# Thema des Seminars

Vorname Nachname

***Zusammenfassung*—Die Ausarbeitung beginnt mit einer kurzen Zusammenfassung.**

## I. Introduction

Hier beginnt der Text...

## II. Ein paar Hinweise

Vor eine Subsection gehören immer noch ein paar einleitende Worte!

### A. *Absätze, etc.*

Ein neuer Absatz sollte nicht durch einen Zeilenumbruchs-Befehl, sondern durch eine Leerzeile im Code erzeugt werden.

Das hier ist richtig.
Das hier nicht.

### B. *Formeln*

So können Formeln gesetzt und referenziert werden:

$$a = b + c\,. \tag{1}$$

Laut (1) ist $a = b + c$. Formeln sind Teil des Fließtextes und sollten deshalb korrekt mit Punkten und Kommata interpunktiert werden. Das `\,` fügt dabei einen kleinen Abstand zwischen Formel und Interpunktion ein.

Mehrzeiliger Formelsatz mit der *split* Umgebung innerhalb der *equation* Umgebung:

$$\begin{split} a &= b + c\,, \\ a_{ij} &= b_{ij} + c_{ij}\,. \end{split}$$

Funktionen sollen in Formeln *nicht* mit mathematischer Schrift gesetzt werden. Dazu gibt es in LaTeX für fast alle Funktionen schon Makros, z.B.

$$y = \sin(x)\,,$$

nicht

$$y = sin(x)$$

benutzen. Für nicht vorhandene Funktionen kann *operatorname* eingesetzt werden:

$$y = \operatorname{spur}(X)\,.$$

### C. *Bilder*

So werden Bilder eingebunden (als pdf, jpg oder png):

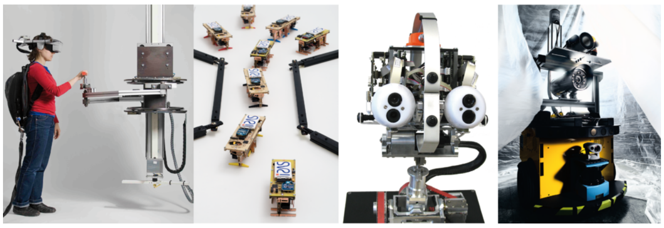

Abbildung 1: Hier kommen weitere Erklärungen zum Bild.

Auf diese Abbildung wird dann mit Abb. 1 verwiesen.

### D. *Zitate*

Immer korrekt zitieren [**?**]!

### E. *LaTeX Hilfe*

Diese Website ist sehr nützlich:
http://en.wikibooks.org/wiki/LaTeX

## III. Zusammenfassung und Ausblick

1) State estimation in dynamic systems (e.g in robotic )
   a) context about what estimation is wrt. to a system
   b) state is not directly observable and we need measurement in order to estimate the state
   c) noisy measurements
   d) linear and non-linear systems (e.g. Kalman Filter in linear case and Unscented Kalman Filter in non-linear)
2) prediction and filtering steps in the estimation
3) problems in estimation
   a) closed form representation of the desired density function are most often not possible (in non-linear case) → approximation of probability density
   b) storing all information and propagate it every time step is computational expensive → progressive
4) Short overview of different methods that focuses on non-linear dynamic systems (state of the art)
   a) Methods using ODE (track true density via ODE)
   b) Other methods
5) Give an outlook on the seminar objective (not quite sure). Either one ODE method and one other non-linear filter method or a different ODE methods

Sources: [5] [6] [3] [4]

- system and measurment quations (nonlinear mapping)
- specification of noise → white noise
- First order markov process

  $$p(x_k|x_{0:k-1}) = p(x_k|x_{k-1})$$

  with $x_{0:k-1} := \{x_0, x_1, \dots, x_{k-1}\}$
- the general estimation process has two steps
  - prediction or time update → prior probability density $p(x_{k+1}|y_{0:k}, u_{0:k})$
  - filtering step or measurement update → posterior probability density $p(x_{k+1}|y_{0:k+1}, u_{0:k})$

  - How are they computed and what does they mean
- Progressive bayes

System and measurment equation:

$$x_{k+1} = a_k(x_k, u_k, w_k)\,,$$
$$y_k = h_k(x_k, v_k)\,.$$

Connection between prediction and filtering in sense of Bayes Theorem

$$p(x_{k+1}|y_{0:k+1}, u_{0:k}) = \frac{p(y_k|x_k)p(x_{k+1}|y_{0:k}, u_{0:k})}{p(y_{k+1}|y_{0:k})}\,,$$

$$\begin{aligned} p(\underline{x}_{k+1}|\underline{Y}_{k+1}) &= p(\underline{x}_{k+1}|\underline{Y}_k, \underline{y}_{k+1}) \\ &= \frac{p(\underline{y}_{k+1}|\underline{x}_{k+1}, \underline{Y}_k)}{p(\underline{y}_{k+1}|\underline{Y}_k)} \\ &= \frac{p(\underline{y}_{k+1}|\underline{x}_{k+1})}{p(\underline{y}_{k+1}|\underline{Y}_k)} \\ &= \frac{p(\underline{y}_{k+1}|\underline{x}_{k+1})p(\underline{x}_{k+1}|\underline{Y}_k)}{p(\underline{y}_{k+1}|\underline{Y}_k)}\,. \end{aligned}$$

