Multidisciplinary
Engineering Technology
COLLEGE OF ENGINEERING

MXET 375 Applied Dynamic Systems Laboratory

Final Project

Double Pendulum

Section 100 - Group 2
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Project Links

Team Folder Link:

https://drive.google.com/drive/folders/1Skka4tQ_YpWwGD7t6B2NOuANbXup-SGk?usp=drive_link

Introduction

- Dynamic systems are systems that involve changes over time. They are modeled using dynamic equations of motion which is a type of mathematical model composed of differential equations. These are typically solved by computers.
- The project is to build a double pendulum dynamic system. This includes the design, modeling, prototyping, and experimentation.
- The time period to construct the project was 7 days. It was highly encouraged to build a double pendulum due to time constraints with complexity.

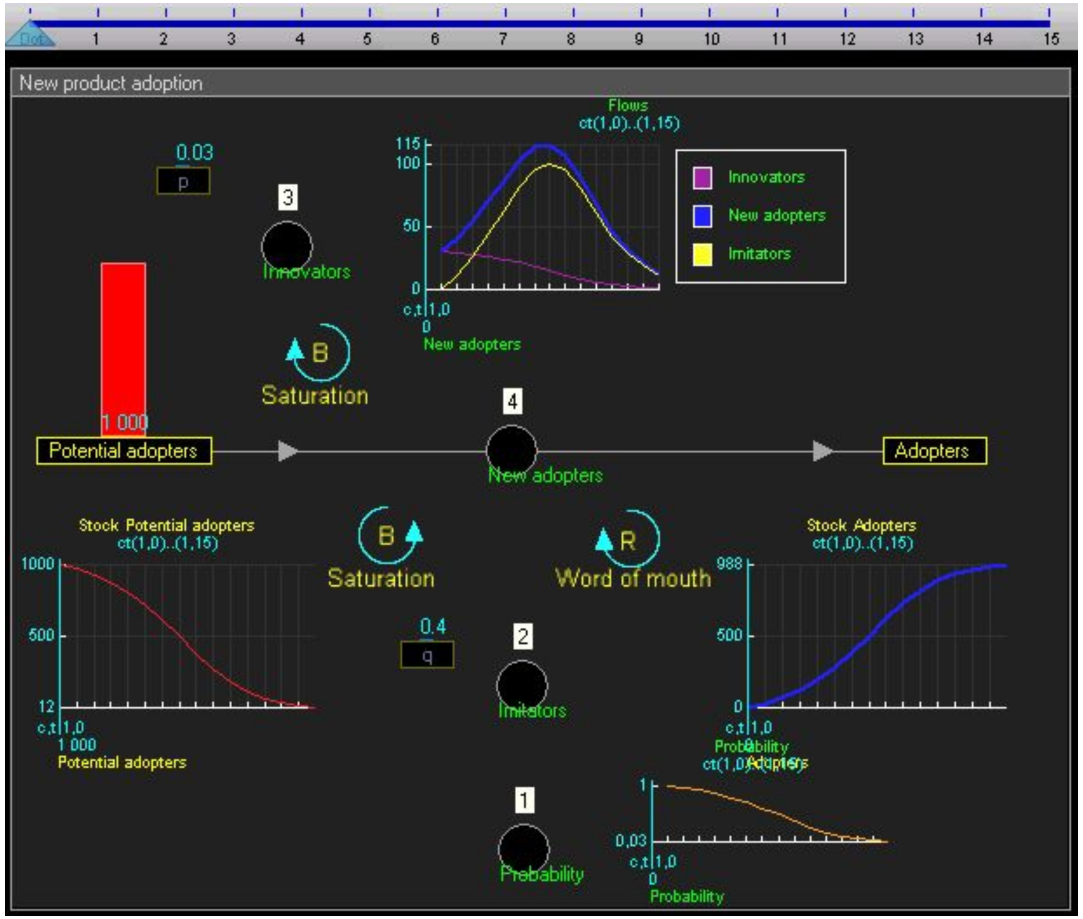


Figure 1: Dynamic system graphic [1]

Project Description

- The choice of a double pendulum was instituted across the lab teams, but this project could be differentiated by its use of a rotary encoder to measure the angular position of the second joint [2].
- The overarching objective was to derive the dynamical system, create a model in Simulink, build a physical prototype, and use the experimental data to fine-tune the model parameters.
- The double pendulum system is a rotational system that is expected to exhibit chaotic behavior.

