



## Formulario Certamen 2

Marcelo Paz  
Investigación de Operaciones

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Versión: 1.0.2

### 1. Líneas de espera

**M/M/1  $\rho < 1$**

$$\rho = \frac{\lambda}{\mu} \quad L_s = \frac{\lambda}{\mu - \lambda} \quad L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \quad W_s = \frac{L_s}{\lambda} = \frac{1}{\mu - \lambda}$$

$$W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu - \lambda)} \quad P_0 = 1 - \frac{\lambda}{\mu} \quad P_n = \rho^n \cdot P_0$$

$$P(W_q > t) = \rho \cdot e^{-\mu \cdot (1 - \rho) \cdot t} \quad P(W_s > t) = e^{-\mu \cdot (1 - \rho) \cdot t}$$

**M/M/S  $\rho < 1$**

$$\rho = \frac{\lambda}{s \cdot \mu} \quad L_s = L_q + \frac{\lambda}{\mu} = \lambda \cdot W_s \quad W_s = W_q + \frac{1}{\mu} = \frac{L_s}{\lambda}$$

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^2 \cdot \lambda \cdot \mu}{(s-1)! \cdot (s \cdot \mu - \lambda)^2} \cdot P_0 = \frac{1}{s!} \cdot \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{\rho}{(1-\rho)^2} \cdot P_0 \quad W_q = \frac{L_q}{\lambda}$$

$$P_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!} \cdot \left(\frac{s \cdot \mu}{s \cdot \mu - \lambda}\right)} \quad P_n = \begin{cases} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} \cdot P_0 & \text{si } n \leq s \\ \frac{\left(\frac{\lambda}{\mu}\right)^n}{s! \cdot s^{n-s}} \cdot P_0 & \text{si } n > s \end{cases}$$

**M/M/1/K**

$$\lambda_{ef} = \lambda \cdot (1 - P_k) \quad \rho = \frac{\lambda}{\mu} \quad L_s = \begin{cases} \frac{\rho}{(1 - \rho)} - \frac{(k + 1) \cdot \rho^{k+1}}{1 - \rho^{k+1}} & \text{si } \rho \neq 1 \\ \frac{k}{2} & \text{si } \rho = 1 \end{cases}$$

$$L_q = L_s - (1 - P_0) = \begin{cases} L_s - \frac{(1 - \rho^k) \cdot \rho}{1 - \rho^{k+1}} & \text{si } \rho \neq 1 \\ \frac{k \cdot (k - 1)}{2 \cdot (k + 1)} & \text{si } \rho = 1 \end{cases} \quad W_s = \frac{L_s}{\lambda_{ef}}$$

$$W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda_{ef}} \quad P_0 = \begin{cases} \frac{1 - \rho}{1 - \rho^{k+1}} & \text{si } \lambda \neq \mu \equiv \rho = \frac{\lambda}{\mu} \neq 1 \\ \frac{1}{1 + k} & \text{si } \lambda = \mu \equiv \rho = \frac{\lambda}{\mu} = 1 \end{cases}$$

$$Largo_q = \lambda_{ef} \cdot W_q \quad P_n = \begin{cases} \rho^n \cdot \frac{(1 - \rho)}{1 - \rho^{k+1}} & \text{si } \lambda \neq \mu \equiv \rho = \frac{\lambda}{\mu} \neq 1 \\ \frac{1}{1 + k} & \text{si } \lambda = \mu \equiv \rho = \frac{\lambda}{\mu} = 1 \end{cases}$$

**Costos en los sistemas de colas**

$$C_t = S \cdot C_s + L \cdot C_w$$