

# Lecture Outline

- Types of sensors
- Sensor performance terminology
- Instrumentation
- Types of sensors:
  - Potentiometers
  - Temperature sensors
  - Strain gauges
  - Load cells
  - Torque sensors
  - Pressure sensors
  - Inductive & capacitive sensors
  - Piezoelectric sensors

# 1. Sensor Performance Terminology (1)

## 1. Range

The range of a sensor indicates the limits between which the input can vary. For example, a thermocouple for the measurement of temperature might have a range of 25-225 °C.

## 2. Span

The span is difference between the maximum and minimum values of the input. Thus, the above-mentioned thermocouple will have a span of 200 °C.

## 3. Error

Error is the difference between the result of the measurement and the true value of the quantity being measured.

$$\text{error} = \text{measured value} - \text{true value}$$

Thus, if a measurement systems gives a temperature reading of 25°C when the actual temperature is 24°C, then the error is +1°C. If the temperature has been 26°C then the error would have been -1°C.

# 1. Sensor Performance Terminology (2)

## 4. Accuracy

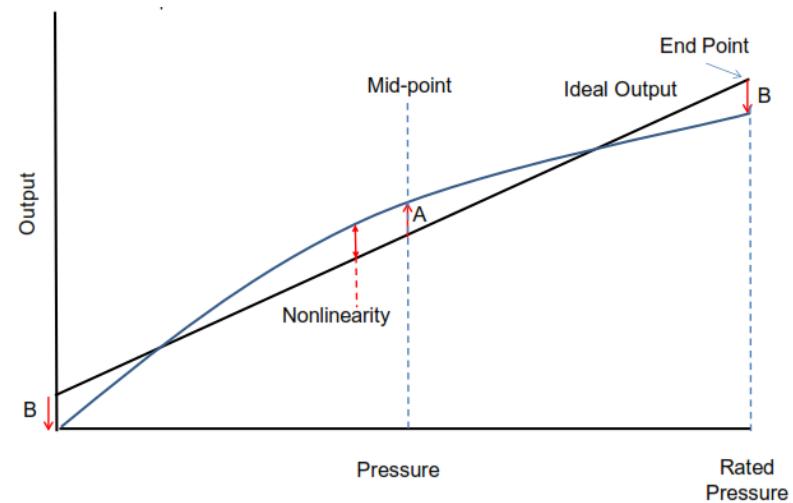
Accuracy is the extent to which the value indicated by a measurement system might be wrong. It is thus the summation of all the possible errors that are likely to occur, as well as the accuracy to which the transducer has been calibrated.

It is often expressed as a percentage of the full range output or full-scale deflection. A piezoelectric transducer used to evaluate dynamic pressure phenomena associated with explosions, pulsations, or dynamic pressure conditions in motors, rocket engines, compressors, and other pressurized devices is capable to detect pressures between 0.1 and 10,000 Psi (0.7 KPa to 70 MPa). If it is specified with the accuracy of about  $\pm 1\%$  full scale, then the reading given can be expected to be within  $\pm 0.7$  MPa.

## 5. Sensitivity

Sensitivity of a sensor is defined as the ratio of change in output value of a sensor to the per unit change in input value that causes the output change. It is frequently used for indicating the sensitivity to inputs other than that being measured. For example, a general purpose thermocouple may have a sensitivity of  $41 \mu\text{V}/^\circ\text{C}$  or a resistance thermometer may have a sensitivity of  $0.5 \Omega/^\circ\text{C}$ .

# 1. Sensor Performance Terminology (3)

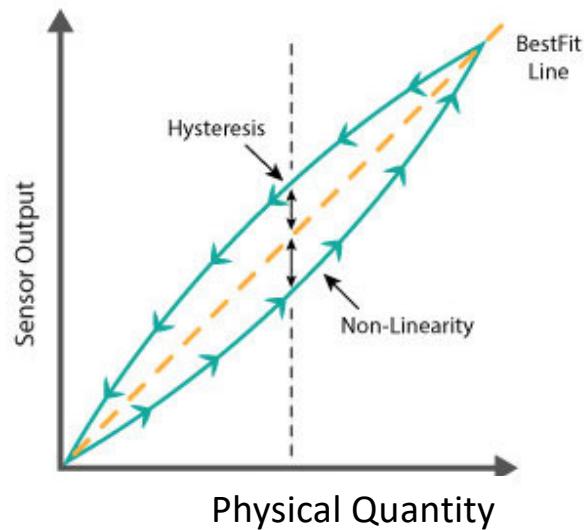


## 6. Non-Linearity

For many transducers a linear relationship between the input and output is assumed over the working range. ie a graph of output plotted against input is assumed to give a straight line. The error is defined as the maximum difference from the straight line shown in the Figure. For example a transducer for the measurement of pressure might be quoted as having a non-linearity error of the  $\pm 0.5\%$  full range.

*Nonlinearity (%) =*  
*Maximum deviation in input / Maximum full scale input*

# 1. Sensor Performance Terminology (4)



## 7. Hysteresis error

Transducers can give different outputs from the same value of quantity being measured according to whether that value has been reached by a continuously increasing change or a continuously decreasing change. This is known as the hysteresis.

Figure shown such an output with the hysteresis error as the maximum difference in output for increasing and decreasing values.

# 1. Sensor Performance Terminology (5)

## 8. Resolution

Resolution is the smallest detectable incremental change of input parameter that can be detected in the output signal. Resolution can be expressed either as a proportion of the full-scale reading or in absolute terms. For example, if a LVDT sensor measures a displacement up to 20 mm and it provides an output as a number between 1 and 100 then the resolution of the sensor device is 0.2 mm.

## 9. Stability

Stability is the ability of a sensor device to give same output when used to measure a constant input over a period of time. The term 'drift' is used to indicate the change in output that occurs over a period of time. It is expressed as the percentage of full range output.

## 10. Dead band/time

The dead band or dead space of a transducer is the range of input values for which there is no output. The dead time of a sensor device is the time duration from the application of an input until the output begins to respond or change.

# 1. Sensor Performance Terminology (6)

## 11. Repeatability

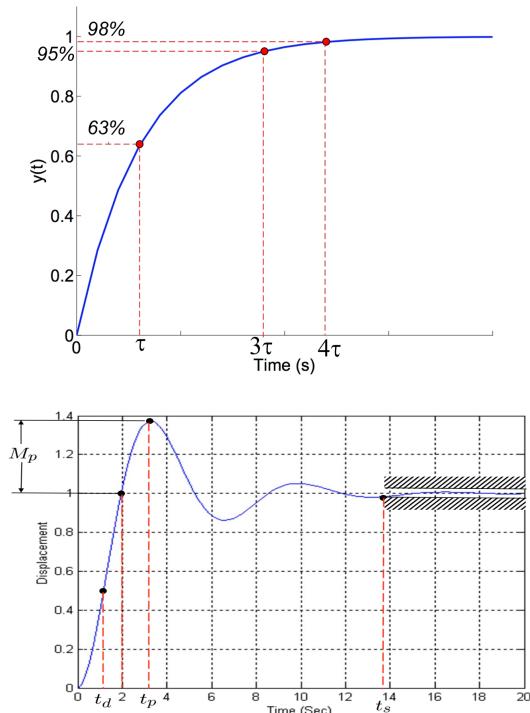
It specifies the ability of a sensor to give same output for repeated applications of same input value. It is usually expressed as a percentage of the full range output:

$$\text{Repeatability} = (\text{maximum} - \text{minimum values given}) \times 100 / \text{full range}$$

## 12. Output impedance

When a sensor is giving an electrical output is interfaced with an electronic circuit it is necessary to know the input impedance since this impedance is being connected either in series or parallel with the circuit. The inclusion of the sensor can thus significantly modify the behavior of the system to which it is connected.

# 1. Static and Dynamic Characteristics of Sensors



- Response time

Response time describes the speed of change in the output on a step-wise change of the measurand. It is always specified with an indication of input step and the output range for which the response time is defined.

- Time constant

This refers to the time taken to reach 63.2% of the steady state value.

- Rise time

This is the time taken for the output to rise to some specified percentage of the steady-state output. Often the rise time refers to the output to rise from 10% of the steady-state value to 90-95% of the steady-state value.

- Settling time

This is the time taken for the output to settle to within some percentage eg: 2% of the steady state value

# 1. Sensor Performance Terminology (7)

## Example:

A strain gauge pressure transducer has the following information in its specification:

Ranges:	70 to 1000 kPa, 2000 to 70 000 kPa
Supply voltage:	10 V dc or ac r.m.s.
Full range output:	40 mV
Non-linearity and hysteresis:	±0.5% full range output
Temperature range:	54°C to +120°C when operating
Thermal zero shift:	0.030% full range output/°C

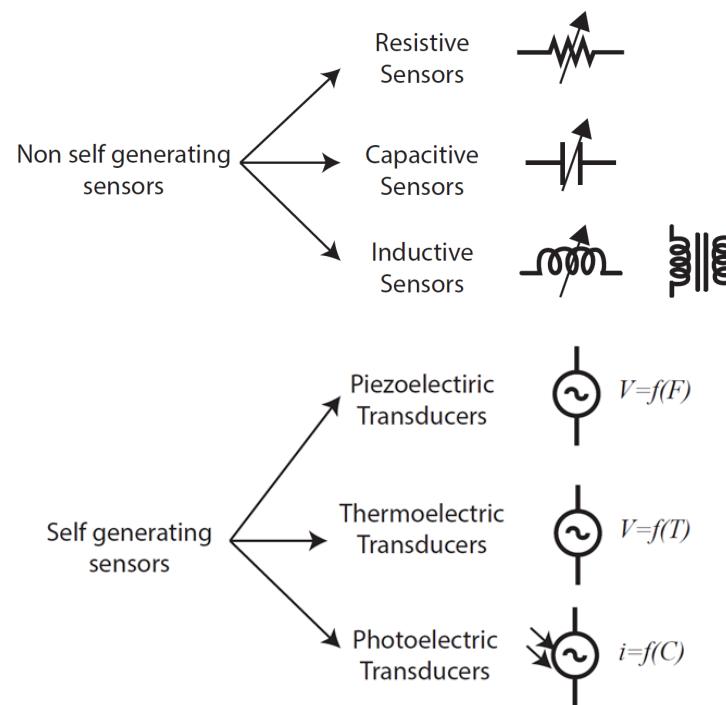
Explain what each of the specifications means?

## 3.1 Types of sensors

- Mechanical measurement systems can be broadly classified to **non-self generating sensors** and **self generating sensors**.
- **Non-self generating sensors**: sensing elements that do not produce a voltage or current signal in response to a measured physical quantity.
- **Self generating sensors**: sensing elements that produce voltage or current signals in response to a measured physical quantity.

## 3.1 Types of sensors

- Depending on the type of the sensing element used, these can be further categorized as follows.



## 3.2 Types of sensors

### **Resistive measurement systems**

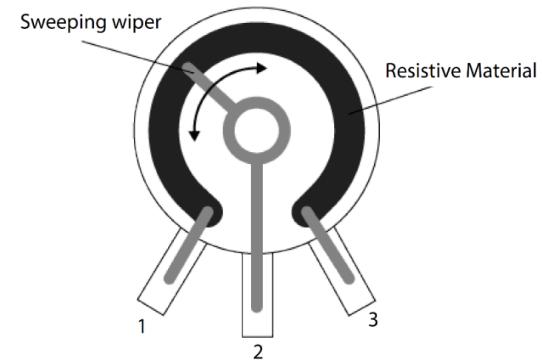
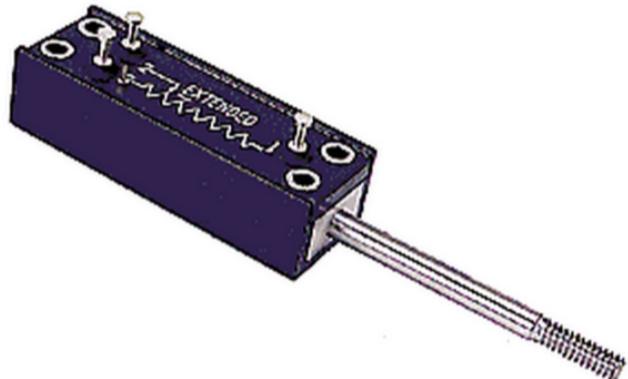
- Can be thought of as variable resistors which changes its resistance in response to some physical variable.
- Resistance of a conductor is governed by the following equation:

$$R = \frac{\rho l}{A}$$

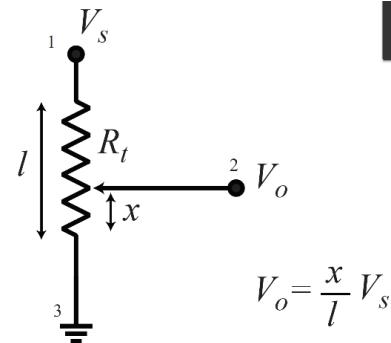
*R* :Resistance  
*ρ* :Resistivity  
*l* :Length of conductor  
*A* :Cross sectional area

### 3.2.1 Measurement of lengths

**Potentiometers:** – measure linear or angular displacement by changing the effective length of a conductor.



**Conditioning circuit:**

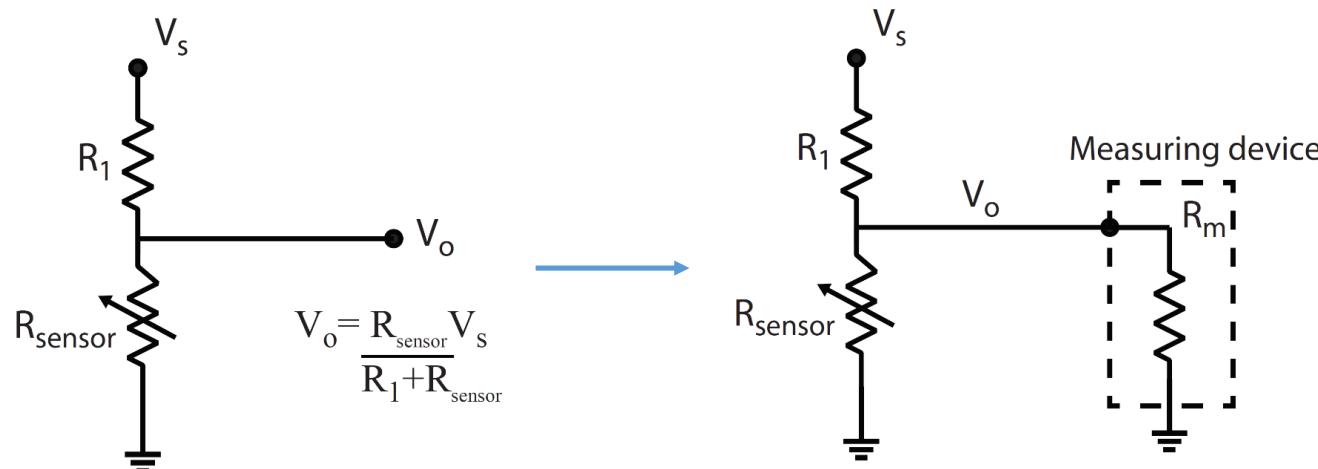


$$V_o = \frac{x}{l} V_s$$

### 3.2.1 Measurement of lengths

#### Inter-stage loading errors

- The resistance changes when the potentiometer is connected to a measuring device. This is termed the loading error.



