

# 3D Scanning & Motion Capture

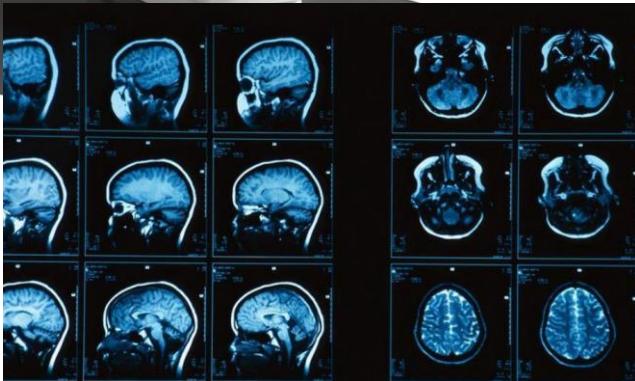
## Rigid Surface Tracking & Reconstruction

Dr. Justus Thies, Dr. Angela Dai



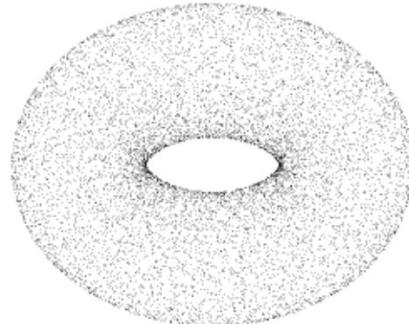
# Last Lecture: How to obtain “3D”?

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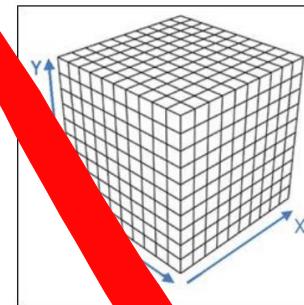


# Last Lecture: Surface Representations

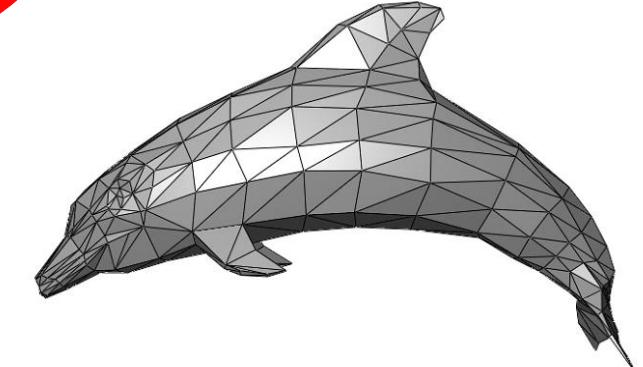
- Point Clouds



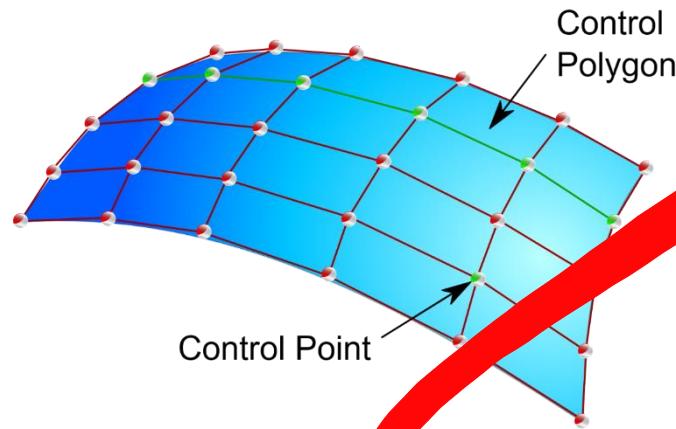
- Voxels



- Polygonal Meshes



- Parametric Surfaces



- Implicit Surfaces

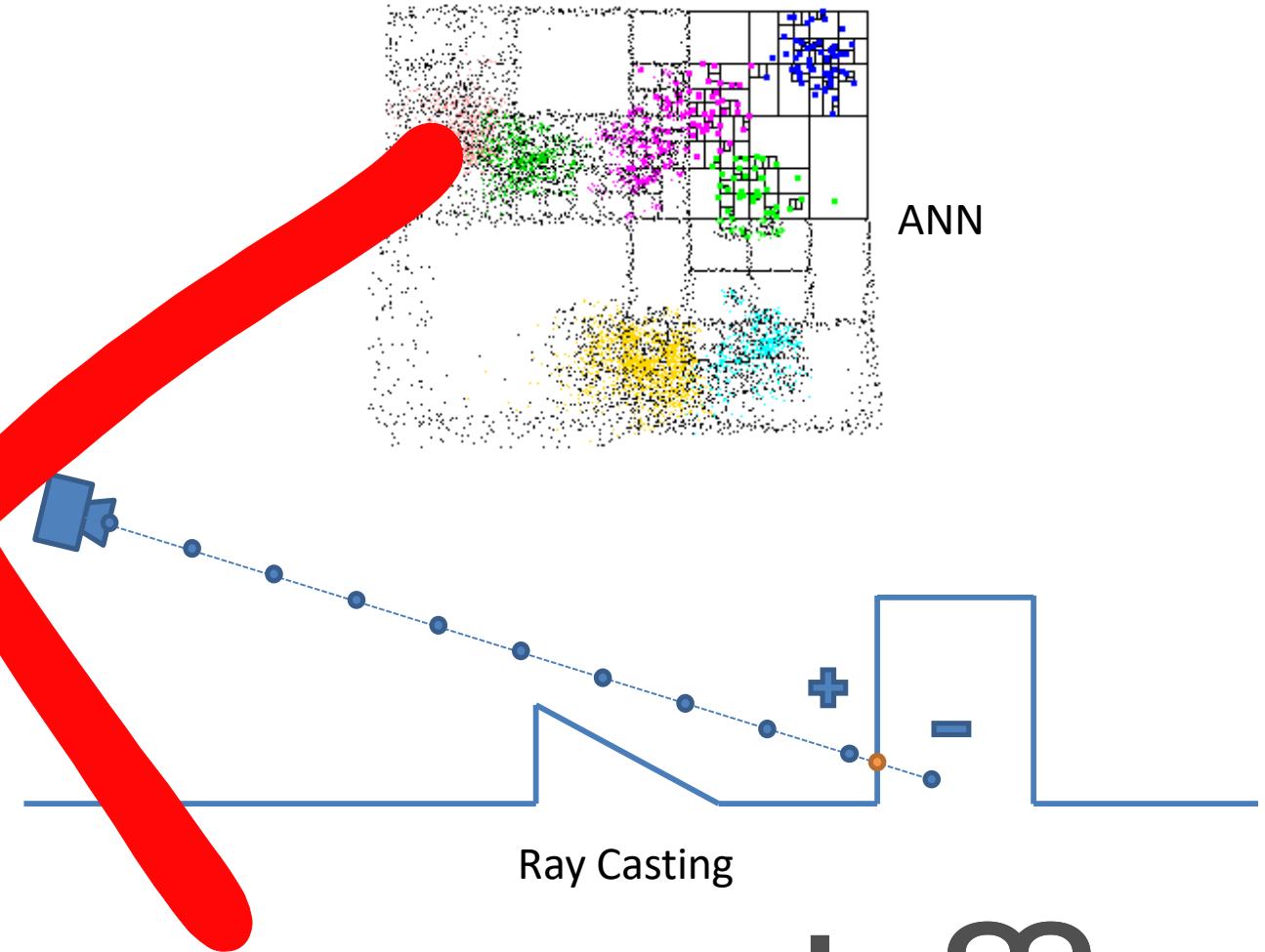
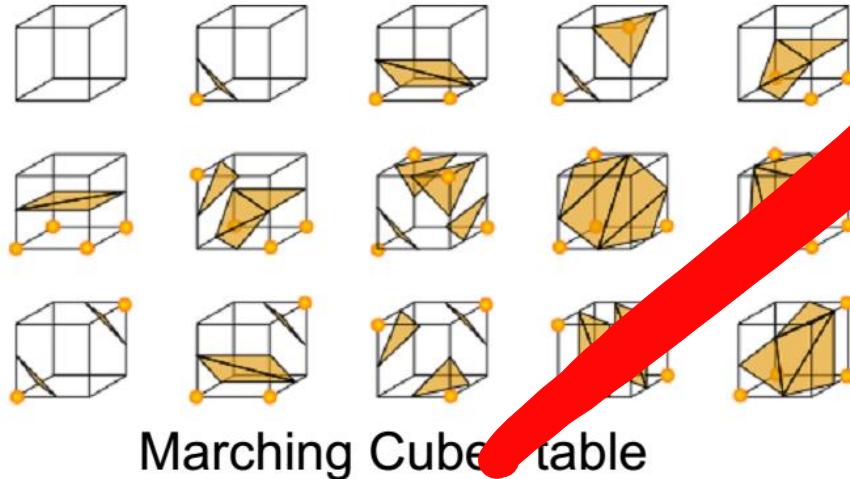


-0.9	-0.4	-0.1	0.2	0.9	1	1	1	1	1
-1	-0.9	-0.2	0.1	0.5	0.9	1	1	1	1
-1	-0.9	-0.3	0.1	0.2	0.8	1	1	1	1
-1	-0.9	-0.4	0.1	0.2	0.8	1	1	1	1
-1	-1	-0.8	-0.1	0.2	0.6	0.8	1	1	1
-1	-0.9	-0.3	0.1	0.3	0.7	0.9	1	1	1
-1	-0.9	-0.4	-0.1	0.3	0.8	1	1	1	1
-0.9	-0.7	-0.5	0.0	0.4	0.9	1	1	1	1
-0.1	-0.9	-0.4	0.1	0.4	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1

Truncated Signed Distance Field (TSDF)

# Last Lecture: Surface Representations

- Important Algorithms
  - Marching Cubes
  - Ray cast
  - Nearest neighbor lookups



# Last Lecture: Correspondence Finding / Matching

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# Last Lecture: Correspondence Finding / Matching



# Last Lecture: Bundle Adjustment (SfM)

- $m$  images
- $n$  points in 3d

- $E_{re-proj}(\mathbf{T}, \mathbf{X}) =$

$$\sum_{i=1}^m \sum_{j=1}^n \left\| \mathbf{x}_{ij} - \pi_i(T_i \cdot \mathbf{X}_j) \right\|^2$$

over images

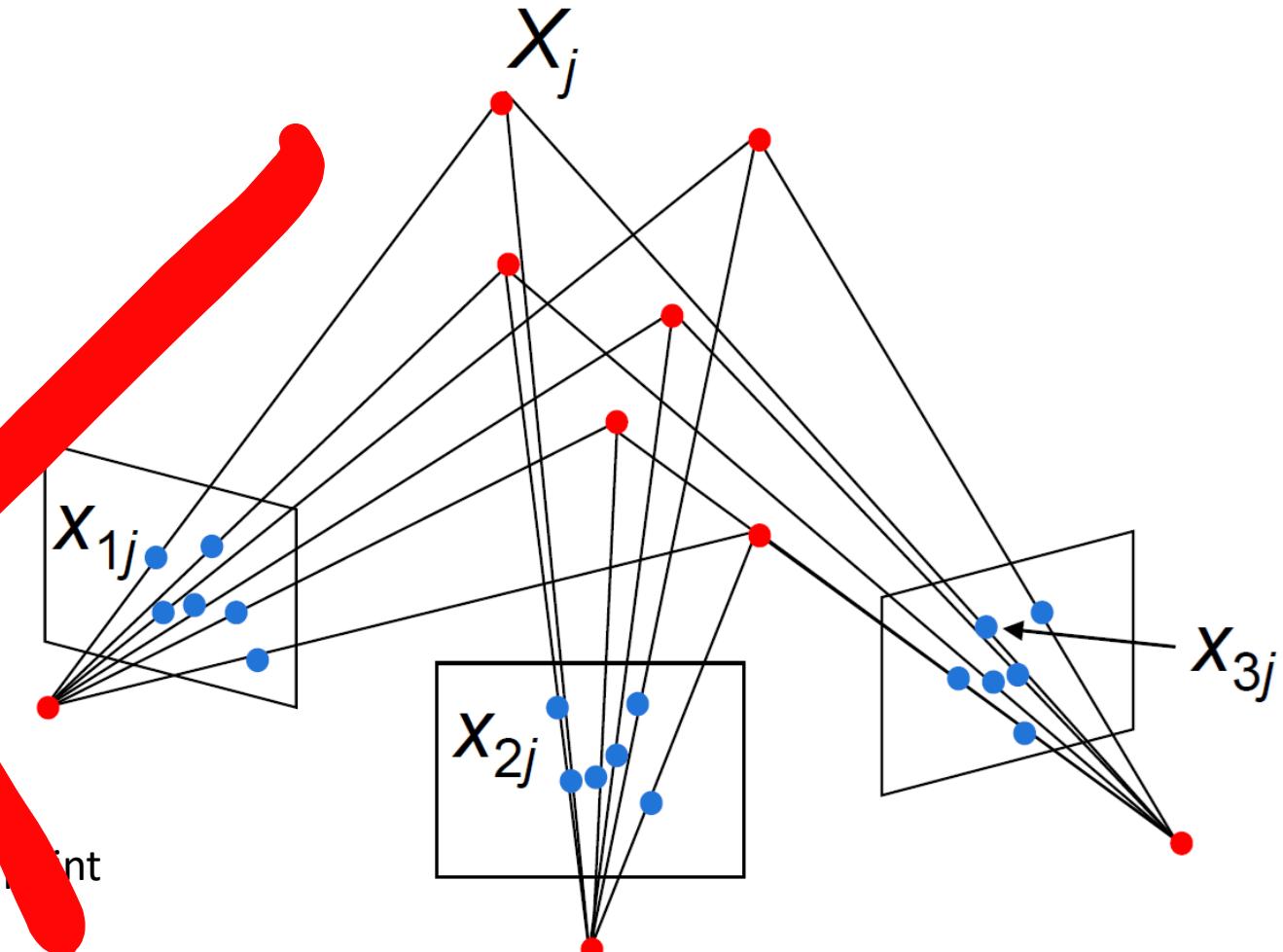
over 3d points

2d keypoint  
locations

3D-2D proj  
(extrinsics)

frame pose  
(extrinsics)

3D point



# Last Lecture: RGB-D “Bundling”

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$$E_{bundle}(T) = \sum_{i,j}^{\text{\#frames}} \sum_{k}^{\text{\#corresp.}} \|T_i p_{ik} - T_j p_{jk}\|_2^2$$

$$E_{depth}(T) = \sum_{i,j}^{\text{\#frames}} \sum_{k}^{\text{\#pixels}} \|(p_k - T_i^{-1} T_j^{-1} d^{-1} (D_j(\pi_d(T_j^{-1} T_i p_k))) \cdot n_k)\|_2^2$$

$$E_{color}(T) = \sum_{i,j}^{\text{\#frames}} \sum_{k}^{\text{\#pixels}} \|\nabla I(\pi_c(T_i p_k)) - \nabla I(\pi_c(T_j^{-1} T_i p_k))\|_2^2$$

# Last Lecture: How do we solve these objectives?

- Bundle Adjustment or RGB-D Bundling

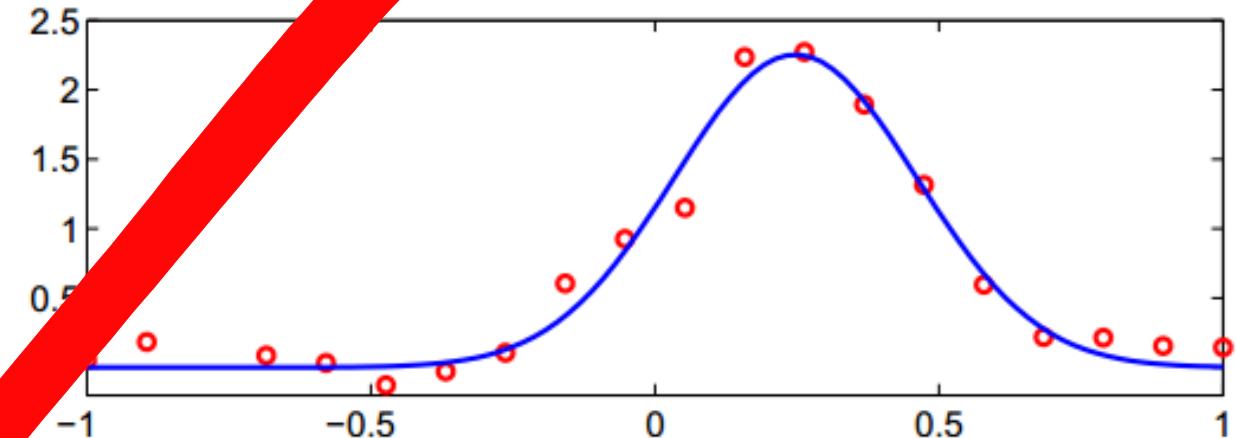
$$E_{re-proj}(T, X) = \sum_{i=1}^m \sum_{j=1}^n \left\| x_j - \pi_i(T_i \cdot X_j) \right\|_2^2$$

$$E_{keypoint}(T) = \sum_{i,j}^{\# feature correspond.} \sum_k \left\| T_i p_{ik} - T_j p_{jk} \right\|_2^2$$

# Last Lecture: Non-Linear Least Squares

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- Find solution that minimizes the sum of squared residuals
  - $f(x) = \sum r_i(x)^2$
  - $r_i$  non linear with respect to  $x$
- Fitting a Gaussian model
- $M(x, t) = x_1 e^{-(t-x_2)^2/(2x_3)^2}$ ,  $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$
- Here:  $r_i(x) = y_i - M(x, t_i)$



# Last Lecture: NNLS -> Gauss-Newton and LM

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- Newton's Method:

$$x_{k+1} = x_k - H_f(x_k)^{-1} \nabla f(x_k)$$

- Gauss-Newton (GN):

$$x_{k+1} = x_k - [2J_F(x_k)^T J_F(x_k)]^{-1} \nabla f(x_k)$$

Solve linear system (again, inverting a matrix is unstable):

$$2(J_F(x_k)^T J_F(x_k)) \underbrace{(x_k - x_{k+1})}_{\text{Solve for delta vector}} = \nabla f(x_k)$$

- Levenberg-Marquardt (LM)

$$(J_F(x_k)^T J_F(x_k) + \lambda \cdot \text{diag}(J_F(x_k)^T J_F(x_k))) \cdot (x_k - x_{k+1}) = \nabla f(x_k)$$

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# Today: Rigid Surface Tracking & Reconstruction

# How do we solve these non-linear terms?

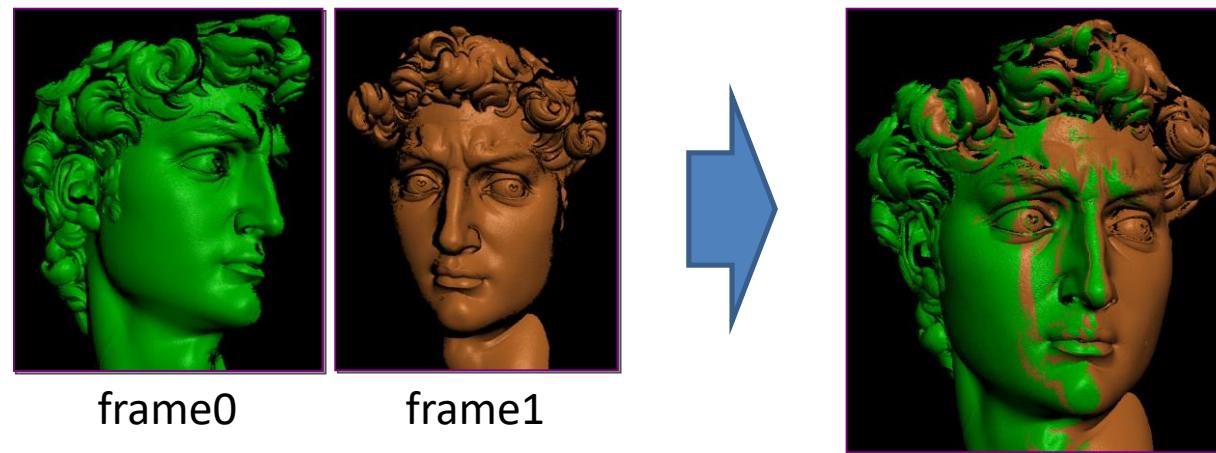
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- Frame-to-frame alignment (RGB-D case)
- $E_{frame-to-frame}(T) = \sum \|p_k - Tq_k\|_2^2$
- How to align two RGB-D frames?
  - Iterative Closest Points (ICP)!

