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Discrete Public Goods

Recall that a public good is non-excludable and non-rival. That is, it is not possible to exclude others from consuming it, and its consumption does not reduce the available quantity. Suppose the public good G takes the value 1 if it is provided and 0 otherwise: $G \in 0, 1$.

- $u_i(x_i, G)$ models the preferences of individuals, where G is a discrete public good, and x_i is a numeraire good that can be used for consumption or for the provision of the public good.

- Endowments $w_i \geq 0$. This endowment can be allocated to x_i or to the provision of the public good g_i. Thus, $w_i = x_i + g_i$.

- The cost of the public good is $c \geq 0$. Hence,

$$
G = \begin{cases} 1, & \text{if } \sum_i g_i \geq c \\ 0, & \text{otherwise.} \end{cases}
$$

- In the case of two individuals, the provision of the public good is Pareto efficient if $g_1 + g_2 \geq c$ and $u_i(w_i - g_i, 1) > u_i(w_i, 0)$ for both individuals.

- The reservation price r_i is the maximum amount of the private good that agent i is willing to give up to obtain the public good: $u_i(w_i - r_i, 1) = u_i(w_i, 0)$.

- The public good is then provided if $r_1 + r_2 > g_1 + g_2 \geq c$.

Exercise 1. Let us consider a scenario where three individuals living in a building are considering the construction of an elevator, which costs $C = 2100$ units of money. The individuals have preferences that depend on the money m^{i}[1], and on x, which takes the value of 1 if there is an elevator and 0 if there is not. The initial wealth that each individual has is w^i for $i = 1, 2, 3$.

(a) Suppose that preferences can be represented by the following Stone-Geary type utility functions:
$$U^i(m^i, x) = \sqrt{m^i}(x + i)^{0.5}, \ i = 1, 2, 3.$$

Find the reservation price of each individual. Then, assuming an endowment of 5000 soles, analyze if the public good should be provided. In the general case, show that this will depend on the wealth distribution $\{w_1, w_2, w_3\}$.

(b) Now suppose that preferences can be represented by quasilinear utility functions:
$$U^i(m^i, x) = m^i + 300(i + 1)(x)^{0.5}, \ i = 1, 2, 3.$$

Find the reservation price of each individual. Assess whether the provision of the public good is efficient or not, and show that, unlike the previous case, your answer does not depend on the distribution of wealth.

(c) What is the individual contribution that maximizes the welfare of agent i given the contributions made by the rest of the community members?

(d) Assuming the costs are shared equally, calculate the net value v_i of the elevator installation for each individual. If majority voting is used as the decision mechanism for the provision of the public good, who votes in favor and who votes against? Is the public good installed?

a) The reservation price r_i is the real number such that

$$\underbrace{U_i(w_i - r_i, 1)}_{\text{utility level paying } r_i \text{ and obtaining the public good}} = \underbrace{U_i(w_i, 0)}_{\text{utility level without paying and without public good}}.$$

Hence,
$$\sqrt{w_i - r_i}(1 + i)^{1/2} = \sqrt{w_i} i^{1/2}.$$

Clearing for r_i,
$$r_i = w_i\left(1 - \frac{i}{i+1}\right) = \frac{w_i}{i+1}.$$

Hence, if $w_i = 5000$, $r_i = \frac{5000}{i+1}$ and

$$\sum_{i=1}^{3} r_i = 5000\left(\sum_{i=1}^{3}\frac{1}{i+1}\right) = \frac{16250}{3} > C = 2100.$$

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[1] Which is used to consume other goods.

Hence, the public good should be provided. In general, the condition that must be satisfied is

$$\sum_{i=1}^{3} \frac{w_i}{1+i} > 2100.$$

b) We repeat our analysis now considering quasilinear utility functions:

$$U_i(w_i - r_i, 1) = w_i - r_i + v_i(1) = w_i + \underbrace{v_i(0)}_{=0}.$$

Thus,

$$r_i = v_i(1).$$

In our model, $v_i(1) = 300(i+1)$. Therefore, $r_1 = 600, r_2 = 900$ and $r_3 = 1200$. Once again, $\sum_i r_i = 2700 > 2100$. Nonetheless, note that, in this case,

$$\underbrace{\sum_i r_i = \sum_i v_i(1)}_{\text{do not depend on } w_i}.$$

c) The contributions are

$$g_i = \begin{cases} c - g_{-i} & \text{if } c - g_{-i} \le r_i \\ 0, & \text{otherwise.} \end{cases}$$

