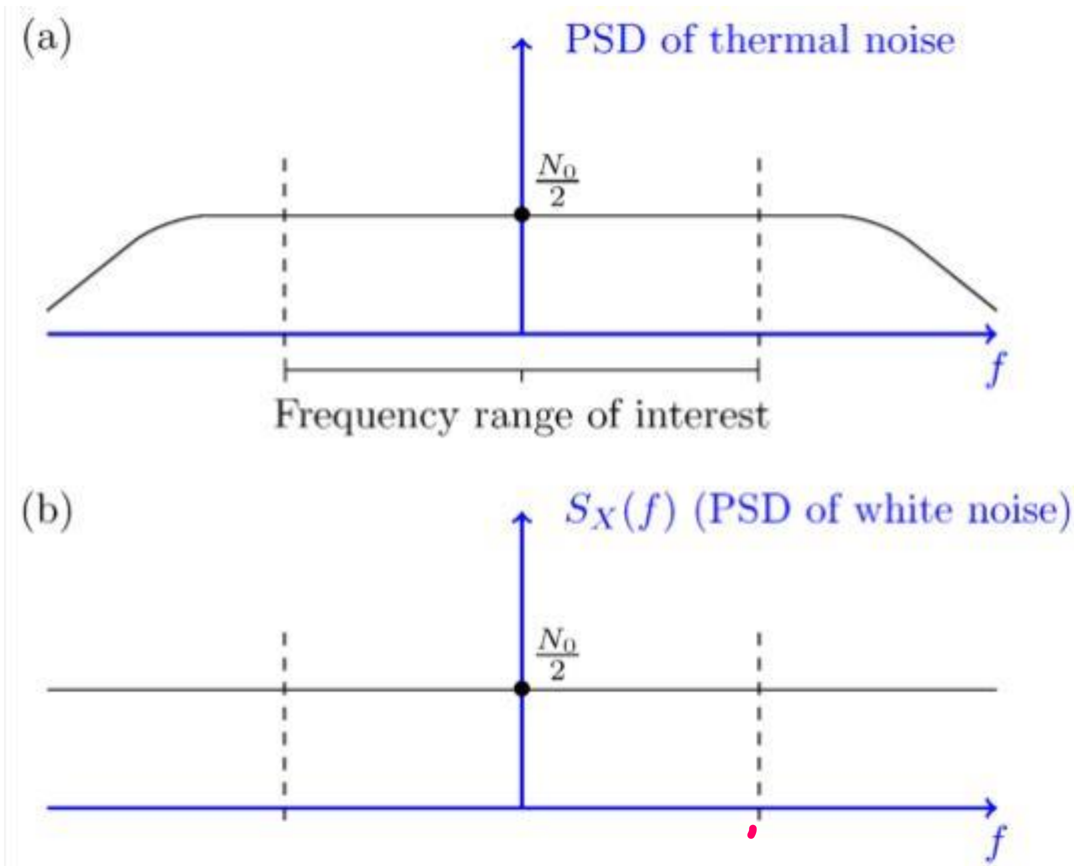


Figure 10.8 - Part (a): PSD of thermal noise; Part (b) PSD of white noise.

- Part (a) in the figure shows what **the real (practical) PSD of a thermal noise** might look like.
- As we see, the **PSD is not constant for all frequencies**; however, it is **approximately constant over the frequency range** that we are **interested in**.
- In other words, **real systems are bandlimited** and work on a limited range of frequencies.
- For the frequency range that we are interested in, the two PSDs **(the PSD in Part (a) and the PSD of the white noise, shown in Part (b)) are approximately the same.**

- The thermal noise in electronic systems is usually modeled as a **white Gaussian noise process**. It is usually assumed that it has **zero mean μ_X=0 and is Gaussian.**
- The random process X(t) is called a **white Gaussian noise** process if X(t) is a stationary Gaussian random process with zero mean, μ_X=0, and flat **power spectral density**:

$$S_X(f) = \frac{N_0}{2}, \quad \text{for all } f.$$

- Since the PSD of a white noise process is given by $S_X(f)=N_0/2$, its **auto correlation function** is given by:

$$R_X(\tau) = \mathcal{F}^{-1}\left\{\frac{N_0}{2}\right\} = \frac{N_0}{2}\delta(\tau),$$

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) e^{j2\pi f\tau} df$$

  - where δ(τ) is the dirac delta function

$$\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & \text{otherwise} \end{cases}$$

The autocorrelation function and the PSD of a white noise process is shown in Figure 1 below.