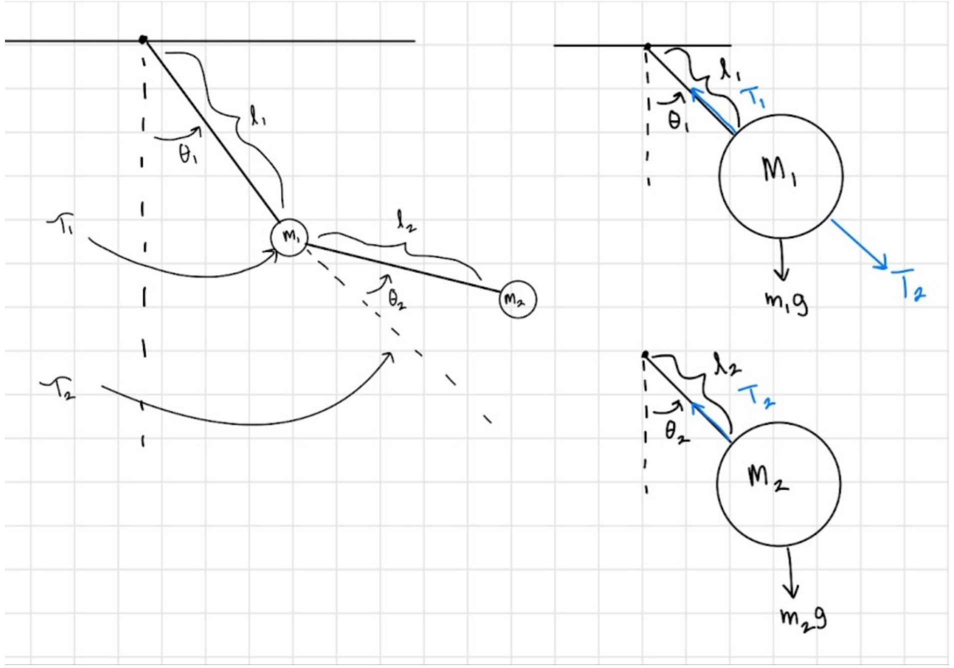


Figure 2: Double Pendulum FBD

Mechanical Design

- The pendulum is a system of 3 bodies:
  - Motor and Stand
  - Top ruler (pendulum 1)
  - Bottom ruler (pendulum 2)
- The bodies are connected by revolving joints. The first being a motor, the second a rotary encoder.
- The motor is connected to the top ruler with a screw clamp.
- The two rulers are made of aluminum, with light cork on one side.

Figure 3: DC motor sleeve holding the ruler in place.

Electrical/Electronic System Design

- The electronic system is controlled by an Arduino Nano [3].
- The Arduino is responsible for providing the input signal to a L298N motor driver to actuate the motor [4]. This provides an input torque to the system.
- The Arduino is also responsible for measuring and recording the rotational position of the two joints in the system. This data is used to plot the motion of the joints.