Hence, for the quasilinear case

$$g_1 = \begin{cases} 2100 - (g_2 + g_3), & \text{si } 2100 - g_2 - g_3 \le 600 \\ 0 & \text{si } 2100 - g_2 - g_3 > 600. \end{cases}$$

$$g_2 = \begin{cases} 2100 - (g_1 + g_3), & \text{si } 2100 - g_1 - g_3 \le 900 \\ 0 & \text{si } 2100 - g_1 - g_3 > 900. \end{cases}$$

$$g_3 = \begin{cases} 2100 - (g_1 + g_2), & \text{si } 2100 - g_1 - g_2 \le 1200 \\ 0 & \text{si } 2100 - g_1 - g_2 > 1200. \end{cases}$$

d) Under the assumption of equally distributed costs, each individual must pay $C/3 = 2100/3 = 700$. Hence, $v_1 = 600 - 700 = -100$, $v_2 = 900 - 700 = 200$ and $v_3 = 1200 - 700 = 500$. If majority voting is used as the decision mechanism for the provision of the public good, individuals 2 and 3 vote in favor and 1 against. Hence, the public good is actually installed.

Exercise 2. Consider a scenario where three individuals are evaluating the provision of a public good with a cost of $C = 330$ monetary units. The individuals have preferences that depend on the amount of money m^i they have to consume other goods and on G (the public good, which takes the value 0 if it is not provided and 1 otherwise):

$$U^i(m^i, G) = m^i + 50(2^{i-1})\sqrt{G}, \quad i = 1, 2, 3.$$

The wealth of each individual is denoted by w^i, $i = 1, 2, 3$.

a) Find the reservation price of each individual. Evaluate whether the provision of the public good is efficient or not.

b) Suppose that if the public good is provided, the cost is shared equally, $s_i = 1/3$. Find the corresponding net value for each individual.

c) If majority voting is used as the decision mechanism for the provision of the public good, who votes in favor and who votes against? Is the public good provided?

d) Suppose that each agent contributes payments in proportion to how much they value the public good, $s_i = \frac{r_i}{\sum r_i}$. Who votes in favor and who votes against? Is the public good provided? What is the problem with this mechanism?

e) Suppose the financing of the public good is based on equal payments. Assume the Groves-Clarke mechanism is applied, such that the public good is provided if the sum of the net reported values of each individual is greater than zero ($\sum_i \tilde{v}_i \geq 0$), and if the good is provided, side payments are given to each individual equal to the sum of the reported valuations of the others ($\sum_{j \neq i} \tilde{v}_j$). Express the profit function of each individual.

a) Since preferences are represented by quasilinear utility functions,

$$r_i = v_i(1) = 50 \cdot 2^{i-1}.$$

Thus, $r_1 = 50, r_2 = 100$ and $r_3 = 200$. Finally, since

$$\sum_i r_i = 350 > 330 = c,$$

it is optimal to supply the public good.

b) Individual net valuations VN_i are given by $r_i - s_i c$. Hence,

$$VN_i = r_i - \frac{c}{3} = \begin{cases} -60, & \text{if } i = 1 \\ -10, & \text{if } i = 2 \\ 90, & \text{if } i = 3. \end{cases}$$

c) Thus, assuming equal shares and majority voting is used as the decision mechanism for the provision of the public good, the public good is not provided. Indeed, individuals 1 and 2 vote against.

d) Let us assume now that $s_i = \frac{r_i}{\sum_i r_i}$. Then,

$$VN_i = r_i - \frac{r_i}{\sum_i r_i} c = \begin{cases} \frac{20}{7}, & \text{si } i = 1 \\ \frac{40}{7}, & \text{si } i = 2 \\ \frac{80}{7}, & \text{si } i = 3. \end{cases}$$

The issue with this mechanism is that individuals have incentives to declare a lower valuation $\tilde{r}_i < r_i$.