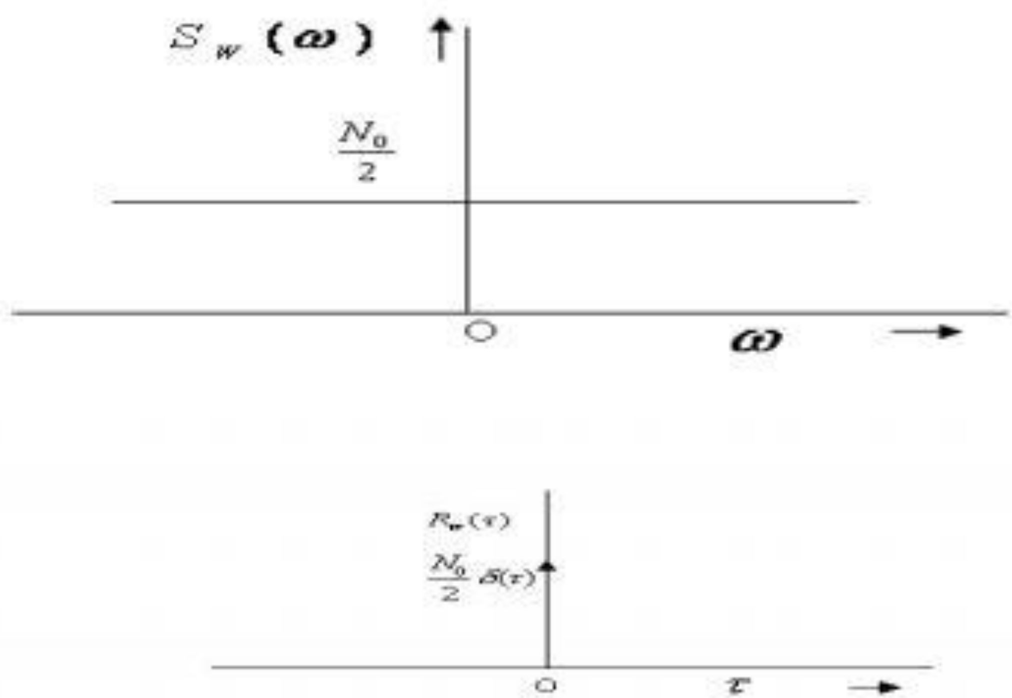


fig: auto correlation and psd of white noise

Effects of Noise

- Noise is an inconvenient feature, which affects the system performance. Following are the effects of noise.
- Noise **limits the operating range of the systems**
  - Noise indirectly places a **limit on the weakest signal** that can be amplified by an amplifier. The oscillator in the mixer circuit may limit its frequency because of noise. A system's operation depends on the operation of its circuits. Noise **limits the smallest** signal that **a receiver is capable of processing**.
- Noise affects the **sensitivity of receivers**
  - Sensitivity is the minimum **amount of input signal necessary to obtain the specified quality output.** Noise affects the sensitivity of a receiver system, **which eventually affects the output**.

Signal to Noise Ratio

- **Signal-to-Noise Ratio (SNR)** is the **ratio of Signal (Power) Amplitude : Noise (Power) Amplitude, referred to as the signal-to-noise ratio (SNR),** is expressed in dB and is calculated using a formula very similar to that for power gain in dB.

$$SNR_{dB} = 10\log_{10}\frac{P_S}{P_N} = 20\log_{10}\frac{V_S}{V_N}$$

- The **higher the value of SNR**, the **greater will be the quality** of the received output.
- If the **signal-to-noise ratio falls below a certain level**, then the information in the signal will be **degraded unacceptably**.
- A **SNR of 0 dB means** that the signal power is **equal** to the noise power and the **signal is unrecoverable at the Rx**.

- **Examples**:

1. The amplitudes of the signal and noise in a transmission link are estimated at 5 V and 2 mV respectively. Estimate the signal-to-noise ratio (SNR).
2. The noise output from a coaxial cable is 0.35 mW with no signal present. What signal power is required if the minimum acceptable signal-to-noise ratio (SNR) is 25 dB?
3. A communications receiver requires an input signal amplitude of 3.2 V and SNR of 36 dB. What is the maximum acceptable value of the noise amplitude?
4. The noise power at the output of a cable link with no signal present is 43.9 μW. The combined noise and signal power is 69.8 mW. Calculate the SNR.

