MXET 375
Applied Dynamic Systems

Multidisciplinary
Engineering Technology
COLLEGE OF ENGINEERING

Final Project

Double Pendulum

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Section 100
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Introduction & Description

The objective of this project is to design, construct, and analyze a physical double pendulum system and to model the system using Simulink. The choice of a double pendulum was instituted across the lab teams, but this project could be differentiated by its use of a rotary encoder to measure the angular position of the second joint [3]. The physical double pendulum will also be set up so that the first joint is provided with input torque from a motor with an Arduino Nano [1] using a L298N [2] motor driver. The torque that is input will generate an angular displacement on the first joint that should cause rotational motion in the second joint. The measured values that are collected from the physical system will be used to parametrize the physical system with the Simulink software. The double pendulum system is a rotational system that is expected to exhibit chaotic behavior.

Implementation Plan and System Design

Dynamics Modeling

To understand the dynamics of the double pendulum before any experimentation, the double pendulum system is derived into a mathematical model [8]. To derive the pendulum's equations of motion it is first described using the free-body diagram so that all the forces can be considered. The free-body diagram drawn to derive the ODE equations is shown in **Figure 1**.

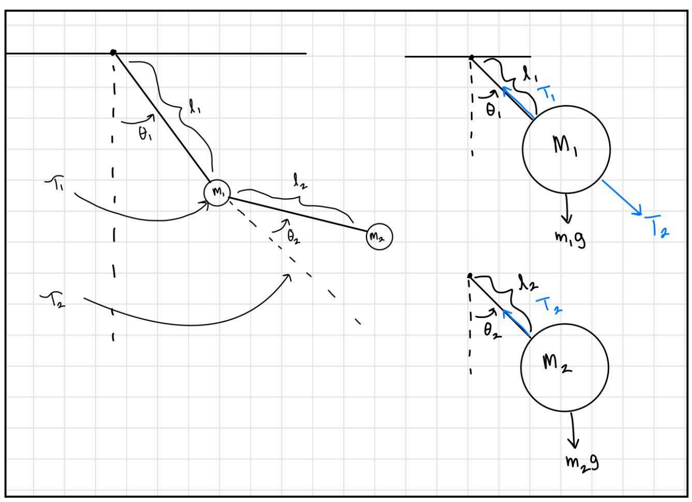

Figure 1. Free body diagram of double pendulum system.

The equations of motion can be obtained by establishing equations for the sum of forces on the two bodies. It is seen that the only forces on the bodies are the tension in the pendulums, T_1 and T_2, and the force due to gravity.

$$\Sigma m\frac{d^2\overline{r_1}}{dt^2} = m_1\overline{g} - T_1\frac{\overline{r_1}}{|\overline{r_1}|} + T_2\frac{\overline{r_2}-\overline{r_1}}{|\overline{r_2}-\overline{r_1}|} \qquad (1)$$

$$\Sigma m \frac{d^2\overline{r_2}}{dt^2} = m_2\overline{g} - T_2 \frac{\overline{r_2}-\overline{r_1}}{|\overline{r_2}-\overline{r_1}|} \tag{2}$$

The tensions in the string are known to be directed in the direction of the pendulum's position vectors. The position vectors can be used to write the system in terms of the angular positions of the pendulums.

$$\overline{r_1} = l_1(sin(\theta_1)\ \overline{i} + cos(\theta_1)\ \overline{j}) \tag{3}$$

$$\overline{r_2} = \overline{r_1} + l_2(sin(\theta_2)\ \overline{i} + cos(\theta_2)\ \overline{j}) \tag{4}$$

After differentiating the position vectors they can be inserted into the equations found using the free-body diagram (equations 1 and 2) to determine the system's rotational motion over time. The equations found for the torques on the two pendulums are shown below.

$$\tau_1 = -\ m_1 l_2 \frac{d^2\theta_2}{dt^2} \frac{1}{sin(\theta_2-\theta_1)} \tag{5}$$

$$\tau_2 = m_1(l_1 \frac{d^2\theta_1}{dt^2} + gsin(\theta_1)) \frac{1}{sin(\theta_2-\theta_1)} \tag{6}$$

Simulink Simulation

The simulation for the double pendulum is created using the multibodies library in Simulink. The model is configured so that it can accurately represent the physical system that is to be implemented. The model is adjusted so that the motor that provides an input torque can be imitated and the angular positions of the joints can be acquired. The model that is established is shown in **Figure 2**.

