Recitation 3

Marcelo Gallardo

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About Test 1

1. Statistics: mean 13.33, range $[7, 18]$.
2. Objectives: $+15$.
3. Good but ...

Distribución de notas sin considerar faltante

Tipo de Nota	Cantidad	Media	Desv. Est.	Mediana	Minimo	Maximo
NORMAL	18	13.33	3.12	14	7	

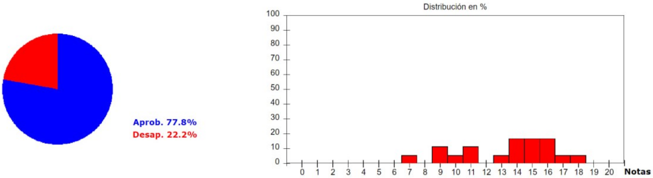


Figura Distribution.

About Test 1

Question by question

1. Question 1 (5 minutes): formalize, **convex** preferences, $\sum_{i=1}^{I} \mathbf{x}_i(\mathbf{p}^*) = \sum_{i=1}^{I} \boldsymbol{\omega}_i$.
2. Question 2 (10 minutes): apply Roy's Identity, use $I = \mathbf{p} \cdot \boldsymbol{\omega}_i$ and clear one market.
3. Question 3 (10 minutes): draw the Edgeworth box (careful with Cobb-Douglas preferences!), obtain Walrasian demands and clear one market. Finally, compute

$$\frac{\partial}{\partial \theta}\left[\frac{p_2^*}{p_1^*}\right].$$

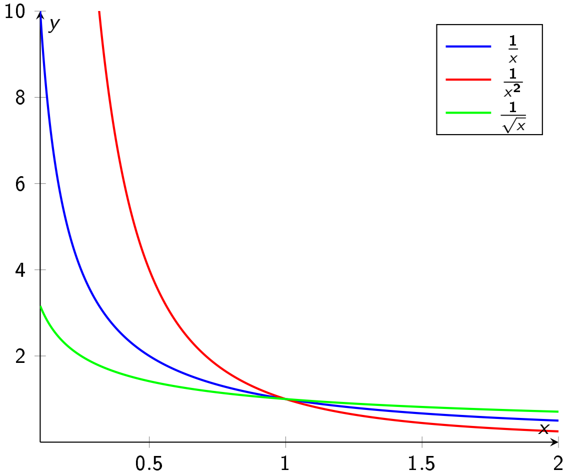
4. Question 4 (20 minutes): don't confuse P.O. with W.E! Finally,

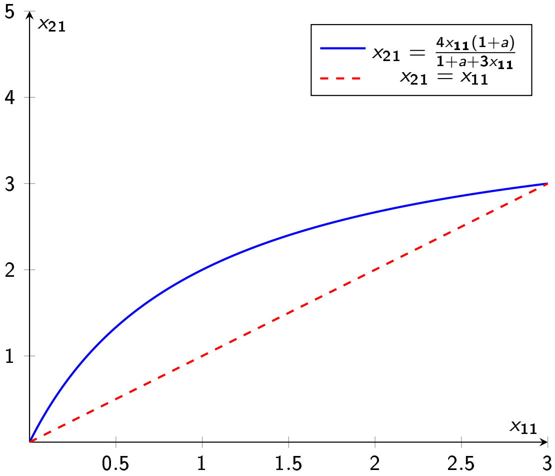
$$\mathcal{P} = \left\{(x_{11}, x_{21}) \in [0, \omega_1] \times [0, \omega_2] : \ x_{21} = \frac{4(1+a)x_{11}}{1+a+3x_{11}}\right\}$$

$$\frac{d}{dx_{11}}\left(\frac{4(1+a)x_{11}}{1+a+3x_{11}}\right) = \frac{4(a+1)^2}{(a+3x_{11}+1)^2}, \quad \frac{d^2}{dx_{11}^2}\left(\frac{4(1+a)x_{11}}{1+a+3x_{11}}\right) = \frac{-24(a+1)^2}{(1+a+3x_{11})^2} < 0.$$

5. Work smarter, not harder

$$MRS_1 = MRS_2 \quad \Leftrightarrow \quad L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\mu})...$$


