Assignment 1, HT2020
Ice on Mars

Assignment in partial fulfilment of the requirements for the course

Optimisation 1TD184

UPPSALA
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Department of Information Technology

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$$\min \|H(x) - H_{obs}(x)\|_2^2$$

Objective Function

Nonlinear relationship between the thickness and mass balance function on the surface

$$a(x) = -\frac{2A}{n+2}(\rho g)^n H(x)^{n+2} \left|\frac{\mathrm{d}h}{\mathrm{d}x}\right|^{n-1} \frac{\mathrm{d}h}{\mathrm{d}x}$$

Eq. (I)

$$\downarrow$$

$$H(x) = \left(- \frac{a(x)(n+2)}{2A(\rho g)^n \left|\frac{\mathrm{dh}}{\mathrm{d}x}\right|^{n-1} \frac{\mathrm{d}h}{\mathrm{d}x}} \right)^{\frac{1}{n+2}}$$

Eq. (II)

```
numerator = a[i]*(n+2)
denominator = 2*A*((rho*g)**n)*dhdx[i]*((abs(dhdx[i]))**(n-1))
h_theoretical[i] = (-(numerator/denominator))**(1/(n+2))
delta_h[i] = h_theoretical[i] - h_obs[i]
sum += delta_h[i]**2
```

Python Implementation

		Python	Matlab
Library		SciPy.org	Optimisation ToolBox
Task 1	Solver:	minimize	fminunc
	Algorithm:	Nelder-Mead	Quasi-Newton (BFGS)

Nelder-Mead (downhill simplex):

1. Gradient-free optimisation algorithm
2. Simplex (polytope of n+1 vertices in n dimensions)
3. Reflecting the worst vertex or shrinking towards the best vertex