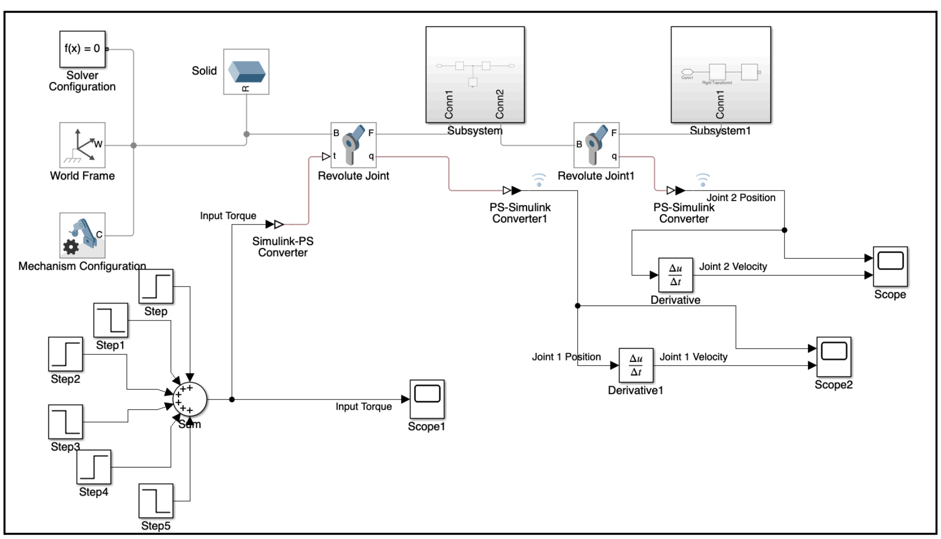

Figure 2. Simulink block diagram of double pendulum system.

The World Frame block in the pendulum system is used to establish the overall reference frame of the system. This means that all other blocks in the system are defined with respect to this frame. This is very important for analysis because it indicates that the base for the pendulum will always remain at rest. The mechanism configuration is used to specify the gravity of the system in the correct coordinate axis according to the block placement.

To produce torque accurate to the actual motor used, Step blocks were used in aggregation to mimic a special kind of square wave. This was achieved by alternating ascending and descending steps. When paired with experimental data, the correct amplitudes and step duration could be determined. This data was transmitted forward as an input torque to the first revolute block.

The first Revolute Joint block in the pendulum system is adjusted so that input torque from the motor can be implemented and the signal for the rotational position of the joint is displayed using a Scope block. The first joint is connected to the rigid body subsystem shown in **Figure 3**, which allows the first pendulum to have rotational motion.

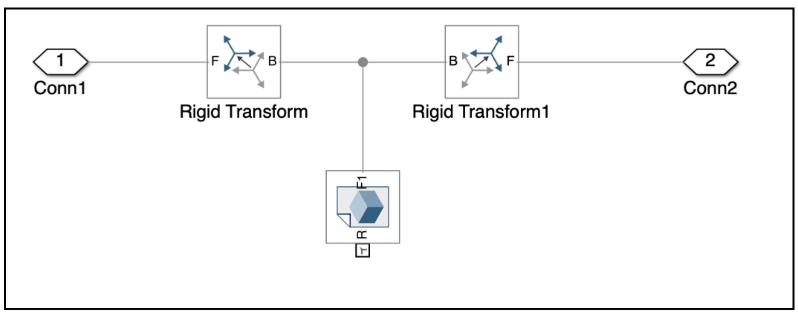

Figure 3. Block diagram of first limb subsystem.