10
8
6
4
2
y
x
0.5
1
1.5
2
$\frac{1}{x}$
$\frac{1}{x^2}$
$\frac{1}{\sqrt{x}}$



5
4
3
2
1
x_{21}
$x_{21} = \frac{4x_{11}(1+a)}{1+a+3x_{11}}$
$x_{21} = x_{11}$
x_{11}
0.5
1
1.5
2
2.5
3

About Test 1

1. Understand the theory, then practice with exercises.
2. Second test: harder than the first.
3. Competencies: solving complex problems, critical thinking, abstract thinking.
4. Plot using Desmos, Wolfram, Tikz, Python or Matlab.
5. Theory: Slides, Mas-Colell et al., or Varian (Microeconomic Analysis).
6. Exercises: Recitation 2, Recitation 3 and Recitation 4.

Outline Recitation 3

1. Welfare Theorems. **Exercise 1.3.**
2. Robinson Crusoe model.
3. Pure Exchange Economies.
4. Private Ownership Economies.
5. Test 2 should consider all of this.

Further content in general equilibrium

1. **Existence and uniqueness of the Walrasian Equilibrium:** Brouwer Fixed Point Theorem (for strictly convex preferences), Kakutani if only convex. See Mas-Colell et al., Stockey and Lucas or Efe A. Ok.
2. **Radner Equilibrium:** A. Lugón book.
3. **Complete and incomplete markets:** Ljungqvist and Sargent (Recursive Macroeconomic Theory).
4. **Infinite goods**: work with linear functionals $\varphi \in \ell_\infty$ instead of vectors **p**. Uses Hahn-Banach separation theorems instead of convex analysis in $\mathbb{R}^n$. Key authors: Aloiso Araujo (IMPA/FGV EPGE), Paulo Klinger (FGVE EPGE).
5. **Differential approach:** famous theorem (Debreu) tell us that when $\succeq$ is rational (sometimes assumed in the definition) and continuous[1], there exists $u(\cdot)$ continuous that represents it. What about differentiability? : surface (manifolds) analysis. See Mas-Colell (The Theory of General Economic Equilibrium: A Differentiable Approach).
6. **Non convexity:** The Second Welfare Theorem with Nonconvex Preferences, Robert M. Anderson (UC Berkeley), Econometrica, Vol. 56, No. 2 (Mar., 1988), pp. 361-382 (22 pages).

[1] $x_n \succeq y_n$ with $x_n \to x$ and $y_n \to y$, then $x \succeq y$.

Welfare Theorems

Welfare Theorems: preliminaries

Definition

An allocation $\mathbf{x} = (\mathbf{x}_1, \cdots, \mathbf{x}_I)$ which is Pareto Optimal is an allocation such that

$$\nexists\, \mathbf{x}' : \sum_{i=1}^{I} \mathbf{x}_i' \leq \overline{\boldsymbol{\omega}} = \sum_{i=1}^{I} \boldsymbol{\omega}_i$$

$$\forall\ i\ \mathbf{x}_i' \succeq_i \mathbf{x}_i \quad \wedge \quad \mathbf{x}_{i_0}' \succ_{i_0} \mathbf{x}_{i_0},\ i_0 \in \{1, ..., I\}.$$

Definition

A Walrasian Equilibrium is an allocation $\mathbf{x}^*$ and a price vector $\mathbf{p}^* \in \mathbb{R}_+^L$ such that:

1. $\mathbf{x}_i^* \in B(\mathbf{p}^*, \mathbf{p}^* \cdot \boldsymbol{\omega}_i)$, and $\mathbf{x}_i^* \succeq_i \mathbf{x}_i$, $\forall\ \mathbf{x}_i \in B(\mathbf{p}^*, \mathbf{p}^* \cdot \boldsymbol{\omega}_i)$.
2. $\sum_{i=1}^{I} \mathbf{x}_i^*(\mathbf{p}^*) = \sum_{i=1}^{I} \boldsymbol{\omega}_i$.

First Welfare Theorem

Theorem

If preferences are locally non-satiated, every W.E. is P.O.