# Problem Statement: RGB-D Tracking

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- Given RGB-D camera, move it over time and we want to align it
- Analog statement: given two point clouds, find rigid transformation between them
- Think about minimizing the distance between two implicit functions; ICP approximates functions



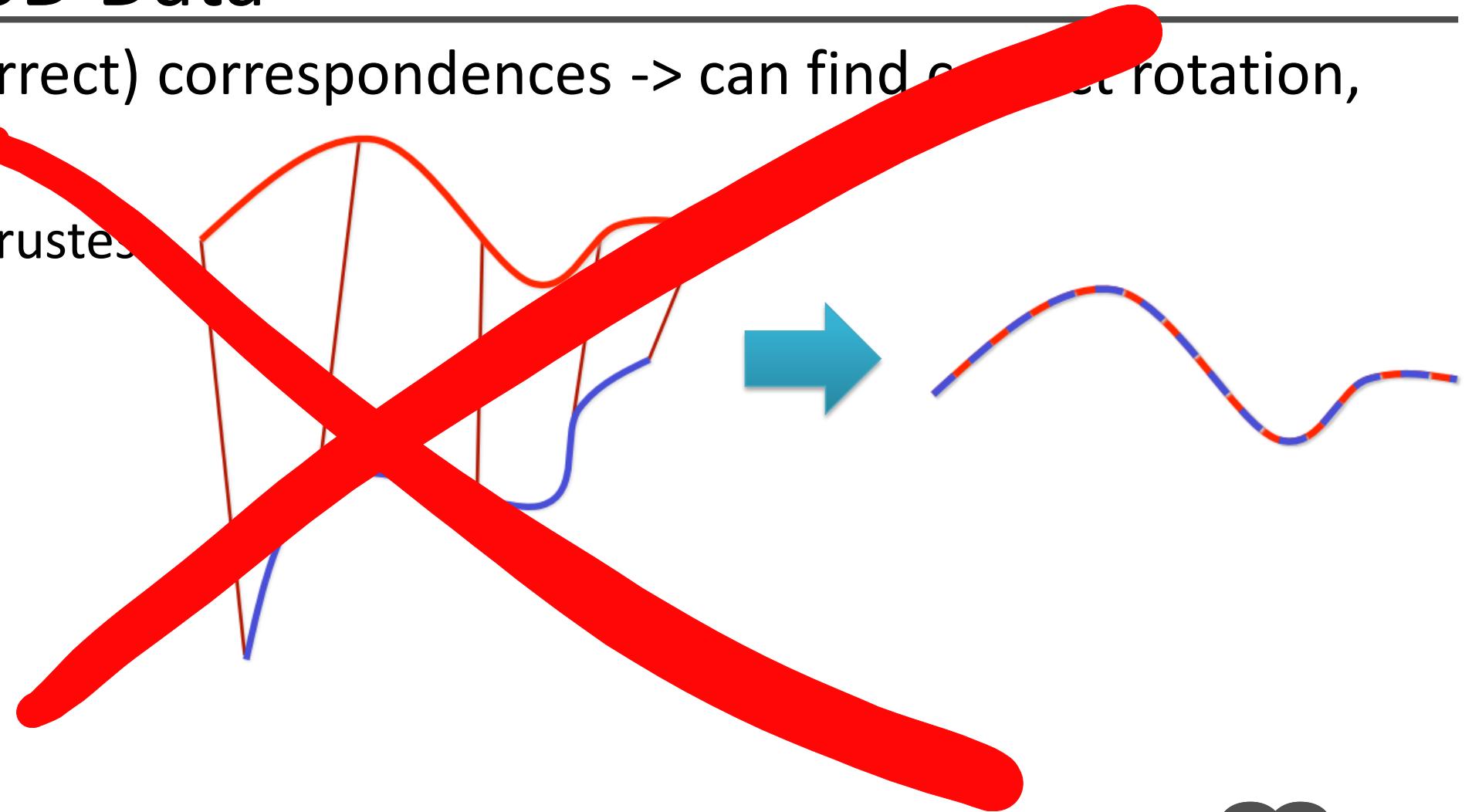
# High-Level ICP Overview

- Data association (Correspondence Search)
- Outlier removal / pruning
- Optimization
- Iterate

# Aligning 3D Data

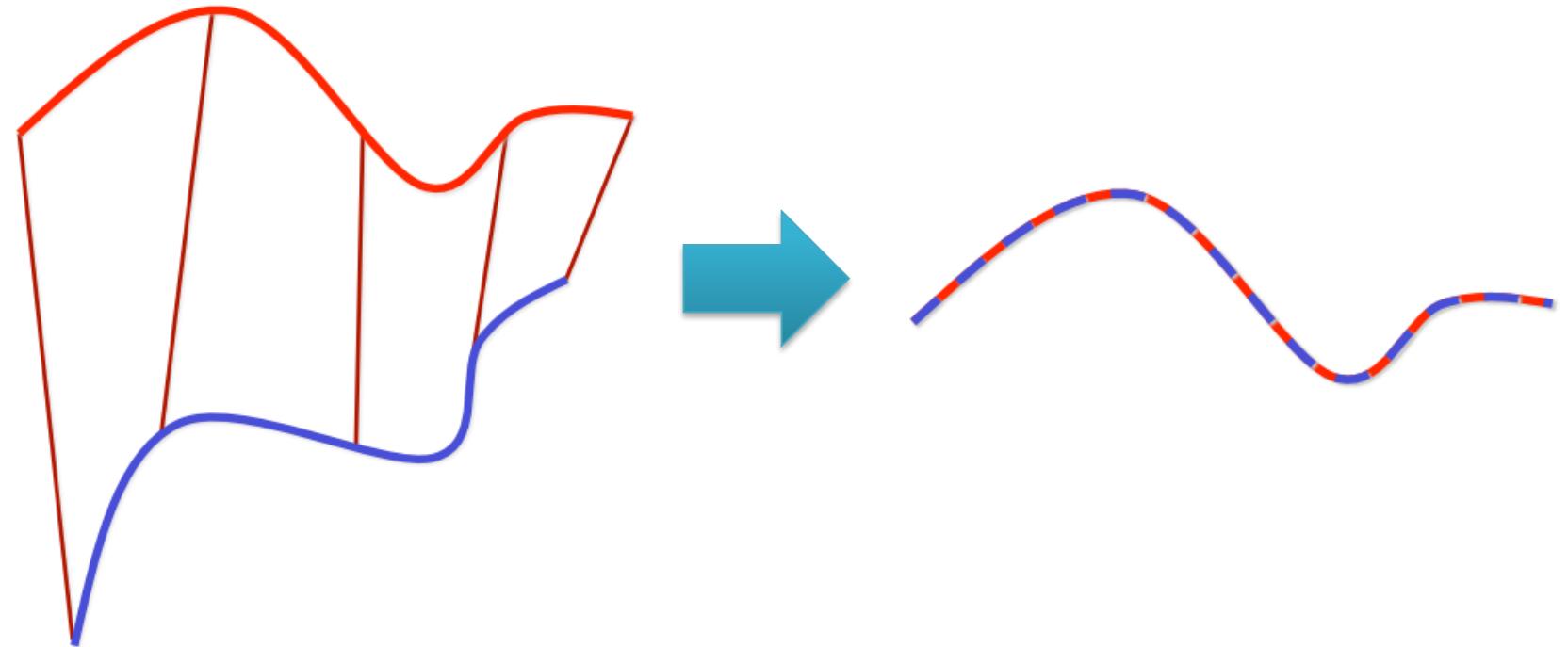
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- Known (correct) correspondences -> can find correct rotation, translation
  - E.g., Procrustes



# Aligning 3D Data

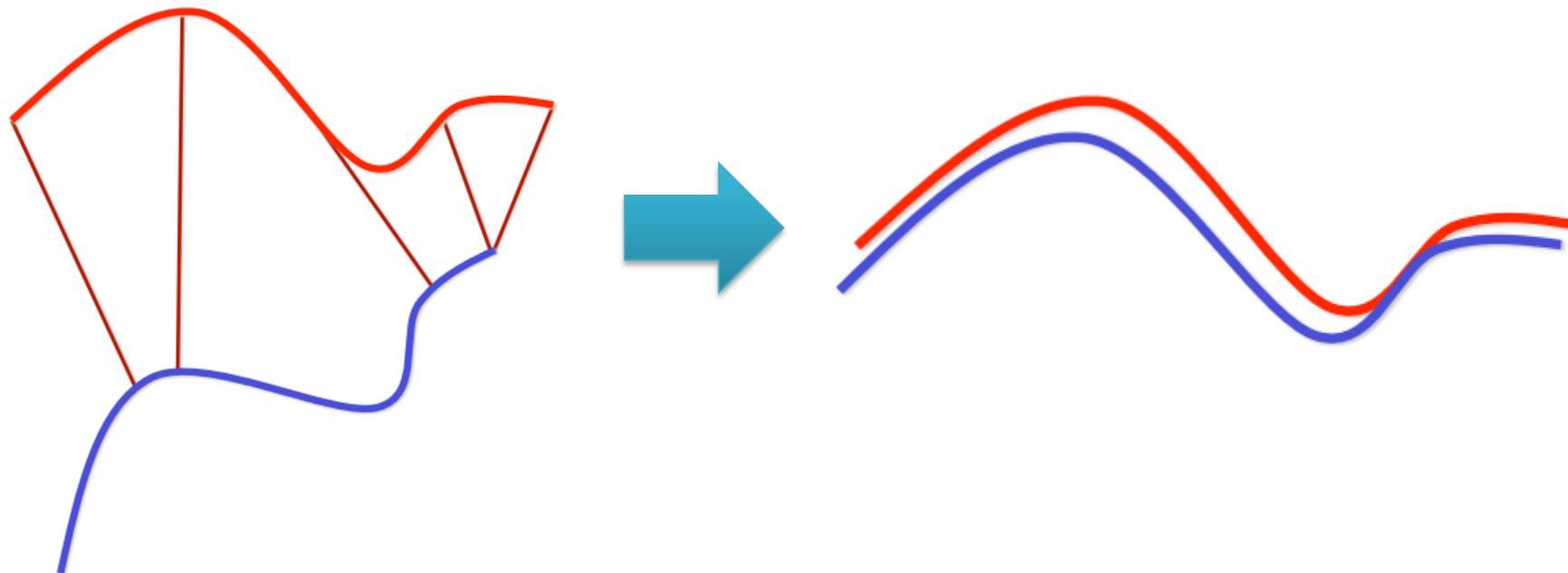
- Known (correct) correspondences -> can find correct rotation, translation
  - E.g., Procrustes



- How to find correspondences?

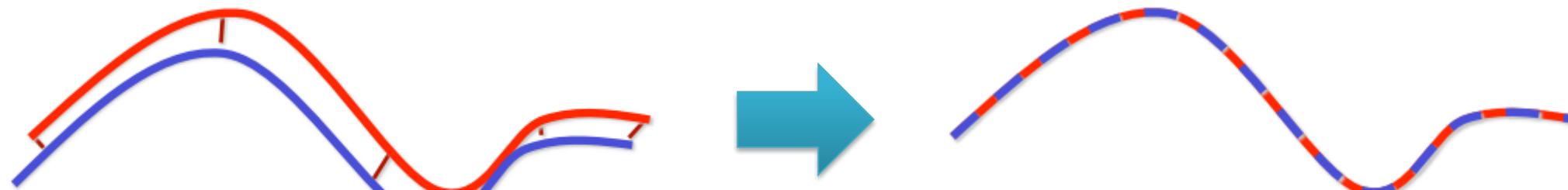
# Iterative Closest Points (ICP) [Besl and McKay 92]

- Assume closest points are corresponding



# Iterative Closest Points (ICP) [Besl and McKay 92]

- Iterate to find alignment
- Converge if poses are “close enough”



# Basic ICP

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- **Select** (e.g., 1000) random points
- **Match** each point to closest point on other scan
  - using efficient search structure, e.g., kd-tree
  - various libraries: ANN, FLANN, nanoflann
- **Reject** outlier pairs with distance  $> k \times \text{median}$
- **Error Function:**

$$E := \sum_i (R\mathbf{p}_i + t - \mathbf{q}_i)^2$$

- **Minimize** (closed form solution in [Horn 87])

# Variants of Basic ICP

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- 
- **Selecting** source points
  - **Matching** to points in other scan
  - **Weighting** the correspondences
  - **Rejecting** outlier point pairs
  - Assigning an **error metric** to the current transform
  - **Minimizing** the error metric w.r.t transformation

# Performance of ICP Variants

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- Analyze:
  - Speed
  - Stability
  - Tolerance to noise and/or outliers
  - Maximum initial displacement
- Comparison of many ICP variants:
  - [Rusinkiewicz and Levoy, 3DIM, 2001]

# Variants of Basic ICP

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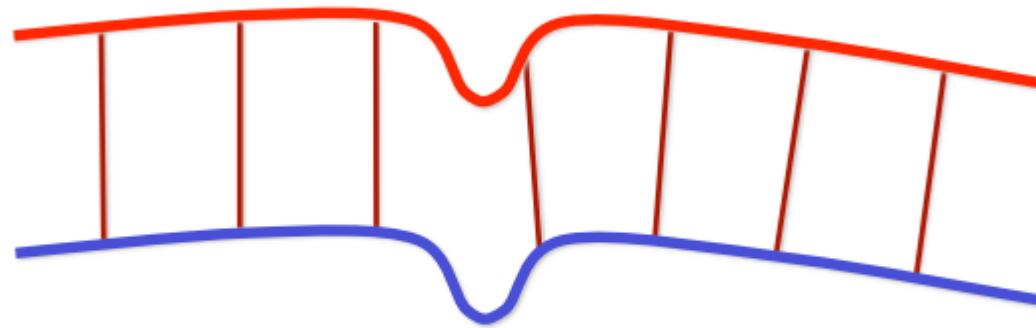
- **Selecting** source points
  - Matching to points in other scan
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  - Rejecting outlier point pairs
  - Assigning an **error metric** to the current transform
  - **Minimizing** the error metric w.r.t transformation

# Selecting Source Points

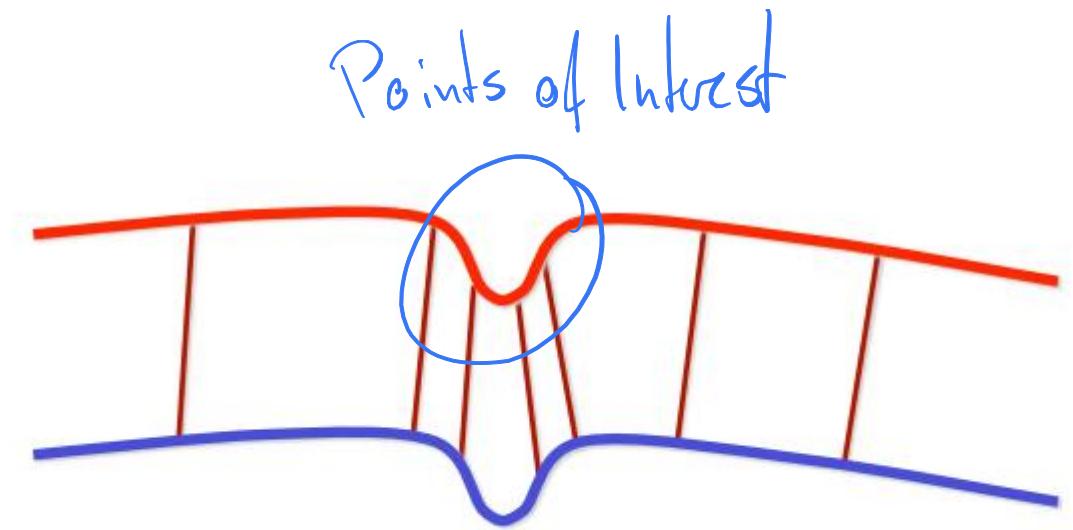
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- Use all points → Only for small #
- Uniform subsampling → ↗ Aliasing
- Random sampling
- Stable sampling [Gelfand et al. 2003]
  - Select samples that constrain all degrees of freedom of the rigid-body transformation

# Stable Sampling



Uniform Sampling



Stable Sampling

# Variants of Basic ICP

---

- **Selecting** source points
  - **Matching** to points in other scan
  - **Weighting** the correspondences
  - **Rejecting** outlier point pairs
- 
- Assigning an **error metric** to the current transform
  - **Minimizing** the error metric w.r.t transformation