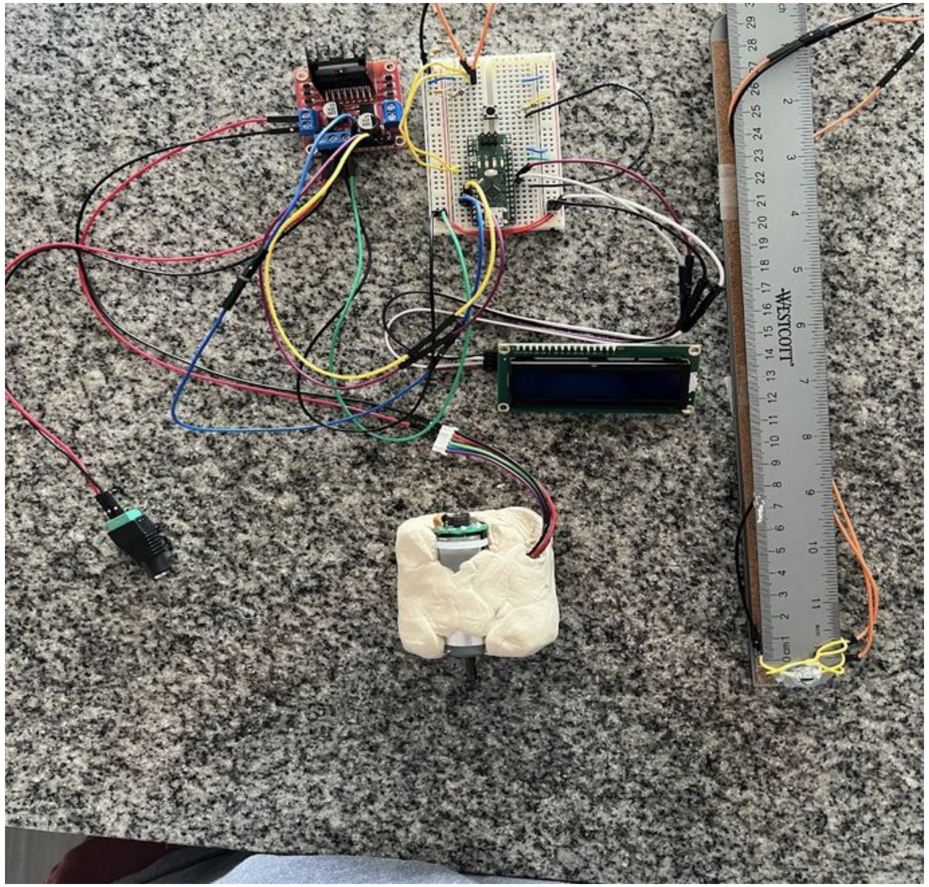


Figure 4: Electrical components in the physical double pendulum system.

Mathematical Model

- The equations of motion for the double pendulum can be obtained by first utilizing the free-body diagram for the system to describe the sum of forces.
- Where m1 is the mass of first pendulum, T1 is the tension in the first pendulum, r1 is the position of the first pendulum, m2 is the mass of the second pendulum, T2 is the tension in the second pendulum, r2 is the position of the second pendulum, and g is the acceleration due to gravity.
- It is seen that the only forces on the bodies are the tension in the pendulums and the force due to gravity. It is also known that the direction of the tension over time is always the same as the direction of the pendulum position vector.

$$\Sigma m_1 \frac{d^2\overline{r_1}}{dt^2} = m_1\overline{g} - T_1\frac{\overline{r_1}}{|\overline{r_1}|} + T_2\frac{\overline{r_2}-\overline{r_1}}{|\overline{r_2}-\overline{r_1}|}$$

$$\Sigma m_2 \frac{d^2\overline{r_2}}{dt^2} = m_2\overline{g} - T_2\frac{\overline{r_2}-\overline{r_1}}{|\overline{r_2}-\overline{r_1}|}$$

Mathematical Model

- To describe the pendulum system in greater detail, the position vectors of both pendulums are established.
- Where r1 is the position of the first pendulum, l1 is the length of the first pendulum, θ1 is angular position of the first joint, r2 is the position of the second pendulum, l2 is the length of the second pendulum, and θ2 is angular position of the second joint.
- The position vectors are shown to be trigonometric in nature, which is expected for an oscillating system such as a pendulum.

$$\overline{r}_1 = l_1(\sin(\theta_1)\,\bar{i} + \cos(\theta_1)\,\bar{j})$$

$$\overline{r}_2 = \overline{r}_1 + l_2(\sin(\theta_2)\,\bar{i} + \cos(\theta_2)\,\bar{j})$$

