

## Recitation 5

Microeconomics 2  
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### 1 Monopoly

**Exercise 1.1.** An economy has two types of consumers and two goods. The agent type A has the following utility function:

$$u_A(x_{1A}, x_{2A}) = 4x_{1A} - \frac{x_{1A}^2}{2} + x_{2A}$$

and the agent type B has the following utility function:

$$u_B(x_{1B}, x_{2B}) = 3x_{1B} - \frac{x_{1B}^2}{2} + x_{2B}.$$

Good 2 is the numeraire, and each consumer has an income of 100. Additionally, the economy has  $N$  consumers of both type A and type B.

1. Identify the type of consumer with high demand and the type with low demand for good  $x_1$ . Compare the marginal willingness to pay for each type of consumer for good  $x_1$ .
2. The monopolist produces good 1 with the following cost function  $C(x_1) = cx_1$  and cannot discriminate prices. Find the optimal price and quantity of good  $x_1$  that the monopolist will choose. For which values of  $c$  will the monopolist choose to sell to both types of consumers?
3. The monopolist engages in second-degree price discrimination by offering a menu of prices and quantities to each type of consumer  $(r_A, x_A)$  and  $(r_B, x_B)$ . Based on this, formulate the monopolist's optimization problem and find the optimal values  $(r_A^*, x_A^*)$  and  $(r_B^*, x_B^*)$ .

4. If the monopolist engages in third-degree price discrimination, what will be the prices and quantities set by the monopolist in the markets for A-type and B-type consumers?
5. If the monopolist engages in first-degree price discrimination, find the quantity produced by the monopolist in the market for good  $x$ . Calculate the consumer surplus and the monopolist's surplus.

**Exercise 1.2.** From [Tirole \(1994\)](#). Consider  $q(p) = p^{-\epsilon}$  and assume constant marginal cost. Prove that the social welfare in competitive equilibrium is

$$\mathcal{W}^s = \frac{c^{1-\epsilon}}{\epsilon - 1}.$$

Then, compute the welfare loss. *Hint:* recall that the total surplus is in the competitive case

$$\int_{p=c'}^{\infty} q(s) ds.$$

So, in this case,  $\int_{p=c}^{\infty} s^{-\epsilon} ds = \frac{c^{1-\epsilon}}{\epsilon-1}$ . For the monopolist case, apply FOC

$$p^m = \frac{c}{1 - \frac{1}{\epsilon}}.$$

Thus, you can conclude that

$$\mathcal{W}^s - \mathcal{W}^m = \left( \frac{c^{1-\epsilon}}{\epsilon - 1} \right) \left[ 1 - \left( \frac{2\epsilon - 1}{\epsilon - 1} \right) \left( \frac{\epsilon}{\epsilon - 1} \right)^{-\epsilon} \right].$$

## 2 Externalities

**Exercise 2.1.** Consider an economy where there are two goods  $(y_1, y_2)$ , two consumers, and a firm. Each consumer has an initial endowment of four units of good 1 and nothing of good 2. Good  $y_1$  is not producible, and good  $y_2$  is produced by the firm using good 1 as input (the quantities of good 1 not directly consumed by individuals) based on the following production function:

$$y_2 = f(z) = z_1^{1/2},$$

where  $y_2$  is the quantity of good 2 produced and  $z_1$  is the amount of good 1 used as input. The profits earned by the firm are equally distributed between the two consumers,  $\theta_j = 1/2$ . Both consumers derive utility from the consumption of the two goods. However, the production of good 2 generates noise and pollution, which negatively affects their well-being. As a result, the utility function of the consumers is given by the following expression:

$$u_i(y_{i1}, y_{i2}, y_2) = y_{i1} + \ln y_{i2} - \frac{1}{2} \ln y_2, \quad i = 1, 2.$$

The superscript refers to the consumer.

- a) Calculate the quantities of good 2 produced and consumed by the two individuals in the general equilibrium, assuming the price of good 1 as the numéraire, whether used as a consumption good or as an input ( $p_1 = w_1 = 1$ ).

- b) Calculate the quantities of good 2 produced and consumed by the two individuals in the efficient allocation, and comment on the results in comparison to the previous part.

**Exercise 2.2.** The company  $S$  produces a certain amount of steel ( $s$ ) and a certain amount of pollution ( $x$ ), which is discharged into a river. The company  $F$  is a fish farm located downstream and is negatively affected by the pollution from company  $S$ . Suppose that the cost function of  $S$  involves both  $s$  and  $x$ . Meanwhile, the company  $F$  depends on  $f$ , representing the collection of fish, and  $x$ , which represents the production of pollution. Additionally, it must be considered that pollution increases the cost of fish production and reduces the cost of steel production.

- a) Formulate the profit maximization problem for both companies.
- b) What are the conditions that characterize profit maximization? Remember that polluting has no price.
- c) How would the efficient production plan of steel and fish in the Pareto sense look? What are the implications of this new scenario for pollution production?

Lima, October 7, 2024.

## References

Tirole, J. (1994). *The Theory of Industrial Organization*. MIT University Press.