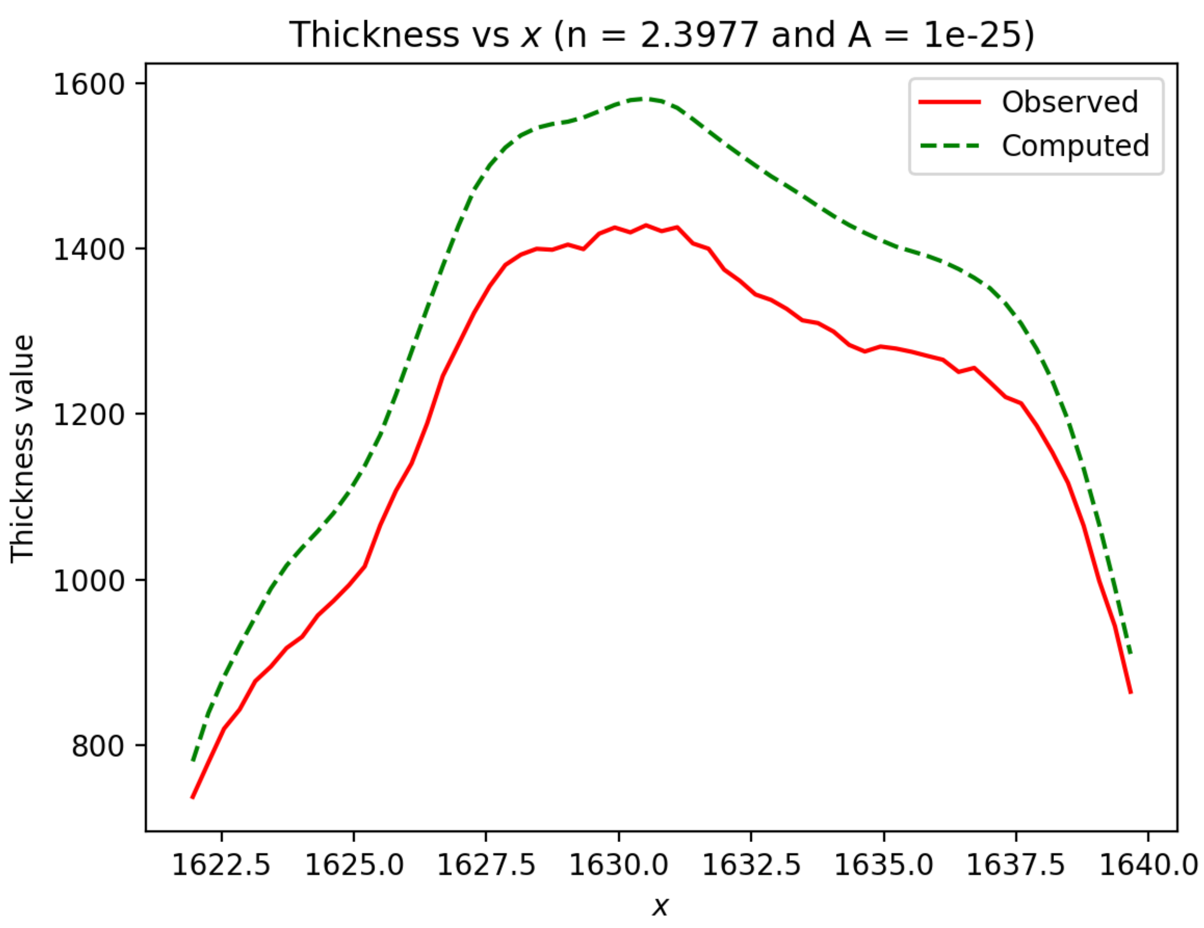
$A = 1 * 10^{(-25)}$

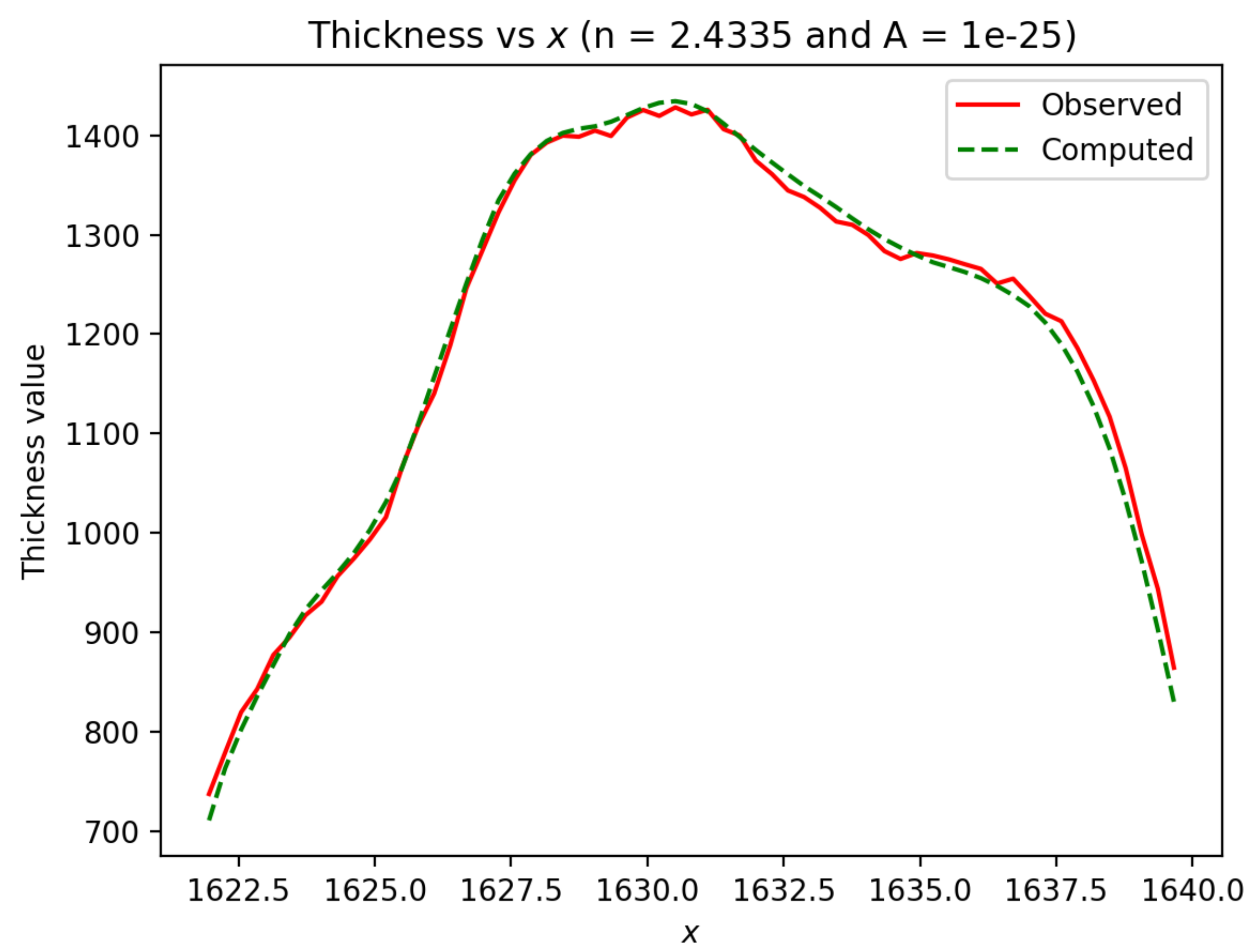
	Start point					
	n = 1		n = 2.5		n = 4	
	Python	Matlab	Python	Matlab	Python	Matlab
Iterations	26	7	19	5	24	5
Function Evaluations	52	22	38	18	48	29
Optimal n	2.4335	2.3977	2.4335	2.4335	2.4335	2.4335
Min. Function Value	$13 * 10^3$	$9.5 * 10^5$	$13 * 10^3$	$13 * 10^3$	$13 * 10^3$	$13 * 10^3$