Mathematical Model

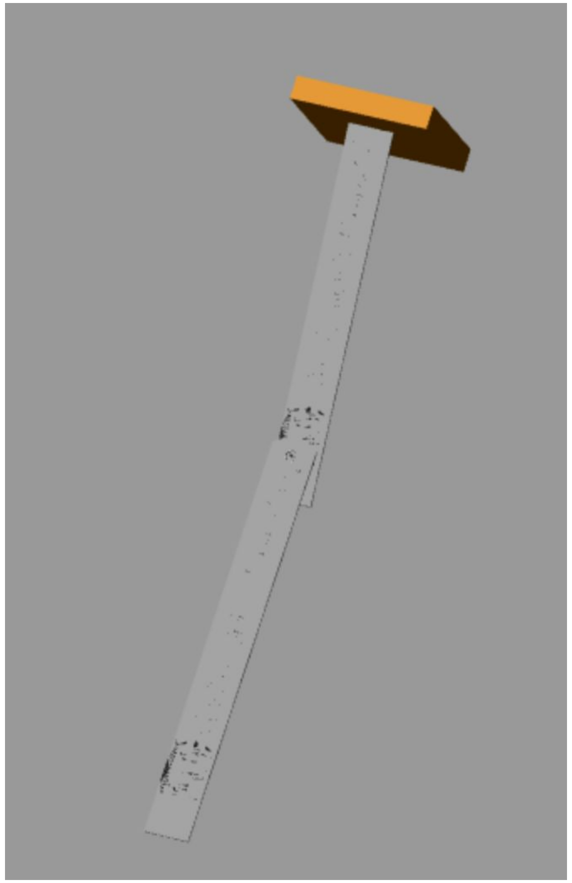
- After differentiating the position vectors they are inserted into the equations found using the free-body diagram. This establishes the equations of motion of the two joints in terms of the torques and angular positions.
- Where $\tau 1$ is the torque of the first pendulum, m1 is the mass of the first pendulum, l1 is the length of the first pendulum, θ1 is angular position of the first joint, $\tau 2$ is the torque of the second pendulum, m2 is the mass of the second pendulum, l2 is the length of the second pendulum, and θ2 is angular position of the second joint.

$$\tau_1 = -\, m_1 l_2 \frac{d^2\theta_2}{dt^2} \frac{1}{sin(\theta_2 - \theta_1)}$$

$$\tau_2 = m_1 (l_1 \frac{d^2\theta_1}{dt^2} + g sin(\theta_1)) \frac{1}{sin(\theta_2 - \theta_1)}$$

Simulation Results

- The simulated pendulum is mainly three solids, connected by two revolute joints.
- The first (orange) body represents the motor and motor sleeve. The second two are the pendulum rulers.
- Input torque is created and input into the first joint to mimic the motor.
- Angular position is sensed from both joints and plotted (on the next slide).



