IDO- Tarea 1

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Problema de la ONU

Enunciado

Considere la situación en que se asignan M poblaciones en N áreas distintas. El tamaño de la población i es p_i y el costo de la asignación de la población i al área j es c_{ij} . Un área seleccionada en la solución óptima debe incluir al menos L personas (L se supone constante). También cada área puede aceptar más de una población. El objetivo del problema trata de la minimización del costo total de las asignaciones. Supongamos que: $x_{ij} = 1$ si se asigna la población i al área j, y 0 en caso contrario $y_j = 1$ si se apunta el área j en la solución, y 0 en caso contrario

Formulación

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{M}\sum_{j=1}^{N} c_{ij}x_{ij} \\
\text{s.a.} \quad & \sum_{j=1}^{N} x_{ij} = 1, \quad i = 1, \dots, M \\
& \sum_{i=1}^{M} p_i x_{ij} \geq L y_j, \quad j = 1, \dots, N \\
& \sum_{i=1}^{M} p_i x_{ij} \leq \left(\sum_{i=1}^{M} p_i \right) y_j \quad j = 1, \dots, N \\
& y_{ij} = \begin{cases} 1 & \text{si el área } j \text{ se utiliza} \\ 0 & \text{en otro caso} \end{cases} \\
& x_{ij} = \begin{cases} 1 & \text{si la población } i \text{ está en el área } j \\ 0 & \text{en otro caso} \end{cases} \\
& x_{ij}, y_{ij} \geq 0, \quad i = 1, \dots, M, \ j = 1, \dots, N
\end{aligned}
$$

Modelo en julia

```
using JuMP, HiGHS
M = 6
N = 7
L = 100
```

```
matrixCostos = [
    10 30 30 40 50 60 80;
    50 25 40 30 60 30 5;
    70 50 15 60 40 50 60;
    30 50   40  30  10  70  45;
    60 30   40  10  50  70  35;
    30 40   50  30  40  50  10;
]

poblacion = [75, 50, 100, 150, 120, 80]

model = Model(HiGHS.Optimizer)

@variable(model, x[1:M, 1:N] >= 0, Bin)
@variable(model, y[1:N] >= 0, Bin)

@constraint(model, [i=1:M], sum(x[i,j] for j=1:N) == 1)
@constraint(model, [j=1:N], sum(x[i,j]*poblacion[i] for i=1:M) >= L*y[j])
@constraint(model, [j=1:N], sum(x[i,j]*poblacion[i] for i=1:M) <= sum(poblacion)*y[j])

@objective(model, Min, sum(matrixCostos[i,j]*x[i,j] for i=1:M, j=1:N))

optimize!(model)
```

```
Running HiGHS 1.6.0: Copyright (c) 2023 HiGHS under MIT licence terms
Presolving model
20 rows, 49 cols, 140 nonzeros
20 rows, 49 cols, 140 nonzeros
Objective function is integral with scale 0.2

Solving MIP model with:
   20 rows
   49 cols (49 binary, 0 integer, 0 implied int., 0 continuous)
   140 nonzeros

         Nodes      |    B&B Tree     |            Objective Bounds              |  Dynamic Co
      Proc. InQueue |  Leaves   Expl. | BestBound       BestSol              Gap |   Cuts   In

         0       0         0   0.00%   0               inf                  inf        0
 R       0       0         0   0.00%   60              100                40.00%       0

30.6% inactive integer columns, restarting
```

```
Model after restart has 18 rows, 32 cols (32 bin., 0 int., 0 impl., 0 cont.), and 88 nonzeros

         0       0         0   0.00%   60              100               40.00%        0
 R       0       0         0   0.00%   60              90                33.33%        0

25.0% inactive integer columns, restarting
Model after restart has 16 rows, 22 cols (22 bin., 0 int., 0 impl., 0 cont.), and 60 nonzeros

         0       0         0   0.00%   60              90                33.33%        0

Solving report
  Status            Optimal
  Primal bound      80
  Dual bound        80
  Gap               0% (tolerance: 0.01%)
  Solution status   feasible
                    80 (objective)
                    0 (bound viol.)
                    0 (int. viol.)
                    0 (row viol.)
  Timing            0.00 (total)
                    0.00 (presolve)
                    0.00 (postsolve)
  Nodes             1
  LP iterations     14 (total)
                    0 (strong br.)
                    0 (separation)
                    0 (heuristics)
```

```julia
solution_summary(model; verbose = true)
```

```
* Solver : HiGHS

* Status
  Result count       : 1
  Termination status : OPTIMAL
  Message from the solver:
  "kHighsModelStatusOptimal"

* Candidate solution (result #1)
  Primal status      : FEASIBLE_POINT
```

```
Dual status        : NO_SOLUTION
Objective value    : 8.00000e+01
Objective bound    : 8.00000e+01
Relative gap       : 0.00000e+00
Primal solution :
  x[1,1] : 0.00000e+00
  x[1,2] : 0.00000e+00
  x[1,3] : 1.00000e+00
  x[1,4] : 0.00000e+00
  x[1,5] : 0.00000e+00
  x[1,6] : 0.00000e+00
  x[1,7] : 0.00000e+00
  x[2,1] : 0.00000e+00
  x[2,2] : 0.00000e+00
  x[2,3] : 0.00000e+00
  x[2,4] : 0.00000e+00
  x[2,5] : 0.00000e+00
  x[2,6] : 0.00000e+00
  x[2,7] : 1.00000e+00
  x[3,1] : 0.00000e+00
  x[3,2] : 0.00000e+00
  x[3,3] : 1.00000e+00
  x[3,4] : 0.00000e+00
  x[3,5] : 0.00000e+00
  x[3,6] : 0.00000e+00
  x[3,7] : 0.00000e+00
  x[4,1] : 0.00000e+00
  x[4,2] : 0.00000e+00
  x[4,3] : 0.00000e+00
  x[4,4] : 0.00000e+00
  x[4,5] : 1.00000e+00
  x[4,6] : 0.00000e+00
  x[4,7] : 0.00000e+00
  x[5,1] : 0.00000e+00
  x[5,2] : 0.00000e+00
  x[5,3] : 0.00000e+00
  x[5,4] : 1.00000e+00
  x[5,5] : 0.00000e+00
  x[5,6] : 0.00000e+00
  x[5,7] : 0.00000e+00
  x[6,1] : 0.00000e+00
  x[6,2] : 0.00000e+00
  x[6,3] : 0.00000e+00
```

```
    x[6,4] : 0.00000e+00
    x[6,5] : 0.00000e+00
    x[6,6] : 0.00000e+00
    x[6,7] : 1.00000e+00
    y[1] : 0.00000e+00
    y[2] : 0.00000e+00
    y[3] : 1.00000e+00
    y[4] : 1.00000e+00
    y[5] : 1.00000e+00
    y[6] : 0.00000e+00
    y[7] : 1.00000e+00

* Work counters
  Solve time (sec)   : 2.63121e-03
  Simplex iterations : 14
  Barrier iterations : -1
  Node count         : 1
```

```
value.(x)
```

```
6×7 Matrix{Float64}:
 0.0  0.0  1.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  1.0
 0.0  0.0  1.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  1.0  0.0  0.0
 0.0  0.0  0.0  1.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  1.0
```

```
value.(y)
```

```
7-element Vector{Float64}:
 0.0
 0.0
 1.0
 1.0
 1.0
 0.0
 1.0
```