Nonlinear Filtering with Homotopy Continuation

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***Abstract*—Non-linear filtering with use of ordinary differential equation.**

I. Introduction

1) State estimation in dynamic systems (e.g in robotic)
   a) context about what estimation is wrt. to a system
   b) state is not directly observable and we need measurement in order to estimate the state
   c) noisy measurements
   d) linear and non-linear systems (e.g. Kalman Filter in linear case and Unscented Kalman Filter in non-linear)
2) prediction and filtering steps in the estimation
3) problems in estimation
   a) closed form representation of the desired density function are most often not possible (in non-linear case) $\rightarrow$ approximation of probability density
   b) storing all information and propagate it every time step is computational expensive $\rightarrow$ progressive
4) Short overview of different methods that focuses on non-linear dynamic systems (state of the art)
   a) Methods using ODE (track true density via ODE)
   b) Other methods
5) Give an outlook on the seminar objective (not quite sure). Either one ODE method and one other non-linear filter method or a different ODE methods

Sources: [4] [5] [2] [3] ...

II. Problem Formulation

- system and measurment quations (nonlinear mapping)
- specification of noise $\rightarrow$ white noise
- First order markov process

$$p(x_k|x_{0:k-1}) = p(x_k|x_{k-1})$$

  with $x_{0:k-1} := \{x_0, x_1, \dots, x_{k-1}\}$
- the general estimation process has two steps
  - prediction or time update $\rightarrow$ prior probability density $p(x_{k+1}|y_{0:k}, u_{0:k})$
  - filtering step or measurement update $\rightarrow$ posterior probability density $p(x_{k+1}|y_{0:k+1}, u_{0:k})$
  - How are they computed and what does they mean
- Progressive bayes

System and measurment equation:

$$x_{k+1} = a_k(x_k, u_k, w_k)\,,$$
$$y_k = h_k(x_k, v_k)\,.$$

Connection between prediction and filtering in sense of Bayes Theorem

$$p(x_{k+1}|y_{0:k+1}, u_{0:k}) = \frac{p(y_k|x_k)p(x_{k+1}|y_{0:k}, u_{0:k})}{p(y_{k+1}|y_{0:k})}\,,$$

Sources: [1]

III. Non-linear Filtering with Homotopy Continuation

- homotopy continuation are used in variety of use cases:
  - Filtering in Polynomial Systems
  - Obtain optimal parameter for the posterior density

Sources: [3] [6]

A. General Idea

- approximate true density with help of intermediate densities using a tracking parameter
- deviation between true and approx. probability density is based on some kind of a deviation measurement
- transform the deviation measure in an ordinary differential equation in order to track the intermediate densities

Source: [4]

B. Ordinary Differential Equation in Estimation

Title may be just Homotopy Continuation as homotopy continuation has the same steps (?)

- Derive how ordinary differential equation can be used in estimation
- Use of Measure of deviation
- Measurment update with ODE

Source: [4]

C. Homotopy Continuation for ...

1) Compare two different uses cases with homotopy continuation and move on to chapter V or
2) present one type of homotopy continuation and move on to chapter IV

IV. Non-linear Filtering with ...

V. Conclusion

- Compare findings of different methods

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