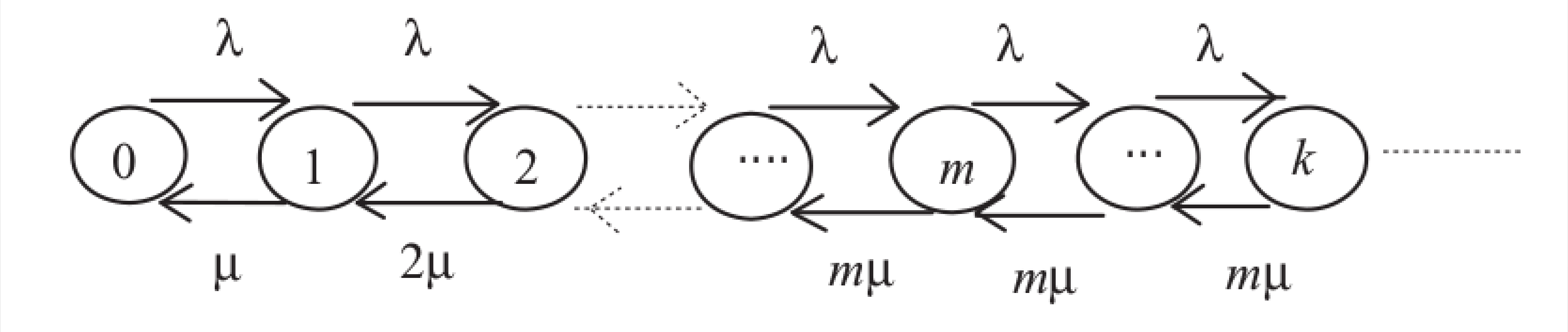
Modelling Multi Serever Queues
(Modelling Queues Part 2)
Lecture 8

Communication Theory III

Eng. (Mrs.) PN Karunanayake

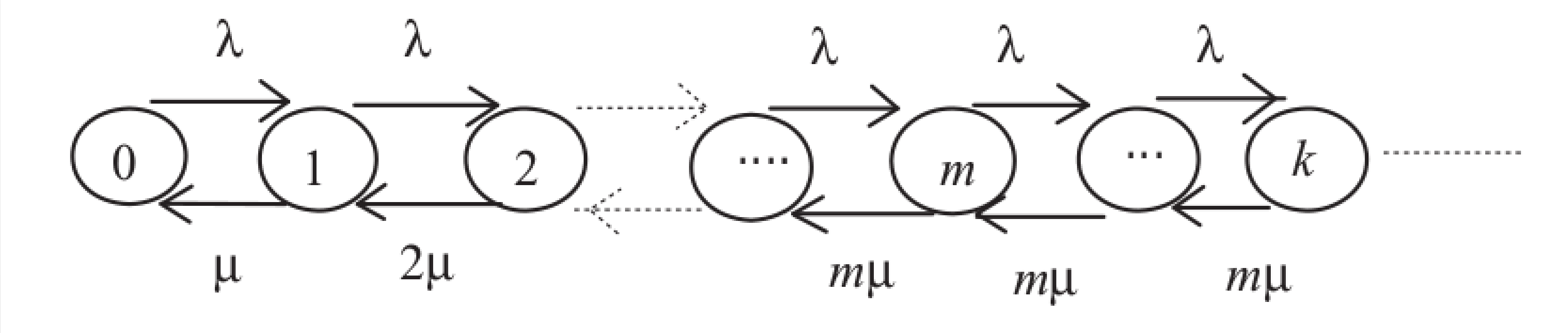
MULTI-SERVER SYSTEMS - M/M/m

- The multi-server queueing system in which the service facility consists of ***m*** identical parallel servers.
- All servers perform the same functions and a customer at the head of the waiting queue can go to any of the servers for service.



MULTI-SERVER SYSTEMS - M/M/m

Ex: When there is one customer, one server is engaged in providing service and the service rate is μ. If there are two customers, then two servers are engaged and the total service rate is 2μ, so the service rate increases until mμ and stays constant thereafter.