Let us briefly recall **Groves-Clarke mechanism.** Consider the following mechanism: each agent reports their net value for the public good ($\tilde{v}_i$). This value may or may not be their true value (v_i). The public good is provided if $\sum_i \tilde{v}_i \geq 0$, and it is not provided if $\sum_i \tilde{v}_i < 0$. Each agent i receives a **side payment** equal to the sum of the net values reported by the other agents $\sum_{j \neq i} \tilde{v}_j$ if the public good is provided (this payment may have a positive or negative value). The profit of agent i takes the following form:

$$\text{profit}_i = \begin{cases} v_i + \sum_{j \neq i} \tilde{v}_j, & \text{if } \tilde{v}_i + \sum_{j \neq i} \tilde{v}_j \geq 0 \\ 0, & \text{if } \tilde{v}_i + \sum_{j \neq i} \tilde{v}_j < 0. \end{cases}$$

1. If $v_i + \sum_{j \neq i} \tilde{v}_j > 0$, then the agent can ensure that the public good is provided by declaring $\tilde{v}_i = v_i$.

2. If $v_i + \sum_{j \neq i} \tilde{v}_j < 0$, then the agent can ensure that the public good is not provided by declaring $\tilde{v}_i = v_i$.

d) Hence,

$$g_1 = \begin{cases} -60 + \tilde{v}_2 + \tilde{v}_3, & \text{si } \tilde{v}_1 + \tilde{v}_2 + \tilde{v}_3 \geq 0 \\ 0 & \text{si } \tilde{v}_1 + \tilde{v}_2 + \tilde{v}_3 < 0. \end{cases}$$

$$g_2 = \begin{cases} -10 + \tilde{v}_1 + \tilde{v}_3, & \text{si } \tilde{v}_1 + \tilde{v}_2 + \tilde{v}_3 \geq 0 \\ 0 & \text{si } \tilde{v}_1 + \tilde{v}_2 + \tilde{v}_3 < 0. \end{cases}$$

$$g_3 = \begin{cases} 90 + \tilde{v}_1 + \tilde{v}_2, & \text{si } \tilde{v}_1 + \tilde{v}_2 + \tilde{v}_3 \geq 0 \\ 0 & \text{si } \tilde{v}_1 + \tilde{v}_2 + \tilde{v}_3 < 0. \end{cases}$$

Continuous Public Goods

Exercise 3. Let G be a public good and x a private good. The utility of the individuals can be expressed as follows:

$$u^h(G, x^h) = \alpha \ln G + \beta \ln x^h, \ h = 1, 2, \ \alpha, \beta \in (0, 1).$$

The production function for the public good is expressed as:

$$G = f(z) = \theta z, \ \theta > 0.$$

Finally, the endowments are w^h, $h = 1, 2$.

a) Find the Pareto optimal level of provision for the public good G.

b) Determine the optimal level of production for the public good. Assume that the price of the good is equal to 1.

c) Will the provision of the public good be efficient in competitive equilibrium? Justify your answer.

d) Does the provision of the public good change when a Lindahl tax is introduced? How does this result compare to what is described in part (c)?

Continuous Public Goods: Pareto Efficient Solution. Let G be a continuous public good and x a private good. Suppose that preferences are represented by continuous and differentiable utility functions $u^h(G, x^h)$, $h = 1, \ldots, n$, and that individuals possess an endowment w^h. Denote by x^h what is consumed and z^h whaty is allocated to the *production* of G. To obtain the levels of production of the optimal good that are Pareto efficient, the following problem is solved:

$$
\begin{cases}
\max_{G, \{x^h\}_{h=1}^n} & \sum_{h=1}^n \alpha_h u^h(G, x^h) \\
\text{s.t.:} & \sum_{h=1}^n x^h + \sum_{h=1}^n z^h \le \sum_{h=1}^n w^h \\
& f\left(\sum_{h=1}^n z^h\right) = G.
\end{cases}
$$

This yields the Samuelson-Lindahl condition:

$$\sum_{h=1}^n \frac{\partial u^h / \partial G}{\partial u^h / \partial x^h} = \sum_{h=1}^n MRTS_h = \frac{1}{f'(\underbrace{z}_{=\sum_{h=1}^n z^h})}$$

Proof.

$$\mathcal{L}(\{x^h\}_{h=1}^n, \{z^h\}_{h=1}^n, \mu) = \sum_{h=1}^n \alpha_h u^h\left(f\left(\sum_{h=1}^n z^h\right), x^h\right) + \mu\left(\sum_{h=1}^n w^h - \sum_{h=1}^n x^h - \sum_{h=1}^n z^h\right).$$