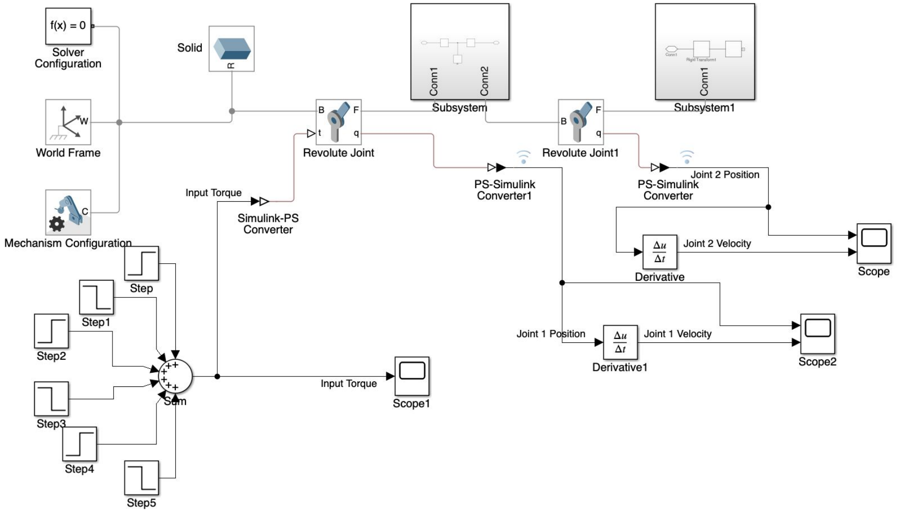


Figure 5 (top): Simulated pendulum block diagram

Figure 6 (left): Still-frame of simulated pendulum

Simulation Results

