

# Probabilistic Risk Analysis and Bayesian Decision Theory

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```
knitr::opts_chunk$set( collapse=TRUE, echo=F, comment=">" )  
library( geodata )  
library( terra )
```

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## Linkages between PRA and BDT

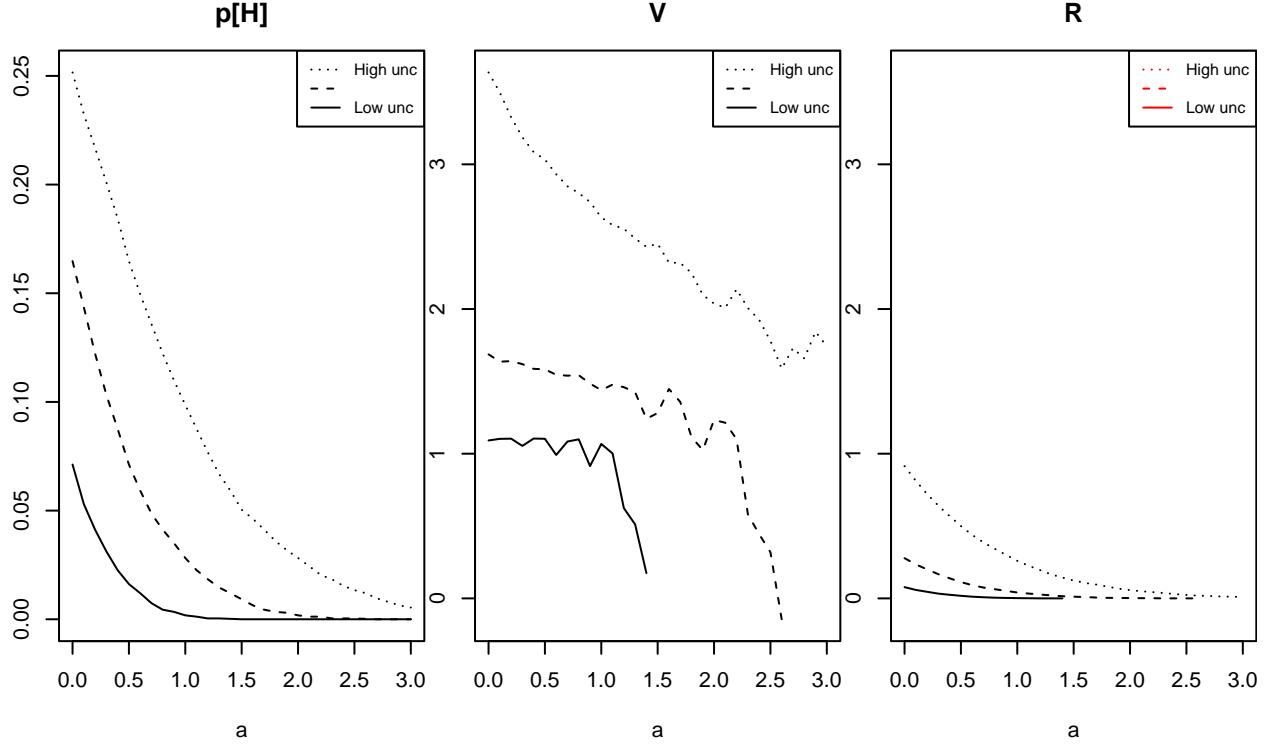


Figure 1: PRAs on data generated for BDT.

