Immersed domain approach for fluid-structure-contact interaction problems

ENUMATH 2025 - MS70 – Recent Advances in Mathematical and Computational Methods for Cardiac Biomechanics

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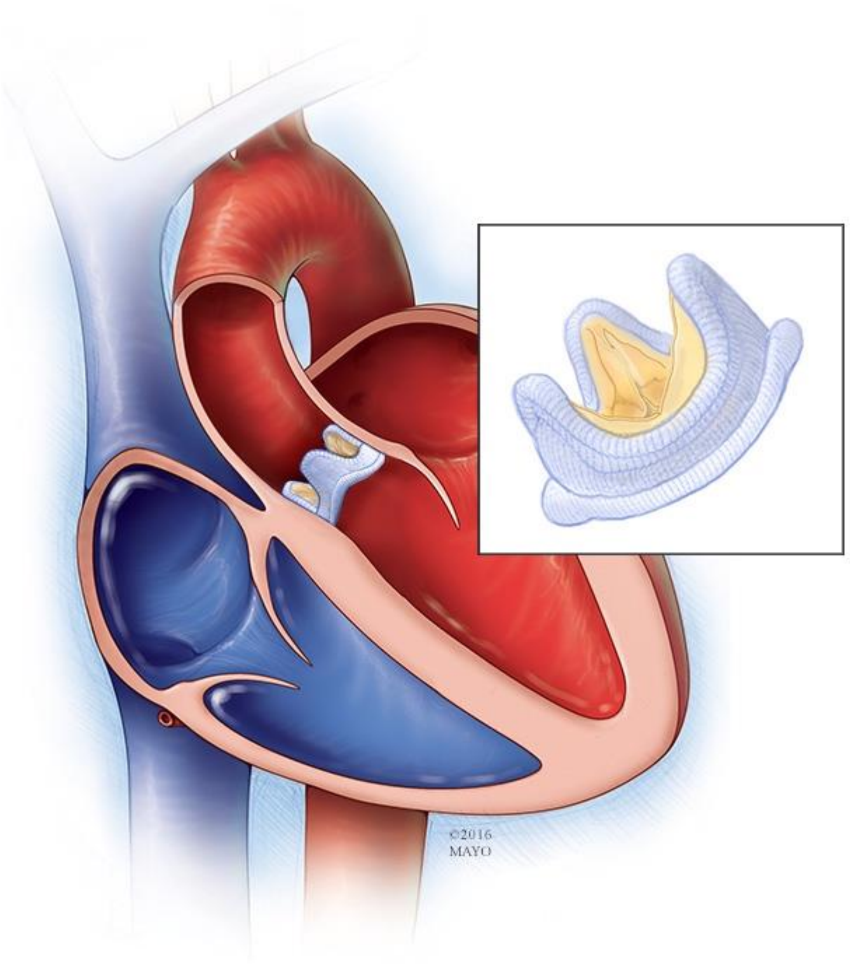
5. **UniBe**, Bern, CH.

COS Small Lecture Hall, September 2nd, 17:20 –17:45, ENUMATH, Heidelberg

Problem formulation

Aim of this work:

- Framework for BHV performance analysis
- Correlation of high-stress regions with design and leaflet material properties



Computational model

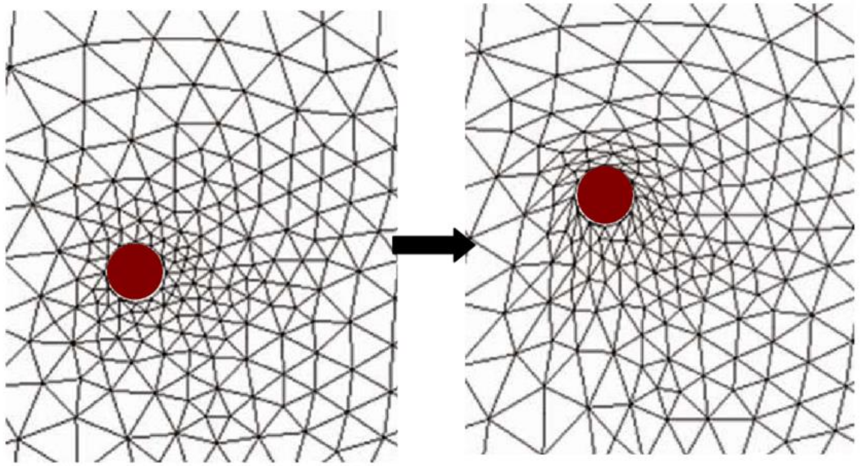
Fully coupled approach for:

- **CONTACT** problems between multiple elastic structures,
- Immersed in a fluid flow (**FSI**)

Using:

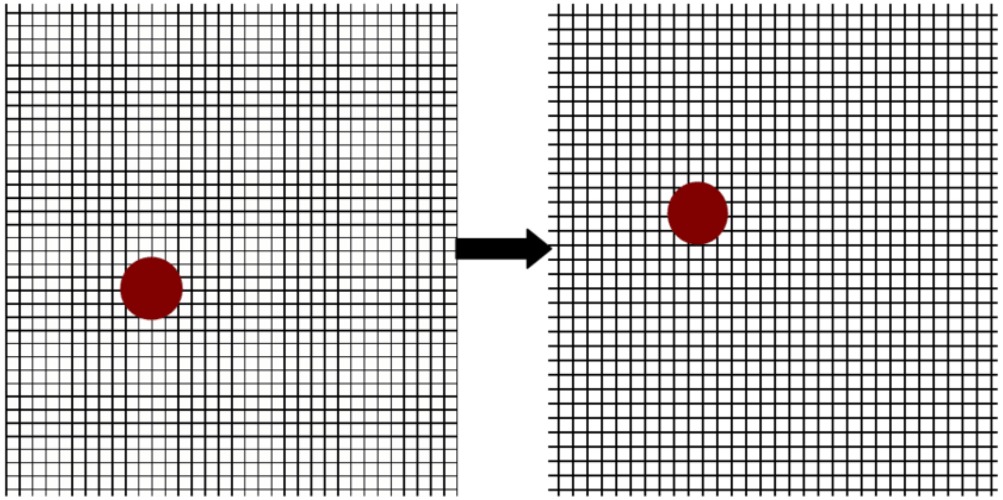
- a fiber-reinforced material for the leaflets,
- a stent,
- a compliant aortic root.

Boundary-fitted methods



- **Matching boundary** of fluid and solid meshes (accurate)
- **Fluid mesh deforms** with solid mesh
- Large displacements → **Distorted fluid grid** → Reduced numerical stability and accuracy

Non Boundary-fitted methods



- **Independent meshes** of fluid and solid meshes
- **Fuzzy** FSI interface → **Higher resolution required** for accurate results
- **Flexible** choice of discretization for the fluid (*e.g.*, FEM, FVM, CVFEM, FDM) and software

Approaches we investigated
Fluid-structure-contact interaction

Project	Application	Fluid	Solid	Coupling	Contact	Article
AV-Flow	Heart-valves	FD	FEM	IB	–	Nestola et al. [2019]
This work	Heart-valves	FEM	FEM	ID	Lagrange Multipliers (LM)	Nestola et al. [2021, 2025]
Fluya	Pumps	CVFEM	FEM	ID	Shifted-Penalty (SP)	In preparation

ID := Immersed Domain, IB := Immersed Boundary

Accepted as a proceeding paper at Domain Decomposition Methods 28 (DD28).

Problem formulation

Fluid: *Navier-Stokes equations*

in Lagrangian coordinates

$$\begin{cases} \rho_f \dfrac{\partial \boldsymbol{u_f}}{\partial t} + \rho_f(\boldsymbol{u_f} \cdot \nabla)\boldsymbol{u_f} - \nabla \cdot \sigma_f(\boldsymbol{u_f}, p_f) = 0 \\ \nabla \cdot \boldsymbol{u_f} = 0 \end{cases} \quad \text{in } \Omega_f$$

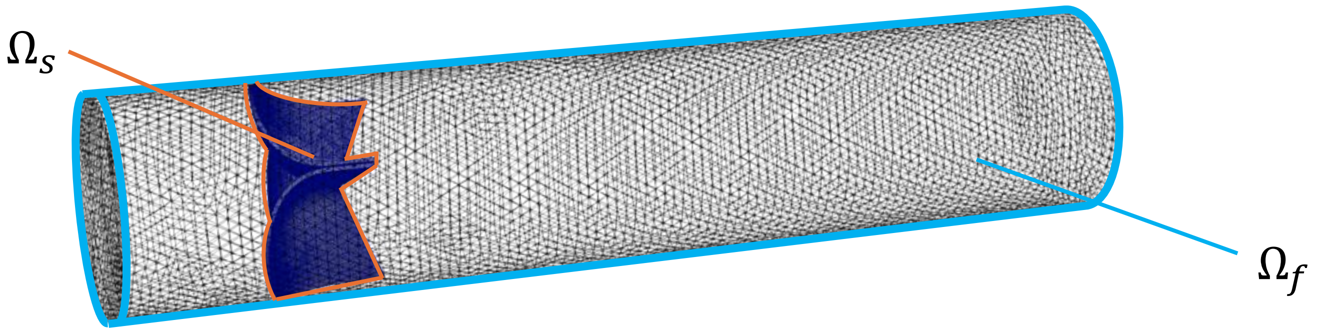
Structure: *Elastodynamics equations*

in Eulerian coordinates

$$\rho_s \frac{\partial^2 \boldsymbol{\eta_s}}{\partial^2 t} - \nabla \cdot \sigma_s(\boldsymbol{\eta_s}) = 0 \quad \text{in } \Omega_s \subset \Omega_f$$

Immersed domain

fluid and solid are coupled in the entire **intersection volume**



Problem formulation

Fluid: *Navier–Stokes equations*

$$\begin{cases} \rho_f \dfrac{\partial \boldsymbol{u_f}}{\partial t} + \rho_f(\boldsymbol{u_f} \cdot \nabla)\boldsymbol{u_f} - \nabla \cdot \sigma_f(\boldsymbol{u_f}, p_f) = 0 \\ \nabla \cdot \boldsymbol{u_f} = 0 \end{cases} \qquad \text{in } \Omega_f$$

Structure: *Elastodynamics equations*

$$\rho_s \frac{\partial^2 \boldsymbol{\eta_s}}{\partial^2 t} - \nabla \cdot \sigma_s(\boldsymbol{\eta_s}) = 0 \qquad \text{in } \Omega_s \subset \Omega_f$$

FSI coupling

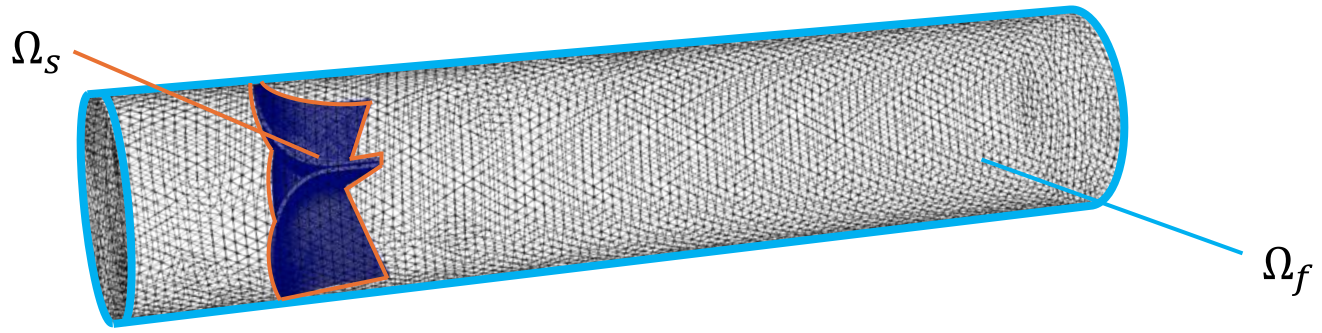
$$\frac{\partial \boldsymbol{\eta_s}}{\partial t} = \boldsymbol{u_f}$$

in

$$I(t) = \Omega_s(t) \cap \Omega_f = \Omega_s(t)$$

Immersed domain

fluid and solid are coupled in the entire **intersection volume**



FSI coupling

$$\boldsymbol{u}_s := \frac{\partial \boldsymbol{\eta}_s}{\partial t} = \boldsymbol{u}_f \longrightarrow \text{Weakly enforced} \longrightarrow \boxed{\text{Distributed Lagrange multipliers}}$$

in $I(t) = \Omega_s(t) \cap \Omega_f = \Omega_s(t)$

Mortar-based Lagrange multipliers

Want to minimize $\boldsymbol{\Phi}(\boldsymbol{u}_f, \boldsymbol{u}_s)$ subjected to $\boldsymbol{u}_s = \boldsymbol{u}_f$ in $I(t)$

$\downarrow$

Define $\boldsymbol{\mathcal{L}}(\boldsymbol{u}_f, \boldsymbol{u}_s, \boldsymbol{\lambda}) = \boldsymbol{\Phi}(\boldsymbol{u}_f, \boldsymbol{u}_s) - \boldsymbol{\lambda}(\boldsymbol{u}_s - \boldsymbol{u}_f)$

$\downarrow$

$$\boxed{\frac{\partial \boldsymbol{\mathcal{L}}}{\partial \boldsymbol{u}_f} = 0, \quad \frac{\partial \boldsymbol{\mathcal{L}}}{\partial \boldsymbol{u}_s} = 0, \quad \frac{\partial \boldsymbol{\mathcal{L}}}{\partial \boldsymbol{\lambda}} = 0}$$

$$\begin{cases} \textbf{\textit{Fluid}}\ \textit{dynamics} \boxed{- \int_{I(t)} \boldsymbol{\lambda}\, \delta \boldsymbol{u}_f \, dV = 0} \\ \textbf{\textit{Structure}}\ \textit{dynamics} \boxed{+ \int_{I(t)} \boldsymbol{\lambda}\, \delta \boldsymbol{u}_s dV = 0} \\ \boxed{\int_{I(t)} (\boldsymbol{u}_s - \boldsymbol{u}_f)\, \delta \boldsymbol{\lambda}\, dV = 0} \end{cases}$$

Weak formulation of the FSI problem

Find $\boldsymbol{u}_f$, p_f, $\boldsymbol{\eta}_s$, $\boldsymbol{\lambda}$, s.t. :

$$\left(\rho_f \frac{\partial \boldsymbol{u}_f}{\partial t} + \rho_f(\boldsymbol{u}_f \cdot \nabla)\boldsymbol{u}_f - \nabla \cdot \sigma_f(\boldsymbol{u}_f, p_f), \delta \boldsymbol{u}_f\right)_{\Omega_f} - (\boldsymbol{\lambda}, \delta \boldsymbol{u}_f)_{I(t)} = 0$$

$$(\nabla \cdot \boldsymbol{u}_f, \delta p_f)_{\Omega_f} = 0$$

$$\left(\rho_s \frac{\partial^2 \boldsymbol{\eta}_s}{\partial^2 t} - \nabla \cdot \sigma_s(\boldsymbol{\eta}_s), \delta \boldsymbol{u}_s\right)_{\Omega_s(t)} + (\boldsymbol{\lambda}, \delta \boldsymbol{u}_s)_{I(t)} = 0$$

$$(\boldsymbol{u}_s - \boldsymbol{u}_f, \delta \boldsymbol{\lambda})_{I(t)} = 0$$

$$\begin{vmatrix} \boldsymbol{A}_f & 0 & -\boldsymbol{D}^T \\ 0 & \boldsymbol{A}_s & \boldsymbol{B}^T \\ -\boldsymbol{D} & \boldsymbol{B} & 0 \end{vmatrix} \begin{vmatrix} \delta \boldsymbol{u}_f \\ \delta \boldsymbol{u}_s \\ \delta \boldsymbol{\lambda} \end{vmatrix} = \begin{vmatrix} \boldsymbol{R}_f \\ \boldsymbol{R}_s \\ \boldsymbol{R}_{FSI} \end{vmatrix}$$

where $\boldsymbol{R}_{FSI}^k = -(\boldsymbol{B}\boldsymbol{u}_s^k - \boldsymbol{D}\,\boldsymbol{u}_f^k)$

$\forall\ \delta \boldsymbol{u}_f, \delta p_f, \delta \boldsymbol{u}_s, \delta \boldsymbol{\lambda}$

where $(\cdot,\cdot)_D \coloneqq L^2$ – inner product over D,
and $\delta \boldsymbol{u}_f, \delta p_f, \delta \boldsymbol{u}_s, \delta \boldsymbol{\lambda}$ are the infinitesimal velocity, pressure, structure velocity, Lagrange multiplier respectively

Weak formulation of the FSI problem

Find $\boldsymbol{u_f},\ p_f,\ \boldsymbol{\eta_s},\ \boldsymbol{\lambda},$ s.t. :

$$\left(\rho_f \frac{\partial \boldsymbol{u_f}}{\partial t} + \rho_f(\boldsymbol{u_f}\cdot\nabla)\boldsymbol{u_f} - \nabla\cdot\sigma_f(\boldsymbol{u_f},p_f),\delta\boldsymbol{u_f}\right)_{\Omega_f} \boxed{-(\boldsymbol{\lambda},\delta\boldsymbol{u_f})_{I(t)}} = 0$$

$$(\nabla\cdot\boldsymbol{u_f},\delta p_f)_{\Omega_f} = 0$$

$$\left(\rho_s \frac{\partial^2 \boldsymbol{\eta_s}}{\partial^2 t} - \nabla\cdot\sigma_s(\boldsymbol{\eta_s}),\delta\boldsymbol{u_s}\right)_{\Omega_s(t)} \boxed{+(\boldsymbol{\lambda},\delta\boldsymbol{u_s})_{I(t)}} = 0$$

$$\boxed{(\boldsymbol{u_s}-\boldsymbol{u_f},\delta\boldsymbol{\lambda})_{I(t)} = 0}$$

$$\forall\ \delta\boldsymbol{u_f},\delta p_f,\delta\boldsymbol{u_s},\delta\boldsymbol{\lambda}$$

FSI system

$$\begin{vmatrix} \boldsymbol{A_f} & 0 & -\boldsymbol{D}^T \\ 0 & \boldsymbol{A_s} & \boldsymbol{B}^T \\ -\boldsymbol{D} & \boldsymbol{B} & 0 \end{vmatrix} \begin{vmatrix} \delta\boldsymbol{u_f} \\ \delta\boldsymbol{u_s} \\ \delta\boldsymbol{\lambda} \end{vmatrix} = \begin{vmatrix} \boldsymbol{R_f} \\ \boldsymbol{R_s} \\ \boldsymbol{R_{FSI}} \end{vmatrix}$$

where $\boldsymbol{R}^k_{FSI} = -(\boldsymbol{B}\ \boldsymbol{u}^k_s - \boldsymbol{D}\ \boldsymbol{u}^k_f)$

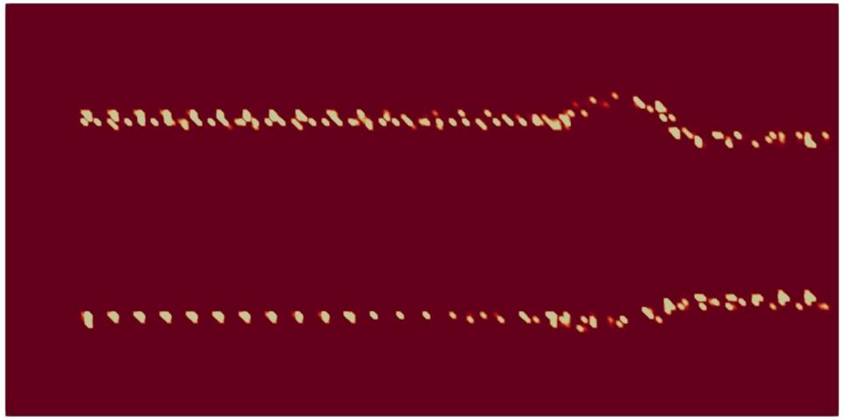
where $(\cdot,\cdot)_D \coloneqq L^2 -$ inner product over D,
and $\delta\boldsymbol{u_f},\delta p_f,\delta\boldsymbol{u_s},\delta\boldsymbol{\lambda}$ are the infinitesimal velocity, pressure, structure velocity, Lagrange multiplier respectively

Coupling and resampling → data transfer between **non-matching** fluid and solid **meshes**

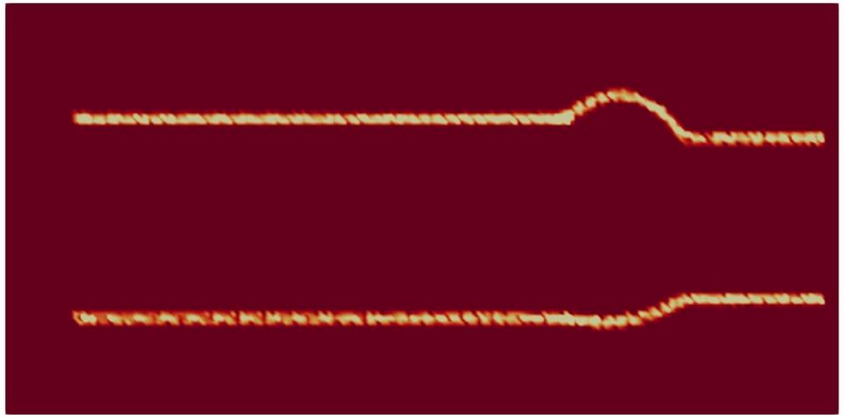
- Variants of **mortar-method** [b] for the coupling between fluid and structure

  - **Variational approach**
  - **Higher accuracy** in transferring interface quantities,
  - **Conservative**: avoids artificial gain/loss of energy,
  - **Stable** even for large mesh mismatches

Standard interpolation *(etc.)*



Variational approach



[a] Baliga and Patankar [1983]
[b] Bernardi et al. [2005]

Coupling and resampling

Solve: $\int_{I(t)} \left(\sum_i \frac{\partial \eta_i}{\partial t} \boldsymbol{\phi_i} - \sum_j \boldsymbol{u_j} \boldsymbol{\psi_j} \right) \cdot \boldsymbol{\mu_k} \, dV = 0, \ \forall \boldsymbol{\mu_k} \in \boldsymbol{\Lambda}$

Elements with affine faces

↓

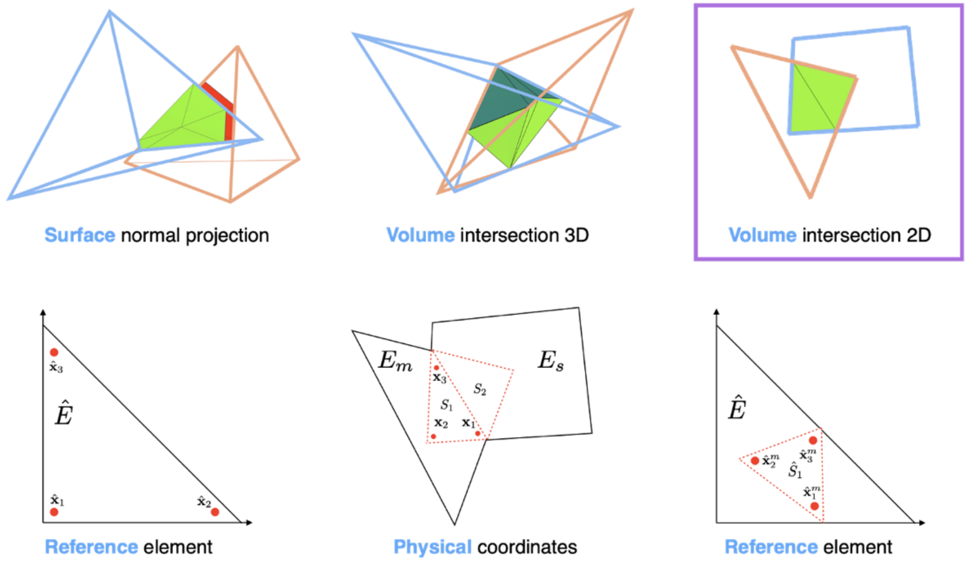
Intersections
"exact" quadrature

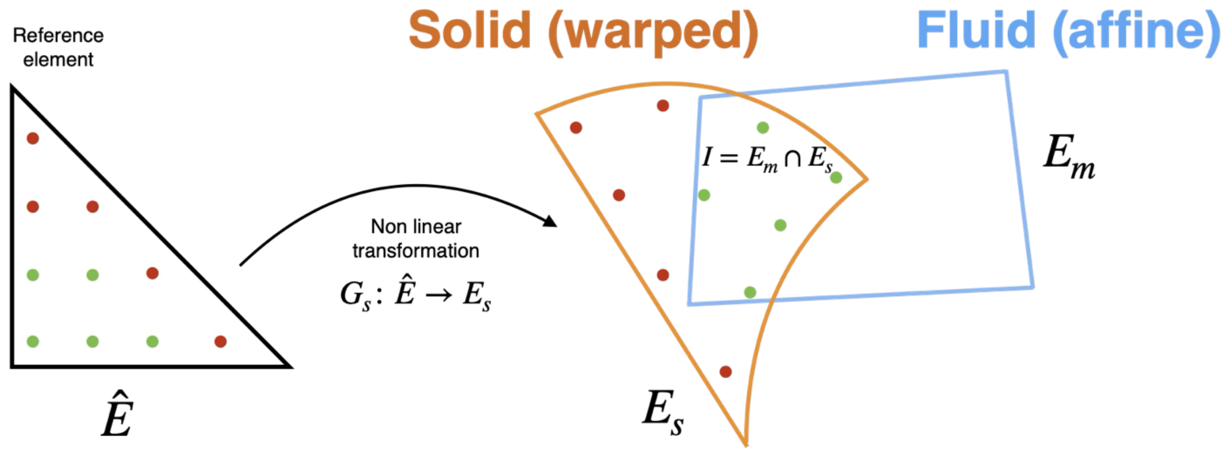
Warped faces

↓

Adaptive sampling strategy, inexact

[E. Griffith and Luo, 2017, Boffi et al., 2024]

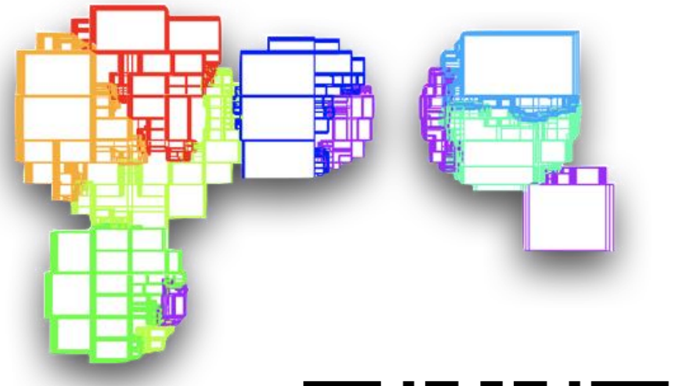


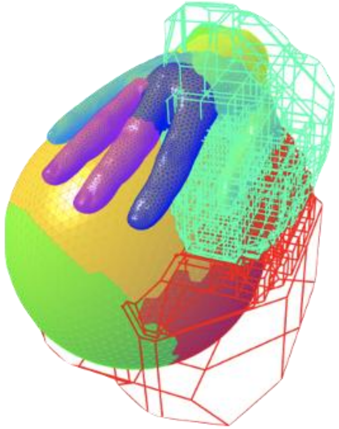


Parallel coupling procedure

Coupling types

- FSI Volumetric coupling
- Contact conditions Surface coupling





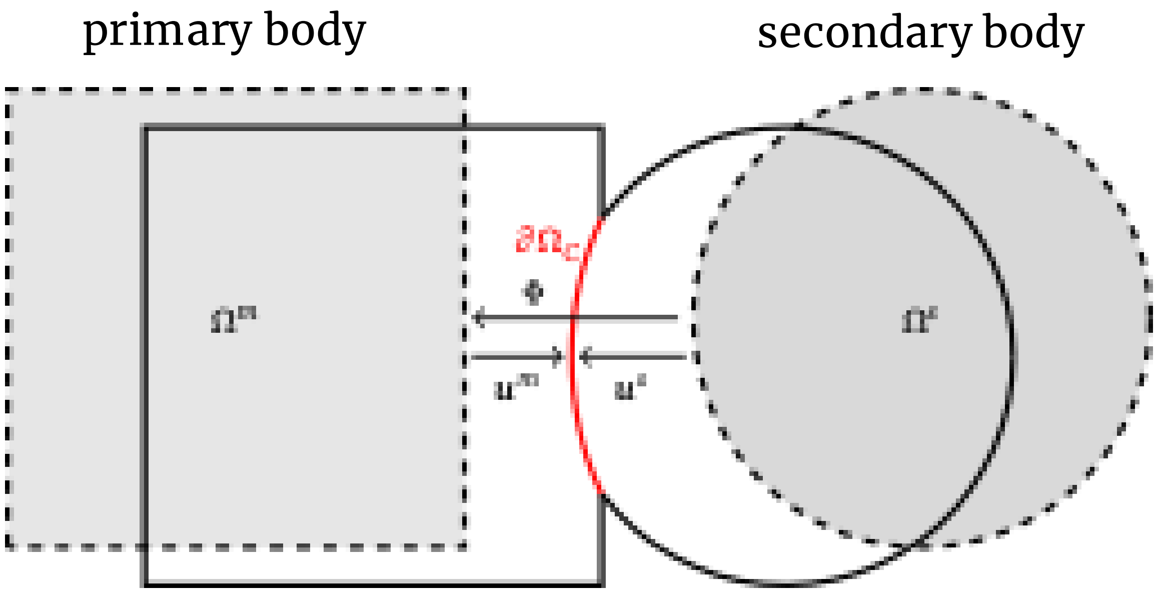
Krause and Zulian [2016]

Geometric operations

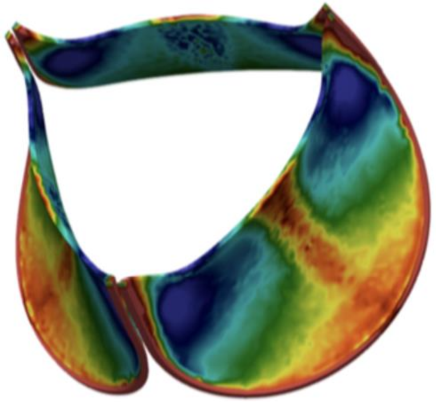
- Structure considered in the deformed configuration
- Intersection mesh for numerical quadrature of the coupling conditions fluid-structure and structure-structure

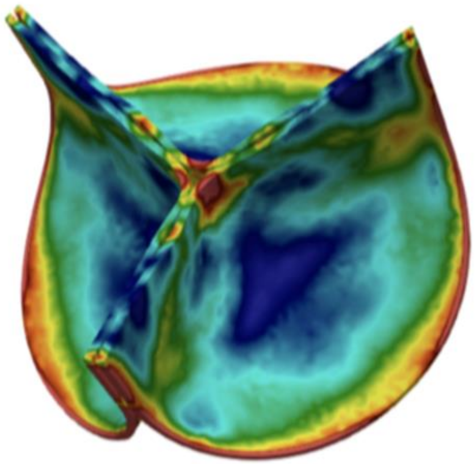
Two-body CONTACT problem

- Elastic bodies $\Omega^m, \Omega^s \in \mathbb{R}^d, d \in \{2, 3\}$
- Lipschitz continuous boundaries Γ^m, Γ^s
- A priori unknown contact boundary $\partial\Omega_c = \Gamma^c$
- Gap between the two bodies g_c



Cardiac valve **open** during ventricular systole



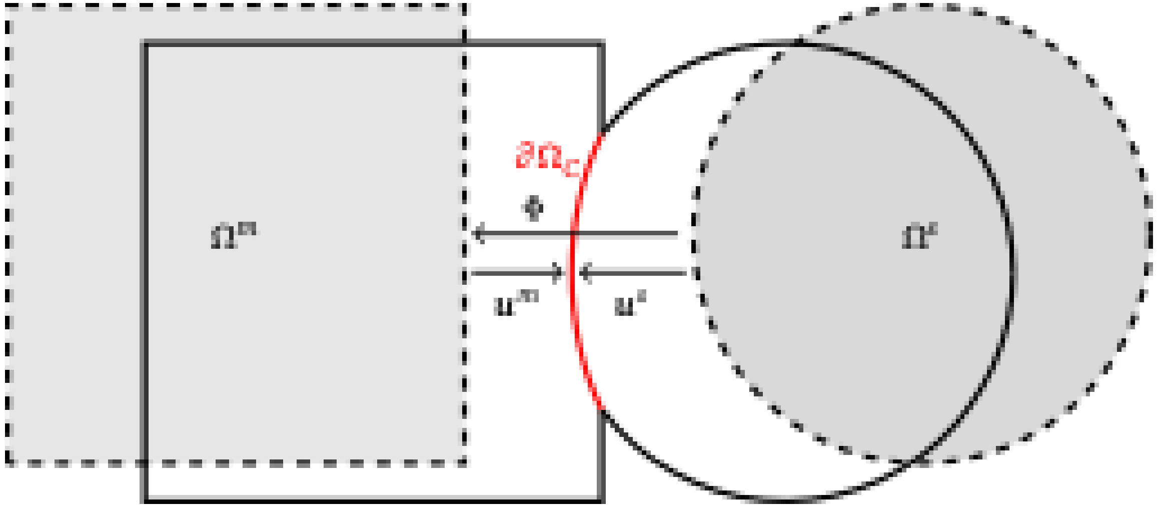


Cardiac valve **closed** during diastole

Contact between elastic leaflets

Two-body CONTACT problem

primary body   secondary body



Mortar-based approach

- Distributed **Lagrange multipliers**
- Large deformations
- No penalty tuning

Contact conditions:

Non-penetration condition: $\int_{\partial\Omega_c} (\llbracket\eta\rrbracket \cdot \boldsymbol{n}_\Phi - g_c)\, \psi_c \, dS \leq 0$

Positiveness of Lagrange multiplier : $\boldsymbol{\lambda_c} \geq 0$

Complementary condition: $\lambda_c\, (\llbracket\eta\rrbracket \cdot \boldsymbol{n}_\Phi - g_c) = 0$

Tangential contact stress: $\tau_t(\eta^s) = 0$

Vector field of normal directions: $n_\Phi\colon \Gamma^s \to \mathbb{S}^2$

$$n_\Phi(x) = \begin{cases} \dfrac{\Phi(x) - x}{|\Phi(x) - x|} & \text{if } \Phi(x) \neq x \\ n^s(x) & \text{otherwise} \end{cases}$$

Jump of the solution in n_Φ

$$\llbracket\eta\rrbracket := \eta^s - \eta^m \circ \Phi$$

Solution algorithm – Staggered approach

- Fluid and structure sub-problems solved **separately** within a Picard iteration
- Multibody **contact** problem is solved with a non-smooth sub-structuring method

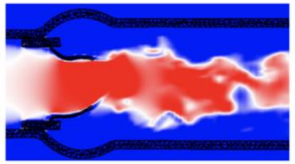
Transfer velocity $u_s = \dot{\eta}_s$

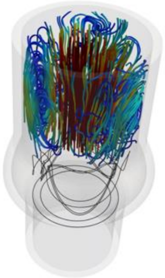
fluid $\Longleftarrow$ solid

FEM (+SUPG), with MOOSE framework
[Peterson et al., 2018]

Backward Differentiation (BDF2)

Solve fluid problem

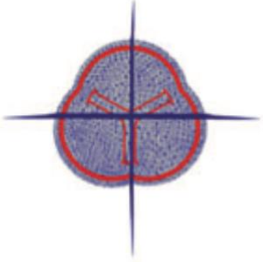


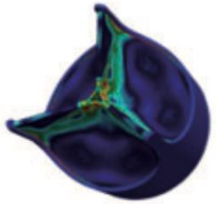


with
velocity constraints

$u_f = u_s$

Solve solid problem





with force R_{FSI} and
contact conditions

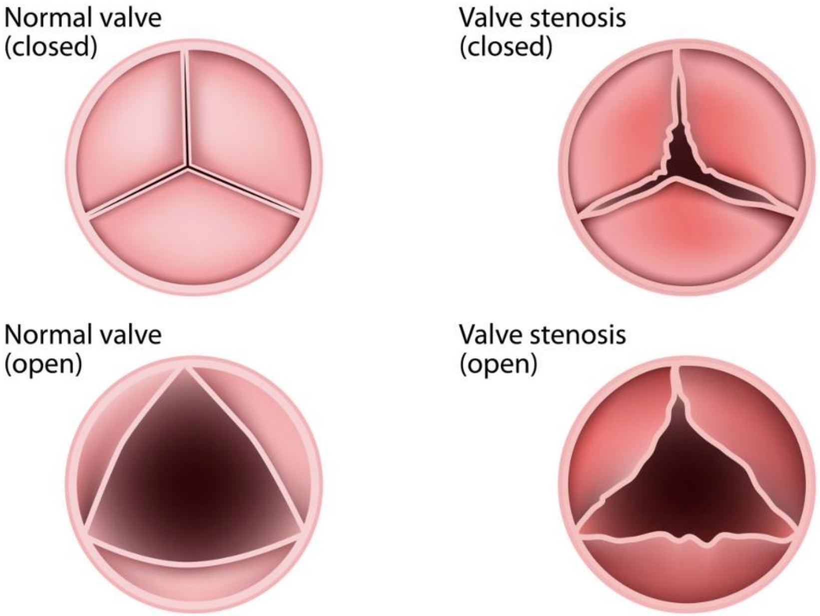
FEM

Contact-stabilized-Newmark Scheme
[Krause and Walloth, 2012]

Transfer reaction force R_{FSI}

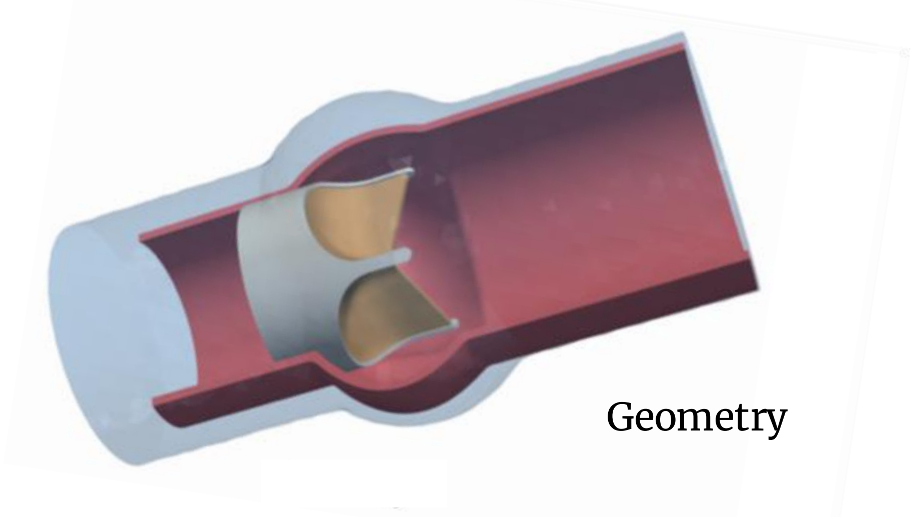
fluid $\Longrightarrow$ solid

Bio-prosthetic heart-valve simulation



Aortic valve stenosis

- Prevalent valvular pathology in Western countries
- Progressive thickening of the valve
- Results in severe impairment of the valve motion
  ⟶ Replacement with bioprosthetic valve



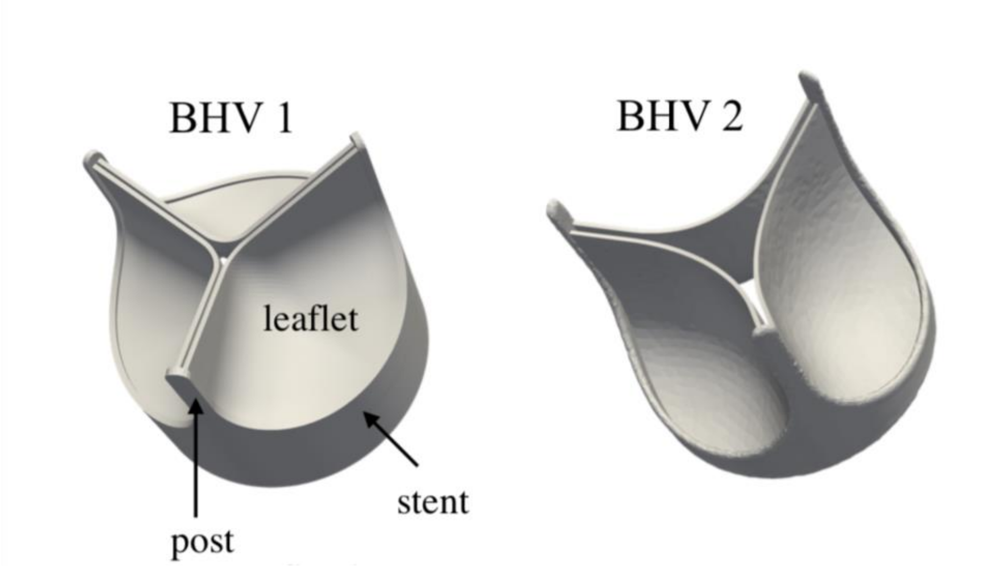
Geometry

Bioprosthetic heart valve

- Limited durability
- Numerical simulations for studying valve design

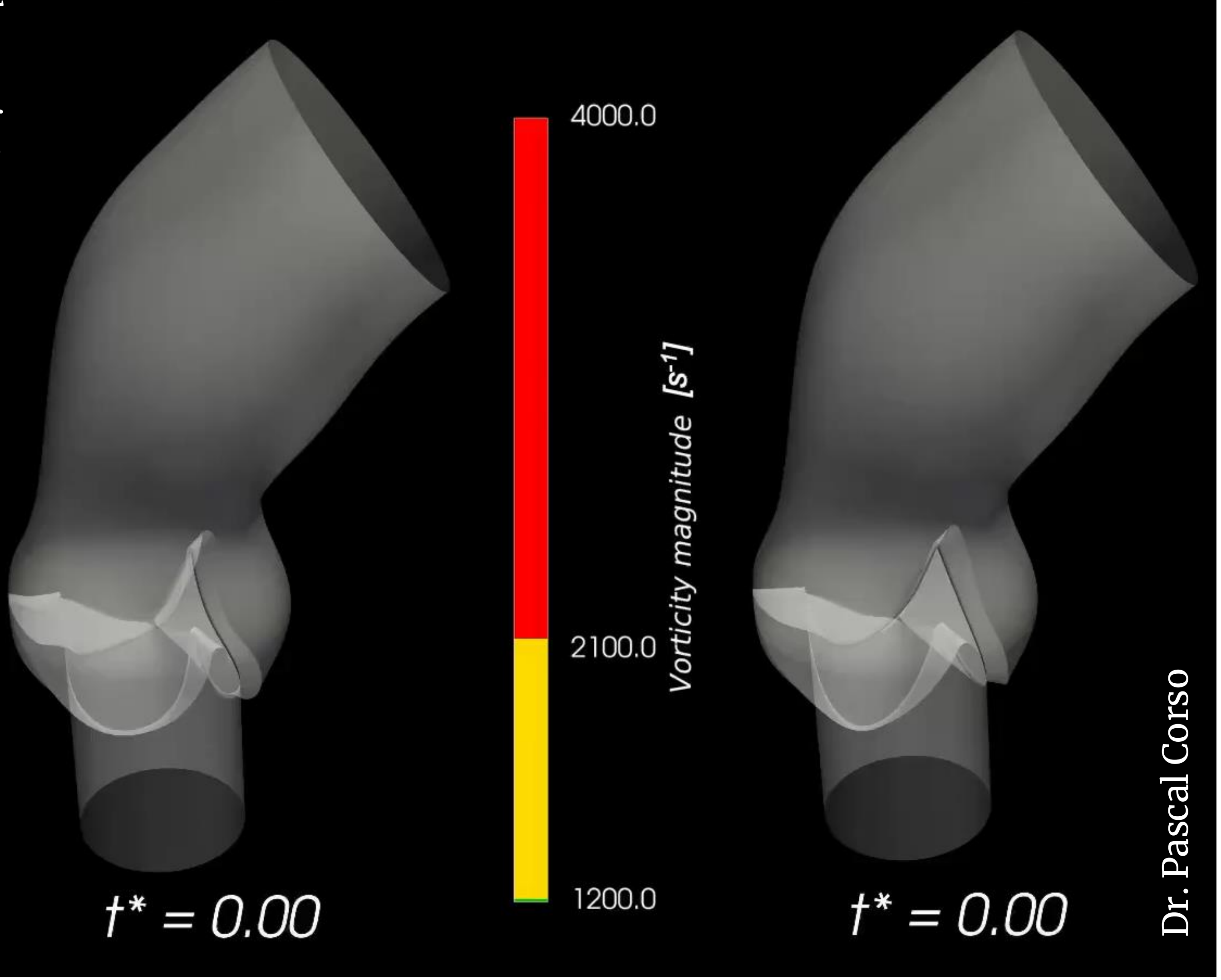
Difficulties in simulation

Contact occurring among the leaflets during the valve closure



AV-Flow project

Blood–soft tissue interaction



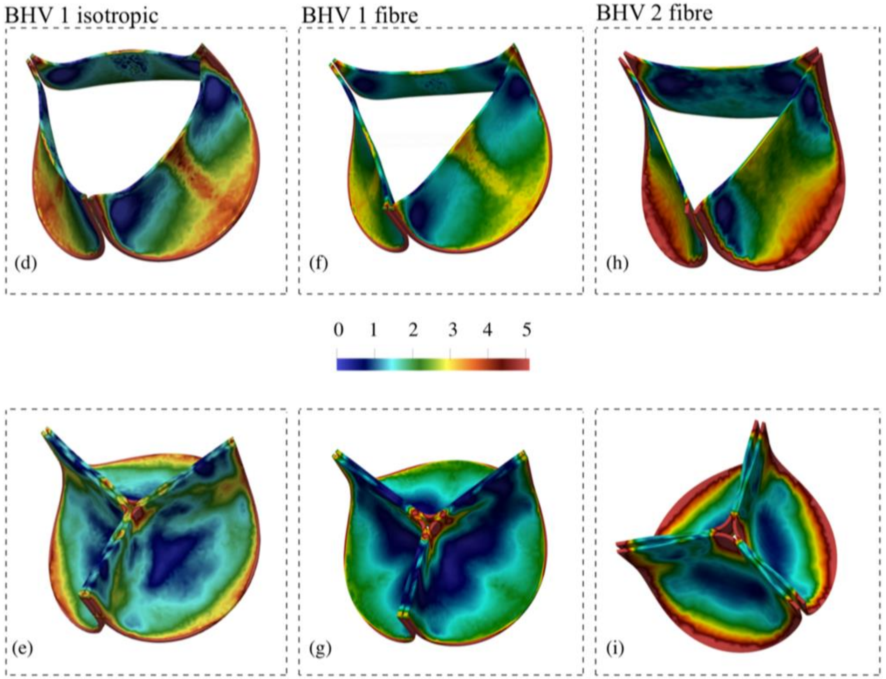
Dr. Pascal Corso

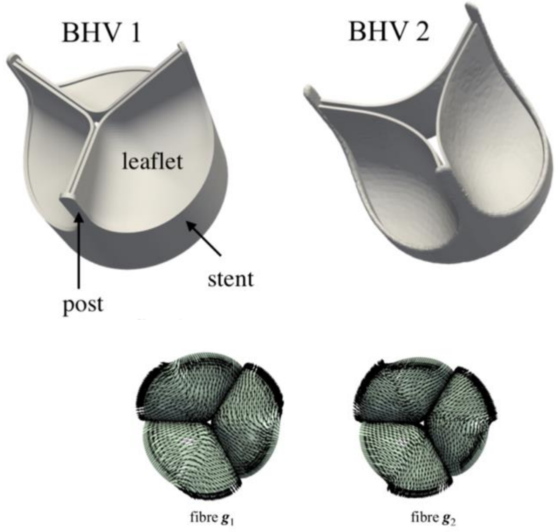
High-fidelity simulations of the coupled blood and valve dynamics during systole

Bio-prosthetic heart-valve simulation

BHV model

- Holzapfel fiber-reinforced material
- Two valve designs
- With and without **fibers**





Purely structure simulation of the BHV

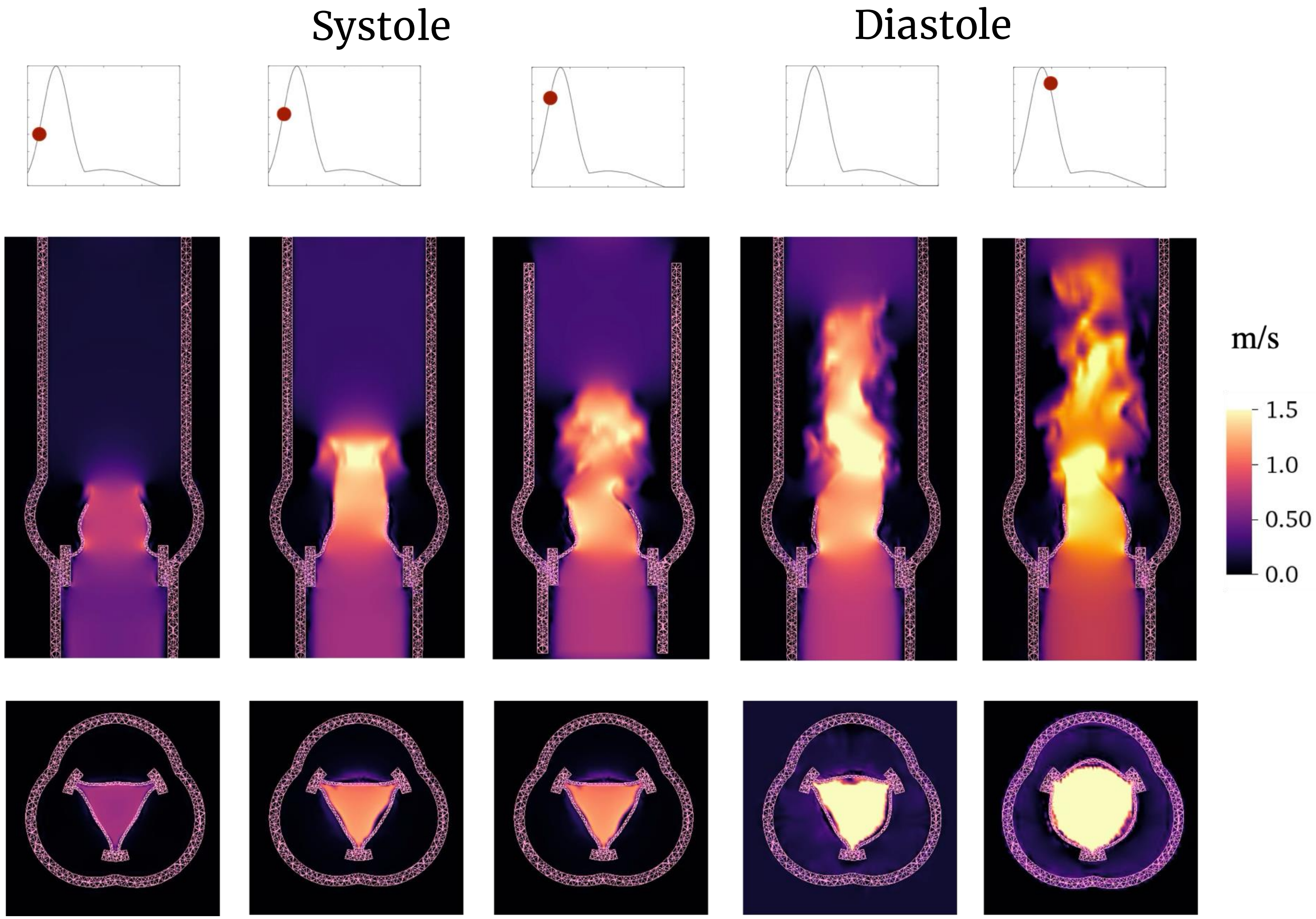
- Pressure profile imposed on the structure
- **VonMises stresses** are lower in the fiber-reinforced BHV model

Bio-prosthetic heart-valve simulation

Fiber reinforced BHV 1 performance

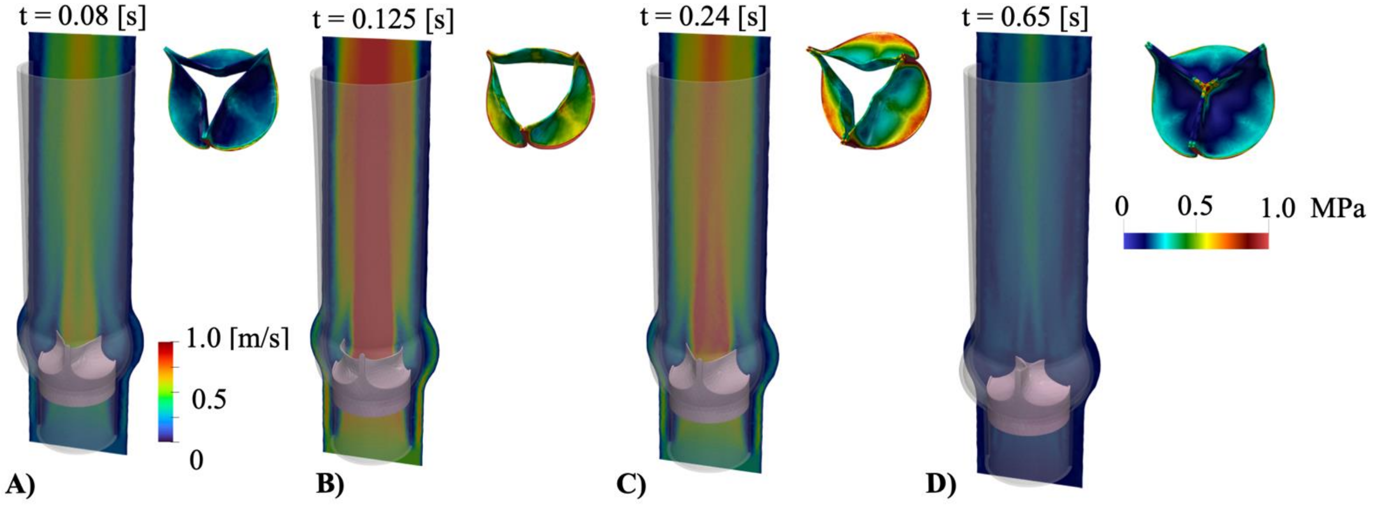
- Mechanical and haemodynamic performance
- Windkessel model for pressure gradient between 80 and 120 mmHg

More details in Nestola, Zulian, Gaedke -Merzhäuser, and Krause [2021]



Main results

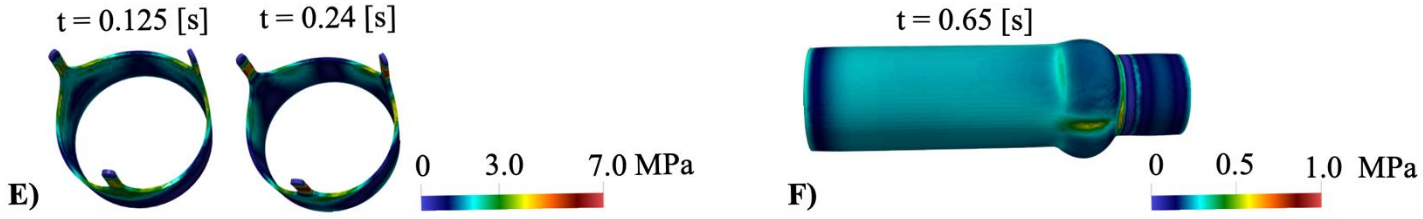
- **Stresses reduced** if leaflets are modelled as **fiber-reinforced material**
- **Valve design** may influence the **stress patterns**
- Stress concentration in the **central region** of the leaflets in the systolic phase and **close to the attachment** between leaflets and stent during the valve closure



agree very well with previous studies

Sigüenza J et al. *Fluid-structure interaction of a pulsatile flow with an aortic valve model: a combined experimental and numerical study*. Int J Numer Meth Biomed Eng [2018]

Wu MC et al. *An anisotropic constitutive model for immersogeometric fluid–structure interaction analysis of bioprosthetic heart valves*. J Biomech [2018]



Our computational framework allows the estimation of **abnormal flow patterns** and **recirculating areas**

Preliminary results

Contact with **fluid boundary**

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Fluid blocking

FSI

Overconstrained problem

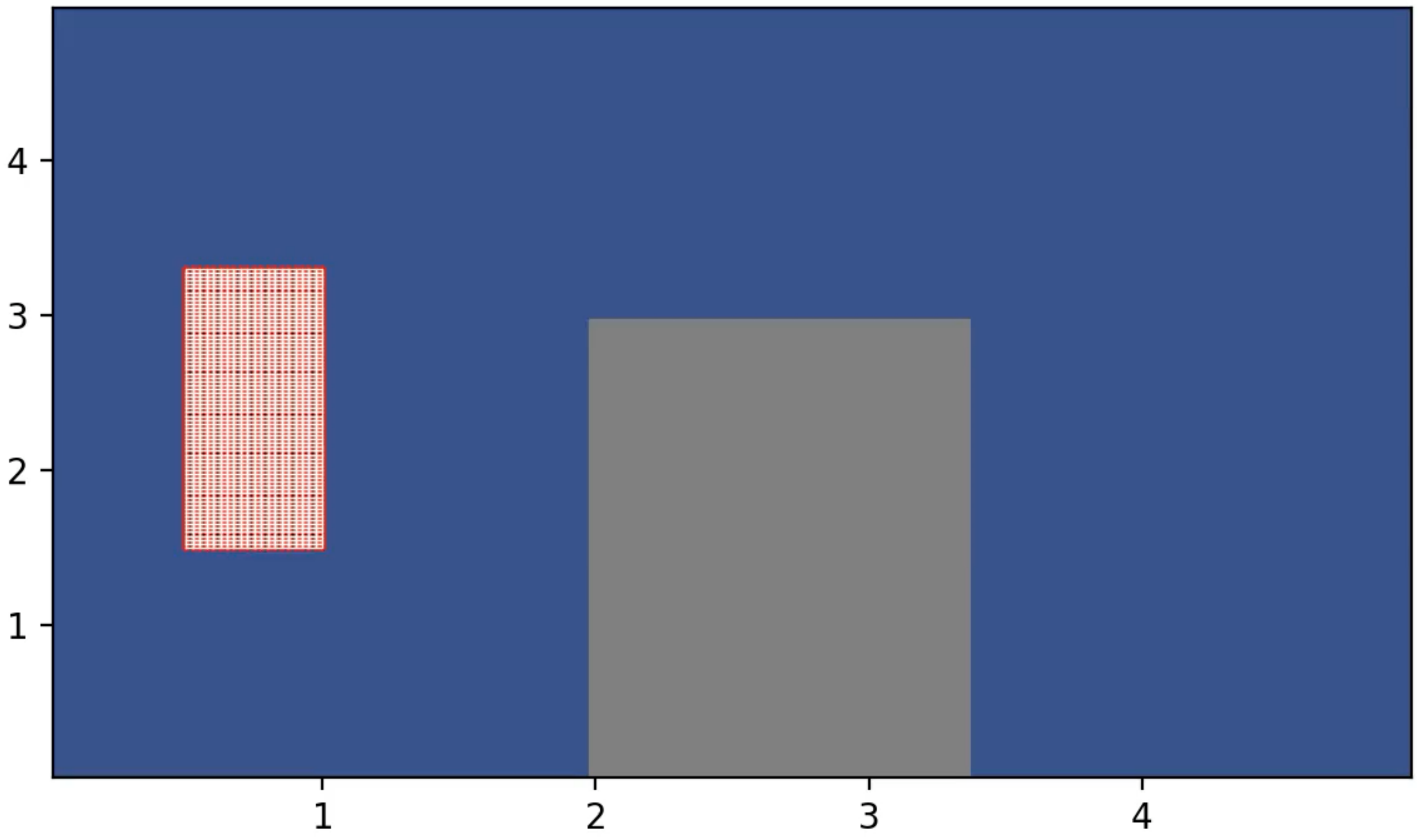
↓

Smooth coupling

approach for FSCI

Easy setup on a challenging scenario

Corner contact case



Application to liquid diaphragm pumps

Thank you for your attention!

Summary

- Immersed approach to FSI with contact between structures
- Example numerical applications
- Open-source libraries (BSD 3-clause license)
  - https://github.com/mfem/mfem (branch "moonolith_h1_bugfix" PR accepted)
  - https://bitbucket.org/zulianp/utopia
  - https://bitbucket.org/zulianp/par_monolith

Future work

- Focus on FSCI models and large scale FSI and FSCI
- Hybrid matrix-free and matrix-based algorithm on GPU
- Preconditioning techniques exploiting semi-structured operators

Acknowledgments

- Innosuisse project 48321.1 IP-ENG
- Swiss National Fund (SNF)
  - Immersed methods for fluid-structure-contact-interaction simulations and complex geometries
  - Stress-based methods for variational inequalities in Solid Mechanics
- UniDistance Suisse and USI-FIR
- PASC 2025-2028 – XSES-FSI

CMIS 21-24 April 2026

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