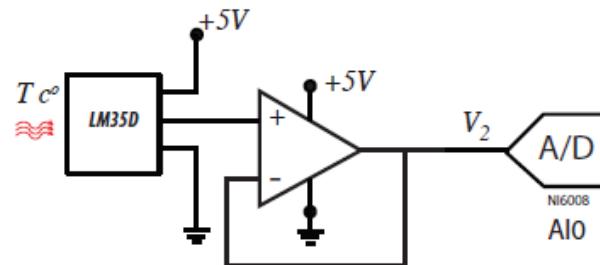
*Exercise:* A Potentiometer which has a total resistance of  $500\Omega$  is connected to a measuring device which has an input impedance of  $10000\Omega$ . The potentiometer is adjusted to 50% and supply voltage is 10V. What is the loading error of the system as a percentage of full scale output?

### 3.2.1 Measurement of lengths

#### Inter-stage loading errors

- A voltage follower can be used to minimize the effects of inter stage loading.
- The Op-amp has a very high input impedance which draws a very low current from the connecting circuit, minimizing any loading effects.

**Exercise 1:** A voltage follower is used to minimize the loading effects evident when the sensor is directly interfaced to a microcontroller.

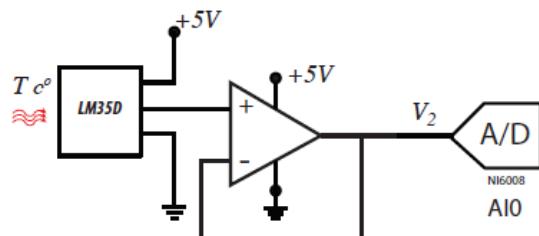


### 3.2.2 Measurement of Temperature

#### Temperature ICs:

- Integrated-circuit (IC) temperature devices directly output a voltage which is linearly proportional to temperature.

**Exercise 2:** The following table summarizes the specifications of a LM35 Temperature measurement IC. Find the total accuracy of the measurement system in  $^{\circ}C$  using the provided data.



Sensitivity	10 mV/ $^{\circ}C$
LM 35 Accuracy	$\pm 1.5^{\circ}C$
Resolution of DAQ	9.766 mV
Precision error	$\pm 0.5^{\circ}C$

## 3.1.2 Measurement of Temperature

### RTDs – Resistance temperature detectors

- RTDs are sensing elements which undergoes a change in resistance in response to changing temperature.
- Temperature sensors are calibrated using the standard fixed point temperatures (ex: triple point of water)

**Table 8.1** Temperature Fixed Points as Defined by ITS-90

Defining Suite	Temperature <sup>a</sup>	
	K	°C
Triple point of hydrogen	13.8033	-259.3467
Liquid-vapor equilibrium for hydrogen at 25/76 atm	≈17	≈-256.15
Liquid-vapor equilibrium for hydrogen at 1 atm	≈20.3	≈-252.87
Triple point of neon	24.5561	-248.5939
Triple point of oxygen	54.3584	-218.7916
Triple point of argon	83.8058	-189.3442
Triple point of water	273.16	0.01
Solid-liquid equilibrium for gallium at 1 atm	302.9146	29.7646
Solid-liquid equilibrium for tin at 1 atm	505.078	231.928
Solid-liquid equilibrium for zinc at 1 atm	692.677	419.527
Solid-liquid equilibrium for silver at 1 atm	1234.93	961.78
Solid-liquid equilibrium for gold at 1 atm	1337.33	1064.18
Solid-liquid equilibrium for copper at 1 atm	1357.77	1084.62

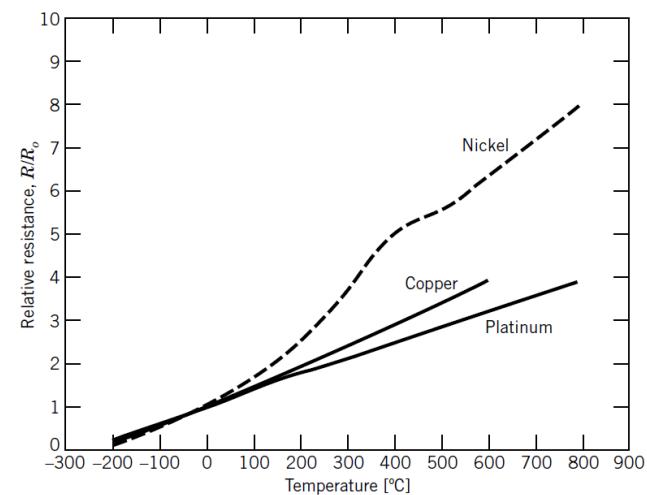
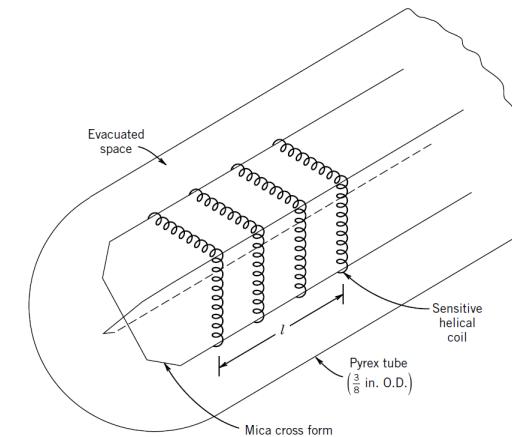
### 3.1.2 Measurement of Temperature

#### RTDs

- The change in resistance of metals are attributed to thermal expansion and the change in resistivity.

$$R = \frac{\rho l}{A}$$

- The design attempts to remove stress of the wires to reduce errors.
- Graph shows resistance vs temperature for common sensing elements.



### 3.1.2 Measurement of Temperature

#### RTDs

- The change in resistance of a RTD is best described by the R-T table.

°C	0	1	2	3	4	5	6	7	8	9	°C
0	100.0000	100.3907	100.7814	101.1719	101.5623	101.9526	102.3427	102.7328	103.1227	103.5125	0
10	103.9022	104.2918	104.6813	105.0706	105.4599	105.8490	106.2380	106.6269	107.0156	107.4043	10
20	107.7928	108.1813	108.5696	108.9578	109.3458	109.7338	110.1216	110.5094	110.8970	111.2845	20
30	111.6718	112.0591	112.4463	112.8333	113.2202	113.6070	113.9937	114.3802	114.7667	115.1530	30
40	115.5392	115.9254	116.3113	116.6972	117.0830	117.4686	117.8541	118.2395	118.6248	119.0100	40
50	119.3951	119.7800	120.1648	120.5495	120.9341	121.3186	121.7030	122.0872	122.4713	122.8554	50
60	123.2392	123.6230	124.0067	124.3902	124.7737	125.1570	125.5402	125.9233	126.3063	126.6891	60
70	127.0718	127.4545	127.8370	128.2194	128.6016	128.9838	129.3658	129.7478	130.1296	130.5113	70
80	130.8928	131.2743	131.6556	132.0369	132.4180	132.7990	133.1799	133.5606	133.9413	134.3218	80
90	134.7022	135.0825	135.4627	135.8428	136.2227	136.6026	136.9823	137.3619	137.7414	138.1207	90
100	138.5000	138.8791	139.2582	139.6371	140.0159	140.3945	140.7731	141.1515	141.5299	141.9081	100

R-T Table – PT100 RTD

Polynomial fits are used as approximate calibration models

$$R = R_0 + \alpha * T$$

$R$ : Resistance at temperature  $T$

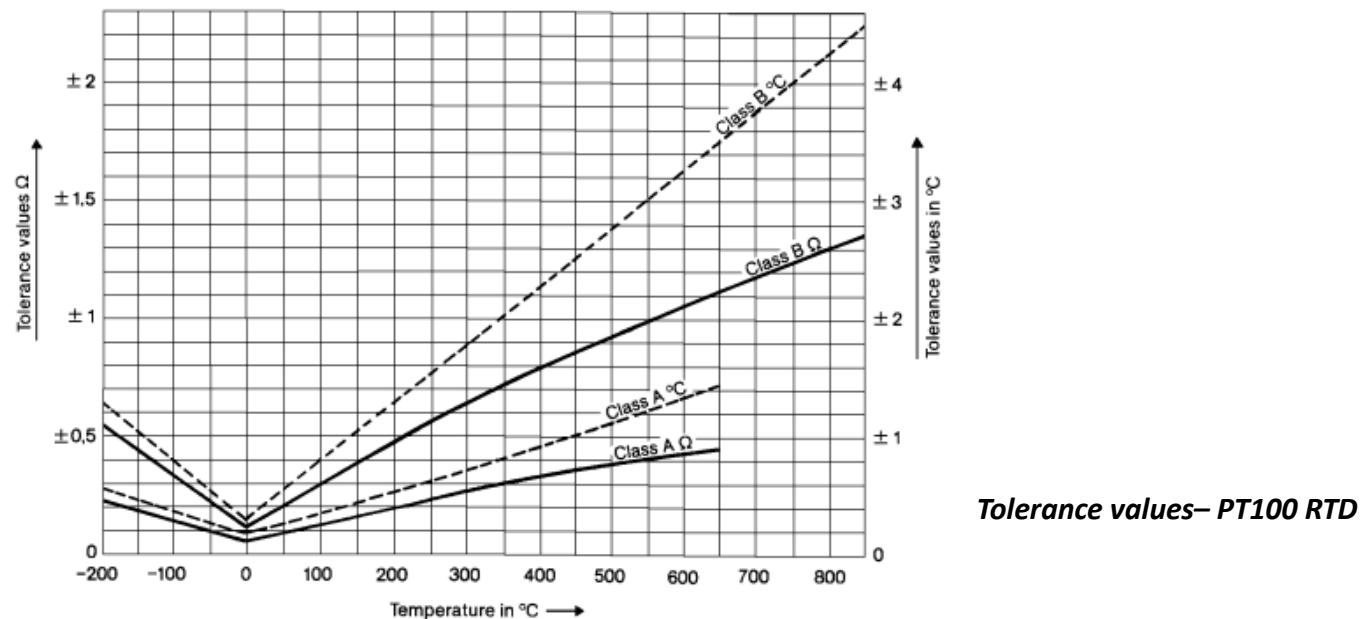
$T$ : Measured temperature in Celsius

$R_0$  : Reference resistance at 0 °C

### 3.1.2 Measurement of Temperature

#### RTDs

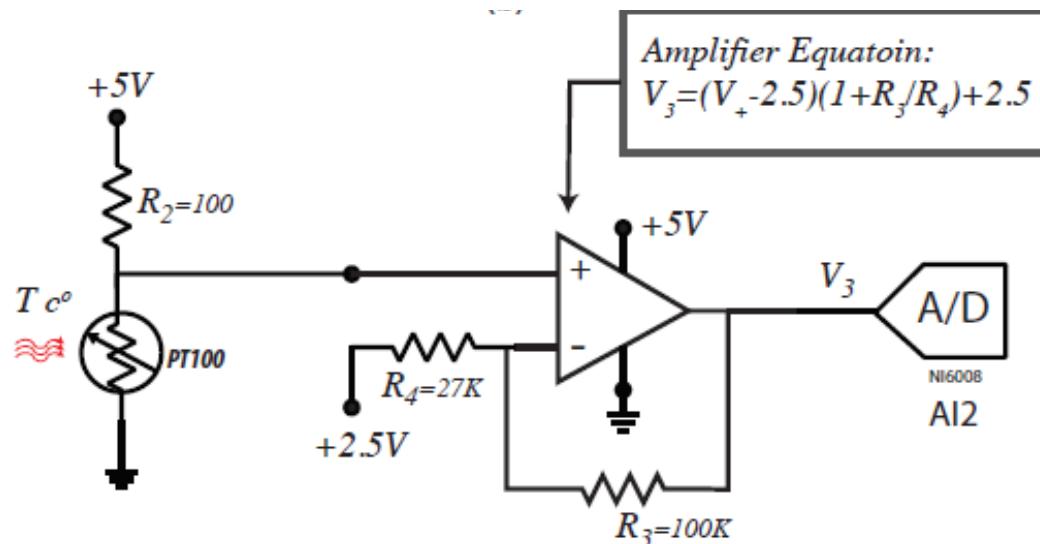
- The accuracy of RTDs depends on the tolerance class of the sensor and temperature range that is measured.



