FACULTY OF SOCIAL SCIENCES
ECONOMICS MAJOR

Exercises Session 4

Microeconomics 2
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1 Monopoly

We closely follow Tirole (1994).

Exercise 1.1. Consider the classical problem of the monopolist. Prove that, if $q = D(p)$ is the demand for the good produced by the monopolist, then the optimal pricing for the monopolist satisfies

$$p^m - C'(D(p^m)) = -\frac{D(p^m)}{D'(p^m)}$$

where $C = C(q)$ is the cost function of the monopolist.

Exercise 1.2. Let ε be the demand elasticity at p^m. Then, prove that

$$\underbrace{\frac{p^m - C'}{p^m}}_{\text{relative markup}} = \underbrace{\frac{1}{\epsilon}}_{\text{Lerner Index}}.$$

Exercise 1.3. Prove that if $q(p) = kp^{-\epsilon}$ with $k, \epsilon > 0$, then Lerner index is constant.

Exercise 1.4. Prove that the monopoly price is a nondecreasing of the marginal cost $C'(\cdot)$.

Hint: assume that $C_2'(q) > C_1'(q)$ for every q. Denote (p_1^m, q_1^m) and (p_2^m, q_2^m) the optimal pricing-quantities of a monopolist with marginal cost C_1 and C_2 respectively. Then, note that

$$p_1^m q_1^m - C_1(q_1^m) \geq p_2^m q_2^m - C_1(q_2^m) \tag{1}$$

and

$$p_2^m q_2^m - C_2(q_2^m) \geq p_1^m q_1^m - C_2(q_1^m). \tag{2}$$

Add (1) and (1). Then, applying the fundamental theorem of calculus, conclude that

$$\int_{q_2^m}^{q_1^m} (C_2'(q) - C_1'(q))dq.$$

Since $C_2' > C_1'$, $q_1^m > q_2^m$ and therefore $p_2^m > p_1^m$.

Exercise 1.5. Compare the monopolist surplus with the competitive pricing surplus.[1]

Exercise 1.6. Consider $q(p) = p^{-\epsilon}$ and assume constant marginal cost. Prove that the social welfare in competitive equilibrium is

$$\mathcal{W}^s = \frac{c^{1-\epsilon}}{\epsilon - 1}.$$

Then, compute the welfare loss. Recall that the total surplus is in the competitive case

$$\int_{p=c'}^{\infty} q(s)ds.$$

So, in this case, $\int_{p=c}^{\infty} s^{-\epsilon}ds = \frac{c^{1-\epsilon}}{\epsilon-1}$. For the monopolist case, apply FOC

$$p^m = \frac{c}{1 - \frac{1}{\epsilon}}.$$

Thus, you can conclude that

$$\mathcal{W}^s - \mathcal{W}^m = \left(\frac{c^{1-\epsilon}}{\epsilon - 1}\right)\left[1 - \left(\frac{2\epsilon - 1}{\epsilon - 1}\right)\left(\frac{\epsilon}{\epsilon - 1}\right)^{-\epsilon}\right].$$

Exercise 1.7. Consider one possible policy (taxation) in order to restore the social optimum in presence of a monopoly. For this, assume that the government taxes monopoly output at rate $t > 0$.

1. Justify that the maximization problem of the monopolist

$$\max_p\{pD(p+t) - C(D(p+t))\}.$$

2. Prove that $t = \frac{D(p^c)}{D'(p^c)} < 0$.
3. Interpret that $t < 0$.

Exercise 1.8. Let $\tilde{c} = c + t$ be the monopolist marginal cost considering a tax. Let $p^m(\tilde{c})$ the corresponding monopoly price. Prove that $\frac{d\tilde{p}}{dc}$ is equal to

1. $\frac{1}{1-1/\epsilon}$ for $p = q^{-1/\epsilon}$
2. $\frac{1}{1+\delta}$ for $p = \alpha - \beta q^{\delta}$
3. 1 for $p = a - b\ln q$.

[1]Monopolist profits is $p^m q^m - C(q^m)$. Consumer surplus is $\int_0^{q^m} p(q)dq - p^m q^m$. On the other hand, when $p = p^c$, profits are zero and consumer surplus is $\int_0^{q^c} p(q)dq - p^c q^c$.

Exercise 1.9. Prove that if the demand function $D = D(p)$ is concave, then the marginal revenue is decreasing. This is, setting $R(p) = pD(p)$, $R''(p) < 0$.

