Kakutani fixed point theorem

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Definition 1. Convex set. A set $S \subset \mathbb{R}^n$ is convex if for every $x, y \in S$

$$tx + (1-t)y \in S, \ \forall \ t \in [0,1].$$

Definition 2. Correspondence. A correspondence is a map such that $F : X \to 2^Y$, i.e., to each element in x, F associates a subset of Y.

Definition 3. Upper hemi-continuous correspondence. A correspondence $F : X \to 2^Y$ is u.h.c. if for every sequence $\{x_n\} \in X$ such that $x_n \to x$, for every $y \in F(x)$, there exists a sequence $\{y_n\}$ with $y_n \in F(x_n)$ such that $y_n \to y$.

Theorem 4. Let S be a compact and convex set in $\mathbb{R}^n$ and $F : S \to 2^S$ an u.h.c. correspondence such that $F(x)$ is non empty, convex and compact for ever $x \in S$. Then, $\exists \ x_0 \in S$ such that $x_0 \in F(x_0)$.