### 3.1.2 Measurement of temperature

#### Exercise 3:

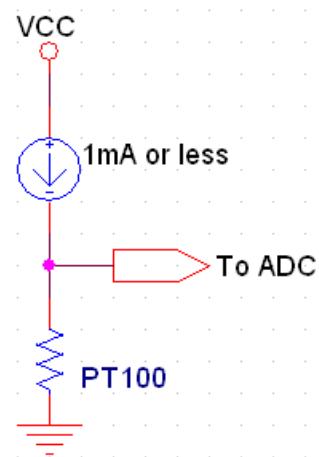
The figure shows a temperature measurement system which uses a PT100 RTD. A voltage divider convert the resistance value to a voltage value and a non inverting amplifier improves the sensitivity of the system. Find the output  $V_3$  as a function of the measure and  $T^{\circ}C$ .



### 3.1.2 Measurement of Temperature

#### Practical Considerations:

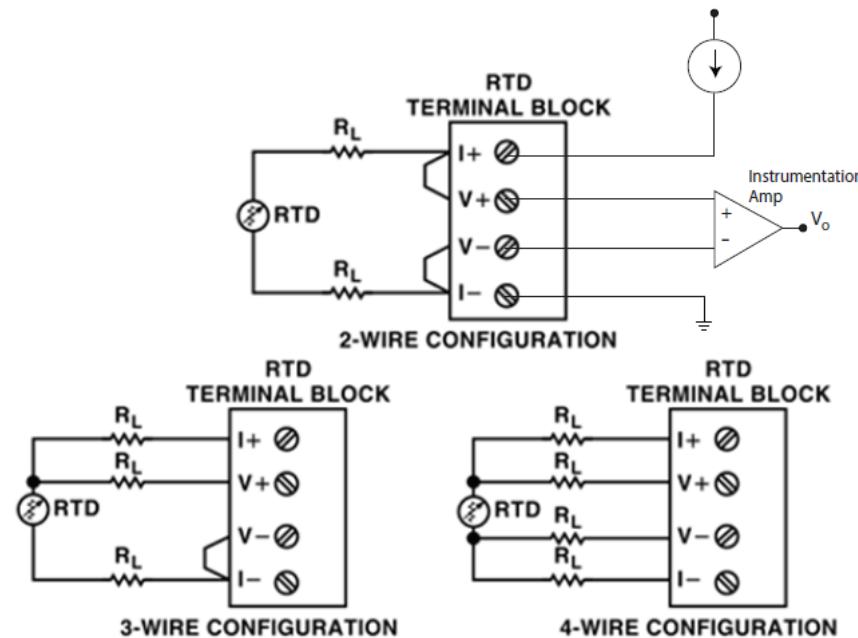
- Use of a Voltage divider for a RTD makes the output voltage nonlinear.  
Use of a current source is recommended for linear response.
- Self heating of the sensor adversely affects the sensor reading. Using a very low current to excite the RTD is recommended to minimize this effect.



### 3.1.2 Measurement of Temperature

#### Practical Considerations:

- Lead wire resistance is a major source of error. This is compensated by 3 wire or 4 wire circuits.



### 3.1.2 Measurement of Temperature

**Thermistors** – semiconductor sensors which change resistance in response to temperature change.

- Highly sensitive and fast response than RTD's, but lesser range of operation.
- High resistance eliminates lead wire errors.
- When the coefficients are positive the devices are categorized as PTC's (Positive Temperature Constant), if negative as NTC's.
- Has a nonlinear relationship between resistance and temperature. Thermistors are typically NTC devices.

$$R = R_0 e^{\beta(1/T - 1/T_0)}$$

$R$ : Resistance of the device at Temperature  $T$   $K^o$

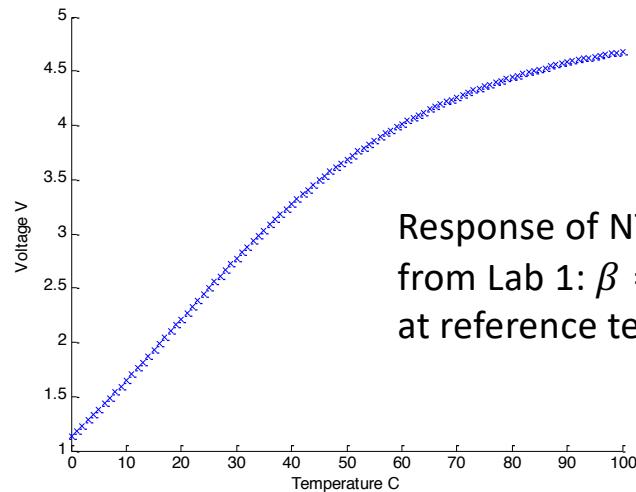
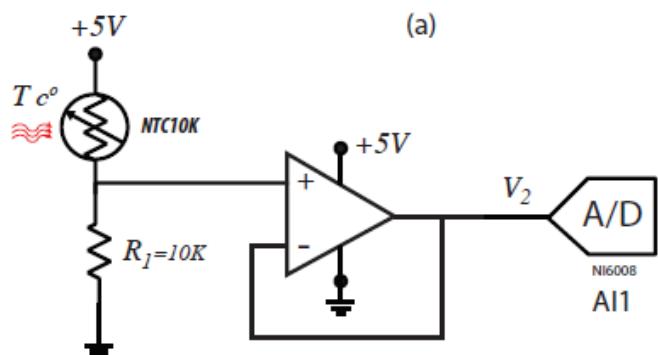
$R_0$  : Resistance of the device at  $T_o$   $K^o$

$\beta$ : The beta value of the thermistor

### 3.1.2 Measurement of temperature

#### Thermistors

- Voltage divider circuits are used to perform signal conditioning.
- Can be designed to achieve linear response for a specific range.



Response of NTC thermistor circuit  
from Lab 1:  $\beta = 3984^{\circ}K$ ,  $R_{25} = 10K$   
at reference temperature  $25^{\circ}C$

# Summary

## Summary of temperature measurement methods

**Temperature Measurement Comparison Chart**

Criteria	Thermocouple	RTD	Thermistor
Temp Range	-267°C to 2316°C	-240°C to 649°C	-100°C to 500°C
Accuracy	Good	Best	Good
Linearity	Better	Best	Good
Sensitivity	Good	Better	Best
Cost	Best	Good	Better

### **Web Resources:**

Temperature measurement: <http://www.ni.com/temperature/>

Strain measurement : <http://www.ni.com/strain/>

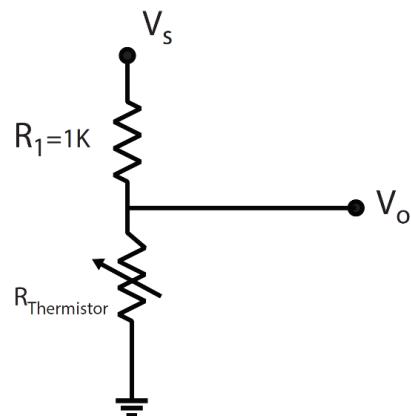
Displacement measurement : <http://www.micro-epsilon.com/displacement-position-sensors/index.html>

# Questions

*Q3.* A thermistor is placed in a  $100C^\circ$  environment, and its resistance measured as  $20,000\Omega$ . The material constant,  $\beta$ , for this thermistor is  $3650C^\circ$ . If the thermistor is then used to measure a particular temperature, and its resistance is measured as  $500\Omega$ , determine the thermistor temperature. (Use the Steinhart?Hart equation)

(Use the beta equation in Kelvin scale)

*Q4.* The figure shows a thermistor attached to a voltage divider. The thermistor has a resistance of  $R_{25} = 2000\Omega$  at the reference temperature  $25C^\circ$ . The material constant  $\beta$  for the thermistor is  $3528K^\circ$ . The voltage divider has a series resistance of  $1K\Omega$  and a supply voltage of  $10V$ . Find the output voltage of the circuit at  $25C^\circ$ ,  $50C^\circ$ , and  $75C^\circ$ .



### 3.1.3 Strain Gauge Basics 1

Expand resistance R into a Taylor Series

- Ignore higher order terms

$$R = R_0 + \Delta R$$

$$\Delta R = \frac{\partial R}{\partial L} \Delta L + \frac{\partial R}{\partial A} \Delta A + \frac{\partial R}{\partial \rho} \Delta \rho$$

- Taking partials,

$$\Delta R = \frac{\rho}{A} \Delta L - \frac{\rho L}{A^2} \Delta A + \frac{L}{A} \Delta \rho$$

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} - \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho}$$



Resistance of wire:

$$R = \frac{\rho L}{A}$$

$\rho$ : Resistivity

$L$ : Length of wire

$A$ : Cross sectional area

$\nu$ : Poisons ratio

$\varepsilon$ : Strain

### 3.1.3 Strain Gauge Basics 2

- Poisson's ratio ( $\nu$ ) is the signed ratio of lateral strain to axial strain is also a material property. This property is called Poisson's ratio, defined as,

$$\nu = \frac{|\text{Lateral strain}|}{|\text{Axial strain}|} = \frac{\varepsilon_L}{\varepsilon_a}$$

- For small values of these changes,  $\nu$  is the amount of transversal expansion divided by the amount of axial compression.
- It could be shown that,

$$\nu = \frac{\Delta L}{L} - \frac{\Delta A}{A}$$

- Then the above equation becomes,

$$\frac{\Delta R}{R} = (1 + 2\nu)\varepsilon + \frac{\Delta \rho}{\rho}$$

$$\frac{\Delta R}{\varepsilon} = 1 + 2\nu + \frac{1}{\varepsilon} \frac{\Delta \rho}{\rho}$$

### 3.1.3 Strain Gauge Basics 3

- In the obtained expression,

$$\frac{\Delta R}{R} = 1 + 2\nu + \frac{1}{\varepsilon} \frac{\Delta \rho}{\rho}$$

Resistance Change  
due to  
Change of Length

Resistance Change due to  
Change of area (0 to 0.5  
for all material)

Resistance Change due  
to piezo resistance  
effect

[ Very small for  
metallic strain gauges]

$$G_f = 1 + 2\nu + \frac{1}{\varepsilon} \frac{\Delta \rho}{\rho}$$

$$\frac{\Delta R}{R} = G_f \varepsilon$$

Where,  $G_f$  is gauge factor

### 3.1.3 Strain Gauge Basics 4

#### Metallic Strain Gauges

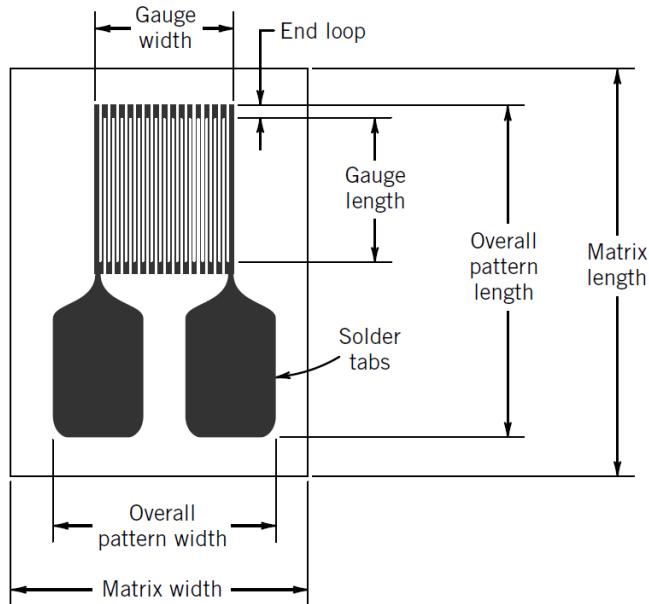
- Gauge Factor:  $G_f = 2.0 \sim 2.2$
- Initial resistance:  $R_0 = 120 \pm 1 \Omega$
- Change in resistance:  $\Delta R = -2.4 \sim 4.8 \Omega$
- Maximum gauge current
  - 15 mA to 100 mA
- Relatively low sensitivity to temperature variation
- Equipment is commercially available that can measure changes in gauge resistance of less than 0.0005 V (0.000001  $\mu\epsilon$ ).

#### Semiconductor Strain Gauge

- Silicon (Si) doped with phosphorous (P), arsenic (As) or boron (B)
- Gauge Factor:  $G_f = 100 \sim 175$
- Initial resistance:  $R_0 = 120 \pm 5 \Omega$
- High sensitivity to temperature variation

### 3.1.4 Measurement of Force

**Strain gauges:** – changes resistance in response to the applied strain.



$$dR/R = d\rho/\rho + dl/l - dA/A$$

$$\frac{dR}{R} = \frac{dL}{L} (1 + 2v_p) + \frac{d\rho_e}{\rho_e}$$

$$G_f = \frac{\Delta R/R}{\Delta l/l} \quad G_f \varepsilon = \frac{\Delta R}{R}$$

Typical values:

80% Ni, 20% Cr, G = 2

45% Ni, 55% Cu, G = 2

Platinum, G = 4.8

95% Pt, 5% Ir, G = 5.1

Semiconductor, G = 70 to 135

### 3.1.4 Measurement of Force

**EX 5:** A steel specimen is subjected to  $1KNm^{-2}$  of stress. This is instrumented with a strain gauge with  $R = 100\Omega$ ,  $G_f = 2$  along the direction of principal stress. What is the change in resistance  $\Delta R$  of the strain gauge? ( $E_{steel} = 200GPa$ )

- The devices have micro sensitivity  $= 10^{-6}\Omega/KNm^{-2}$
- Require a micro sensitive conditioning circuit such as the Wheatstone bridge shown below:

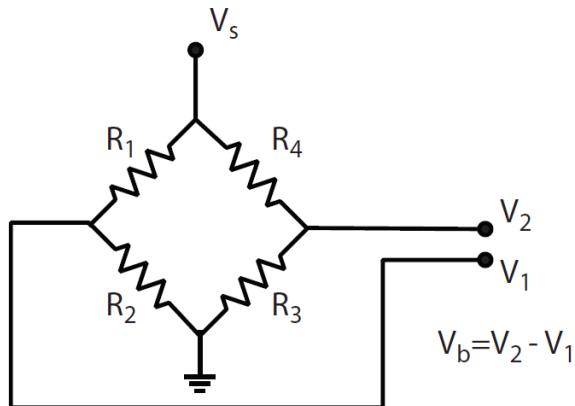


Figure 2: The Wheatstone bridge

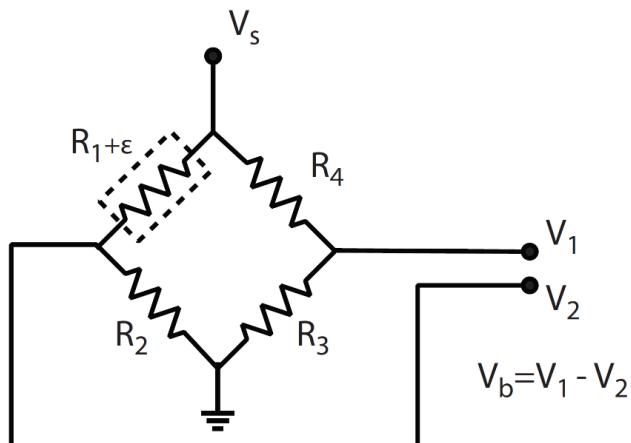
$$V_b = V_2 - V_1 = \left( \frac{R_3}{R_3 + R_4} - \frac{R_2}{R_1 + R_2} \right) V_s$$
$$V_b = \left( \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} \right) V_s$$

Bridge produces zero voltage (is balanced) when:

$$\frac{R_2}{R_1} = \frac{R_3}{R_4}$$

### 3.1.4 Measurement of Force

- Assume a resistive sensor is attached to  $R_1$  and the bridge is initially balanced by using equal resistances. The change in resistance  $\Delta R_1$  produces an imbalance of the bridge voltage output.



$$V_b = \left( \frac{R_3}{R_3 + R_4} - \frac{R_2}{R_1 + \Delta R_1 + R_2} \right) V_s$$
$$R_1 = R_2 = R_3 = R_4 = R$$

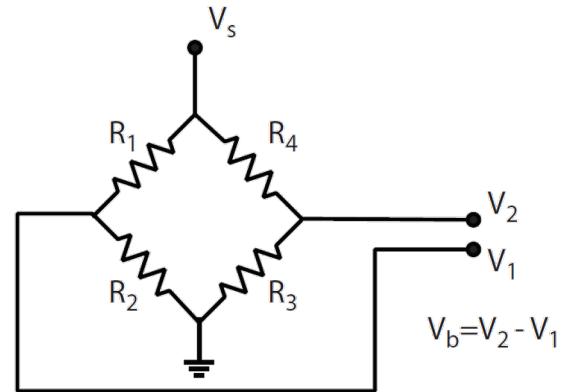
$$V_b = \left( \frac{1}{2} - \frac{R}{2R + \Delta R_1} \right) V_s$$

$$V_b = \frac{\Delta R_1 / R}{4 + 2\Delta R_1 / R} V_s$$

### 3.1.4 Measurement of Force

- The  $\Delta R$  quantities of the denominator can be neglected when its small.
- If all bridge arms change their resistances an approximate solution for output voltage using  $\Delta R$ 's can be found as follows:

$$V_b = \frac{1}{4} \left( \frac{\Delta R_1}{R} - \frac{\Delta R_2}{R} + \frac{\Delta R_3}{R} - \frac{\Delta R_4}{R} \right) V_s$$

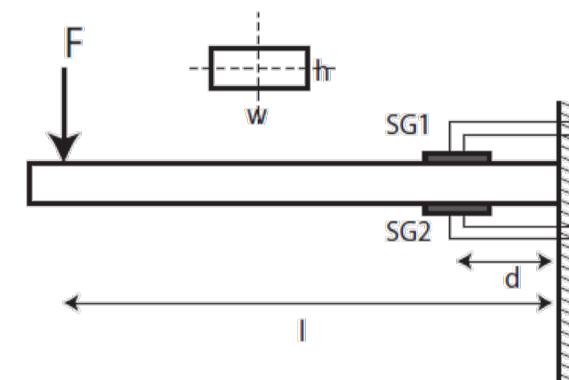
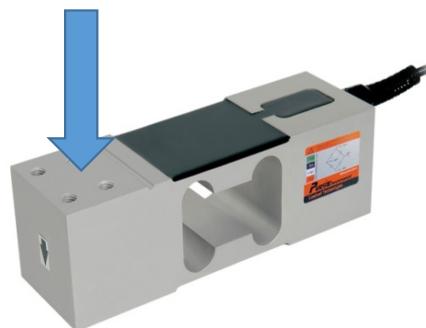
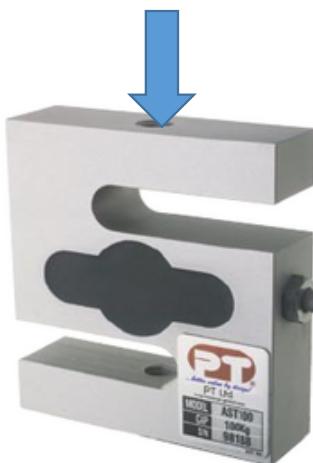


EX 6: Consider a Wheatstone bridge, which initially has all arms of the bridge equal to  $100\Omega$ , with a resistive temperature sensor connected to  $R_1$ . The input supply voltage of the bridge is 10V. If the temperature of  $R_1$  changes which produces an output of 0.569V, what is the resistance of the sensor? What is the error if a linearized approximation is used? What is the current drawn, and power dissipated by the resistors when the bridge balanced?

### 3.1.4 Measurement of Force

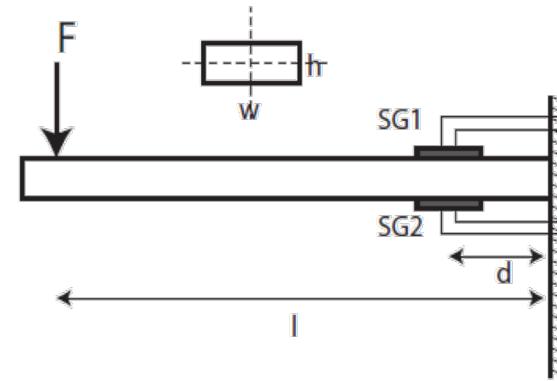
#### Strain gauge applications – Load cells

- Load cells are devices which are designed to measure the force using structural members instrumented with strain gauges.
- Cell types:



### 3.1.4 Measurement of Force

- The cell design determines the following:
  - Sensitivity of the device
  - Range of the device
  - Dynamic response
- The bridge configuration determines the following:
  - Sensitivity of the device (bridge constant)
  - Temperature compensation
  - Axial, bending, load compensation
- The amplifier relates to the gain, and the inter-stage loading effects.



### 3.1.4 Measurement of Force

#### Bridge configurations: 1. Quarter bridge

- The configuration has a sensitivity of  $\frac{V_s G_f}{4}$ . ( $\kappa = 1$ )
- No temperature compensation. I.e, output voltage changes in response to changing  $G_f$ .
- No compensation for axial loads.

#### Bridge configurations: 1. Quarter bridge

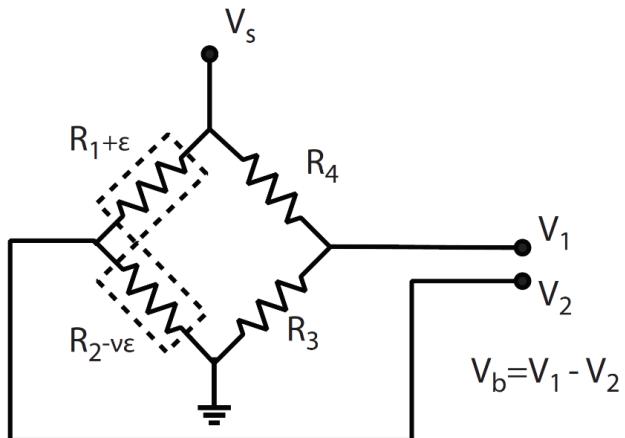
- The configuration has a sensitivity of  $\frac{V_s G_f}{4}$ . ( $\kappa = 1$ )
- No temperature compensation. I.e, output voltage changes in response to changing  $G_f$ .
- No compensation for axial loads.

$$V_b = \frac{1}{4} G_f \varepsilon V_s$$

### 3.1.4 Measurement of Force

#### Bridge configurations: 2.Half bridge

- The configuration has a sensitivity of  $\frac{V_s G_f}{4} (1 + \nu)$ . ( $\kappa = 1 + \nu$ )
- Compensation for temperature effects.
- No compensation for axial loads.



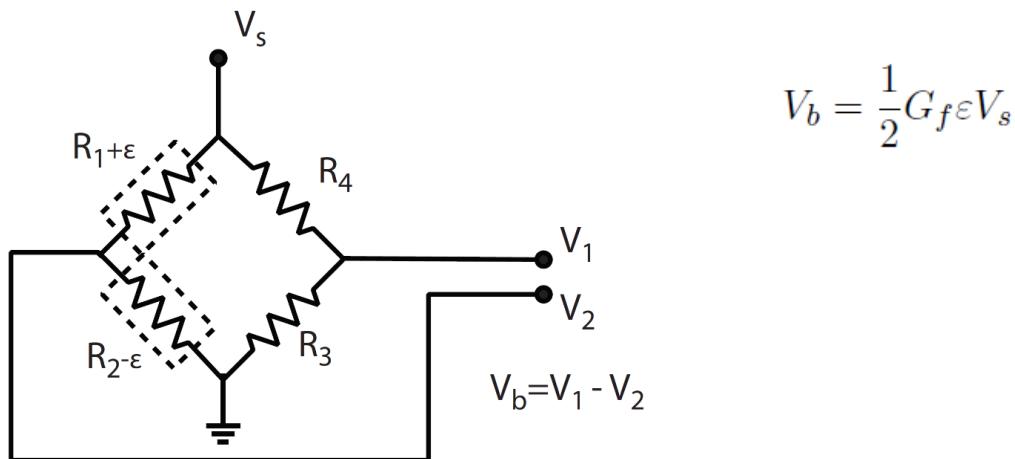
$$V_b = \frac{1}{4} G_f \varepsilon (1 + \nu) V_s$$

$$V_b = V_1 - V_2$$

### 3.1.4 Measurement of Force

#### Bridge configurations: 2.Half bridge

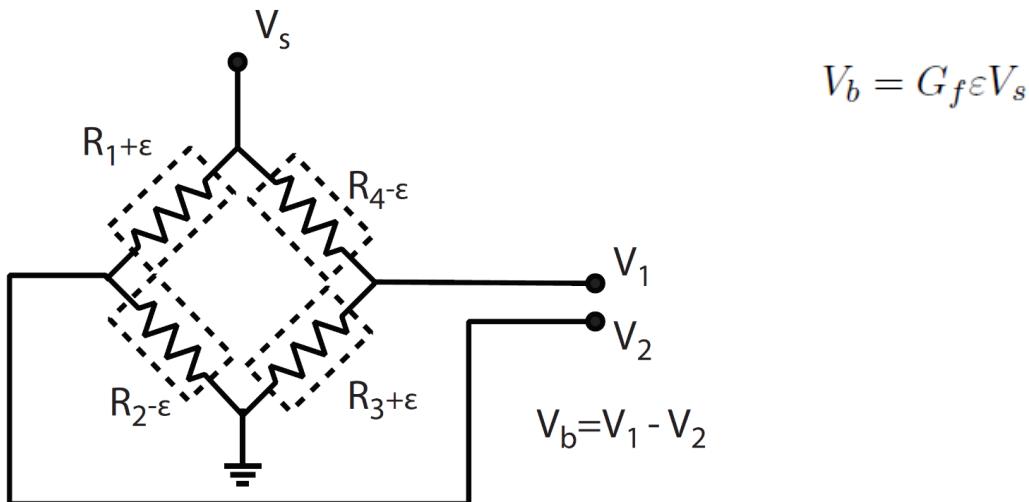
- The configuration has a sensitivity of  $\frac{V_s G_f}{2}$ . ( $\kappa = 2$ )
- Compensation for temperature effects.
- Compensation for axial loads.



### 3.1.4 Measurement of Force

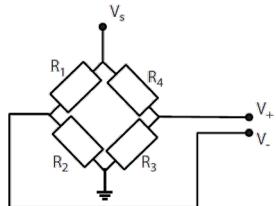
#### Bridge configurations: 4. Full bridge

- The configuration has a sensitivity of  $V_s G_f$ . ( $\kappa = 4$ )
- Compensation for temperature.
- Compensation for axial loads.