The homotopy used in the procedure [2] is given in (2) and is similar to (**??**) except the additional normalization constant $c(\lambda)$. Furthermore, the main assumption for this approach is that $\tilde{f}^e(\underline{x})$, $f^p(\underline{x})$ and the importance density $q_\tau(\underline{x})$ are Gaussian densities.

$$\tilde{f}^e(\underline{x}, \lambda) = c(\lambda) f^p(\underline{x}) f^L(\underline{x})^\lambda\,. \tag{2}$$

The first step is primarily aimed to create an importance density which is then used to update the weights of the particles in the second step. In order to derive the importance density for the $\tau^{\text{th}}$ progression step, the first objective is to predict the displacement of the particle set $\left[\underline{x}_{\tau+1}^{(1)}, \dots, \underline{x}_{\tau+1}^{(L)}\right]$ which is caused during the progression. The following equation delivers the desired particle set [2]:

$$\underline{x}_{\tau+1}^{(i)} = \mathbf{F}_\tau \begin{bmatrix} \underline{x}_\tau^{(i)} \\ \underline{x}_{\tau-1}^{(i)} \end{bmatrix}\,,$$

with F being a transition matrix that adjusts the particles $\underline{x}_\tau^{(i)}$ and $\underline{x}_{\tau-1}^{(i)}$ via coefficients $T = \frac{\Delta\lambda_\tau}{\Delta\lambda_{\tau-1}}$[2]. These particles are obtained from the Dirac mixture approximation of the respective interim posterior densities $\tilde{f}^e(\underline{x}, \lambda)$. An approximation for a specific $\tau$-th progression step is given by

$$f^e(\underline{x}, \lambda_\tau) = \sum_{i=1}^{L} w^{(i)} \delta(\underline{x} - \underline{x}_\tau^{(i)})\,,$$

including equally weighted weights. Furthermore, the particles for calculating $\underline{x}_{\tau+1}^{(i)}$ are selected in such way that minimizes the second-order Wasserstein distance between $f^e(\underline{x}, \lambda_\tau)$ and $f^e(\underline{x}, \lambda_{\tau-1})$ [2]. The importance density $q_\tau(\underline{x})$ is then calculated based on the predicted particle set via moment matching.

The next step leverages the importance density $q_\tau(\underline{x})$ similar to (**??**) and combines it with the homotopy (2) resulting in the interim posterior

$$\tilde{f}^e(\underline{x}, \lambda_\tau) = c(\lambda) f^L(\underline{x})^{\lambda_\tau} \frac{f^p(\underline{x})}{q_\tau(\underline{x})} q_\tau(\underline{x})\,. \tag{3}$$

The predicted particles are also used to simplify the equation (3) by using a Dirac mixture for $q_\tau(\underline{x})$ with dirac components equal to the particles (see section **??**). This leads to the final equation for the interim posterior $\tilde{f}^e(\underline{x}, \lambda_\tau)$

$$\tilde{f}^e(\underline{x}, \lambda_\tau) = c(\lambda) \sum_{i=1}^{L} w^{(i)} f^L(\underline{x}^{(i)})^{\lambda_\tau} \frac{f^p(\underline{x}^{(i)})}{q_\tau(\underline{x}^{(i)})}\,.$$

Lastly, the weights are being updated according to

$$\hat{w}^{(i)} = f^L(\underline{x}^{(i)})^{\lambda_\tau} \frac{f^p(\underline{x}^{(i)})}{q_\tau(\underline{x}^{(i)})}\,,$$

and their normalized weights $\bar{w}^{(i)}$ are calculated from the $\hat{w}^{(i)}$ [2]. The procedure is repeated until the the final interim posterior $\tilde{f}^e(\underline{x}, \lambda = 1)$ is reached which is the desired posterior $\tilde{f}(\underline{x})$.

Sources: [1]

## LITERATUR


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[2] Christof Chlebek, Jannik Steinbring, and Uwe D. Hanebeck. Progressive Gaussian filter using importance sampling and particle flow. In *2016 19th International Conference on Information Fusion (FUSION)*, pages 2043–2049, July 2016.

[3] F. Daum. Nonlinear filters: Beyond the Kalman filter. *IEEE Aerospace and Electronic Systems Magazine*, 20(8):57–69, August 2005. Conference Name: IEEE Aerospace and Electronic Systems Magazine.

[4] Jonas Hagmar, Mats Jirstrand, Lennart Svensson, and Mark Morelande. Optimal parameterization of posterior densities using homotopy. In *14th International Conference on Information Fusion*, pages 1–8, July 2011.

[5] Uwe D. Hanebeck, Kai Briechle, and Andreas Rauh. Progressive Bayes: A new framework for nonlinear state estimation. In Belur V. Dasarathy, editor, *AeroSense 2003*, page 256, Orlando, FL, April 2003.

[6] Marco F. Huber and Uwe D. Hanebeck. Gaussian Filter based on Deterministic Sampling for High Quality Nonlinear Estimation. *IFAC Proceedings Volumes*, 41(2):13527–13532, 2008.