The first limb subsystem is what establishes the first rigid pendulum. The Rigid Transform blocks in the subsystem ensure the geometry of the solid is unaffected. The first Rigid Transform is given an initial translation of 1 in in the negative Z direction so the initial position of the first pendulum is correct. The second Rigid Transform is given an initial translation of 12.7 in in the negative X direction so that the second pendulum is directly below the first pendulum. The file solid allows for the cad file of the mass to be implemented as the rigid body. This ensures that the geometry of the system is correct.

The second Revolute Joint block is only adjusted so that the angular position can be displayed with a Scope block. This is because the second Revolute block will rotate due to the resultant torque produced by the first pendulum's motion and does not receive other inputs. The second joint is connected to the rigid body subsystem shown in **Figure 4**, which allows the second pendulum to have rotational motion.

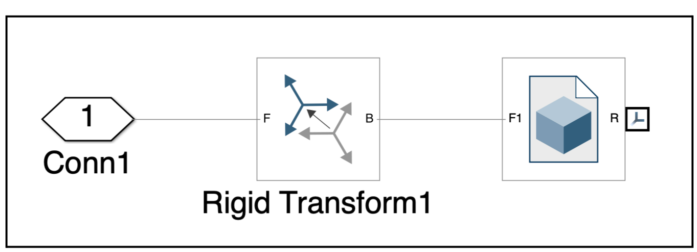

Figure 4. Block diagram of second limb subsystem.

The second limb subsystem is what establishes the second rigid pendulum. The Rigid Transform is given an initial translation of 2 in in the negative Z direction so the initial position of the first pendulum is correct. The solid view of the pendulum system is shown in **Figure 5**.

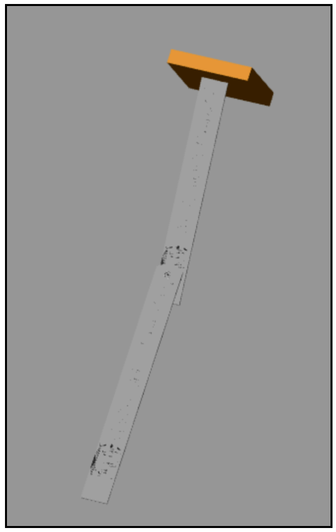

Figure 5. Mechanical view of the Simulink model.

Physical Results and Discussion

With the block schematic for the double pendulum system completed, specific materials are required to build the physical experiment. Two 12 $inch$ rulers are utilized as the rigid bodies that would be moving as pendulums. The mass of the rulers is a known constant of 0.090 kg according to the manufacturer datasheet [5]. A Taiss rotary encoder is used to connect the two pendulums so that the angular displacement of the second joint can be measured by the Arduino [1] [4]. The rotary encoder is known to have 20 $digits/revolution$, meaning that the encoder can measure an angular displacement in increments of 0.314 $rads/digit$ [3]. To help prevent debouncing in the encoder's switches, two simple RC filter circuits are implemented to ensure accurate data collection.

The motor that is attached to the top joint is driven by the L298N motor driver [2]. The motor driver is controlled by the Arduino and it is programmed to provide a torque to the pendulum using a 12 V source. The motor is set to rotate at max speed in the positive angular direction for the first 500 ms of the experiment, wait for 150 ms, and then rotate in the negative angular direction for 100 ms. A 16x2 liquid crystal display using the I2C interface is also connected to the Arduino so the measurements can be seen by an observer [6]. All of the electrical components utilized in the physical system are shown in **Figure 7**.