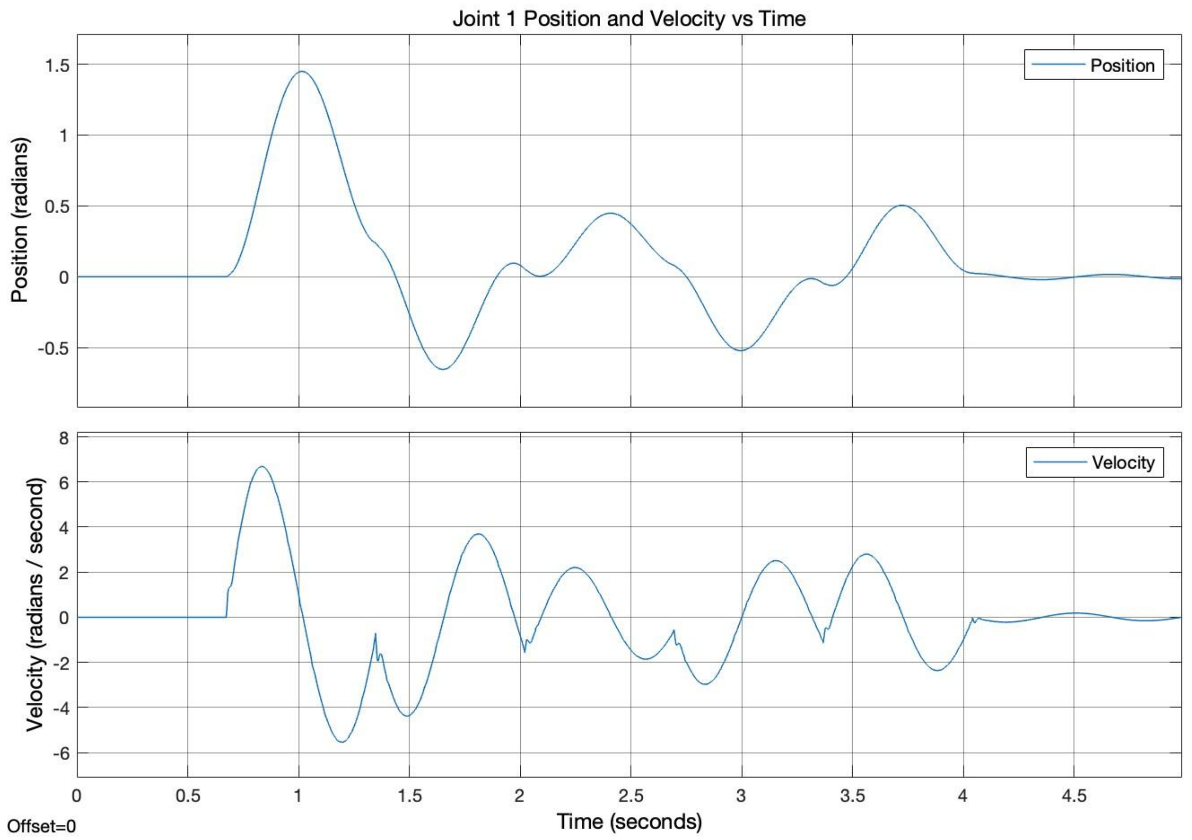


Figure 7: Simulated position and velocity of the first joint

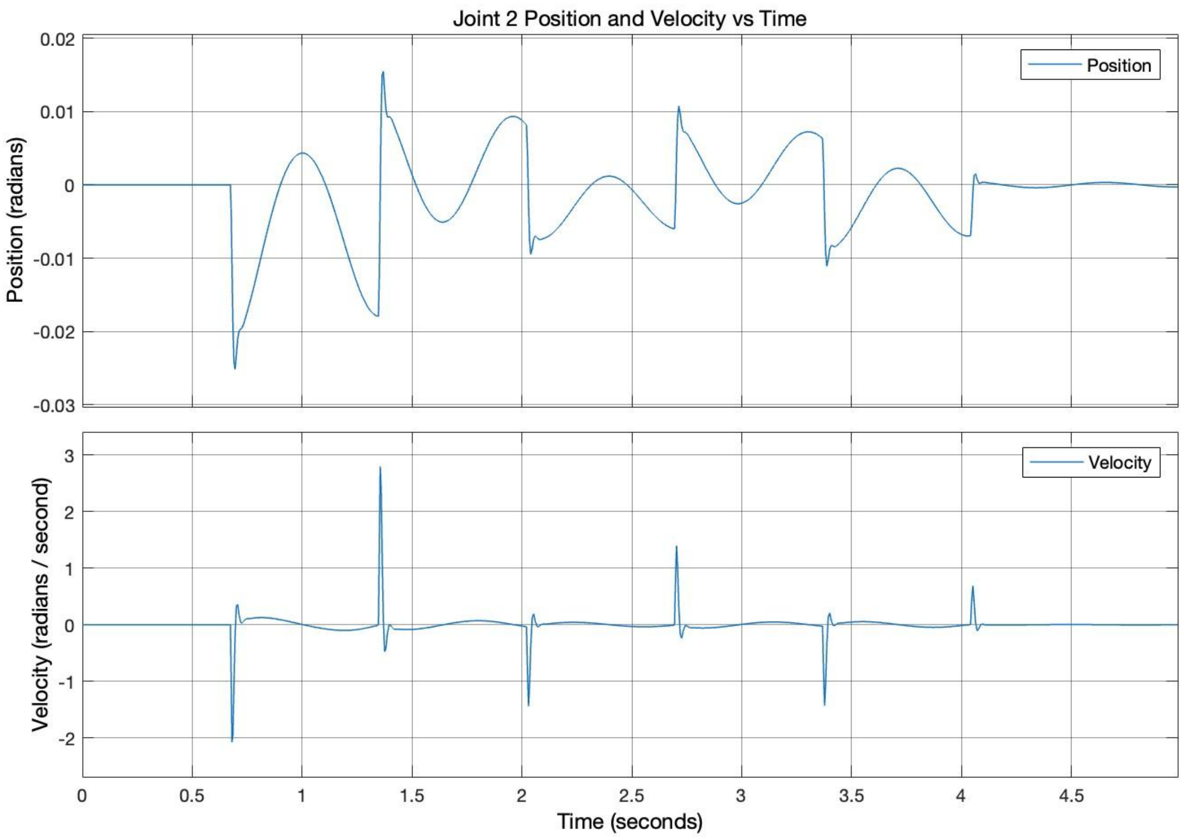
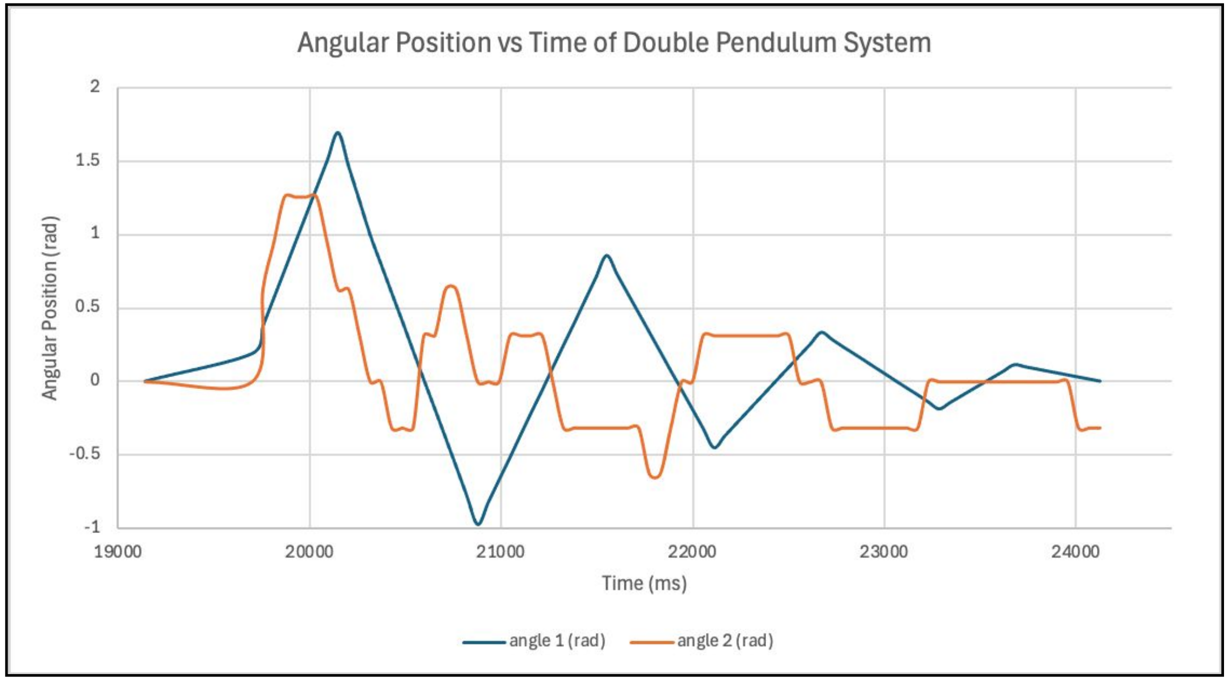