By First-Order Conditions,

$$\frac{\partial \mathscr{L}}{\partial x^h} = \alpha^h \frac{\partial u^h}{\partial x^h} - \mu = 0$$

$$\frac{\partial \mathscr{L}}{\partial z^h} = \sum_{j=1}^{n} \alpha_j \frac{\partial u^j}{\partial G} \frac{\partial f}{\partial z^h} - \mu = 0.$$

$$\alpha^1 \frac{\partial u^1}{\partial x^1} = \alpha^2 \frac{\partial u^2}{\partial x^2} = \cdots = \alpha^n \frac{\partial u^n}{\partial x^n}.$$

$$\frac{\partial f}{\partial z^1} \sum_{j=1}^{n} \alpha_j \frac{\partial u^j}{\partial G} = \frac{\partial f}{\partial z^2} \sum_{j=1}^{n} \alpha_j \frac{\partial u^j}{\partial G} = \cdots = \frac{\partial f}{\partial z^n} \sum_{j=1}^{n} \alpha_j \frac{\partial u^j}{\partial G}.$$

That is,

$$\frac{\partial f}{\partial z^1} = \frac{\partial f}{\partial z^2} = \cdots = \frac{\partial f}{\partial z^n} = f'(\cdot)$$

Also,

$$\alpha_1 \frac{\partial u^1}{\partial G} \frac{\partial f}{\partial z^1} + \cdots + \alpha_1 \frac{\partial u^n}{\partial G} \frac{\partial f}{\partial z^n} = \mu$$

$$\frac{\alpha_1 \frac{\partial u^1}{\partial G} \frac{\partial f}{\partial z^1}}{\mu} + \cdots + \frac{\alpha_n \frac{\partial u^n}{\partial G} \frac{\partial f}{\partial z^n}}{\mu} = \frac{\mu}{\mu}$$

$$\frac{\alpha_1 \frac{\partial u^1}{\partial G} \frac{\partial f}{\partial z^1}}{\alpha_1 \frac{\partial u^1}{\partial x^1}} + \cdots + \frac{\alpha_n \frac{\partial u^n}{\partial G} \frac{\partial f}{\partial z^n}}{\alpha_n \frac{\partial u^n}{\partial x^n}} = 1$$

$$\sum_{h=1}^{n} \frac{\partial u^h / \partial G}{\partial u^h / \partial x^h} = \frac{1}{f'(\cdot)}$$

$$\sum_{h=1}^{n} MRTS_h = \frac{1}{f'(\cdot)}.$$

$\square$

a, b) We have

$$u^h(G, x^h) = \alpha \ln G + \beta \ln x^h, \quad h = 1,2, \quad \alpha, \beta \in (0,1)].$$

Also, consider $f(z) = z$. We have that

$$MRTS_h = \frac{\alpha / G}{\beta / x^h} = \frac{\alpha x^h}{\beta G}.$$

Thus,

$$\frac{\alpha x^1}{\beta G} + \frac{\alpha x^2}{\beta G} = \frac{1}{f'(z)} = \frac{1}{\theta}.$$

That is, $x^1 + x^2 = \frac{\beta G}{\theta \alpha}$. Using the fact that

$$\underbrace{x^1 + x^2}_{=\beta G/\theta\alpha} + \underbrace{z^1 + z^2}_{=G/\theta} = w^1 + w^2.$$

Thus,

$$G^* = \frac{\theta\alpha(w^1 + w^2)}{\beta + \alpha}. \tag{1}$$

Thus, (1) is the Pareto optimal solution.

c) With respect to the market perspective, we adress the following problem (note the similarity with Private Ownership Economies)

1. First, we solve

$$\max_z pf(z) - z \implies p = \frac{1}{f'(z)}.$$

2. Then, we solve

$$\begin{cases} \max & u^h(\underbrace{g^h + G^{-h}}_{=G}, x^h) \\ s.a. & x^h + pg^h = w^h \end{cases}$$

for $h = 1, ..., n$.

