

Trabalho7

June 27, 2025

```
[23]: library(pacman)
      p_load(dplyr, tidyr, splines, glarma)
```

```
[24]: setwd('C:\\Users\\Marcelo\\OneDrive\\Área de Trabalho\\ts\\trabalho 2 - TS\\Trabalho7')
      getwd()
```

'C:/Users/Marcelo/OneDrive/Área de Trabalho/ts/trabalho 2 - TS/Trabalho7'

Função de Verossimilhança da Professora:

```
[25]: Like <- function(Parametros, y, nb, np, nq, lamb, x, Ind) {

  ny <- length(y)

  # Separating parameters: beta, fi, theta
  if (nb > 0) {
    beta <- Parametros[1:nb]
  }

  ar_ma_start <- nb + 1

  if (np > 0 && nq == 0) {
    fi <- Parametros[ar_ma_start:(ar_ma_start + np - 1)]
  }

  if (nq > 0 && np == 0) {
    theta <- Parametros[ar_ma_start:(ar_ma_start + nq - 1)]
  }

  if (np > 0 && nq > 0) {
    fi <- Parametros[ar_ma_start:(ar_ma_start + np - 1)]
    theta <- Parametros[(ar_ma_start + np):(ar_ma_start + np + nq - 1)]
  }

  # Initialize variables
  z <- rep(0, ny)
```

```

w      <- rep(0, ny)
mi     <- rep(0, ny)
erro  <- rep(0, ny)

# First observation: compute w[1]
if (nb == 0) {
  w[1] <- 0
} else if (nb == 1) {
  w[1] <- beta[1]
} else {
  w[1] <- beta[1] + as.numeric(x[1, 1:(nb - 1)] %*% beta[2:nb])
}

mi[1]   <- exp(w[1])
erro[1] <- (y[1] - mi[1]) / (mi[1]^lamb)
L       <- y[1] * w[1] - exp(w[1])

# Loop over t = 2 to ny
for (t in 2:ny) {

  # AR component
  if (np > 0) {
    for (k in 1:min(np, t - 1)) {
      z[t] <- z[t] + fi[k] * (z[t - k] + erro[t - k])
    }
  }

  # MA component
  if (nq > 0) {
    for (k in 1:min(nq, t - 1)) {
      z[t] <- z[t] + theta[k] * erro[t - k]
    }
  }

  # Compute w[t] based on z[t] and x[t,]
  if (nb == 0) {
    w[t] <- z[t]
  } else if (nb == 1) {
    w[t] <- beta[1] + z[t]
  } else {
    w[t] <- beta[1] + as.numeric(x[t, 1:(nb - 1)] %*% beta[2:nb]) + z[t]
  }

  mi[t]   <- exp(w[t])
  erro[t] <- (y[t] - mi[t]) / (mi[t]^lamb)
  L       <- L + y[t] * w[t] - exp(w[t])
}

```

```

L <- -L # negative log-likelihood

valores <- cbind(erro, z, mi)

if (Ind == 0) return(L)
if (Ind == 1) return(valores)
}

```

Função que adiciona faz a estimação, calcula o AIC dado o numero de bases para cada covariável não linear:

```

[26]: MLE_com_splines <- function(y, z_list, k_list,
                                other_covariates = NULL,
                                np = 0, nq = 0, lamb = 0.5,
                                Ind = 0) {

  nz <- length(z_list)

  # Construindo as bases splines com seus respectivos ks.
  spline_list <- list()
  spline_sizes <- integer(nz)

  for (i in seq_along(z_list)) {
    k_i <- k_list[[i]]
    spline_i <- ns(z_list[[i]], df = k_i) #bases splines para a variável não
    ↳linear numero i
    spline_list[[i]] <- spline_i #lista com as bases
    spline_sizes[i] <- ncol(spline_i) #numero de bases a variavel não linear i
  }

  # juntando as bases splines usando cbind:
  names(spline_list) <- names(z_list)
  spline_basis <- do.call(cbind, spline_list)
  n_spline <- sum(spline_sizes)

  # Juntando com as outras covariáveis:
  if (!is.null(other_covariates)) {
    x <- cbind(spline_basis, other_covariates)
    n_other <- ncol(other_covariates)
  } else {
    x <- spline_basis
    n_other <- 0
  }

  nb <- 1 + n_spline + n_other # intercepto + splines + numero betas das outras
  ↳cov
}