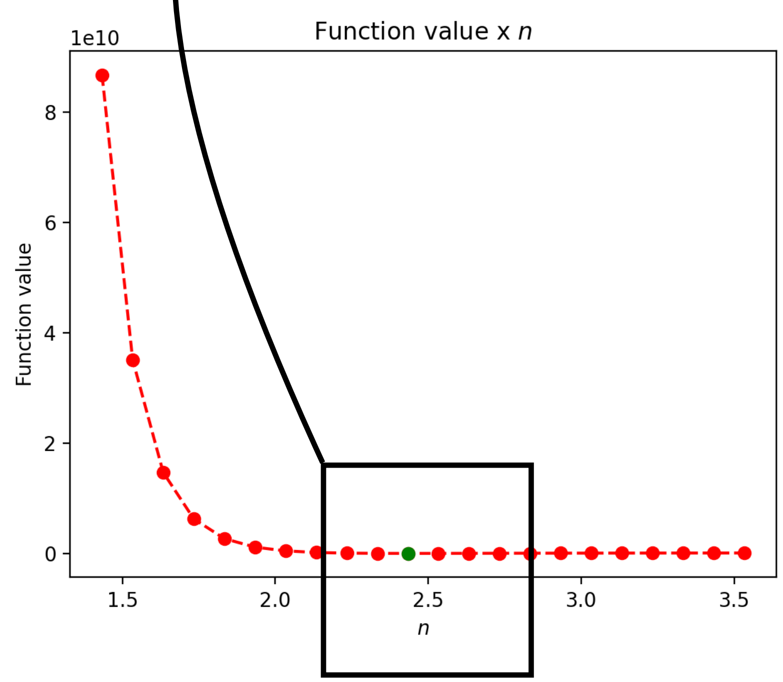
Optimisation converged successfully

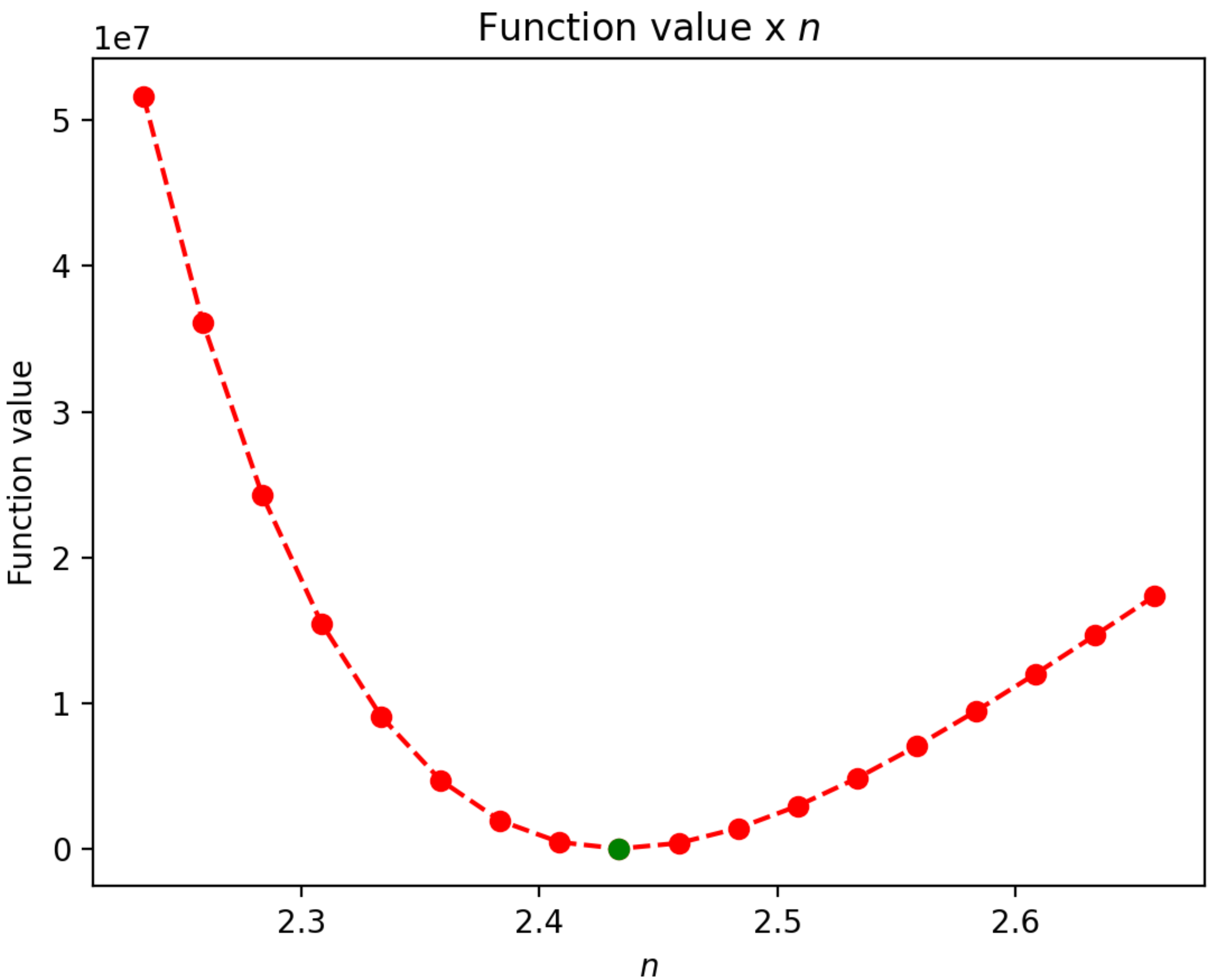
Local minimum found: size of gradient < value of optimality tolerance

Task 1


Thickness vs x (n = 2.3977 and A = 1e-25)
Observed
Computed
Thickness value
1600
1400
1200
1000
800
1622.5
1625.0
1627.5
1630.0
1632.5
1635.0
1637.5
1640.0
x


Thickness vs x (n = 2.4335 and A = 1e-25)
Observed
Computed
Thickness value
1400
1300
1200
1100
1000
900
800
700
1622.5
1625.0
1627.5
1630.0
1632.5
1635.0
1637.5
1640.0
x


Function value x n
1e10
8
6
4
2
0
Function value
1.5
2.0
2.5
3.0
3.5
n


Function value x n
1e7
5
4
3
2
1
0
Function value
2.3
2.4
2.5
2.6
n

		Python	Matlab
Library		SciPy.org	Optimisation ToolBox
Task 1	Solver:	minimize	fminunc
	Algorithm:	Nelder-Mead	Quasi-Newton (BFGS)
Task 2	Solver:	least-squares	lsqnonlin
	Algorithm:	Trust Region Reflective	Trust Region Reflective

$$A = 1 * 10^{(-25)}$$

	Start point					
	n = 1		n = 2.5		n = 4	
	Python	Matlab	Python	Matlab	Python	Matlab
Iterations	26	34	11	15	16	20
Function Evaluations	36	70	19	32	25	42
Optimal n	2.4335	2.4335	2.4335	2.4335	2.4335	2.4335
Min. Function Value	$13 * 10^3$	$13 * 10^3$	$13 * 10^3$	$13 * 10^3$	$13 * 10^3$	$13 * 10^3$

Optimisation converged successfully

Local minimum found: size of gradient < value of optimality tolerance

1. Most converged to same optimum n = 2.4335
2. Most converged to same minimum function value = $13 * 10^3$
3. Differ in number of iterations and function evaluations depending on initial guess and algorithm chosen

Task 1: Quasi-Newton (BFGS) < Nelder-Mead
Convergence of BFGS depends on initial guess & value of optimality tolerance