### Risk management

**The relationship between utility maximisation in BDT and risk assessment in PRA:  $R_c$ :**

$$R = E[u|\neg H] - E[u]. \quad (1)$$

$$\begin{aligned} R_c &= R - E[u|\neg H] \\ &= R + E[ka|\neg H] \times a - E[ky \times y|\neg H]. \end{aligned} \quad (\#eq : R_c) \quad (2)$$

**Simplified accounting for both benefits and costs of the action:  $R_b$**

$$R_b = R + E[ka] \times a - E[ky] \times E[y]. \quad (\#eq : R_b) \quad (3)$$

**Only correcting for costs:  $R_a$**

$$R_a = R + E[ka] \times a. \quad (4)$$

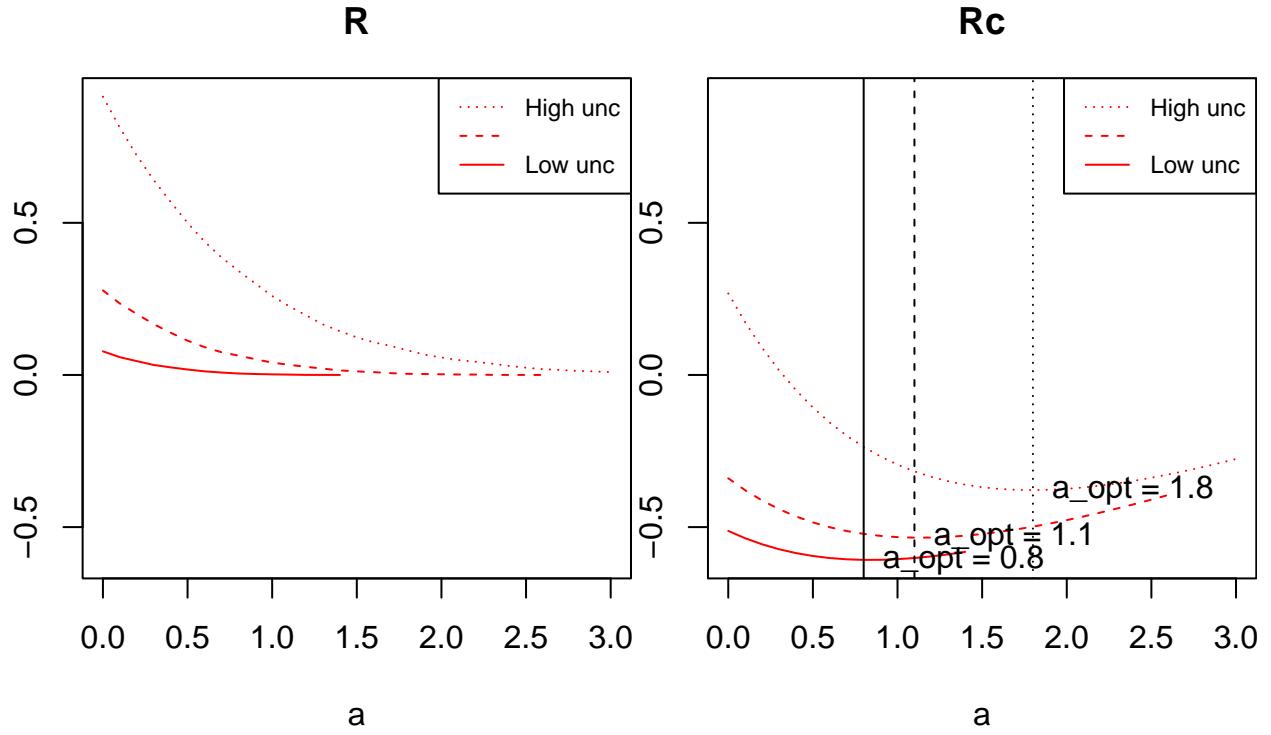


Figure 2: Impact of action  $a$  on risk calculated from data generated for BDT. Left: uncorrected risk  $R$ , always decreasing with increasing  $a$ . Right: corrected risk  $R_c$ , reaching an uncertainty-dependent minimum at finite  $a$ .

