Pontificia Universidad Católica del Perú
Economics Major

November 28, 2024

Test 4
ECO 263

Professor: Pavel Coronado
TA's: Marcelo Gallardo, Fernanda Crousillat

Exercise 1. **8 points.** For the first item, answer with full detail. For the second, third and fourth, analyze whether the statement is true or false. For the fifth and sixth, solve. **Justify.**

1. The state intends to privatize a public electric utility company. However, those responsible for the privatization process have determined that there is an issue of excess personnel, but they are hesitant to propose a voluntary resignation scheme (where the company pays workers a certain amount in exchange for them resigning) due to concerns about the potential problem of adverse selection. **What would the adverse selection problem entail in this case?**

2. If the relative risk aversion coefficient is constant and different from 1, then $u(x) = A \ln x + B, \quad A, B \in \mathbb{R}$.

3. Manuel has the following Bernoulli utility function $u_1(x) = \sqrt{\ln x}$, while Carlo's is $u_2(x) = \ln(\ln x)$. Then, Manuel is more risk-adverse than Carlos.

4. In the market for lemons, the informed agent forms rational expectations about the quality of the car, but is unaware of the distribution of cars quality in the market.

5. The absolute risk tolerance (ART) is defined as the inverse of the absolute risk aversion. Show that, for $u(x) = \delta\left(\eta + x/\gamma\right)^{-\gamma}$, the ART is linear in x.

6. A consumer has an expected utility function given by $u(w) = -1/w$. They are offered the possibility to participate in a game where they would receive a wealth of w_1 with a probability of p and w_2 with a probability of $1 - p$. How much wealth would they need to be indifferent between keeping their current wealth and accepting this game?

Solution:

1. The problem of adverse selection arises when there is asymmetric information between the company and its employees regarding their productivity or willingness to leave the company. In this case, the company offers a voluntary resignation scheme, where employees are paid a compensation amount if they choose to resign. The employees best positioned to find alternative employment or those with higher skills and productivity are more likely to accept the offer, as they can secure equivalent or better opportunities elsewhere. Conversely, less productive employees or those with fewer employment options may choose to remain with the company, as they perceive the compensation offered under the scheme as insufficient relative to their limited opportunities. As a result, the company risks retaining a disproportionately high number of less productive employees while losing its most valuable workforce. This scenario undermines the intended goal of reducing excess personnel while maintaining operational efficiency, and can make the privatization process less attractive to potential buyers.

2. False. In the case of constant relative risk aversion, we have

$$R(c) = -c\frac{v''(c)}{v'(c)} = \rho$$

Thus,

$$A(c) = -\frac{v''(c)}{v'(c)} = \frac{\rho}{c}$$

Then we can solve,

$$v'(c) = \exp\left[\int -\frac{\rho}{c}dc\right] = \exp(-\rho \ln c + B)$$

So, $v(c) = \int v'(c)dc$, thus,

$$v_2(c) = \frac{Ac^{1-\rho}}{1-\rho} + B,\ \rho \neq 1$$

3. False.
Absolute Risk Aversion for Manuel:

$$A_1(x) = -\frac{u_1''(x)}{u_1'(x)}.$$

$$A_1(x) = -\frac{-\frac{1}{2x^2\sqrt{\ln x}} - \frac{1}{4x^2(\ln x)^{3/2}}}{\frac{1}{2x\sqrt{\ln x}}}.$$

$$A_1(x) = \frac{\frac{1}{2x^2\sqrt{\ln x}} + \frac{1}{4x^2(\ln x)^{3/2}}}{\frac{1}{2x\sqrt{\ln x}}}.$$

$$A_1(x) = \frac{1}{x} + \frac{1}{2x \ln x}.$$

Absolute Risk Aversion for Carlo:

$$A_2(x) = -\frac{u_2''(x)}{u_2'(x)}.$$

$$A_2(x) = -\frac{-\frac{1}{x^2 \ln x} - \frac{1}{x^2 (\ln x)^2}}{\frac{1}{x \ln x}}.$$

$$A_2(x) = \frac{\frac{1}{x^2 \ln x} + \frac{1}{x^2 (\ln x)^2}}{\frac{1}{x \ln x}}.$$

$$A_2(x) = \frac{1}{x} + \frac{1}{x \ln x}.$$

Thus,

$$A_2(x) > A_1(x) \quad \text{(for all } x > 1\text{)},$$

4. False, the **informed** agent (seller) **knows** the quality of the car they have. Note that the buyers also know the distribution, but not how that is materialized in the market (who has what).