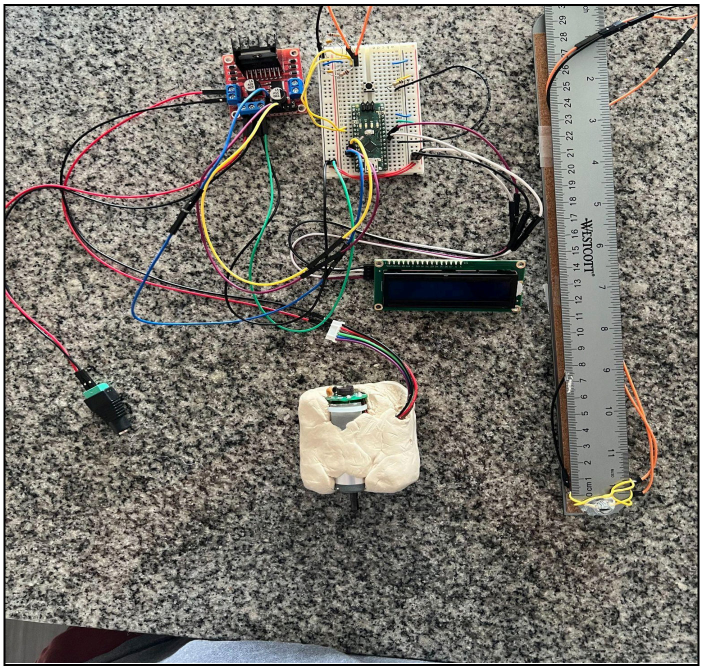

Figure 7. Electrical components in the physical double pendulum system.

To help provide stability, air-dry clay is molded around the motor, and the wires are attached to it in the shape of a cube. To ensure that solid contact is made between the motor and the pendulum a sleeve is placed onto the DC motor shaft and screwed in as shown in **Figure 8**. This ensures that all the torque generated by the motor is input into the pendulum system and not wasted.

Figure 8. DC motor sleeve holding the ruler in place.

The results that were obtained from the physical system are exported to Excel via the Arduino serial monitor so that the motion can be plotted. The plots that are produced from the results of the experiment are shown in **Figure 9**.

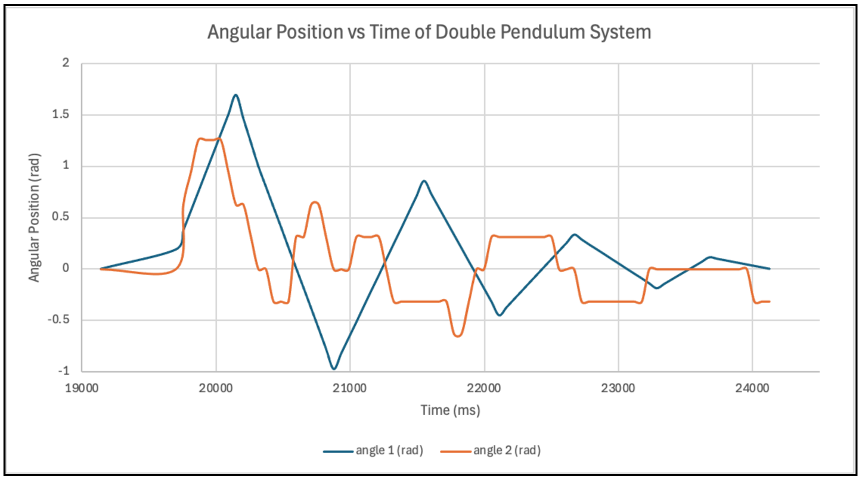

Figure 9. Angular Position vs Time of the Double Pendulum System

The physical experiment results shown in **Figure 9** show the second joint displays erratic behavior, as expected from a double pendulum system. It is seen that Angle 1 exhibits less chaotic motion than Angle 2, oscillating between peak amplitudes of approximately 1.75 and -1 rad while Angle 2 is non-repeating and more complex. Angle 1 exhibits less chaotic motion because the position of the first angle is trigonometrically related to the input torque of the motor to the first joint in the system (equation 5). With damping it is expected that the angle would slowly decrease over time which is exactly what is seen for Angle 1. The equations of motion for the double pendulum system are very nonlinear but it is more pronounced in the second pendulum's angular position. The amplified pattern of motion is due to the compounded effects of the forces acting on it and the energy transferring from the first to the second pendulum [8].