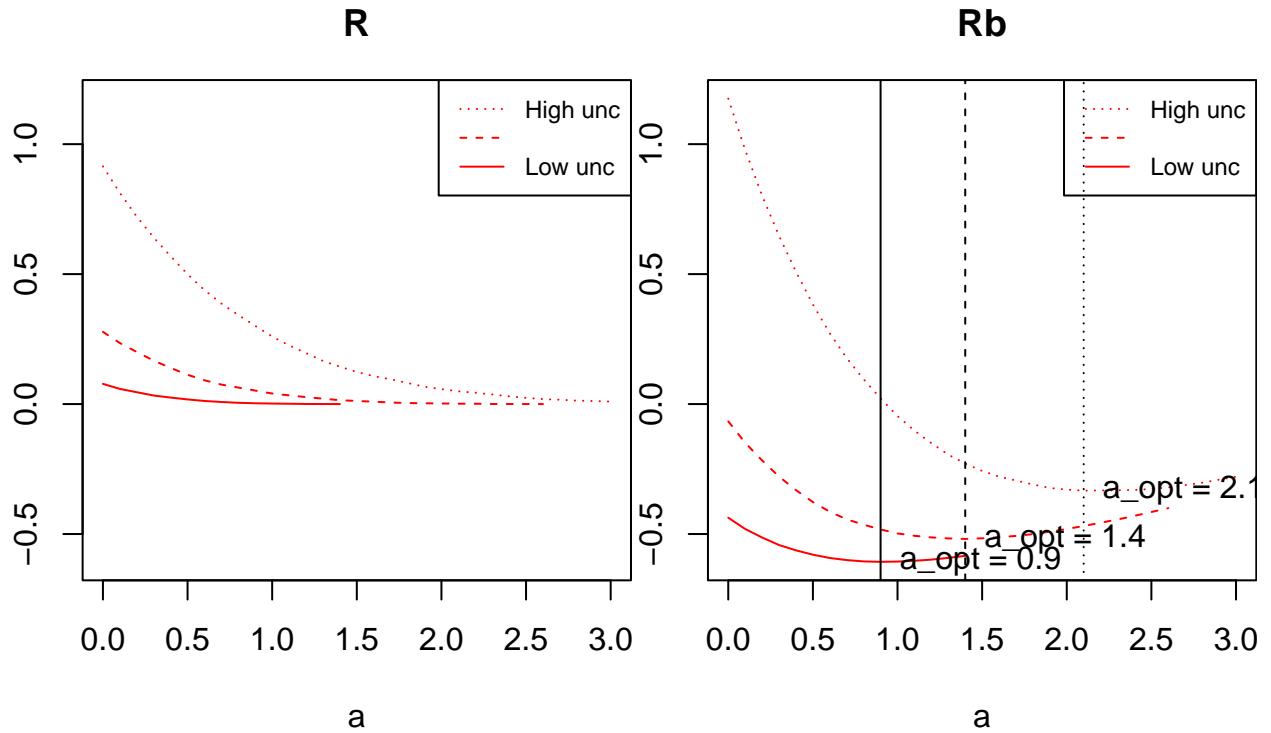


Figure 3: Cost- and benefit-adjusted risk  $R_b$  using data generated for BDT.