# Closest Compatible Point

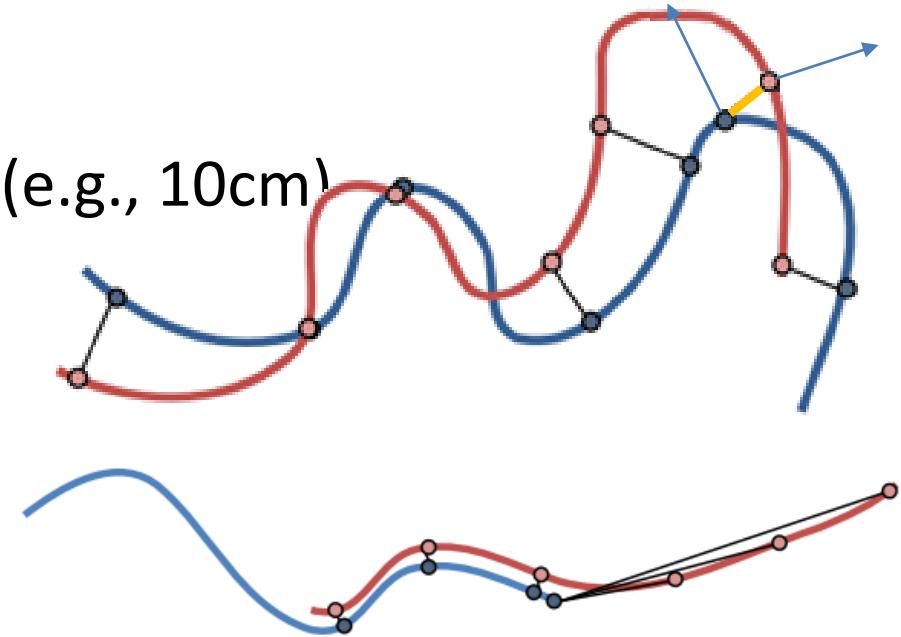
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- Closest points are often bad as corresponding points
- Improve matching effectiveness by restricting to **compatible** points
  - Color compatibility [Godin et al. 94]
  - Normal compatibility [Pulli 99]
  - Other possibilities: curvature, higher-order derivatives, other local features

# ICP: Data Association

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- Nearest neighbor (use ANN, FLANN, nanoflann)
- Projective (often much better and faster!!)
  - Only works with structured data; e.g., depth images (not on raw point clouds)
- Rejection based on outliers
  - Distance between depth values  $>$  threshold (e.g., 10cm)
  - Variation between normals
- Pruning
  - Prune correspondences to borders



# Variants of Basic ICP

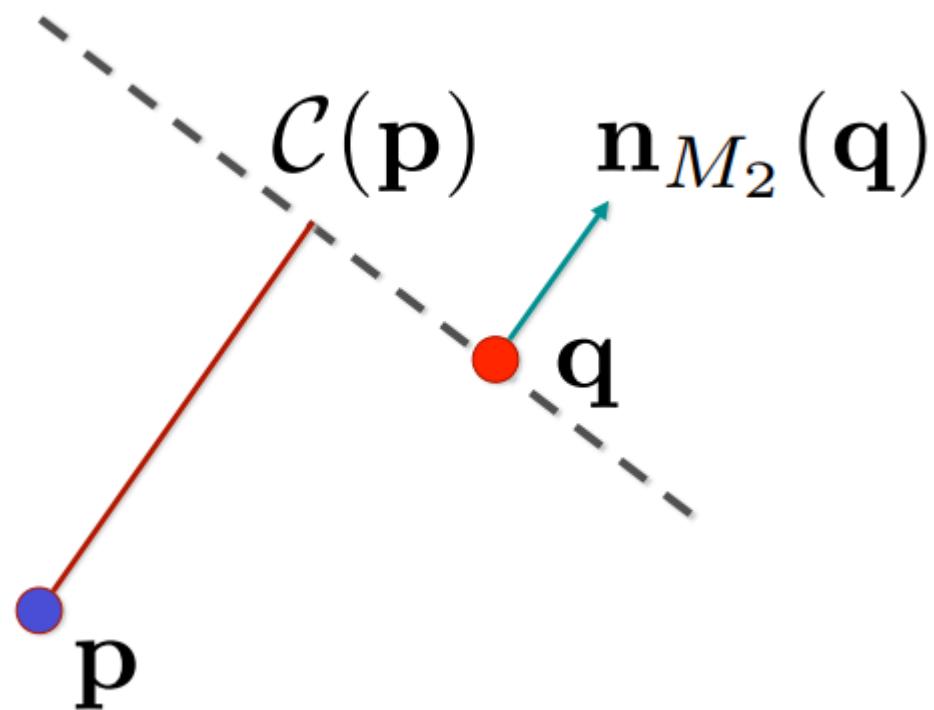
---

- Selecting source points
- Matching to points in other scan
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# Point-to-Plane Error Metric

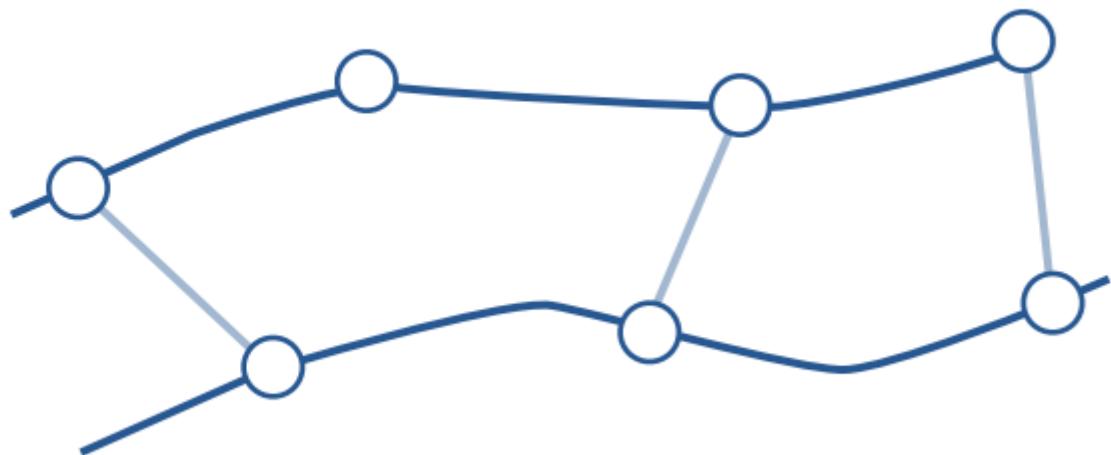
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- Allows flat regions to slide along each other, in contrast to point-to-point distance

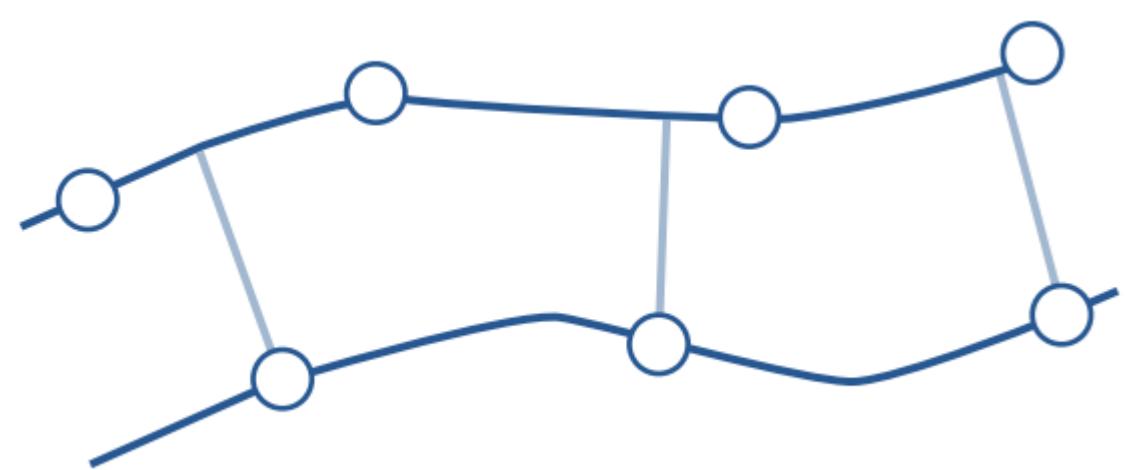


# Point-to-Plane Error Metric

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Point-to-Point



Point-to-Plane

# Variants of Basic ICP

---

- Selecting source points
- Matching to points in other scan
- Weighting the correspondences
- Rejecting outlier point pairs
- Assigning an error metric to the current transform
- **Minimizing the error metric w.r.t transformation**

# Point-to-Plane Error Metric - Optimization

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- Error function:

$$E := \sum_i ((R\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i) \cdot \mathbf{n}_i)^2$$

where  $R$  is a rotation matrix,  $t$  is a translation vector

*(camera frames only small differences!)*

- Linearize (i.e., assume  $\sin \theta \approx \theta, \cos \theta \approx 1$ ):

$$E \approx \sum_i ((\mathbf{p}_i - \mathbf{q}_i) \cdot \mathbf{n}_i + \mathbf{r} \cdot (\mathbf{p}_i \times \mathbf{n}_i) + \mathbf{t} \cdot \mathbf{n}_i)^2 \quad \text{where } \mathbf{r} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

# Point-to-Plane Error Metric - Optimization

---

- Overconstrained linear system

$$\mathbf{A}x = b,$$

$$\mathbf{A} = \begin{pmatrix} \leftarrow & \mathbf{p}_1 \times \mathbf{n}_1 & \rightarrow & \leftarrow & \mathbf{n}_1 & \rightarrow \\ \leftarrow & \mathbf{p}_2 \times \mathbf{n}_2 & \rightarrow & \leftarrow & \mathbf{n}_2 & \rightarrow \\ & \vdots & & & \vdots & \end{pmatrix}, x = \begin{pmatrix} r_x \\ r_y \\ r_z \\ t_x \\ t_y \\ t_z \end{pmatrix}, b = \begin{pmatrix} -(p_1 - q_1) \cdot n_1 \\ -(p_2 - q_2) \cdot n_2 \\ \vdots \end{pmatrix}$$

- Solve using least squares

$$\mathbf{A}^T \mathbf{A}x = \mathbf{A}^T b$$

# Point-to-Plane Error Metric - Optimization

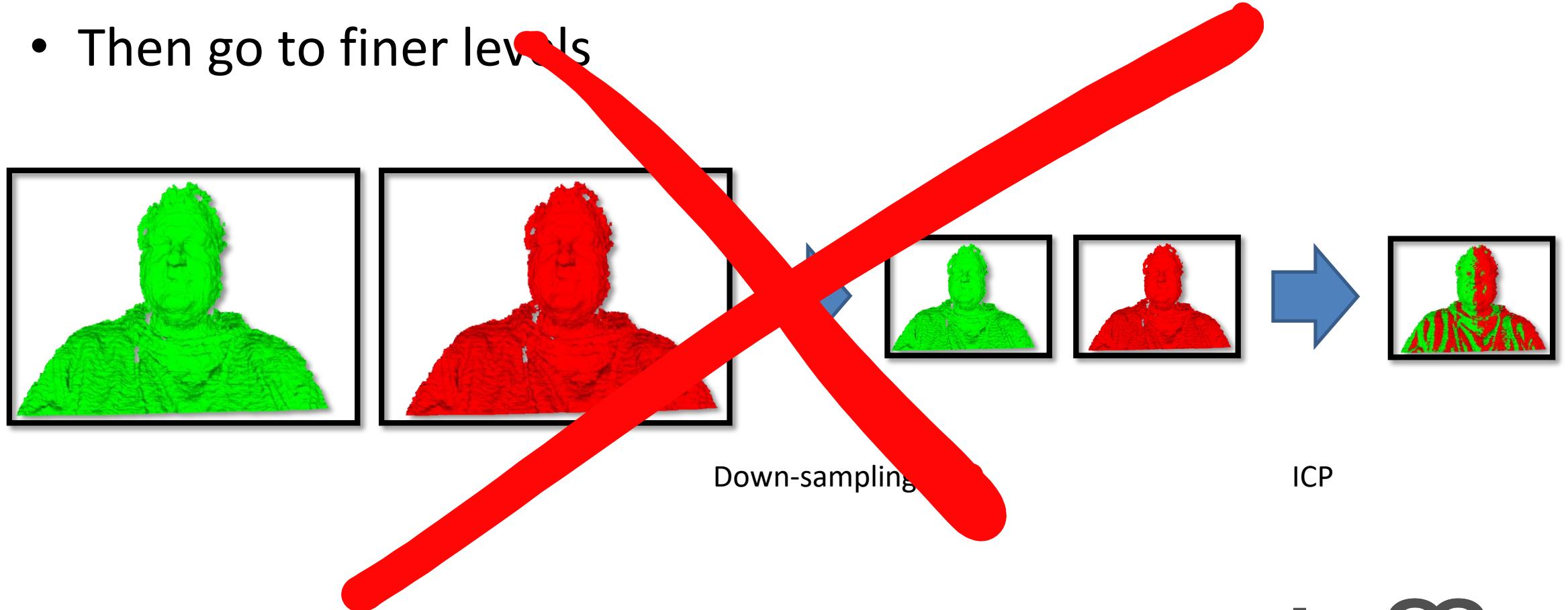
---

- Linearized Point-to-Plane Optimization [Low]
  - Only valid for small rotations (linear approximation of the rotation!)
- If you don't know what to do, always use Levenberg-Marquardt!
  - 2<sup>nd</sup> order method → **quadratic convergences** -> rules them all

$$E_{frame-to-frame}(T) = \sum_i [(Tp_i - q_i) \cdot n_i]^2$$

# Coarse-to-fine Alignment

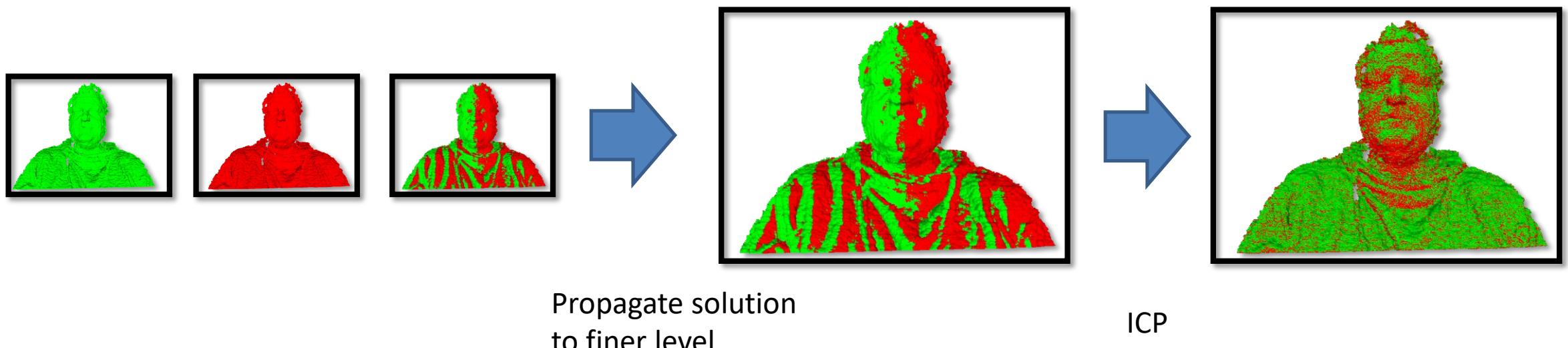
- Start on low-resolution depth map
- Then go to finer levels



# Coarse-to-fine Alignment

---

- Start on low-resolution depth map
- Then go to finer levels



# Coarse-to-fine Alignment

- Start on low-resolution depth map
- Then go to finer levels

- Faster convergence
- Avoids local minima (down-sampling → smoothing)
- Allows faster motions

# ICP: Conclusion

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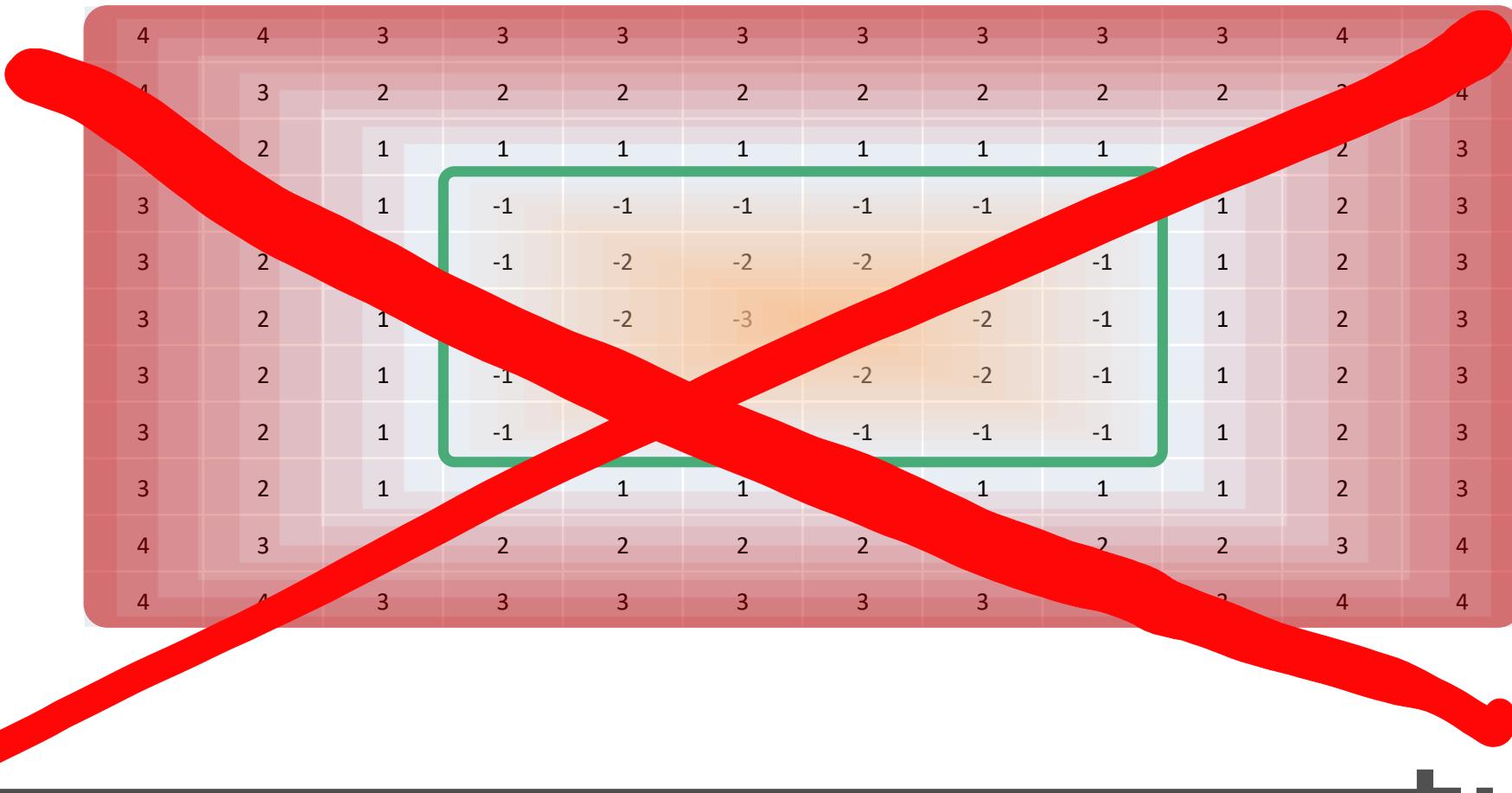
- Because the correspondence finding is not differentiable, there is the hack of alternating
- Ideally, you would want to directly align a continuous surface, and differentiate it
- Many ICP variants (e.g., adding a color term)
  - Can also do similar algorithm for RGB-only alignment

