Stationary Processes and Ergodic Processes

Random Signals & Processes
Lecture 5
Eng. (Mrs.) PN Karunanayake

Stationary Process

- If the statistical characterization of a process is independent of the time at which observation of the process is initiated the process is stationary process.

- A stationary process arises from a stable physical phenomenon that has evolved into a steady state mode of behaviour.

- A nonstationary process arises from an unstable phenomenon.

Strictly Stationary

- Consider random Process $X(t)$, initiated at $t = -\infty$.
- Let $X(t_1), X(t_2), X(t_3), \ldots, X(t)$ denote the random variables obtained by observing the random process $X(t)$ at times $t_1, t_2, \ldots, t_k$ respectively.
- The joint distribution function of this set of random variables is $F_{X(t1), \ldots, X(tk)}(x_1, \ldots, x_k)$.
- Shift all observation times by a fixed amount of τ,

  $X(t_1+\tau), X(t_2+\tau), X(t_3+\tau), \ldots, X(t+\tau)$.
- The joint distribution function $F_{X(t1+\tau), \ldots, X(tk+\tau)}(x_1, \ldots, x_k)$

- The random process X(t) is said to stationary in the strict sence if the following condition holds:

$$F_{X(t1+\tau), \ldots, X(tk+\tau)}(x_1, \ldots, x_k) = F_{X(t1), \ldots, X(tk)}(x_1, \ldots, x_k)$$

  - For all the time shifts τ, and for all k and all possible observation times.

- The random process X(t) is said to stationary in the strict sence if the joint distribution of any set of random variables obtained by observing the random process X(t) is invarient with respect to the location of the origin t=0.

Jointly Strictly Stationary

- Two random processes X(t) & Y(t)are jointly strictly stationary if the joint finite- dimensional distributions of the two sets of random variable $X(t_1)$, $X(t_2)$, ... $X(t_k)$ and $Y(t'_1)$, $Y(t'_2)$, ... $Y(t'_j)$ are invariant with respect to the origin t=0 for all k and j and all choices of observation of times $t_{1,}$ t_2, … , t_k and $t'_{1,}$ t'_2, … , $t'_{j.}$

Mean

- Mean of the SSP is a constant.

$\mu_x(t) = \mu_x$

Autocorrelation function of SSP

- $R_x(t_1, t_2) = R_x(t_2 - t_1)$ for all t_1 and t_2
- Depends only on the time difference $t_2 - t_1$

Autocovariance function of a SSP

- $C_x(t_1, t_2) = E[(X(t_1) – \mu x)(X(t_2)-\mu_x)]$

  $= R_x(t_2-t_1) – \mu_x^2$
- Depends only on the time difference t_2-t_1

Properties of the Autocorrelation Function

- Let X(t) be a stationary process.

  $R_X(\tau) = E[X(t+\tau)X(t)]$

- The mean square value of the process may be obtained from $R_X(\tau)$ by putting $\tau=0$.
- The autocorrelation function $R_X(\tau)$ is an even function of τ

  $R_X(\tau) = R_X(-\tau)$

- The autocorrelation function $R_X(\tau)$ has its maximum magnitude at $\tau=0$.

Significant of Autocorrelation function

- Provides a means of describing the interdependence of two random variables obtained by observing a random process $X(t)$ at times τ seconds apart.