Exercise 1.10. Multi-product monopolist problem. Consider now a monopolist producing $q_1, \cdots, q_n$ goods. He charges prices $p_1, \cdots, p_n$. Assume that his associated cost function is separable

$$C(q_1, \cdots, q_n) = \sum_{i=1}^{n} C_i(q_i).$$

Finally, let $D_i = D_i(p)$ the demand for good i.

1. Set the monopolist maximization problem.[2]

2. Prove that FOC provide

$$\left(D(p_i) + p_i \frac{\partial D_i}{\partial p_i}\right) + \sum_{j \neq i} p_j \frac{\partial D_j}{\partial p_i} - \sum_{j} \frac{\partial C}{\partial q_i} \frac{\partial D_j}{\partial p_i} = 0, \ \forall\, i.$$

3. Define $\varepsilon_{ii} = \frac{p_i}{D_i} \frac{\partial D_i}{\partial p_i}$ and $\varepsilon_{ij} = -\frac{\partial D_j}{\partial p_i} \frac{p_i}{D_j}$. Prove that

$$\frac{p_i - C_i'}{p_i} = \frac{1}{\varepsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j') D_j \varepsilon_{ij}}{R_i \varepsilon_{ii}}$$

   where $R_i = p_i D_i$.

Exercise 1.11. Consider the problem of the monopolist in two periods

$$p_1 D(p_1) - C(D(p_1)) + \delta(p_2 D(p_2) - C(D(p_2))). \tag{3}$$

Set (3) as multi-product monopolist problem. Find $\frac{\partial D_1}{\partial p_2}$.

Exercise 1.12. Learning by doing. *In some industries, cost reductions are achieved over time simply because of learning. Learning by doing is especially apparent in industrial activity. This is for instance the case of the military aircraft production.* Consider a single-good monopolist producing at dates $t = 1, 2$. Assume that $q_t = D(p_t)$. The total cost at $t = 1$ is $C_1(q_1)$ and at $t = 2$, $C_2(q_1, q_2)$ where $\frac{\partial C_2}{\partial q_1} < 0$ (why?). Set the monopolist maximization problem as in (3), and prove that $p_1^* < p_m$. Interpret.

Exercise 1.13. Assume that a monopolist has a unit-cost function such that $c = c(\omega(t))$ where $\omega(t)$ is the firm's experience at time t.

1. Explain why it is logical to assume that, denoting by $q = q(t)$ the output at time t, $\frac{d\omega}{dt} = q$.

2. Consider the monopolist maximization problem:

$$\begin{aligned} \max_{q(t) \in \mathbb{R}_+} \int_0^\infty & [R(q(t)) - c(\omega(t)) q(t)] e^{-rt} dt \\ \text{s. t. } & w'(t) = q(t) \\ & w(0) = w_0. \end{aligned}$$

[2]Note that $C(q_1, \cdots, q_n) = C(D_1(p_1), \cdots, D_n(p_n))$.

Prove that

$$R'(q(t)) = c(\omega(t)) + \int_t^{\infty} c'(\omega(s))q(s)e^{-(s-t)}ds.$$

Hint: apply the Maximum Principle. Note that the current value Hamiltonian is

$$\mathcal{H}(\omega(t), q(t), \psi(t), t) = R(q(t)) - c(\omega(t))q(t) + \psi(t)q(t).$$

Exercise 1.14. Argue under what circunstantes the cost structure satisfies subadditivity

$$C\left(\sum_{i=1}^{k} q_i\right) < \sum_{i=1}^{k} C(q_i).$$

Exercise 1.15. Analyze the following statements:

1. Monopolies usually have large fixed costs.
2. Monopolist do not exhibit (often) cost subadditivity.
3. The monopolist operates in the inelastic part of the demand, $|\varepsilon| < 1$.

Exercise 1.16. Consider a quasilinear economy with an inverse demand function $p_d(q) = 5 - 0.4q$ and a single firm with a marginal cost $Cmg(q) = q$. The firm behaves as a monopolist. What is the monopoly price?

Exercise 1.17. Suppose that good q is produced by a monopolistic firm in the short run. If the market demand is given by $q^D = 100 - 0.5p$ and the firm's cost curve is $C(q) = 2q^2 + 10q + 4$,

a) Find the quantity produced, the price of the good, and the monopolist's profits.

b) Find the quantity produced, the price of the good, and the monopolist's profits if we consider a new cost curve given by $C(q) = 2q^2 + 10q$.