# 3D Reconstruction

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1. Compute poses between all frames
  - e.g., frame-to-frame ICP, or frame-to-model ICP
2. Integrate data into a shared model
  - e.g., accumulate in a distance field
3. Extract Mesh from implicit representation

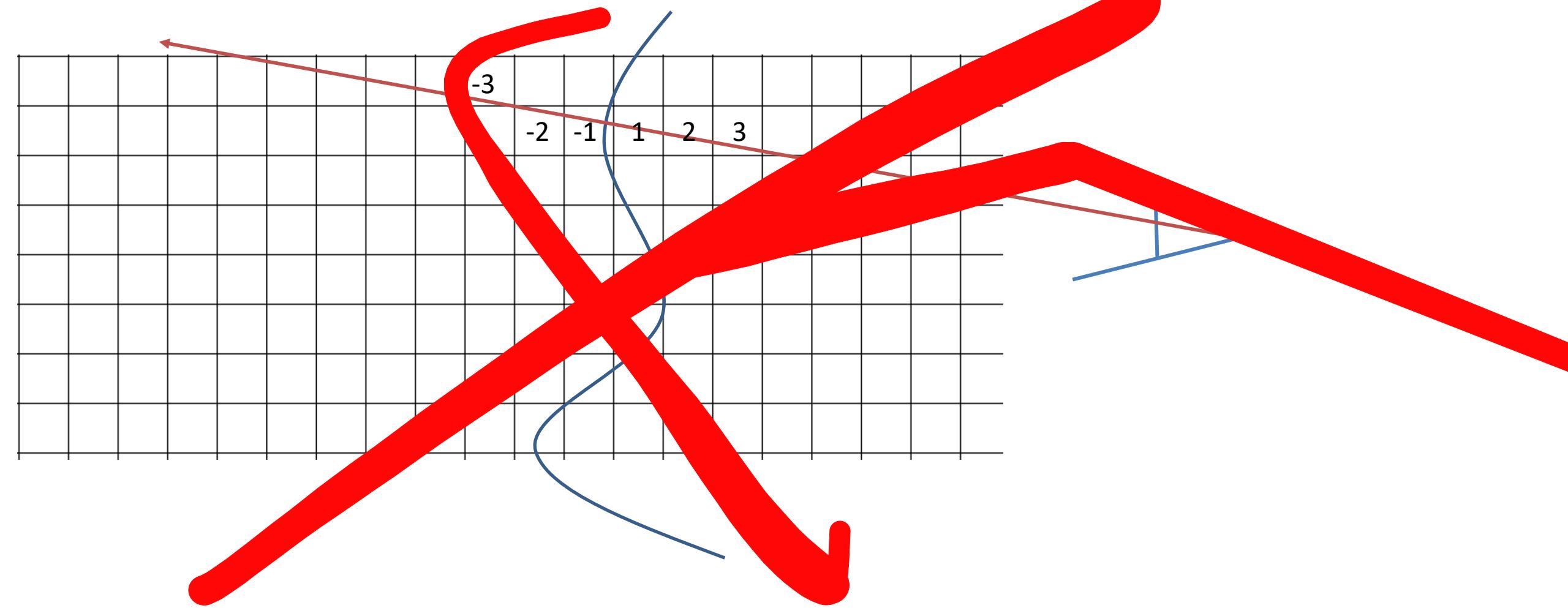
# Signed Distance Field (SDF)



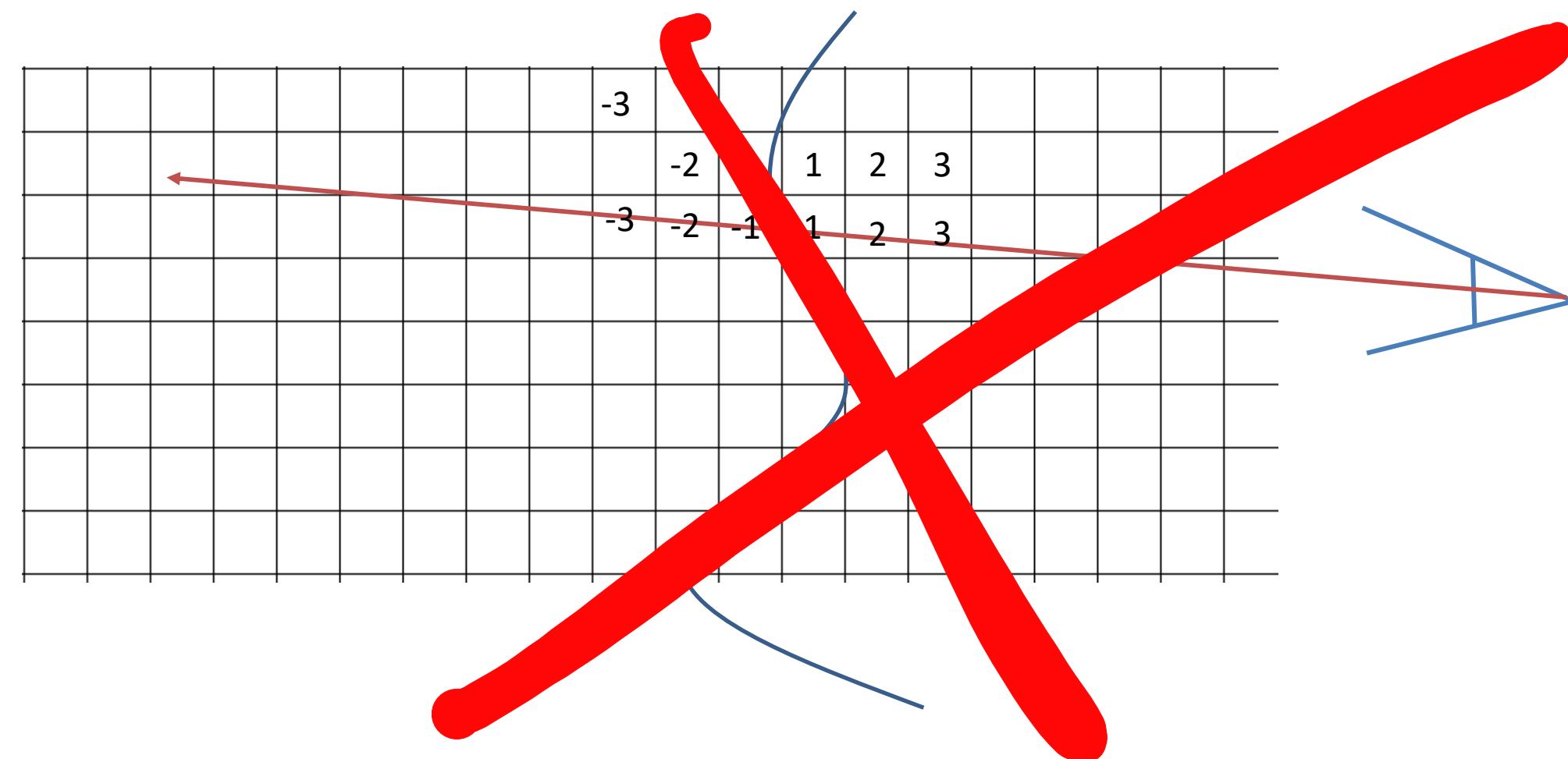
# Truncated Signed Distance Field (TSDF)

N/A											
N/A											
N/A	N/A	1		1	1	1	1		1	N/A	N/A
N/A	N/A	1	-1	-1	-1	-1	-1		1	N/A	N/A
N/A	N/A	1	-1	N/A	N/A	N/A	N/A	-1	1	N/A	N/A
N/A	N/A	1	-1	N/A	N/A	N/A	N/A	-1	1	N/A	N/A
N/A	N/A	1	-1	N/A	N/A	N/A	N/A	-1	1	N/A	N/A
N/A	N/A	1	-1	-1	N/A	N/A	N/A	-1	1	N/A	N/A
N/A	N/A	1	-1	-1	-1	N/A	N/A	-1	1	N/A	N/A
N/A											
N/A											

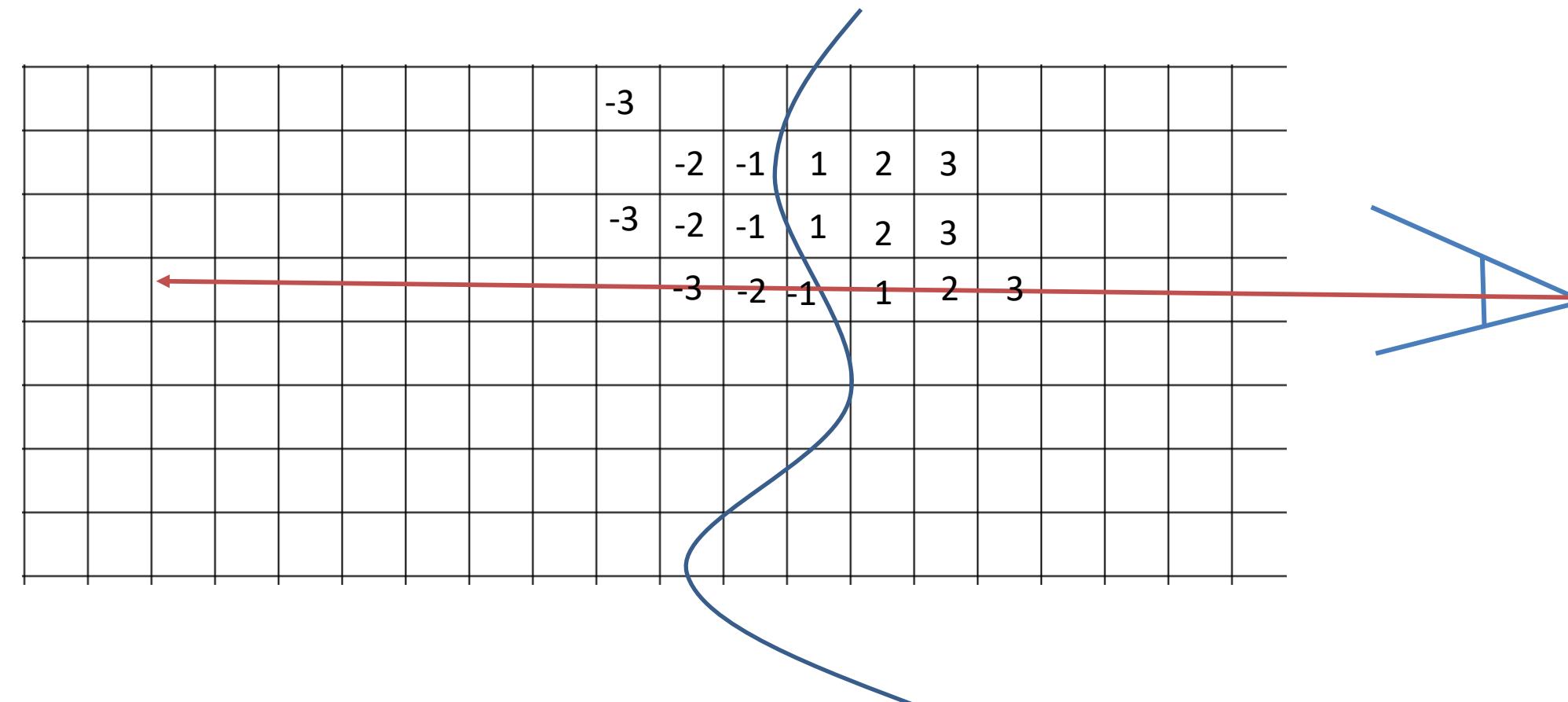
# Volumetric Fusion: Surface Integration



# Volumetric Fusion: Surface Integration

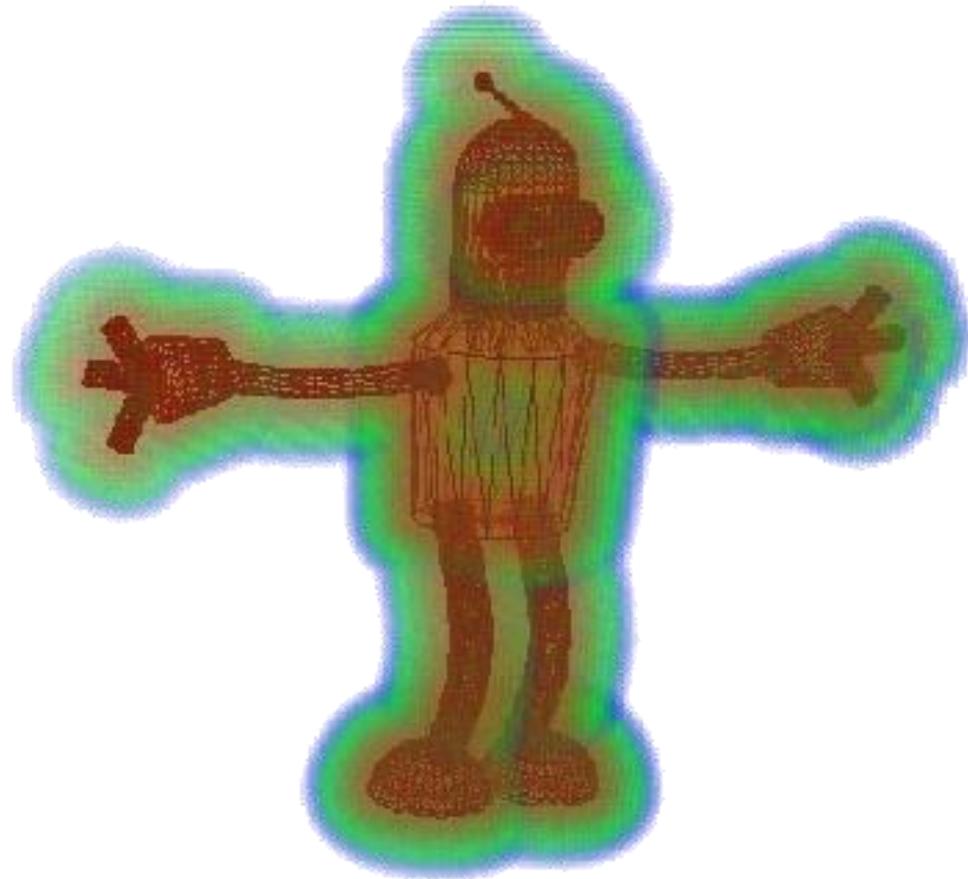


# Volumetric Fusion: Surface Integration



Obviously SDF calculation is in floats

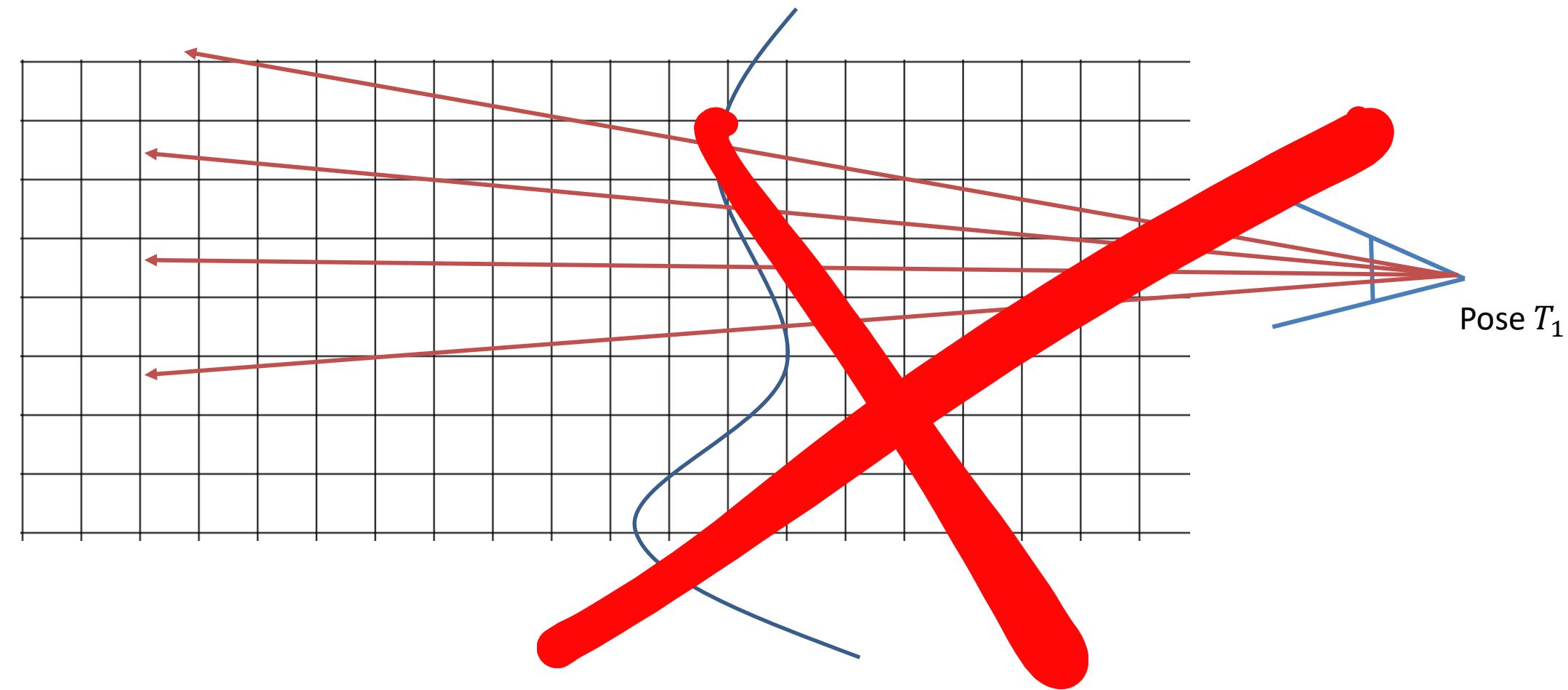
# Truncated Signed Distance Field (TSDF)



-0.9	-0.4	-0.1	0.2	0.9	1	1	1	1	1
-1	-0.9	-0.2	0.1	0.5	0.9	1	1	1	1
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-1	-0.9	-0.3	-0.6	0.3	0.7	0.9	1	1	1
-1	-0.9	-0.4	-0.1	0.3	0.8	1	1	1	1
-0.9	-0.7	-0.5	0.0	0.4	0.9	1	1	1	1
-0.1	0.0	0.0	0.1	0.4	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1

Truncated Signed Distance Field (TSDF)