Task 2: Python implementation < Matlab implementation for same initial guess

Overall: n = 2.5 < n = 4 < n = 1

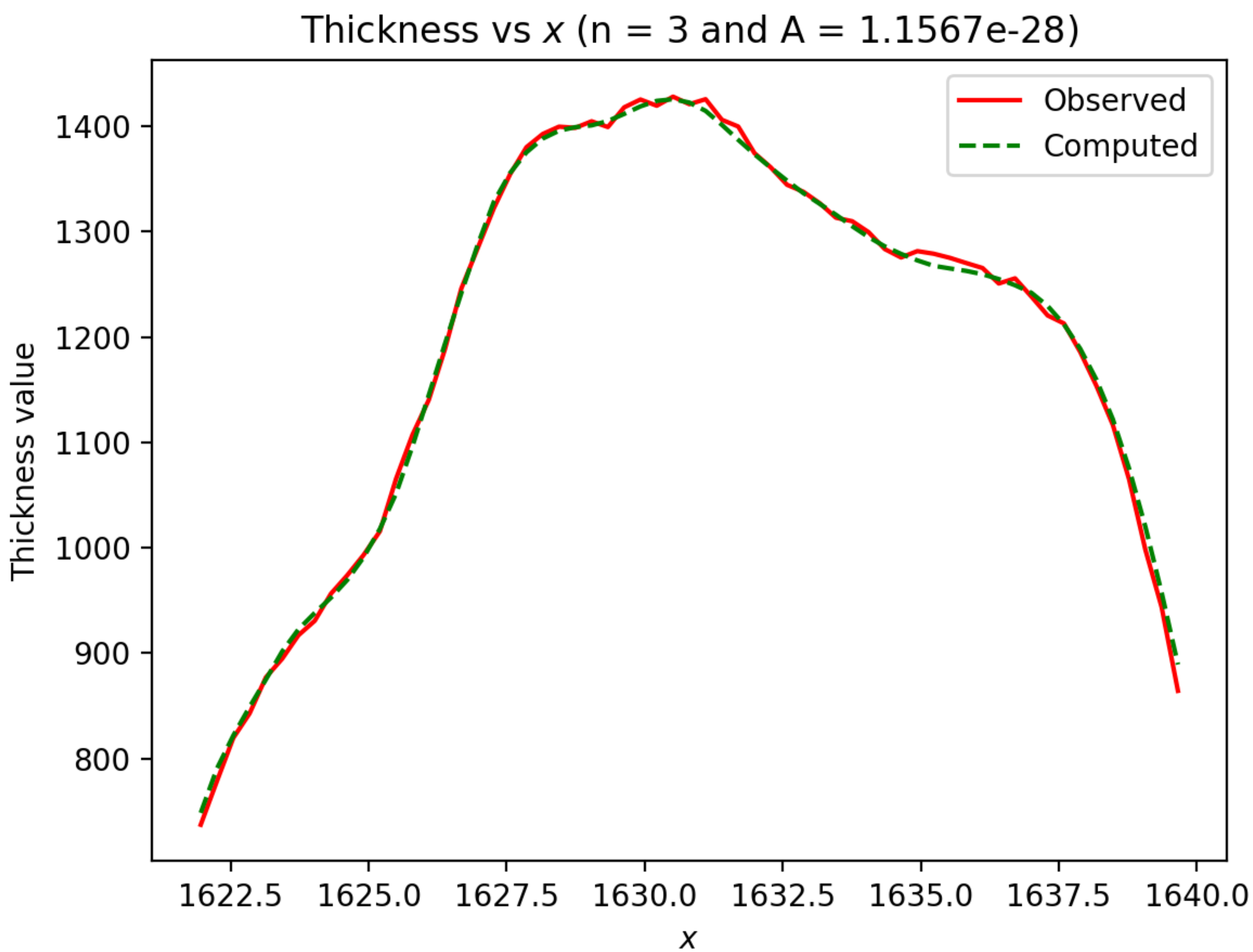
		Python	Matlab
Library		SciPy.org	Optimisation ToolBox
Task 1	Solver:	minimize	fminunc
	Algorithm:	Nelder-Mead	Quasi-Newton (BFGS)
Task 2	Solver:	least-squares	lsqnonlin
	Algorithm:	Trust Region Reflective	Trust Region Reflective
Task 4	Solver:	minimize	-
	Algorithm:	Nelder-Mead	

n = 3

	Start point		
	$A = 1*10^{(-25)}$	$A = 1*10^{(-28)}$	$A = 1*10^{(-31)}$
Iterations	36	14	31
Function Evaluations	72	28	62
Optimal A	$1.16*10^{(-28)}$	$1.16*10^{(-28)}$	$1.16*10^{(-28)}$
Min. Function Value	$3.577 * 10^3$	$3.577 * 10^3$	$3.577 * 10^3$