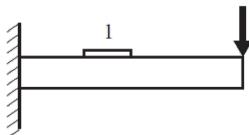


### 3.1.4 Measurement of Force

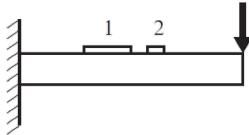
- Same analysis can be used for different loading (axial)



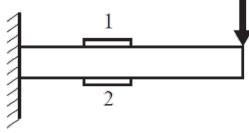
$$V_b = V_s/4(\Delta R_1/R - \Delta R_2/R + \Delta R_3/R - \Delta R_4/R)$$



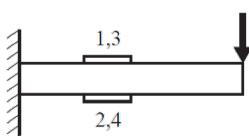
$$V_b = V_s G_f \varepsilon / 4$$



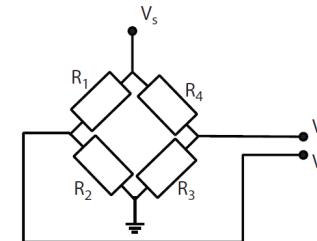
$$V_b = V_s G_f \varepsilon (1+\nu) / 4$$



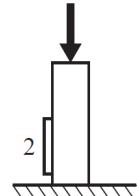
$$V_b = V_s G_f \varepsilon / 2$$



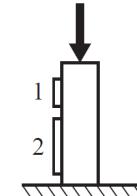
$$V_b = V_s G_f \varepsilon$$



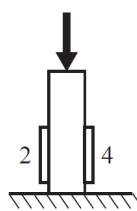
$$V_b = V_s/4(\Delta R_1/R - \Delta R_2/R + \Delta R_3/R - \Delta R_4/R)$$



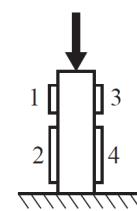
$$V_b = V_s G_f \varepsilon / 4$$



$$V_b = V_s G_f \varepsilon (1+\nu) / 4$$



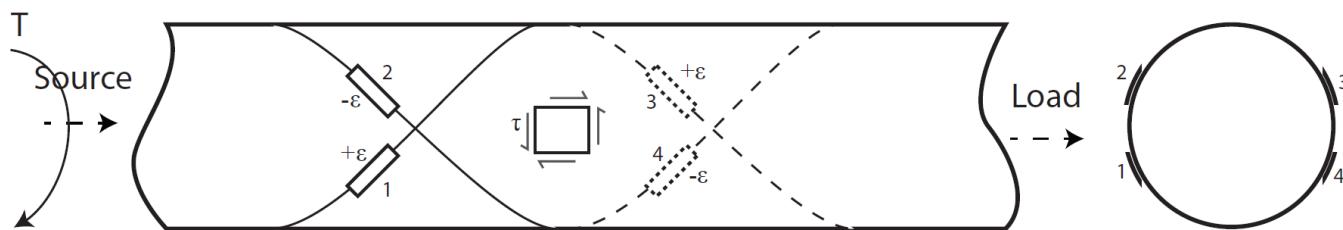
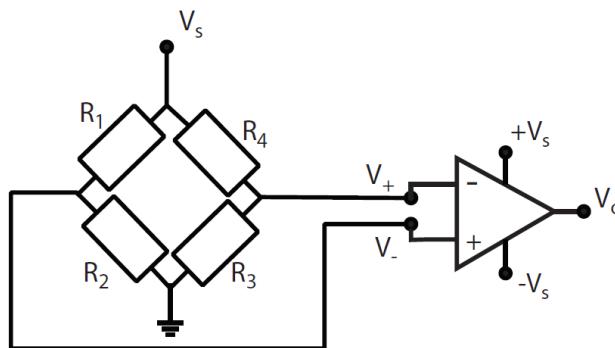
$$V_b = V_s G_f \varepsilon / 2$$



$$V_b = V_s G_f \varepsilon (1+\nu) / 2$$

### 3.1.4 Measurement of Force

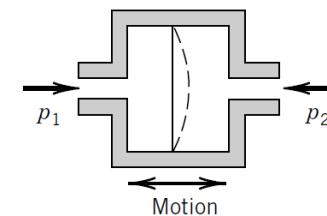
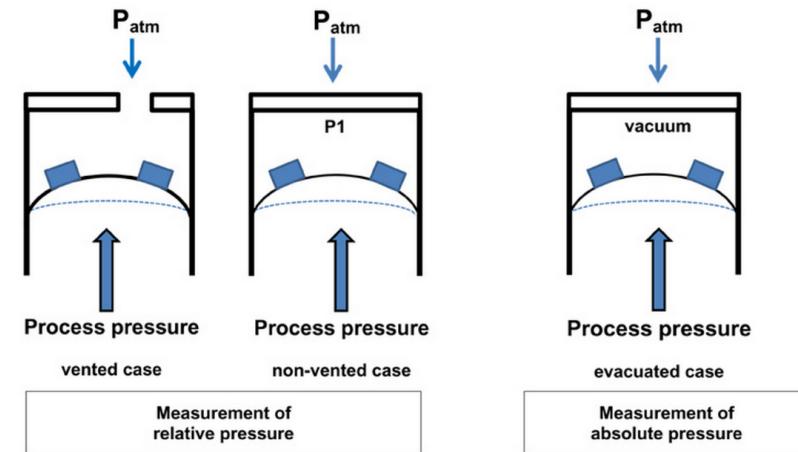
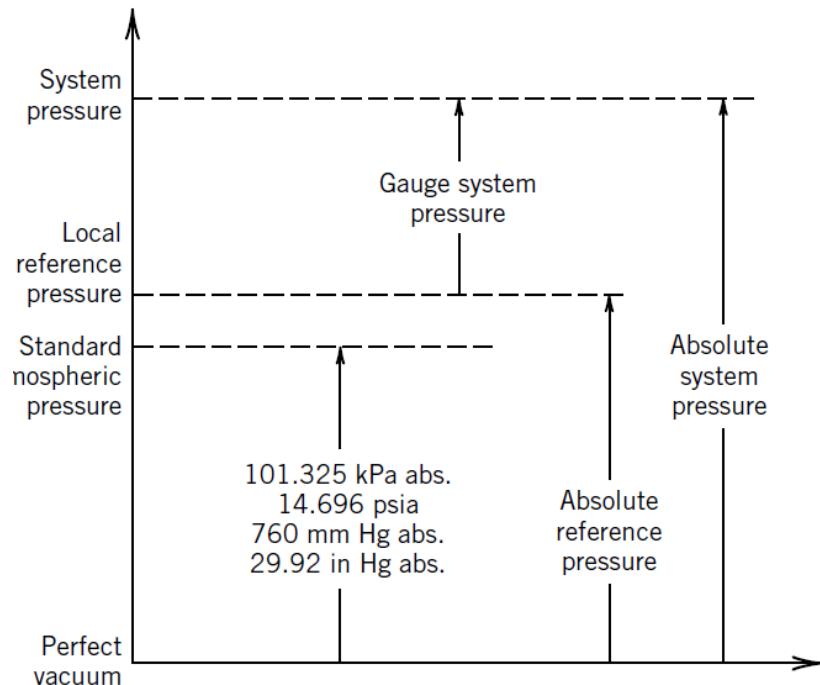
- Torque measurement



$$V_b = \frac{1}{4} \left( \frac{\Delta R_1}{R} - \frac{\Delta R_2}{R} + \frac{\Delta R_3}{R} - \frac{\Delta R_4}{R} \right) V_s$$

### 3.1.5 Measurement of pressure

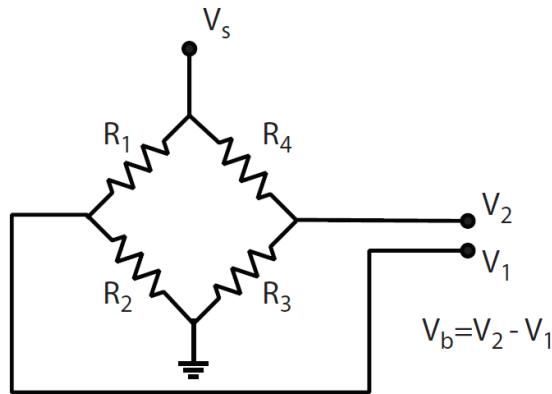
- Pressure measurement



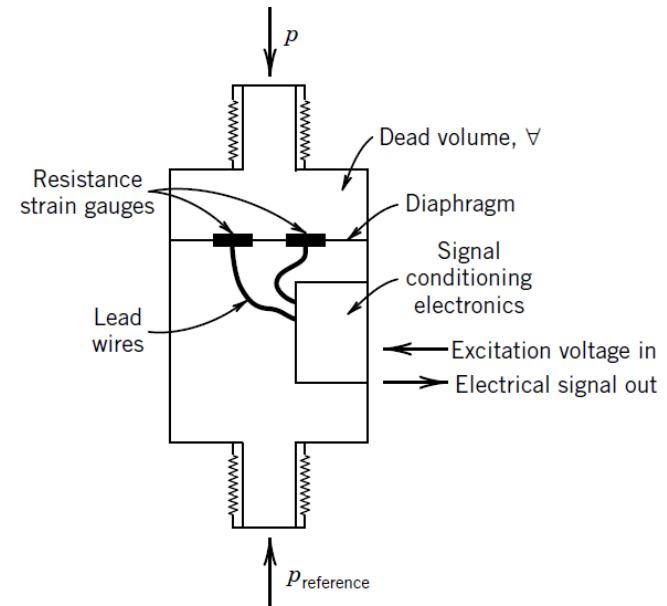
Diaphragm

### 3.1.5 Measurement of pressure

Exercise: Determine the connection for positive voltage output for positive gauge pressure



$$V_b = \frac{1}{4} \left( \frac{\Delta R_1}{R} - \frac{\Delta R_2}{R} + \frac{\Delta R_3}{R} - \frac{\Delta R_4}{R} \right) V_s$$



# Questions

**EX 7:** A steel cantilever beam is fixed at one end and free to move at the other. A load  $F$  of  $980N$  is applied to the free end. Four axially aligned strain gauges ( $G_f = 2$ ) are mounted to the beam a distance  $L$  from the applied load, two on the upper surface,  $R_1$  and  $R_3$ , and two on the lower surface,  $R_2$  and  $R_4$ . The bridge deflection output is passed through an amplifier (gain,  $K = 1000$ ) and measured. For a cantilever, the relationship between applied load and strain is

$$\varepsilon = \frac{6FL}{Ebh^2}$$

$h$  is the beam thickness, and  $b$  is the beam width. Estimate the measured output for the applied load if  $L = 0.1m$ ,  $b = 0.03m$ ,  $h = 0.01m$ , and the bridge excitation voltage is  $5V$ . ( $E_{steel} = 200GPa$ ).

### 3.2.1 Capacitive sensors

#### Capacitive measurement systems

The capacitance between two plates are governed by the following equation:

$$C = \frac{\epsilon_0 K A}{d}$$

$C$  Capacitance

$\epsilon_0$  Permittivity of free space

$K$  dielectric constant

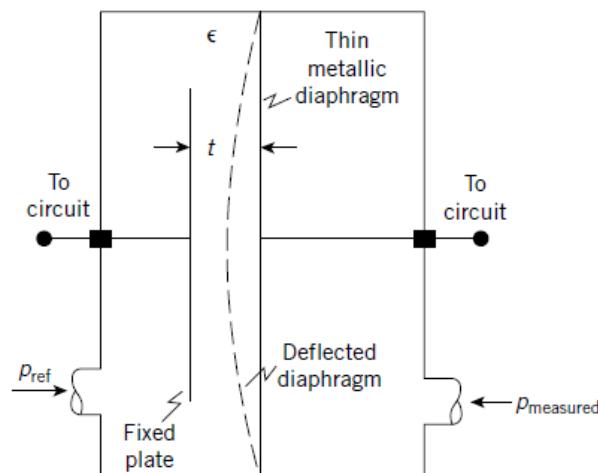
$A$  Area of plates

$d$  separation between plates

- By changing the distance between the plates or the effective area, the capacitance can be changed.

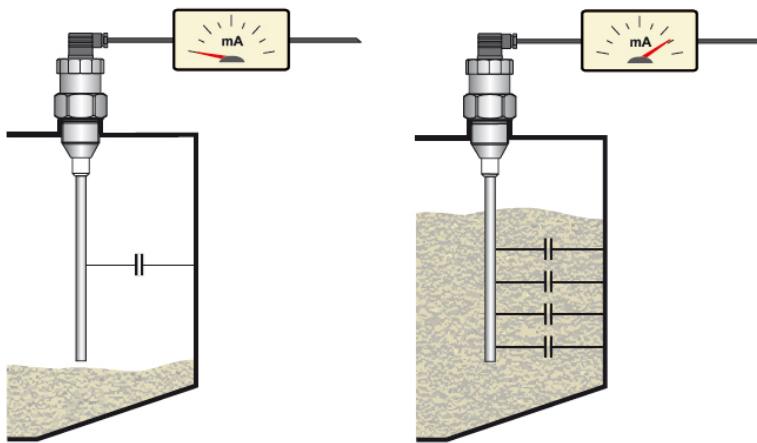
### 3.2.1 Capacitive sensors

Estimate the theoretical capacitance of a sensor similar to that of Figure 9.13 if the plate area is  $1 \text{ mm}^2$  and the instantaneous plate separation is 1 mm. What is the sensor sensitivity? Assume the plates are separated by air.



### 3.2.1 Capacitive sensors

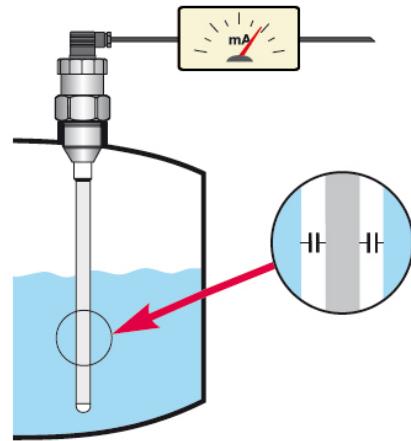
#### Capacitive level sensing



- Non conducting materials in a metallic tank: the tank itself forms one plate of the capacitor and a probe forms the second surface of the capacitor.
- Together forms a sensing element which changes its capacitance with the quantity of material in the tank

### 3.2.1 Capacitive sensors

#### Capacitive level sensing

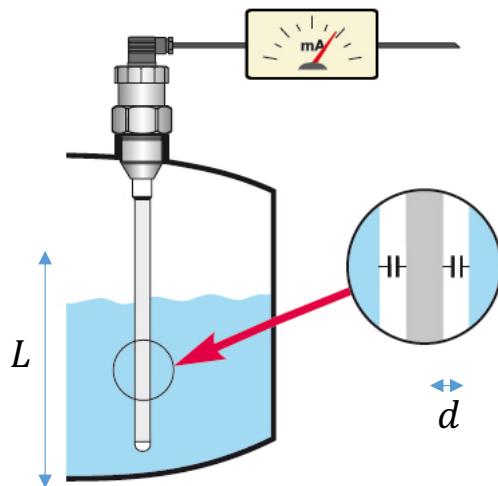


- For conducting materials like water, the probe is made of two insulated cylinders as shown in figure.
- The inner and outer cylinders form a capacitor which changes its capacitance with the amount of liquid between the two cylinders.