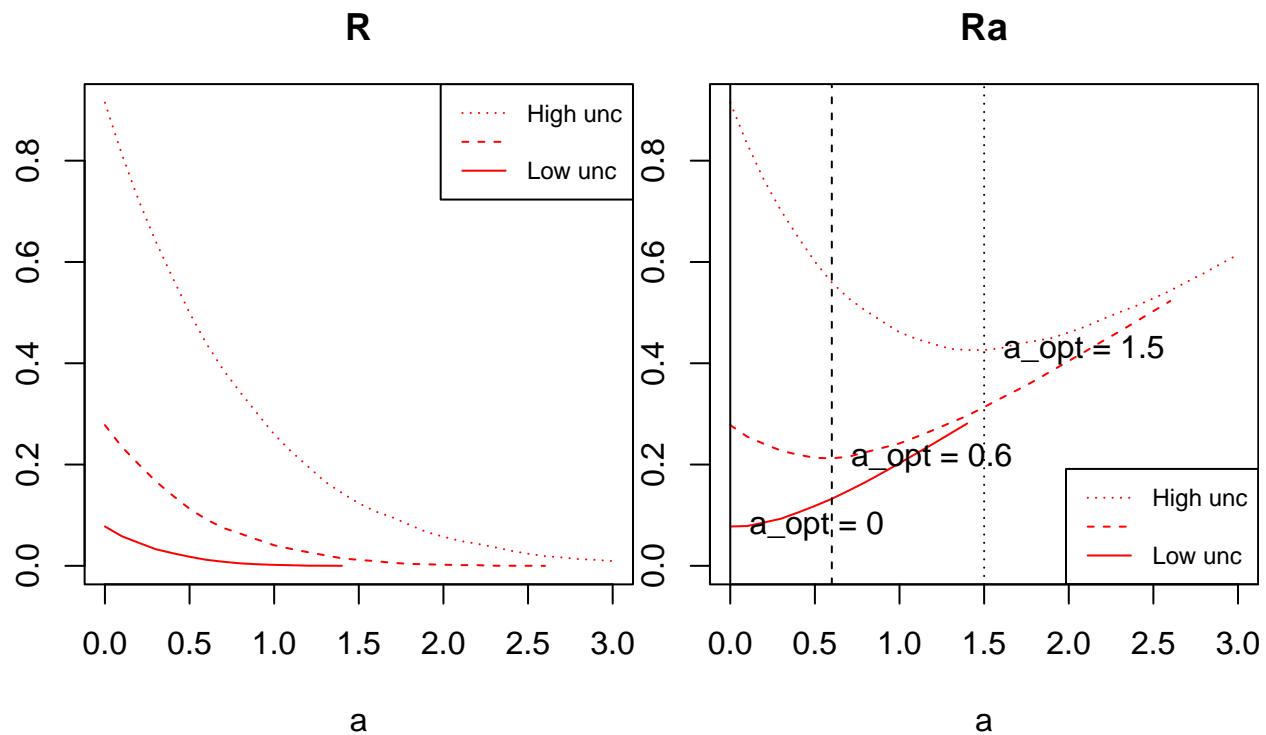


Figure 4: Cost-adjusted risk  $R_a$  using data generated for BDT.

## PRA vs. BDT in the spatial example

```
IRRIG <- 500

s_YC      <- rast( sapply(1:19, function(i){
  xapp( r_alt, s_prec[[i]]           , fun=YC )} ) )
s_YC.IRRIG <- rast( sapply(1:19, function(i){
  xapp( r_alt, s_prec[[i]]+IRRIG, fun=YC )} ) )
```

```
ky       <- 30 ; ka <- 0.1