Receiver Model/Band pass Structure

Band-pass System Structures

- Mixer is usually used to translate the IF frequency to the RF frequency, or vice versa.

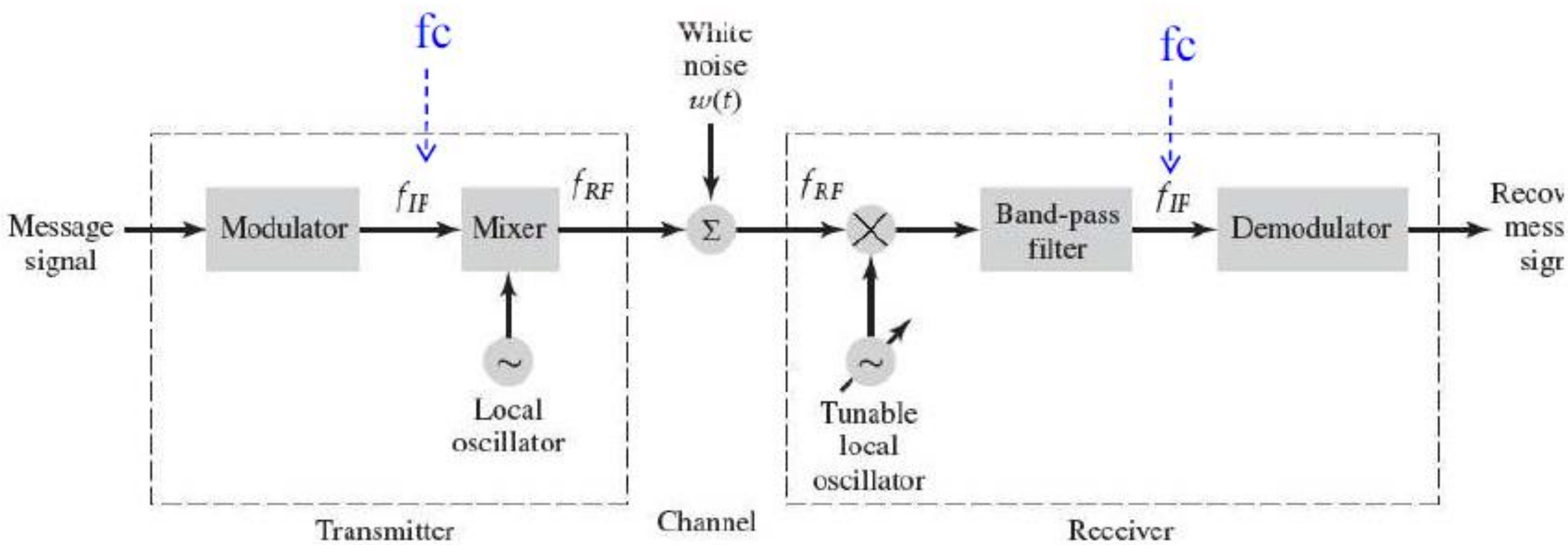
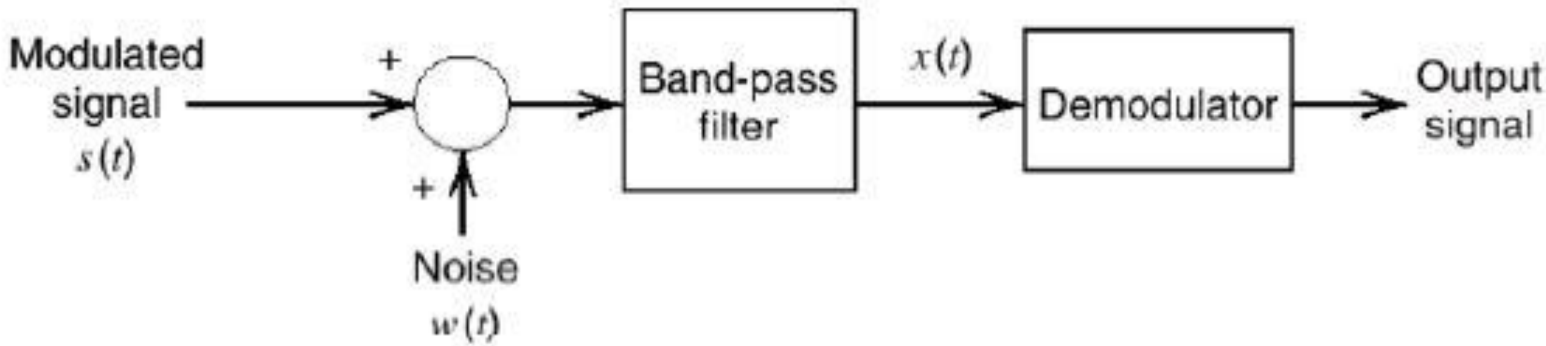