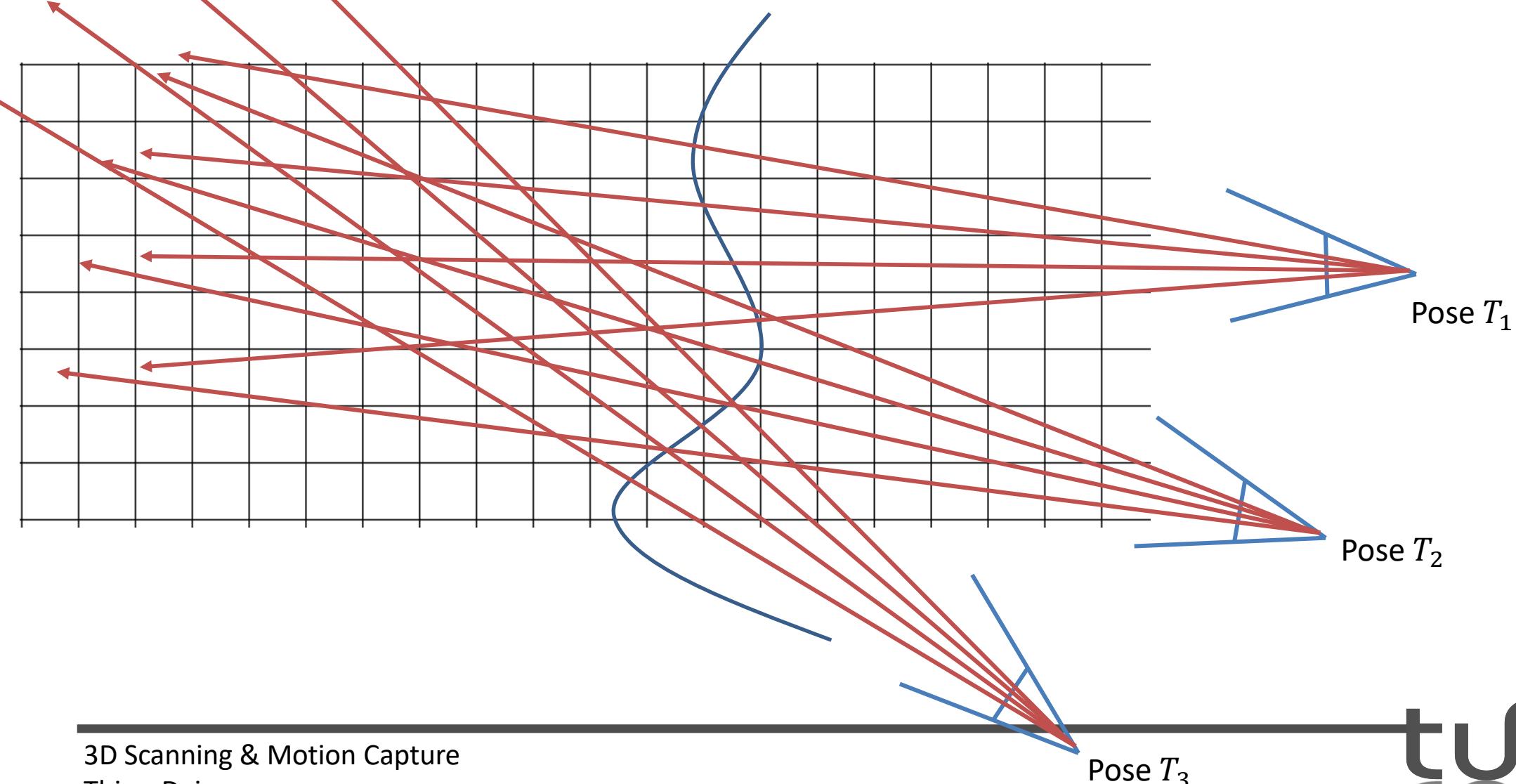
# Volumetric Fusion: Surface Integration



# Volumetric Fusion: Surface Integration



# Volumetric Fusion: Surface Integration



# Volumetric Fusion

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- Weighted average of signed distance values
- $D(x) = \frac{\sum w_i(x)d_i(x)}{\sum w_i(x)}$
- $W(x) = \sum w_i(x)$
- Depth value  $d_i(x)$  at point  $x$
- Weight functions  $w_i(x)$ 
  - E.g., depth-based -> further away -> lower weight
  - E.g.,  $w_i(x) = \max(w(1 - depthZeroOne), 1)$
- Isosurface  $D(x) = 0$

# Volumetric Fusion

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- Running average of signed distance values
- $D_{i+1}(x) = \frac{W_i(x)D_i(x)+w_{i+1}(x)d_{i+1}(x)}{W_i(x)+w_{i+1}(X)}$
- $W_{i+1}(x) = W_i(x) + w_{i+1}(x)$
- $W_i(x)$  and  $D_i(x)$  are cumulative signed distanced and weight functions after integrating the i-th range image

# Volumetric Fusion

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- Surface Integration [*Curless and Levoy 96*]

```
Voxel {  
    distance;  
    color;  
    weight;  
}
```

$$D(v) = \frac{\sum_i w_i(v)d_i(v)}{\sum_i w_i(v)}$$

$$W(v) = \sum_i w_i(v)$$

# Volumetric Fusion

---

- Surface Integration [*Curless and Levoy 96*]

```
Voxel {  
    distance;  
    color;  
    weight;  
}
```

$$D'(v) = \frac{W(v)D(v) + w_k(v)d_k(v)}{W(v) + w_k(v)}$$

$$W'(v) = W(v) + w_k(v)$$

# Volumetric Fusion

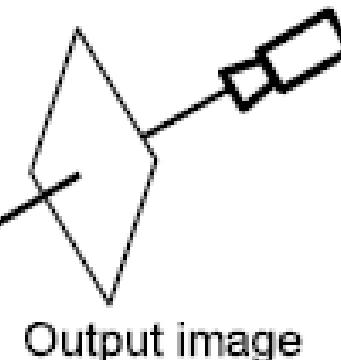
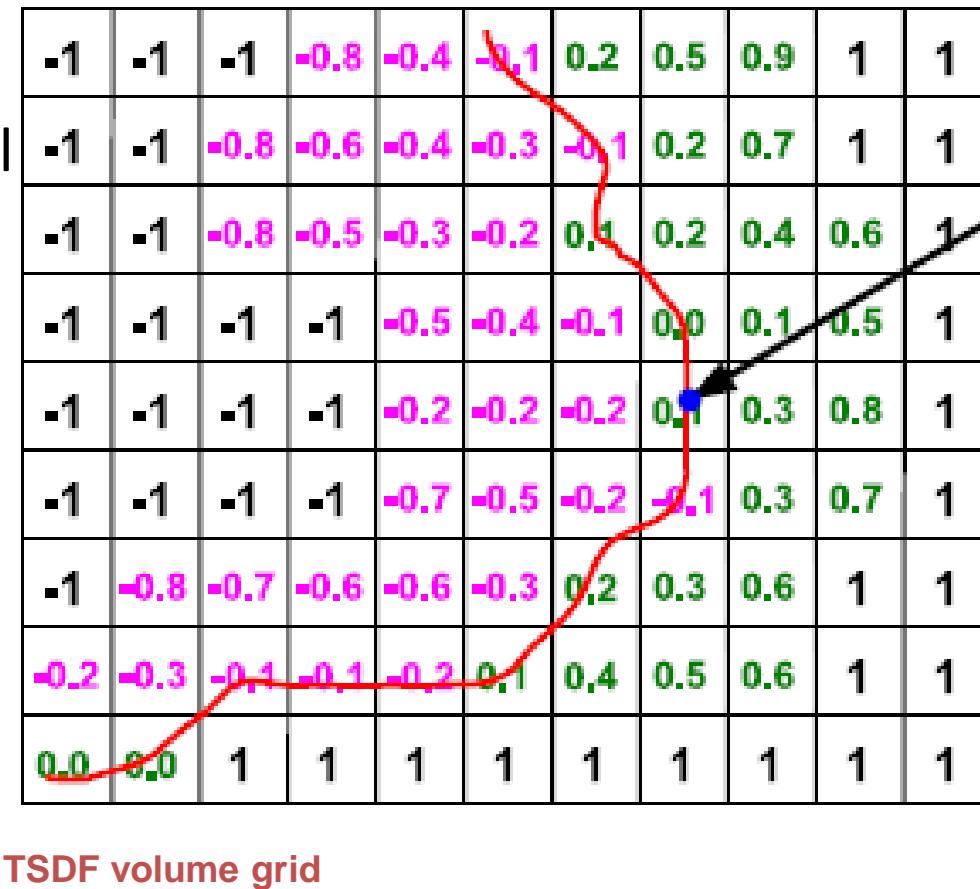
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```
void integrate(const mat4f& intrinsic, const mat4f& cameraToWorld, const DepthImage32& depthImage) {  
  
    const mat4f worldToCamera = cameraToWorld.getInverse();  
    BoundingBox3<int> voxelBounds = computeFrustumBounds(intrinsic, cameraToWorld, depthImage.getWidth(), depthImage.getHeight());  
  
#pragma omp parallel for  
    for (int k = voxelBounds.getMinZ(); k <= voxelBounds.getMaxZ(); k++) {  
        for (int j = voxelBounds.getMinY(); j <= voxelBounds.getMaxY(); j++) {  
            for (int i = voxelBounds.getMinX(); i <= voxelBounds.getMaxX(); i++) {  
  
                //transform to current frame  
                vec3f p = worldToCamera * voxelToWorld(vec3i(i, j, k));  
  
                //project into depth image  
                p = skeletonToDepth(intrinsic, p);  
  
                vec3i pi = math::round(p);  
                if (pi.x >= 0 && pi.y >= 0 && pi.x < (int)depthImage.getWidth() && pi.y < (int)depthImage.getHeight()) {  
                    float d = depthImage(pi.x, pi.y);  
  
                    //check for a valid depth range  
                    if (d >= m_depthMin && d <= m_depthMax) {  
  
                        //update free space counter if voxel is in front of observation  
                        if (p.z < d) {  
                            (*this)(i, j, k).freeCtr++;  
                        }  
  
                        //compute signed distance; positive in front of the observation  
                        float sdf = d - p.z;  
                        float truncation = getTruncation(d);  
  