```

```

n_params <- nb + np + nq
start_vals <- rep(0, n_params)

# ajustando o modelo dado um numero fixo de bases para cada covariável não
→linear:
opt <- optim(par = start_vals,
             fn = Like,
             y = y,
             nb = nb,
             np = np,
             nq = nq,
             lamb = lamb,
             x = x,
             Ind = Ind,
             method = "BFGS",
             hessian = FALSE)

logLik <- -opt$value
AIC <- 2 * n_params - 2 * logLik

best_coef <- opt$par
beta0 <- best_coef[1]
beta_spline_all <- best_coef[2:(1 + n_spline)]

# separando os coeficientes das bases splines para cada variável não linear:
beta_spline_list <- list()
idx <- 1
for (i in seq_along(z_list)) {
  varname <- names(z_list)[i]
  len <- spline_sizes[i]
  beta_spline_list[[varname]] <- beta_spline_all[idx:(idx + len - 1)]
  idx <- idx + len
}

# separando os outros coeficientes:
beta_other <- if (n_other > 0)
  {best_coef[(2 + n_spline):(1 + n_spline + n_other)]}
  else {numeric(0)}

fi <- if (np > 0)
  {best_coef[(nb + 1):(nb + np)]}
  else {numeric(0)}

theta <- if (nq > 0)
  {best_coef[(nb + np + 1):(nb + np + nq)]}
  else {numeric(0)}

```

```

fit <- list(
  AIC = AIC,
  logLik = logLik,
  beta0 = beta0,
  beta_spline = beta_spline_list,
  beta_other = beta_other,
  fi = fi,
  theta = theta,
  k_list = k_list
)

return(fit)
}

```

```

[27]: select_best_k_multi <- function(y, z_list, k_ranges,
                                     other_covariates = NULL,
                                     np = 0, nq = 0, lamb = 0.5,
                                     Ind = 0) {

  # Gerando um grid com os possíveis números de bases para cada variável spline:

  k_grid <- expand.grid(k_ranges)

  best_AIC <- Inf
  best_fit <- NULL
  best_k_list <- NULL

  #calculando o AIC para cada combinação de número de bases:
  for (i in seq_len(nrow(k_grid)))
  {
    current_k <- as.list(k_grid[i, ])

    result <- tryCatch({
      MLE_com_splines(
        y = y,
        z_list = z_list,
        k_list = current_k,
        other_covariates = other_covariates,
        np = np,
        nq = nq,
        lamb = lamb,
        Ind = Ind
      )
    },
    error = function(e) {return(NULL)}
  )
}

```

```

    if (!is.null(result) && result$AIC < best_AIC) {

      best_AIC <- result$AIC
      best_fit <- result
      best_k_list <- current_k
    }
  }

  best_fit$k_list <- best_k_list
  return(best_fit)
}

```

Pegando o dataset de Doença respiratórias:

```
[28]: df <- read.csv("DR.csv",header = TRUE,sep=',')
```

```
[29]: summary(df)
```

Data	Atendimentos	tempmed	O3
Length:72	Min. : 1.0	Length:72	Length:72
Class :character	1st Qu.:12.0	Class :character	Class :character
Mode :character	Median :21.5	Mode :character	Mode :character
	Mean :24.1		
	3rd Qu.:33.0		
	Max. :91.0		
CO	N02	S02	
Length:72	Length:72	Length:72	
Class :character	Class :character	Class :character	
Mode :character	Mode :character	Mode :character	

```
[30]: df<- df %>% select(Atendimentos,tempmed,O3,CO,N02,S02)
```

Limpendo os dados:

```
[31]: df[] <- lapply(df, function(x) {
  if (is.character(x)) as.numeric(gsub(",", ".", x)) else x
})
```

```
[32]: summary(df)
```