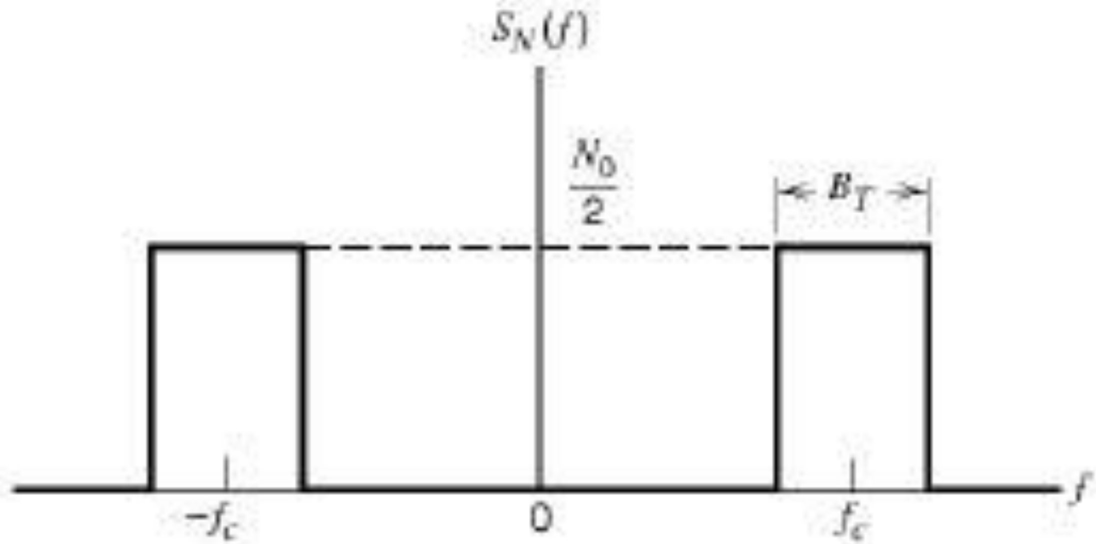
FIGURE 9.5 Block diagram of band-pass transmission showing a superheterodyne receiver.

Receiver Model



- $s(t)$ denotes the incoming modulated signal.
- w(t) denotes front-end receiver noise. The **power spectral density of the noise** w(t) is denoted by ***N*₀/2,** defined for both positive and negative frequencies.
- ***N*₀** is the average noise power **per unit bandwidth measured at the front end of the receiver**.
- The bandwidth of this **band-pass filter is just wide enough to pass the modulated signal without distortion**.

- Assume the **band-pass filter is ideal**, having a bandwidth equal to the **transmission Bandwidth B_T** of the **modulated signal s(t),** and a mid transmission bandwidth , and a **mid-band frequency** equal to the carrier frequency fc , fc >> B_T.



- The **filtered noise *n(t)*** may be treated as a **narrow band noise** represented in the canonical form:

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

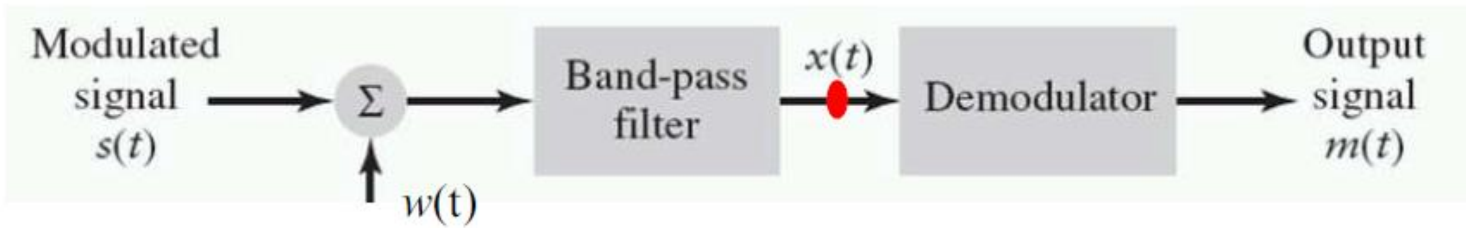
- where $n_I(t)$ is the **in-phase** noise component and $n_Q(t)$ is the **quadrature** noise component, both **measured with respect to the carrier wave** $A_c\cos(2\pi f_c t)$.
- The **filtered signal *x(t)*** available for demodulation is defined by:

$$x(t) = s(t) + n(t)$$

- The **average noise** power at the **demodulator input is equal to the total area under the curve of the power** spectral density $S_N(f)$:

$$P_{\text{avg-noise}} = 2 \times B_T \times \frac{N_0}{2} = B_T N_0$$

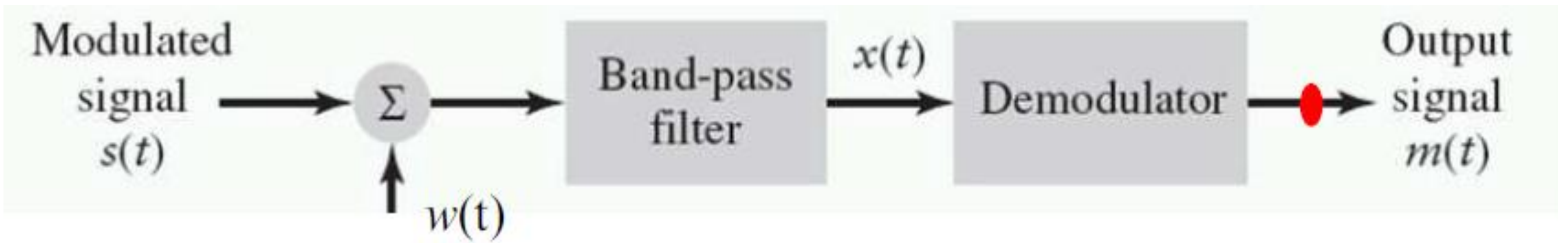
Input signal-to-noise ratio *(SNR)I* : ***Pre Detection SNR***



- At the input to the demodulator: x(t)=s(t)+n(t)
- ***Input signal-to-noise ratio*** *(SNR)I :or* ***Pre Detection SNR*** *(at the demodulator)* is defined as:

$$\mathrm{SNR}_{\mathrm{pre}} = \frac{\text{power of modulated signal}}{\text{power of the narrowband noise}}$$

Output signal-to-noise ratio (SNR)*o: or* ***Post Detection SNR***

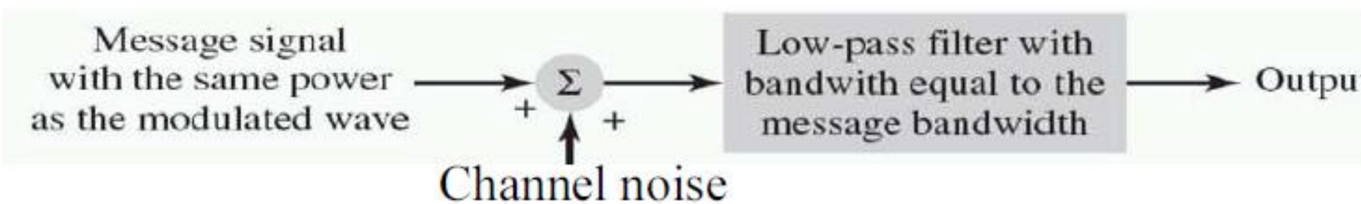


- The output of the demodulator: recovered message with noise.
- The bandwidth of the output signal is usually the bandwidth of the message signal m(t), denoted as W.
- ***Post Detection SNR*** *(after the demodulator)* is defined as:

$$\mathrm{SNR}_{\mathrm{post}} = \frac{\text{Recovered message power}}{\text{Output noise power}}.$$

Channel or Reference signal-to-noise Ratio

- To compare noise performances of different modulation systems, we need a reference baseband transmission model, which transmits the message directly without any modulation.
- For fair comparison, the transmitted power should be the same as that in a bandpass modulation system.
- The bandwidth of the LPF at the receiver equals to the message bandwidth.