Simulation Results and Discussion

With the results from the physical experimentation, a parameter estimation can be conducted. Parameter estimation is the process by which unknown variables used throughout the Simulink block parameters are optimized to fit experimental data. From the output generated by the Arduino, CSV files are imported into the "Parameter Estimator" application within Simulink. The measured data for both joints was paired with the angular position sensed by each Revolute Joint block. The variables *b1* and *b2* represent the dampness coefficient for joints 1 and 2, respectively. Similarly, the variables *k1* and *k2* are for the spring stiffness values.

Table 1. Dynamic equation values

Block	Parameter	Value
Revolute Joint 1	Spring Stiffness	0.0256 $N \cdot m/rad$
Revolute Joint 1	Dampness Coefficient	0.522 $kN \cdot m/(rad/s)$
Revolute Joint 2	Spring Stiffness	0.513 $N \cdot m/rad$

Revolute Joint 2	Dampness Coefficient	2.822 $mN \cdot m/(rad/s)$

Table 1 displays the result of the estimation, each variable relevant to the system equations. Some other variables can be seen in the estimation plot **Figure 10**, namely *amp* and *freq*, which were used to match the torque input.

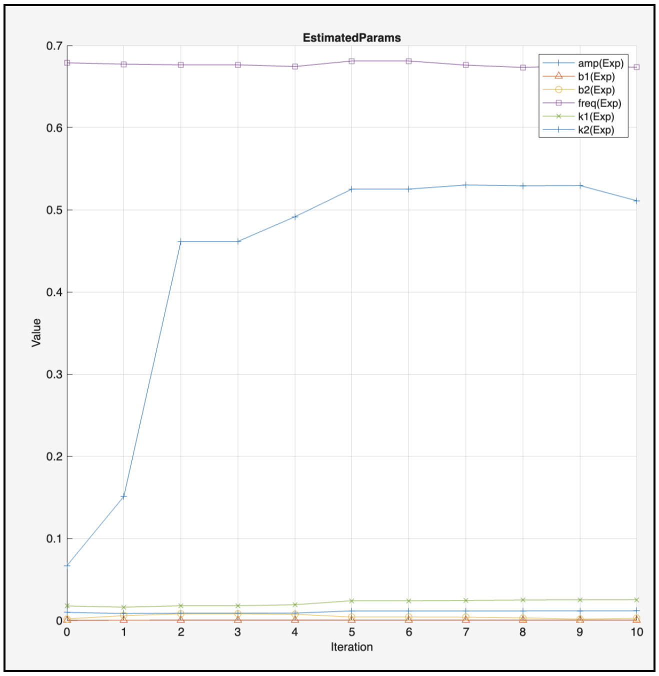

Figure 9. One example round of estimation.

As the variables operate at different scales, it is difficult to tell the progress made through **Figure 10** alone. The fit of the simulated results to the measured results can be seen in **Figure 11** below.