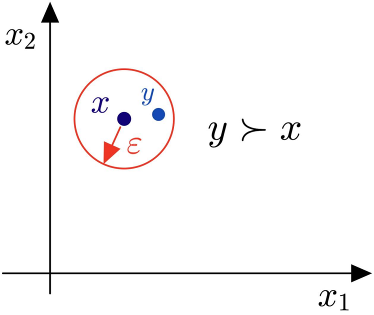


Figura $\succeq$ l.n.s.

First Welfare Theorem

Prueba.

Let $(\mathbf{x}^*, \mathbf{p}^*)$ be a W.E. Proceeding by contradiction, suppose that the allocation $\mathbf{x}^*$ is not P.O. In this case, there must exist a feasible allocation $\mathbf{x} = \{\mathbf{x}_i\}_{i=1}^{I}$, such that for each $i = 1, \ldots, I$, $\mathbf{x}_i \succeq_i \mathbf{x}_i^*$, and at least for some $i_0 \in \{1, \ldots, I\}$, $\mathbf{x}_{i_0} \succ_{i_0} \mathbf{x}_{i_0}^*$. We will prove that for such an allocation, the inequality

$$\sum_{i=1}^{I} \mathbf{x}_i > \overline{\boldsymbol{\omega}},$$

holds, which contradicts the fact that $\mathbf{x}$ is feasible. First, the condition $\mathbf{x}_i \succeq_i \mathbf{x}_i^*$ implies that $\mathbf{p}^* \cdot \mathbf{x}_i \geq \mathbf{p}^* \cdot \boldsymbol{\omega}_i$. Indeed, if $\mathbf{p}^* \cdot \mathbf{x}_i < \mathbf{p}^* \cdot \boldsymbol{\omega}_i$, then there exists $\epsilon > 0$, such that for all $z \in \mathcal{B}(\mathbf{x}_i; \epsilon)$, $\mathbf{p}^* \cdot z < \mathbf{p}^* \cdot \boldsymbol{\omega}_i$; and since preferences are locally non-satiated, $\exists\ z_0 \in \mathcal{B}(\mathbf{x}_i; \epsilon)$ such that $z_0 \succ_i \mathbf{x}_i \succeq_i \mathbf{x}_i^*$. However, this contradicts the maximality of $\mathbf{x}_i^*$. On the other hand, the condition $\mathbf{x}_{i_0} \succ \mathbf{x}_{i_0}^*$ implies that $\mathbf{p}^* \cdot \mathbf{x}_{i_0} > \mathbf{p}^* \cdot \boldsymbol{\omega}_{i_0}$. Indeed, the contrary inequality, that is, $\mathbf{p}^* \cdot \mathbf{x}_{i_0} \leq \mathbf{p}^* \cdot \boldsymbol{\omega}_{i_0}$, contradicts the maximality of $\mathbf{x}_{i_0}^*$. Thus, we conclude that

$$\sum_{i=1}^{I} \mathbf{p}^* \cdot \mathbf{x}_i = \sum_{i \neq i_0} \mathbf{p}^* \cdot \mathbf{x}_i + \mathbf{p}^* \cdot \mathbf{x}_{i_0} > \sum_{i \neq i_0} \mathbf{p}^* \cdot \boldsymbol{\omega}_i + \mathbf{p}^* \cdot \boldsymbol{\omega}_{i_0} = \sum_{i=1}^{I} \mathbf{p}^* \cdot \boldsymbol{\omega}_i.$$

Since $\mathbf{p}^* \in \mathbb{R}_+^L - \{0\}$, this equation implies that we cannot have $\sum_{i=1}^{I} \mathbf{x}_i \leq \boldsymbol{\omega}$. That is, it must hold that $\sum_{i=1}^{I} \mathbf{x}_i > \overline{\boldsymbol{\omega}}$, as we wanted to show. $\square$

Second Welfare Theorem

Definition

A **Walrasian equilibrium with transfers** is a tuple $(\mathbf{x}, \boldsymbol{p}, \mathbf{T})$, where $\mathbf{x} \in \mathbb{R}_+^{IL}$, $\boldsymbol{p} \in \mathbb{R}_+^L$ (a price vector), and $\mathbf{T} = (T_i)_{i=1}^I \in \mathbb{R}^I$ (a vector of net transfers), such that:

(i) for all $i = 1, \ldots, I$, $\mathbf{x}_i \in B(\boldsymbol{p}, M_i)$, and $\mathbf{x}_i' \in B(\boldsymbol{p}, M_i) \Rightarrow \mathbf{x}_i \succeq \mathbf{x}_i'$, where $M_i = \boldsymbol{p} \cdot \boldsymbol{\omega}_i + T_i$ (consumers optimize by choosing $\mathbf{x}_i$ in their budget sets);

(ii) $\sum_{i=1}^I \mathbf{x}_i = \sum_{i=1}^I \boldsymbol{\omega}_i$ (demand equals supply);

(iii) $\sum_{i=1}^I T_i = 0$ (net transfers are balanced).

Theorem

Second Welfare Theorem. Let $\mathcal{E} = (\succeq_i, \boldsymbol{\omega}_i)_{i=1}^I$ be a Pareto optimal allocation in which each preference $\succeq_i$ is strongly monotonic, convex, and continuous. If $\mathbf{x}^*$ is a Pareto optimal allocation such that $\sum_{i=1}^I \mathbf{x}_i^* > 0$, then there exists a price vector $\boldsymbol{p}^* \in \mathbb{R}_+^L$ and transfers $\mathbf{T} = (T_i)_{i=1}^I$ such that $(\mathbf{x}^*, \boldsymbol{p}^*, \mathbf{T})$ is a Walrasian equilibrium with transfers.

Second Welfare Theorem

Proof? Too long: see Echenique (2015) or Varian (1992). Key ideas:

1. Convexity of preferences: hyperplane separation theorem. The price vector comes from this.
2. Continuity and monotonicity: redistribute resources.
3. You need to find a price vector (candidate), then prove that lies in $\mathbb{R}^L_+$. Finally, prove that it lies in $\mathbb{R}^L_{++}$.

Pure Exchange Economy

Exercise 1.3

Imagine an exchange economy composed of two individuals, A and B. The preferences of these individuals are represented by the following utility functions:

$$u_A(x_A, y_A) = x_A y_A^{1/2}$$
$$u_B(x_B, y_B) = x_B^{1/2} y_B.$$

The endowments are $\omega_1 = (100, 0)$ and $\omega_2 = (0, 150)$.

- Find and characterize the Pareto set.
- Compute the Walrasian equilibrium of this economy given the initial endowments indicated in the statement. Show that the allocation found belongs to the Pareto set. Link this with the 1st Welfare Theorem.
- Choose any other point in the Pareto set, and indicate a way to reach it through competitive equilibrium by proposing transfers between the individuals that make it possible. Link this with the 2nd Welfare Theorem.

To find the P.O. allocations, we can use the result $MRS_A = MRS_B$ since the utility functions are C^1 and satisfy Inada's condition (interior solution). We obtain

$$\begin{aligned}
\frac{u_{x_A}}{u_{y_A}} &= \frac{u_{x_B}}{u_{y_B}} \\
\frac{2y_A}{x_A} &= \frac{y_B}{2x_B} \\
\frac{2y_A}{x_A} &= \frac{150 - y_B}{2(100 - x_B)} \\
y_A &= \frac{150x_A}{400 - 3x_A}.
\end{aligned}$$

Then, the demands are (identifying the Cobb-Douglas structure)

$$\begin{aligned}
x_A^* &= \frac{2}{3}\frac{100p_x}{p_x} \\
y_A^* &= \frac{1}{3}\frac{100p_x}{p_y} \\
x_B^* &= \frac{1}{3}\frac{150p_y}{p_x} \\
y_B^* &= \frac{2}{3}\frac{150p_y}{p_y}.
\end{aligned}$$

Applying Walras' Law:

$$\frac{200}{3} + 50\frac{p_y}{p_x} = 100 \implies \frac{p_y}{p_x} = \frac{2}{3}.$$

Substituting into the demands:

$$x_A^* = \frac{200}{3}$$
$$y_A^* = 50$$
$$x_B^* = \frac{100}{3}$$
$$y_B^* = 100.$$

We easily observe that this belongs to the Pareto set, thus verifying the 1st Welfare Theorem (1WT): the preferences satisfy the hypotheses of the 1WT; monotonicity implies locally non-satiated: take $\mathbf{y} = \mathbf{x} + \frac{\epsilon}{2L}\mathbf{1}$.

