Assignment 3, HT2020
Linear Programming in Staff Planning

Assignment in partial fulfilment of the requirements for the course

Optimisation 1TD184

UPPSALA
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1. Salary of workers who start at 18:00 and 02:00 is **50%** higher
2. Salary of workers who start at 22:00 is **100%** higher
3. 8-hours salary unit (day-shift) = u

Worker Group (WG)	Start	Finish	Salary
1	06:00	14:00	100 %
2	10:00	18:00	100 %
3	14:00	22:00	100 %
4	18:00	02:00	150 %
5	22:00	06:00	200 %
6	02:00	10:00	150 %

Worker Group (WG)	Start	Finish	Salary
1	06:00	14:00	100 %
2	10:00	18:00	100 %
3	14:00	22:00	100 %
4	18:00	02:00	150 %
5	22:00	06:00	200 %
6	02:00	10:00	150 %

1. Objective Function: $Z = u * (WG1 + WG2 + WG3) + 1.5u * (WG4 + WG6) + 2u * (WG5)$

2. Matrix Form: $Z = c^T \mathrm{x},\ c^T = u * [1 \quad 1 \quad 1 \quad 1.5 \quad 2 \quad 1.5],\ x = \begin{bmatrix} WG1 \\ WG2 \\ WG3 \\ WG4 \\ WG5 \\ WG6 \end{bmatrix}$

Worker Group (WG)	Start	Finish	Salary
1	06:00	14:00	100 %
2	10:00	18:00	100 %
3	14:00	22:00	100 %
4	18:00	02:00	150 %
5	22:00	06:00	200 %
6	02:00	10:00	150 %

Table 1

Time Period	Min Workers
06:00 - 10:00	700
10:00 - 14:00	500
14:00 - 18:00	600
18:00 - 22:00	300
22:00 - 02:00	100
02:00 - 06:00	50

Table 3

Worker Group (WG)	06:00	10:00	14:00	18:00	22:00	02:00
1						
2						
3						
4						
5						
6						

Table 2

Constraints (from Table 2 and Table 3):

$$WG6 + WG1 \geq 700$$
$$WG1 + WG2 \geq 500$$
$$WG2 + WG3 \geq 600$$
$$WG3 + WG4 \geq 300$$
$$WG4 + WG5 \geq 100$$
$$WG5 + WG6 \geq 50$$
$$WG_i \geq 0, \ for \ i = 1, \ldots, 6$$

Constraints (from Table 2 and Table 3): → Matrix form:

$$WG6 + WG1 \geq 700$$
$$WG1 + WG2 \geq 500$$
$$WG2 + WG3 \geq 600$$
$$WG3 + WG4 \geq 300$$
$$WG4 + WG5 \geq 100$$
$$WG5 + WG6 \geq 50$$

$$A\mathrm{x} \geq b,\ A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix},\ b = \begin{bmatrix} 700 \\ 500 \\ 600 \\ 300 \\ 100 \\ 50 \end{bmatrix}$$

$$WG_i \geq 0,\ for\ i = 1, \ldots, 6$$

$$\begin{bmatrix} WG1 \\ WG2 \\ WG3 \\ WG4 \\ WG5 \\ WG6 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \equiv \mathrm{x} \geq 0$$

Task 1

Staff Planning Modelling

$$\min_{x} c^T x \text{ such that } \begin{cases} Ax \geq b, \\ x \geq 0 \end{cases}$$

`x = linprog(c,-A,-b,[],[],lb,[],options)`

Linprog

$$\min_{x} f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

`x = linprog(f,A,b,Aeq,beq,lb,ub,options)`

```
%% Task 1
nshifts = 6; % Number of shifts in a 24-hours period = worker groups
u = 10; % 8-hours salary unit (day-shift)

c = u*[1 1 1 1.5 2 1.5]'; % objective function z = c'x

% constraints Ax >= b:
b = [700 500 600 300 100 50]';
A = eye(nshifts);
for row = nshifts:-1:1
    if row == 1
        A(1,nshifts) = 1;
        break;
    end
    A(row,row - 1) = 1;
end

lb = zeros(nshifts,1)'; % constraint x >= 0

options = optimoptions('linprog','Algorithm','dual-simplex','Diagnostics','on','Display','iter');
[x,fval,exitflag,output,lambda] = linprog(c, -A, -b, [], [], lb, [], options);
```