                        //if (std::abs(sdf) < truncation) {  
                        if (sdf > -truncation) {  
                            Voxel& v = (*this)(i, j, k);  
                            v.sdf = (v.sdf * (float)v.weight + sdf * (float)m_weightUpdate) / (float)(v.weight + m_weightUpdate);  
                            v.weight = (uchar)std::min((int)v.weight + (int)m_weightUpdate, (int)std::numeric_limits<unsigned char>::max())  
                        }  
                }  
        }  
    }  
}
```

# Surface Rendering: Ray Casting

- Cast ray through focal point of each pixel
- Traverse voxels along the ray  
(truncation step size)
- Find surface by checking for sign change
- Compute intersection  
e.g., bilinear search  
compute normal by  $\nabla TSDF$

surface

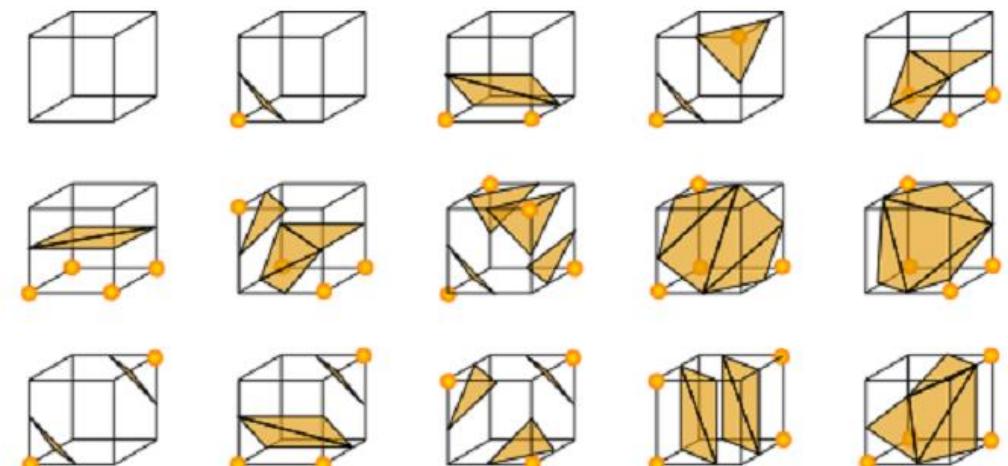


# Surface Extraction: Marching Cubes

-1	-1	-1	-0.8	-0.4	-0.1	0.2	0.5	0.9	1	1
-1	-1	-0.8	-0.6	-0.4	-0.3	-0.1	0.2	0.7	1	1
-1	-1	-0.8	-0.5	-0.3	-0.2	0.1	0.2	0.4	0.6	1
-1	-1	-1	-1	-0.5	-0.4	-0.1	0.0	0.1	0.5	1
-1	-1	-1	-1	-0.2	-0.2	-0.2	0.1	0.3	0.8	1
-1	-1	-1	-1	-0.7	-0.5	-0.2	-0.1	0.3	0.7	1
-1	-0.8	-0.7	-0.6	-0.6	-0.3	0.2	0.3	0.6	1	1
-0.2	-0.3	-0.1	0.1	0.2	0.1	0.4	0.5	0.6	1	1
0.0	0.0	1	1	1	1	1	1	1	1	1

TSDF volume grid

surface (zero-crossing)

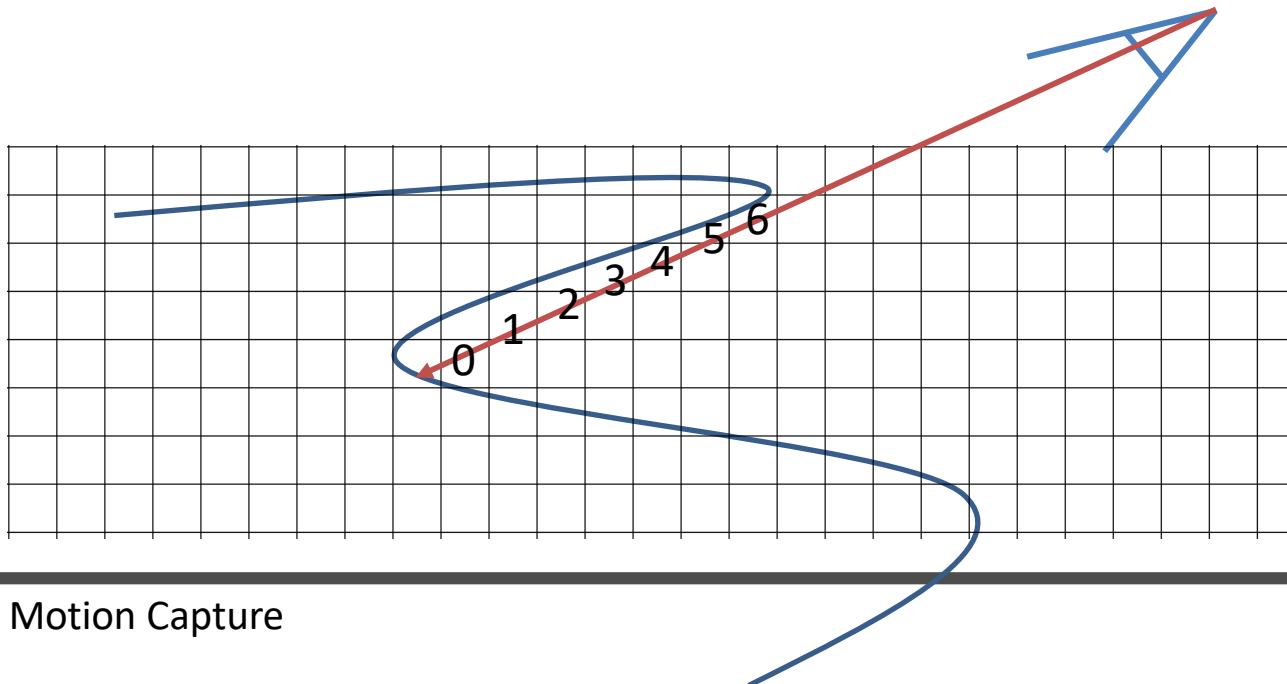


Marching Cubes table

# Volumetric Fusion

---

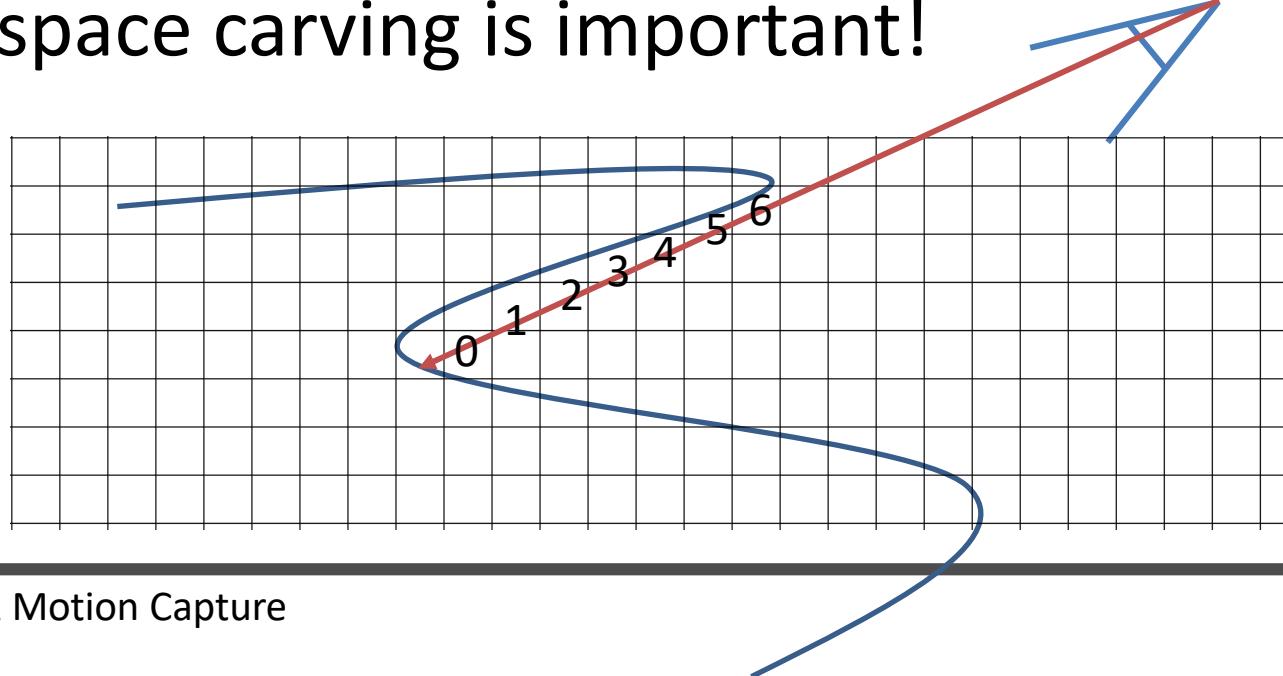
- Projective integration is not a ‘true’ SDF
  - Less accurate the further away from the surface
  - More accurate with more different views
  - Could apply distance transform but misses data



# Volumetric Fusion

- Interesting property though:
  - ‘0’ -> surface
  - ‘negative’ -> unknown
  - ‘positive’ -> free space
- Free space carving is important!

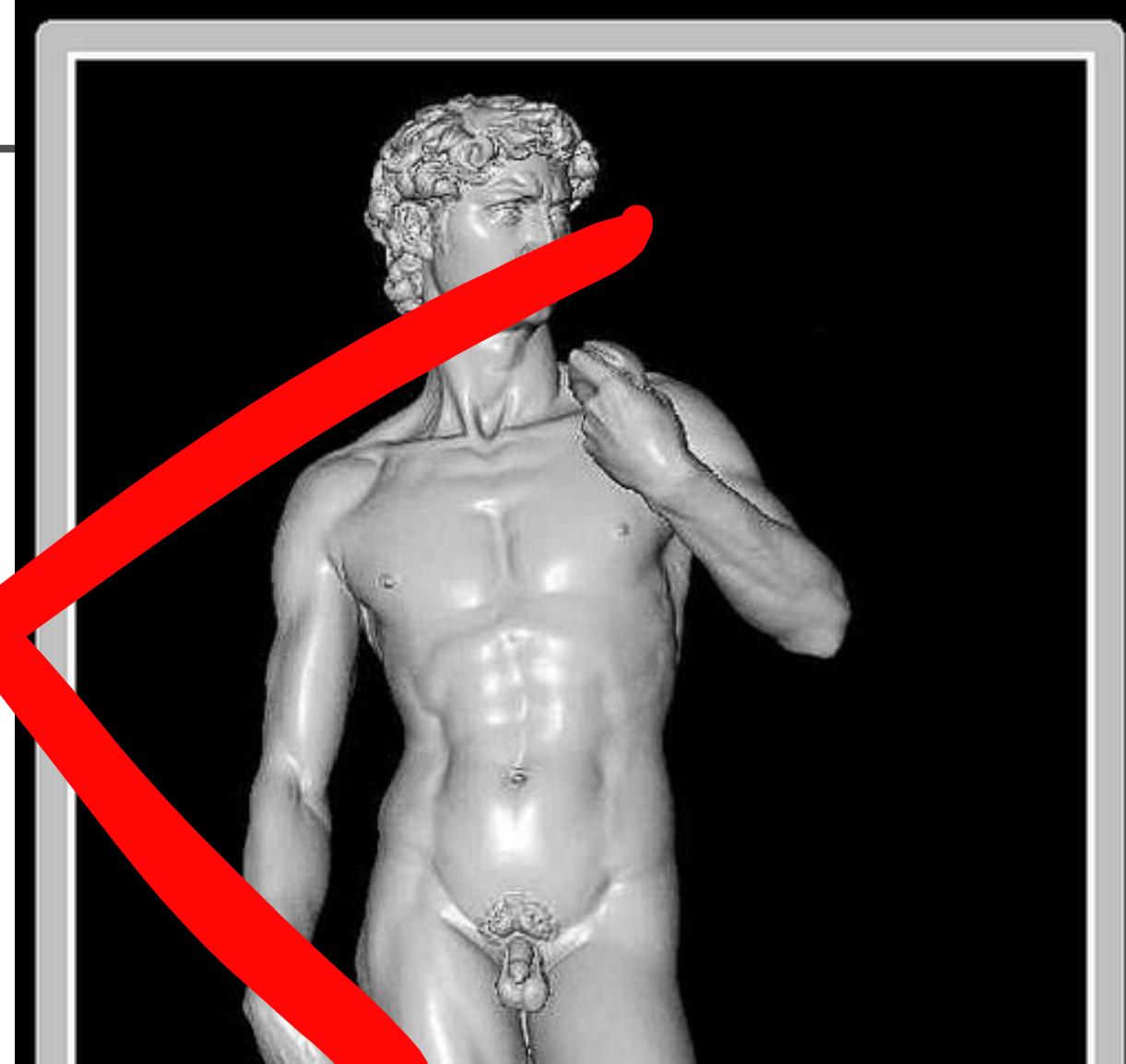
-1	-1	-1	-0.8	-0.4	-0.1	0.2	0.5	0.9	1	1
-1	-1	-0.8	-0.6	-0.4	-0.3	-0.1	0.2	0.7	1	1
-1	-1	-0.8	-0.5	-0.3	-0.2	0.1	0.2	0.4	0.6	1
-1	-1	-1	-1	-0.5	-0.4	-0.1	0.0	0.1	0.5	1
-1	-1	-1	-1	-0.2	-0.2	-0.2	0.1	0.3	0.8	1
-1	-1	-1	-1	-0.7	-0.5	-0.2	-0.1	0.3	0.7	1
-1	-0.8	-0.7	-0.6	-0.6	-0.3	0.2	0.3	0.6	1	1
-0.2	-0.3	-0.1	-0.1	-0.2	0.1	0.4	0.5	0.6	1	1
0.0	0.0	1	1	1	1	1	1	1	1	1



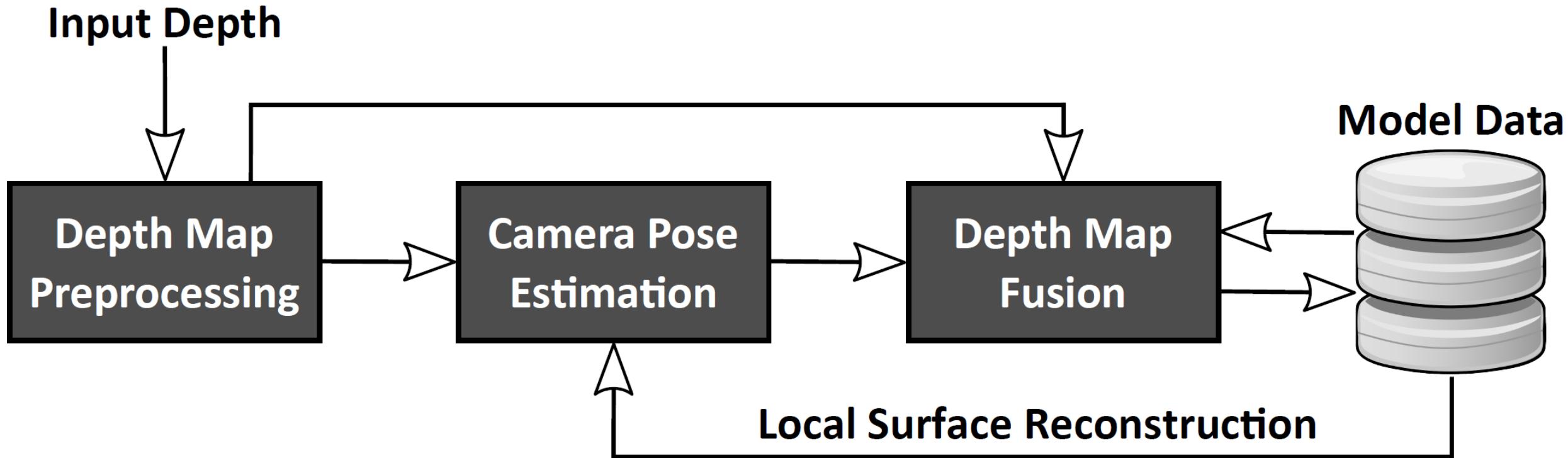
# Volumetric Fusion



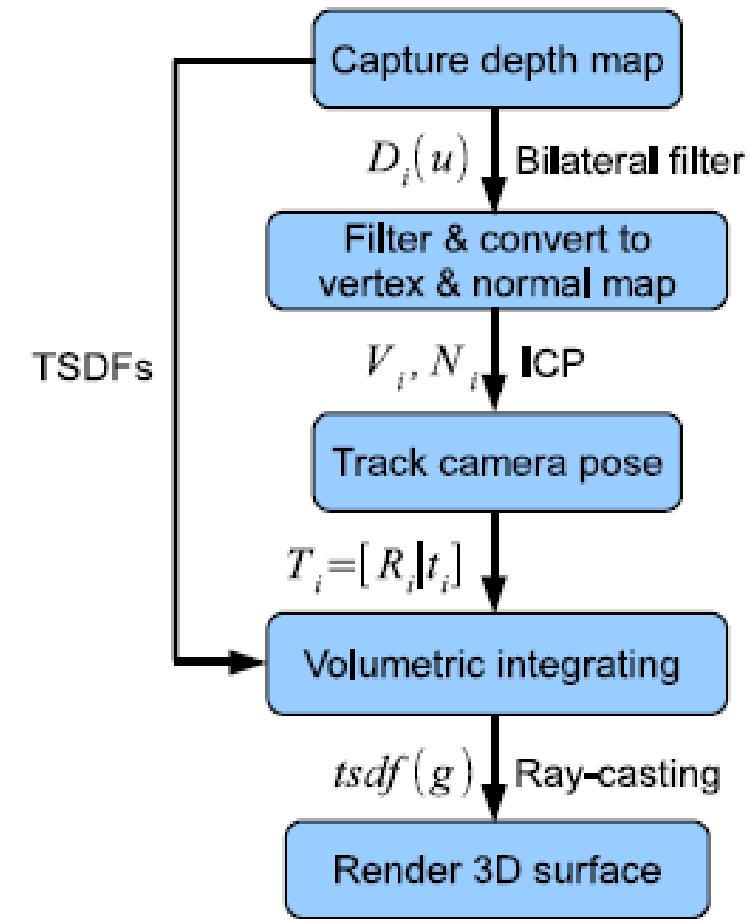
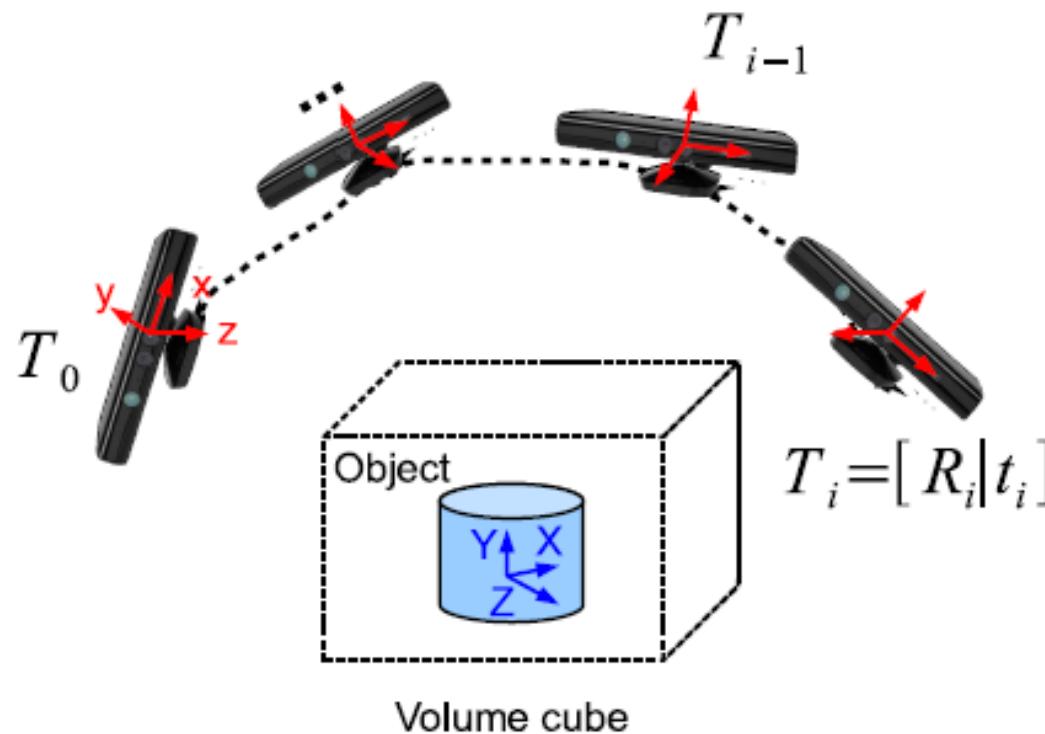
Stanford Bunny



# Online 3D Reconstruction



# Online 3D Reconstruction



# Online 3D Reconstruction

---

- Often frame-to-model tracking rather than frame-to-frame
  - Use ray-casted normal map -> less noise!

More robust



Input normal map



Ray-casted normal map from SDF



# Prominent Literature

---

- A volumetric method for building [...] Durless and Levoy 96]
  - TSDF integration
- Szymon Rusinkiewicz
  - Efficient Variants of ICP
  - Real-time Model Acquisition <- Siggraph 2002!
- KinectFusion [Newcombe/Izadi et al. 11]

# Real-time 3D Model Acquisition

---

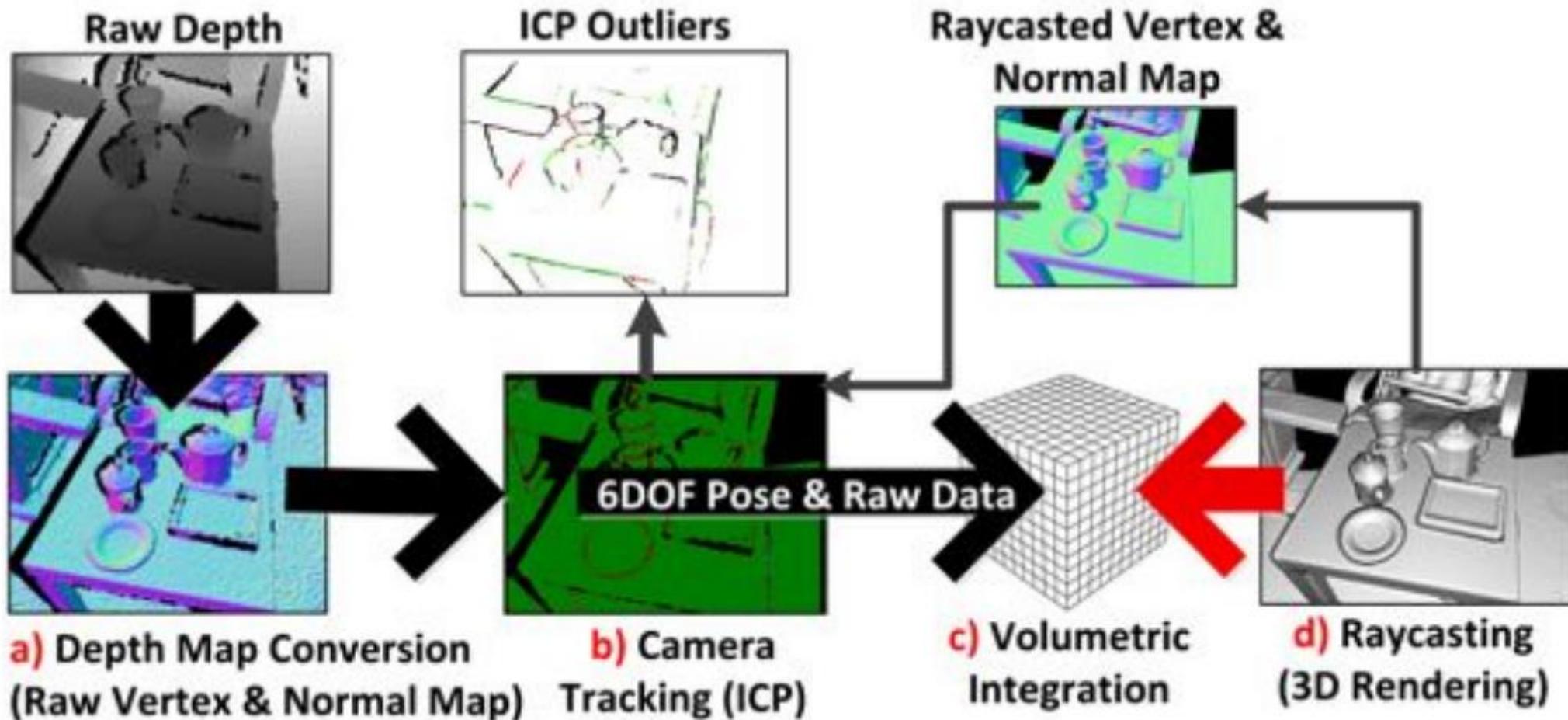


# Kinect Fusion

---

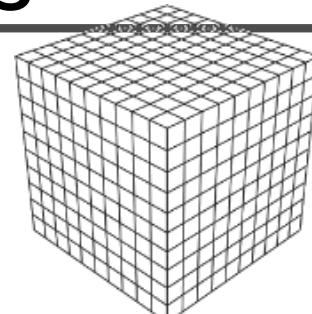


# Kinect Fusion

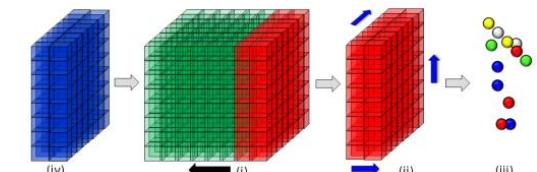


# Spatial Representations

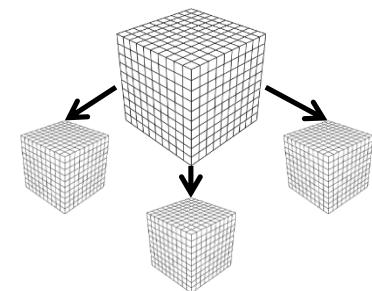
- Kinect Fusion -> Dense Grid
  - [Izadi/Newcombe et al. 11]



- Extended Kinect Fusion
  - [Roth et al. 12; Whelan et al. 12]



- Hierarchical Fusion
  - [Stueckler et al. 12; Chen et al. 13]

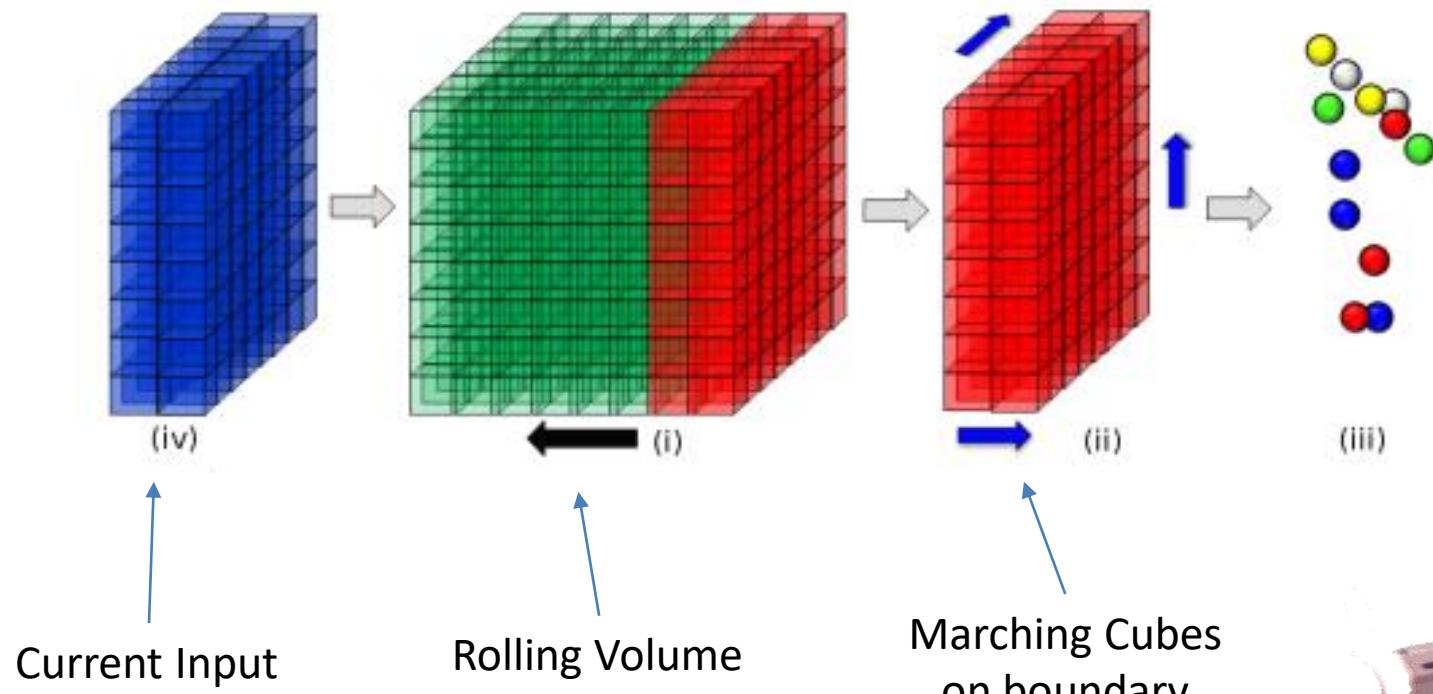


- Voxel Hashing
  - [Niessner et al. 13]



# Extended Kinect Fusion

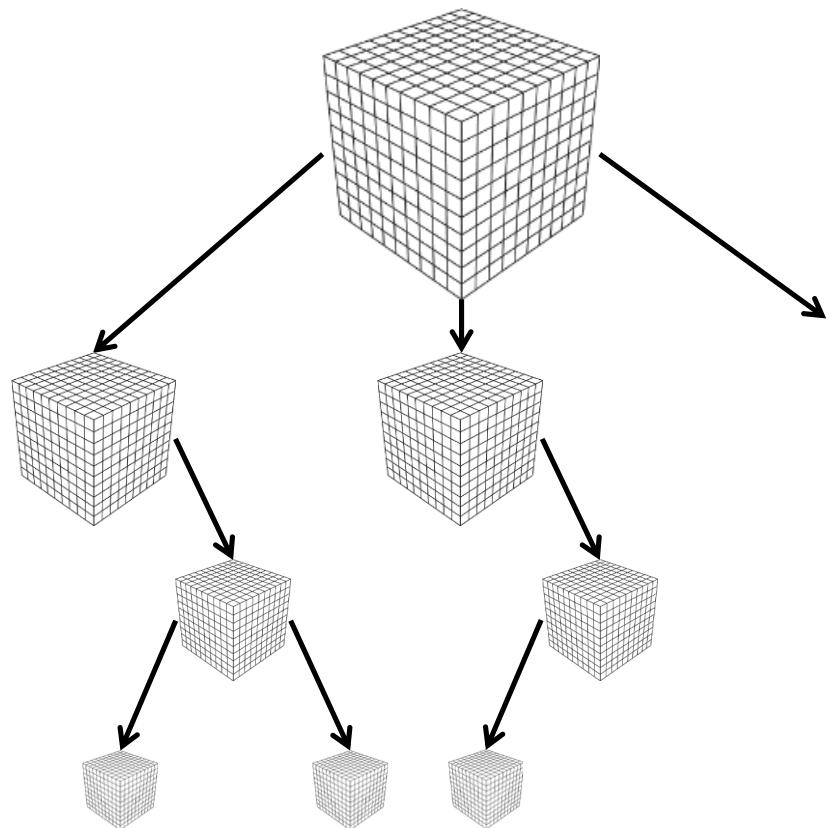
- Moving Volume



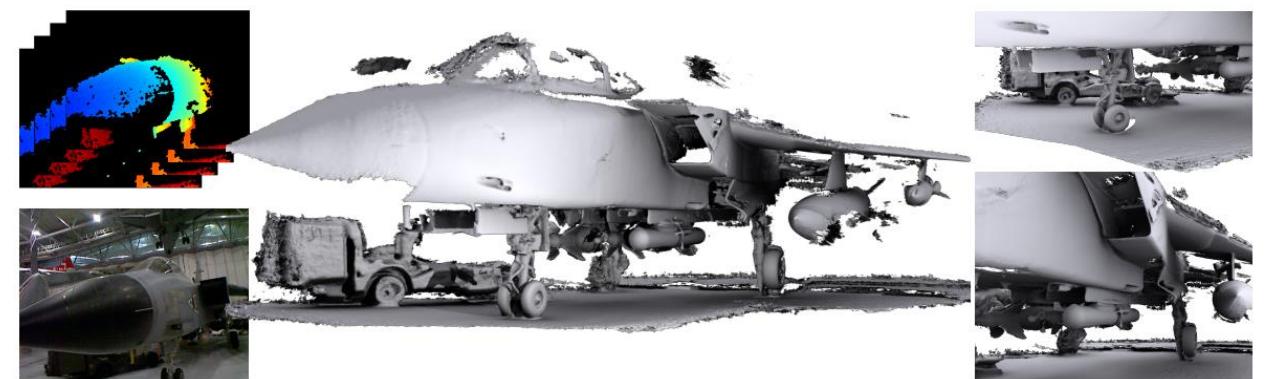
- + very easy to implement
- + unlimited spatial volumes
- cannot integrate into previously regions



# Hierarchical Fusion



Spatial subdivision with tree  
SDF values live only on finest level



- + efficient storage: large spatial extent, high local res
- + fast lookups
- costly updates during integration (need tree update)  
e.g., insertion, removal, etc.

# Voxel Hashing

Spatial Hash Function:

$$H(x, y, z) = (x \cdot p_1 \oplus y \cdot p_2 \oplus z \cdot p_3) \bmod n$$

