

Descomposición en Valores Singulares Para el Método del Espacio Nulo

A $m \times n$

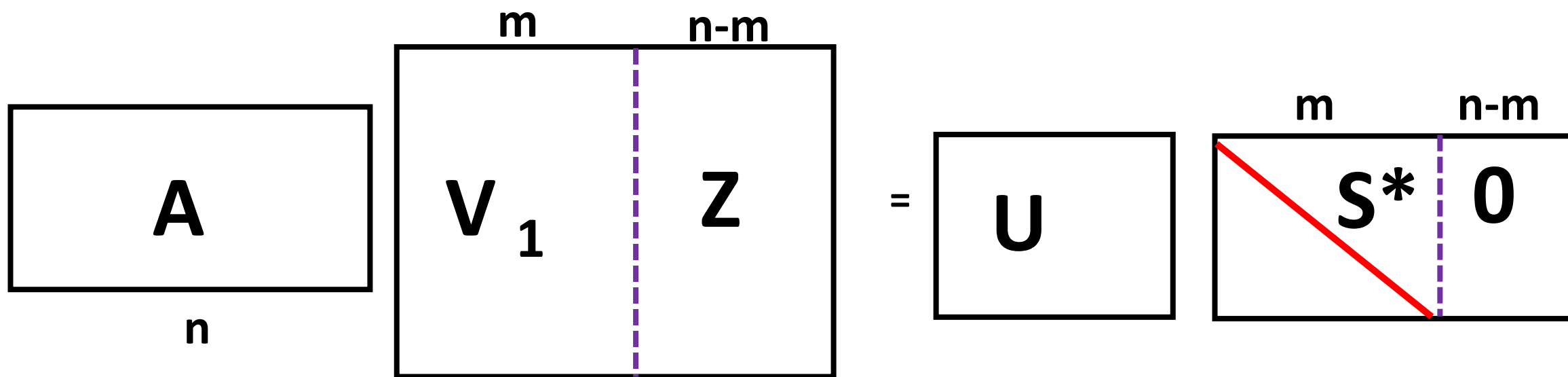
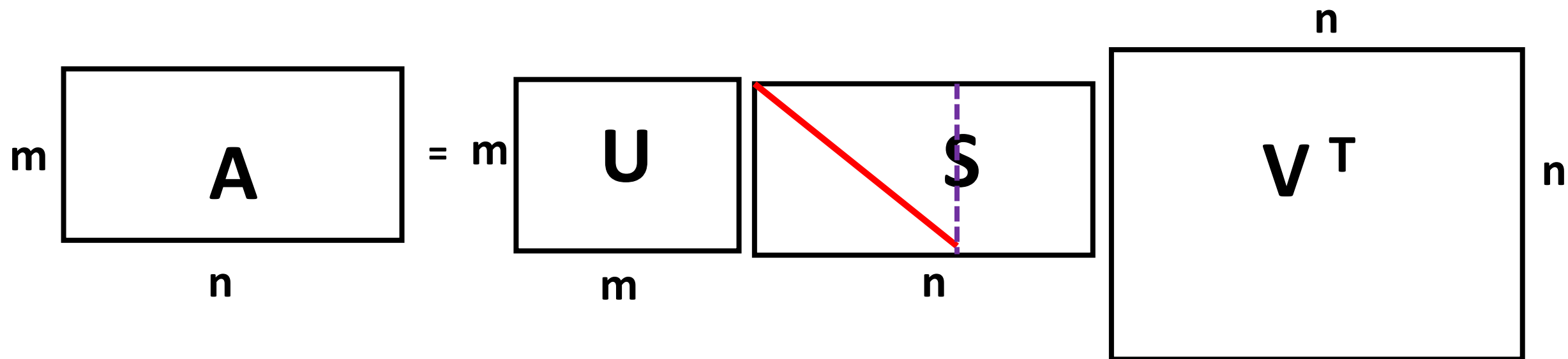
$$A = U S V^T$$

donde U es $m \times m$ V es $n \times n$

$$U^T U = I_m \quad V^T V = I_m$$

S = diagonal de $m \times n$

**Una Base Ortonormal de $\text{Null}(A)$
Con la Descomposición de Valores
Singulares**



$$\begin{array}{|c|c|} \hline AV_1 & AZ \\ \hline \end{array} = \begin{array}{|c|c|} \hline \overset{m}{US^*} & \overset{n-m}{U^*0} \\ \hline \end{array}$$

$$AZ = 0$$

Columnas de Z son una base ortonormal de Null(A)

**Una solución particular del sistema
lineal**

$$**Ax = b**$$

**Con descomposición en valores
singulares**

$$\begin{array}{c} m \\ \boxed{A} \end{array} = \begin{array}{c} m \\ \boxed{U} \end{array} \begin{array}{cc} m & n-m \\ \boxed{\begin{array}{c|c} S^* & 0 \end{array}} & \end{array} \begin{array}{c} n \\ \boxed{V^T} \end{array} \begin{array}{c} n \end{array}$$

Diagram illustrating the SVD decomposition of matrix A (size $m \times n$) into matrices U (size $m \times m$), S^* (size $m \times n$), and V^T (size $n \times n$). The matrix S^* is partitioned into a top-left block of size $m \times m$ and a top-right block of size $m \times (n-m)$, with a red diagonal line and a dashed purple vertical line indicating the partitioning.

$$\begin{array}{c} n \\ \boxed{A^*} \end{array} = \begin{array}{c} \boxed{V} \end{array} \begin{array}{c} \boxed{\begin{array}{c|c} (S^*)^{-1} & 0 \end{array}} \end{array} \begin{array}{c} \boxed{U^T} \end{array}$$

Diagram illustrating the SVD decomposition of matrix A^* (size $n \times m$) into matrices V (size $n \times n$), $(S^*)^{-1}$ (size $n \times m$), and U^T (size $m \times m$). The matrix $(S^*)^{-1}$ is partitioned into a top-left block of size $n \times m$ and a bottom-right block of size $(n-m) \times (n-m)$, with a red diagonal line and a dashed purple horizontal line indicating the partitioning.

$$A A^* = (USV^T) (V S_{\text{inv}} U^T)$$

$$= US(V^T V) S_{\text{inv}} U^T$$

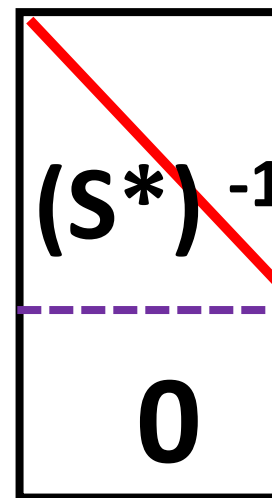
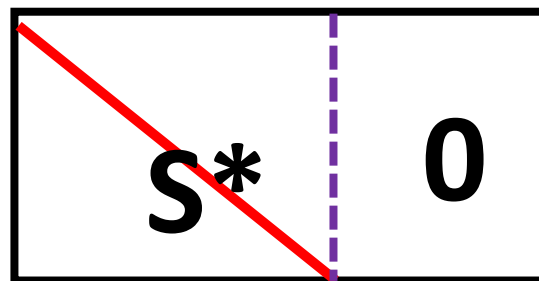
$$= US(I_n) S_{\text{inv}} U^T$$

$$= U (S S_{\text{inv}}) U^T$$

$$= U (I_m) U^T$$

$$= U U^T$$

$$= I_m$$



$$= S^* (S^*)^{-1}$$

$$A A^* = I_m$$

$$A A^* b = I_m b = b$$

$$x_p = A^* b$$

$$x_p = (V S_{\text{inv}} U^T) b$$

$$x_p = \begin{array}{|c|c|} \hline V_1 & Z \\ \hline \end{array} \begin{array}{|c|} \hline (S^*)^{-1} \\ \hline 0 \\ \hline \end{array} (U^T b)$$

$$x_p = V_1 (S^*)^{-1} (U^T b)$$