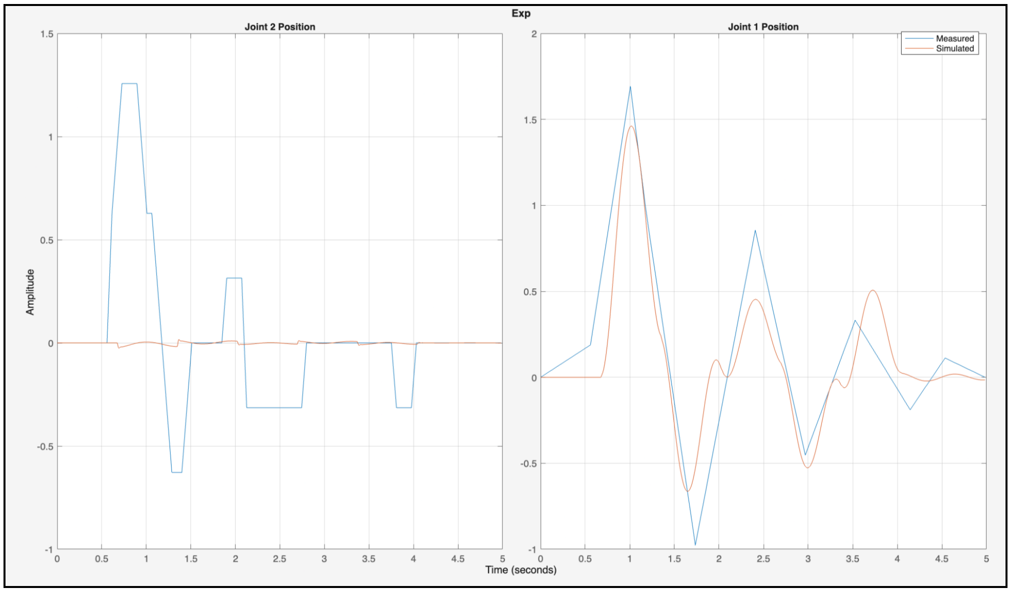

Figure 11. Simulated results compared to measured results.

While the first joint position is accurate between measured and simulated values, the position of the second joint was not able to be accurately approximated. This is likely due to the dependence of the second joint on the first. As the first joint wasn't perfect, it was beneficial for the estimator to reduce error by keeping the second joint position as constant as possible. That was achieved by increasing the Damping Coefficient quite a bit, as can be seen in **Table 1**.

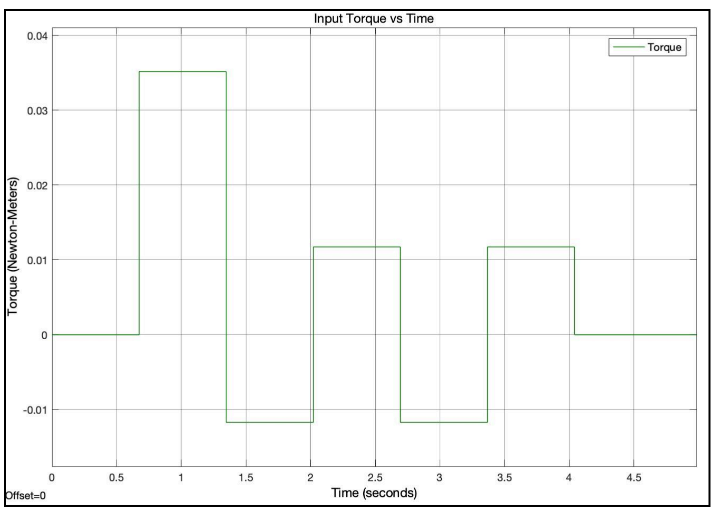

Figure 12. The torque, amplitude, and step size were determined by estimation.

The first joint, physically, was completely controlled by the motor. The simulated model, however, must take a torque as an input. With a preconfigured step input torque, aided by the parameter estimator, the input torque over time, **Figure 12**, was determined. To more accurately model the torque provided from the motor an ammeter could be used to measure the current that flowed into the motor at any given moment. The current provides a value that is known to have a linear relationship with the torque of the motor [9].