Atendimentos	tempmed	O3	CO
Min. : 1.0	Min. :18.85	Min. :16.76	Min. : 295.2
1st Qu.:12.0	1st Qu.:22.61	1st Qu.:26.34	1st Qu.: 645.7
Median :21.5	Median :23.97	Median :32.22	Median : 829.8
Mean :24.1	Mean :24.17	Mean :32.72	Mean : 826.8
3rd Qu.:33.0	3rd Qu.:26.02	3rd Qu.:38.44	3rd Qu.:1018.4

Max.	:91.0	Max.	:29.14	Max.	:48.15	Max.	:1616.3
	N02		S02				
Min.	:11.19	Min.	: 5.959				
1st Qu.:	19.33	1st Qu.:	9.361				
Median	:24.45	Median	:11.140				
Mean	:24.42	Mean	:11.809				
3rd Qu.:	28.70	3rd Qu.:	13.695				
Max.	:38.72	Max.	:20.420				

```
[33]: df<- df %>% drop_na()
```

Montando o X:

```
[34]: X <- as.matrix(df[, setdiff(names(df), "Atendimentos")])
```

Ajustando o modelo sem os splines:

```
[35]: fit =glarma(df$Atendimentos, X=X , phiLags = c(1) ,thetaLags = NULL, type = "Poi")
summary(fit)
```

```
Call: glarma(y = df$Atendimentos, X = X, type = "Poi", phiLags = c(1),
  thetaLags = NULL)
```

Pearson Residuals:

	Min	1Q	Median	3Q	Max
	-5.1470	-1.5949	-0.2822	1.8875	11.4303

GLARMA Coefficients:

	Estimate	Std.Error	z-ratio	Pr(> z)
phi_1	0.064316	0.007945	8.095	6.66e-16 ***

Linear Model Coefficients:

	Estimate	Std.Error	z-ratio	Pr(> z)
tempmed	0.0626788	0.0080746	7.763	8.44e-15 ***
O3	0.0417608	0.0038306	10.902	< 2e-16 ***
CO	0.0018250	0.0001737	10.508	< 2e-16 ***
N02	-0.0078196	0.0055289	-1.414	0.157
S02	-0.0994414	0.0113291	-8.778	< 2e-16 ***

```
Null deviance: 795.53 on 71 degrees of freedom
Residual deviance: 612.48 on 66 degrees of freedom
AIC: 943.4291
```

Number of Fisher Scoring iterations: 30

LRT and Wald Test:

Alternative hypothesis: model is a GLARMA process

Null hypothesis: model is a GLM with the same regression structure

	Statistic	p-value
LR Test	75.71	< 2e-16 ***
Wald Test	65.53	5.55e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
[36]: fit = glarma(df$Atendimentos, X=X[,c('tempmed','03','CO','SO2')] , phiLags =  
      ↪ c(1,2) ,thetaLags = NULL, type = "Poi")  
summary(fit)
```

Call: glarma(y = df\$Atendimentos, X = X[, c("tempmed", "03", "CO",
"SO2")], type = "Poi", phiLags = c(1, 2), thetaLags = NULL)

Pearson Residuals:

	Min	1Q	Median	3Q	Max
	-5.2529	-1.7941	-0.1555	1.4626	8.5222

GLARMA Coefficients:

	Estimate	Std.Error	z-ratio	Pr(> z)
phi_1	0.082932	0.008198	10.116	< 2e-16 ***
phi_2	0.046149	0.008895	5.188	2.13e-07 ***

Linear Model Coefficients:

	Estimate	Std.Error	z-ratio	Pr(> z)
tempmed	0.0639349	0.0083392	7.667	1.75e-14 ***
03	0.0364563	0.0032981	11.054	< 2e-16 ***
CO	0.0013487	0.0001142	11.814	< 2e-16 ***
SO2	-0.0720963	0.0118371	-6.091	1.12e-09 ***

Null deviance: 795.53 on 71 degrees of freedom
Residual deviance: 578.17 on 66 degrees of freedom
AIC: 917.5981

Number of Fisher Scoring iterations: 30

LRT and Wald Test:

Alternative hypothesis: model is a GLARMA process