3. Finally, we do $\sum_i x_i + g_i = \sum_i w_i$.

Thus, FOC lead to (when the production function is linear, we can conclude that $p\theta = 1$, so $p = \theta^{-1}$)[2]

$$TMS_h = \frac{\frac{\alpha}{g^h + G^{-h}}}{\beta/x_h} = \frac{\frac{\alpha}{G}}{\beta/x_h} = p = \frac{1}{f'(z)} = \frac{1}{\theta} \quad h = 1, 2, .$$

Hence

$$\frac{1}{\theta} = \frac{\frac{\alpha}{G}}{\beta/x_1} = \frac{\frac{\alpha}{G}}{\beta/x_2},$$

$$\frac{\alpha x_1}{\beta G} = \frac{\alpha x_2}{\beta G} = \frac{1}{\theta} \implies x_1 + x_2 = \frac{2\beta G}{\theta\alpha}$$

Applying the third condition,

$$\underbrace{(x_1 + g_1) + (x_2 + g_2)}_{x_1 + x_2 + G} = w_1 + w_2,$$

we have

$$\frac{2\beta G}{\theta\alpha} + G = \frac{2\beta + \theta\alpha}{\alpha}G = w_1 + w_2.$$

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[2] Otherwise, there is no market equilibrium.

Clearing G,

$$G = \frac{\theta\alpha(w_1 + w_2)}{2\beta + \theta\alpha}.$$

Note that this is less than the Pareto Optimal solution[3].

d) **Continuous Public Goods: Lindahl Taxes or Equilibrium.** We introduce weighting coefficients λ^h such that $g^h = \lambda^h G$ and $\sum_h \lambda^h = 1$. Thus, for each $h = 1, \ldots, n$, one solves:

$$\begin{cases} \max & u^h(\underbrace{g^h + G^{-h}}_{=G}, x^h) \\ \text{s.t.} & x^h + p\lambda^h G = w^h. \end{cases}$$

From there, we have:

$$MRTS_h = \frac{\partial u^h/\partial G}{\partial u^h/\partial x^h} = p\lambda^h.$$

Summing over all h:

$$\sum_h MRTS_h = \sum_h p\lambda^h = p = \frac{1}{f'(z)}.$$

Thus, we recover the Pareto optimality condition.

Let us conclude by applying this last to our example. We will have

$$\frac{\alpha x_1}{\beta G} + \frac{\alpha x_2}{\beta G} = \frac{1}{\theta}$$

and

$$x_1 + x_2 = w_1 + w_2 - G.$$

Thus, we recover the Pareto optimal provision level,

$$G^* = \frac{\theta\alpha(w^1 + w^2)}{\beta + \alpha}.$$

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[3]In competitive equilibrium, individuals contribute to the public good based on their own private valuation of its marginal benefit. Since public goods are non-excludable, individuals have an incentive to under-contribute, hoping to *free-ride* on the contributions of others. This leads to less total contribution and a smaller quantity of the public good being provided compared to the socially optimal level.

Optional problems

For game theory concepts, see *Game Theory for Applied Economists* from Robert Gibbons.

Exercise 4. Consider two consumers $i = 1, 2$, each one with an income $w_1 = w_2 = w > 0$ to allocate between two goods. Good 1 provides a unit of consumption to its purchase and $\alpha \in (0, 1)$ units of consumption to the other consumer. Each consumer has a quasilinear utility function $u_i = \ln x_1^i + x_2^i$.

a) Provide an interpretation of the parameter α.

b) Assume that good 2 is a private good. Find the optimal level of consumption assuming that the price of both goods is equal to one.

c) By maximizing the sum of utilities, show that the equilibrium is Pareto efficient if $\alpha = 0$ but inefficient for all other values of α.

d) Assume that good 2 provides 1 unit of consumption to its purchaser and $\alpha \in (0, 1)$ to the other consumer. Obtain the Nash equilibrium and show that it is efficient for all values of α.

Exercise 5. **The Commons Problem.** Consider n farmers in a village. Each summer, the farmers take their livestock to graze. Let g_i denote the number of animals owned by farmer i. Thus, the total number of animals is $G = \sum_{1 \leq i \leq n} g_i$. The cost of maintaining an animal is c and does not depend on the number of animals the farmer already owns. The value of raising an animal when there are G animals grazing is $v(G)$[4]. Given that the animals need a minimum amount of food, there is a maximum number of animals that can coexist: $G_{\max}$. Thus, $v(G) > 0$ for $G < G_{\max}$ and $v(G) = 0$ if $G \geq G_{\max}$. Additionally, since the animals compete for food, we assume $v'(G) < 0$ for $G < G_{\max}$ and $v''(G) < 0$ (adding an animal initially has little impact on the others, but as more animals are added, each additional one has a greater negative impact on the rest). During spring, the farmers decide how many animals to acquire. For simplicity, we assume that this number is perfectly divisible.