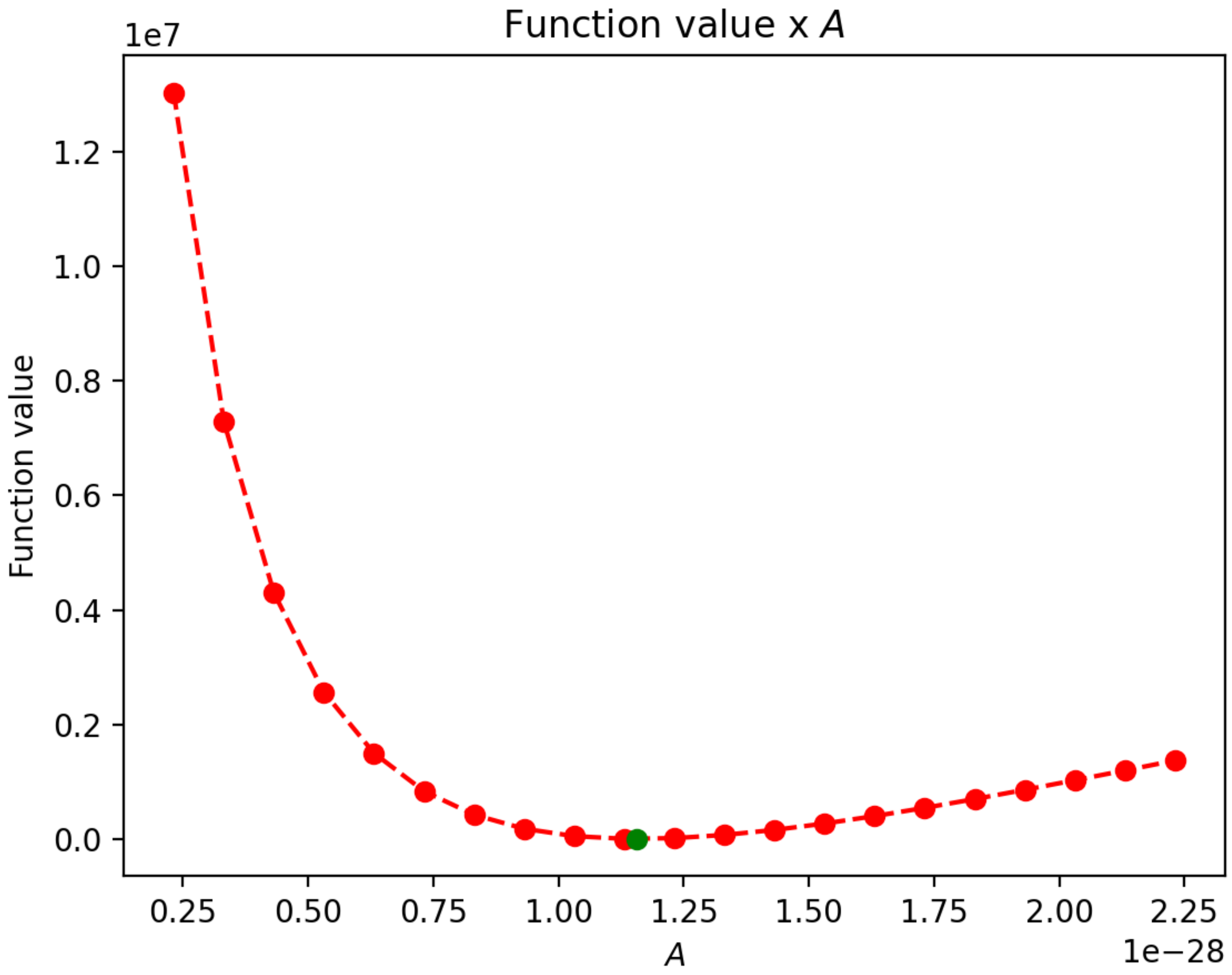
Optimisation converged successfully

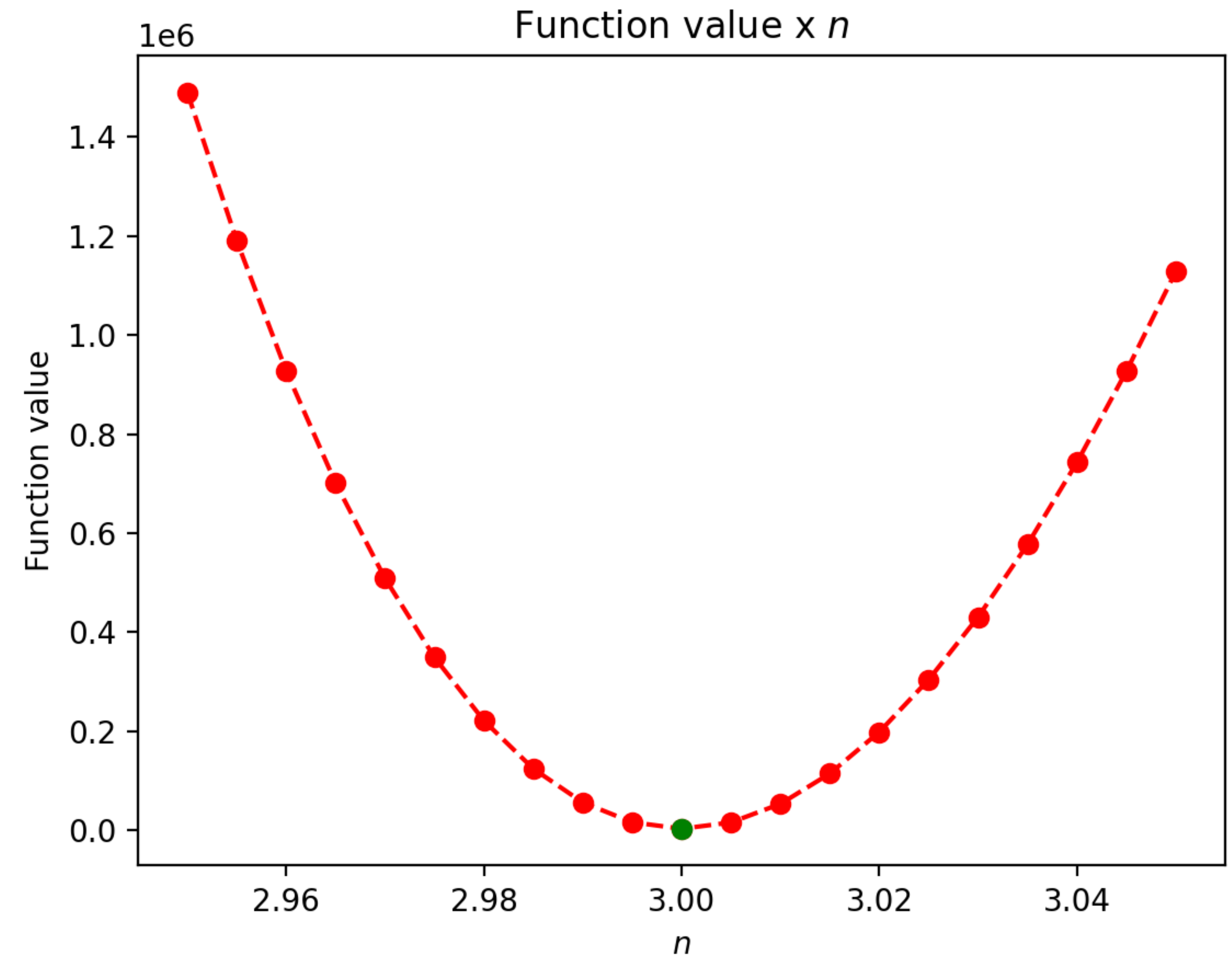
Task 4


Thickness vs x (n = 3 and A = 1.1567e-28)
Observed
Computed
Thickness value
1400
1300
1200
1100
1000
900
800
1622.5
1625.0
1627.5
1630.0
1632.5
1635.0
1637.5
1640.0
x

$n = 3 \longrightarrow A = 1.16*10^{(-28)}$

$A = 1.16*10^{(-28)} \longrightarrow n = 3$





		Python	Matlab
Library		SciPy.org	Optimisation ToolBox
Task 1	Solver:	minimize	fminunc
	Algorithm:	Nelder-Mead	Quasi-Newton (BFGS)