5. The absolute risk tolerance (ART) is defined as the inverse of the absolute risk aversion (ARA). The absolute risk aversion, denoted as $A(x)$, is given by:

$$A(x) = -\frac{u''(x)}{u'(x)},$$

where $u(x)$ is the utility function. The ART is therefore:

$$\text{ART}(x) = \frac{1}{A(x)} = -\frac{u'(x)}{u''(x)}.$$

The first derivative of $u(x)$ is:

$$u'(x) = -\frac{\delta\gamma}{\gamma}\left(\eta + \frac{x}{\gamma}\right)^{-\gamma-1} = -\delta\gamma^{-1}\left(\eta + \frac{x}{\gamma}\right)^{-\gamma-1}.$$

The second derivative is:

$$u''(x) = \delta\gamma^{-2}(\gamma+1)\left(\eta + \frac{x}{\gamma}\right)^{-\gamma-2}.$$

Thus, ART is:

$$\text{ART}(x) = -\frac{u'(x)}{u''(x)}.$$

$$\text{ART}(x) = -\frac{-\delta\gamma^{-1}\left(\eta + \frac{x}{\gamma}\right)^{-\gamma-1}}{\delta\gamma^{-2}(\gamma+1)\left(\eta + \frac{x}{\gamma}\right)^{-\gamma-2}}.$$

$$\text{ART}(x) = \frac{\gamma\left(\eta + \frac{x}{\gamma}\right)}{\gamma+1}.$$

$$\text{ART}(x) = \frac{\gamma\eta + x}{\gamma+1}.$$

$$\text{ART}(x) = \frac{\gamma\eta}{\gamma+1} + \frac{x}{\gamma+1}.$$

6. The consumer's expected utility from the game is:

$$U^e = p \cdot u(w_1) + (1-p) \cdot u(w_2).$$

Since $u(w) = -\frac{1}{w}$, we have:

$$U^e = p \cdot \left(-\frac{1}{w_1}\right) + (1-p) \cdot \left(-\frac{1}{w_2}\right).$$

$$U^e = -\left(\frac{p}{w_1} + \frac{1-p}{w_2}\right).$$

The certainty equivalent, denoted as CE, is the wealth level such that the utility of CE equals the expected utility of the game:

$$u(CE) = U^e.$$

Then, $u(CE) = -\frac{1}{CE}$:

$$-\frac{1}{CE} = -\left(\frac{p}{w_1} + \frac{1-p}{w_2}\right).$$

$$\frac{1}{CE} = \frac{p}{w_1} + \frac{1-p}{w_2}.$$

$$CE = \frac{1}{\frac{p}{w_1} + \frac{1-p}{w_2}}.$$

The certainty equivalent of the game is:

$$CE = \frac{1}{\frac{p}{w_1} + \frac{1-p}{w_2}}.$$

$$CE = \frac{w_1 + w_2}{w_2 p + w_1(1-p)}.$$

Exercise 2. **4 points.** Consider an investor with an initial wealth w. There is a risky asset that provides a return of $z \in \mathbb{R}$ per dollar invested. Let F be the cumulative distribution function (CDF) of z, which is continuous. Let α denote the amount invested in the risky asset, and $u(\cdot)$ the investor's basic (Bernouilli) utility function. You are asked to:

1. Determine the investor's expected utility.
2. Consider the case where the expected net return is non-positive, i.e., $\mathbb{E}[z] - 1 \leq 0$. Show that, in this case, the optimal investment is $\alpha^* = 0$.
3. Consider the case where the expected net return is positive, i.e., $\mathbb{E}[z] - 1 > 0$. Show that, in this case, the investment $\alpha = 0$ is not optimal.

