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3D Computer Vision
Exercise Sheet 1

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Exercise 1 (Homography Definition).
In the lecture a homography was introduced as $h : \mathbb{P}^2 \to \mathbb{P}^2$. Define it for $h : \mathbb{P}^n \to \mathbb{P}^n$. How many degrees of freedom has the homography?

Solution (to Exercise 1).
A Homography is an invertible, line-preserving projective mapping $h : \mathbb{P}^n \to \mathbb{P}^n$. It is represented by a square $(n+1) \times (n+1)$ matrix with $(n+1)^2 - 1$ degrees of freedom.

Exercise 2.
A Homography between two images taken with the same camera can be used to compute the relative rotation R_{rel} (in case the camera has only undergone a rotation between the shots). The relative rotation tells how the camera was placed between the two shots. For the following tasks assume a camera with given intrinsic parameters $(\alpha_x, \alpha_y, x_0, x_0, s)$. Use Matlab as a calculator and explain your intermediate steps.

Solution.
See file `Homography.m`

Exercise 3.
A homography between a plane in the world coordinate system and a camera image can be used to compute rotation R and translation t of the camera. Assume a camera with given $(\alpha_x, \alpha_y, x_0, y_0, s)$ and a homography H3, that was computed from the corners of a (fully visible) chessboard. The chessboard lies in the xy-plane of the world coordinate system centered around the origin. Compute R and t. Use Matlab as a calculator and explain your intermediate steps.

Solution. See file `Homography.m`. It is displayed in the console.

Exercise 4.

1. Illustrate the meaning of $t = -RC$ in a camera pose $[R|t]$, i.e. from where to where does this vector point? Hint: What is linked by $[R|t]$? Try applying $[R|t]$ to the origin of the world.

2. The third element of t in exercise 3 might be negative. What does this mean in this particular case (consider the location of the chessboard corners)? Why can this happen?

Solution.

1. The vector $t = -RC$ illustrates the rotation and translation of the camera in world coordinates. The coordinates C are the of the camera in world coordinates. Applying R to them leads to the rotation. The $-$ in front of the vector makes it pointing from the camera c to the origin of the world.
   If we apply $[R|t]$ to the origin of the world we get again the origin of the world.

2. The third element of the vector t in the third exercise can be interpreted as the position of the camera relative to the XY-plane. If the third entry of t is positive the camera is above the chessboard, whereat it is beneath the chessboard if the entry is negative. The corners of the chessboard remains the same.

Exercise 5.
Let $x_1, x_2, x_3 \in \mathbb{P}^2$ be three points on a line. Show that a homography H preserves this property. Hint: Use the implicit definition of a line $ax + by + c = 0$, thus $l = (a, b, c)$.

By definition we have for a line

$$ax + by + c = 0.$$

In matrix vector notation we have for $l = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$l^T x = 0 \text{ for } x = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Furthermore we have by definition, that each Homography H is invertible and line preserving. We define

$$\tilde{x} := Hx.$$

Using the line-preserving property and the non-singularity of H we get

$$0 = \underbrace{l^T H^{-1}}_{=:K} \tilde{x},$$

which is again a line equation.

Solution.