### 3.2.1 Capacitive sensors

#### Exercise

The level of ethyl alcohol is to be measured in a tank from 0 to 5m using a capacitive system. The following specifications defines the system:



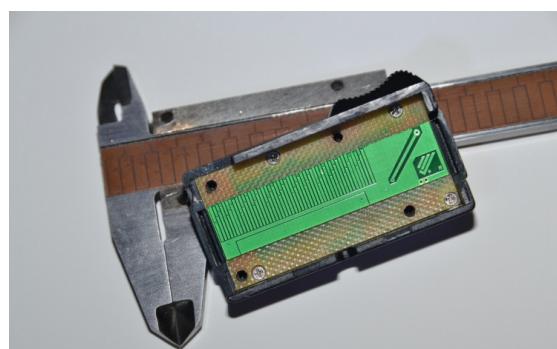
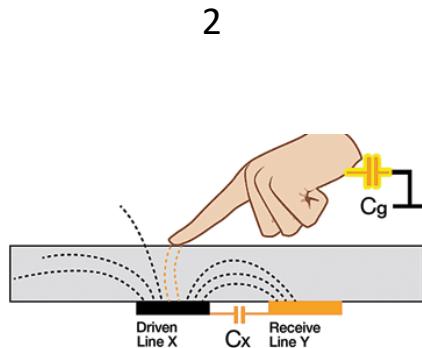
Dielectric constant  $K = 26$   
 $\epsilon_0 = 8.85 \text{ pF/m}$   
Effective plate area  $A = \pi D L$   
 $D = \text{average diameter} = 11.5\text{cm}$   
Cylinder separation  $d = 0.5 \text{ cm}$

Find the range of capacitance as the liquid level changes from 0 to 5m.

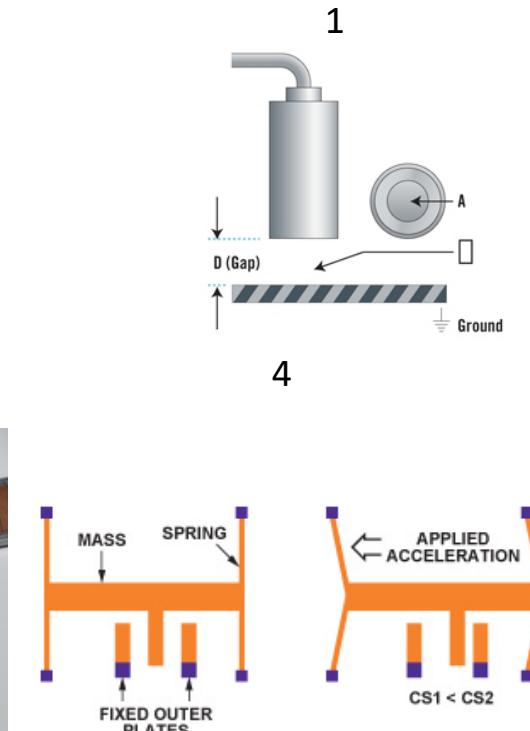
### 3.2.1 Capacitive sensors

#### More applications of capacitive sensing

1. Non contact distance sensing
2. Capacitive touch sensing
3. Digital caliper
4. Capacitive elements in pressure sensors and accelerometers.

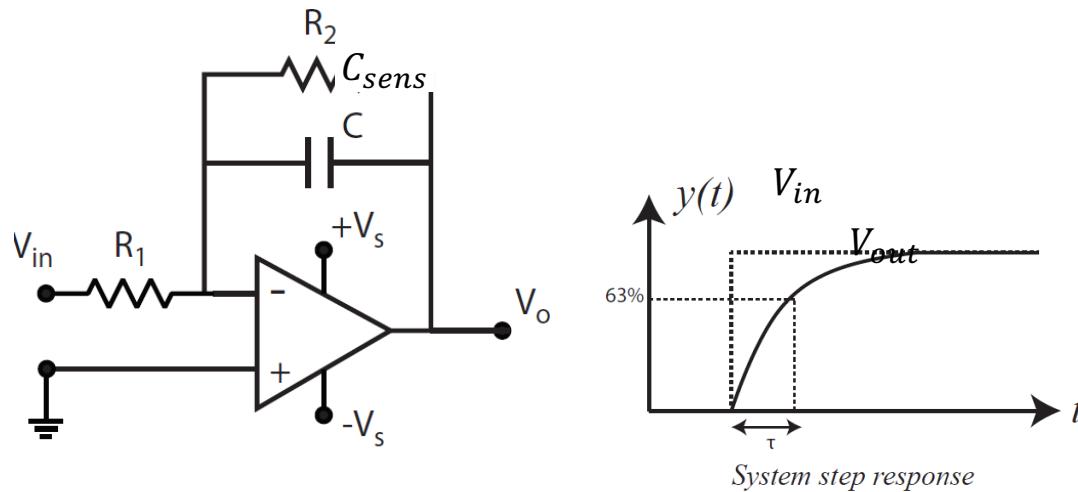


3



### 3.2.1 Capacitive sensors

#### Measuring capacitance



- Simple method: Connect the capacitor to a low pass filter and measure the time constant using a step response.

### 3.2.1 Capacitive sensors

#### Exercise

Capacitance of a level sensor attached to a tank can be modelled using the following equation:

$$C_{sens} = 0.55h + 0.15 \quad \begin{aligned} C_{sens} &: \text{Capacitance in } pF \\ h &: \text{material height of the tank in cm} \end{aligned}$$

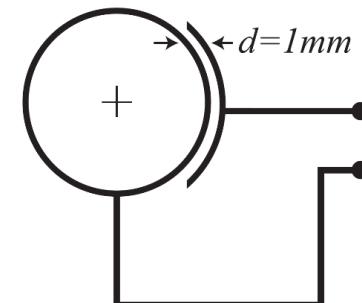
The capacitor is connected to a first order inverting Low-pass filter with  $R_1 = R_2 = 1M\Omega$ .

- (a) Find the range of time constants of the output step response if the level of the tank changes from  $10cm$  to  $100cm$ .
- (b) Design a comparator to generate a positive edge when the output of the low pass filter reaches a value corresponding to the time constant of the filter.

### 3.2.1 Capacitive sensors

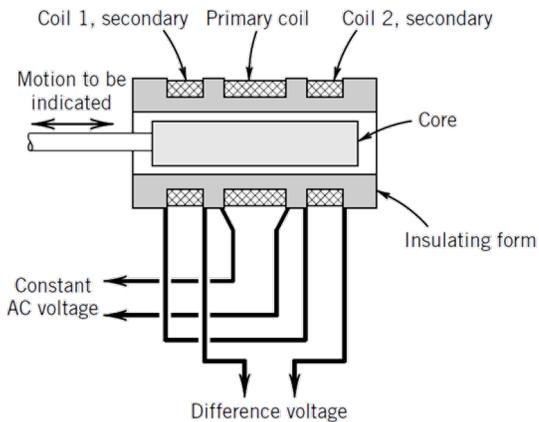
#### Exercise

A Capacitive displacement sensor is used to measure the wobble of a rotating shaft. The capacitance of the sensing element is  $880\text{pF}$  when there is no wobble of the shaft. Find the range of capacitance for a shaft wobble ranging from  $-.02\text{mm}$  to  $+.02\text{mm}$ .



### 3.3.1 Inductive sensors

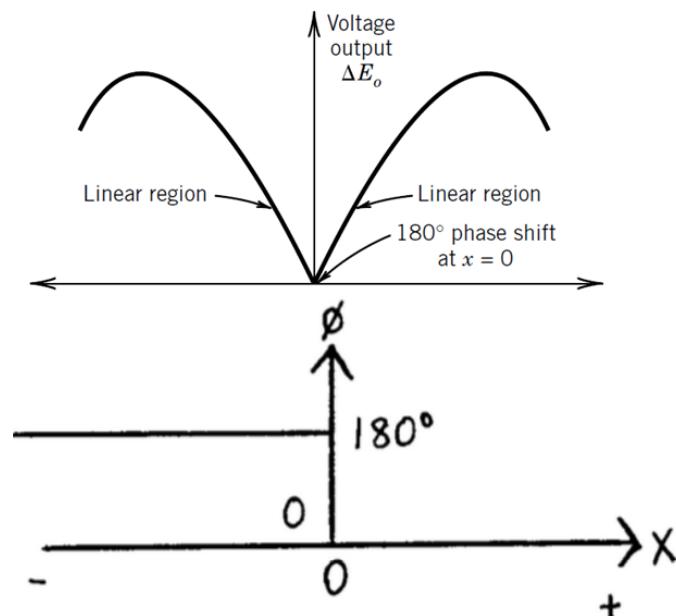
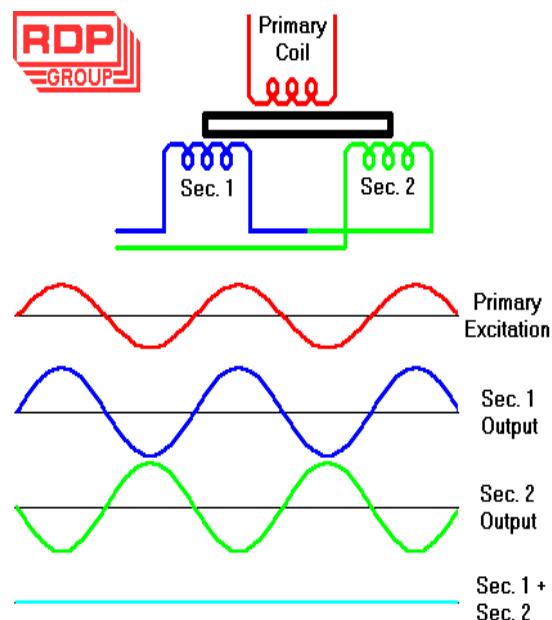
#### Inductive measurement systems – LVDT



- Linear Variable Differential Transformers (LVDTs) are used for precision distance measurement
- The sensor is a transformer which has a moving core.
- The moving core affects the magnetic coupling between a primary and a secondary coil.

### 3.3.1 Inductive sensors

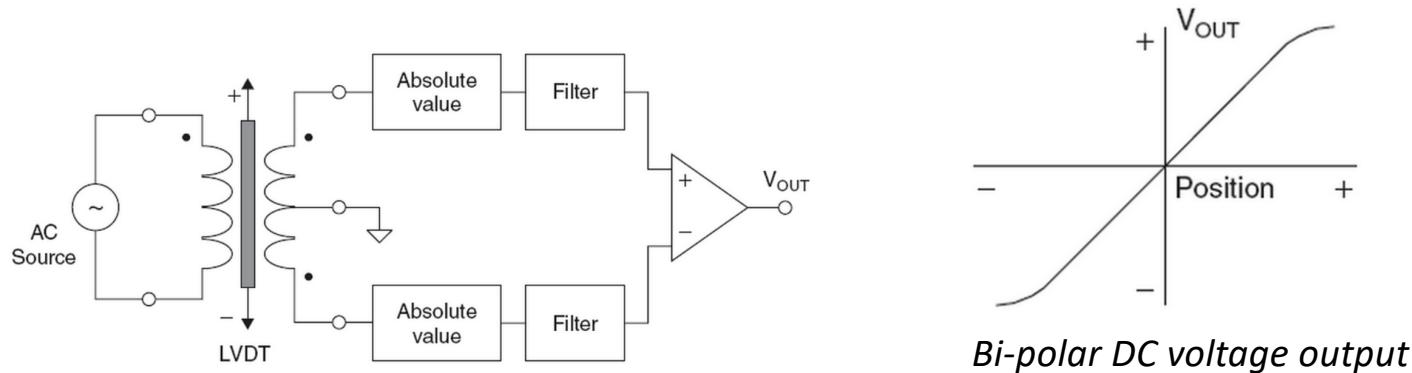
- The voltage difference between the two secondary coils change proportionately with the core movement



Link to animation: <https://hackscape.wordpress.com/2009/11/27/mclvdt/>

### 3.3.1 Inductive sensors

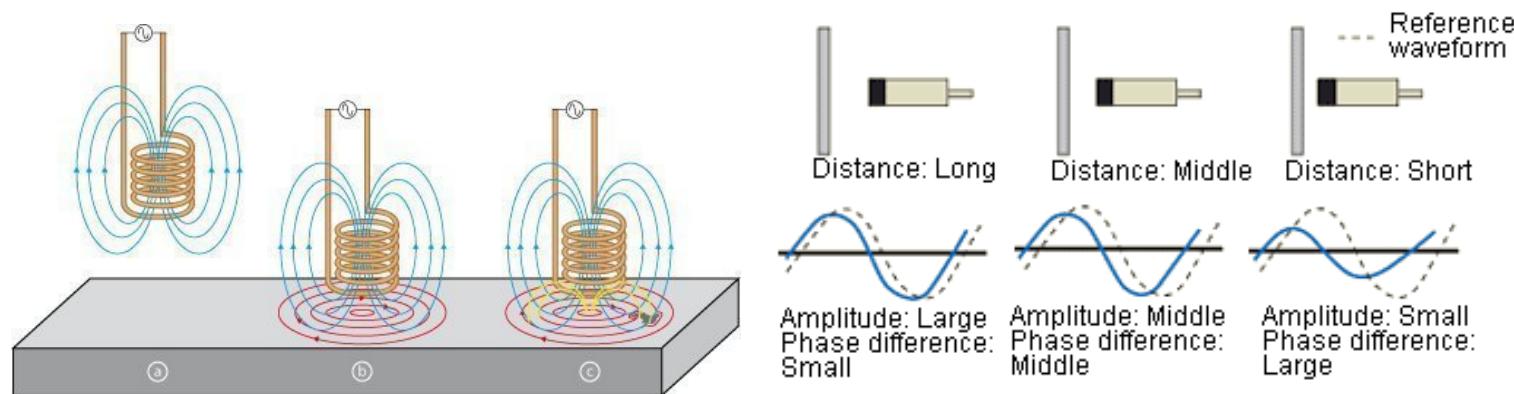
- A Phase sensitive demodulator is used to produce an output signal which can be used to distinguish the direction of motion.



- AD 598 – LVDT signal conditioner can be used as a integrated device which can be used to drive and generate a bipolar DC voltage signal from an LVDT.

### 3.3.1 Inductive sensors

#### Eddy current sensor



- The driver creates an alternating current in the sensing coil which creates an alternating magnetic field with induced small currents in the target material; these currents are called eddy currents.
- This creates an opposing field which is measured by a receiving coil. The induced AC current of the receiving coil changes with the distance to a metallic material.