Solution:

1. The investor's expected utility is given by

$$U^e(\alpha) = \int v(w + \alpha(z-1))dF(z)$$

2. If $\mathbb{E}[z] \leq 1$,

$$\int zdF(z) \leq 1$$

Thus,

$$\begin{aligned} \frac{dU^e}{d\alpha} &= \int v'(w + \alpha(z-1))(z-1)dF(z) \\ &= \int zv'(w + \alpha(z-1))dF(z) - \int v'(w + \alpha(z-1))dF(z) = 0 \end{aligned}$$

In $\alpha = 0$,

$$\left.\frac{dU^e}{d\alpha}\right|_{\alpha=0} = v'(w)\left[\int zdF(z) - \int dF(z)\right] = v'(w)(\mathbb{E}[z] - 1) \leq 0$$

3. If $\mathbb{E}[z] > 1$,

$$\left.\frac{dU^e}{d\alpha}\right|_{\alpha=0} = v'(w)\left[\int zdF(z) - \int dF(z)\right] = v'(w)(\mathbb{E}[z] - 1) > 0$$

Exercise 3. **4 points.** Given the high probability of a global crisis, an investor has decided to accumulate wealth by purchasing artwork and keeping it in his home. His art collection is valued at x; however, by accumulating wealth in this manner, the investor faces a probability of theft π, which, if it occurs, would reduce the value of his collection by y, where $y \leq x$. Faced with this issue, a friend of his has recommended he purchase theft insurance. This insurance reduces the net loss from theft to y' (where $y > y' > 0$). This insurance has a cost $c > 0$, but the investor must also pay an additional amount, $d > 0$, as a deductible in the event of theft. The investor has a utility function $u(x) = \sqrt{x}$, where x represents his wealth.

1. Define the expressions for the individual's expected utility both if he decides to take the insurance and if he decides not to.
2. Prove that if the individual were risk-neutral, he would decide to take the insurance only if $\pi \cdot (y - y' - d) \geq c$.

Solution:

1. We build the table with payments and probabilities:

	Gets robbed	**Doesn't get robbed**
Without insurance	$x - y$	x
With insurance	$x - y' - d - c$	$x - c$
Probabilities	π	$1 - \pi$

Expected utility with insurance

$$U^e_{\text{with insurance}} = \pi(x - y' - d - c)^{1/2} + (1 - \pi)(x - c)^{1/2}$$

Expected utility without insurance

$$U^e_{\text{without insurance}} = \pi(x - y)^{1/2} + (1 - \pi)(x)^{1/2}$$

2. A risk-neutral individual maximizes expected payoffs, not utility. Let us calculate the expected wealth (W) in both scenarios:

Without insurance:

$$E[W_{\text{without insurance}}] = (1 - \pi)(x) + \pi(x - y) = x - \pi y.$$

With insurance:

$$E[W_{\text{with insurance}}] = (1 - \pi)(x - c) + \pi(x - c - d - y') = x - c - \pi(d + y').$$

A risk-neutral individual will choose to purchase insurance if:

$$E[W_{\text{with insurance}}] \geq E[W_{\text{without insurance}}].$$

Substituting the expressions for expected wealth:

$$x - c - \pi(d + y') \geq x - \pi y.$$

$$-c - \pi(d + y') \geq -\pi y.$$

Rearranging:

$$\pi(y - y' - d) \geq c.$$

Thus, a risk-neutral individual would purchase the insurance only if:

$$\pi(y - y' - d) \geq c.$$

Exercise 4. **4 points.** Consider the following Bernoulli utility function

$$u(x) = 50 + 4x - \frac{5}{2}x^2,$$

and the following monetary payoffs set $X = \{x_1, \cdots, x_n\}$.

1. Show that, for any lottery $L = (p_1, \cdots, p_n) \in \Delta(X)$, the associated expected utility **only depends** on $\mu = \sum_{i=1}^{N} p_i x_i$ and $\sigma^2 = \sum_{i=1}^{N} (x_i - \mu)^2$.
2. Compute $d\mu/d\sigma$ and comment.

Solution:

1.

$$U_e = \sum_{i=1}^{N} p_i u(x_i)$$

$$= \sum_{i=1}^{N} p_i \left(u(\mu) + u'(\mu)(x_i - \mu) + \frac{u''(\mu)}{2!}(x_i - \mu)^2 \right)$$

$$= u(\mu) + \frac{u''(\mu)}{2}\sigma^2 = 50 + 4\mu - \frac{5}{2}(\mu^2 + \sigma^2).$$

2.

$$\frac{d\mu}{d\sigma} = \frac{5\sigma}{4 - 5\mu},$$

which implies that increases in σ generate more than proportional increases in μ. This is a consequence of risk aversion.

Viernes económicos (1 point, 0.33 per correct word)
Los principales retos de iO son la limitación de **RECURSOS**, la presión por **CRECER** rápidamente y la necesidad de **ESTABLECERSE** en el mercado antes que la competencia.