FACULTY OF SOCIAL SCIENCES
ECONOMICS MAJOR

Exercises Session 2

Microeconomics 2
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Some exercises have been extracted and adapted from Professor Alejandro Lugón's material.

1 2×2 Economy

Exercise 1.1. Suppose that in a 2×2 economy consumer i has Cobb-Douglas preferences $u_i(x_{1i}, x_{2i}) = x_{1i}^{\alpha} x_{2i}^{1-\alpha}$. Furthermore, assume that endowments are $\omega_1 = (1, 2)$ and $\omega_2 = (2, 1)$. Find Pareto optimal assignments and the (a)[1] Walrasian equilibrium.

Exercise 1.2. In each of the following cases, draw the Edgeworth box, some indifference curves for each consumer, the Pareto set and the core (contract curve). Finally, find Walrasian (competitive) equilibrium in each case.

a) $u_1(x_{11}, x_{21}) = 2x_{11}^2 x_{21}$, $u_2(x_{12}, x_{22}) = x_{12} x_{22}^3$, $\omega_1 = (2, 3)$ and $\omega_2 = (1, 2)$.

b) $u_1(x_{11}, x_{21}) = 2x_{11} + x_{21}$, $u_2(x_{12}, x_{22}) = x_{12} x_{22}^3$, $\omega_1 = (2, 3)$ and $\omega_2 = (1, 2)$.

c) $u_1(x_{11}, x_{21}) = x_{11} + \ln x_{21}$, $u_2(x_{12}, x_{22}) = x_{12} + 2\ln x_{22}$, $\omega_1 = (2, 3)$ and $\omega_2 = (1, 2)$.

d) $u_1(x_{11}, x_{21}) = x_{11} x_{21}$, $u_2(x_{12}, x_{22}) = \min\{x_{12}, x_{22}\}$, $\omega_1 = (2, 6)$ and $\omega_2 = (4, 1)$.

e) $u_1(x_{11}, x_{21}) = \min\{2x_{11}, x_{21}\}$, $u_2(x_{12}, x_{22}) = \min\{x_{12}, 2x_{22}\}$, $\omega_1 = (1, 2)$ and $\omega_2 = (3, 4)$.

Identify whenever it is possible the type (Cobb-Douglas, CES, Leontief, linear...) of the utility function.

[1] We don't know if it is unique or no! However, under some conditions over the preferences, which are satisfied in this exercise, existence is ensured.

Exercise 1.3. From Mas-Colell et al. (1995). Consider a 2×2 economy in which consumers preferences are monotonic. Prove that ($\omega_\ell = \omega_{1\ell} + \omega_{2\ell}$)

$$p_1 \left(\sum_{i=1}^{2} x_{1i}(p_1, p_2) - \omega_1 \right) + p_2 \left(\sum_{i=1}^{2} x_{2i}(p_1, p_2) - \omega_2 \right) = 0.$$

Exercise 1.4. From Mas-Colell et al. (1995). Consider and Edgeworth box economy in which each consumer has Cobb-Douglas preferences

$$u_1(x_{11}, x_{21}) = x_{11}^{\alpha} x_{21}^{1-\alpha}$$
$$u_2(x_{12}, x_{22}) = x_{12}^{\beta} x_{22}^{1-\beta},$$

with $\alpha, \beta \in (0, 1)$. Consider endowments $(\omega_{1i}, \omega_{2i}) > 0$ for $i = 1, 2$. Solve for the equilibrium price ratio and allocation.

Exercise 1.5. Under some conditions over the preferences, in a 2×2 economy, every Pareto Optimal allocation can be characterized as the solution of the following maximization problem

$$\begin{aligned} &\max\ u_1(x_1) \\ &\text{s. t. } u_2(x_2) \geq k \\ &\qquad x_1 + x_2 = \omega_1 + \omega_2, \end{aligned}$$

where $k \in \mathbb{R}$. Find the aforementioned conditions over the preferences.

Exercise 1.6. There are two consumers, A and B, with the following utility functions,

$$u_A(x_A^1, x_A^2) = a \ln x_A^1 + (1 - a) \ln x_A^2,\ \omega_1 = (0, 1)$$
$$u_B(x_B^1, x_B^2) = \min\{x_B^1, x_B^2\},\ \omega_2 = (1, 0).$$

Compute the prices and quantities that clear the market. Interpret.

Exercise 1.7. From Varian (1992). Consider two individuals in a pure exchange economy whose indirect utilities are

$$v_1(p_1, p_2, w) = \ln w - a \ln p_1 - (1 - a) \ln p_2$$
$$v_2(p_1, p_2, w) = \ln w - b \ln p_1 - (1 - b) \ln p_2$$

Endowments are $\omega_1 = (1, 1)$ and $\omega_2 = (1, 1)$. Obtain the prices that clean the market.

Exercise 1.8. Prove that if $\succeq$ is monotone, then it is locally non satiated. Here $\succeq$ represents a preference relation over $\mathbb{R}_+^L$.

2 Additional exercises

Exercise 2.1. From Chavez and Gallardo (2024). Consider an economy with N consumers, two goods, and preferences given by

$$u_i(x_{1i}, x_{2i}) = x_{1i}^2 + x_{2i}^2.$$

Endowments are $\omega_i = (1, 1)$. If N is even, find, if it exists, a Walrasian equilibrium. What if N is odd?

Exercise 2.2. Prove 1st Welfare theorem for a 2×2 economy. This is, if preferences are locally non satiated, then, every Walrasian equilibrium is Pareto optimal. Can you generalize this for a pure exchange economy with N consumers and L goods? You can guide yourself from Echenique (2005).

Lima, September 2, 2024.

References

Chavez, J. and Gallardo, M. (2024). *Algebra Lineal y Optimization para el Análisis Económico*. Prepublished.

Echenique, F. (2005). Lecture notes general equilibrium theory.

Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). *Microeconomic Theory*. Oxford University Press, New York.

Varian, H. R. (1992). *Microeconomic Analysis*. W. W. Norton & Company, New York, 3rd edition.