FACULTY OF SOCIAL SCIENCES
ECONOMICS MAJOR

PUCP

Recitation 2

Microeconomics 2
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1 2×2 Economy

Exercise 1.1. In each of the following cases, draw the Edgeworth box, some indifference curves for each consumer and find Walrasian (competitive) equilibrium in each case. **Later on, you should be able to find the Pareto set and the core (contract curve).**

a) $u_1(x_{11}, x_{21}) = 2x_{11}^2 x_{21}$, $u_2(x_{12}, x_{22}) = x_{12}x_{22}^3$, $\omega_1 = (2, 3)$ and $\omega_2 = (1, 2)$.

b) $u_1(x_{11}, x_{21}) = x_{11} + x_{21}$, $u_2(x_{12}, x_{22}) = \min\{x_{12}, x_{22}\}$, $\omega_1 = (1, 2)$ and $\omega_2 = (3, 4)$.

c) $u_1(x_{11}, x_{21}) = x_{11} + \ln x_{21}$, $u_2(x_{12}, x_{22}) = x_{12} + 2\ln x_{22}$, $\omega_1 = (2, 3)$ and $\omega_2 = (1, 2)$.

d) $u_1(x_{11}, x_{21}) = x_{11}x_{21}$, $u_2(x_{12}, x_{22}) = \min\{x_{12}, x_{22}\}$, $\omega_1 = (2, 6)$ and $\omega_2 = (4, 1)$.

e) $u_1(x_{11}, x_{21}) = \min\{2x_{11}, x_{21}\}$, $u_2(x_{12}, x_{22}) = \min\{x_{12}, 2x_{22}\}$, $\omega_1 = (1, 2)$ and $\omega_2 = (3, 4)$.

f) $u_1(x_{11}, x_{21}) = 3x_{11} + x_{21}$, $u_2(x_{12}, x_{22}) = x_{12} + 3x_{22}$, $\omega_1 = (2, 2)$ and $\omega_2 = (2, 2)$.

Identify whenever it is possible the type (Cobb-Douglas, CES, Leontief, linear...) of the utility function.

Exercise 1.2. From Mas-Colell et al. (1995). Consider a 2×2 economy in which consumers preferences are monotonic. Prove that (here below $\omega_\ell = \omega_{1\ell} + \omega_{2\ell}$)

$$p_1 \left(\sum_{i=1}^{2} x_{1i}(p_1, p_2) - \omega_1 \right) + p_2 \left(\sum_{i=1}^{2} x_{2i}(p_1, p_2) - \omega_2 \right) = 0.$$

Use this to explain Walras law, *if one market clears the other too*. Generalize this result to I consumers and L goods.

Exercise 1.3. From Mas-Colell et al. (1995). Consider and Edgeworth box economy in which each consumer has Cobb-Douglas preferences

$$u_1(x_{11}, x_{21}) = x_{11}^{\alpha} x_{21}^{1-\alpha}$$
$$u_2(x_{12}, x_{22}) = x_{12}^{\beta} x_{22}^{1-\beta},$$

with $\alpha, \beta \in (0, 1)$. Consider endowments $(\omega_{1i}, \omega_{2i}) > 0$ for $i = 1, 2$. Solve for the equilibrium price ratio and allocation.

Exercise 1.4. There are two consumers, A and B, with the following utility functions,

$$u_A(x_A^1, x_A^2) = a \ln x_A^1 + (1-a) \ln x_A^2,\ \omega_1 = (0, 1)$$
$$u_B(x_B^1, x_B^2) = \min\{x_B^1, x_B^2\},\ \omega_2 = (1, 0).$$

Compute the prices and quantities that clear the market. Interpret.

Exercise 1.5. Consider two individuals in a pure exchange (2×2) economy whose indirect utilities are

$$v_1(p_1, p_2, w) = \frac{w}{p_1 + p_2}$$
$$v_2(p_1, p_2, w) = \frac{abw}{bp_1 + ap_2},\ a, b > 0.$$

Endowments are $\omega_1 = (1, 1)$ and $\omega_2 = (1, 1)$. Obtain the equation that prices which clear the market must satisfy. *Hint*: apply Roy's identity.

2 Additional exercises

Exercise 2.1. Suppose that in a 2×2 economy consumer i has Cobb-Douglas preferences $u_i(x_{1i}, x_{2i}) = x_{1i}^{\alpha} x_{2i}^{1-\alpha}$. Furthermore, assume that endowments are $\omega_1 = (1, 2)$ and $\omega_2 = (2, 1)$. Find the (a)[1] Walrasian equilibrium. **Later on, you should be able to find the optimal Pareto assignments.**

Exercise 2.2. For when you've seen Pareto Optimality in class. Under some conditions over the preferences, in a 2×2 economy, every Pareto Optimal allocation can be characterized as the solution of the following maximization problem (you should try to prove it)

$$\begin{aligned} &\max\ u_1(\mathbf{x}_1) \\ &\text{s. t. } u_2(\mathbf{x}_2) \geq k \\ &\qquad \mathbf{x}_1 + \mathbf{x}_2 = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2, \end{aligned}$$

where $k \in \mathbb{R}$. Find the aforementioned conditions over the preferences. Generalize this result for pure exchange economies with I individual and L goods.

Exercise 2.3. Medium-difficulty. From Chavez and Gallardo (2024). Consider an

[1] We don't know if it is unique or no! However, under some conditions over the preferences, which are satisfied in this exercise, existence is ensured.

economy with N consumers, two goods, and preferences given by

$$u_i(x_{1i}, x_{2i}) = x_{1i}^2 + x_{2i}^2.$$

Endowments are $\omega_i = (1, 1)$. If N is even, find, if it exists, a Walrasian equilibrium. What if N is odd?

Exercise 2.4. Mandatory to know. Prove that if $\succeq$ is monotone, then it is locally non satiated. Here $\succeq$ represents a preference relation over $\mathbb{R}_+^L$.

Exercise 2.5. Medium difficulty. For when you've seen Pareto Optimality in class. Prove 1st Welfare theorem for a 2×2 economy. This is, if preferences are locally non satiated, then, every Walrasian equilibrium is Pareto optimal. Can you generalize this for a pure exchange economy with N consumers and L goods? You can guide yourself from Echenique (2015).

Lima, September 2, 2024.

References

Chavez, J. and Gallardo, M. (2024). *Algebra Lineal y Optimization para el Análisis Económico*. Prepublished.

Echenique, F. (2015). Lecture notes general equilibrium theory.

Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). *Microeconomic Theory*. Oxford University Press, New York.