Figure 8: Simulated position and velocity of the second joint

Experimental Results



- Angle 1 has less chaotic motion than Angle 2, oscillating between -1 and 1.75 rad.
- Angle 2 is non-repeating and more complex.
- Position of Angle 1 is trigonometrically related to the input torque of the system explaining the oscillatory behavior.
- Angle 2 has an amplified pattern of motion due to compounded effects of forces and energy transfer.
- The amplitude of the position of Angle 1 decreases over time due to system damping.

Experimental Results

- Parameter estimation was used to optimize the variables of the simulation to fit the experimental data.
- The position of Joint 2 was not accurately approximated due to its dependence on Joint 1, and the imprecisions of joint 1's approximation.

Table 1: Dynamic equation values

Block	Parameter	Value
Revolute Joint 1	Spring Stiffness	0.0256 Nm/rad
Revolute Joint 1	Dampness Coefficient	0.522 kNm/(rad/s)
Revolute Joint 2	Spring Stiffness	0.513 Nm/rad
Revolute Joint 2	Dampness Coefficient	2.822 mNm/(rad/s)

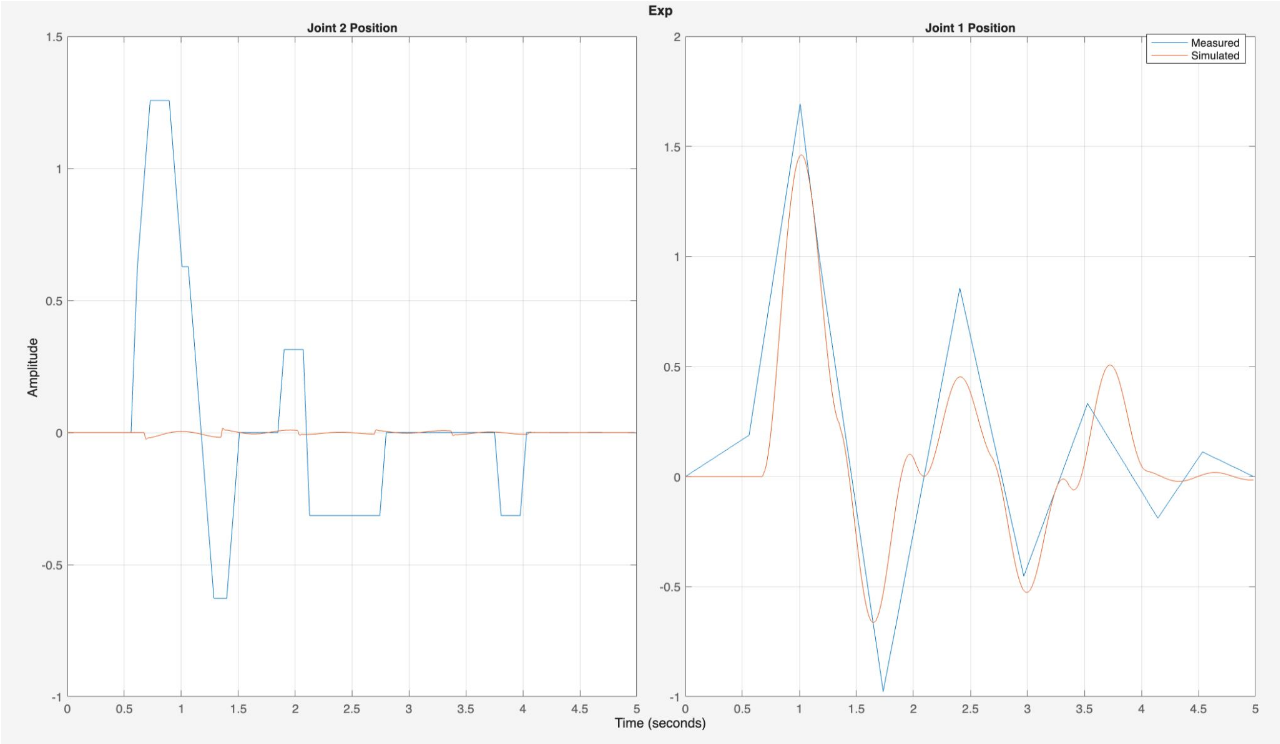


Figure 10: Comparison of measured and simulated angular positions

Experiment Video:

Experiment Video:

Final Project Demonstration Video.MOV

Conclusion

- Practical challenges were faced when creating the physical experiment such as determining materials to use and securing pieces to each other.
- Using the results from the simulation and the physical experiment, parameter estimation was used to optimize the Simulink block parameters so that they fit the experimental results.
- To improve the parameter estimation for the simulation, factors like friction, motor back-emf, motor current, and air resistance could be considered.
- Likewise, a larger measurement sample size could be utilized to improve the estimation.

References

[1]