- The reference SNR is defined as:

$$\mathrm{SNR}_{\mathrm{ref}} = \frac{\text{Average power of modulated signal}}{\text{Average noise power in the message bandwidth}}$$

Figure of Merit

- For the purpose of **comparing different continuous-wave (CW) modulation systems**, we normalize the receiver performance by dividing the **output signal-to-noise ratio** by the **channel (reference) signal-to-noise ratio**.
- The **higher the value of the figure of merit, the better will the noise performance** of the receiver be.
- **The figure of merit** may equal one, be less than one, or be greater than one, depending on the type of modulation used.
- **Figure of merit** of a receiver is:

$$F = \frac{(SNR)_O}{(SNR)_C}$$

or

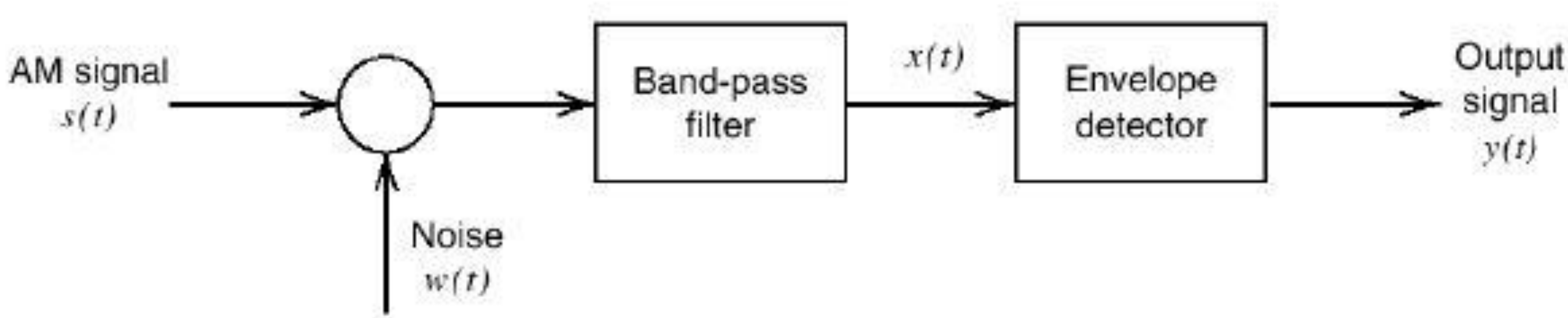
$$\text{Figure of Merit} = \frac{\text{Postdetection SNR}}{\text{Reference SNR}}$$

Noise in AM Receivers Using Envelope Detection

- A standard equation of AM signal is given by:

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

- where $A_c\cos(2\pi f_c t)$ is the carrier wave, $m(t)$ is the message signal and bandwidth is W, k_a is a constant (Sensitivity) that determines the percentage modulation.
- We would like to perform noise analysis for an AM system using an **envelope detector.**



- We perform the noise analysis of the AM receiver by first determining the **channel (ref) signal-to-noise ratio,** and then the **output signal-to-noise ratio**.
- We can easily obtain average power of the AM signal.

$$s(t) = A_c \cos(2\pi f_c t) + A_c k_a m(t) \cos(2\pi f_c t)$$

$$P_s = \frac{1}{2} A_c^2 + \frac{1}{2} A_c^2 k_a^2 P$$

- The **average power of noise in the message band width** is *WN0 (*same as the DSB-SC system)
- The **channel or reference signal-to-noise** ratio for AM is therefore:

$$(SNR)_C = \left. \frac{\text{average power of the modulated signal}}{\text{average power of noise in the message BW}} \right|_{\text{measured at the receiver input}}$$

$$(\mathrm{SNR})_{C,\mathrm{AM}} = \frac{A_c^2 \left(1 + k_a^2 P\right)}{2WN_0}$$