- Determine the strategy space for each agent.

- Show that in a Nash equilibrium,

$$v(G^*) + \frac{1}{n} G^* v'(G^*) - c = 0.$$

- Analyze whether the Nash equilibrium solution differs from the socially optimal outcome.

Exercise 6. **Public Goods and Groves Mechanism.** Consider an economy with I consumers, whose utility functions are quasi-linear, given by $u_i = V_i(x, \theta_i) + t_i$, where

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[4]This depends on how much it eats.

t_i is the (monetary) income of the consumer, x is the quantity of a public good, $\theta_i \in \Theta_i$ is a parameter commonly referred to as the *type* of the consumer, and V_i is the consumer's gross surplus that depends on x and their type. The cost of producing x is $C(x)$. Assume that V_i is strictly concave in x, that C is strictly convex and increasing, that both functions are twice differentiable, and that $\partial^2 V_i / \partial x \partial \theta_i > 0$.

1. Argue why the socially efficient decision on the level of production of x solves

$$\max_x \left\{ \sum_{i=1}^{I} V_i(x, \theta_i) - C(x) \right\}. \tag{2}$$

2. Now consider the following *revelation game*: consumers are asked simultaneously to report their type θ_i. They announce $\hat{\theta}_i$, and $x^*(\hat{\theta}_1, \ldots, \hat{\theta}_I)$ is determined by solving (2) for the parameter configuration $\hat{\theta}_1, \ldots, \hat{\theta}_I$[5]. Then, each consumer receives

$$t_i(\hat{\theta}_1, \ldots, \hat{\theta}_I) = \sum_{j \neq i} V_j(x^*(\hat{\theta}_1, \ldots, \hat{\theta}_I), \theta_j) - C(x^*(\hat{\theta}_1, \ldots, \hat{\theta}_I)).$$

   • Model this situation as a static game of complete information. Specifically, determine the players, their strategy spaces, and their utility functions.

   • Show that telling the truth, that is, revealing their true type $\hat{\theta}_i = \theta_i$, is a strictly dominant strategy for each $i = 1, \ldots, I$.

Exercise 7. **From Mas-Colell, Whinston and Green (11.D.7).** A continuum of individuals can build their houses in one of two neighborhoods, A or B. It costs c_A to build a house in neighborhood A and $c_B < c_A$ to build in neighborhood B. Individuals care about the prestige of the people living in their neighborhood. Each individual has a level of prestige, denoted by the parameter θ, where θ is uniformly distributed across the population on the interval $[0, 1]$.

The prestige of neighborhood k (where $k = A, B$) is represented by the average prestige value of individuals in that neighborhood, denoted by θ_k. If an individual i with prestige parameter θ chooses to build their house in neighborhood k, their derived utility, net of building costs, is given by

$$(1 + \theta)(1 + \theta_k) - c_k.$$

Thus, individuals with higher prestige value living in a prestigious neighborhood more. Assume that c_A and c_B are both less than 1 and that the cost difference $(c_A - c_B)$ falls within the interval $\left(\frac{1}{2}, 1 \right)$.

(a) Show that in any building-choice equilibrium (technically, the Nash equilibrium of the simultaneous-move game where individuals simultaneously choose where to build their house), both neighborhoods must be occupied.

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[5]Both $C(\cdot)$ and V_i are public knowledge.

(b) Show that in any equilibrium where the prestige levels of the two neighbor-hoods differ, every resident of neighborhood A must have at least as high a prestige level as every resident of neighborhood B. This implies the existence of a cutoff prestige level $\hat{\theta}$ such that all types $\theta \geq \hat{\theta}$ build in neighborhood A, and all $\theta < \hat{\theta}$ build in neighborhood B. Characterize this cutoff level.

(c) Show that in any equilibrium of the type identified in part (b), a Pareto improvement can be achieved by adjusting the cutoff value θ slightly and allowing transfers between individuals.