Formulario Certamen 2

Marcelo Paz

Investigación de Operaciones

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Versión: 1.0.1

1. Lineas de espera

M/M/1

$$\rho = \frac{\lambda}{\mu} \qquad L_s = \frac{\lambda}{\mu - \lambda} \qquad L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \qquad W_s = \frac{L_s}{\lambda} = \frac{1}{\mu - \lambda}$$

$$W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu - \lambda)} \qquad P_0 = 1 - \frac{\lambda}{\mu} \qquad P_n = \rho^n \cdot P_0$$

$$P(W_q > t) = \rho \cdot e^{-\mu \cdot (1 - e) \cdot t} \qquad P(W_s > t) = e^{-\mu \cdot (1 - e) \cdot t}$$

M/M/S

$$\rho = \frac{\lambda}{s \cdot \mu} \qquad L_s = L_q + \frac{\lambda}{\mu} = \lambda \cdot W_s \qquad W_s = W_q + \frac{1}{\mu} = \frac{L_s}{\lambda}$$

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^2 \cdot \lambda \cdot \mu}{(s-1)! \cdot (s \cdot \mu - \lambda)^2} \cdot P_0 = \frac{1}{s!} \cdot \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{\rho}{(1 - \rho)^2} \cdot P_0 \qquad W_q = \frac{L_q}{\lambda}$$

$$P_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!} \cdot \left(\frac{s \cdot \mu}{s \cdot \mu - \lambda}\right)} \qquad P_n = \begin{cases} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} \cdot P_0 & \text{si } n \leq s \\ \frac{\left(\frac{\lambda}{\mu}\right)^n}{s! \cdot s^{n-s}} \cdot P_0 & \text{si } n \geq s \end{cases}$$

M/M/1/K

$$\lambda_{ef} = \lambda \cdot (1 - P_k) \qquad \rho = \frac{\lambda}{\mu} \qquad L_s = \begin{cases} \dfrac{\rho}{(1-\rho)} - \dfrac{(k+1)\cdot \rho^{k+1}}{1-\rho^{k+1}} & \text{si } \rho \neq 1 \\ \dfrac{k}{2} & \text{si } \rho = 1 \end{cases}$$

$$L_q = L_s - (1 - P_0) = \begin{cases} L_s - \dfrac{(1-\rho^k)\cdot \rho}{1-\rho^{k+1}} & \text{si } \rho \neq 1 \\ \dfrac{k \cdot (k-1)}{2 \cdot (k+1)} & \text{si } \rho = 1 \end{cases} \qquad W_s = \frac{L_s}{\lambda_{ef}}$$

$$W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda_{ef}} \qquad P_0 = \begin{cases} \dfrac{1-\rho}{1-\rho^{k+1}} & \text{si } \lambda \neq \mu \equiv \rho = \frac{\lambda}{\mu} \neq 1 \\ \dfrac{1}{1+k} & \text{si } \lambda = \mu \equiv \rho = \frac{\lambda}{\mu} = 1 \end{cases}$$

$$Long_q = \lambda_{ef} \cdot W_q \qquad P_n = \begin{cases} \rho^n \cdot \dfrac{(1-\rho)}{1-\rho^{k+1}} & \text{si } \lambda \neq \mu \equiv \rho = \frac{\lambda}{\mu} \neq 1 \\ \dfrac{1}{1+k} & \text{si } \lambda = \mu \equiv \rho = \frac{\lambda}{\mu} = 1 \end{cases}$$

Costos en los sistemas de colas

$$C_t = S \cdot C_s + L \cdot C_w$$