Task 2	Solver:	least-squares	lsqnonlin
	Algorithm:	Trust Region Reflective	Trust Region Reflective
Task 4	Solver:	minimize	-
	Algorithm:	Nelder-Mead	
Task 5 & Task 6	Solver:	minimize	-
	Algorithm:	Nelder-Mead	

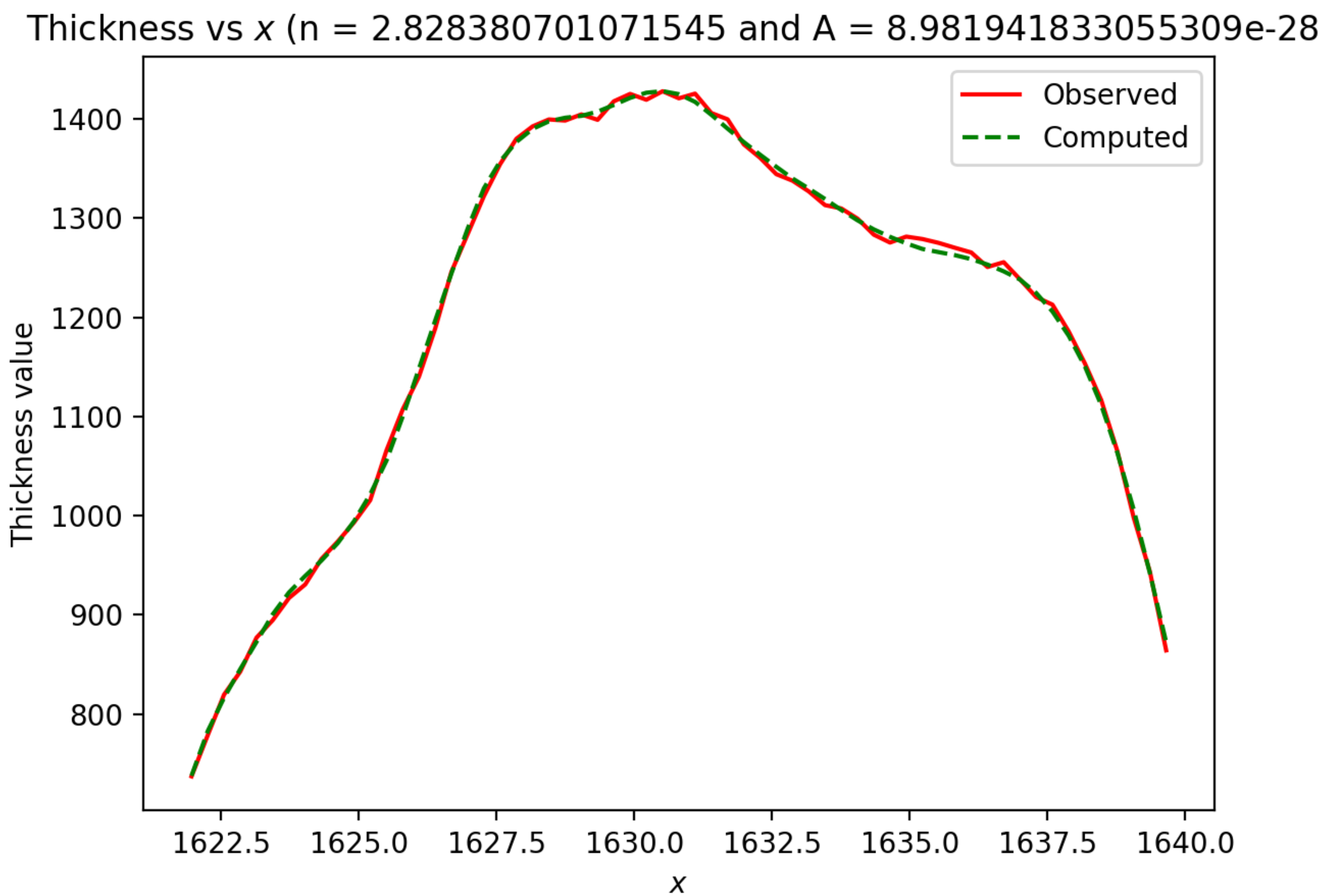
	Start point	
	$A = 1*10^{(-25)}$ $n = 1$	$A = 1*10^{(-25)}$ $n = 3$
Iterations	200	200
Function Evaluations	370	360
Optimal A	$5.2261*10^{(-27)}$	$3.9370*10^{(-27)}$
Optimal n	2.6807	2.7045
Min. Function Value	$3.2959 * 10^3$	$2.8608 * 10^3$