# Summary

## Summary of displacement measurement methods

Characteristics	Potentiometers	AC-operated LVDTs
<b>Range</b>	<b>0.1 – 20"</b> <b>0.2 (2.5 – 500mm)</b>	<b>0.02 – 20"(0.5-500mm)</b>
<b>Accuracy</b>	<b>Moderate</b>	<b>Very Good</b>
<b>Resolution</b>	<b>Moderate</b>	<b>Excellent</b>
<b>Repeatability</b>	<b>Fair</b>	<b>Excellent</b>
<b>Temperature Resistance</b>	<b>Fair</b>	<b>Excellent</b>
<b>Linearity</b>	<b>Moderate</b>	<b>Good</b>
<b>Cost</b>	<b>Low</b>	<b>Moderate</b>

Principle	Eddy current	Capacitive
Measuring range	mm	0.4 - 80
Linearity	µm	8
Reachable resolution	µm	0.02
Bandwidth	kHz	up to 100
Sampling rate	kHz	up to 50
Temperature range	°C	-50 ... 350
Temperature stability	FSO / °C	< ± 0.015
		< 0.0005

Contact sensing



Non Contact sensing



### 3.4.1 Piezo electric sensors

This presentation looks at capacitive, Inductive and piezoelectric sensing elements

#### **Piezo electric Transducers**

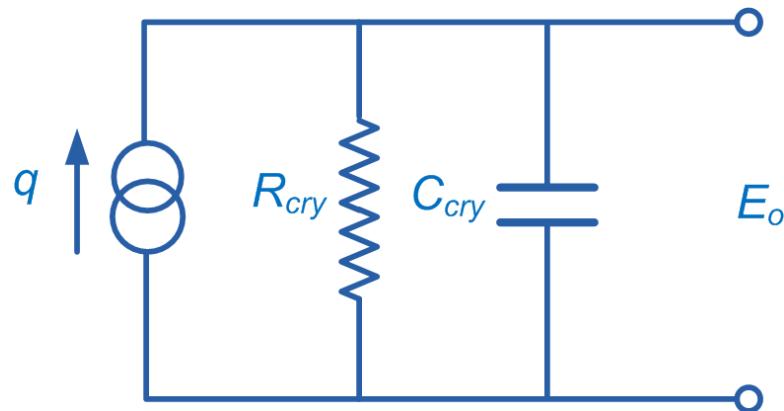
- Piezoelectric Effect is the ability of certain materials to generate an electric charge in response to applied mechanical stress.
- piezoelectric effect is reversible, meaning that materials generate electricity when stress is applied also can generate stress when an electric field is applied.
- Commonly used in Load sensors, accelerometers, and pressure sensors (ex: ultrasonic sensing, generation)

### 3.4.1 Piezo electric sensors

- Materials that exhibit piezoelectric properties include,
  - Natural – Quartz, Rochelle salt
  - Synthetic – Lithium Sulfate, Ammonium dihydrogen phosphate
  - Polarized ferroelectric crystals – Barium Titanate, Lead Zirconate-titanate

### 3.4.1 Equivalent Circuit of a Piezoelectric Crystal

- If a force or pressure is applied to this crystal it could induce a charge.
- The equivalent circuit of a piezoelectric crystal could be given by,



$R_{cry}$  – Leakage Resistance ( $10^{12} \Omega$ )

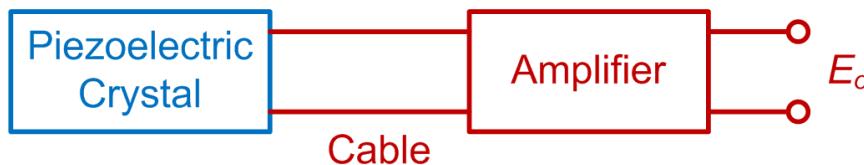
$C_{cry}$  – Crystal Capacitance

$q$  – Charge generated

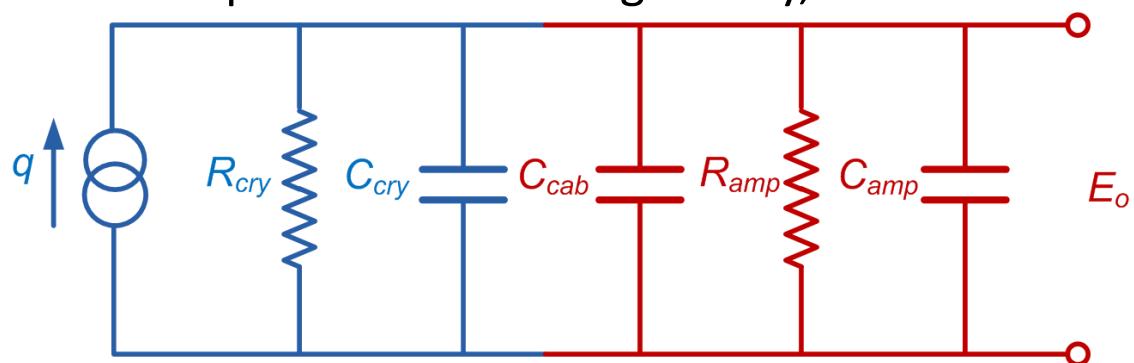
$E_o$  – Voltage Generated

### 3.4.1 Piezo electric sensors

- Usually the piezoelectric sensors require amplification for interfacing.



- Therefore, in the equivalent circuit input impedance of the cable and amplifier should be considered.
- The equivalent circuit is given by,



$C_{cab}$  – Cable Capacitance

$R_{amp}$  – Amplifier Resistance

$C_{amp}$  – Amplifier Capacitance

### 3.4.1 Piezo electric sensors

- The equivalent resistance and the capacitance of the circuit is given by,

$$\text{Where } C = C_{cry} + C_{cab} + C_{amp} \quad R = R_{cry} \parallel R_{amp}$$

- The induced voltage is given by,

$$e_0 = iR = \left( \frac{dq}{dt} - C \frac{de_0}{dt} \right) R$$

- By rearranging terms,

$$RC \frac{de_0}{dt} + e_0 = R \frac{dq}{dt}$$

- The transfer function of the system is given by,

$$\frac{e_o}{q}(s) = \frac{Rs}{RCs + 1}$$

### 3.4.1 Piezo electric sensors

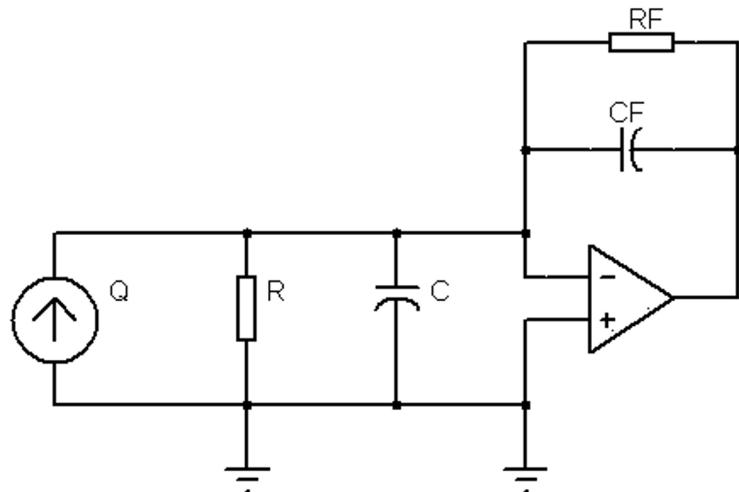
- If a force ( $f$ ) is applied on the piezoelectric crystal, charge induced due to this force is given by,

$$q = d \times f \quad \text{Where } p \text{ is a piezo electric constant}$$

$$\therefore \frac{e_0}{f}(s) = \frac{dRs}{RCs + 1} = \frac{(d/c)\tau s}{1 + \tau s} \quad \text{Where } \tau = RC$$

- The above transfer function represents a high pass filter.
- Measurement of a static force is not possible with this type of a sensor.
- Static sensitivity ( $d/C$ ) will change with the change in any of the values in  $C_{cab}$  or  $C_{amp}$ .
- Therefore, we will use a charge amplifier for measuring these quantities.

### 3.4.1 Measurement using a Charge Amplifier



$C_f$  – Filter Capacitance

$R_f$  – Filter Resistance

From circuit analysis,

$$\frac{dq}{dt} = -\frac{e_0}{R_f} - C_f \frac{de_0}{dt}$$

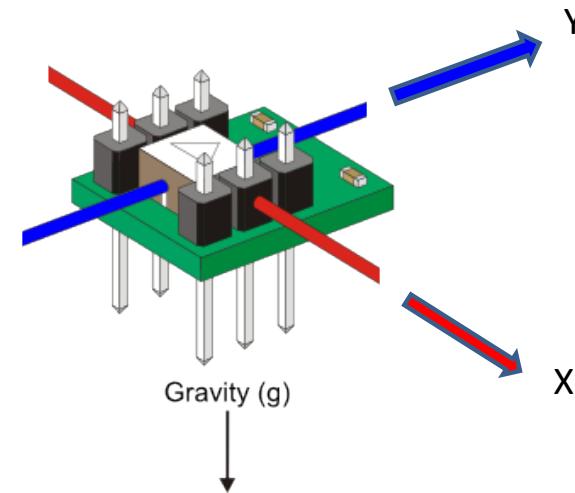
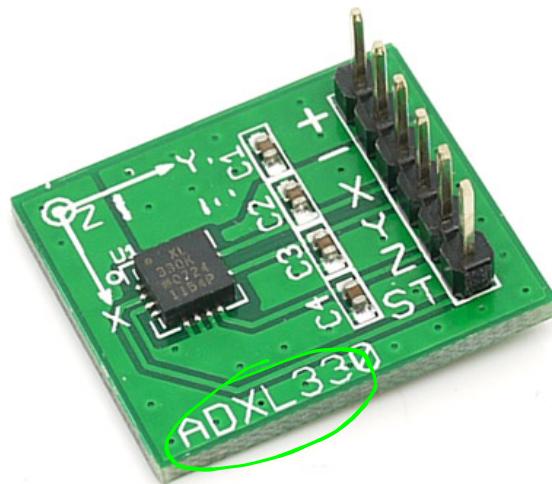
$$R_f C_f \frac{de_0}{dt} + e_0 = -R_f \frac{dq}{dt}$$

$$\frac{e_0}{q}(s) + e_0 = \frac{-R_f s}{1 + R_f C_f s}$$

- The above circuit also cannot measure static force but it removes the dependence on cable capacitance and amplifier impedance.

### 3.4.2 Acceleration measurement

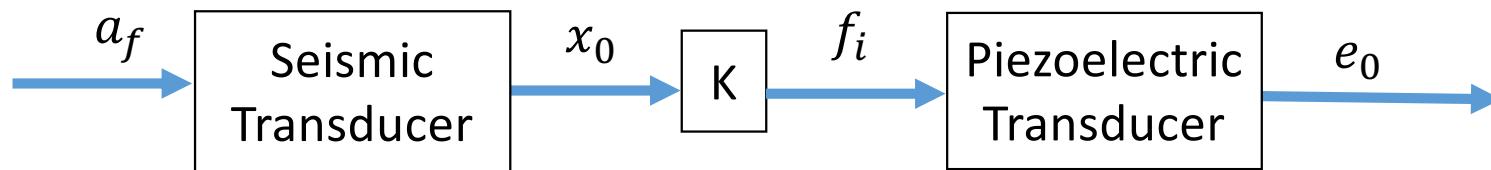
#### Accelerometer



Measurement range is given in 'g's.

### 3.4.2 Piezoelectric Accelerometers (1)

- Basic block diagram of a piezo electric accelerometer



$K$  – Stiffness of the crystal

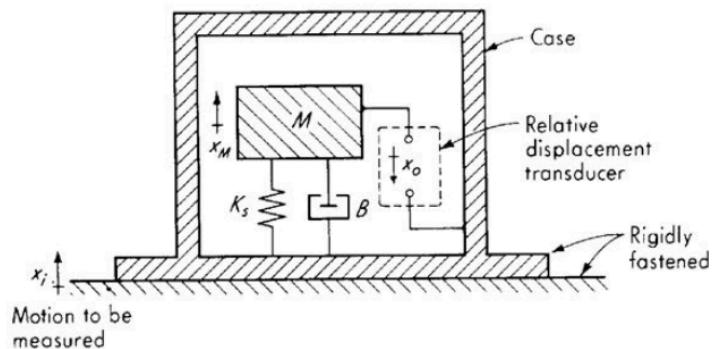
- For a piezo electric accelerometer,

$$\frac{x_0}{a_f}(s) = \frac{1}{s^2 + 2\xi\omega_n + \omega_n^2}$$

Where,

$\xi$  – Damping factor                     $x_0$  – displacement

$\omega_n$  – Natural frequency        $a_i$  – acceleration



### 3.4.2 Piezoelectric Accelerometers (2)

- For a piezoelectric transducer we can show that,

$$\checkmark \quad \frac{e_0}{f_i}(s) = \frac{(d/c)\tau s}{1 + \tau s} \quad \text{Where,} \quad \tau = RC$$

$d = \text{sensitivity}$

- We can write,

$$\begin{aligned}\frac{e_0}{a_i}(s) &= \frac{e_0}{f_i} \times \frac{f_i}{x_0} \times \frac{x_0}{a_i} \\ &= \cancel{\frac{(d/c)\tau s}{1 + \tau s}} \times K \times \frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2}\end{aligned}$$