M/M/m with an Infinite Waiting Queue

$$P_k = \begin{cases} P_0 \dfrac{(m\rho)^k}{k!} & k \le m \\ P_0 \dfrac{m^m \rho^k}{m!} & k \ge m \end{cases}$$

Performance Measures of M/M/m Systems

(1) **The probability of delay**

This is the probability that an arriving customer finds all servers busy and is forced to wait in the queue. This situation occurs when there are more than m customers in the system.

This probability is often referred to as the **Erlang C formula** or the **Erlang Delay formula** and is often written as C(λ/m, m). Most of the parameters of interest can be expressed in terms of this probability.

$$P_d = P[delay] = \sum_{k=m}^{\infty} P_k$$

$$= \frac{P_o(m\rho)^m}{m!} \sum_{k=m}^{\infty} \rho^{k-m}$$

$$= \frac{P_0(m\rho)^m}{m!(1-\rho)}$$

Performance Measures of M/M/m Systems

(2) **The number of customers waiting in the queue**

$$N_q = \sum_{k=0}^{\infty} kP_{m+k} = P_0 \frac{(m\rho)^m}{m!} \sum_{k=0}^{\infty} k\rho^k$$

$$= P_0 \frac{(m\rho)^m}{m!} \frac{\rho}{(1-\rho)^2}$$

$$= \frac{\rho}{1-\rho} P_d = \frac{\lambda}{m\mu - \lambda} P_d$$

Performance Measures of M/M/m Systems

(3) The time spent in the waiting queue:

$$W = \frac{N_q}{\lambda} = \frac{1}{m\mu - \lambda} P_d$$

(4)The time spends in the queueing system

$$T = W + \frac{1}{\mu} = \frac{P_d}{m\mu - \lambda} + \frac{1}{\mu}$$

Performance Measures of M/M/m Systems

(5) The number of customers in the queueing system:

$$N = \lambda T = \frac{\rho}{1-\rho} P_d + m\rho$$

ERLANG'S LOSS QUEUEING SYSTEMS – M/M/m/m SYSTEMS

- **<u>Case 1</u>**
- M/M/m/m system where there is no waiting queue. This particular model has been widely used in evaluating the performance of circuit-switched telephone systems.
- It corresponds to the classical case where all the trunks of a telephone system are occupied and connection cannot be set up for further calls.
- Since **there is no waiting queue,** when a customer arrives at the system and finds all servers engaged, the customer will not enter the system and is considered lost.

ERLANG'S LOSS QUEUEING SYSTEMS – M/M/m/m SYSTEM

$$P_0 = \left[\sum_{k=0}^{m} \frac{(m\rho)^k}{k!} \right]^{-1}$$

$$P_k = \frac{(m\rho)^k / k!}{\left[\sum_{k=0}^{m} \frac{(m\rho)^k}{k!} \right]}$$

Performance Measures of the M/M/m/m

(1) The probability that an arrival will be lost when all servers are busy.

This probability is the blocking probability and is the same as the probability when the system is full. It is commonly referred to as the **Erlang B formula or Erlang's loss formula** and is often written as B(λ/μ,m).

$$P_b = \frac{(m\rho)^m / m!}{\sum_{k=0}^{m} (m\rho)^k / k!} = P_m$$

Performance Measures of the M/M/m/m

(2) The number of customers in the queueing system:

$$N = \sum_{k=0}^{m} kP_k = \sum_{k=1}^{m} k \cdot P_0 \frac{(m\rho)^k}{k!} = (m\rho) \sum_{k=0}^{m-1} P_0 \frac{(m\rho)^k}{k!}$$

$$= m\rho \left[\frac{\sum_{k=0}^{m} \frac{(m\rho)^k}{k!} - \frac{(m\rho)^m}{m!}}{\sum_{i=0}^{m} \frac{(m\rho)^i}{i!}} \right] = (m\rho)(1 - P_m) = m\rho(1 - P_b)$$