Maximum number of iterations has been exceeded
Success: False

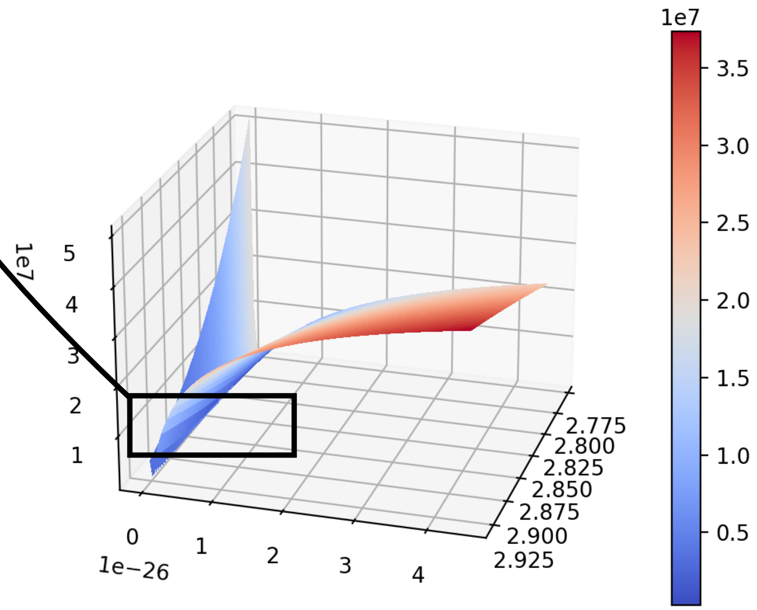
	Start point		
	A = 1*10$^{(-25)}$ n = 1	A = 1*10$^{(-25)}$ n = 3	A = 1*10$^{(-28)}$ n = 3
Iterations	408	340	183
Function Evaluations	754	618	338
Optimal A	8.983*10$^{(-28)}$	8.980*10$^{(-28)}$	8.982*10$^{(-28)}$
Optimal n	2.8284	2.8284	2.8284
Min. Function Value	$1.860 * 10^3$	$1.860 * 10^3$	$1.860 * 10^3$