P is the power of the message signal= $\frac{A_m^{\ 2}}{2}$

W is the message bandwidth

- **The filtered signal *x*(*t*) applied to the envelope detector** in the receiver is given by:

$$x(t) = s(t) + n(t)$$
$$= \left[A_c + A_c k_a m(t) + n_I(t)\right]\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$
$$y(t) = \text{envelope of } x(t)$$
$$= \left\{\left[A_c + A_c k_a m(t) + n_I(t)\right]^2 + n_Q^2(t)\right\}^{1/2}$$

Assume average carrier power >> average noise power

$$y(t) \approx A_c + A_c k_a m(t) + n_I(t)$$ Using the approximation $\sqrt{A^2 + B^2} \approx A$ *when* $A >> B$,

Note: $n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$

- The **dc term or constant term *Ac*** may be **removed simply by means of a blocking capacitor.**
- If we ignore the dc term *Ac*, we find the remainder has a form similar to the output of a DSB-SC receiver using coherent detection.

- **Average power of the recovered message signal** is:

$$P_m = \frac{A_c^{\,2} k_a^{\,2} P}{\text{[redacted]}}$$

- **Average power of noise at the output** is:

$$P_n o = \text{[redacted]}$$ $= 2WN_0$

- The **output signal-to-noise ratio (Post Detection SNR)** of an AM using an envelope detector is approximately

$$(SNR)_{O,AM} = \frac{Average\ Power\ of\ demodulated\ signal}{Average\ Power\ of\ noise\ at\ Output}$$

$$\Rightarrow (SNR)_{O,AM} = \frac{A_c^{\,2} k_a^{\,2} P}{2WN_0}$$

- Substitute, the values in **Figure of merit** of AM receiver formula:

$$F = \frac{(SNR)_{O,AM}}{(SNR)_{C,AM}}$$

$$\Rightarrow F = \left(\frac{A_c^2 k_a^2 P}{2WN_0}\right) / \left(\frac{A_c{}^2\left(1 + k_a{}^2\right)P}{2WN_0}\right)$$

$$\Rightarrow F = \frac{K_a{}^2 P}{1 + K_a{}^2 P}$$

- Therefore, the **Figure of merit of AM receiver is less than one.**

In Class activity

Problem

An amplitude-modulated signal is given by $s(t) = 10\cos(2\pi\, 10^6 t)\,(1 + 0.5 \sin(1{,}000\, \pi\, t))$. Figure of merit of AM receiver to demodulate the above AM wave is

(A) 0.33

(B) 0.25

(C) 1.0

(D) 0.11

Example 6.1 Single-Tone Modulation

- Consider a sinusoidal wave of frequency *fm* and amplitude *Am* as the modulating wave, as shown by:

$$m(t) = A_m \cos(2\pi f_m t)$$

- The corresponding AM wave is

$$s(\mathrm{t}) = A_c\,[1 + \mu\cos(2\pi f_m t)]\,\cos(2\pi f_c t)$$

modulation factor : $\mu = k_a A_m$

- The average power of the modulation wave $m(t)$ is (assuming a load resistor of 1ohm)