2 Price discrimination

Exercise 2.1. Describe the constraints of second degree price discrimination:

$$\begin{aligned}
u_1(x_1) &\geq r_1 \\
u_1(x_1) - r_1 &\geq u_1(x_2) - r_2 \\
u_2(x_2) &\geq r_2 \\
u_2(x_2) - r_2 &\geq u_2(x_1) - r_1.
\end{aligned}$$

Recall that the monopolist maximize

$$\Pi = (r_1 - cx_1) + (r_2 - cx_2).$$

Prove that

$$\begin{aligned}
u_1'(x_1) - c + u_1'(x_1) - u_2'(x_1) &= 0 \\
u_2'(x_2) - c &= 0.
\end{aligned}$$

Exercise 2.2. Prove that third degree price discrimination leads to (FOC)

$$p_1 \left[1 + \frac{dp_1}{dq_1}\frac{q_1}{p_1}\right] = c = p_2 \left[1 + \frac{dp_2}{dq_2}\frac{q_2}{p_2}\right].$$

Exercise 2.3. An economy has two types of consumers and two goods. The agent type Anakin has the following utility function:

$$u_A(x_{1A}, x_{2A}) = 4x_{1A} - \frac{x_{1A}^2}{3} + x_{2A}$$

and the agent type Ben Kenobi has the following utility function:

$$u_B(x_{1B}, x_{2B}) = 3x_{1B} - \frac{x_{1B}^2}{2} + x_{2B}.$$

Good 2 is the numeraire, and each consumer has an income of 100. Additionally, the economy has N consumers of both type Anakin and type Ben Kenobi.

a) Identify the type of consumer with high demand and the type with low demand for good x_1. Compare the marginal willingness to pay for each type of consumer for good x_1.

b) The monopolist produces good 1 with the following cost function $C(x_1) = cx_1$ and cannot discriminate prices. Find the optimal price and quantity of good x_1 that the monopolist will choose. For which values of c will the monopolist choose to sell to both types of consumers?

c) The monopolist engages in second-degree price discrimination by offering a menu of prices and quantities to each type of consumer (r_A, x_A) and (r_B, x_B). Based on this, formulate the monopolist's optimization problem and find the optimal values (r_A^*, x_A^*) and (r_B^*, x_B^*).

d) If the monopolist engages in third-degree price discrimination, what will be the prices and quantities set by the monopolist in the markets for Anakin-type and Ben Kenobi-type consumers?

e) If the monopolist engages in first-degree price discrimination, find the quantity produced by the monopolist in the market for good x. Calculate the consumer surplus and the monopolist's surplus.

Exercise 2.4. Consider a monopolist company that sells COVID-19 vaccines in two markets, the USA and Peru. In Peru, the inverse demand is given by:

$$p(x_{Pe}) = 8 - 2x_{Pe}$$

while in the USA:

$$p(x_{US}) = 10 - x_{US}.$$

The monopolist's cost function is $c(x_{Pe}, x_{US}) = x_{Pe} + x_{US}$, meaning the marginal cost of production is constant and equal to 1.

a) Which market has a higher willingness to pay (i.e., consumers are willing to pay more for a given quantity)?

b) Suppose the monopolist can discriminate prices between the two markets, meaning it can charge different prices in each market. Formulate the monopolist's profit maximization problem. Solve the problem. Calculate the profit-maximizing choices of x_{Pe}^D and x_{US}^D as well as the equilibrium prices p_{Pe}^D and p_{US}^D.

c) Compare the quantities and prices in the two markets. Interpret the results in relation to item a).

d) Now, suppose new legislation makes price discrimination between the two markets illegal. That is, the monopolist must charge the same price $p_{Pe}^M = p_{US}^M = p^M$ in both markets. Set up the new profit maximization problem for the firm.

e) Calculate the profit-maximizing choice of the total quantity produced X^M, the price p^M, and the quantities sold in each market x_{Pe}^M and x_{US}^M.

f) Compare the price p^M with the prices p_{Pe}^D and p_{US}^D under price discrimination. Make a similar comparison for the quantities produced with and without price discrimination.

Lima, September 23, 2024.

References

Tirole, J. (1994). *The Theory of Industrial Organization*. MIT University Press.