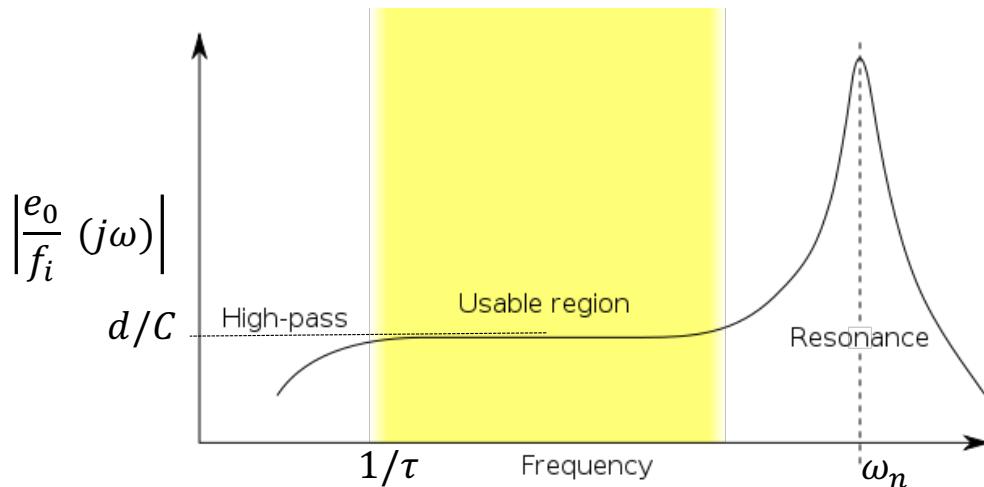
- Considering the Mass ( $M$ ) of the seismic transducer,

$$\frac{e_0}{f_i}(s) = \frac{1}{M} \frac{e_0}{a_i}(s)$$

### 3.4.2 Piezoelectric Accelerometers (3)

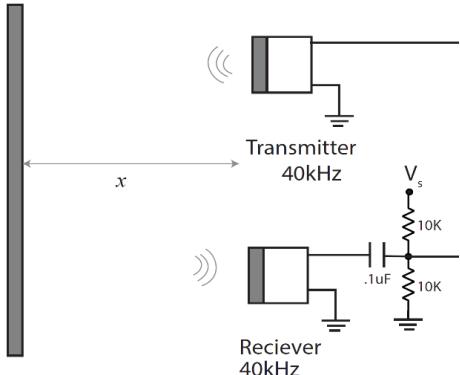
$$\begin{aligned}\frac{e_0}{f_i}(s) &= \frac{1}{M} \frac{e_0}{a_i}(s) = \frac{(d/c)\tau s}{1 + \tau s} \times \left(\frac{K}{M}\right) \times \frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2} \\ &= \frac{(d/c)\tau s}{1 + \tau s} \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}\end{aligned}$$

- The response curve will be,



### 3.4.3 Piezo electric sensors

#### Piezoelectric transducers



$$x = \frac{C_{air} dt}{2}$$

$x$  Distance  
 $C_{air}$  Speed of sound in air  
 $dt$  Return trip time of signal

#### Air Ultrasonic Ceramic Transducers



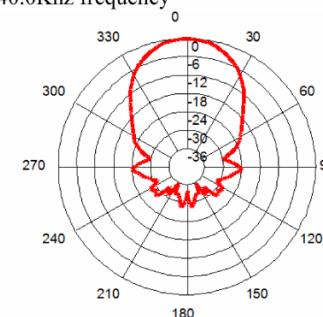
#### Specification

400ST160	Transmitter
400SR160	Receiver
Center Frequency	40.0±1.0KHz
Bandwidth (-6dB)	4.00ST160 2.0KHz 400SR160 2.5KHz
Transmitting Sound Pressure Level	120dB min.
at 40.0KHz, 0dB re 0.0002μbar per 10Vrms at 30cm	
Receiving Sensitivity	-61dB min.
at 40.0KHz 0dB = 1 volt/μbar	
Capacitance at 1KHz	2400 pF
±20%	
Max. Driving Voltage (cont.)	20Vrms
Total Beam Angle	-6dB
55° typical	
Operation Temperature	-30 to 70°C
Storage Temperature	-40 to 80°C

All specification taken typical at 25°C  
Closer frequency tolerance can be supplied upon request.

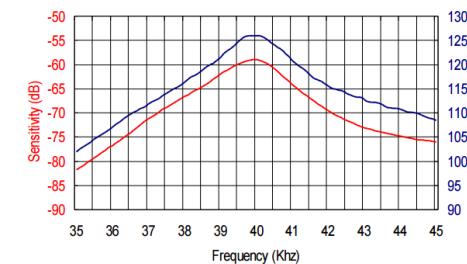
#### Beam Angle

Tested at 40.0Khz frequency



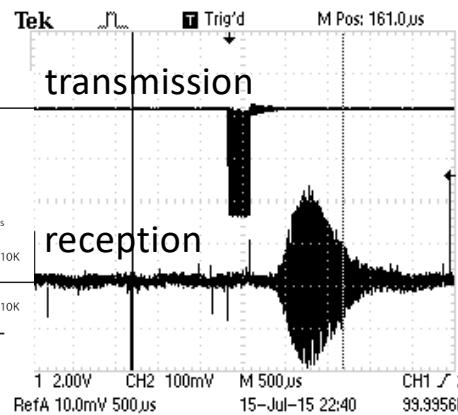
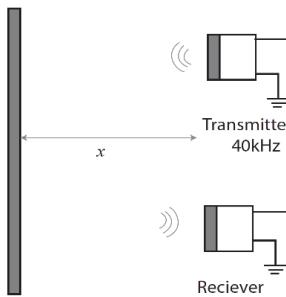
#### Sensitivity/Sound Pressure Level

Tested under 10Vrms @30cm

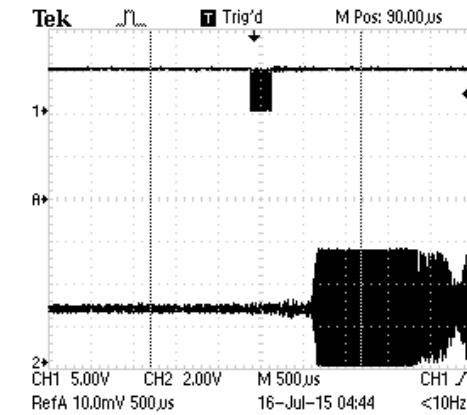


### 3.4.3 Piezo electric sensors

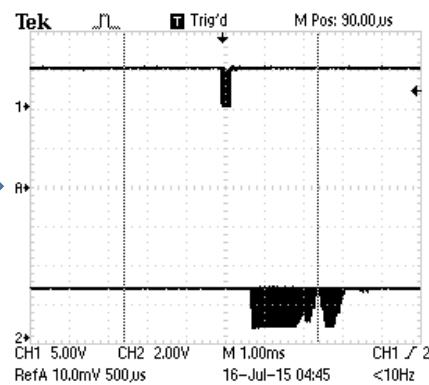
#### Signal processing



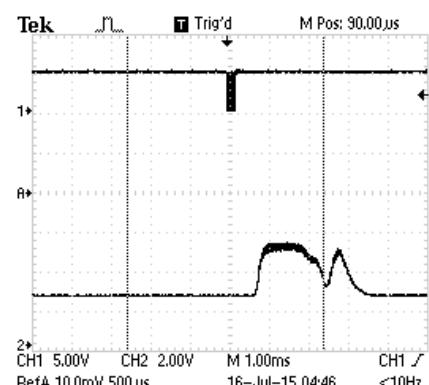
1. Received signal



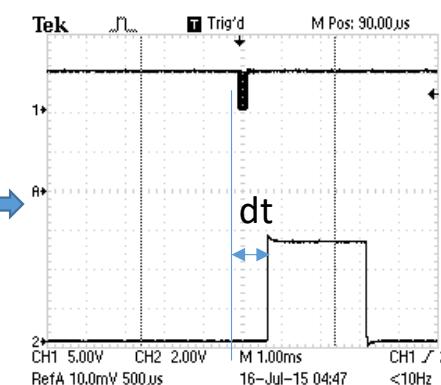
2. Filter+amplify ~100x



3. Invert signal



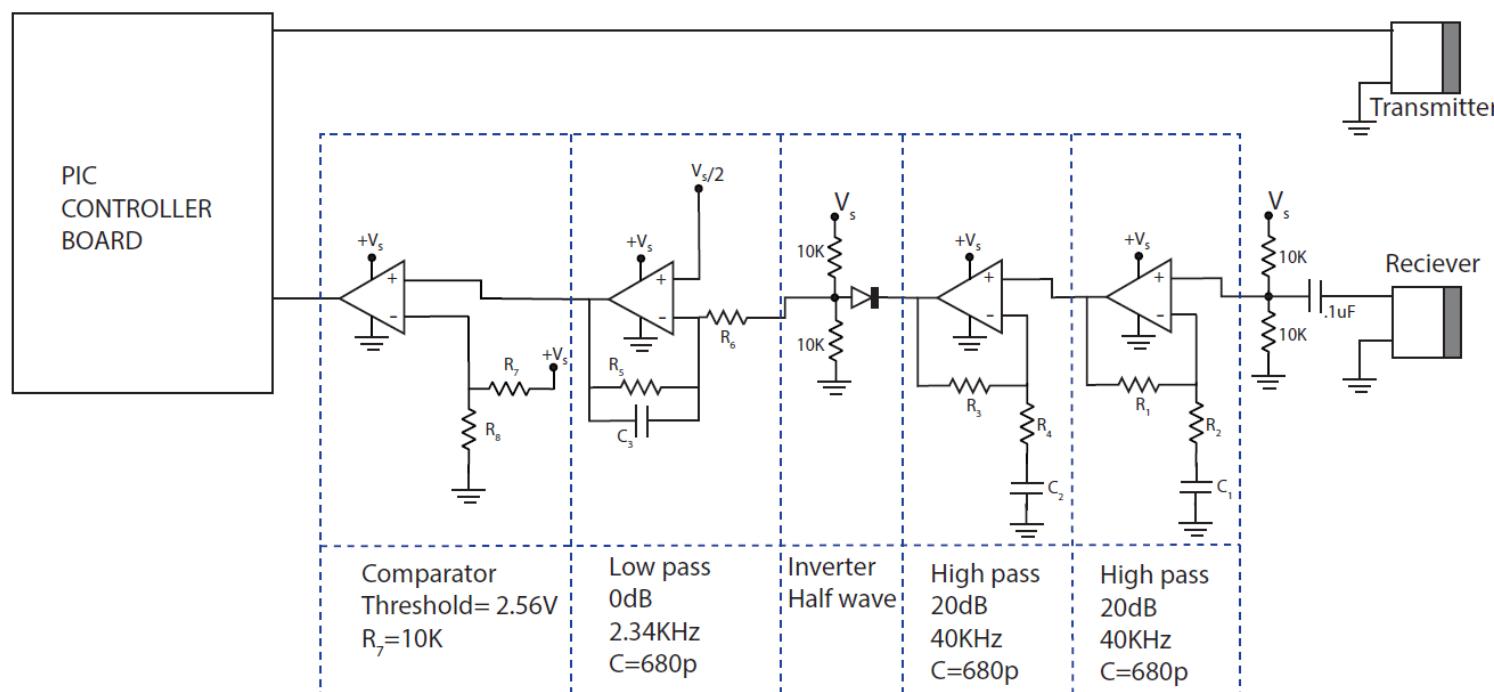
4. Low pass filter



5. Comparison

### 3.4.3 Piezo electric sensors

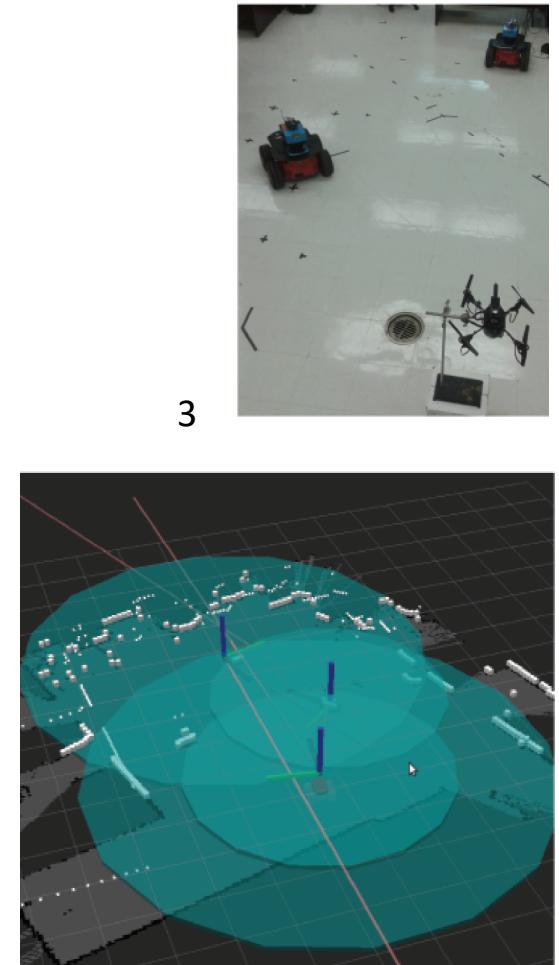
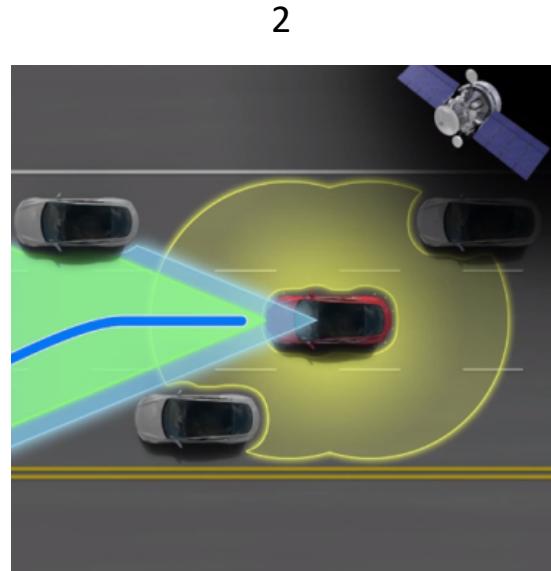
The signal processing circuit for interfacing the 400ST160, 400SR160 ultrasonic sensors.



### 3.4.3 Piezo electric sensors

#### Ultrasonic sensors – applications

1. Obstacle sensors
2. Omnidirectional sensing
3. Beacon sensing



END