Worker Group (WG)	Employees (Task 1)
1	650
2	0
3	600
4	100
5	0
6	50

Table 4

Constraints (from Table 2 and Table 4):

$$WG6 + WG1 = 700 \geq 700$$
$$WG1 + WG2 = 650 \geq 500$$
$$WG2 + WG3 = 600 \geq 600$$
$$WG3 + WG4 = 700 \geq 300$$
$$WG4 + WG5 = 100 \geq 100$$
$$WG5 + WG6 = 50 \geq 50$$
$$WG_i \geq 0, \; for \; i = 1, \ldots, 6$$

$$\longrightarrow \quad Z = 1475 * u$$

Task 2

Worker Group (WG)	Employees (Task 1)	Employees (Task 2)
1	650	650
2	0	0
3	600	600
4	100	100
5	0	0
6	50	50

Table 5

Time Period	Min Workers
06:00 - 10:00	700
10:00 - 14:00	500 - 250
14:00 - 18:00	600
18:00 - 22:00	300
22:00 - 02:00	100
02:00 - 06:00	50

Table 6

Constraints (from Table 5 and Table 6):

$$WG6 + WG1 = 700 \geq 700$$
$$WG1 + WG2 = 650 \geq (500 - 250)$$
$$WG2 + WG3 = 600 \geq 600$$
$$WG3 + WG4 = 700 \geq 300$$
$$WG4 + WG5 = 100 \geq 100$$
$$WG5 + WG6 = 50 \geq 50$$
$$WG_i \geq 0, \ for \ i = 1, \ldots, 6$$

$$Z = 1475 * u$$

COMMAND WINDOW

```
>> disp(lambda.ineqlin)
    10
     0
    10
     0
    15
     5

>> |
```

Task 3

Worker Group (WG)	Employees (Task 1)	Employees (Task 2)	Employees (Task 3)
1	650	650	657.2809
2	0	0	141.3821
3	600	600	458.6179
4	100	100	92.7191
5	0	0	7.2809
6	50	50	42.7191

Table 7

Constraints (from Table 2 and Table 7):

$$
\begin{aligned}
&WG6 + WG1 = 700 \geq 700 \\
&WG1 + WG2 = 798.66 \geq 500 \\
&WG2 + WG3 = 600 \geq 600 \\
&WG3 + WG4 = 551.337 \geq 300 \\
&WG4 + WG5 = 100 \geq 100 \\
&WG5 + WG6 = 50 \geq 50 \\
&WG_i \geq 0,\ for\ i = 1, \ldots, 6
\end{aligned}
$$

$$Z = 1475 * u$$

Task 3

Objective function has **several optima**

Several solutions with **same Objective function value**

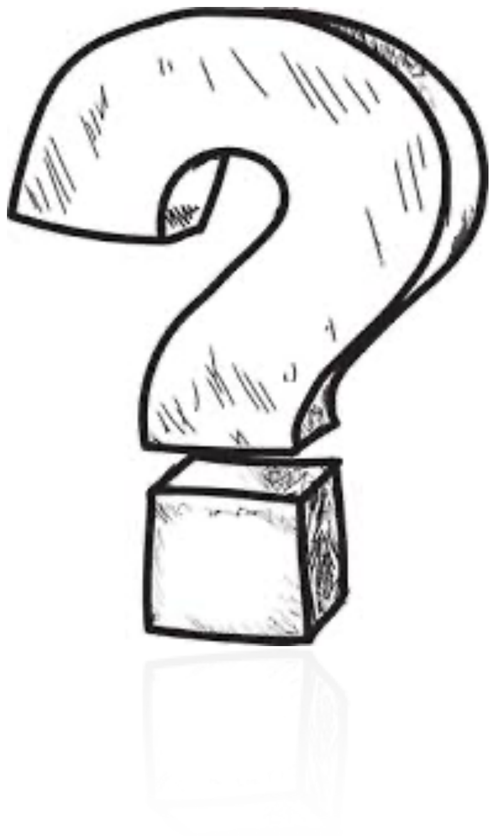
1. **Simplex Algorithm:** optima solution is always an extreme point. Method starts from corner point and move to other corner points until no improvement.
2. **Interior-point Algorithm:** Does not use corner points as start point. It may end up in a different optima that not a corner point.

Potential strengths with Interior-Point Algorithms

1. Low memory usage
2. Ability to solve large problems quickly

Potential inaccuracies with Interior-Point Algorithms

1. Barrier function **iterates away** from inequality constraint **boundaries**
2. Minimum found that **satisfies the constraints** (vs. optimal solution found in simplex)
3. Specify **smaller** StepTolerance, OptimalityTolerance, and possibly ConstraintTolerance



Thank you