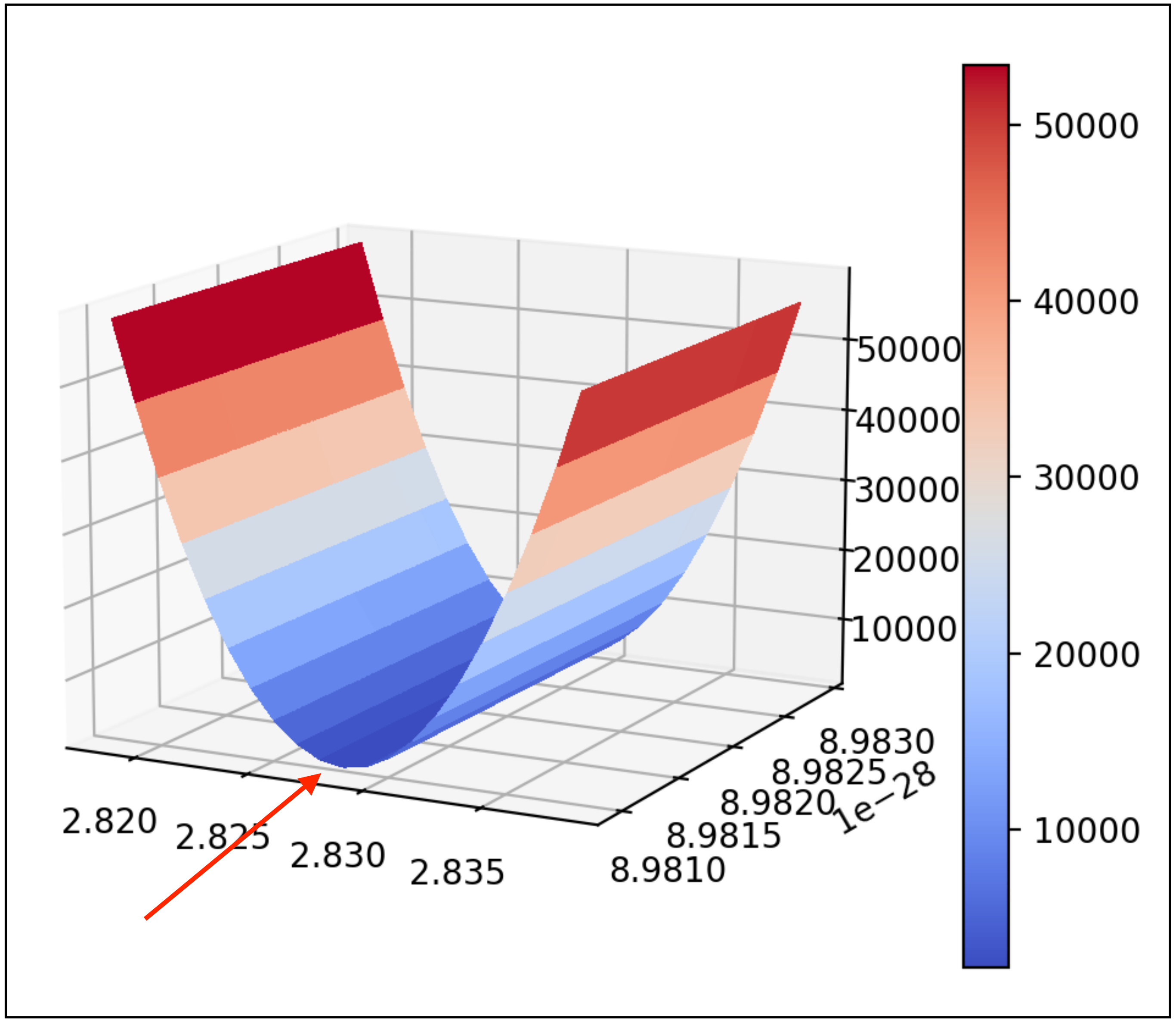
Optimisation converged successfully

Task 5 & Task 6



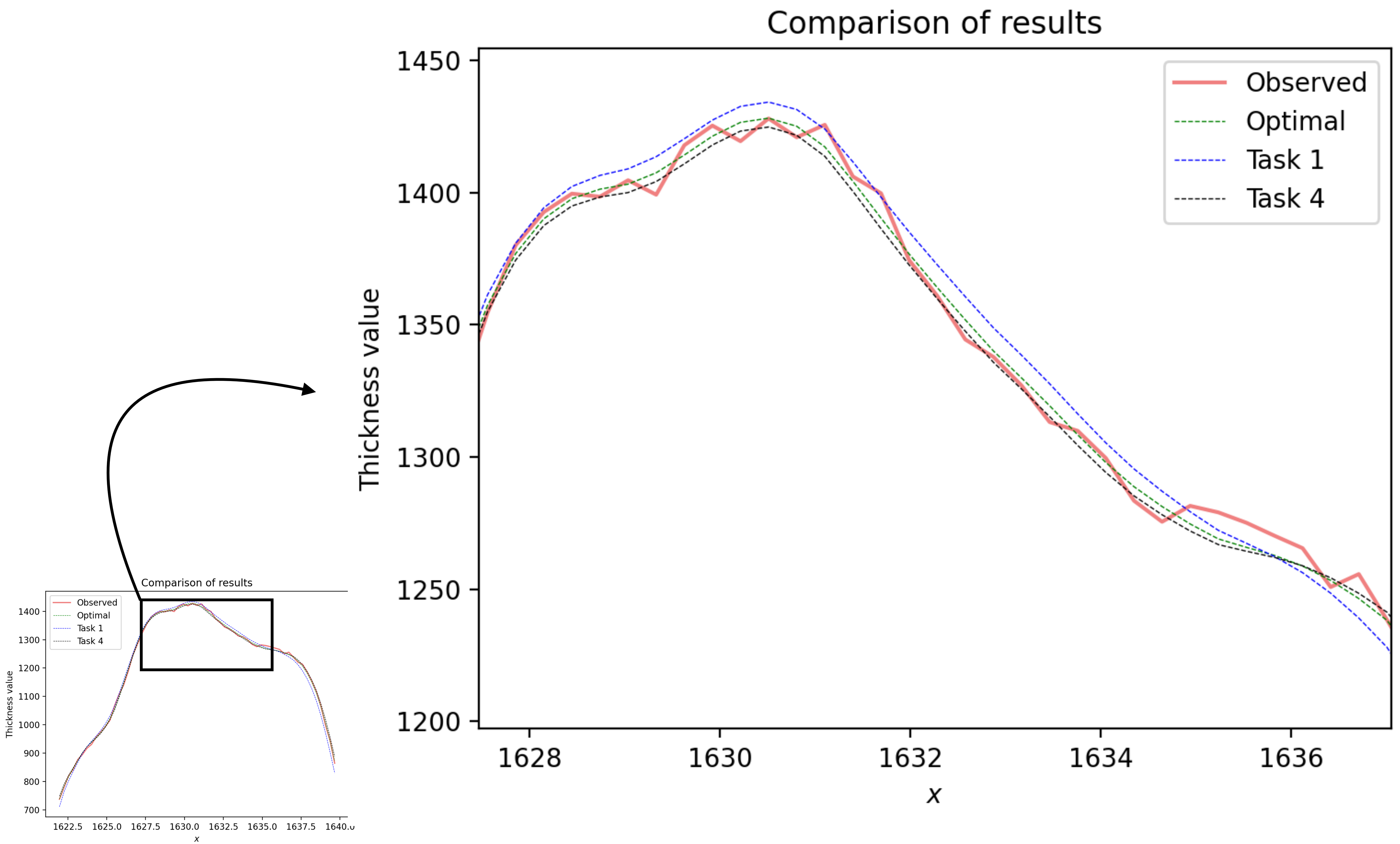
Thickness vs x (n = 2.828380701071545 and A = 8.981941833055309e-28)
Observed
Computed
Thickness value
x
1400
1300
1200
1100
1000
900
800
1622.5
1625.0
1627.5
1630.0
1632.5
1635.0
1637.5
1640.0

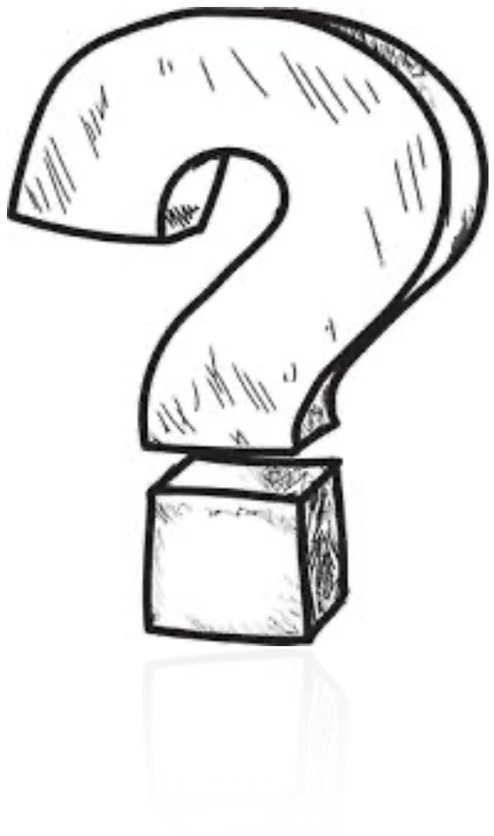




Local Analysis

Observation





Thank you