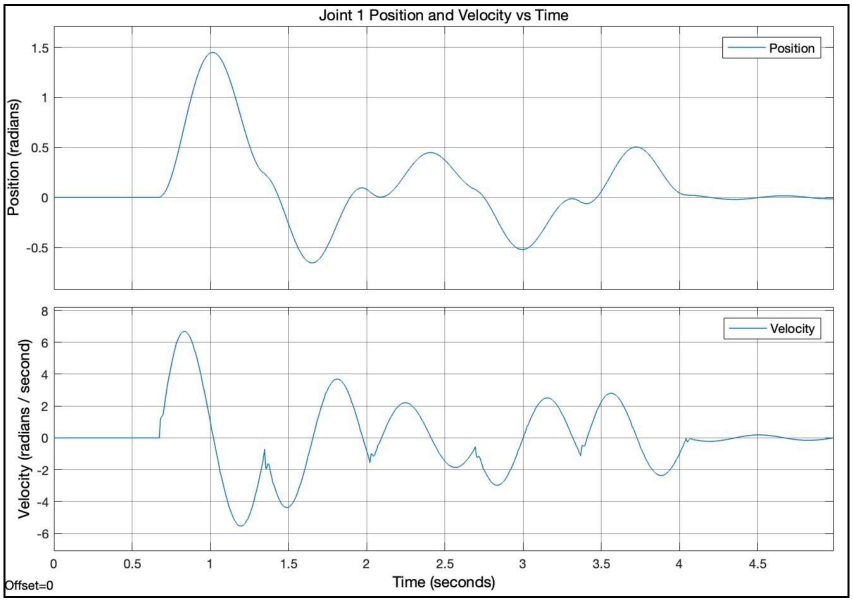

Figure 13. The position and velocity of the first joint over time, simulated.

Figure 13 above depicts the simulated positions and velocities of the first joint over time. While not configured this way, the position resembles a sine wave. This confirms the close approximation depicted in **Figure 11**, as the equations in the intro include sine functions to simulate the trigonometric relationship for the first torque (equation 5).

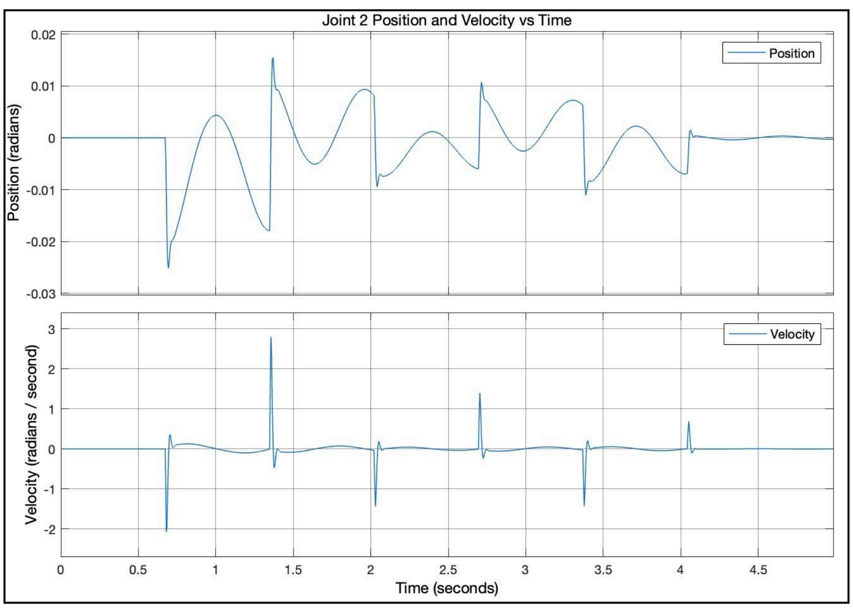

Figure 14. The position and velocity of the second joint over time, simulated.

The second joint position shown in **Figure 14** is quite sporadic and unpredictable. This is typical of a chaotic double pendulum, where a small change in input causes a large change in its output. As the position of the second joint depends on the position of the first, much of the distortion is propagated [8]. With a larger sample size and more accurate torque representation, better parametrization might be accomplished in a future experiment.

Conclusion

The goal of this project was to create a physical double pendulum system, simulate the system in Simulink, and compare the physical and simulated results. The system was planned using a free-body diagram and then derived into a mathematical model. Afterward, a simulation of the system was completed in Simulink, and the physical experiment was finalized. Practical challenges were faced when creating the physical experiment such as determining materials to use and securing pieces to each other. Using the results from the simulation and the physical experiment, parameter estimation was used to optimize the Simulink block parameters so that they fit the experimental results. To improve the parameter estimation for the simulation, factors like friction, motor back-emf, motor current, and air resistance could be considered. Likewise, a larger measurement sample size could be utilized to improve the estimation. Overall, the final results confirm the chaotic nature of double pendulum systems as shown in both the simulated and physical results. This project provided experience with data analysis, a tool directly applicable to many fields of engineering. This project was also the first chance for students to apply the knowledge of mechatronics first-hand and acquire experimental results for analysis.

References

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