$$P = \frac{1}{2}A_m^2$$

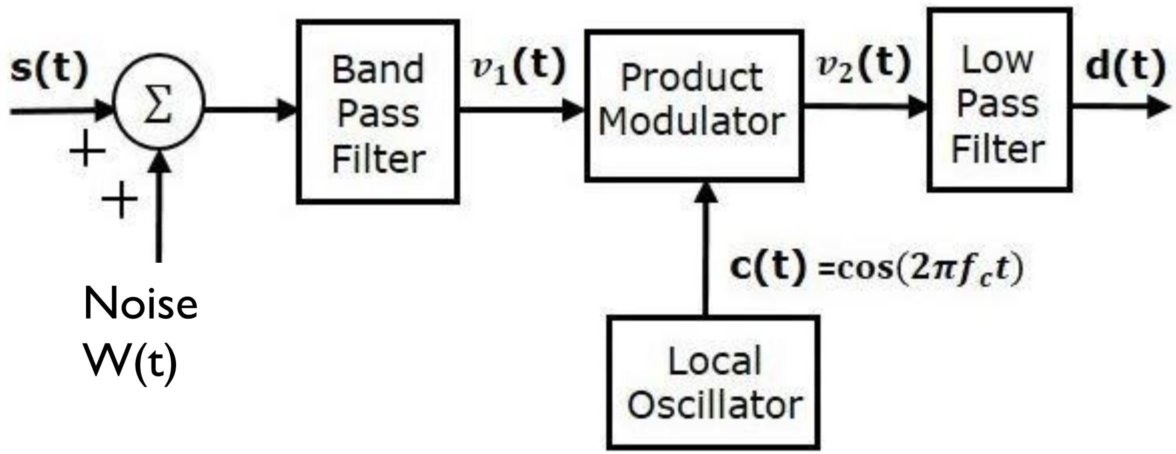
- We obtain the figure of merit

$$\left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{\mathrm{AM}} = \frac{\frac{1}{2}k_a^2 A_m^2}{1 + \frac{1}{2}k_a^2 A_m^2} = \frac{\mu^2}{2 + \mu^2} \qquad (6.18)$$

- When $\mu = 1$ (100% modulation using envelope detection), we get a figure of merit = 1/3.
- This means that, other factors being equal, an AM system (using envelope detection) must transmit three times as much average power as a suppressed-carrier system (using coherent detection) in order to achieve the same quality of noise performance.

Noise in DSBSC Receivers (Coherent Detection)

- Consider the following receiver model of DSBSC system to analyze noise.



- We know that the DSBSC modulated wave is

$$s(t) = A_c m(t) \cos(2\pi f_c t)$$

- Average power of DSBSC modulated wave is

$$\frac{A_c^{\,2} P}{2}$$

- Average power of noise in the **message bandwidth** is:

$$P_{nc} = WN_0$$

- Substitute, these values in **channel SNR** formula.

$$(SNR)_{C,DSBSC} = \frac{Average\ Power\ of\ DSBSC\ modulated\ wave}{Average\ Power\ of\ noise\ in\ message\ bandwidth}$$

$$\Rightarrow (SNR)_{C,DSBSC} = \frac{A_c{}^2 P}{2WN_0}$$

- Assume the **band pass noise is mixed with DSBSC modulated wave** in the channel as shown in the above figure. Hence, the **input of this product modulator(Input of coherent detector)** is

$$v_1(t) = s(t) + n(t)$$

$$\Rightarrow v_1(t) = A_c m(t)\cos(2\pi f_c t) + [n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)]$$

$$\Rightarrow v_1(t) = [A_c m(t) + n_I(t)]\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

- **Local oscillator generates the carrier signal**

c(t)=cos(2πfct). This signal **is applied as another input to the product modulator**. Therefore, the product modulator produces an output, which is the **product of v1(t) and c(t).**

$$v_2(t) = v_1(t)\, c(t)$$

Substitute, $v_1(t)$ and $c(t)$ values in the above equation.

$$\Rightarrow v_2(t) = ([A_c m(t) + n_I(t)] \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)) \cos(2\pi f_c t)$$

$$\Rightarrow v_2(t) = [A_c m(t) + n_I(t)] \cos^2(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$$

$$\Rightarrow v_2(t) = [A_c m(t) + n_I(t)] \left(\frac{1 + \cos(4\pi f_c t)}{2}\right) - n_Q(t) \frac{\sin(4\pi f_c t)}{2}$$

- When the above signal is applied as an input to low pass filter, we will get the output of low pass filter as

$$d(t) = \frac{[A_c m(t) + n_I(t)]}{2}$$

- Average power of the **demodulated signal** is

$$P_{avg} = \frac{A_c^2 P}{4}$$

- Average power of **noise at the output** is

$$= \frac{1}{2} W N_0$$

- Substitute, these values in **output SNR: Post Detection SNR** formula.

$$(SNR)_{O,DSBSC} = \frac{Average\ Power\ of\ demodulated\ signal}{Average\ Power\ of\ noise\ at\ Output}$$

$$\Rightarrow (SNR)_{O,DSBSC} = \qquad = \frac{A_c{}^2 P}{2WN_0}$$

- Substitute, the values in **Figure of merit** of DSBSC

$$F = \frac{(SNR)_{O,DSBSC}}{(SNR)_{C,DSBSC}}$$

$$\Rightarrow F = \left(\frac{A_c{}^2 P}{2WN_0}\right) / \left(\frac{A_c{}^2 P}{2WN_0}\right)$$

$$\Rightarrow F = 1$$

- Therefore, the Figure of merit of DSBSC receiver is 1.