Performance Measures of the M/M/m/m

(3)The system time and other parameters:

Since there is no waiting, the system time (or delay) is the same as service time and has the same distribution:

$$T = 1/\mu$$

$$F_T(t) = P[T \le t] = 1 - e^{-\mu t}$$

$$N_q = 0 \quad \text{and} \quad W = 0$$

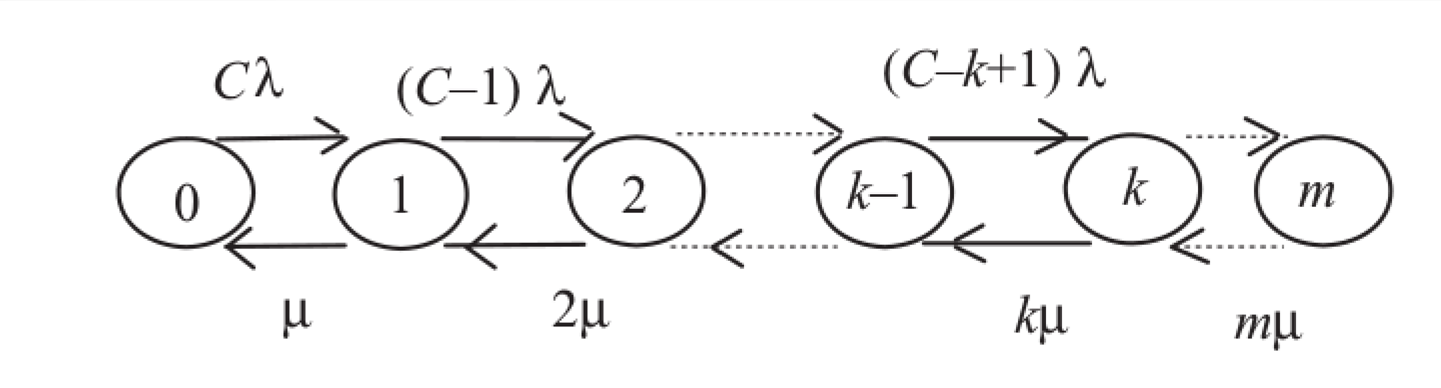
ENGSET'S LOSS SYSTEMS

Case 2

Finite population version of Erlang's loss system. **This is a good model for a time-sharing computer system with a group of fully-occupied video terminals.**

- The jobs generated by each video terminal are assumed to be a Poisson process with rate λ.
- When a user at a video terminal has submitted a job, he is assumed to be waiting for the reply (answer) from the central CPU, and hence the total jobs' arrival rate is reduced.

ENGSET'S LOSS SYSTEMS



ENGSET'S LOSS SYSTEMS

$$P_0 = \frac{1}{\sum_{k=0}^{m} \binom{C}{k} \rho^k}$$

$$P_k = \frac{\binom{C}{k} \rho^k}{\sum_{i=0}^{m} \binom{C}{i} \rho^i}$$

Performance Measures of M/M/m/m with Finite Customer Population

(1) The probability of blocking (the probability of having all m servers engaged)

$$P_b = P_m = \frac{\binom{C}{m}\rho^m}{\sum_{k=0}^{m}\binom{C}{k}\rho^k}$$

Performance Measures of M/M/m/m with Finite Customer Population

(2) The number of customers in the queueing system.

$$N = \frac{\rho}{1+\rho}C - \frac{\rho}{1+\rho}(C-m)P_b$$

What is OMNET ++

OMNET ++ is C++ based discrete event simulator for modeling communication n

Its results are very close to the real world scenario.

www.omnetpp.org

OMNET ++ Main Components

- **Omnetpp.ini**
- Specify: Network, simulation speed, output-scaler file, network area (x,y), number of nodes, other parameters
- **Module.ned**
- Defines the network and the modules inside it, module parameters (gates and connections)
- **Module.cc, module.h**
- Defines the functionality of each module, mainly includes:
- \- initialize ()
- \- handleMessage(cMessage *msg)
- \- finish()


Thank You!