```
struct HashEntry {  
    short position[3];  
    short offset;  
    int pointer;  
};
```

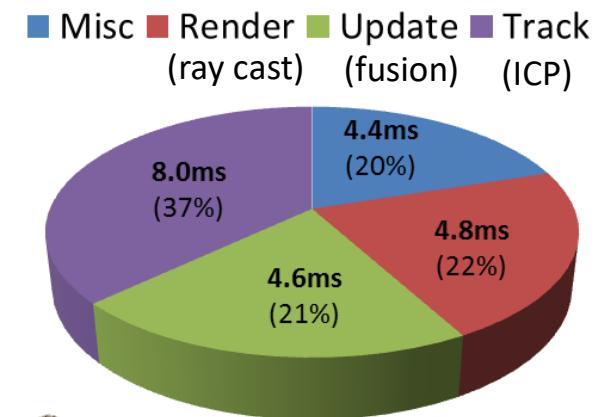
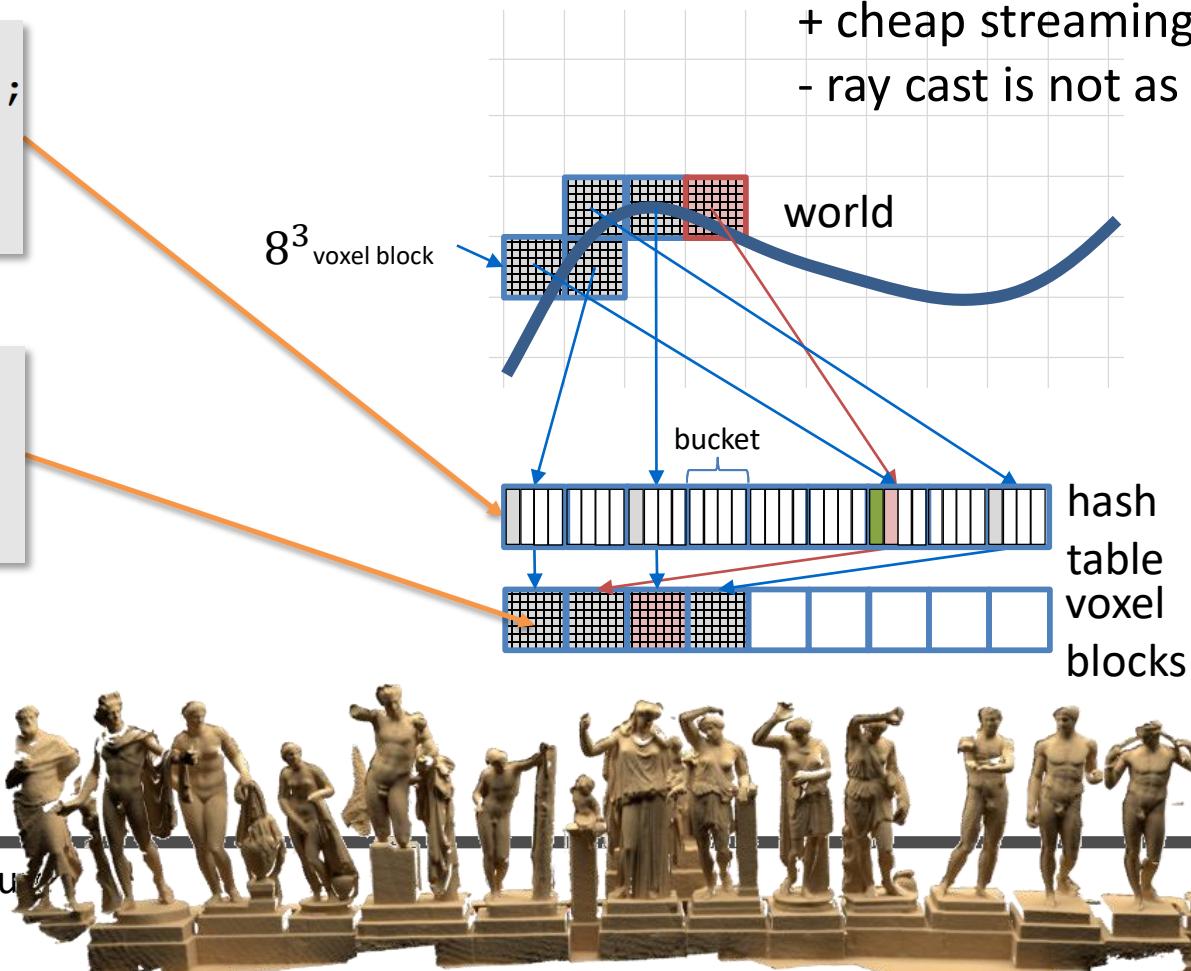
```
struct Voxel {  
    float sdf;  
    uchar colorRGB[3];  
    uchar weight;  
};
```

+ efficient storage: large spatial extent, high local res  
+ extremely fast updates:

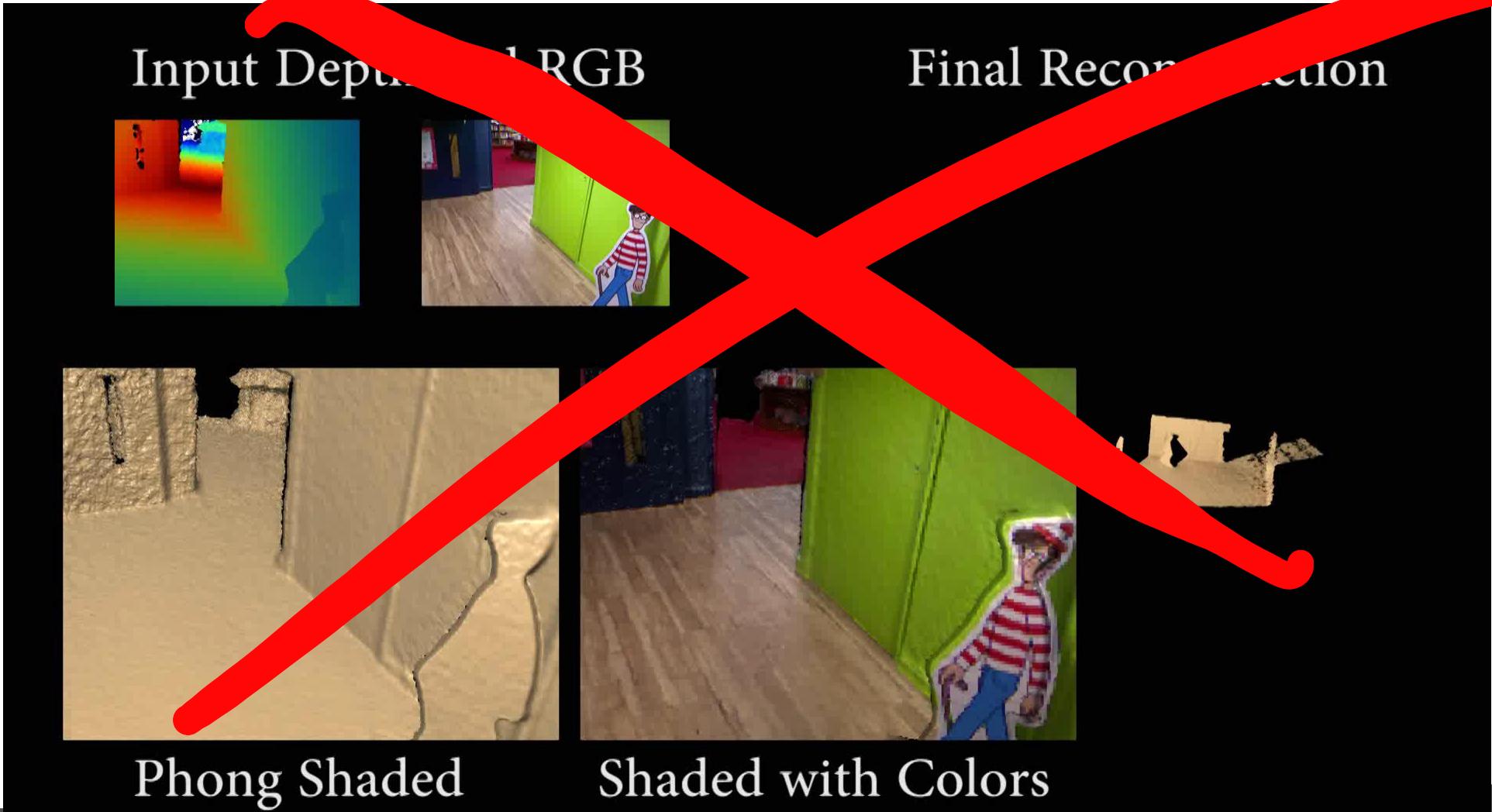
e.g., insertion, removal, etc. is  $O(1)$

+ cheap streaming

- ray cast is not as fast (need to check voxel by voxel)

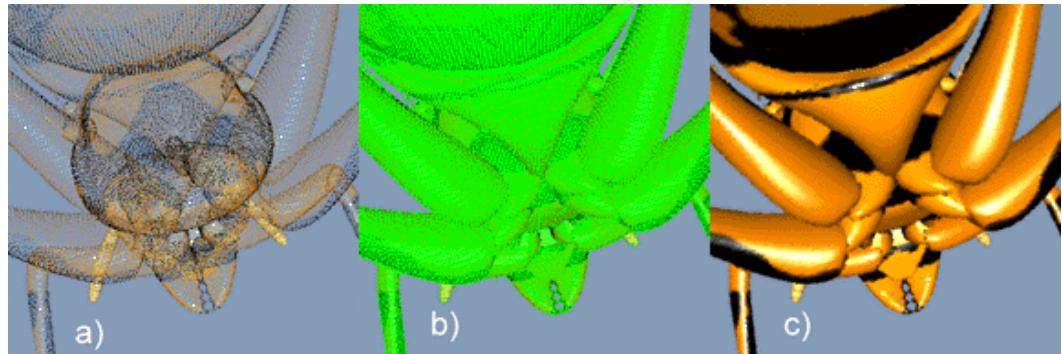


# Voxel Hashing



# Other Representations: Point-based Fusion

- Surfel representation
  - Point + normal + radius
- Rendering through splatting
  - Use graphics pipeline

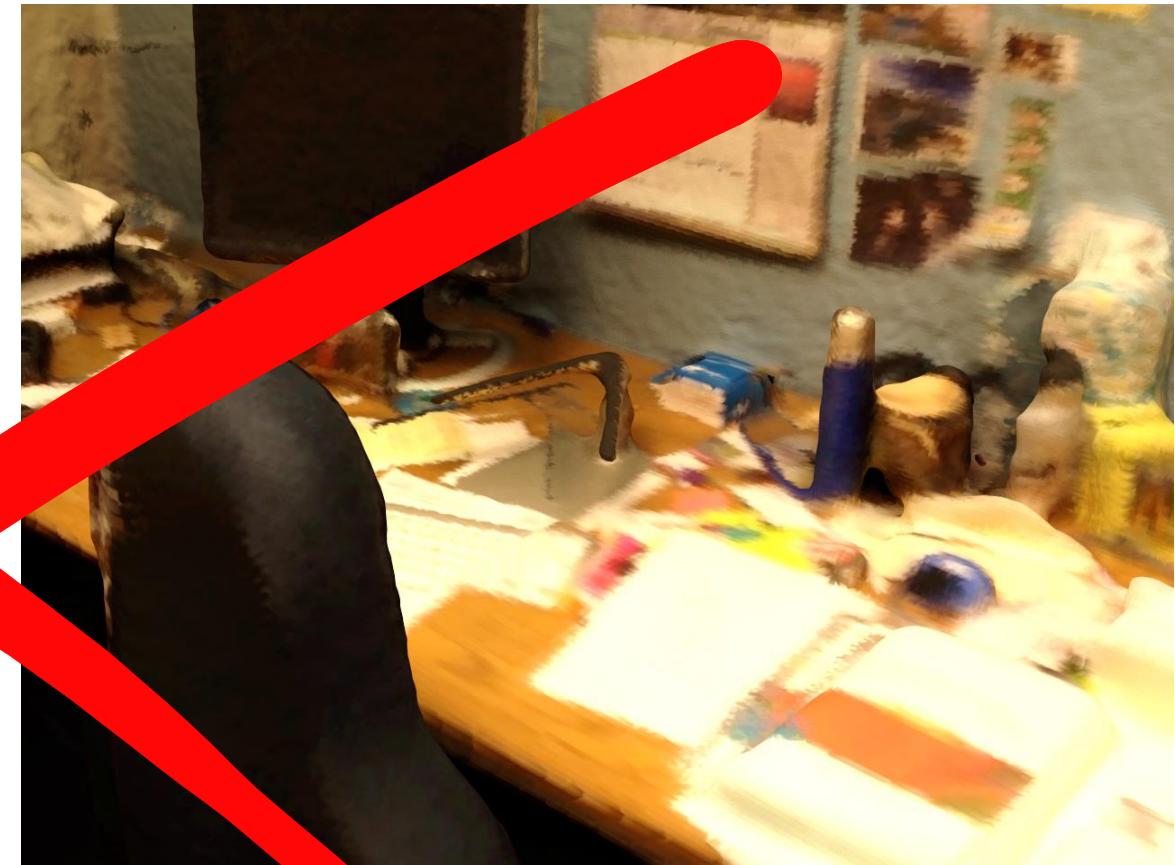


+ unstructured storage -> large extent and high-res  
+ in a way easier for dynamics (although not quite there)  
- surface quality slightly less than volumetric SDF

# Other Representations: Point-based Fusion



Surfel-based representation



TSDF Implicit Surface

# Drift and Loop Closure Problems

---



# Elastic Fusion: Surfel-based + Loop Closure

---

- Point-based representation
- Frame-to-model tracking
- Explicitly detect loop closures through localization
- Non-rigidly warp representation to close loops



# Elastic Fusion: Surfel-based + Loop Closure

---

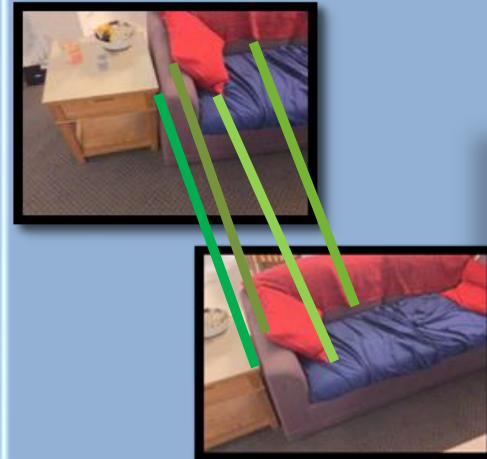


# Bundle Fusion

## RGB-D



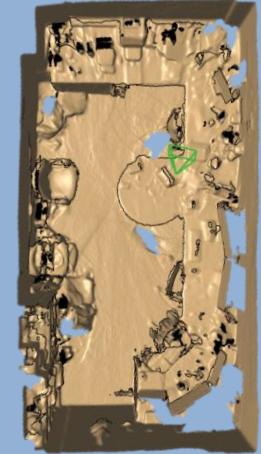
## Correspondence Search



## Global Pose Optimization



## Dynamic Scene Update



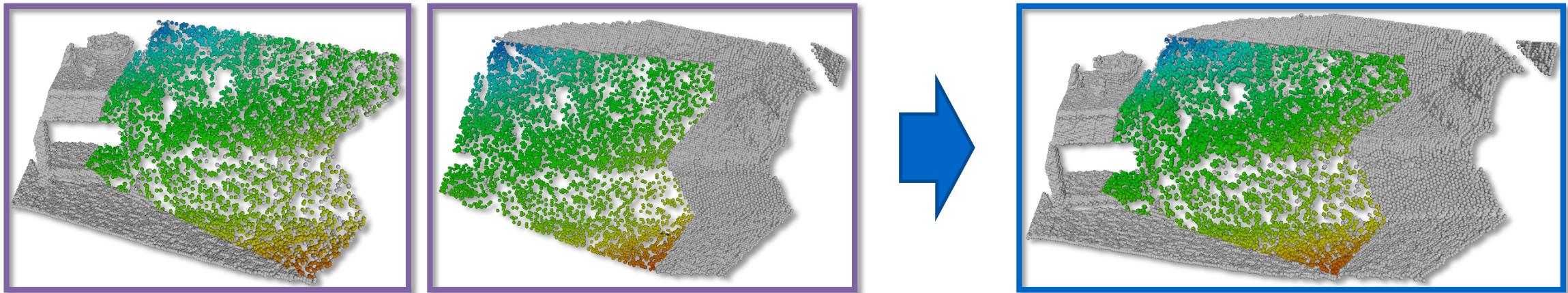
$$E(T) = w_{sparse} E_{sparse}(T) + w_{dense} E_{dense}(T)$$

$$E_{sparse}(T) = \sum_{i,j}^{\#frames \#corresp.} \sum_k \|T_i p_{ik} - T_j p_{jk}\|_2^2$$

# Bundle Fusion: Dense Terms

$$E_{dense}(T) = w_{depth} E_{depth}(T) + w_{color} E_{color}(T)$$

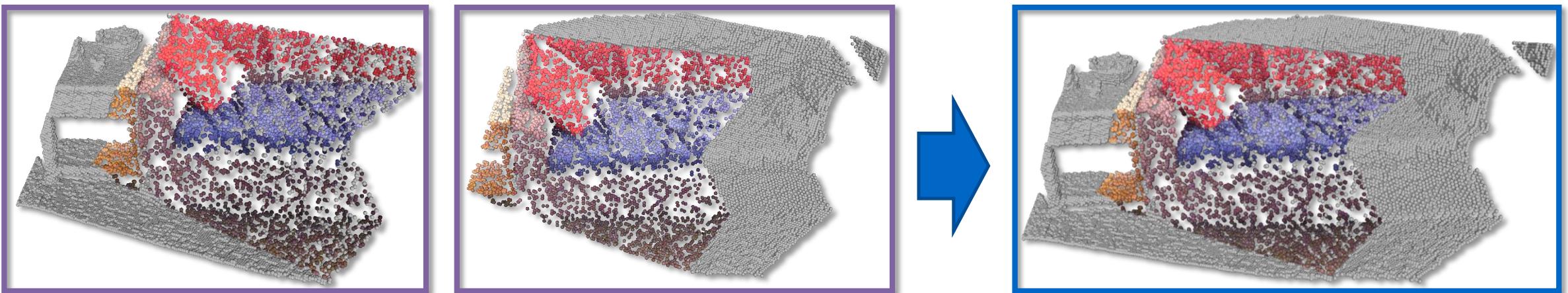
$$E_{depth}(T) = \sum_{i,j}^{\text{#frames}} \sum_k^{\text{#pixels}} \left\| (p_k - T_i^{-1} T_j \pi_d^{-1} (D_j(\pi_d(T_j^{-1} T_i p_k)))) \cdot n_k \right\|_2^2$$



# Bundle Fusion: Dense Terms

$$E_{dense}(T) = w_{depth} E_{depth}(T) + w_{color} E_{color}(T)$$

$$E_{color}(T) = \sum_{i,j}^{\text{#frames}} \sum_k^{\text{#pixels}} \left\| \nabla I(\pi_c(p_k)) - \nabla I(\pi_c(T_j^{-1} T_i p_k)) \right\|_2^2$$



# Bundle Fusion: On-the-fly Scene Updates

---

- Surface Integration [*Curless and Levoy 96*]

```
Voxel {  
    distance;  
    color;  
    weight;  
}
```

$$D(v) = \frac{\sum_i w_i(v)d_i(v)}{\sum_i w_i(v)}$$

$$W(v) = \sum_i w_i(v)$$

# Bundle Fusion: On-the-fly Scene Updates

---

- Surface Integration [*Curless and Levoy 96*]

```
Voxel {  
    distance;  
    color;  
    weight;  
}
```

$$D'(\nu) = \frac{W(\nu)D(\nu) + w_k(\nu)d_k(\nu)}{W(\nu) + w_k(\nu)}$$

$$W'(\nu) = W(\nu) + w_k(\nu)$$

# Bundle Fusion: On-the-fly Scene Updates

- Surface De-integration: remove  $d_k$  from weighted average

```
Voxel {  
    distance;  
    color;  
    weight;  
}
```

$$D'(v) = \frac{W(v)D(v) - w_k(v)d_k(v)}{W(v) - w_k(v)}$$

$$W'(v) = W(v) - w_k(v)$$

# Bundle Fusion: On-the-fly Scene Updates

---



# Bundle Fusion: On-the-fly Scene Updates



Integrated  
(wrong pose)

tum<sup>3</sup>B

# Bundle Fusion: On-the-fly Scene Updates

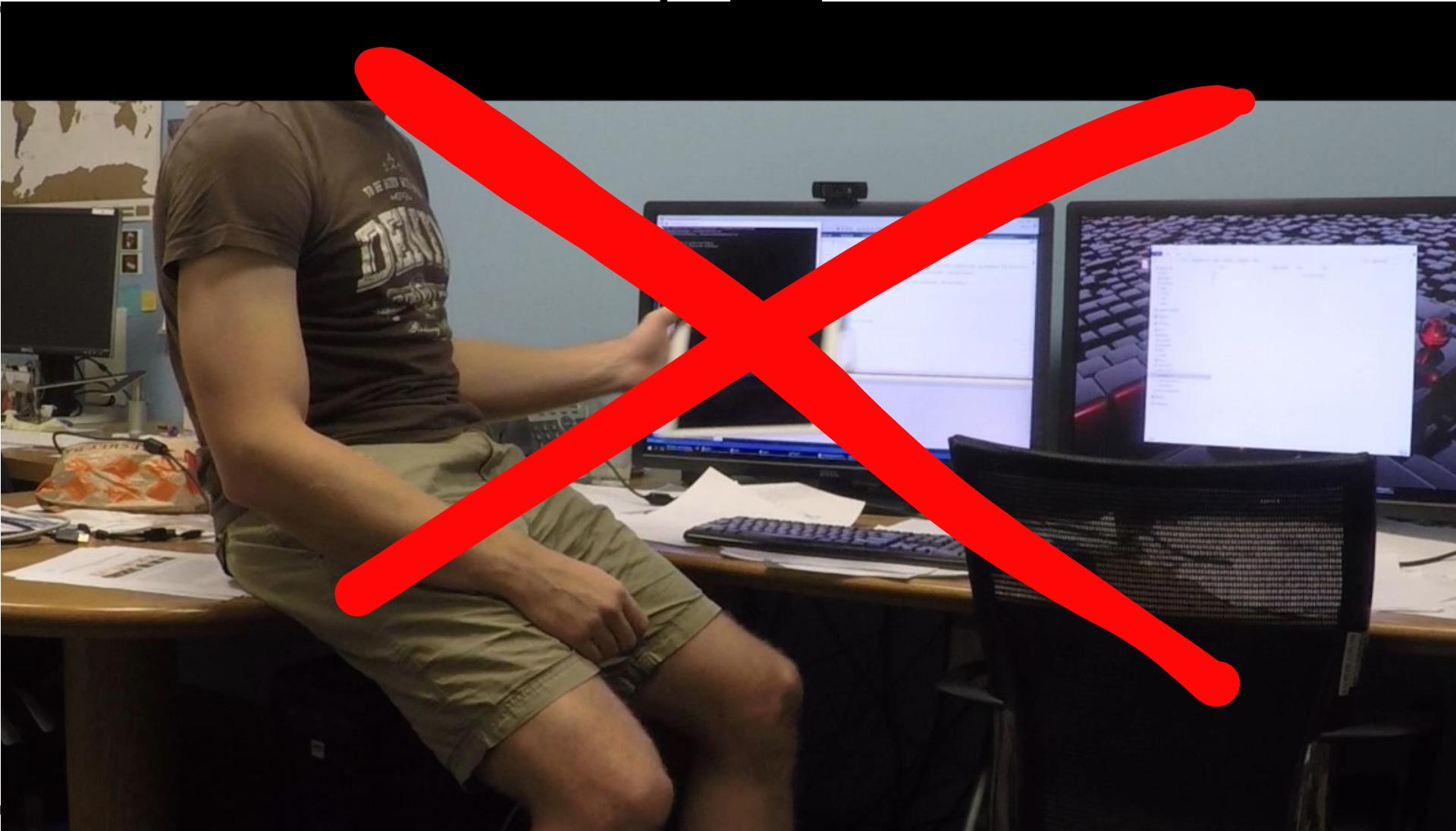


De-integrated

# Bundle Fusion: On-the-fly Scene Updates



# Bundle Fusion: Live Capture



# Bundle Fusion: Loop Closure



3D Scanning  
Thies, Dai

3D

# Bundle Fusion: Loop Closure



# Bundle Fusion: Re-Localization



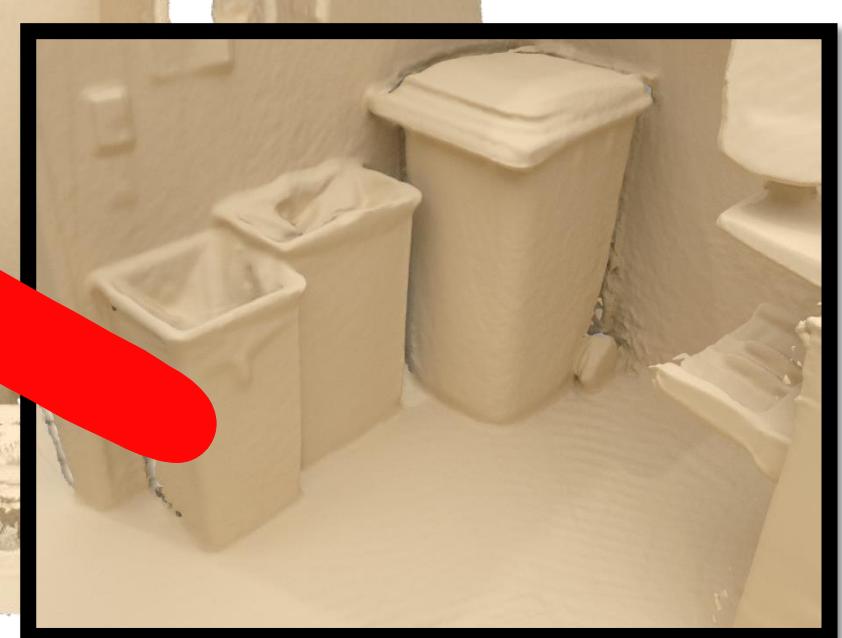
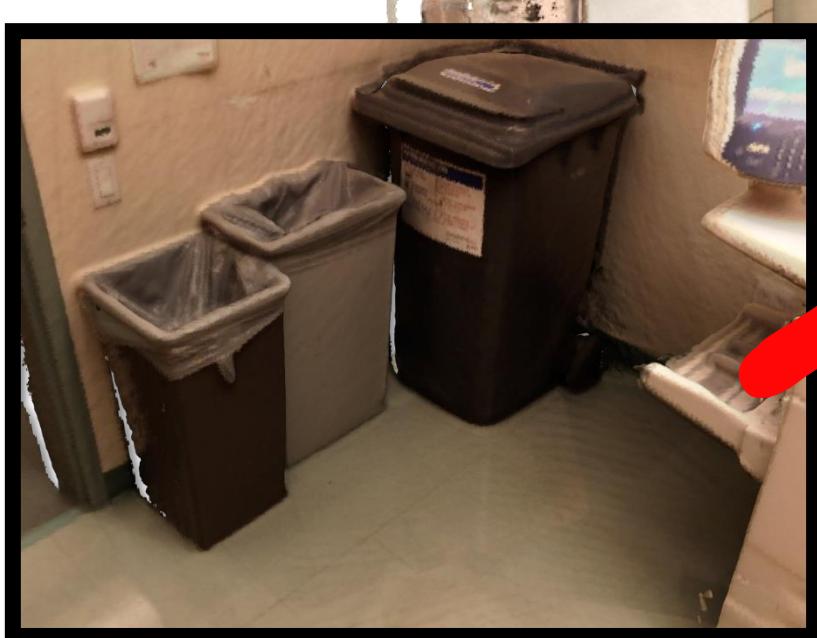
# Bundle Fusion: Re-Localization



# BF: Results

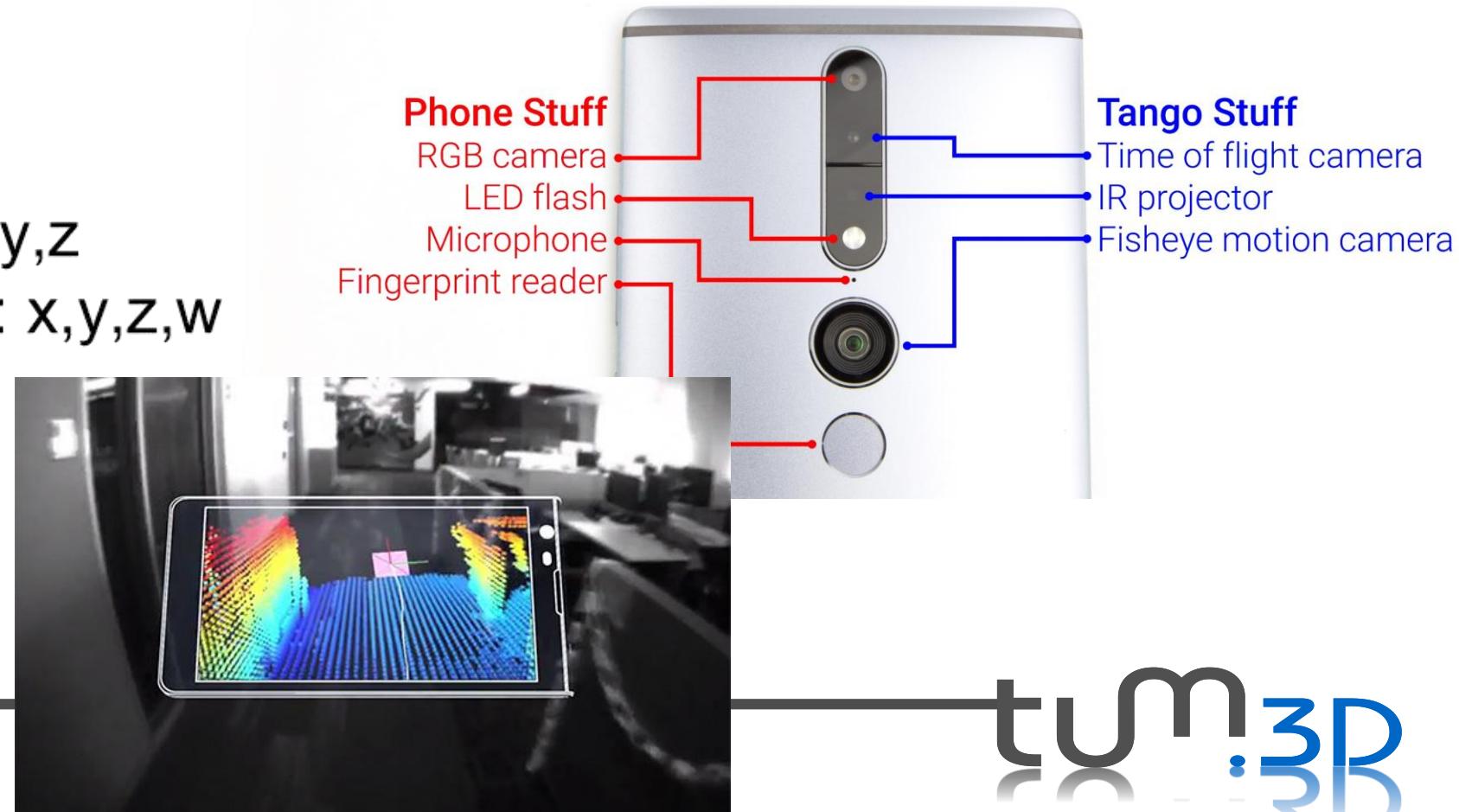


# BF: Results



# Other Things (e.g., how to scan a white wall?)

- Sensor fusion IMU + sparse feature tracking (+ ICP)
  - E.g., Google Tango or Microsoft HoloLens



# Administrative

- Tutorials
  - Tomorrow 16:00
    - Assignment 4
    - Project Proposals due
- Next Lecture:
  - Non-rigid Surface Tracking & Reconstruction