# Exercise 1

Deadline: November 25, 2016

**Please send your solutions to threedcv@dfki.uni-kl.de**

# Theory

## 1. Properties of Rotation Matrices

Rotation matrices are orthogonal matrices with determinant 1.

1. Show, that $U^T = U^{-1}$ holds for a *general* rotation matrix $U \in \mathbb{R}^{3\times3}$. Note that by definition $UU^T = I \Leftrightarrow U^{-1} = U^T$.

2. What is the geometric interpretation of the determinant of a square 3x3 matrix A? Why does a rotation matrix have determinant 1? Hint: Compare the determinants of base transformations (e.g. a rotation, translation,...).

## 2. Transformation Chain

Describe the transformation chain for mapping a point from the world coordinate system to the pixel coordinate system of an intrinsically and extrinsically calibrated camera. Why are homogeneous coordinates used for transforming points between coordinate systems? Use formulas and explain the intermediate steps in words.

# Implementation

The zip file for this exercise contains images of a chessboard that were used for calibrating a camera with high radial distortion. The results of the calibration (intrinsics of the camera and extrinsics for each board) are stored in "Calib_Results.m". You are asked to

1. Write a function "projectPoints" for projecting all 3D-points defined by a chessboard (in the world coordinate system) to the 2D pixel coordinate system. It should optionally regard the radial distortion (k1, k2, k5). The function takes as input a vector of 3D world points, the camera's intrinsic and extrinsic parameters and a flag for considering the distortion. It returns the projected 2D-points.

2. Write a function "projectAndDraw" that takes an image and projects and draws all chessboard points into that image using "projectPoint". The function takes as input an image, a flag for considering the distortion and a color for drawing the projections.

3. Hand in image 1 with the following information:
   - projected points without correction of the distortion in red
   - projected points with correction of the radial distortion (k1, k2 and k5) in blue

**Remarks:** The needed data is found in the following variables (the variables are found in Calib_Results.mat):

- KK: camera intrinsic matrix
- X_i world points for image i-1 (they are equal for all images)
- Rc_i: rotation matrix for image/chessboard i-1
- Tc_i: translation vector for image/chessboard i-1

- kc: distortion coefficients

Distortion coefficient k5 influences the points in the following way:
$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_d \\ y_d \end{pmatrix} \cdot (1 + k_1 r^2 + k_2 r^4 + k_5 r^6)$
Distortion must be applied in the normalized image coordinate system. $K^{-1} \cdot (x, y, 1)^T$ transforms the point $(x, y)^T$ from the pixel coordinate system to the normalized image coordinate system. After the distortion was applied, the point needs to be transformed back to the pixel coordinate system (multiplication by K).

# Good Luck!