s_U      <- s_YC * ky
s_U.IRRIG <- s_YC.IRRIG * ky - IRRIG * ka
```

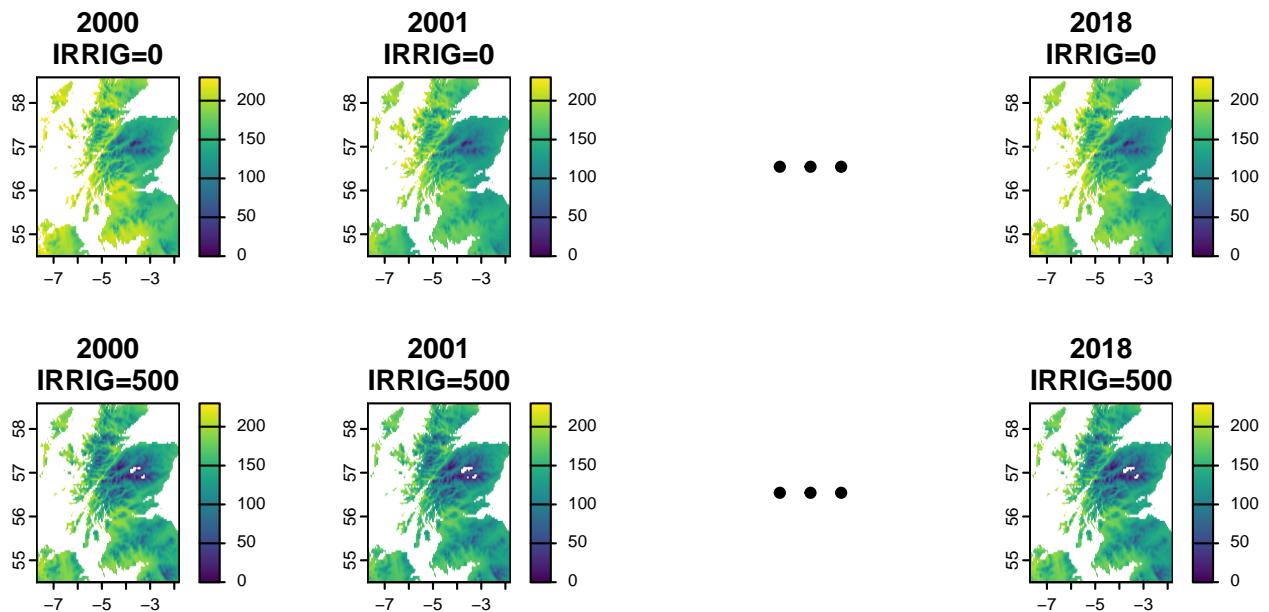


Figure 5: Utility in 2000, 2001 and 2018. Top row: no irrigation. Bottom row: irrigation at  $500 \text{ mm } y^{-1}$ .

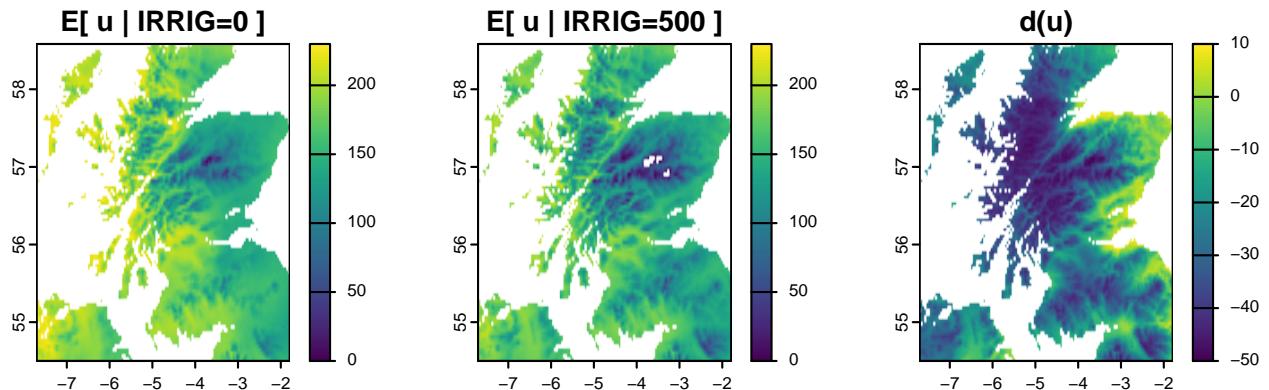


Figure 6: Mean utility. Left: no irrigation. Middle: irrigation =  $500 \text{ mm } y^{-1}$ . Right: utility-gain from irrigation.

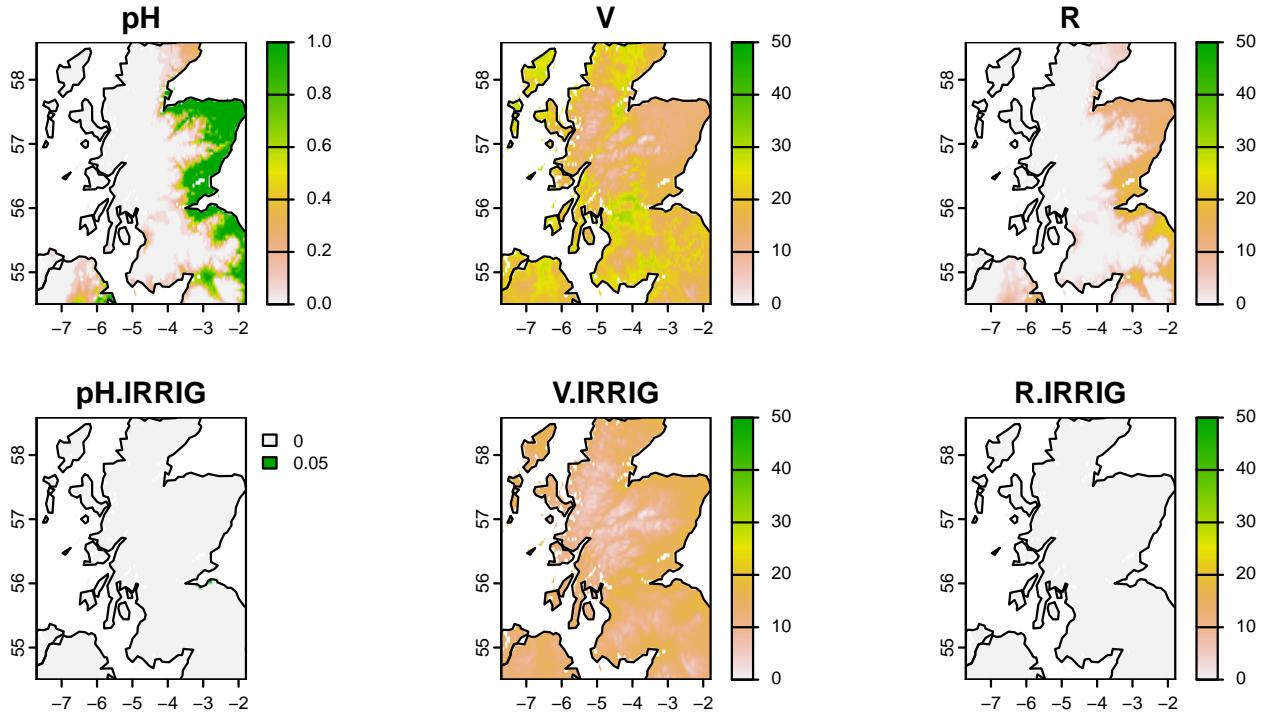


Figure 7: PRA based on data for 2000-2018. Top row: no irrigation. Bottom row: irrigation =  $500 \text{ mm } y^{-1}$ .

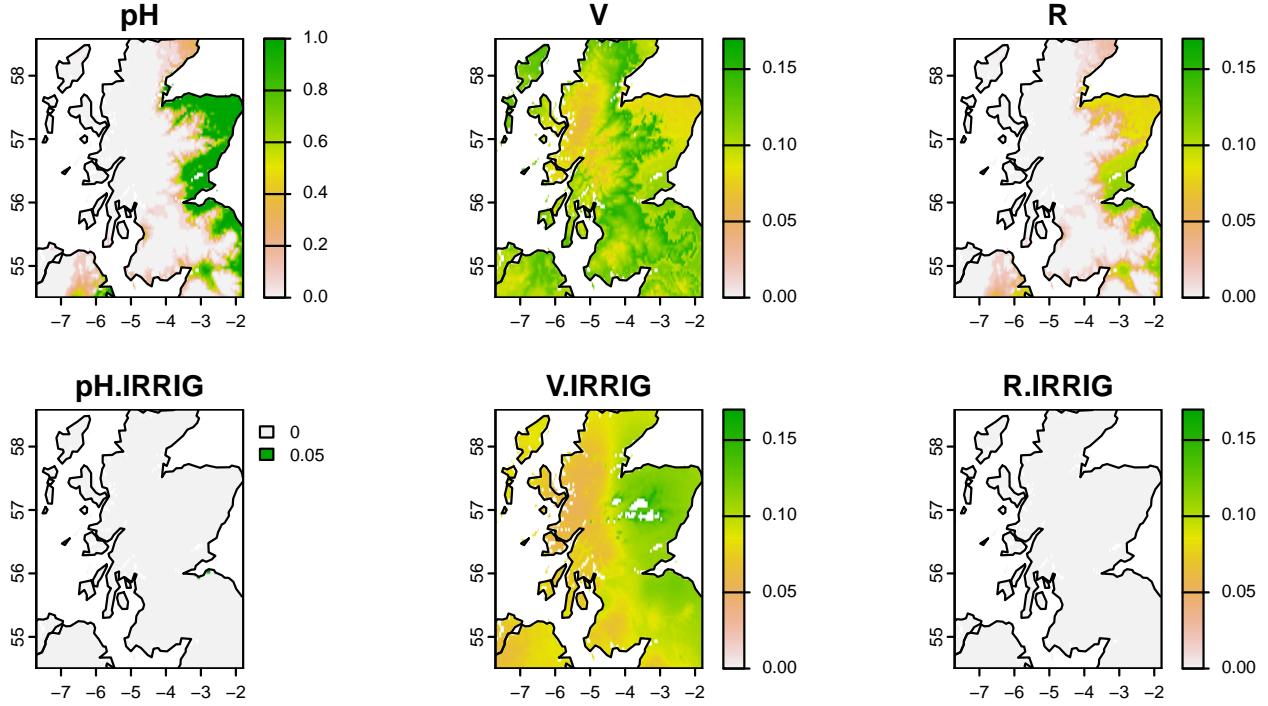


Figure 8: Relative PRA based on data for 2000-2018. Top row: no irrigation. Bottom row: irrigation =  $500 \text{ mm } y^{-1}$ .

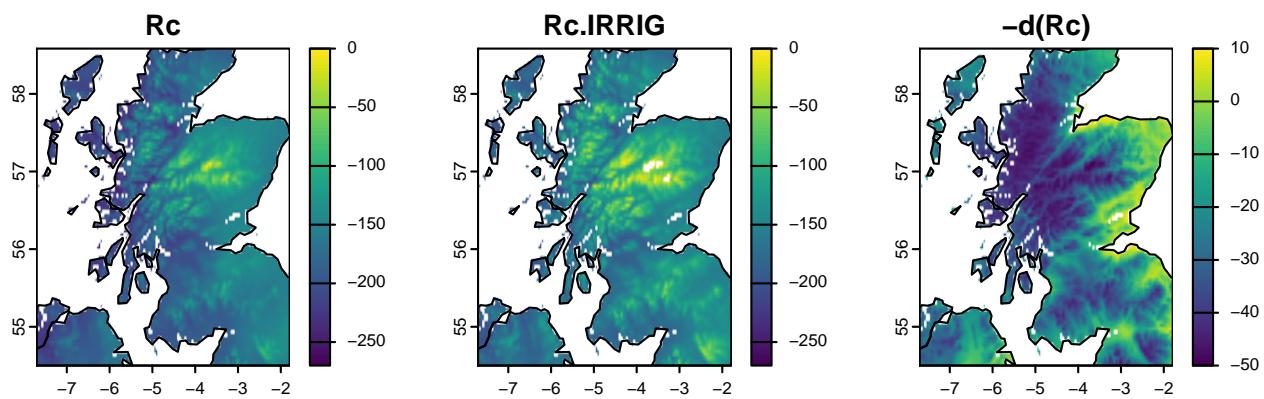
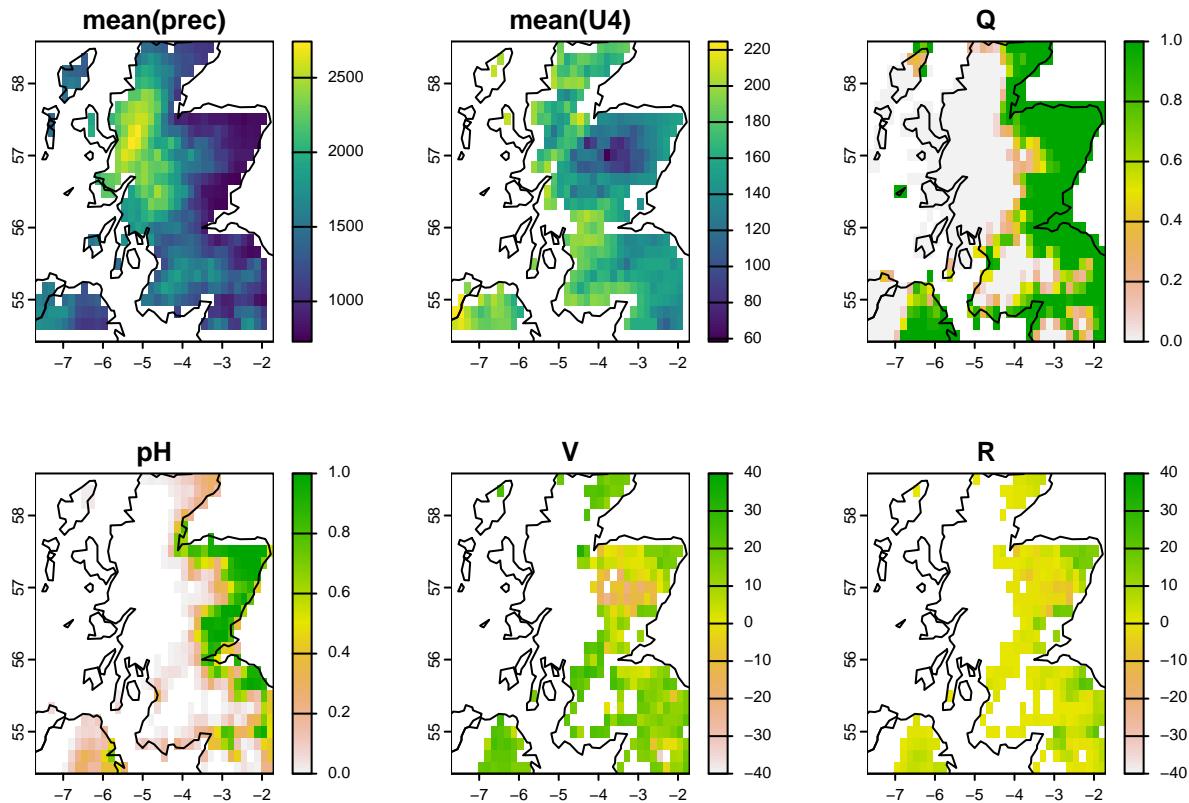


Figure 9: Corrected risk for utility. Left: no irrigation. Middle: irrigation as above. Right: decrease in corrected risk due to irrigation.

## Three-component PRA in the spatial example

```
exposure <- function( pH, ... ){  
  pH <- pH[!is.na(pH)] ; n <- length(pH)  
  nE <- length( pH[pH>1/21] ) ; Q <- nE/n  
  return( Q ) }
```



## **Discussion**

**PRA and its application**

**Data and computational demand of PRA**

**BDT**

**Computational demand of BDT**

**PRA as a tool for simplifying and elucidating BDT**

**Parameter and model uncertainties**

**Modelling and decision-support for forest response to hazards**

**Spatial statistics**

## References