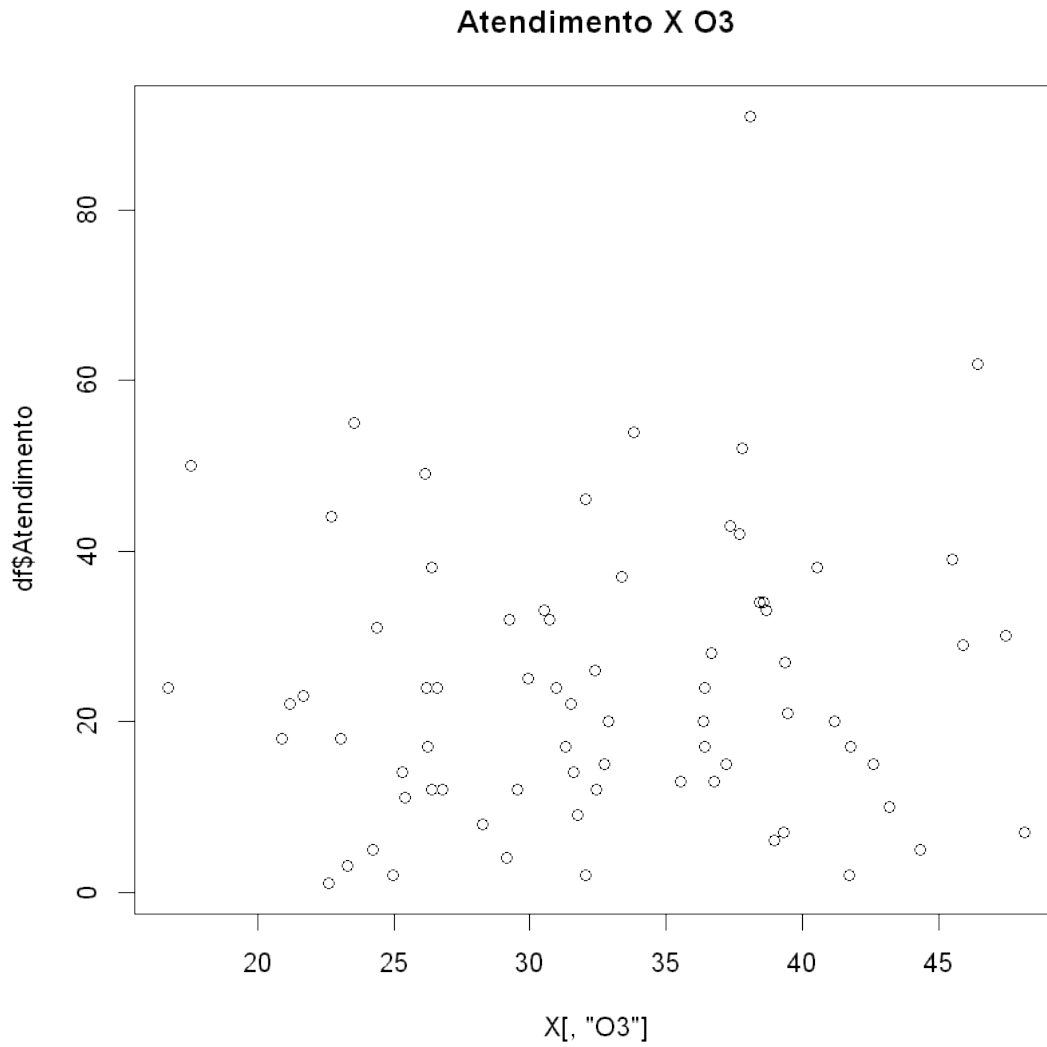
Null hypothesis: model is a GLM with the same regression structure

	Statistic	p-value
LR Test	106.0	<2e-16 ***
Wald Test	112.5	<2e-16 ***

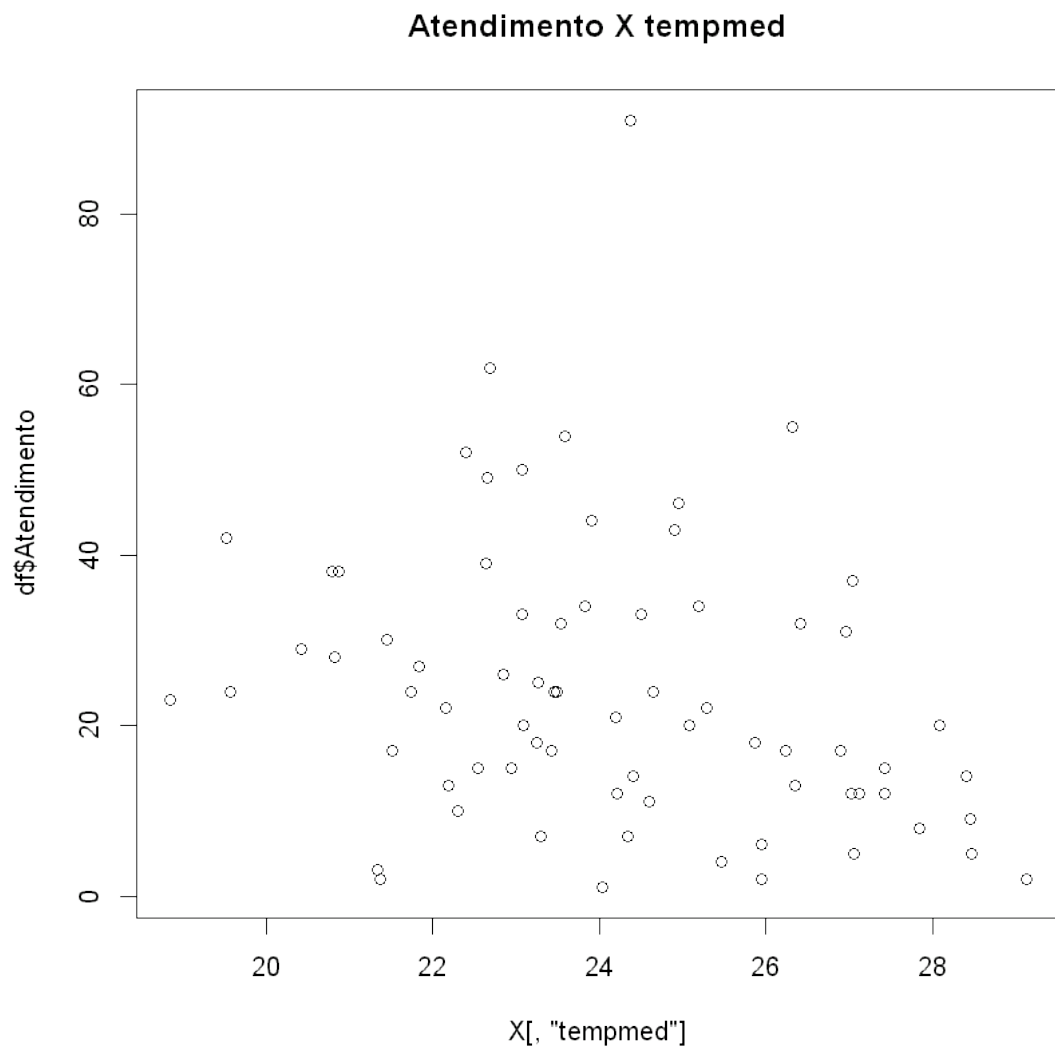
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

A partir do AR(2) os coeficientes do phi não são significativos em todos os níveis de confiança.

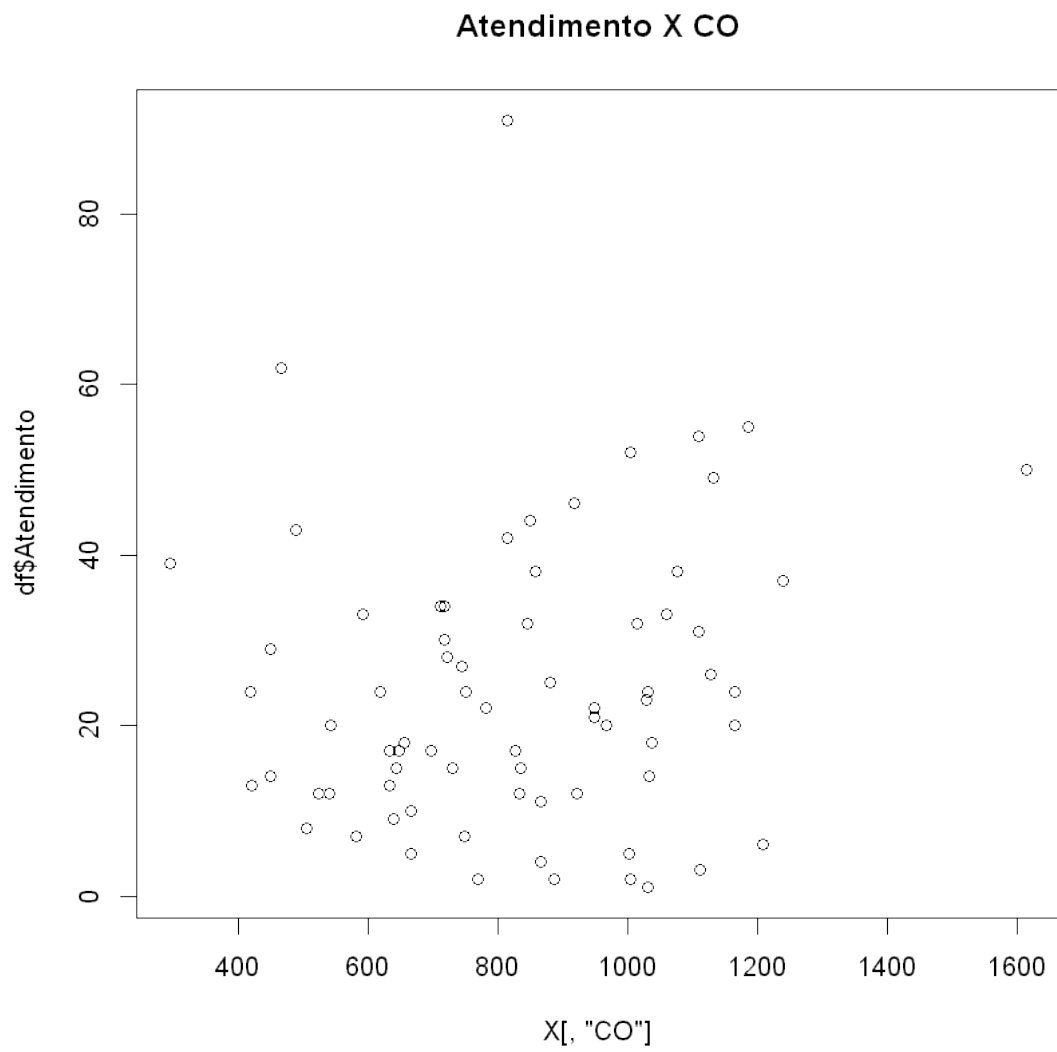
```
[37]: plot(X[, "O3"], df$Atendimento, main = "Atendimento X O3")
```



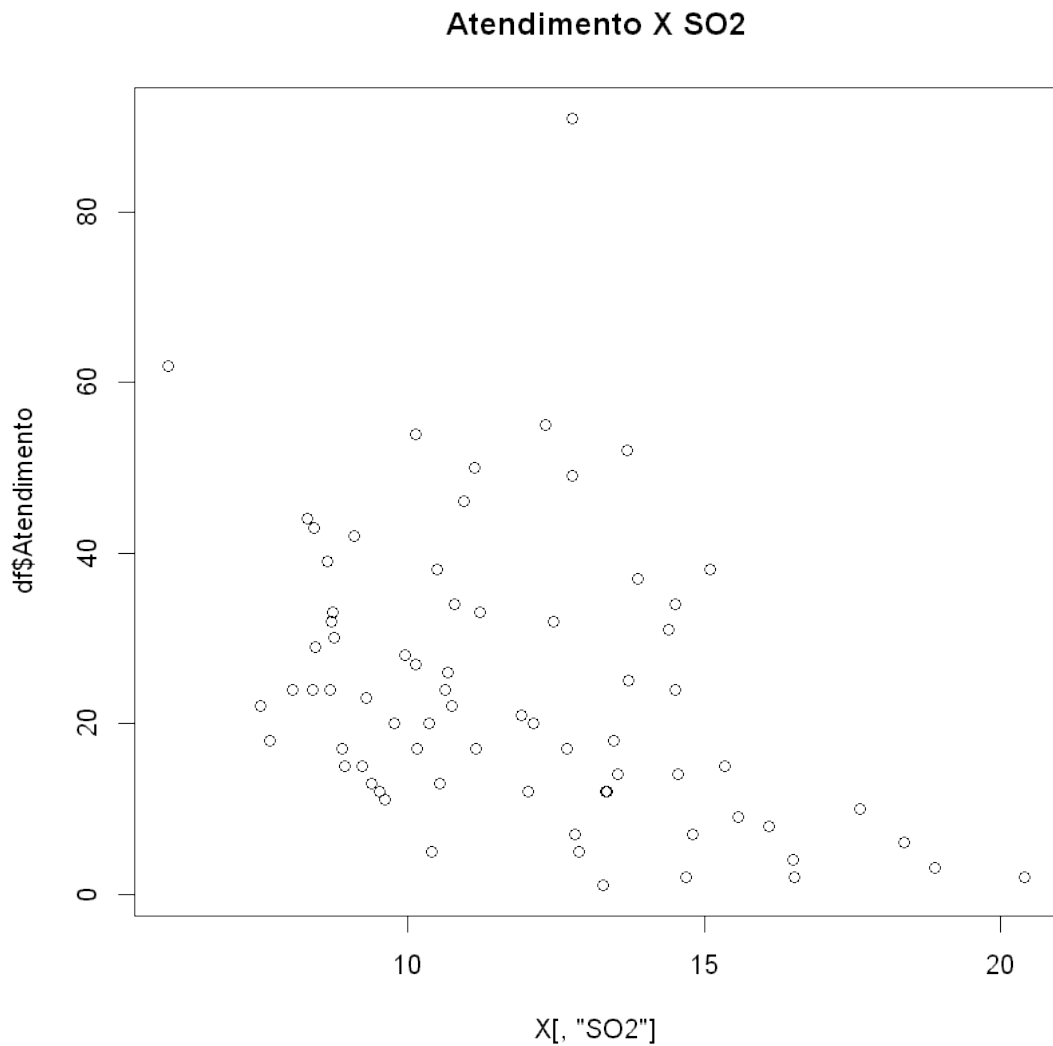
```
[38]: plot(X[, "tempmed"], df$Atendimento, main = "Atendimento X tempmed")
```



