FACULTY OF SOCIAL SCIENCES
ECONOMICS MAJOR

Second Test

Microeconomics 2
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- You have 115 minutes.
- Be clear in your solutions. Justify each step.
- Up to 22 points.
- Closed books and all electronic devices are forbidden.
- You may assume any result covered in class.

Exercise 1 (4 points). For items (1) and (2), analyze if the statement is true or not and **justify**. For (1), (3) and (4), give the Pareto optimal allocations as a detailed draw.

1. In a 2×2 economy, if preferences are represented by $u_i(x_{1i}, x_{2i}) = \exp(x_{1i}^2 + x_{2i}^2)$, then the Pareto set does not exist.

2. Alice and Bob's utilities are

$$U^A(x_1^A, x_2^A) = x_1^A, \quad U^B(x_1^B, x_2^B) = x_2^B.$$

   Then, $\mathbf{x} = \{(3,3),(0,0)\}$ is a Pareto optimal allocation.

3. Alice and Bobs' utilities are $u_A(x^A, y^A) = \ln x^A + y^A$ and $u_B(x^B, y^B) = x^B$. Find Pareto optimal allocations.

4. Obtain the contract curve for a 2×2 economy in which

$$\underbrace{u_i(x_i, y_i) = (x_i - 1)^{\beta_1}(y_i - 1)^{\beta_2}}_{\text{Stone-Geary utility function}}, \ \beta_i > 0$$

   and $\boldsymbol{\omega}_1 = (3,2)$, $\boldsymbol{\omega}_2 = (2,3)$.

Exercise 2 (4 points.). Consider a Robinson Crusoe economy where

$$u(\ell_o, c) = \sqrt{\ell_o c}$$
$$f(\ell_t) = \sqrt{\ell_t}$$
$$\overline{\ell} = 24.$$

Remember that $\ell_t + \ell_o = \overline{\ell}$.

1. Solve the problem in a centralized manner. This involves directly substituting the constraints into the optimization problem, all in terms of ℓ_t. Be clear why the solution is or (is not) interior.

2. Solve the problem from a market perspective.

Exercise 3 (6 points). Consider an economy called Sommerville, consisting of two consumers (Carlos and Brik), two goods, and a firm. The agents consume two goods: papers (x) and books (y). However, the agents only have initial endowments of papers, $\boldsymbol{\omega}_1 = (3, 0)$ and $\boldsymbol{\omega}_2 = (2, 0)$ respectively. On the other hand, the only firm produces books with the following technology

$$Y = \{(x, y) \in \mathbb{R}^2 | x \leq 0,\ y \leq \sqrt{-x}\}.$$

Moreover, the preferences of the consumers are represented by $u_1(x_1, y_1) = \sqrt{x_1 y_1}$ and $u_2(x_2, y_2) = 2 \ln x_2 + \ln y_2$, respectively. Shares are $\boldsymbol{\theta} = (\theta_1, \theta_2) = (0.5, 0.5)$.

a) Find the firm's input demand function for papers (x^d), the firm's supply function (y^s), and the profits π^*.

b) Find the demands for goods x and y for each consumer.

c) Find the Walrasian equilibrium, this is, the quantities consumed by each agent for each good, the input quantity used (x), and the firm's production (y).

Exercise 4 (4 points). In the Economics Department at PUCP, the only seller of algorithms (x) , Manuel, faces a demand curve given by $x = a - bp$, where $a, b > 0$ and p is the price per algorithm sold. We assume that an algorithm is a perfectly divisible good, so $x \in \mathbb{R}_+$. Manuel has a quadratic cost function $C(x) = 2x^2 + 10x + \overline{c}$ in the number of algorithms sold ($\overline{c} > 0$ is a parameter).

1. Find the quantity of algorithms that Manuel sells (x^m) and the price at which he sells them (p^m). Remember that Manuel, given the context, operates as a monopolist. Your answer will depend on a and b. For your answer to make sense ($x^m \geq 0$), which is the relation that a and b must satisfy?

2. What happens with x^m and p^m if b increases?

3. If the fixed cost changes to $2\overline{c}$, do any of the previous answers changes? Why?

Exercise 5 (2 points). Give an example of a **weak** Pareto optimal allocation which is not a Pareto optimum. Consider only continuous and monotone preferences.

Viernes económicos (2 points)

a) El Perú es el ________ país líder en el mundo en la emisión de créditos de carbono forestal.

b) COFIDE, al ser el Banco de Desarrollo del Perú, busca financiar proyectos que no tengan solo un impacto ________, sino que también tengan un impacto ________.