Patrhoue, *English: Dynamic Stock and flow diagram of Adoption model.* 2009. Accessed: Jul. 01, 2024. [Online]. Available: https://commons.wikimedia.org/wiki/File:Adoption_SFD_ANI.gif

[2]

Taiss 5PCS Rotary Encoder Switch Push Button EC11 360 Degree 5 Pins 20 Detents Points Digital Potentiometer with Knob Cap: Amazon.Com: Industrial & Scientific, www.amazon.com/Taiss-Detents-Points-Encoder-Diameter/dp/B07F24TRYG. Accessed 1 July 2024.

[3]

"Nano." Docs.Arduino.Cc, docs.arduino.cc/hardware/nano/. Accessed 30 June 2024.

"L298 Motor Driver." *L298 Motor Driver - Arduino Reference*, www.arduino.cc/reference/en/libraries/l298-motor-driver/. Accessed 30 June 2024. [4]

Distribution of Work

Name	Assignments
Louis Birla	1. Connect model to inputs and outputs
	2. Connect arduino to pendulum system
	3.Collection of experimental data
Heath Marchese	1. Derivation of equations of system
	2. Establish foundations of block diagram
	3. Building of pendulum system
Titilayomi Okunade	1. Modeling of CAD pieces
	2. Procurement of rigid bodies and miscellaneous materials
	3. Bootstrapping of deliverables

Timeline

Task #	TASK TITLE	TASK OWNER	START DATE	DUE DATE	Week 1				
1	Initiation and Modeling				1	2	3	4	5
1.1	Derivation of equations of system	Heath Marchese	06/24	06/24					
1.2	Connect arduino to pendulum system	Louis Birla	06/25	06/27					
1.3	Bootstrapping of deliverables	Titilayomi Okunade	06/24	06/28					
2	Simulation								
2.1	Modeling of CAD pieces	Titilayomi Okunade	06/24	06/24					
2.2	Establish foundations of block diagram	Heath Marchese	06/24	06/25					
2.3	Connect model to inputs and outputs	Louis Birla	06/25	06/26					
3	Experiment								
3.1	Procurement of rigid bodies and miscellaneous materials	Titilayomi Okunade	06/25	06/25					
3.2	Building of pendulum system	Heath Marchese	06/25	06/26					
3.3	Collection of experimental data	Louis Birla	06/26	06/27					

Thank You