```
[39]: plot(X[, "CO"], df$Atendimento, main = "Atendimento X CO")
```



```
[40]: plot(X[, "S02"], df$Atendimento, main = "Atendimento X S02")
```



O3 aparentemente tem relação não linear com o número de atendimentos

Rodando nossa regressão com os splines:

```
[41]: other_covs <- X[,c('tempmed', 'CO', 'SO2')]
```

```
[42]: resposta<-select_best_k_multi(y=df$Atendimentos,z=list(O3=df$O3,tempmed=df$tempmed),other_covar
  => other_covs,
                                     k_ranges=list(O3=1:8,temp_med=1:8),np=3,nq=0,lamb =0.
  =>5,Ind=0)
```

```
[43]: resposta
```

\$AIC -7840.30738737551

\$logLik 3942.15369368775

\$beta0 2.76306610689278

\$beta_spline \$O3 1. 0.351727607570428 2. 0.748339770830553 3. 0.0514625420616618
4. 1.07794204788271 5. -0.521010265442694 6. 0.572686123411616 7. 0.636101002121954

\$tempmed 1. -0.10411575379499 2. -0.342784661481924 3. -0.0206866784576695
4. -0.127113092415123 5. -0.622252363904484 6. 0.0563834096290639
7. -1.64067137163042 8. -1.7303685099724

\$beta_other 1. 0.0284734533890816 2. 0.000740302083696139 3. -0.0866522606064693

\$fi 1. 0.0433467203032375 2. 0.00227101251663183 3. -0.0660772354515024

\$theta

\$k_list \$O3 7

\$temp_med 8

[]: