Assignment 1

For the first part (deadline 27 September, 2024) you have to find and describe solutions of the following 4 problems using the indicated tools or libraries.

1) Magic Factory

Six trucks have to deliver pallets of obscure building blocks to a magic factory. Every truck has a capacity of 8000 kg and can carry at most ten pallets. In total, the following has to be delivered:

- Six pallets of nuzzles, each of weight 800 kg.

- Twelve pallets of prittles, each of weight 405 kg.

- Fifteen pallets of skipples, each of weight 500 kg.

- Eight pallets of crottles, each of weight 2500 kg.

- A number of pallets of dupples, each of weight 600 kg.

Skipples are round and elastic, only two out of the six trucks have facilities for fastening them. Prittles are somewhat valuable: those pallets need to be distributed over *at least* five trucks.

1. Investigate what is the maximum number of pallets of dupples that can be delivered, and show how for that number all pallets may be divided over the six trucks.

2. Do the same, with the extra information that crottles get too cold if there are less than two dupples in the same truck.

2) Poster Printing

A printing company wants to fit eleven posters tightly on a large sheet of canvas. This needs to meet the following constraints:

- Both the width and the height of the canvas is 30.

- The sizes of the eleven posters are 4×5, 4×6, 5×21, 6×9, 6×8, 6×10, 6×11, 7×12, 8×9, 10×11, 10×20, respectively.

- All posters may be turned $90°$, but may not overlap.

- In order to make the first cut easier, there should be a line in the canvas that does not cross through any poster.

1. Give a layout of the posters on the canvas following the above requirements. Also indicate the line.

2. What if the line needs to be at least 10 length units away from the edge of the canvas?

3. And what if there must be a second line that is perpendicular to the first line? (requirement (b) does not need to be applied here)

3) Food Supply

Three non-self-supporting villages A, B and C consume one food package each per time unit. The required food packages are delivered by a truck, having a capacity of 130 food packages. The truck has to pick up its food packages at location S containing an unbounded supply. The locations of this supply location and the villages and the roads between them are shown in the following picture. Here every number indicates the number of time units the truck needs to travel from one village to another, including loading and delivering. There is often a lot of traffic going from B to C, hence it takes a bit longer than the journey from C to B. The villages only have a limited capacity to store food packages: for A and C this capacity is 90, for B it is 120. Initially, the truck is in S and is fully loaded, and in each of the four villages there are 60 food packages.
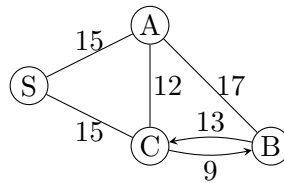
1. Show that it is impossible to deliver food packages in such a way that each of the villages consumes one food package per time unit forever. Hint: after a certain number of steps all runs end in starvation.

2. Show that this is possible if the capacity of the truck is increased to 150 food packages. Hint: since this problem has finite size, it is sufficient to look for a "lasso-shaped" finite schedule $s_0, \ldots, s_i, \ldots, s_n$ such that $s_i = s_n$ for some i. Make sure your schedule can be sustained indefinitely.

4) Program Verification

Consider the following program:

```
a := 1; b := 1;
for i := 1 to 10 do
    if ? then {
        a := a + 2b; b := b + 3;
    } else {
        a := a + i; b := b − a;
    }
if b = 210 + n then crash
```

Here '?' is an unknown test that may yield false or true in any situation. Note that the test on crash is outside the loop, so is only tested at the end.

Establish for which values of $n = 1, 2, \ldots, 10$ it is safe, that is, will not reach 'crash'. Show for one of the non-safe values of n how $b = 210 + n$ can be reached.