Descomposición en Valores Singulares
Para el Método del Espacio Nulo

A_{mxn}

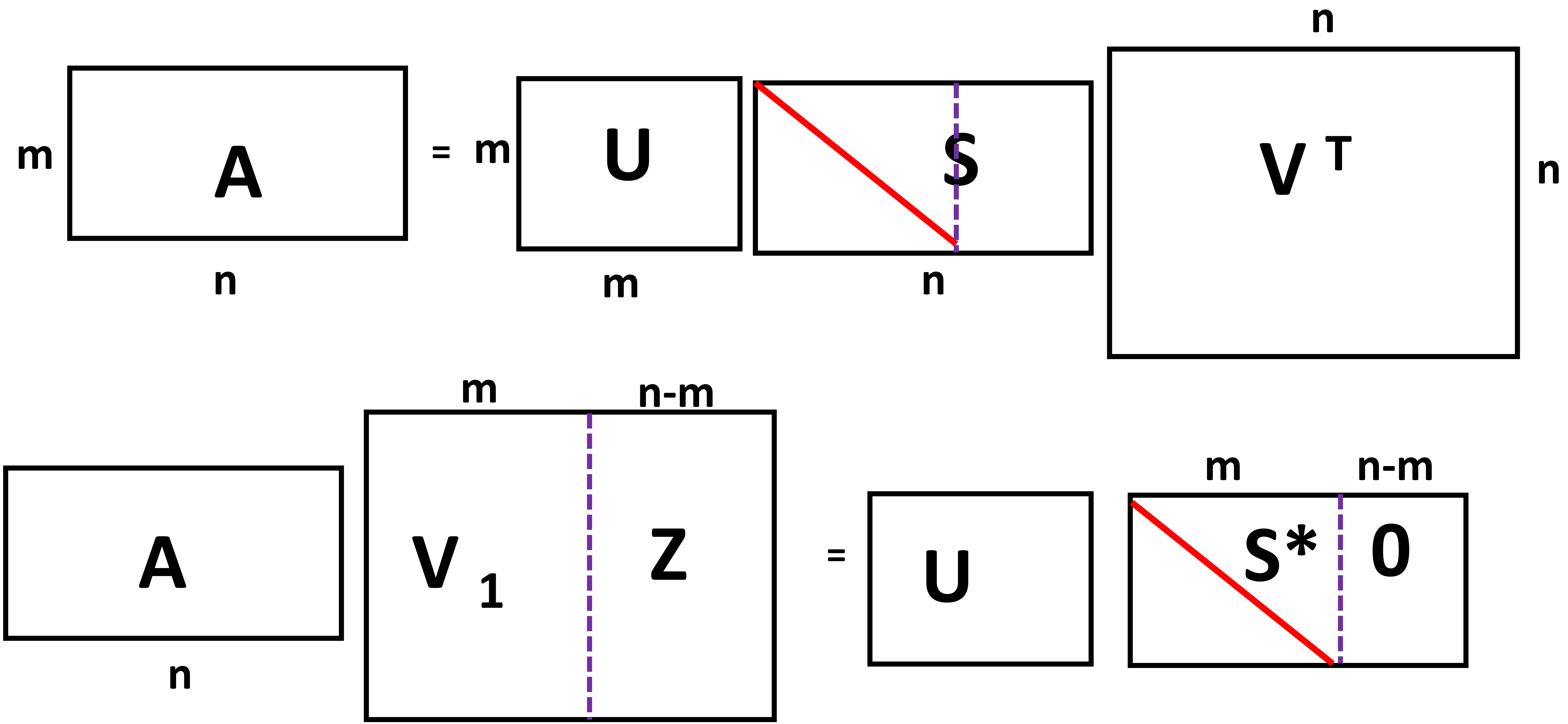
$$A = U S V^T$$

donde U es mxm V es nxn

$$U^T U = I_m \qquad V^T V = I_m$$

S = diagonal de mxn

Una Base Ortonormal de Null(A) Con la Descomposición de Valores Singulares


m
A
n
=
m
U
m
S
n
n
V
T
n
A
n
m
n-m
V
1
Z
=
U
m
n-m
S*
0

$$\left[\begin{array}{c|c} AV_1 & AZ \end{array}\right] = \begin{array}{c|c} m & n-m \\ \hline US^* & U^*0 \end{array}$$

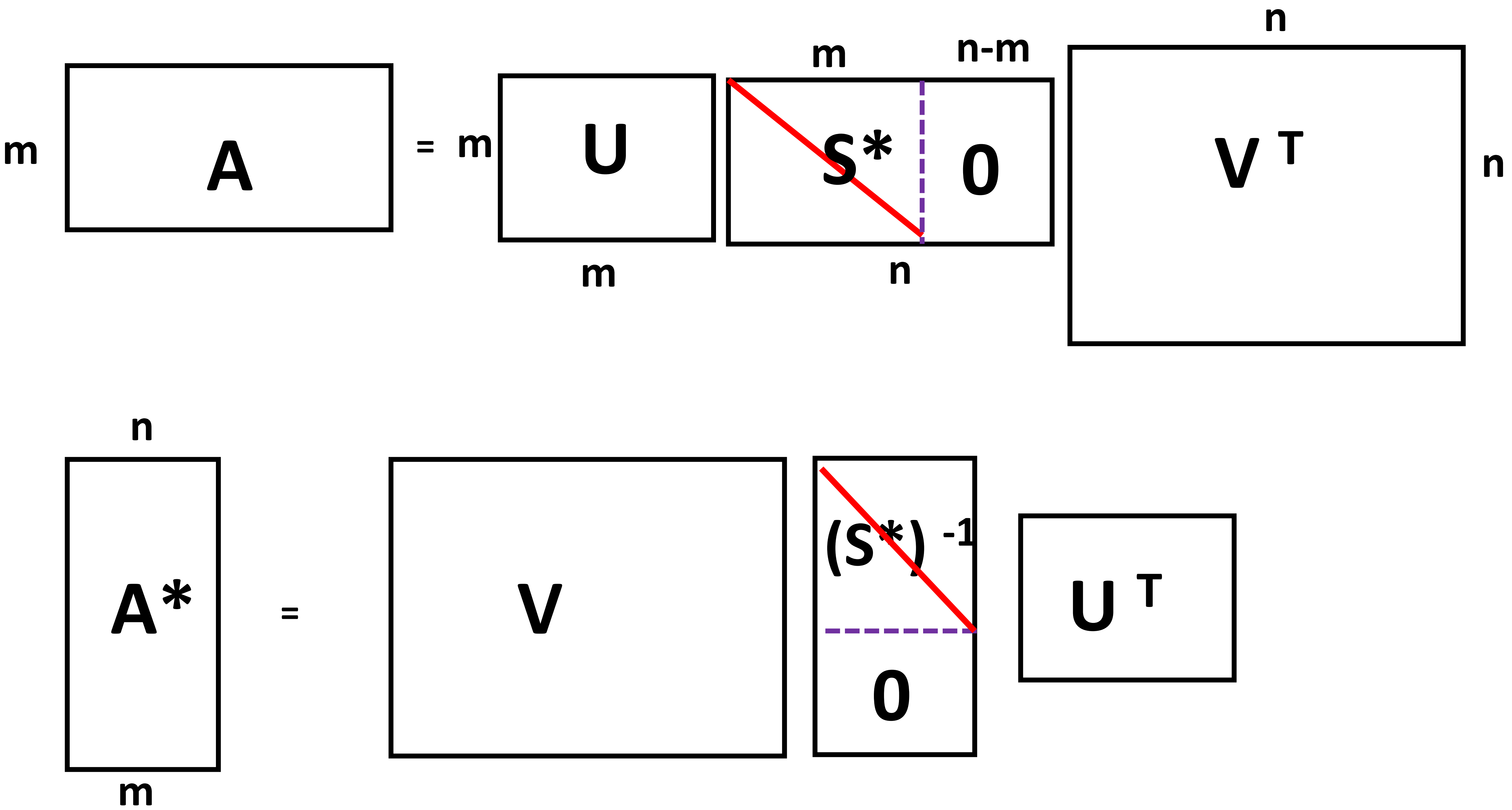
$$AZ = 0$$

Columnas de Z son una base ortonormal de Null(A)

Una solución particular del sistema lineal

$Ax = b$

Con descomposición en valores singulares


m
A
=
m
U
m
m
n-m
S*
0
n
n
V T
n
n
A*
m
=
V
(S*) -1
0
U T

$$A\,A^* = (USV^T)\,(V\,S_{inv}\,U^T)$$

$$= US(V^T\;V)\,S_{inv}\,U^T)$$

$$= US(\,I_n\,)\,S_{inv}\,U^T)$$

$$= U\,(S\,S_{inv})U^T$$

$$= U\,(I_m)U^T$$

$$= U\,U^T$$

$$= I_m$$

$$\left[\begin{array}{c|c} S^* & 0 \end{array}\right]\left[\begin{array}{c} (S^*)^{-1} \\ \hline 0 \end{array}\right] = S^*(S^*)^{-1}$$

$$A\,A^* = I_m \qquad\qquad A\,A^*\, b = I_m\, b = b$$

$$xp = A^*\, b$$

$$xp = (\,V\, S_{inv}\, U^T)\; b$$

$$xp = \left[\begin{array}{c|c} V_1 & Z \end{array}\right] \left[\begin{array}{c} (S^*)^{-1} \\ \hline 0 \end{array}\right] (U^T\; b\,)$$

$$Xp = V_1\, (S^*)^{-1}\;\; (U^T\; b\,)$$