

Form factor of a two-boson bound state

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1 Introduction

The bound state is a quantum state of a particle subject to a potential such that the particle has a tendency to remain localized in one or more regions of space.

In this exercise, we would like to study the properties of bound states of two bosons. We assume that the bound state problem was solved for the OBE using several cutoffs. Thereby, the interactions have been tuned so that our two-boson system corresponds to the deuteron bound state in nuclear physics and reproduce the experimentally known binding energy of $E = -2.225$ MeV.

2 Solution of questions

Question No 2

$$F(\vec{q}^2) = \int d^3 q' \psi^*(\vec{p}') \psi(\vec{p}' - \frac{1}{2}\vec{q}) \quad (1)$$

To verify,

$$F(\vec{q}^2) = 2\pi \int dp p'^2 \int_{-1}^1 dx Y_{lm_z}(\hat{p}') Y_{lm_z}(\vec{p}' - \frac{1}{2}\vec{q}) \psi_{lm_z}^*(\vec{p}') \psi_{lm_z}(\vec{p}' - \frac{1}{2}\vec{q}) \quad (2)$$

We have to use the momentum $\vec{p}' = (\vec{p}'\sqrt{1-x^2}, 0, p'x)$, $\vec{q} = q\hat{e}_z$ and the solid angle integration can be simplified to the integration over $x = \cos(\theta)$. Also, we have

$$\psi(\vec{p}) = \sum_{l,m} \psi_l(|p|) Y_{lm}(\hat{p}) \quad (3)$$

Since the angular momentum is conserved all other sum parts except for given l and m_z vanishes. Also notice that we have no ϕ dependance, so we can integrate easily over ϕ : i.e.

$$F(\vec{q}^2) = \int d^3 \vec{p}' \psi_{lm_z}^*(|p'|) Y_{lm_z}(\hat{p}') \psi_{lm_z}(|\vec{p}' - \frac{1}{2}\vec{q}|) Y_{lm_z}(|\vec{p}' - \frac{1}{2}\vec{q}|) \quad (4)$$

$$= \int_0^\infty d\vec{p} p'^2 \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta \psi_{lm_z}^*(|p'|) Y_{lm_z}(\hat{p}') \psi_{lm_z}(|\vec{p}' - \frac{1}{2}\vec{q}|) Y_{lm_z}(|\vec{p}' - \frac{1}{2}\vec{q}|) \quad (5)$$

$$= 2\pi \int_0^\infty dp' p'^2 \int_{-1}^1 dx \psi_{lm_z}^*(|p'|) Y_{lm_z}(\hat{p}') \psi_{lm_z}(|\vec{p}' - \frac{1}{2}\vec{q}|) Y_{lm_z}(|\vec{p}' - \frac{1}{2}\vec{q}|) \quad (6)$$

Question No 4

To check the numerical of our result, accuracy We have used the wave function for $\lambda = 1200$ MeV and selected momentum transfers $|q|$ in the range up to 10 fm^{-1} . Especially with respect to the number of grid points used for angular integration we got the plot as in figure 1.

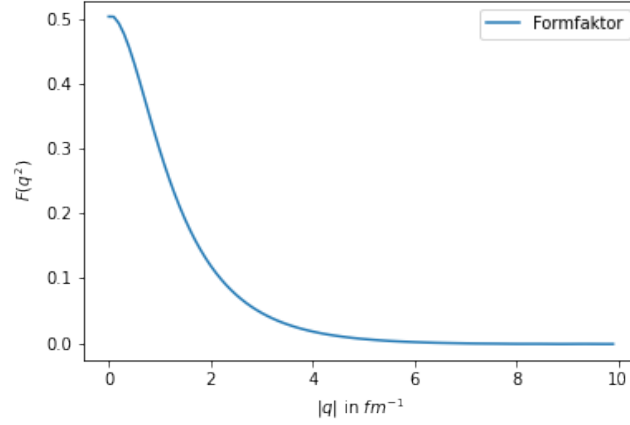


Figure 1: The distribution of form factor.

We can see that this plot is not quite correct because for $q = 0$ the form factor should be 1. We also vary the grid points such that the change in value is only at the third significant position.

Question No 4

In the following we plot the form factor for different cutoffs Λ . Which you can see in 2.

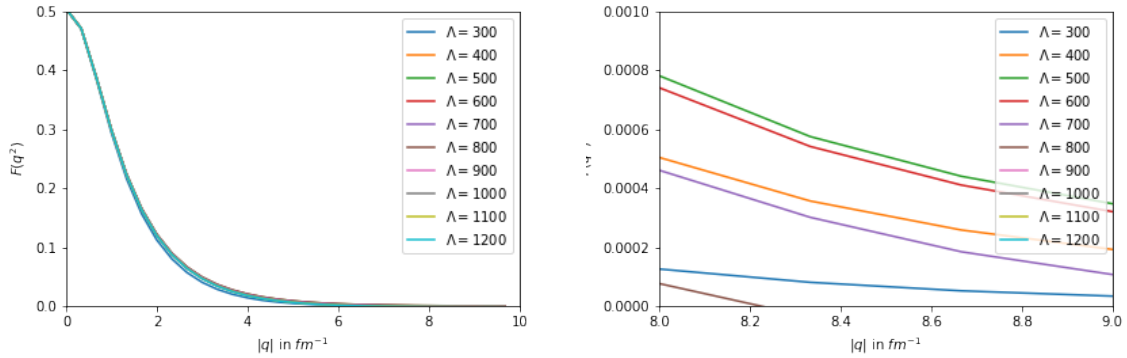


Figure 2: Form factors for different cutoffs

We can see some differences but they are quite low and random. So we cannot really say one is bad or good. This of course can be explained by the fact that sth is still wrong in the calculation of the form factor.

References

- [1] Thomas Luu, Andreas Nogga, Marcus Petschlies and Andreas Wirzba, Exercise-sheet, 2020.