Consider, for example, $x_A = 50 \implies y_A = 30$, a Pareto-efficient allocation. The goal is to determine transfers so that this allocation becomes the new Walrasian equilibrium (E.W.). We have $x_B = 50$ and consequently $y_B = 120$.

$$\begin{aligned} T_A &= p_x \Delta x_A + p_y \Delta y_A \\ &= \left(50 - \frac{200}{3}\right) \cdot \underbrace{1} = p_x + \underbrace{\frac{2}{3}} = p_y(30 - 50) \\ &= -30 \quad T_B \qquad\qquad = 30. \end{aligned}$$

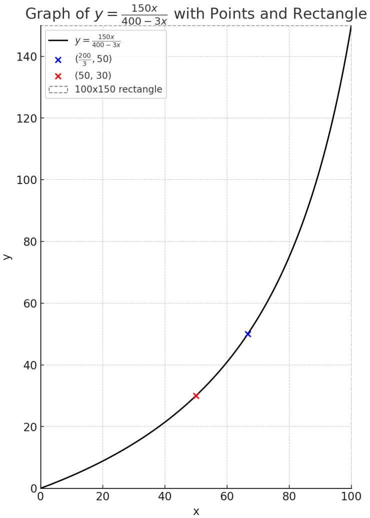


Figura Pareto set and key allocations.

Robinson Crusoe

Exercise 2.1.

Consider a Robinson Crusoe economy where

$$u(\ell_o, c) = \ell_o^2 c$$
$$f(\ell_t) = \sqrt{\ell_t}$$
$$\bar{\ell} = 10.$$

ℓ_t denotes the hours worked and ℓ_0 the leisure hours.

1. Solve the problem in a centralized manner.
2. Solve the problem from the market approach.

The centralized problem is the following:

$$\begin{cases} \max & u(\ell_o, c) = \ell_o^2 c \\ \text{s.t.} & c = f(\ell_t) = \sqrt{\ell_t} \\ & \ell_t + \ell_o = \underbrace{\bar{\ell}} = 10 \\ & \ell_t, \ell_o, c \geq 0. \end{cases}$$

It is easy to see that in the optimum, $\ell_o, \ell_t, c > 0$. Therefore, we apply the FOCs to

$$L(\ell_o, c, \lambda) = \underbrace{\ell_o^2 c} = u(\ell_o, c) + \lambda \left(\underbrace{f(\ell_t)}_{=f(\bar{\ell}-\ell_o)} - c \right).$$

Applying the chain rule, we obtain (thanks Paolo Jove for pointing out a typo here which has been already corrected):

$$\underbrace{\frac{2\ell_o c}{\ell_o^2}}_{\partial_\ell u/\partial_c u} = -\frac{d}{d\ell_o}(f(\bar{\ell} - \ell_o)) = f'(\ell_t) = \frac{1}{2\sqrt{\bar{\ell} - \ell_o}}.$$

Taking into account that $c = f(\bar{\ell} - \ell_o) = \sqrt{10 - \ell_o}$, we solve for ℓ_o

$$\begin{aligned}
\frac{2\ell_o\sqrt{10 - \ell_o}}{\ell_o^2} &= \frac{1}{2\sqrt{10 - \ell_o}} \\
\frac{2(10 - \ell_o)}{\ell_o} &= \frac{1}{2}\ 4(10 - \ell_o) \\
&= \ell_o\ 40 \\
&= 5\ell_o \\
\ell_o &= 8.
\end{aligned}$$

Thus, the solution to the centralized problem is: $\ell_t = 2$, $\ell_o = 8$, and $c^* = \sqrt{2}$.

Now we solve the problem from the market approach. In this formulation, we have on one side the firm and the consumer. The firm solves:

$$\max \left\{ \underbrace{pc - w\ell_t}_{=\text{profits}} \right\}$$
$$c = f(\ell_t)$$
$$\ell_t, c \geq 0.$$

Incorporating the technology constraint:

$$\max_{\ell_t \geq 0} \left\{ p\sqrt{\ell_t} - w\ell_t \right\}.$$

Applying first-order conditions (since the function is strictly concave over $\mathbb{R}+$), we obtain:

$$\underbrace{\frac{p}{2\sqrt{\ell_t}} - w = 0}_{f'(\ell_t^d) = w/p}.$$

Solving for ℓ_t:

$$\ell_t^d = \frac{p^2}{4w^2}.$$

Thus, $c_o = \frac{p}{2w}$ and $\Pi^* = \frac{p^2}{4w}$.

Incorporating this information into the consumer's problem, we arrive to:

$$\begin{cases} \max & u(\ell_o, c) = \ell_o^2 c \\ \text{s.t.} & pc + w\ell_o = \underbrace{w\bar{\ell} + \Pi^*}_{=10w+\frac{p^2}{4w}} \\ & 0 \leq \ell_o \leq \bar{\ell} \\ & 0 \leq c. \end{cases} \tag{1}$$

Because of the Cobb-Douglas structure, it is clear that

$$\ell_o^d = \frac{20}{3} + \frac{p^2}{6w^2}.$$

Hence, donde $\ell_o^d + \ell_t^d = \bar{\ell}$

$$\frac{20}{3} + \frac{1}{6}\left(\frac{p}{w}\right)^2 + \frac{1}{4}\left(\frac{p}{w}\right)^2 = 10.$$

It follows that $\ell_o = 8$, $\ell_t^d = 2$, and $c = \sqrt{2}$. As expected, the solutions coincide.

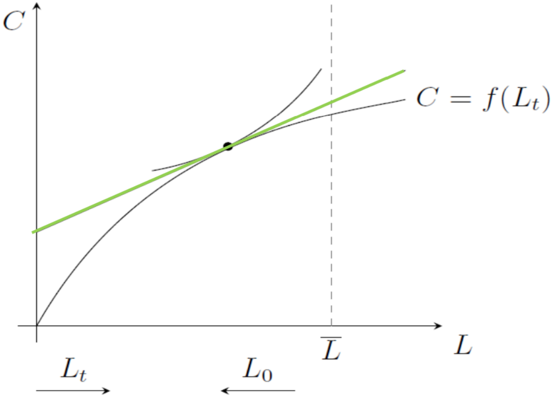


Figura Robinson Crusoe.

Private Ownership Economy

Consider an economy with two goods, two consumers (Obi-Wan and Palpatine), and a firm (Sereno). Obi-Wan has preferences represented by $u_1(x_1, y_1) = \sqrt{x_1 y_1}$, with an initial endowment $\omega_1 = (1, 0)$ and $\theta_1 = 0.3$. Palpatine has quasilinear preferences $u_2(x_2, y_2) = x_2 + \ln(y_2)$, with an initial endowment $\omega_2 = (2, 0)$ and $\theta_2 = 0.7$. On the other hand, the firm's technology is

$$Y = \left\{(x, y) \in \mathbb{R}^2 : x \leq 0, y \leq \frac{Ax}{x-1}\right\}$$

where $A > 0$ is a productivity factor.

1. Find the supply function of Sereno.
2. Find the demand correspondence for Obi-Wan and Palpatine. Obtain the Walrasian equilibrium.
3. Study the effect of the productivity factor A on the price ratio.

Preliminaries: production set

1. Technology $Y \subset \mathbb{R}^L$.
2. $y = (-1, 0, 2, 3, -4) \in Y$.
3. Convex, closed, compact, free-disposal.

Example

Let $Y \subset \mathbb{R}^n$ be a technology. We will say that the technology exhibits non-increasing returns to scale if: $\forall\ \mathbf{y} \in Y$, $\alpha \mathbf{y} \in Y$, $\forall\ \alpha \in [0,1]$. On the other hand, we will say that the technology is additive if given $\mathbf{y}, \mathbf{y}' \in Y$, $\mathbf{y} + \mathbf{y}' \in Y$. Prove that a technology exhibits non-increasing returns to scale and is additive if and only if it is a convex cone.

Example

It is said that a technology $Y \subset \mathbb{R}^L$ has the property of free disposal if given $\mathbf{y} \in Y$ and $\mathbf{y}' \leq \mathbf{y}$, then $\mathbf{y}' \in Y$. Prove that if a technology is closed (i.e., Y is a closed set), convex, and such that $-\mathbb{R}^L_+ \subset Y$, then it satisfies the property of free disposal.

As usual, we start with firm's maximization problem

$$\max_{(x,y)\in Y} \{p_x x + p_y y\}.$$

1. Solution lies in ∂Y: **why?**
2. Problem becomes:

$$\max \left\{ \underbrace{p_x x + \frac{A p_y x}{x-1}}_{=\Pi} \right\}$$

$$\text{s.a. } x \leq 0.$$

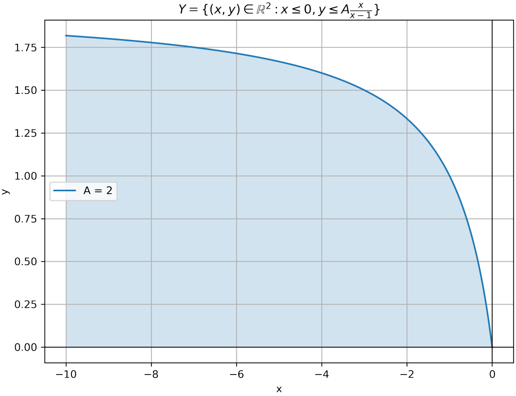


Figura Technology.

$f''(x) = \frac{2}{(x-1)^3} < 0$, for $x \leq 0$, being $f(x) = \frac{Ax}{x-1}$. Thus

1. Unique solution.
2. In the boundary of the technology.

We solve the maximization problem via KKT

$$\mathscr{L}(x, \lambda) = p_x x + \frac{Ap_y x}{x-1} + \lambda(-x).$$

FOC provide

$$x^d = \begin{cases} 0, & \text{si } \sqrt{\frac{Ap_y}{p_x}} \leq 1 \\ 1 - \sqrt{\frac{Ap_y}{p_x}}, & \text{si } \sqrt{\frac{Ap_y}{p_x}} > 1. \end{cases}$$

If $x^d = 0$, certainly y^O=0.

Otherwise,

$$
\begin{aligned}
y^O &= \frac{Ax^d}{x^d - 1} \\
&= \frac{A\left(1 - \sqrt{\frac{Ap_y}{p_x}}\right)}{1 - \sqrt{\frac{Ap_y}{p_x}} - 1} \\
&= A - \sqrt{\frac{Ap_x}{p_y}}.
\end{aligned}
$$

Note that

$$
A - \sqrt{\frac{Ap_x}{p_y}} > 0
$$

when $\sqrt{\frac{Ap_y}{p_x}} > 1$. Therefore,

$$
y^O = \begin{cases} 0, & \text{si } \sqrt{\frac{Ap_y}{p_x}} \leq 1 \\ A - \sqrt{\frac{Ap_x}{p_y}}, & \text{si } \sqrt{\frac{Ap_y}{p_x}} > 1. \end{cases}
$$

If $\sqrt{\frac{Ap_y}{p_x}} > 1$:

$$\begin{aligned}
\Pi &= \underbrace{p_y y^O + p_x x^d}_{I-C} \\
&= p_x\left(1 - \sqrt{\frac{Ap_y}{p_x}}\right) + p_y\left(A - \sqrt{\frac{Ap_x}{p_y}}\right) \\
&= Ap_y + p_x - 2\sqrt{Ap_x p_y}.
\end{aligned}$$

Thus,

$$\Pi = \begin{cases} 0, & \text{si } \sqrt{\frac{Ap_y}{p_x}} \leq 1 \\ Ap_y + p_x - 2\sqrt{Ap_x p_y}, & \text{si } \sqrt{\frac{Ap_y}{p_x}} > 1. \end{cases}$$

With respect to consumers, they solve

$$\begin{aligned} \max \ & u^i(x) \\ s.a.\ & px \leq p\omega_i + \sum_{j=1}^{J} \theta_{ij} \underbrace{p \cdot y^j(p)}_{\Pi_j^*} \\ & x \geq 0 \end{aligned}$$

In this case, adjusting the notation

$$\begin{aligned} \max \ & \sqrt{x_1 y_1} \\ s.a.\ & p_x x_1 + p_y y_1 \leq p_x + 0.3\Pi \\ & x_1, y_1 \geq 0 \end{aligned}$$

and

$$\begin{aligned} \max \ & x_2 + \ln y_2 \\ s.a.\ & p_x x_2 + p_y y_2 \leq 2p_x + 0.7\Pi \\ & x_2, y_2 \geq 0 \end{aligned}$$

We easily compute

$$x_1^d = \frac{p_x + 0.3\Pi}{2p_x}$$
$$y_1^d = \frac{p_x + 0.3\Pi}{2p_y}.$$

On the other hand,

$$y_2^d = \frac{p_x}{p_y}$$

$$x_2^d = 1 + \frac{0.7\Pi}{p_x}.$$

Definition

In a POE, the excess of demand function is

$$Z(p) = \sum_{i=1}^{I} x_i(p) - \sum_{j=1}^{J} y_j(p) - \sum_{i=1}^{I} \omega^i.$$

Therefore, using this definition:

$$\begin{aligned} Z(p_x, p_y) &= \begin{bmatrix} x_1^d(p_x, p_y) + x_2^d(p_x, p_y) - x^d - 3 \\ y_1^d(p_x, p_y) + y_2^d(p_x, p_y) - y^O \end{bmatrix} \\ &= \begin{bmatrix} \frac{p_x + 0.3\Pi}{2p_x} + 1 + \frac{0.7\Pi}{p_x} - \left(1 - \sqrt{\frac{Ap_y}{p_x}}\right) - 3 \\ \frac{p_x + 0.3\Pi}{2p_y} + \frac{p_x}{p_y} - A + \sqrt{\frac{Ap_x}{p_y}} \end{bmatrix}. \end{aligned}$$

If you assume $\Pi = 0$, we fall into a contradiction. Indeed, we get

$$\underbrace{\frac{1}{2} + 1 - 1 + \sqrt{\frac{Ap_y}{p_x}}}_{\leq 1} - 3 = 0.$$

Hence, we use $\Pi = Ap_y + p_x - 2\sqrt{Ap_x p_y}$. Walras law is (again) satisfied:

$$\begin{aligned}
p \cdot Z(p) &= p_x\left(\frac{p_x + 0.3\Pi}{2p_x}\right) + p_x\left(1 + \frac{0.7\Pi}{p_x}\right) - 3p_x - p_x\left(1 - \sqrt{\frac{Ap_y}{p_x}}\right) \\
&+ p_y\left(\frac{p_x + 0.3\Pi}{2p_y}\right) + p_y\left(\frac{p_x}{p_y}\right) - p_y\left(A - \sqrt{\frac{Ap_x}{p_y}}\right) \\
&= \frac{p_x + 0.3\Pi}{2} + p_x + 0.7\Pi - 3p_x - p_x + \sqrt{Ap_y p_x} + \frac{p_x + 0.3\Pi}{2} + p_x - Ap_y + \sqrt{Ap_x p_y} \\
&= p_x + \Pi + p_x - 3p_x - p_x + 2\sqrt{Ap_x p_y} + p_x - Ap_y \\
&= -p_x - Ap_y + 2\sqrt{Ap_x p_y} + \Pi \\
&= 0.
\end{aligned}$$

Therefore, we only need to equilibrate one market. Doing so, we find

$$Z_1(p_x, p_y) = \frac{p_x + 0.3\Pi}{2p_x} + 1 + \frac{0.7\Pi}{p_x} - \left(1 - \sqrt{\frac{Ap_y}{p_x}}\right) - 3 = 0.$$

Numerical computation leads to $p_y \simeq \frac{3.47673}{A} > 0$. Finally,

$$\frac{\partial}{\partial A}\left(\frac{p_y}{p_x}\right) = -\frac{C}{A^2} < 0.$$

This means that the price ratio falls as A increases. This makes sense because, if the technology is more productive, the good y becomes **relatively cheaper**.

What is next?

1. Power market (monopoly).
2. Externalities and public goods.
3. Uncertainty.
4. Adverse selection,.
5. Moral hazard.